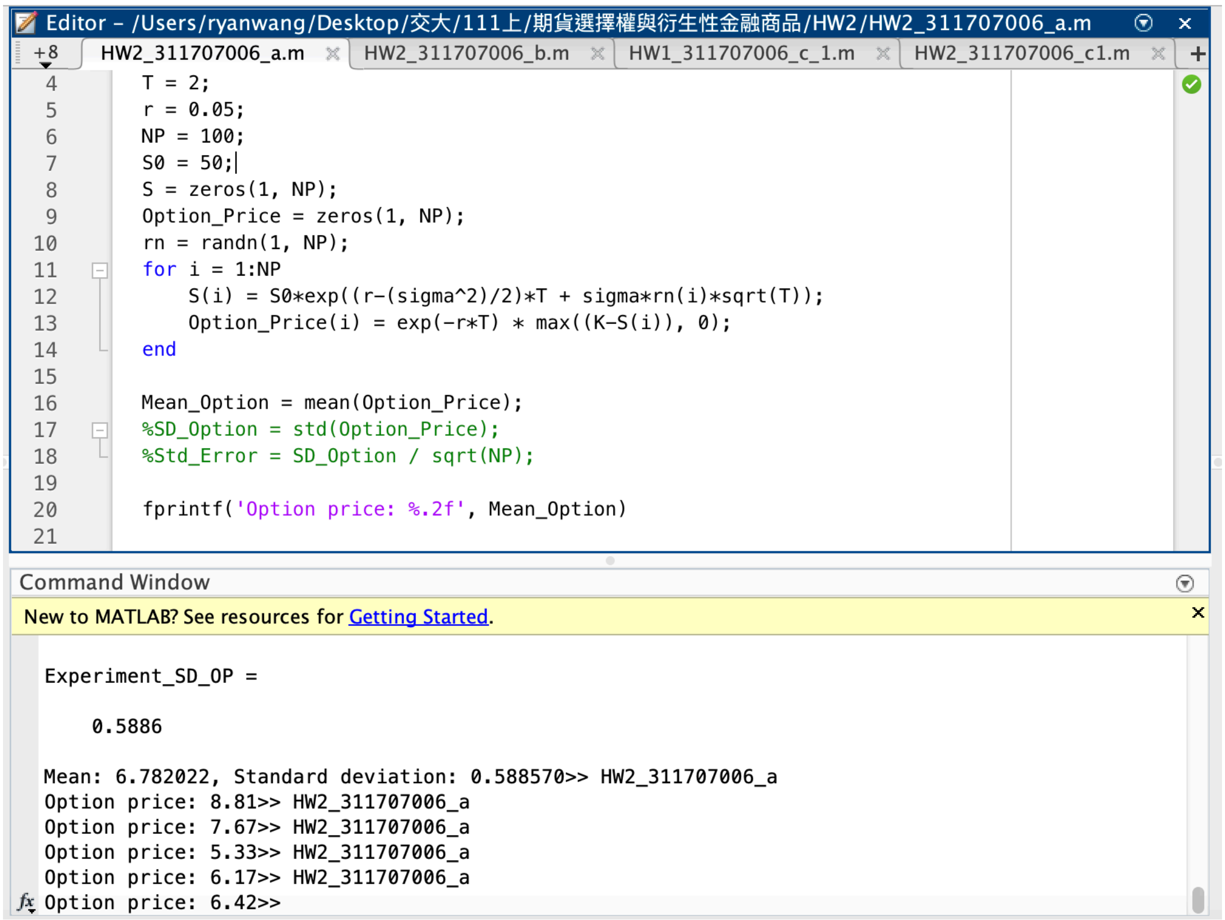
期貨與選擇權 – 作業二

311707006 汪文豪

(50%) I. Standard Error and Number of Trials

(a) Please use Monte Carlo simulation to compute the European put option prices by sampling 100 random paths (NP=100). For each sample path, you can obtain a payoff of the put option at maturity. Next compute the estimate of the value of put option by averaging sample payoffs of these 100 sample paths (NP=100). Discount this estimate at the risk-free rate. This is the estimate of the put option value of your experiment.

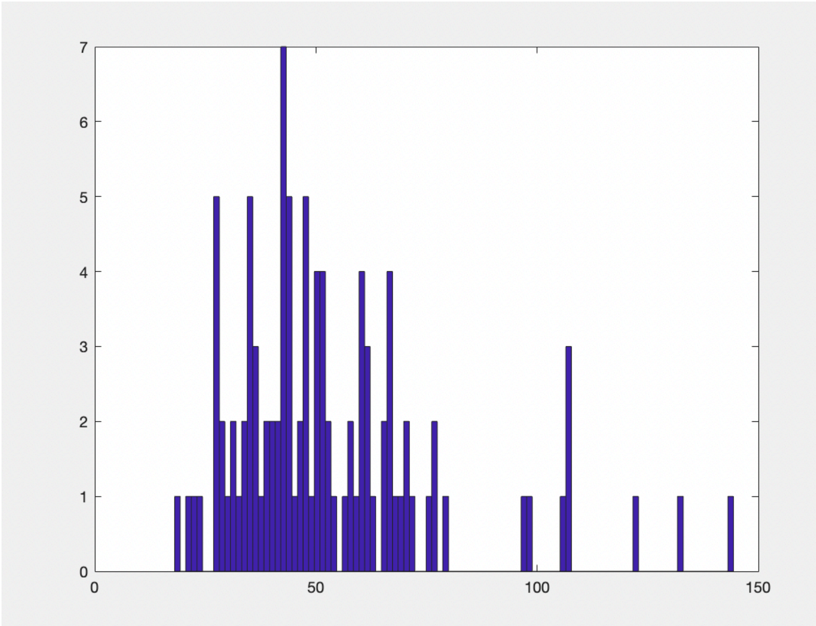


圖一、**HW2_311707006_a.m**執行結果

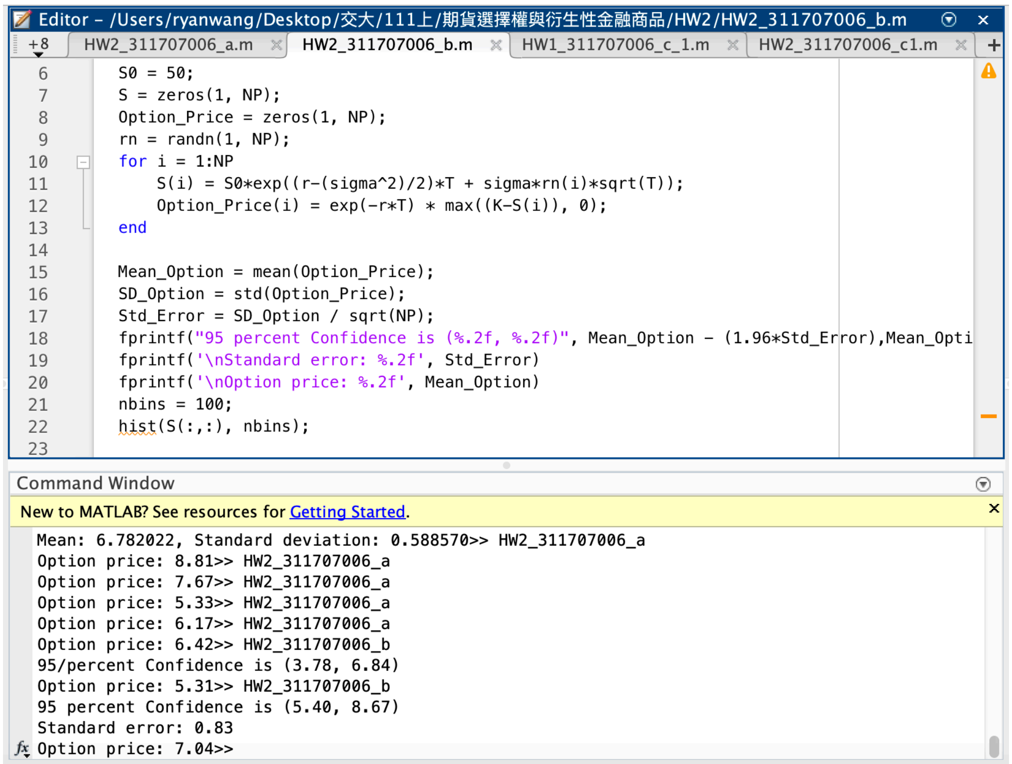
本次執行結果:

Option price為6.42

(b) Compute and report the standard error of the estimated put option value as well as the 95% confidence interval. Please also plot a histogram of the terminal value of stock prices. Please set the number of bins to 100.



圖二、**HW2_311707006_b.m**執行結果圖表



圖三、**HW2_311707006_b.m**執行結果

本次執行結果：

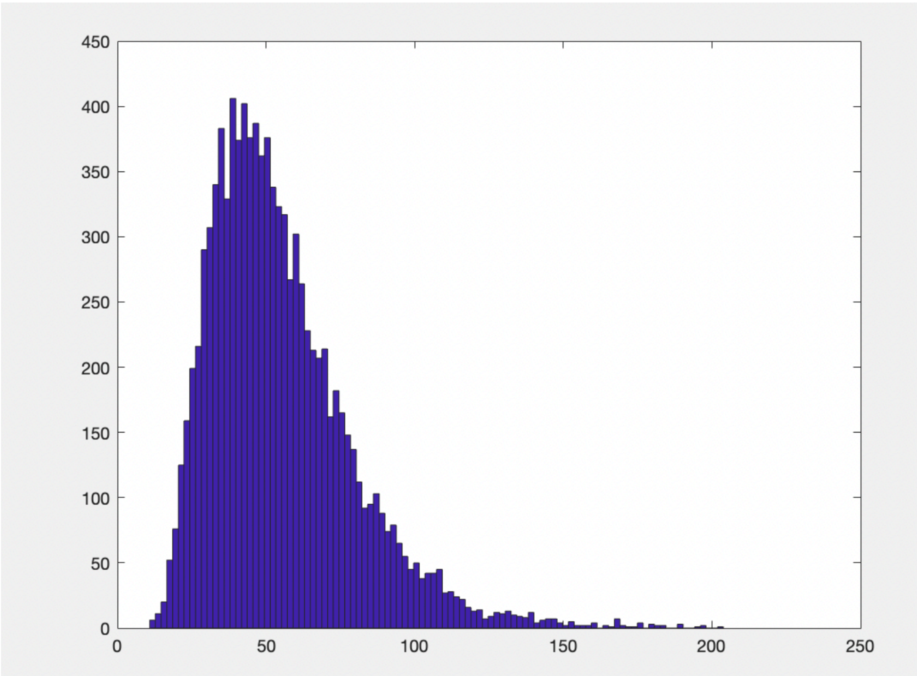
95%信賴區間為(5.40, 8.67)

Standard error為 0.83

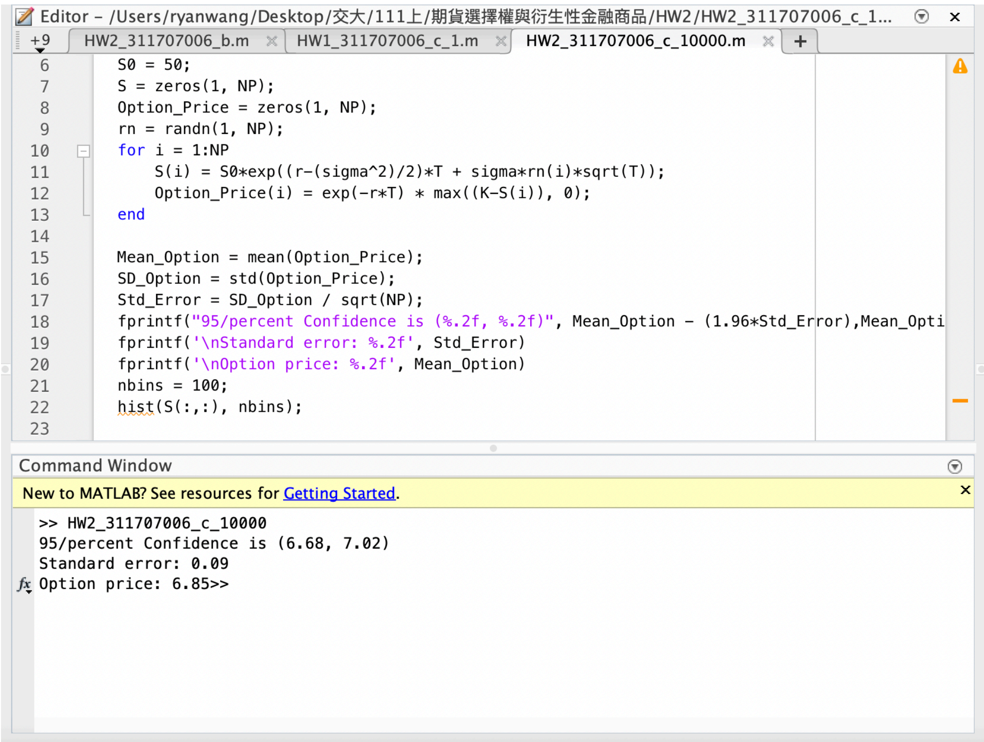
Option price為 7.04

(c) Next, change the number of sample paths (NP) to 10,000 and 1,000,000, redo the experiment, and report your results. Please still keep the number of bins of histogram as 100 in (b). Compare your results with the option value of the Black-Scholes formula (see section 15.8) and briefly explain your findings.

Put option price by Black-Scholes formula: 6.76014



圖四、HW2_311707006_c_10000.m執行結果圖表



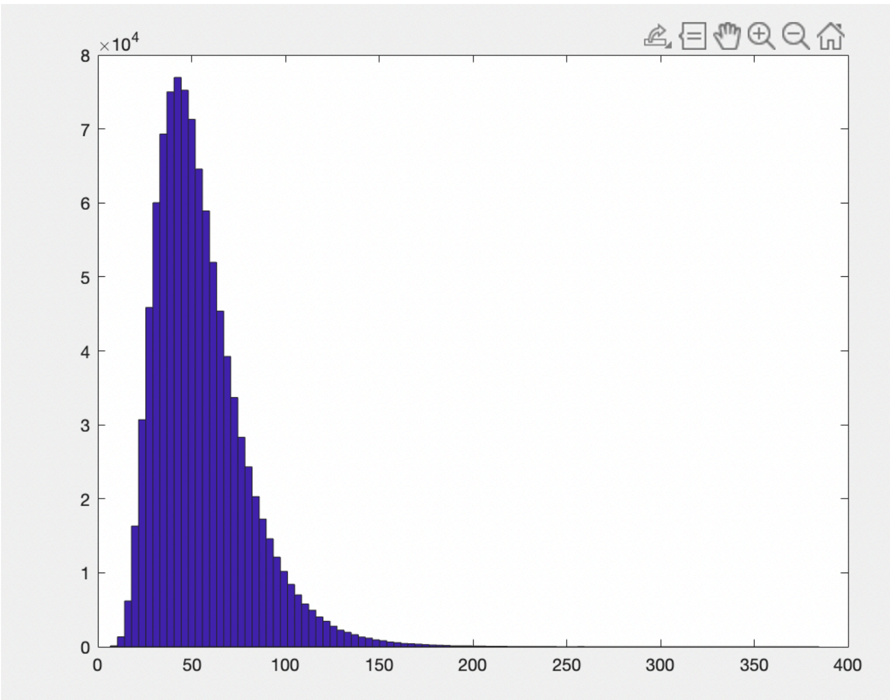
圖五、HW2_311707006_c_10000.m執行結果

本次執行結果：

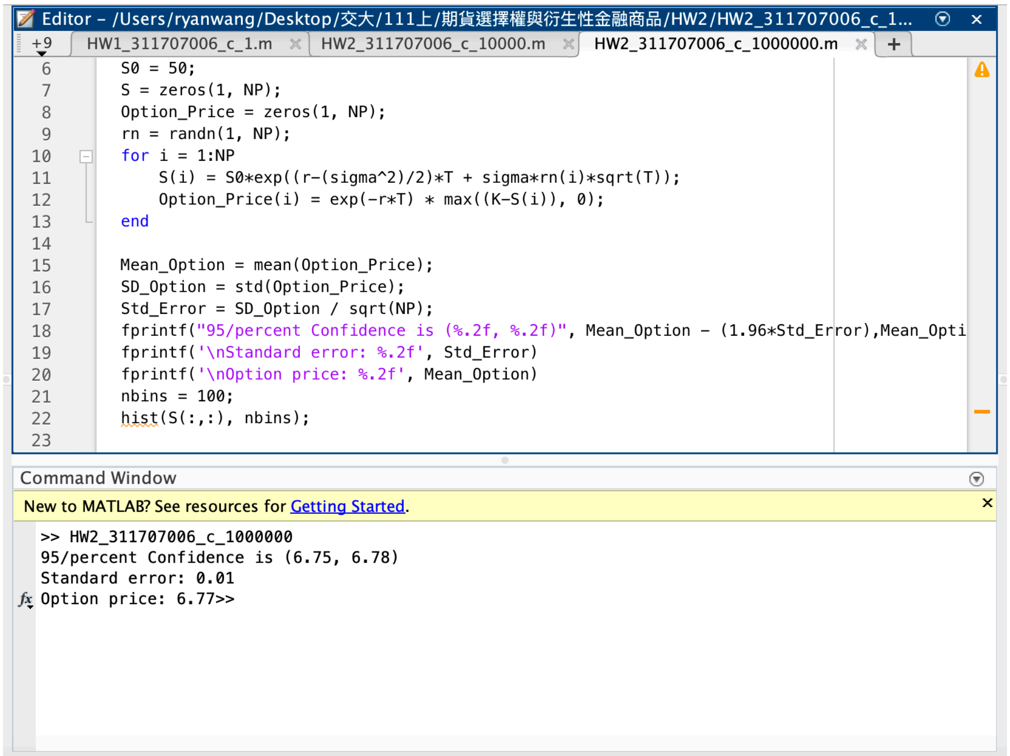
95%信賴區間為(6.68, 7.02)

Standard error為 0.09

Option price為 6.85



圖六、HW2_311707006_c_1000000.m執行結果圖表



圖七、HW2_311707006_c_1000000.m執行結果

本次執行結果：

95%信賴區間為(6.75, 7.78)

Standard error為 0.01

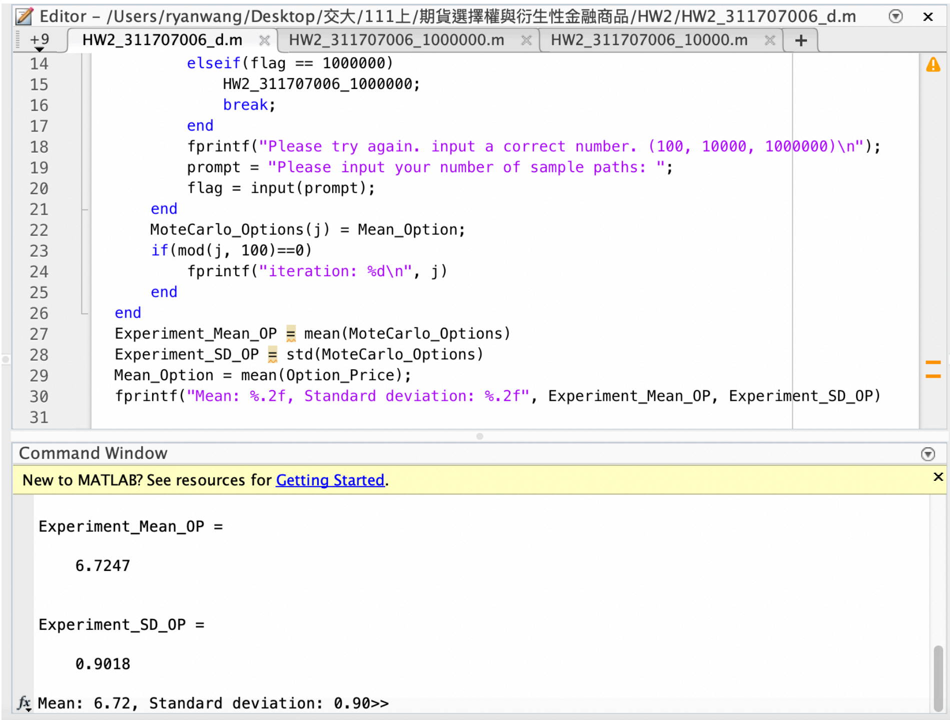
Option price為 6.77

Findings:

當模擬次數越多時，從直方圖來看，會逐漸收斂至log normal distribution的型態。此外，Standard error與信賴區間會逐漸縮小。再與BS模型計算結果的數字比較，也可以發現執行1000000次的模擬結果之6.77與BS模型的6.76有著極小的差距，結果近乎一樣。

(50%) II. Standard Deviation of Estimated Option Values

(d) Repeat the experiment in (a) for 500 times (NE), and compute the mean and standard deviation of the estimated put option prices of these 500 experiments.



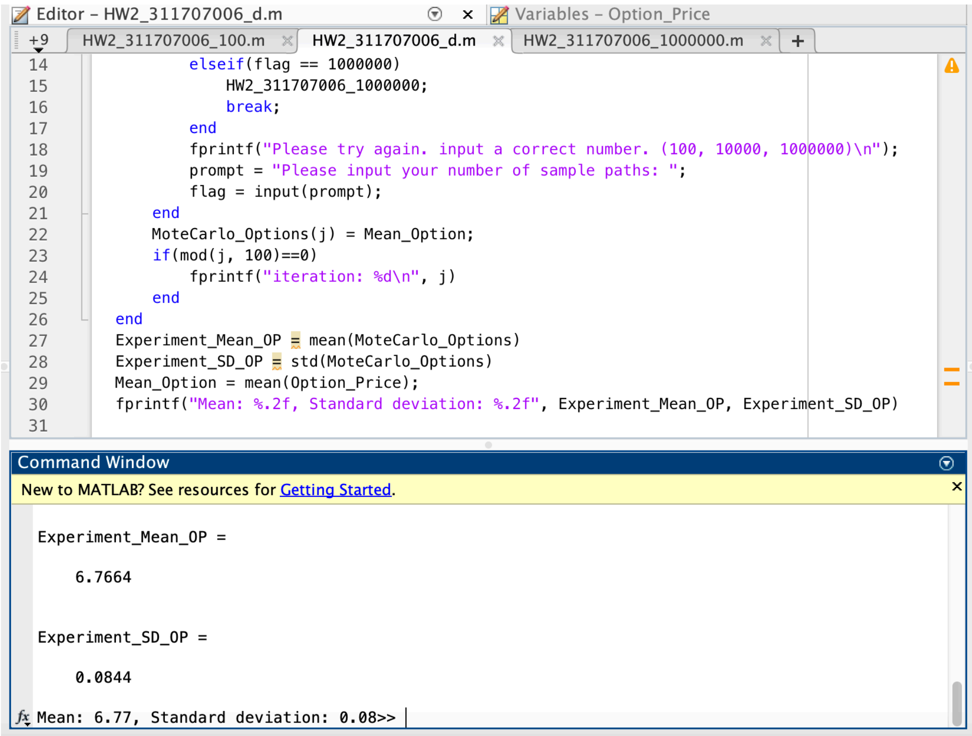
圖八、**HW2_311707006_d.m**，輸入**100**之執行結果

本次執行結果：

Mean: 6.72

Standard deviation: 0.9

(e) Redo (d) and increase the number of sample paths (NP) of each experiment in (a) to 10,000 and 1,000,000 (still repeat the experiment for 500 times (NE))

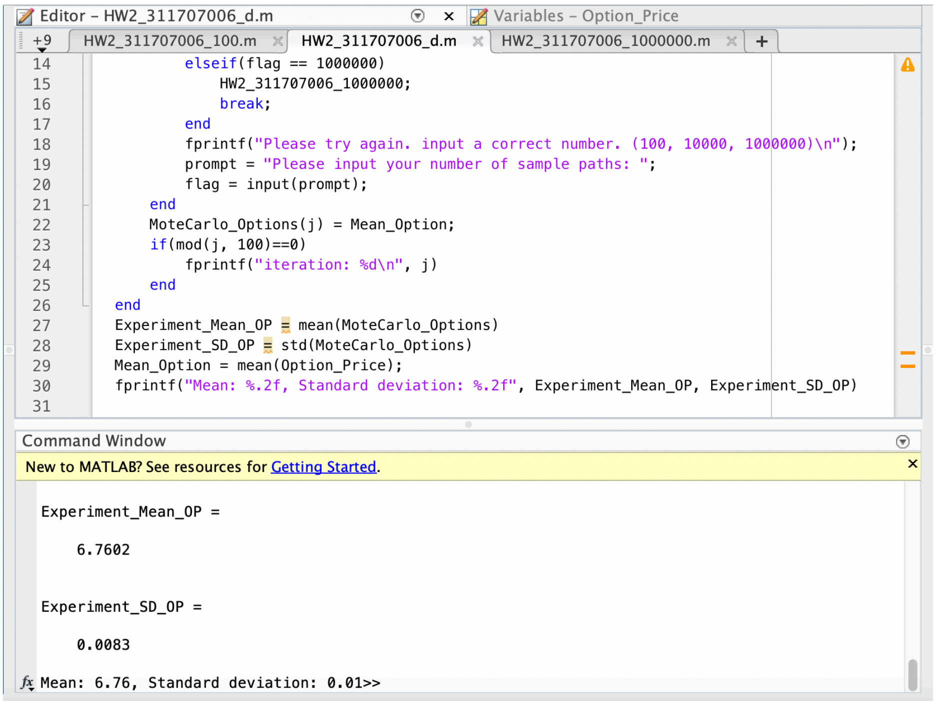


圖九、**HW2_311707006_d.m**，輸入**10000**之執行結果

本次執行結果：

Mean: 6.77

Standard deviation: 0.08



圖十、**HW2_311707006_d.m**，輸入**1000000**之執行結果

本次執行結果：

Mean: 6.76

Standard deviation: 0.01

*** Please report the first 30 results (the estimates of European put values in (a)) under these three alternative settings, namely, (i) 100 (ii) 10,000 (iii) and 1,000,000 sample paths. Next, summarize in a table the means and standard deviations of the put option values under these three settings. Briefly explain your results**

	NP = 100	NP = 10000	NP = 1000000
Mean	6.72	6.77	6.76
Standard Deviation	0.9	0.08	0.01
Estimated Put Value	7.04	6.85	6.77
1	7.54486040319188	6.85289352011084	6.76208065848992
2	5.53605717791108	6.66541283465147	6.74559144827448
3	5.94052241133643	6.81990872326744	6.75085161862373
4	7.13406132016823	6.69386806340658	6.76602905376110

	NP = 100	NP = 10000	NP = 1000000
5	9.52508313809112	6.72234510190085	6.77027619360948
6	8.91886968771589	6.87831923794357	6.75417629672670
7	6.99252495852533	6.70137791365737	6.74660804366215
8	7.73082788846195	6.69824117812888	6.75191808217856
9	8.02689043844842	6.61570462693351	6.76596462502912
10	7.42758521642434	6.81198477081495	6.75072541078720
11	6.42301126342703	6.75592655065757	6.76339165202896
12	5.22604096537042	6.83668993177552	6.76114944029600
13	6.49112641616823	6.76905601900347	6.75956231456137
14	5.17064458974158	6.73355018892933	6.75406967833823
15	6.82106953809522	6.67867070116804	6.77239784417824
16	6.54232271454966	6.81531272059189	6.75360621830470
17	7.00153911720156	6.72506049893224	6.76093238853250
18	6.89368387558941	6.78029336062131	6.75796649957939
19	5.93284501208138	6.79269979139111	6.75819670818378
20	5.70895897344422	6.61078986878872	6.75903708808789
21	5.93413684022908	6.70953614879340	6.76350852626809
22	7.07269162668602	6.81162042499896	6.75007931367978
23	6.25177639842722	6.66426098330129	6.76337426869881
24	7.72834403488155	6.88915060682542	6.76405993069245
25	6.65216693271050	6.72876698628513	6.76427679411590
26	6.61060348489502	6.60037348170284	6.76727112137078
27	6.50076307474364	6.63957438636645	6.76066878329306
28	8.78203577657597	6.82901824412307	6.76096525929221
29	8.00229079125185	6.68315114290004	6.74976578601486
30	7.01796850424053	6.67220223179037	6.75381095461645

Explain:

隨著NP的次數增加，值會逐漸收斂與穩定(Standard deviation從NP=100的0.9減少到NP=1000000的0.01)，再從NP=1000000的均值觀察，幾乎與BS-Model

所推出的價值一樣。此外，再從五百筆中的前三十筆資料觀察，也可以看到隨著NP次數的增加，值也變得穩定許多。再來也因為模擬實驗五百次的關係，可以看到與第一題所模擬出來的選擇權價格更為接近BS-Model所推出的價值。