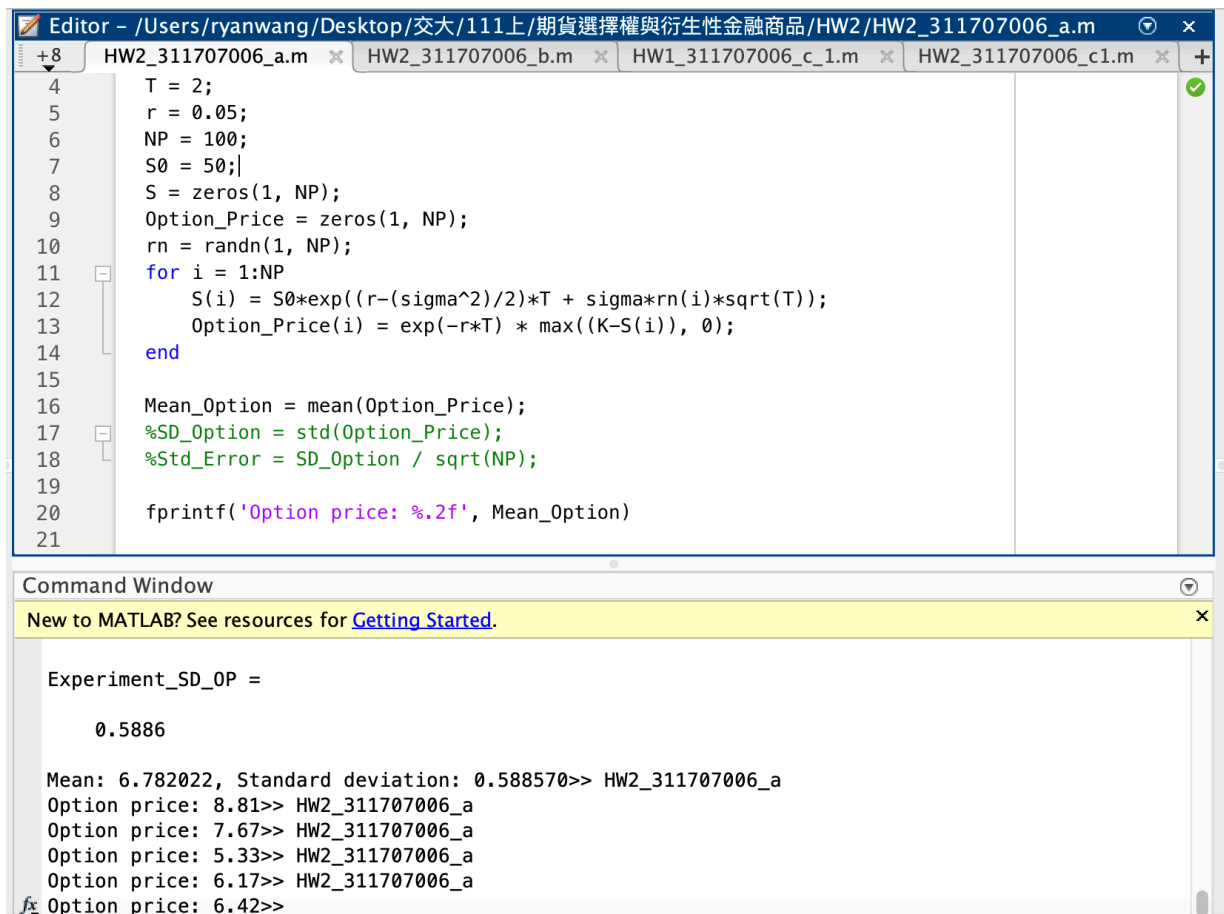


期貨與選擇權 – 作業二

311707006 汪文豪

(50%) I. Standard Error and Number of Trials

(a) Please use Monte Carlo simulation to compute the European put option prices by sampling 100 random paths (NP=100). For each sample path, you can obtain a payoff of the put option at maturity. Next compute the estimate of the value of put option by averaging sample payoffs of these 100 sample paths (NP=100). Discount this estimate at the risk-free rate. This is the estimate of the put option value of your experiment.



```
Editor - /Users/ryanwang/Desktop/交大/111上/期貨選擇權與衍生性金融商品/HW2/HW2_311707006_a.m
+8 HW2_311707006_a.m HW2_311707006_b.m HW1_311707006_c_1.m HW2_311707006_c1.m +
4 T = 2;
5 r = 0.05;
6 NP = 100;
7 S0 = 50;
8 S = zeros(1, NP);
9 Option_Price = zeros(1, NP);
10 rn = randn(1, NP);
11 for i = 1:NP
12     S(i) = S0*exp((r-(sigma^2)/2)*T + sigma*rn(i)*sqrt(T));
13     Option_Price(i) = exp(-r*T) * max((K-S(i)), 0);
14 end
15
16 Mean_Option = mean(Option_Price);
17 %SD_Option = std(Option_Price);
18 %Std_Error = SD_Option / sqrt(NP);
19
20 fprintf('Option price: %.2f', Mean_Option)
21

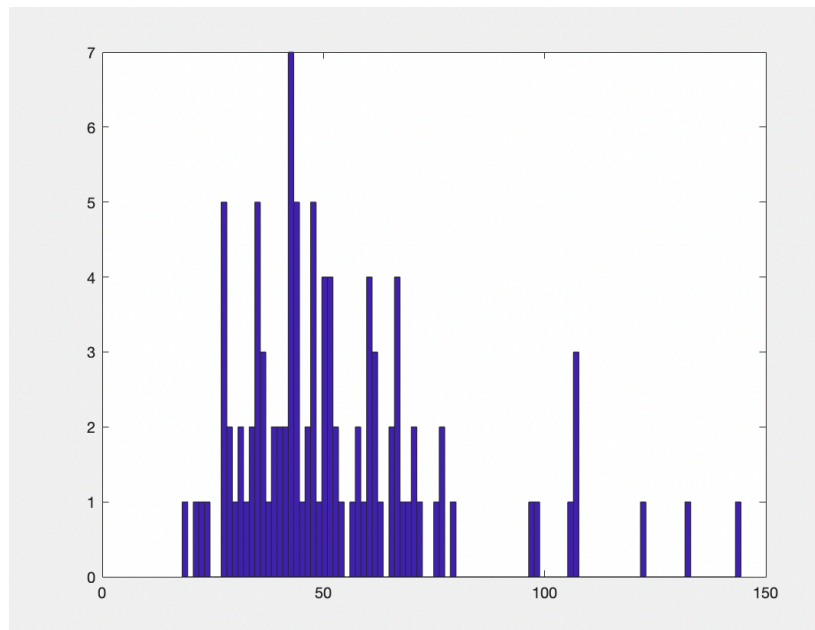
Command Window
New to MATLAB? See resources for Getting Started.
Experiment_SD_OP =
    0.5886
Mean: 6.782022, Standard deviation: 0.588570>> HW2_311707006_a
Option price: 8.81>> HW2_311707006_a
Option price: 7.67>> HW2_311707006_a
Option price: 5.33>> HW2_311707006_a
Option price: 6.17>> HW2_311707006_a
fx Option price: 6.42>>
```

圖一、HW2_311707006_a.m執行結果

本次執行結果:

Option price為6.42

(b) Compute and report the standard error of the estimated put option value as well as the 95% confidence interval. Please also plot a histogram of the terminal value of stock prices. Please set the number of bins to 100.



圖二、HW2_311707006_b.m執行結果圖表

```

Editor - /Users/ryanwang/Desktop/交大/111上/期貨選擇權與衍生性金融商品/HW2/HW2_311707006_b.m
+8 HW2_311707006_a.m HW2_311707006_b.m HW1_311707006_c_1.m HW2_311707006_c1.m
6 S0 = 50;
7 S = zeros(1, NP);
8 Option_Price = zeros(1, NP);
9 rn = randn(1, NP);
10 for i = 1:NP
11     S(i) = S0*exp((r-(sigma^2)/2)*T + sigma*rn(i)*sqrt(T));
12     Option_Price(i) = exp(-r*T) * max((K-S(i)), 0);
13 end
14
15 Mean_Option = mean(Option_Price);
16 SD_Option = std(Option_Price);
17 Std_Error = SD_Option / sqrt(NP);
18 fprintf('95 percent Confidence is (%.2f, %.2f)', Mean_Option - (1.96*Std_Error), Mean_Option + (1.96*Std_Error));
19 fprintf('\nStandard error: %.2f', Std_Error)
20 fprintf('\nOption price: %.2f', Mean_Option)
21 nbins = 100;
22 hist(S(:, :), nbins);
23
Command Window
New to MATLAB? See resources for Getting Started.
Mean: 6.782022, Standard deviation: 0.588570>> HW2_311707006_a
Option price: 8.81>> HW2_311707006_a
Option price: 7.67>> HW2_311707006_a
Option price: 5.33>> HW2_311707006_a
Option price: 6.17>> HW2_311707006_a
Option price: 6.42>> HW2_311707006_b
95/percent Confidence is (3.78, 6.84)
Option price: 5.31>> HW2_311707006_b
95 percent Confidence is (5.40, 8.67)
Standard error: 0.83
Option price: 7.04>>

```

圖三、HW2_311707006_b.m執行結果

本次執行結果：

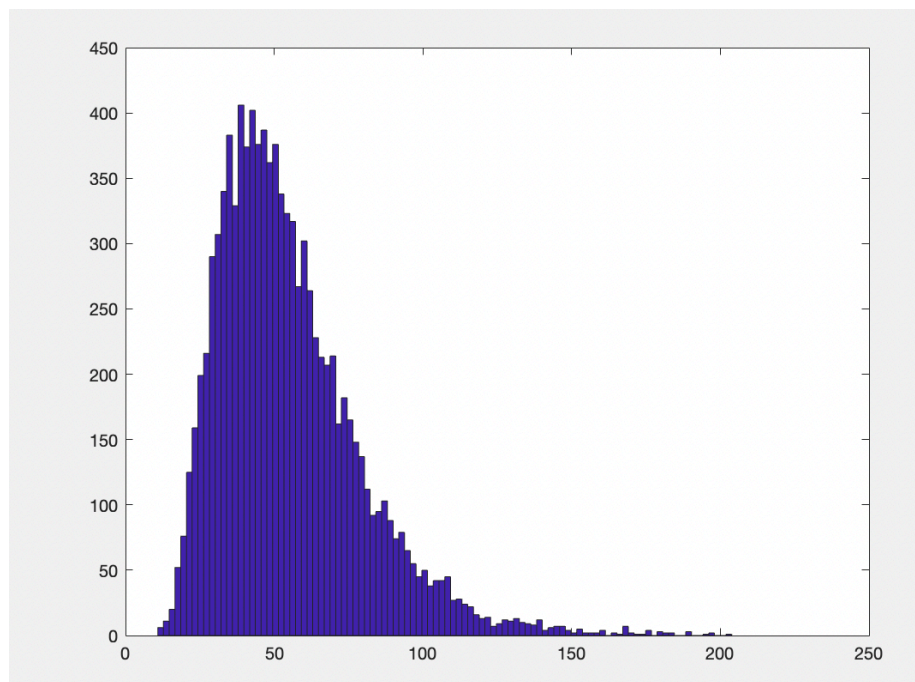
95%信賴區間為(5.40, 8.67)

Standard error為 0.83

Option price為 7.04

(c) Next, change the number of sample paths (NP) to 10,000 and 1,000,000, redo the experiment, and report your results. Please still keep the number of bins of histogram as 100 in (b). Compare your results with the option value of the Black-Scholes formula (see section 15.8) and briefly explain your findings.

Put option price by Black-Scholes formula: 6.76014



圖四、HW2_311707006_c_10000.m執行結果圖表

```
Editor - /Users/ryanwang/Desktop/交大/111上/期貨選擇權與衍生性金融商品/HW2/HW2_311707006_c_1...
+9 HW2_311707006_b.m x HW1_311707006_c_1.m x HW2_311707006_c_10000.m x +
6 S0 = 50;
7 S = zeros(1, NP);
8 Option_Price = zeros(1, NP);
9 rn = randn(1, NP);
10 for i = 1:NP
11     S(i) = S0*exp((r-(sigma^2)/2)*T + sigma*rn(i)*sqrt(T));
12     Option_Price(i) = exp(-r*T) * max((K-S(i)), 0);
13 end
14
15 Mean_Option = mean(Option_Price);
16 SD_Option = std(Option_Price);
17 Std_Error = SD_Option / sqrt(NP);
18 fprintf("95/percent Confidence is (%.2f, %.2f)", Mean_Option - (1.96*Std_Error), Mean_Option + (1.96*Std_Error));
19 fprintf("\nStandard error: %.2f", Std_Error);
20 fprintf("\nOption price: %.2f", Mean_Option);
21 nbins = 100;
22 hist(S(:, :), nbins);
23
```

```
Command Window
New to MATLAB? See resources for Getting Started.
>> HW2_311707006_c_10000
95/percent Confidence is (6.68, 7.02)
Standard error: 0.09
Option price: 6.85>>
```

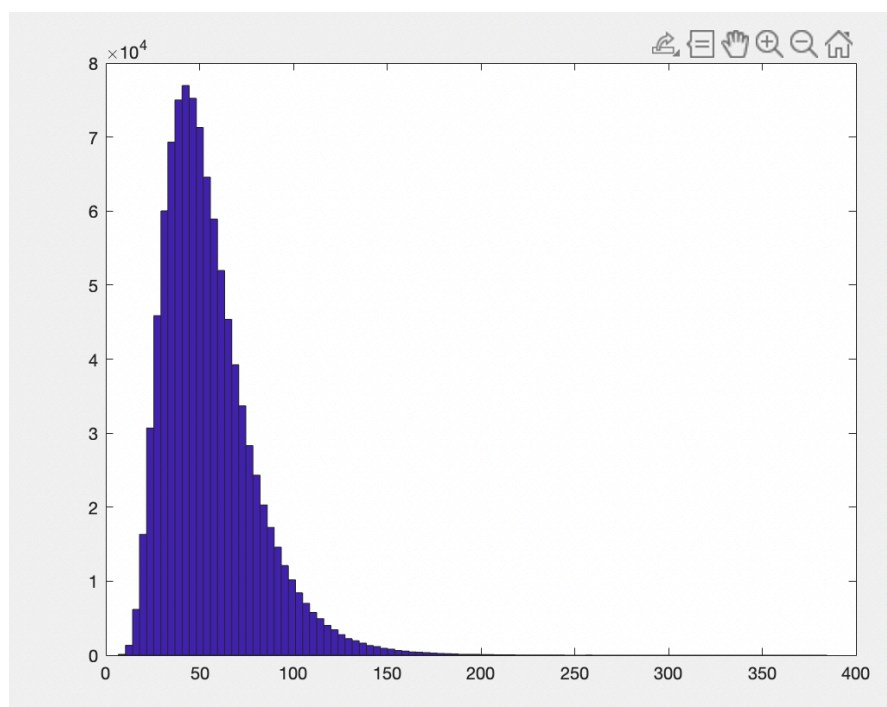
圖五、HW2_311707006_c_10000.m執行結果

本次執行結果：

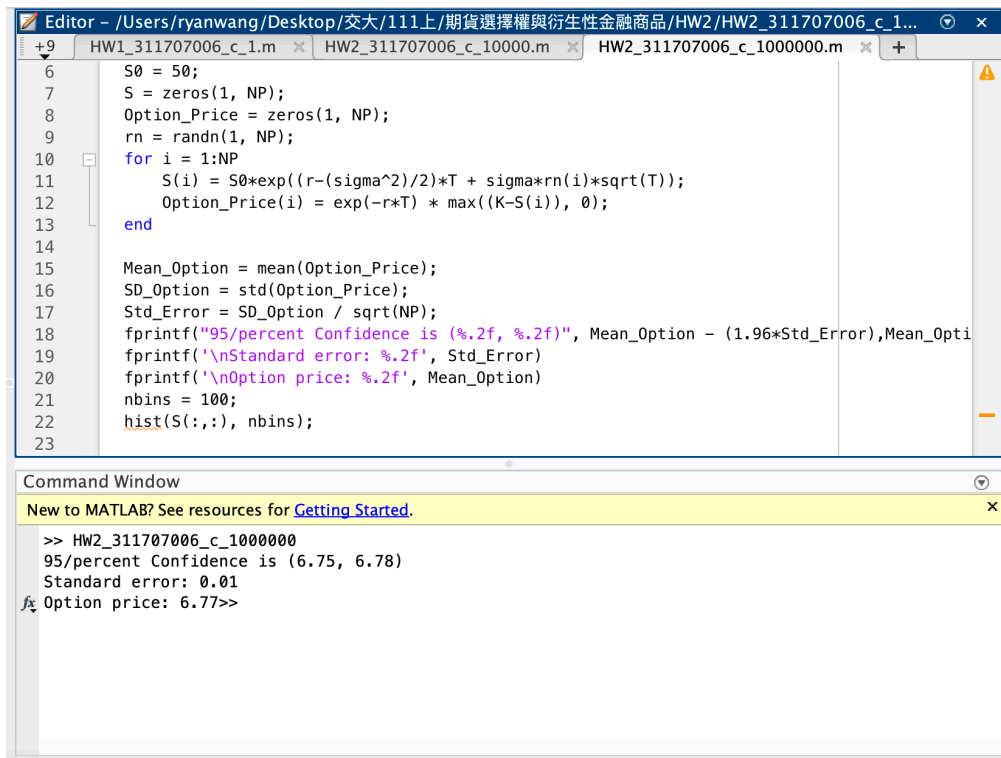
95%信賴區間為(6.68, 7.02)

Standard error為 0.09

Option price為 6.85



圖六、HW2_311707006_c_1000000.m執行結果圖表



The image shows a MATLAB Editor window with three tabs: HW1_311707006_c_1.m, HW2_311707006_c_10000.m, and HW2_311707006_c_1000000.m. The active tab is HW2_311707006_c_1000000.m, which contains the following code:

```
6 S0 = 50;
7 S = zeros(1, NP);
8 Option_Price = zeros(1, NP);
9 rn = randn(1, NP);
10 for i = 1:NP
11     S(i) = S0*exp((r-(sigma^2)/2)*T + sigma*rn(i)*sqrt(T));
12     Option_Price(i) = exp(-r*T) * max((K-S(i)), 0);
13 end
14
15 Mean_Option = mean(Option_Price);
16 SD_Option = std(Option_Price);
17 Std_Error = SD_Option / sqrt(NP);
18 fprintf('95/percent Confidence is (%.2f, %.2f)', Mean_Option - (1.96*Std_Error), Mean_Option + (1.96*Std_Error));
19 fprintf('\nStandard error: %.2f', Std_Error)
20 fprintf('\nOption price: %.2f', Mean_Option)
21 nbins = 100;
22 hist(S(:, :), nbins);
23
```

Below the editor is the Command Window, which displays the following output:

```
>> HW2_311707006_c_1000000
95/percent Confidence is (6.75, 6.78)
Standard error: 0.01
Option price: 6.77>>
```

圖七、HW2_311707006_c_1000000.m執行結果

本次執行結果：

95%信賴區間為(6.75, 6.78)

Standard error為 0.01

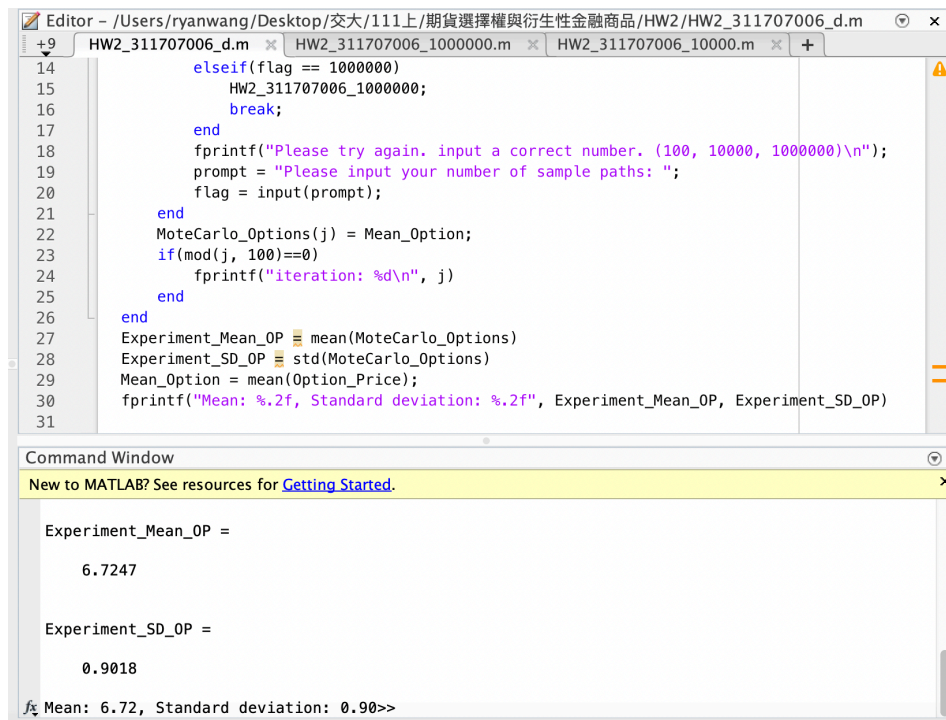
Option price為 6.77

Findings:

當模擬次數越多時，從直方圖來看，會逐漸收斂至log normal distribution的型態。此外，Standard error與信賴區間會逐漸縮小。再與BS模型計算結果的數字比較，也可以發現執行1000000次的模擬結果之6.77與BS模型的6.76有著極小的差距，結果近乎一樣。

(50%) II. Standard Deviation of Estimated Option Values

- (d) Repeat the experiment in (a) for 500 times (NE), and compute the mean and standard deviation of the estimated put option prices of these 500 experiments.



```
14         elseif(flag == 1000000)
15             HW2_311707006_1000000.m;
16             break;
17         end
18         fprintf("Please try again. input a correct number. (100, 10000, 1000000)\n");
19         prompt = "Please input your number of sample paths: ";
20         flag = input(prompt);
21     end
22     MoteCarlo_Options(j) = Mean_Option;
23     if(mod(j, 100)==0)
24         fprintf("iteration: %d\n", j)
25     end
26 end
27 Experiment_Mean_OP = mean(MoteCarlo_Options)
28 Experiment_SD_OP = std(MoteCarlo_Options)
29 Mean_Option = mean(Option_Price);
30 fprintf("Mean: %.2f, Standard deviation: %.2f", Experiment_Mean_OP, Experiment_SD_OP)
31
```

Command Window

New to MATLAB? See resources for [Getting Started](#).

Experiment_Mean_OP =
6.7247

Experiment_SD_OP =
0.9018

Mean: 6.72, Standard deviation: 0.90>>

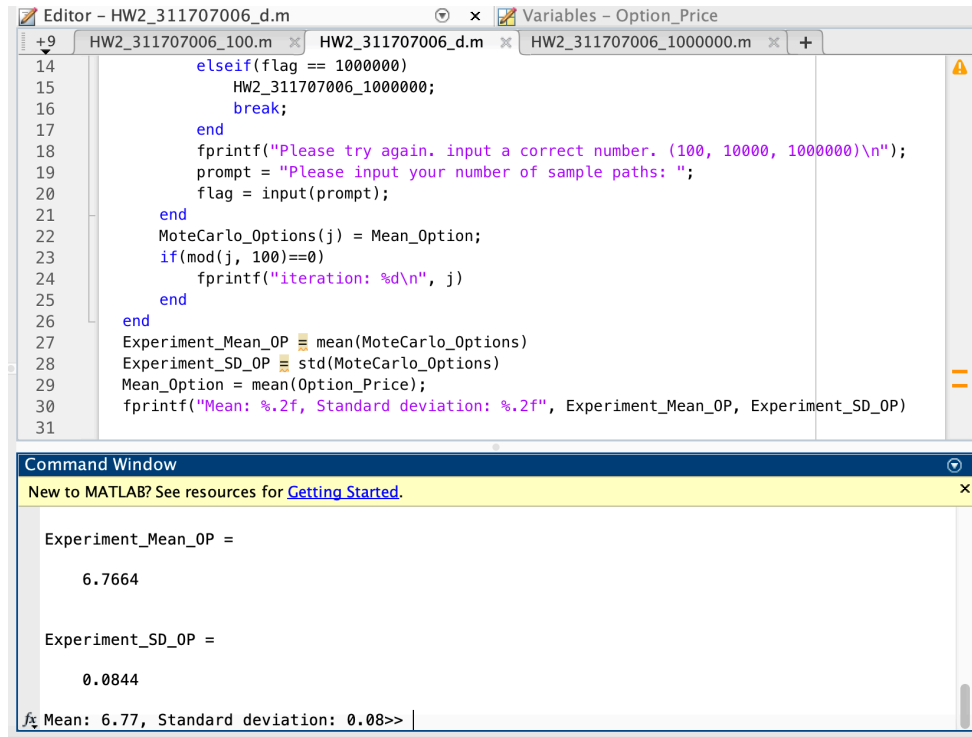
圖八、HW2_311707006_d.m，輸入100之執行結果

本次執行結果：

Mean: 6.72

Standard deviation: 0.9

- (e) Redo (d) and increase the number of sample paths (NP) of each experiment in (a) to 10,000 and 1,000,000 (still repeat the experiment for 500 times (NE))



The image shows a MATLAB environment with the Editor window displaying the script `HW2_311707006_d.m`. The script includes a loop for input validation and a Monte Carlo simulation. The Command Window shows the results of the execution for 10,000 sample paths.

```
14 elseif(flag == 1000000)
15     HW2_311707006_1000000;
16     break;
17 end
18 fprintf("Please try again. input a correct number. (100, 10000, 1000000)\n");
19 prompt = "Please input your number of sample paths: ";
20 flag = input(prompt);
21 end
22 MonteCarlo_Options(j) = Mean_Option;
23 if(mod(j, 100)==0)
24     fprintf("iteration: %d\n", j)
25 end
26 end
27 Experiment_Mean_OP = mean(MonteCarlo_Options)
28 Experiment_SD_OP = std(MonteCarlo_Options)
29 Mean_Option = mean(Option_Price);
30 fprintf("Mean: %.2f, Standard deviation: %.2f", Experiment_Mean_OP, Experiment_SD_OP)
31
```

Command Window output:

```
New to MATLAB? See resources for Getting Started.

Experiment_Mean_OP =
    6.7664

Experiment_SD_OP =
    0.0844

f3 Mean: 6.77, Standard deviation: 0.08>> |
```

圖九、HW2_311707006_d.m，輸入10000之執行結果

本次執行結果：

Mean: 6.77

Standard deviation: 0.08

The image shows a MATLAB Editor window with the file `HW2_311707006_d.m` open. The code includes a loop that prompts the user for the number of sample paths. The Command Window shows the results of the execution:

```

Experiment_Mean_OP =
    6.7602

Experiment_SD_OP =
    0.0083

fx Mean: 6.76, Standard deviation: 0.01>>

```

圖十、HW2_311707006_d.m，輸入1000000之執行結果

本次執行結果：

Mean: 6.76

Standard deviation: 0.01

*** Please report the first 30 results (the estimates of European put values in (a)) under these three alternative settings, namely, (i) 100 (ii) 10,000 (iii) and 1,000,000 sample paths. Next, summarize in a table the means and standard deviations of the put option values under these three settings. Briefly explain your results**

	NP = 100	NP = 10000	NP = 1000000
Mean	6.72	6.77	6.76
Standard Deviation	0.9	0.08	0.01
Estimated Put Value	7.04	6.85	6.77
1	7.54486040319188	6.85289352011084	6.76208065848992
2	5.53605717791108	6.66541283465147	6.74559144827448
3	5.94052241133643	6.81990872326744	6.75085161862373
4	7.13406132016823	6.69386806340658	6.76602905376110

	NP = 100	NP = 10000	NP = 1000000
5	9.52508313809112	6.72234510190085	6.77027619360948
6	8.91886968771589	6.87831923794357	6.75417629672670
7	6.99252495852533	6.70137791365737	6.74660804366215
8	7.73082788846195	6.69824117812888	6.75191808217856
9	8.02689043844842	6.61570462693351	6.76596462502912
10	7.42758521642434	6.81198477081495	6.75072541078720
11	6.42301126342703	6.75592655065757	6.76339165202896
12	5.22604096537042	6.83668993177552	6.76114944029600
13	6.49112641616823	6.76905601900347	6.75956231456137
14	5.17064458974158	6.73355018892933	6.75406967833823
15	6.82106953809522	6.67867070116804	6.77239784417824
16	6.54232271454966	6.81531272059189	6.75360621830470
17	7.00153911720156	6.72506049893224	6.76093238853250
18	6.89368387558941	6.78029336062131	6.75796649957939
19	5.93284501208138	6.79269979139111	6.75819670818378
20	5.70895897344422	6.61078986878872	6.75903708808789
21	5.93413684022908	6.70953614879340	6.76350852626809
22	7.07269162668602	6.81162042499896	6.75007931367978
23	6.25177639842722	6.66426098330129	6.76337426869881
24	7.72834403488155	6.88915060682542	6.76405993069245
25	6.65216693271050	6.72876698628513	6.76427679411590
26	6.61060348489502	6.60037348170284	6.76727112137078
27	6.50076307474364	6.63957438636645	6.76066878329306
28	8.78203577657597	6.82901824412307	6.76096525929221
29	8.00229079125185	6.68315114290004	6.74976578601486
30	7.01796850424053	6.67220223179037	6.75381095461645

Explain:

隨著NP的次數增加，值會逐漸收斂與穩定(Standard deviation從NP=100的0.9減少到NP=1000000的0.01)，再從NP=1000000的均值觀察，幾乎與BS-Model

所推出的價值一樣。此外，再從五百筆中的前三十筆資料觀察，也可以看到隨著NP次數的增加，值也變得穩定許多。再來也因為模擬實驗五百次的關係，可以看到與第一題所模擬出來的選擇權價格更為接近BS-Model所推出的價值。