Programming Part

Problem 2 - Zig-Zag Permutation (50 pts)

Problem Description

For an array **a** with N integers $\mathbf{a}[1], \mathbf{a}[2], \cdots, \mathbf{a}[N]$, we say this array is **beautiful** if it satisfies the following condition:

$$(\mathbf{a}[n+1] - \mathbf{a}[n])(\mathbf{a}[n+2] - \mathbf{a}[n+1]) < 0, \text{ for all } 1 \leq n \leq N-2.$$

Consider the permutations of any given integer array **a**, please output all the **beautiful** permutations of **a**.

Input

The first line includes an integer N, representing the length of the array. The second line includes N integers, representing the elements of the array $\mathbf{a}[1], \mathbf{a}[2], \ldots, \mathbf{a}[N]$. All integers are separated by spaces. *Note that* **a** *can contain repeated elements and is not guaranteed to be ordered.*

Output

In the first line, output a number M that indicates the number of *different* **beautiful** permutations of **a**. Note that $M = 0$ if there is no such permutation.

For each of the following M lines, output the *different* **beautiful** permutations in lexicographical order, from the smallest to the largest.[1] The lexicographical order defines a length-N array **p** to be smaller than another same-length array **q** if and only if $\mathbf{p}[n] < \mathbf{q}[n]$ for the first n in $1, 2, \ldots, N$ such that $\mathbf{p}[n] \neq \mathbf{q}[n]$. The **beautiful** permutation should be outputted with N integers, separated by spaces.

Constraints

- $1 \leq N \leq 20$
- $-10^9 \leq \mathbf{a}[n] \leq 10^9$ for every $n \in \{1, 2, \ldots, N\}$
- The number of **beautiful** permutations (M) is no more than 2×10^5.

[1] https://en.wikipedia.org/wiki/Lexicographic_order

Subtasks

Subtask 1 (10 pts)

- $1 \leq N \leq 10$

Subtask 2 (20 pts)

- $-10^4 \leq \mathbf{a}[n] \leq 10^4$ for every $n \in \{1, 2, \ldots, N\}$

Subtask 3 (20 pts)

- no other constraints

Sample Testcases

Sample Input 1

```
3
3 2 1
```

Sample Output 1

```
4
1 3 2
2 1 3
2 3 1
3 1 2
```

Sample Input 2

```
7
7 7 7 7 7 14 49
```

Sample Output 2

```
0
```

Sample Input 3

```
3
7 7 49
```

Sample Output 3

```
1
7 49 7
```

Sample Input 4

```
2
1 2
```

Sample Output 4

```
2
1 2
2 1
```

Hints

- Any array with length $N \leq 2$ satisfies all the beauty constraints and hence should be considered **beautiful**.
- You may get TLE (Time Limit Exceeded) if you only enumerate every permutation in a brute-force manner without considering any cut-off. That is, you should try to stop spending time on permutations that are **not** beautiful.
- While $\mathbf{a}[n]$ does not exceed the range of 4-byte integers, their difference and multiplication may not stay within 4 bytes. So using some longer integer format such as `long long` can be helpful.