$$\delta I = \int_a^b f(z, \dot{z}, x) dx = 0$$

$$z \to z + \epsilon n$$

$$\frac{dI}{d\epsilon} = \int_a^b \left(\frac{\partial f}{\partial z}\frac{dz}{d\epsilon} + \frac{\partial f}{\partial \dot{z}}\frac{d\dot{z}}{d\epsilon} + \frac{\partial f}{\partial x}\frac{dx}{d\epsilon} \right) dx$$

$$\frac{dI}{d\epsilon}|_{\epsilon=0} = \int_a^b \left(\frac{\partial f}{\partial z}\eta + \frac{\partial f}{\partial \dot{z}}\eta' \right) dx$$

$$\int_a^b \left(\frac{\partial f}{\partial z} - \frac{d}{dx}\frac{\partial f}{\partial \dot{z}}\right)\eta dx + \left[\frac{\partial f}{\partial \dot{z}}\eta\right]_a^b \to 0$$

$$0 = \int_a^b \left(\frac{\partial f}{\partial z} = \frac{d}{dx} \frac{\partial f}{\partial \dot{z}} \eta \right) dx$$

$$\boxed{\frac{\partial f}{\partial z} = \frac{d}{dx}\frac{\partial f}{\partial \dot{z}} = 0}$$

$$f(x)$$

$$g(x) = c$$

$$\mathcal{L}(x, \lambda) = f(x) + \lambda g(x)$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = \lambda \frac{\partial f}{\partial x}$$

$$\mathcal{L} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + m_1 g x_1 + m_2 g x_2$$

$x_1 + x_2 = l =$

$$m_1\ddot{x}_1 - m_1 g = \lambda$$

$$m_2\ddot{x}_2 - m_2 g = \lambda$$

$$\ddot{x}_1 = -\ddot{x}_2;$$

$$m_1\ddot{x}_1 - m_1 g = -m_2\ddot{x}_1 - m_2 g$$

$$\rightarrow \boxed{\ddot{x}_1 = \frac{(m_1 - m_2)g}{m_1 + m_2}}$$

μ

$$\mathcal{H}(q,p) = p\dot{q} - \mathcal{L}$$

$$\frac{\partial \mathcal{H}}{\partial p} = \dot{q}$$

$$\frac{\partial \mathcal{H}}{\partial q} = -\dot{p}$$

$$\frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}$$

$$\{A, B\}_{q,p} = \frac{\partial A}{\partial q}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial q}$$

$$\frac{df}{dt} = \{f, \mathcal{H}\} + \frac{\partial f}{\partial t}$$

$$I = \int r^2 dm = \int r^2 \rho(x) dV$$

$$I_{ij} = \int \rho(x)[\delta^i_j \left|\mathbf{x}\right|^2 - x_i x_j] d^3x$$

$$I_{parallel} = I_{cm} + MR^2$$

$$I = mr^2$$

$$I = \frac{1}{12}mL^2$$

$$I = \frac{1}{3}mL^2$$

$$I = \frac{mr^2}{2}$$

$$I = \frac{2}{5}mr^2$$

$$\boxed{\left.\frac{d}{dt}\right|_I = \left.\frac{d}{dt}\right|_B + \omega \times (\quad)}$$

$$\left.\frac{d}{dt}\right|_I \mathbf{L} = \left.\frac{d}{dt}\right|_B \mathbf{L} + \omega \times \mathbf{L}$$

$$\to \mathbf{\Gamma}^{(e)} = \mathbf{I} \cdot \frac{\omega}{dt} + \omega \times \mathbf{I}'\omega'$$

$$\Gamma_1^{(e)} = I_1 \frac{d\omega_1}{dt} + \omega_2 I_3 \omega_3 - \omega_3 I_2 \omega_2$$

$$\Gamma_1^{(e)} = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$$

$$\boxed{I_i \dot{\omega}_i = \omega_j \omega_k (I_j - I_k) + \Gamma_i^{(e)}}$$

Γ

$\hat{z}$

α

$$y \neq y'$$

$\hat{y}'$

β

$\hat{z}'$

γ

$$\boxed{\omega_1 = -\dot{\alpha} \sin \beta \cos \gamma + \dot{\beta} \cos \gamma}$$

$$\boxed{\omega_2 = \dot{\alpha} \sin\beta \sin\gamma + \dot{\beta} \cos\gamma}$$

$$\boxed{\omega_3 = \dot{\alpha}\cos\beta + \dot{\gamma}}$$

$\dot{\alpha} \cos \beta$

$$\dot{\beta}\cos\alpha$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

$$d\sigma = 2\pi b db$$

$$d\Omega = 2\pi \sin \theta d\theta$$

TODO: DIAGRAM

$$\mathcal{L}_g = -\rho(\mathbf{x},t)\phi(\mathbf{x},t) - \frac{1}{8\pi G}(\nabla\phi(\mathbf{x},t))^2$$

$$\rightarrow -\rho = \frac{2}{8\pi G}\nabla^2\phi$$

$$\boxed{\nabla^2\phi = 4\pi G\rho}$$

$$\boxed{\mathbf{g} = -\nabla\phi}$$

$$\nabla \cdot \mathbf{g} = -4\pi G\rho$$

$$\oint \nabla \cdot \mathbf{g} dV$$

$$= \oint \mathbf{g} \cdot d\sigma = -4\pi GM$$

$$p^\mu = \{E, p^x, p^y, 0\}$$

$$v^{\mu}_{obs} = \{\frac{dt}{d\tau}, \frac{dx}{d\tau}\} = \{\frac{dt}{d\tau}, \gamma v_x\}$$

$$E = p^{\mu} V_{\mu} = \omega \frac{dt}{d\tau} - \omega \cos \theta v_x \gamma$$

$$\omega' = \omega(\gamma - \cos\theta\gamma v_x)$$

$$\boxed{\frac{\omega'}{\omega} = \frac{1 - v\cos\theta}{\sqrt{1 - v^2}}}$$

$$\boxed{\frac{\omega'}{\omega} = \sqrt{\frac{1-v}{1+v}}}$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 + \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2 = -dt^2 + \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2$$

$\rightarrow$

$$r = \sinh\theta$$

$$r = \sin\theta$$

$$w = \frac{1}{2} \int_{v} \rho(x) \Phi(x) d^3x,$$

$$\mathbf{F} = \int \frac{\sigma^2}{2\epsilon_0} d\mathbf{a}$$

$$\mathbf{s} = \frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right)$$

$$\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{s}$$

$$\partial_\mu s^\mu = 0$$

$$\partial_\mu J^\mu = 0$$

$$\mathcal{P}_{em} = \frac{1}{c^2}\mathbf{s}$$

$$(\nabla^2 + k^2)\psi = -\delta(r - r_0)$$

$$\int dV[\psi(\nabla^2 + k^2)\phi - \phi(\nabla^2 + k^2)\psi]$$

$$\int d\mathbf{s} \cdot (\psi \nabla \phi - \phi \nabla \psi)$$

$$\mathbf{J} = qN\mathbf{v}_{drift} = \frac{n\mathbf{I}}{a}$$

$$\sigma = \frac{J}{\epsilon}$$

$$\sigma = -\epsilon_0 \frac{\partial \phi}{\partial} \eta$$

$$q' = \frac{-a}{y} q$$

$$\frac{q'}{y'} = \frac{-q}{a}$$

$$\Phi = \sum_i \phi_i$$

TODO: Image Needed Here

$$\theta = \tan^{-1} n$$

$$U = -\mathbf{mu} \cdot \mathbf{B}$$

$$U = -\mu\sigma \cdot \mathbf{B}$$

σ

$$\mathbf{F}_m = \int dq(\mathbf{v} \times \mathbf{B}) = \int d\mathbf{l}\lambda(\mathbf{v} \times \mathbf{B})$$

$$\lambda \mathbf{V} = \mathbf{I};$$

$$\boxed{\int (\mathbf{I} \times \mathbf{B}) \cdot d\mathbf{l} = \mathbf{L}}$$

$$\boxed{\mathbf{F} = \int (\mathbf{J} \times \mathbf{B}) d\tau}$$

$$\psi(r') = \frac{-ik}{2\pi} \frac{e^{ik(r_0+r')}}{r_0 r'} \int_\sigma dx dy e^{i\mathbf{q}\cdot\mathbf{x}}$$

$$I = \left|\psi(r')\right|^2$$

$$\sim \frac{e^{ikr}}{r^2}$$

$$\nabla \times \mathbf{E} = \frac{\partial^2 \mathbf{B}}{\partial t^2} \cdot d\mathbf{a}$$

$$\oint \nabla \times \mathbf{E} \cdot d\mathbf{a} = -\int \frac{\partial^2 \mathbf{B}}{\partial t^2} \cdot d\mathbf{a}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint \mathbf{B} \cot d\mathbf{a}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{r}}{r^2} dl'$$

$$= \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{r}}{r^2}$$

$$\mathbf{P} = \int r' \rho(r') d\tau' = \sum_{i=1} q_i \mathbf{r}'_i$$

$$V_{dipole}(r) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{P}\cdot\hat{r}}{r^2}$$

$$\mathbf{P} = \mathcal{N} < \mathbf{P} >$$

$$\mathbf{E}_{dip}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\mathbf{p}\cdot\hat{r})\hat{r} - \mathbf{p}\right]$$

$$\sigma_b = \dot{\mathbf{p}} \hat{n}$$

$$\rho_b = -\nabla \cdot \mathbf{p}$$

$$\rho_T = \nabla \cdot (\epsilon_0 \mathbf{E} - \mathbf{P})$$

$$\epsilon_0 \mathbf{E} - \mathbf{P} = \mathbf{D}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\iint \nabla \times \mathbf{B} \cdot d\mathbf{a} = \mu_0 \iint \mathbf{J} \cdot d\mathbf{a} + \frac{1}{c^2} \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{a}$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \iint \mathbf{J} \cdot d\mathbf{a} + \frac{1}{c^2} \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{a}$$

∞

$$\frac{Q}{\epsilon_0} = \oiint \mathbf{E} \cdot d\mathbf{a} = \iiint \nabla \cdot \mathbf{E} \cdot d\mathbf{V}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \frac{\mathbf{m}\times\mathbf{r}}{r^3}$$

B

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{m}\cdot\mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3}\right) = \frac{\mu_0}{4\pi}\frac{1}{r^3}(3(\mathbf{m}\cdot\hat{r})\hat{r} - \mathbf{m})$$

$$\mathbf{m} \times \mathbf{B} = \mathbf{\Gamma}$$

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$

$$\mathbf{m} = I \int d\mathbf{a} = I\mathbf{a}$$

$$\mathbf{M} = \mathcal{N}\mathbf{m}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

$$T_{ij} = \epsilon_0 \left[E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right] +$$

$$\frac{1}{\mu_0}\left[B_i B_j - \frac{1}{2}\delta ij B^2\right]$$

$$\frac{\partial^2 \mathcal{P}_{mech} + \mathcal{P}_{em}}{\partial t^2} = \nabla \cdot \mathbf{T}$$

$$\mathcal{P}_{em} = \mu_0 \epsilon_0 \mathbf{s}$$

$$dV = \frac{dq}{C} \rightarrow qdV = dU = \frac{qdq}{C}$$

$$U = \frac{q^2}{2C} = \frac{1}{2}vq = \boxed{\frac{1}{2}cv^2 = U}$$

$(V = L\dot{I})I;$

$$IV = IL\dot{I} = P$$

$$P = LI\frac{dI}{dt}$$

$$\int dPdE = \int LIdI$$

$$\boxed{U = \frac{1}{2} L I^2}$$

$$w = \frac{\epsilon_0}{2}\epsilon_r E^2 d\tau = \frac{1}{2}\int \mathbf{DE} d\tau$$

$$w = \frac{1}{2\mu_0} \int \mathbf{HB} d\tau$$

$$u_{em} = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$$

Electric and Magnetic field relations:

$$\mathbf{k} \times \mathbf{E} = \mu\omega\mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\epsilon\omega\mathbf{E}$$

$$\mathbf{k} \cdot \mathbf{E} = 0$$

$$\mathbf{k} \cdot \mathbf{H} = 0$$

$$\mathbf{B} = \mp \frac{i\mathbf{E}}{c}$$

$$E_1 = E cos\theta$$

$$I_1 = Icos^2\theta$$

$$\mathbf{k} \cdot \mathbf{r} = \mathbf{k}'' \cdot \mathbf{r}$$

$$H^\dagger = (H^*)^{-1} = H$$

$$U^{-1}U = UU^{-1} = \mathbb{1}$$

$$O^\dagger O = \mathbb{1}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbb{1} = \delta_{ij}$$

$$\sigma_i \sigma_j = i \sigma_k$$

$$\sigma_i^2 = \mathbb{1}$$

$$\sigma = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z}$$

$$\rho = \sum_i w_i \left|a^i\right\rangle \left\langle a^i\right|$$

$$Tr(\rho) = 1$$

$$Tr(\rho A) = \langle A \rangle$$

ρ

$(\rho^2 = \rho)$

$$a^{\pm} = \left(\frac{m\omega}{2\hbar}\right)^{1/2} x \mp i\left(\frac{1}{2\hbar\omega m}\right)^{1/2} p$$

$$x = c_1(a + a^\dagger); p = c_2(a - a^\dagger)$$

$$a\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a^\dagger \left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

$$a^\dagger a \left| n \right\rangle = n \left| n \right\rangle$$

$$\mathcal{Z} = \sum_{n=0}^{\infty} (e^{\mu\beta} e^{-E\beta})^N$$

$$\rightarrow \boxed{\frac{1}{1 - e^{(E-\mu)\beta}} = \mathcal{Z}}$$

$$n = \frac{1}{\beta}\frac{\partial^2 ln(\mathcal{Z})}{\partial \mu^2} \rightarrow$$

$$\langle n \rangle = \Big[\exp{(E - \mu)\beta} - 1 \Big]^{-1}$$

$$\mathcal{Z} = e^{0\mu\beta - 0\beta} + e^{\mu\beta - \epsilon\beta}$$

$$= 1 + e^{(\mu-\epsilon)\beta}$$

$$N = kT \frac{\partial}{\partial \mu} ln \mathcal{Z}$$

$$\boxed{= \frac{1}{e^{(\epsilon-\mu)\beta}} + 1}$$

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$$

$$[\mathbf{J}^2, J_k] = 0$$

$$J_{\pm} = J_x \pm iJ_y[J_+, J_-] = 2\hbar J_z$$

$$[J_z, J_\pm] = \pm\hbar J_\pm$$

$$J^2 |j, m\rangle = j(j+1)\hbar^2 |j, m\rangle$$

$$J_z \left| j, m \right\rangle = m\hbar \left| j, m \right\rangle$$

$$J_{\pm}|j,m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}\hbar\,|j,m \pm 1\rangle$$

$$H_0 + H', E = E_0 + \lambda E_1 + \lambda^2 E_2 ...$$

$$|\psi\rangle = |n_0\rangle + \lambda\,|n_1\rangle + \lambda^2\,|n_2\rangle \, ...$$

$$E_1 = \langle n_0 | \, H' \, | n_0 \rangle$$

$$|n_1\rangle = \sum_{m=1, m\neq n}^{\infty} \frac{\langle n_0| H' |n_0\rangle}{E_n^{(0)} - E_m^{(0)}} |m_0\rangle$$

$$E2 = \sum_{m=1, m \neq n}^{\infty} \frac{|\langle m_0 | H' | n_0 \rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\left|\psi^{+}\right\rangle = |i\rangle + \frac{1}{E_i - H_0 + i\hbar\epsilon} V \left|\psi^{+}\right\rangle$$

$$\psi^{\pm}(\mathbf{x}, t) = \phi^{\pm}(\mathbf{x}, t) - \frac{m}{2\pi\hbar^2} \int_{\infty}^{\infty} d\mathbf{x}'$$

$$\times \frac{e^{\pm ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|} V(\mathbf{x}'\psi^{\pm}(\mathbf{x}', t)$$

$$|\mathbf{x} - \mathbf{x}'| \to r$$

r

$$f^{(1)}(k,k') = \frac{-m}{\hbar^2 2\pi} \int_{\infty}^{\infty} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} V(\mathbf{r}) d^3\mathbf{r}$$

$$f^{(1)}(\theta) = \frac{-2m}{\hbar^2 q} \int_0^\infty r dr V(r) sin(qr)$$

$$\frac{d\sigma}{d\Omega} = \left|(f^{(1)}(\theta)\right|^2$$

$$w_{ij} = \frac{2\pi}{\hbar} \left| \langle \psi_f | V | \psi_i \rangle \right|^2 \rho(E_i)$$

$$E = \frac{\langle\alpha|\, H \,|\alpha\rangle}{\langle\alpha|\alpha\rangle}$$

α

$$\frac{dE}{d\alpha} = 0 \iff E = E_0;$$

$$\dot{c}_m(t) = -c_m(t) \langle m; t| \left[\frac{\partial}{\partial t} |m; t\rangle \right]$$

$$-\sum_n c_n(t)e^{i(\theta_n-\theta_m)}\frac{\langle m;t|\,H\,|n;t\rangle}{E_n-E_m}$$

$$i\frac{\partial}{\partial s}U(t,t_0) = \frac{H}{\hbar/T}U(t,t_0) = \frac{H}{\hbar\Omega}U(t,t_0)$$

$$T \to 0, \hbar\Omega >> H$$

$$U(t, t_0) \to 1$$

$T \to 0$

$$|\psi(t)\rangle_s$$

$$e^{iHt}\,|\psi\rangle_s = |\psi\rangle_H$$

$$U^\dagger A U = A_H$$

$$U^{\dagger} A U$$

$$|\psi\rangle_I = e^{iH_0t}\,|\psi(t)\rangle_s$$

$$e^{iH_0t}Ae^{-iH_0t} = A_I$$

$$V_I$$

$$+\mu N$$

$$P = \frac{1}{\mathcal{Z}} e^{-E\beta}$$

$$e^{-F\beta} = \int \frac{1}{h^n c} e^{-E\beta} dp_1 ... dq_n = \sum_i e^{-E_i \beta} = \mathcal{Z}$$

Ω

$$S = k \ln \Omega$$

$$\boxed{\text{fugacity } f = e^{\mu\beta}}$$

$$\boxed{\mathcal{Z} = \sum_i f_i e^{-\epsilon_i \beta}}$$

$$= \frac{1}{2}kT$$

$$U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$$

$$\frac{1}{2}mv_i^2, \frac{1}{2}I\omega_i^2, \frac{1}{2}k_s x^2,$$

$$\Delta S = \int \frac{dQ}{T}$$

$$S = \beta(\langle E \rangle - \langle F \rangle)$$

$$S = -\sum_{i} \mathcal{P}_i \ln \mathcal{P}_i$$

$$S = -kTr(\rho \ln \rho)$$

$$\langle (\Delta E)^2 \rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial \beta^2}$$

$$C_v = \frac{\partial \langle E \rangle}{\partial T} = \frac{\beta}{T} \langle (\Delta E)^2 \rangle$$

$$\mathcal{Z} = \sum_n e^{-\beta\hbar\omega n} = \frac{1}{1 - e^{-\beta\hbar\omega}}$$

$$d^3n = \frac{V}{h^3}\int d^3p = \frac{4\pi V}{h^3}\hbar^3\int k^2dk$$

$$(p = \hbar k, \omega = ck)$$

$$\hbar = c = 1$$

$$u = -\frac{\partial \ln \mathcal{Z}}{\partial \beta}$$

$$\rightarrow U = (2)\frac{4\pi V}{(2\pi)^3}\int \frac{\omega k^2}{1-e^{-\beta\omega}}dke^{-\beta\omega}$$

$$\frac{U}{V} = \frac{8\pi}{(2\pi)^3} \int \frac{k^3 dk}{1 - e^{-\beta k}} e^{-\beta k}$$

$$\beta k = x, \beta dk = dx;$$

$$\int \frac{x^3 dx}{e^x - 1} = ...$$

$$\boxed{\frac{U}{V} = \frac{8\pi}{(2\pi)^3}(kT)^4 \int \frac{x^3 dx}{e^x - 1}}$$

$$\boxed{= \frac{\pi^2 (kT)^4}{15\hbar^3 c^3} \equiv \sigma T^4}$$

$$f = e^{\mu\beta}$$

$$\mathcal{Z} = \sum_{N=0}^{\infty} \Omega \left[f e^{-\beta E} \right]^N$$

$$\boxed{\langle n \rangle = \frac{1}{\beta} \frac{\partial \ln \mathcal{Z}}{\partial \mu}}$$

$$dU = TdS - pdV$$

$$ddU = 0 = dT \wedge dS - dp \wedge dV$$

$$\boxed{dT \wedge dS = dp \wedge dV}$$

$$dT = \frac{\partial T}{\partial S} dV \wedge dS + \frac{\partial T}{\partial V} dV$$

$$\frac{\partial T}{\partial V} dV \wedge dS = \frac{\partial p}{\partial S} dS \wedge dV$$

$$\rightarrow \boxed{\frac{\partial T}{\partial V} = -\frac{\partial p}{\partial S}}$$

$$\mathcal{Z} = \frac{V}{h^3} \int d^3p \left[e^{-\beta p^2/2m} \right] = \frac{V}{h^3} \sqrt{\frac{2\pi m}{\beta}}^3$$

$$\mathcal{Z} = \frac{V^N}{h^{3N}} \left(\frac{2\pi m}{\beta} \right)^{3N/2} = V^N \left(\frac{2\pi m}{h^2 \beta} \right)^{3N/2}$$

$$\mathcal{Z} = \frac{V^N}{\lambda_{th}^{3N}}$$

$$A = -kT \ln \mathcal{Z}$$

$$S = -\frac{\partial A}{\partial T}$$

C_p

C_v

α_p

k_T

$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\alpha_P = -\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$C_P = -T\left(\frac{\partial S}{\partial T}\right)_P$$

$$C_V = -T\left(\frac{\partial S}{\partial T}\right)_V$$

$$C_v = C_p - \frac{v\alpha_p^2}{k_T}$$

$$k_T = -\frac{1}{v}\frac{\partial v}{\partial p}\Big|_p$$

$$\alpha_p = -\frac{1}{v}\frac{\partial v}{\partial T}\Big|_p$$

$$dW = -PdV$$

$$W = -\int_{v_i}^{v_f} PdV$$

$$= -\int_{v_i}^{v_f} \frac{NT}{V} dV$$

$$\boxed{W = NT \ln \frac{v_i}{v_f}}$$

$$U = \frac{f}{2}NT \rightarrow dU = \frac{f}{2}NdT$$

$$\frac{f}{2}NdT = -PdV \rightarrow \frac{f}{2}NdT = -\frac{NT}{V}dV$$

$$\frac{f}{2}\frac{dT}{T} = -\frac{dV}{V} \rightarrow VT^{f/2} = V_0 T_0^{f/2}$$

$$\boxed{PV^{\gamma} = \text{ const; } \gamma = \frac{f+2}{f}}$$

$\mathbb{1}$

$$\mathbb{1} = A (\forall A \in G)$$

$\in$

$$A^{-1}$$

$$AA^{-1} = A^{-1}A = \mathbb{1}$$

$$\nu \neq 0; R = a\rho^{\nu} + b\rho^{-\nu}$$

$$\psi = A \cos \nu\phi + B \sin \nu\phi$$

$$\nu = 0; R = a_0 + b_0 \ln \rho$$

$$\psi = A_0 + B_0\phi$$

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}(\rho\frac{\partial R}{\partial\rho}) = \nu^2$$

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\nabla^2\Phi = 0$$

$$\Phi = \sum_{l,m} Y_l^m(\Omega)\left[A_l r^l + \frac{B_l}{r^{l+1}}\right]$$

$$= \sum_{l,m} P_l(\cos\theta) e^{im\phi} \left[A_l r^l + \frac{B_l}{r^{l+1}} \right]$$

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{2}\left(3\cos^2\theta - 1\right)$$

$$\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{2n+1}\delta_{mn}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}}$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy$$

$$= \int_0^\infty dr \int_0^{2\pi} r d\theta e^{-r^2}$$

$$u = r^2, du = 2rdr$$

$$\boxed{\int_{\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}}$$

$$x \to \sqrt{\alpha}x', dx = \sqrt{\alpha}dx'$$

$$\boxed{\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx$$

$$= -\int_{-\infty}^{\infty} \frac{d}{d\alpha} e^{-\alpha x^2} dx$$

$$= -\frac{d}{d\alpha} \int_{-\infty}^{\infty} e^{-\alpha x^2} dx$$

$$= -\frac{d}{d\alpha}\sqrt{\frac{\pi}{\alpha}}$$

$$\boxed{\int_{-\infty}^{\infty} dx e^{-\frac{a}{2}x^2 + Jx} = \sqrt{\frac{2\pi}{\alpha}} e^{J^2/2\alpha}}$$

$$\int_{-\infty}^{\infty} dx e^{-\frac{\alpha}{2}+Jx}$$

$$-\frac{\alpha}{2}x^2 + Jx = -\frac{\alpha}{2}\left(x^2 - \frac{2Jx}{\alpha}\right)$$

$$= -\frac{\alpha}{2}\left(x - \frac{J}{\alpha}\right)^2 + \frac{J^2}{2\alpha}$$

$$\to \int_{-\infty}^{\infty} e^{-\frac{\alpha}{2}(x-\frac{J}{\alpha})^2} dx e^{J^2/2\alpha}$$

$$u = x - \frac{J}{\alpha}, du = dx$$

$$\boxed{\int_{-\infty}^{\infty} dx e^{-\frac{\alpha}{2}+iJx} = \sqrt{\frac{2\pi}{\alpha}} e^{-J^2/2\alpha}}$$

$$J \to iJ$$

$$\boxed{\int_{-\infty}^{\infty} dx e^{i\frac{\alpha}{2}+iJx} = \sqrt{\frac{2\pi i}{\alpha}}e^{-J^2/2\alpha}}$$

$$\alpha \to -i\alpha$$