

Homework 3

Binomial Trees

I. Basics

(a) Derivation of (Eq.13.2 and Eq.13.3) (15%)

In the binomial model, suppose that the initial stock price is S_0 , and the life of the option is T . S_0 can either move up from S_0 to a new level, S_0u , where $u > 1$, or down to a new level, S_0d , where $0 < d < 1$. Suppose the payoff from option is f_u in the up state, and is f_d in the down state. Denote the risk-free rate by r .

Please construct a riskless portfolio in a one-step tree and show **in detail**

that $f = e^{-rT} [pf_u + (1-p)f_d]$ where $p = \frac{e^{rT} - d}{u - d}$

(b) (10%) End-of-Chapter exercise 21.7.

II. Computing Option Prices Using Binomial Model

Consider a non-dividend-paying stock with current stock price $S_0 = \$50$, volatility $\sigma = 0.3$, strike price $K = \$52$, time to maturity $T = 2$ years, interest rate $r = 5\%$.

Please use binomial model to price European put options. You may refer to the materials on page 475 of the textbook. Consider the following three alternative settings of time steps: $\Delta t = 1$ month ($12 \cdot T$ steps); 1 week ($52 \cdot T$ steps); and 1 day ($252 \cdot T$ steps).

- (a) (5%) First compute the up step size u , the down step size d , and the probability of up move p under these three settings.
- (b) (30%) Use binomial model to compute the put option prices under these three settings. Report your results and compare them with that of the Black-Scholes formula. Briefly explain your findings.
- (c) (10%) Change the number of time steps from 1 to 2 to 3 all the way to 252. Plot your results as well as the Black-Scholes closed form solution. Briefly explain your findings.
- (d) (10%) For 6, 12, and 52 time steps, compute the terminal stock prices as well as their corresponding probabilities. Plot the terminal stock price distribution. Briefly explain your findings.
- (e) (20%) Modify your program in (b) to compute the American put option values. Report your result.

Matlab function and syntax:

1. To plot terminal stock distribution, you may use matlab function plot().

e.g.

```
plot(ST,Prob,'-o');
```

where ST is a vector of terminal stock prices and Prob is a vector of their corresponding probabilities. '-o' is the LineSpec option that specifies the line type, marker symbol and color: LineStyle is Solid line ('-') and marker type is circle ('o').

2. nchoosek(): Binomial coefficient or all combinations

*You have to submit your homework and **programs by e3**. Your computer program is part of this assignment.