Ryan Riley
CSCI4446 - Problem Set 1
20 January 2017

Part 1 - Intro

In this assignment, the behavior of the logistic map is explored. The logistic map is a polynomial mapping that is frequently used to display chaotic behavior from a simple system and is described by the following equation:

$$x_{n+1} = Rx_n(1 - x_n)$$

Part 2 - Experimenting with values

By manipulating the values of the parameters, the behavior of system can be explored. First we start with a plot $x_n \ vs \ n$ of the map with an R value of 2.6 and an initial condition of x_0 = .02 in figure 1:

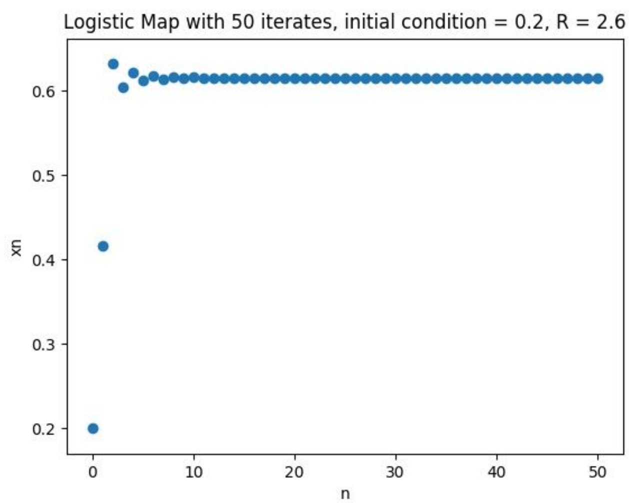

Figure 1

In this plot there is an initial transient that converges to a stable value.

Part 3 - Plots with various axes

Next we can look at a plot of $x_n \ vs \ n$ of the map with an R value of 3.83 and an initial condition of x_0 = .02 in figure 2a:

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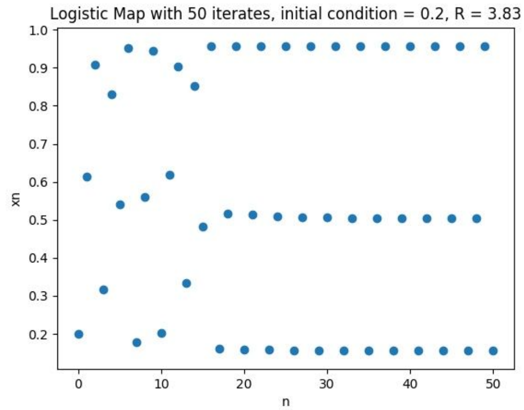

Figure 2a

We can see that there is again an initial transient period before the system stabilizes. This time, however, the system does not converge to a single value but rather oscillates between three different values. All three values seem centered around some point, which is often called an attractor.

The nonlinearity is very apparent when looking at the plot of the same values but of x_{n+1} vs x_n in figure 2b and of x_{n+2} vs x_n in figure 2c:

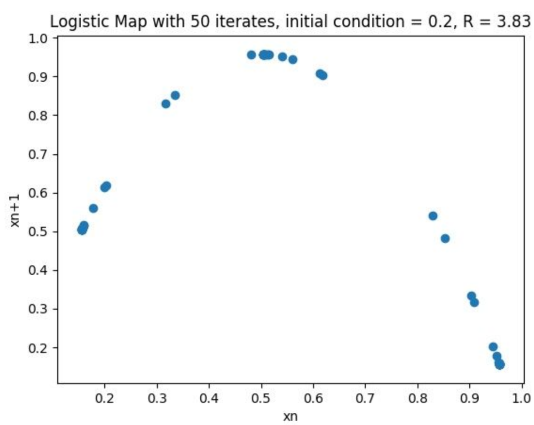

Figure 2b

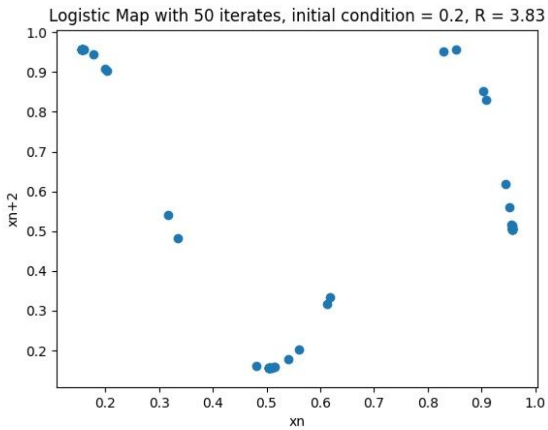

Figure 2c

Part 4 - R > 4

If R > 4, as shown in figure 3, the system quickly diverges and heads to infinity. Python's pyplot library was unable to reasonably display this behavior, so the iterate values have been included after the the plot to display the divergent behavior.

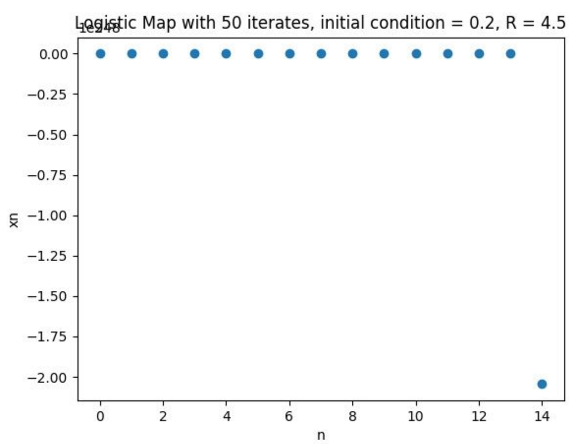

Figure 3

Iterate values:

[0.2, 0.7200000000000001, 0.9071999999999998, 0.3788467200000008, 1.0589484723535882, -0.280905276358845, -1.619158727902878, -19.08375171321125, -1724.7299902411705, -13393882.211523972, -807282423405272.4, -2.932672100125906e+30, -3.870254541085601e+61, -6.740491595757173e+123, -2.0445402128612882e+248, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf, -inf,

-inf, -inf]

Part 5 - Chaos

To display chaotic behavior, we can keep R to a fixed value, in this case 3.99, and alter the initial condition x_0 with by a very small Δ. The resulting variance in plots, as shown in figure 4 displays chaotic behavior:

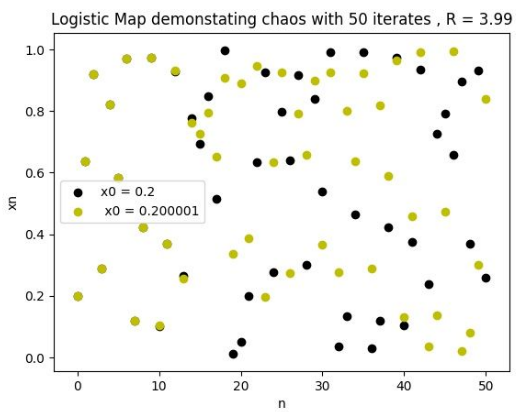

Figure 4

It should be noted that the previous figure (4) is two separate plots, superimposed upon each other. The black and yellow each represent different initial conditions. The variance can be seen after the initial ~20 iterations where the black and yellow no longer share the same values.

Part 6 - Fix R to 2.5

The following three plots explore the impact of different initial conditions on the values when R is kept to a fixed value of 2.5. Figure 5a displays an initial condition of x_0 = 0.2. Figure 5b displays x_0 = 0.55. Figure 5c displays x_0 = 0.76

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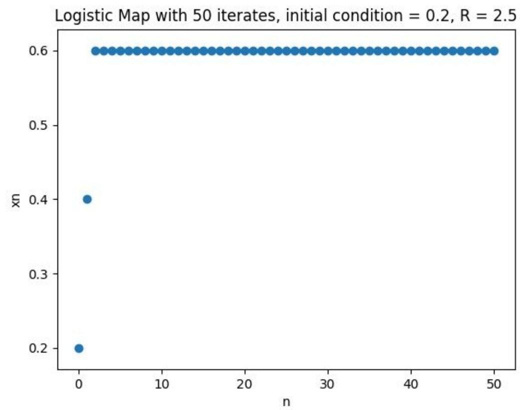

Figure 5a

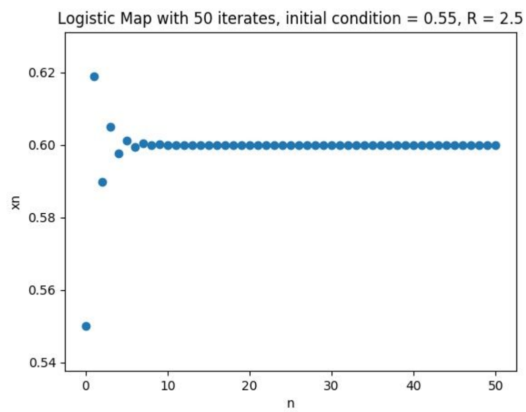

Figure 5b

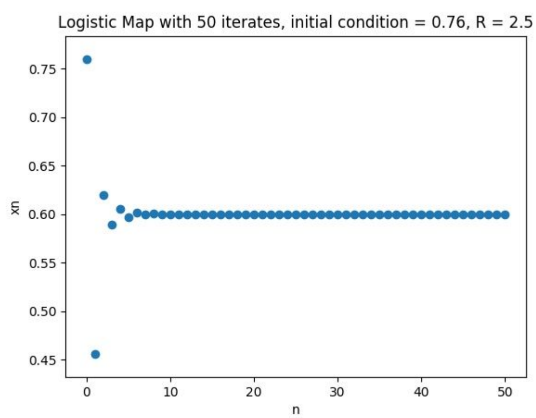

Figure 5c

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The three plots have different initial transient phases but all three converge to a fixed point. This is called an attractor. When various initial conditions all share an attractor, this is called a basin of attraction.