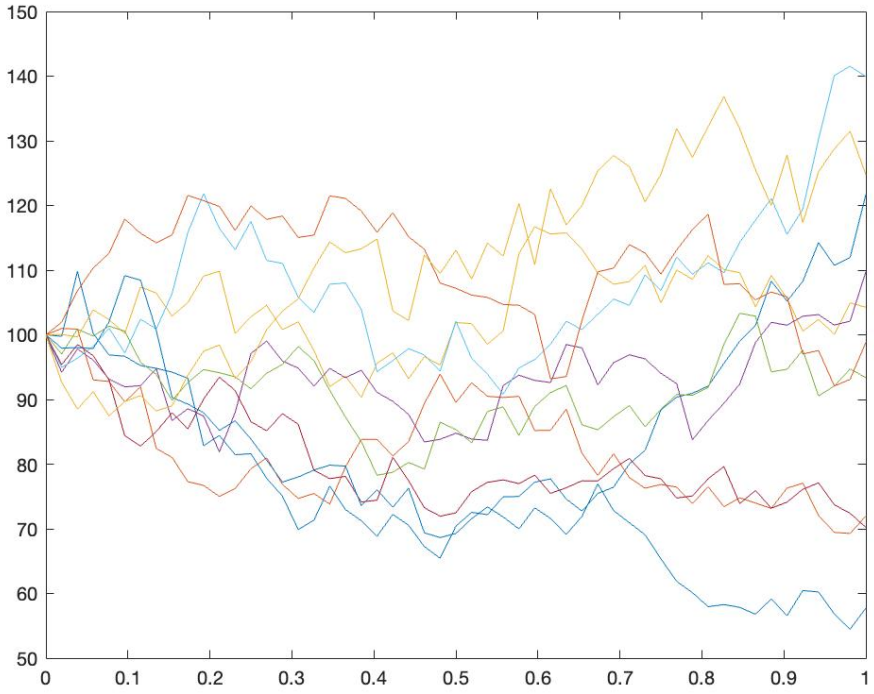
期貨與選擇權 – 作業一

311707006 汪文豪

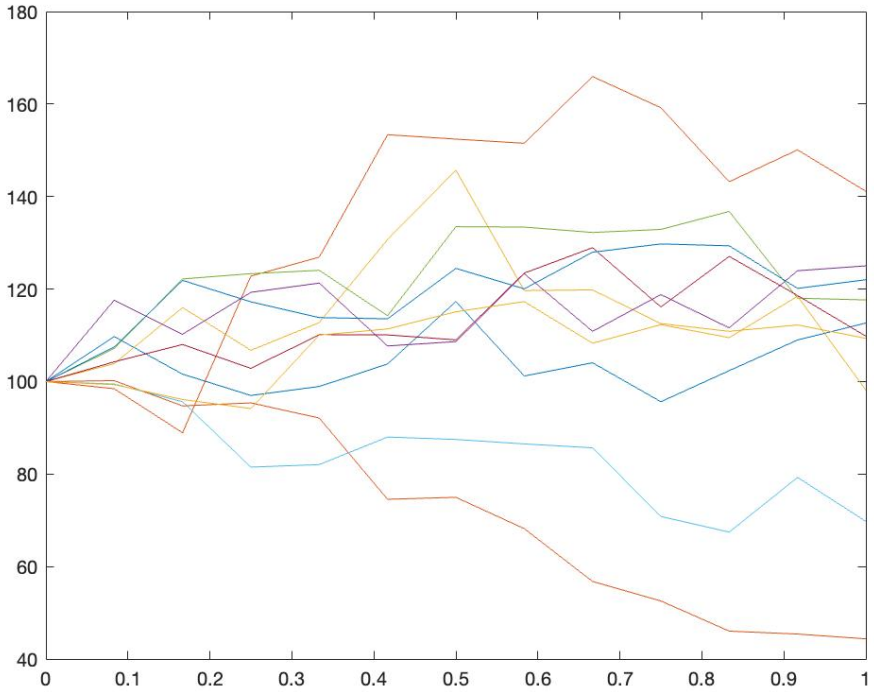
- Computer homework (90%)

(a) (40%) Consider a time interval of 1 week (Δt =1/52), please simulate 10 sample paths of stock prices for 1 year and plot them in a time chart.

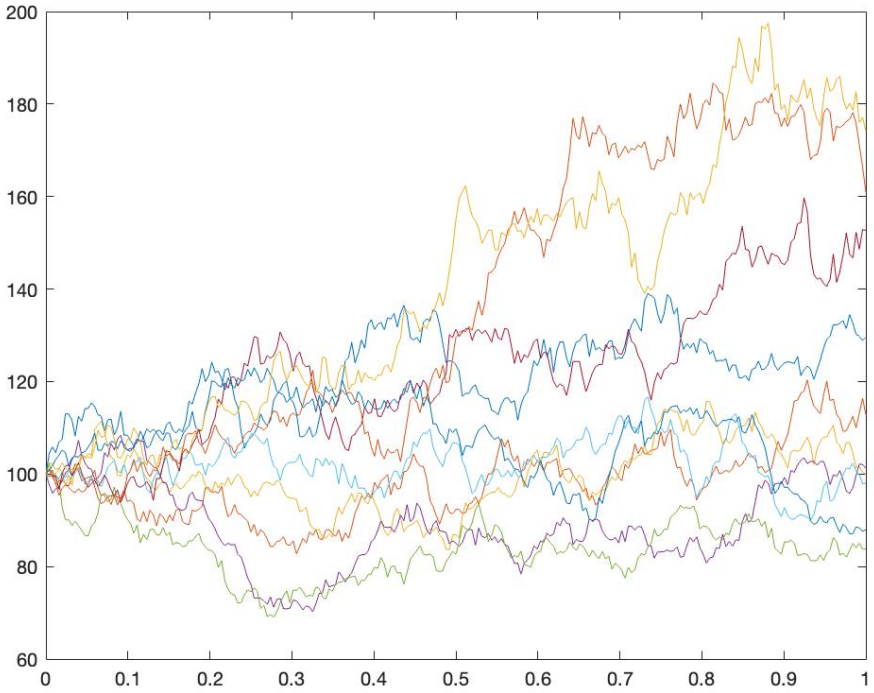


A小題[HW1_311707006_a.m]執行結果

(b) (25%) Consider a time interval of 1 month (Δt =1/12), please simulate 10 sample paths of stock prices for 1 year and plot them in a time chart. Next, consider time interval of 1 trading day (Δt =1/252), again simulate 10 sample paths of stock prices for 1 year and plot them in a time chart. Explain your findings.



b小題（Δt =1/12）[HW1_311707006_b_1.m]執行結果

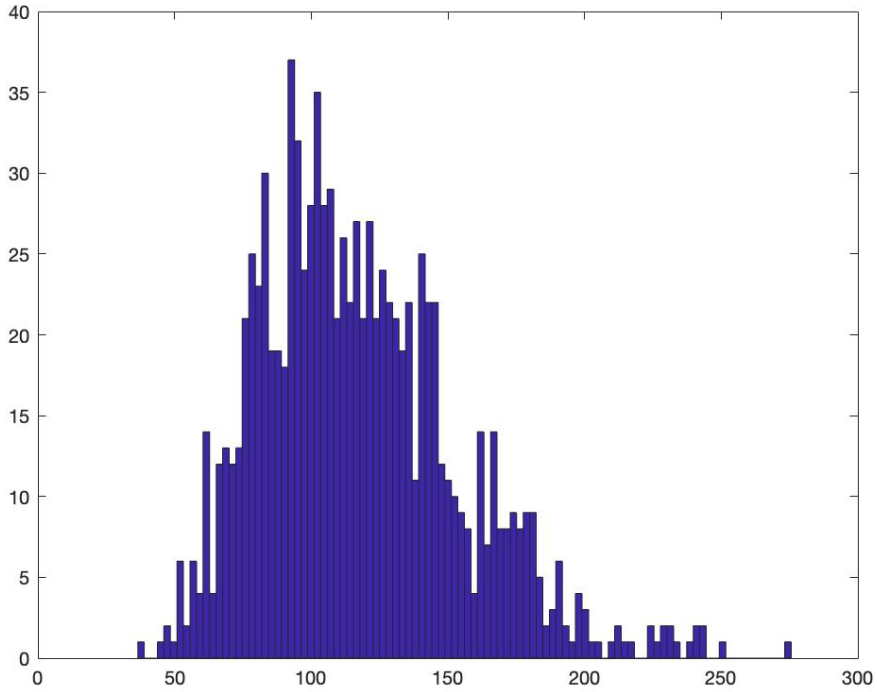


b小題（Δt =1/252）[HW1_311707006_b_2.m]執行結果

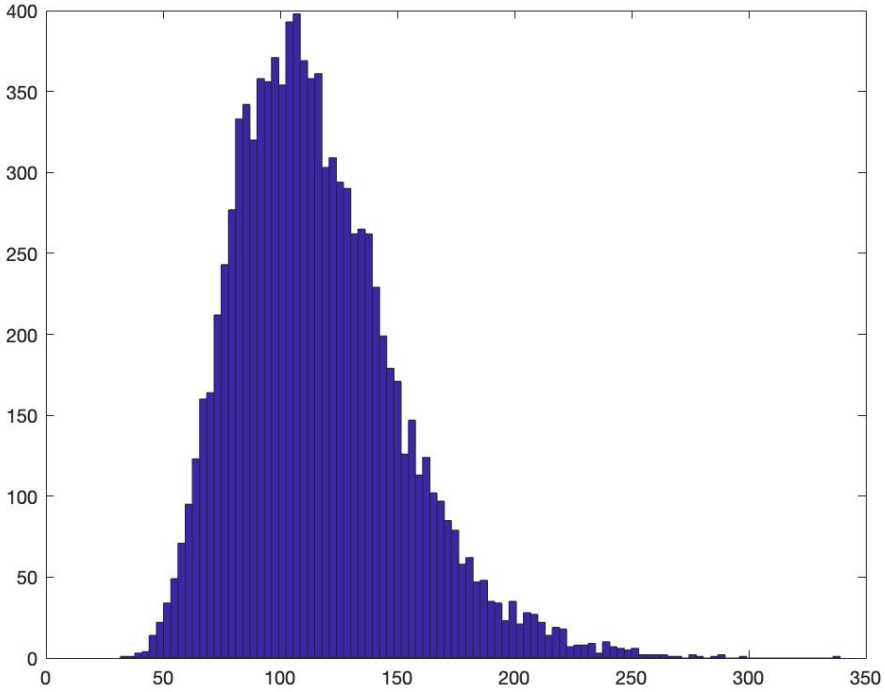
(b)Explain:

透過其Δt = 1/12 -> 1/52 -> 1/252所繪製出的三張圖表觀察，可以很明顯地看到當區間區分的越細時，模擬出來的走勢將會更為曲折呈現鋸齒狀，而在股價波動計算中其dz走勢會隨著根號Δt的值而有所影響，當Δt的時間區分越長時，Δt會越小，而Δt越小開根號後的值會變大，因此最後畫出來的圖就會較為曲折鋸齒。

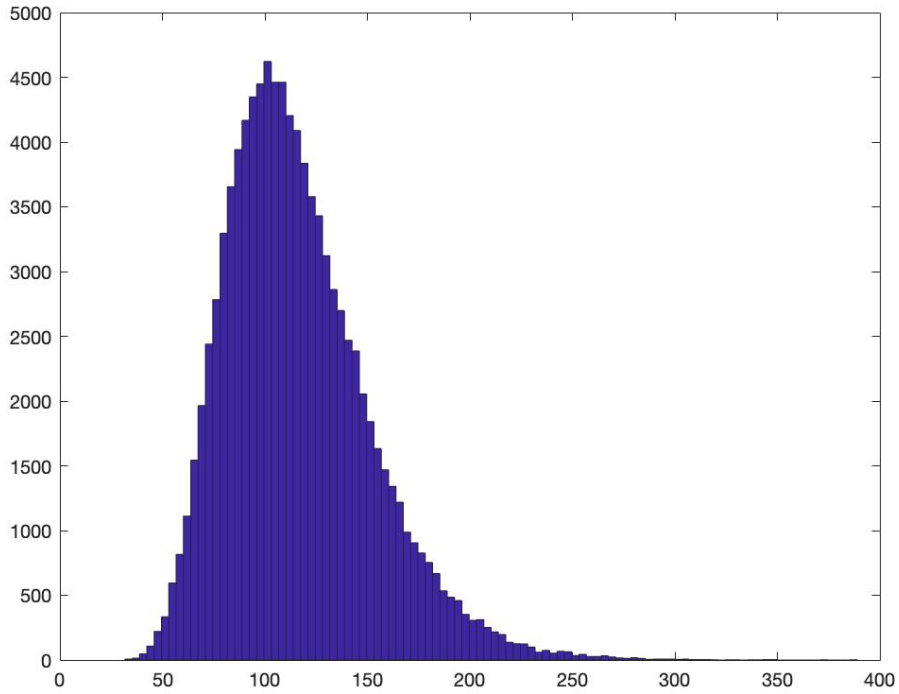
(c) (25%) Please plot a histogram of the terminal value of stock prices. Please set the number of bins to 100, and change the number of sample paths from 1000 to 10,000 and 100,000. Explain your findings



c小題（sample paths = 1000）[HW1_311707006_c_1.m]執行結果



c小題（sample paths = 10000）[HW1_311707006_c_2.m]執行結果



c小題（sample paths = 100000）[HW1_311707006_c_3.m]執行結果

(c)Explain:

從sample paths = 1000 –> 10000 –> 100000所繪製出的三張圖表中可以觀察到，當資料樣本越大時，其資料樣本分布會趨近於log normal distribution.

- **End–of–Chapter Problem 14.4 (10%)**

14.4: $dS = \mu dt + \sigma dz$

第一個3年，$\mu = 2$ & $\sigma = 3$

第二個3年，$\mu = 3$ & $\sigma = 4$

$S_0 = 5$

問 S_6 之機率分布

Solution:

Suppose stock price follow Markov processes, then

六年間 dS 之 $\mu \Rightarrow 2 \times 3 + 3 \times 3 = 15$

$\sigma^2 \Rightarrow 3^2 \times 3 + 4^2 \times 3 = 75$

$\Rightarrow dS \sim N(15, 75)$

$S_0 + dS = S_6$

$\Rightarrow 5 + dS = S_6$

$\because dS \sim N(15, 75)$

$\therefore S_6 \sim N(20, 75)$ #