

Written Hw 1

$$\begin{aligned} 1. \quad & x_1 - 2x_2 + 0x_3 - x_4 + x_5 = 5 \\ & -2x_1 + 4x_2 + 2x_3 + 2x_4 - 10x_5 = 4 \\ & -x_1 + 2x_2 + 2x_3 + 0x_4 - 11x_5 = 2 \end{aligned}$$

Augmented matrix:

(at least) 2 free variables

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 1 & 5 \\ -2 & 4 & 2 & 2 & -10 & 4 \\ -1 & 2 & 2 & 0 & -11 & 2 \end{array} \right] \quad \begin{array}{l} R1(2) + R2 \rightarrow R2 \\ R1 + R3 \rightarrow R3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 1 & 5 \\ 0 & 0 & 2 & 0 & -8 & 14 \\ 0 & 0 & 2 & -1 & -10 & 7 \end{array} \right] \quad R2(\frac{1}{2}) \rightarrow R2$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 1 & 5 \\ 0 & 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 2 & -1 & -10 & 7 \end{array} \right] \quad R2(-2) + R3 \rightarrow R3$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 1 & 5 \\ 0 & 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 0 & -1 & -2 & -7 \end{array} \right] \quad R3(-1) \rightarrow R3$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 1 & 5 \\ 0 & 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 0 & 1 & 2 & 7 \end{array} \right] \quad R3(1) + R1 \rightarrow R1$$

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & b \\ \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3 & 12 \\ 0 & 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 0 & 1 & 2 & 7 \end{array} \right] & \text{REF} & \begin{array}{l} x_2 \text{ is free, has no lead} \\ x_5 \text{ is free, has no lead} \end{array} \end{array}$$

$$\begin{aligned} x_1 - 2x_2 + 3x_5 &= 5 & \rightarrow x_1 &= 2x_2 - 3x_5 + 5 \\ x_3 - 4x_5 &= 7 & \rightarrow x_3 &= 4x_5 + 7 \\ x_4 + 2x_5 &= 7 & \rightarrow x_4 &= -2x_5 + 7 \end{aligned}$$

let $x_2 = s$ and $x_5 = t$. so,

$$(x_1, x_2, x_3, x_4, x_5) = (2s - 3t + 5, s, 4t + 7, -2t + 7, t)$$

2.

$$(A - 2I)^{-1} = \begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 7 & -6 \\ -4 & 3 \end{bmatrix}$$

$$A - 2I = \begin{bmatrix} \frac{7}{3} & -\frac{6}{3} \\ -\frac{4}{3} & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{7}{3} & -\frac{6}{3} \\ -\frac{4}{3} & -1 \end{bmatrix} + 2I$$

Solve for A

Invert both sides (Thm 1.4.5)

$$3(7) - 4(6) = -3$$

$$\begin{vmatrix} 3 & 6 \\ 4 & 7 \end{vmatrix} = -3$$

The size of A and 2I must be the same in order to subtract.

The result must also be the same size. So I must be the 2x2 matrix

$$A = \begin{bmatrix} \frac{7}{3} & -\frac{6}{3} \\ -\frac{4}{3} & -1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{and } 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{13}{3} & -\frac{6}{3} \\ -\frac{4}{3} & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{13}{3} & -\frac{6}{3} \\ -\frac{4}{3} & -1 \end{bmatrix}$$