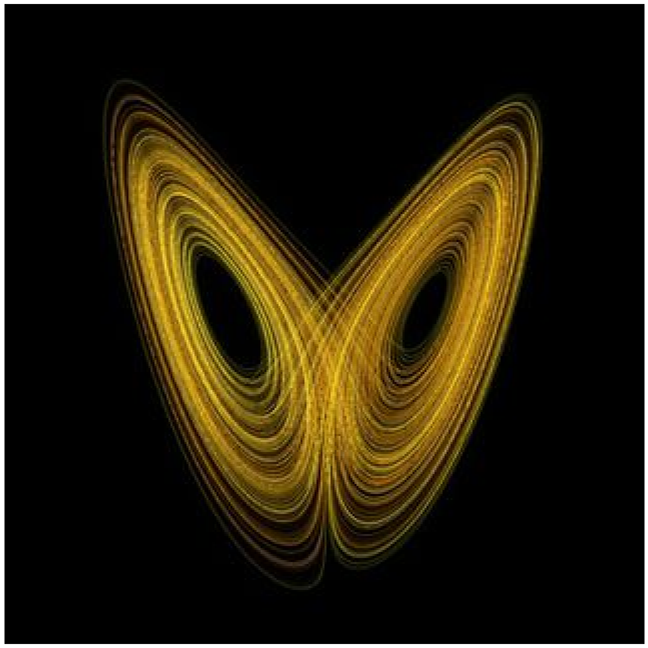

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Problem 2 - Lorenz Systems

(a) Figure 1 below shows a plot of the the Lorenz system with $a = 16,\ r = 15,\ b = 4$ and initial conditions $(x, y, z) = (-13,\ -12,\ 52)$. This three-dimensional plot is projected on the x - z plane. The plot was created with an adaptive Runge Kutta 4th order solver.

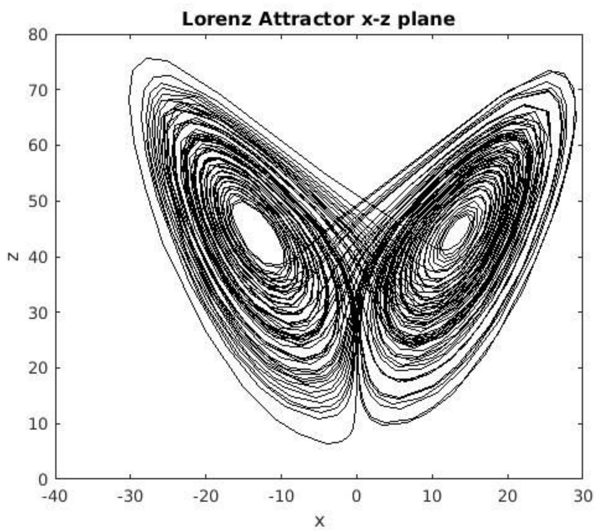

Figure 1

(b) Figure 2 shows the plots of the Lorenz system using a timestep of 0.001. The black plot was generated with the adaptive RK4 and the red with the non-adaptive RK4. The plot was limited to easily demonstrate the variation between to the solvers.

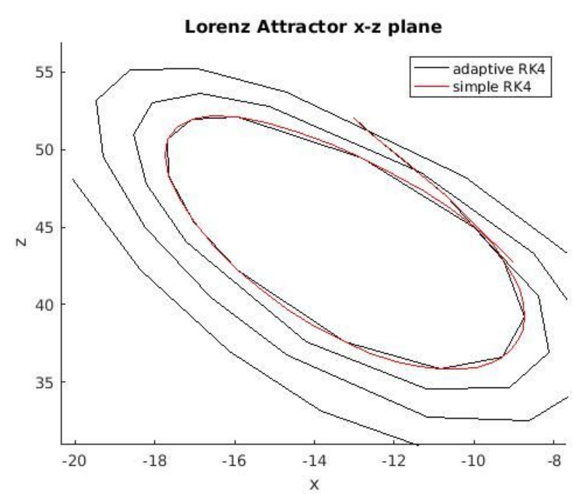

Figure 2

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The solutions agree but the adaptive solver reaches the solutions quicker. The adaptive solvers are not evenly spaced over time. This is precisely the purpose of the adaptive solver - it adjusts its timesteps in order to constrain error.

(c) Next r was varied and the results were observed.

For $0 < r < 1$, the system converges to $(0, 0, 0)$. This behavior was observed across multiple initial conditions.

For $13.5 < r < 14$ the system converges to a stable fixed point near $(0, 0, 0)$. These trajectories are longer than the previous ones created with a smaller r value. Nonetheless, they still converge.

For $23 < r < 30$, similar behavior to the previous behavior was observed until roughly $r \approx 28.944$, at which point chaotic behavior begins. Before this value for r, the system converges on a stable fixed point. Afterwards, it begins to behave chaotically.

Plots were not explicitly requested for this section but I included a plot with $r = 27$, zoomed in on the attractor because I thought the plot was extremely interesting; almost like an optical illusion. Figure 3 below is the aforementioned plot.

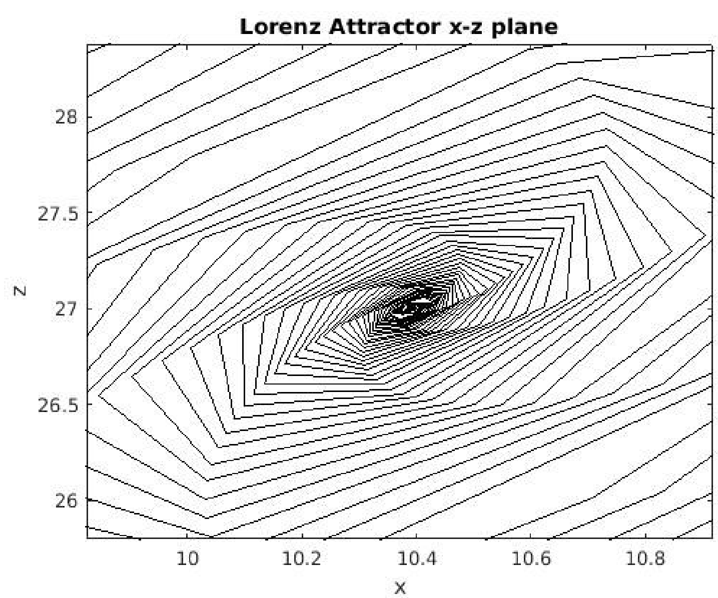

Figure 3

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Problem 3 - Rossler Systems

Figure 4 below displays a plot of the Rossler system with parameters $a = 0.398,\ b = 2,\ c = 4$

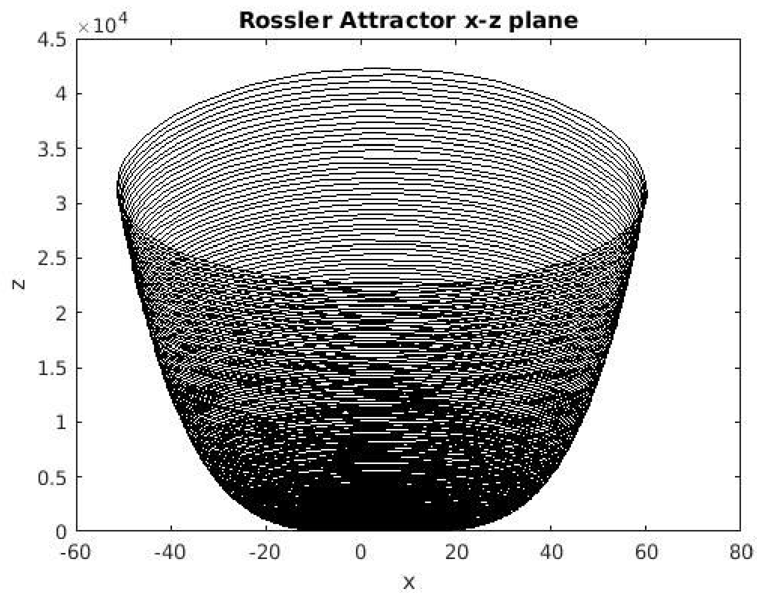

Figure 4

Problem 4 - Timestep Manipulation

Figure 5 and figure 6 display plots of Lorenz systems created by adaptive Runge Kutta methods with varying error thresholds. Figure 5 has a sensible threshold of 0.1 while figure 6 has a threshold of 100.

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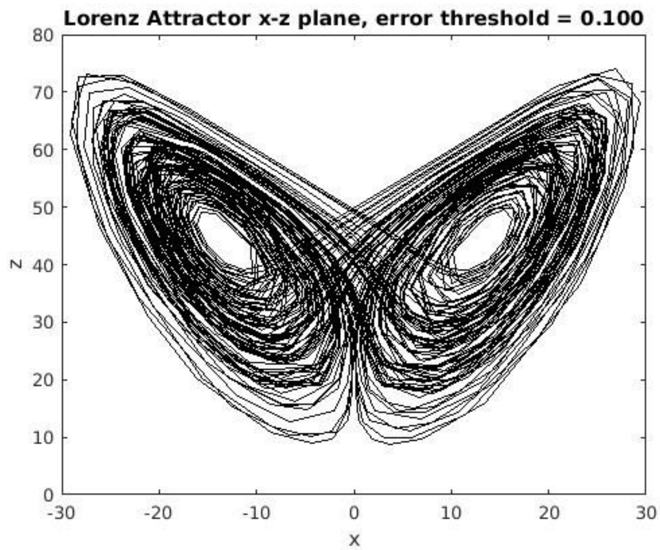

Figure 5

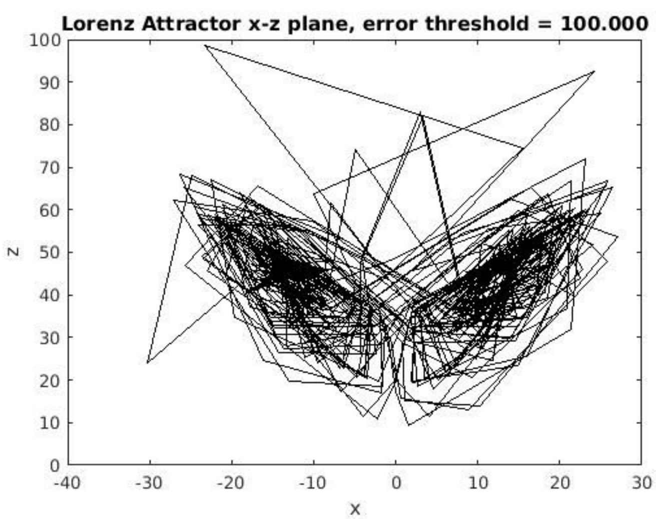

Figure 6

The dynamics are clearly altered by the error threshold. If too much error is permitted, the timestep becomes too large. This is equivalent to the previous assignment in which the the Runge Kutta method overshoots the point in questions. By increasing the error threshold, the timestep never gets small enough and overshoots the point just like in the previous assignment.