Ryan Riley
CSCI4446
Problem Set 4
13 February 2017

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Problem 2 - State-Space Trajectories

A. Figure 1, below, displays the state-space trajectory emanating from the point $[\theta,\ \omega]\ =\ [3,\ 0.1]$ with $\Delta t\ =\ 0.005,\ m\ =\ 0.1kg,\ l\ =\ 0.1m,\ \beta = 0,\ A\ = 0,\ \ \alpha\ =\ 0.$ The initial condition is near an unstable equilibrium point at $\pi,\ 0$

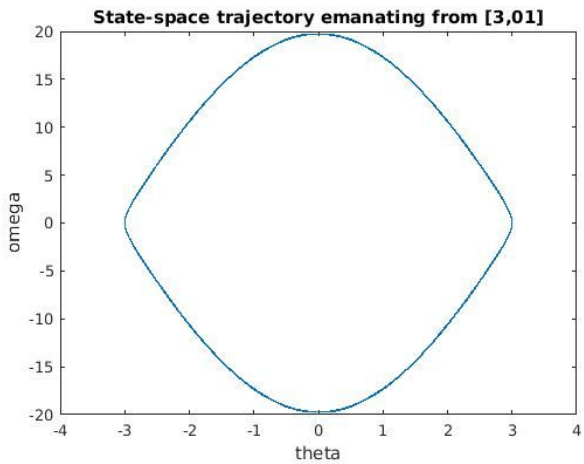

Figure 1

B. Figure 2 displays the state-space trajectory emanating from the point $[\theta,\ \omega]\ =\ [0.01,\ 0]$ with $\Delta t\ =\ 0.00,\ m\ =\ 0.1kg,\ l\ =\ 0.1m,\ \beta = 0,\ A\ = 0,\ \ \alpha\ =\ 0.$

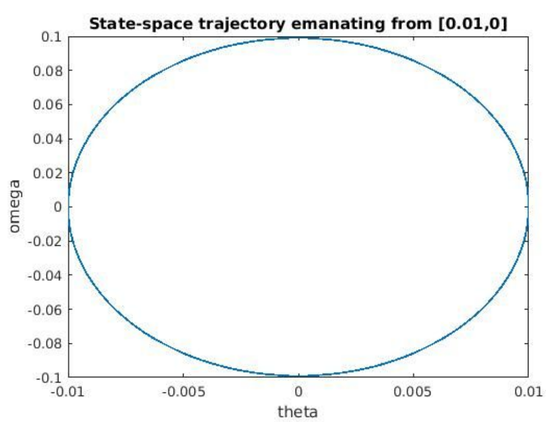

Figure 2

By viewing the graphs, it is apparent that figure 2 is significantly closer to a perfect ellipse than figure 1 .

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Problem 3 - State-Space Portraits

Using the coefficients from part 2, a state-space Portrait was constructed by plotting various initial conditions. Figure 3 below displays the result.

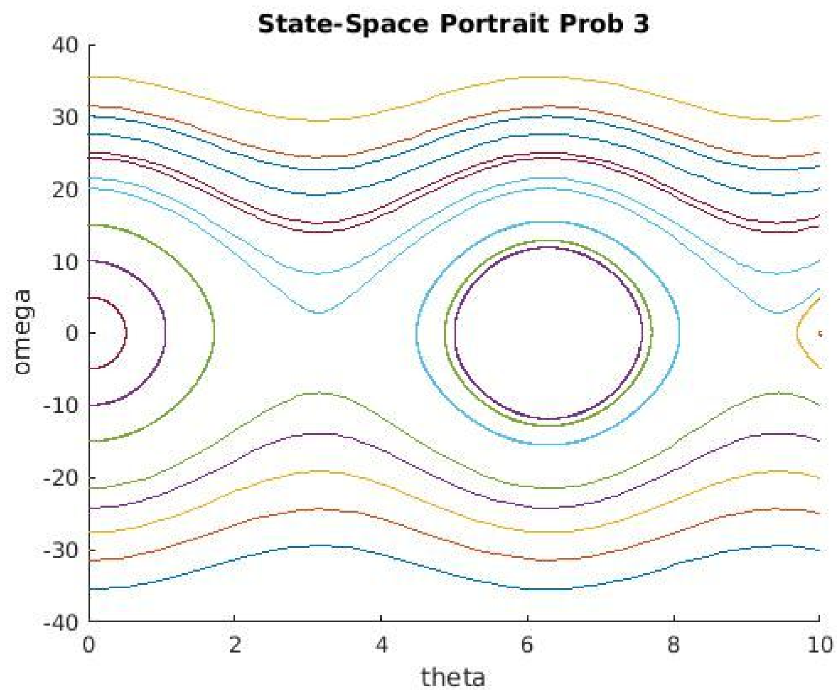

Figure 3

Picking the correct initial conditions took significant effort. I tried many different values and had to annotate the results until I found satisfactory results. One of the hurdles was my xlimit for the x axis. I was initially plotting over a much larger span of theta values and the portrait was obscured.

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Problem 4 - State-Space Portraits Cont'd

Using the same parameters as in problem the portrait was redrawn with $\beta = 0.25$. The result is shown below in figure 4.

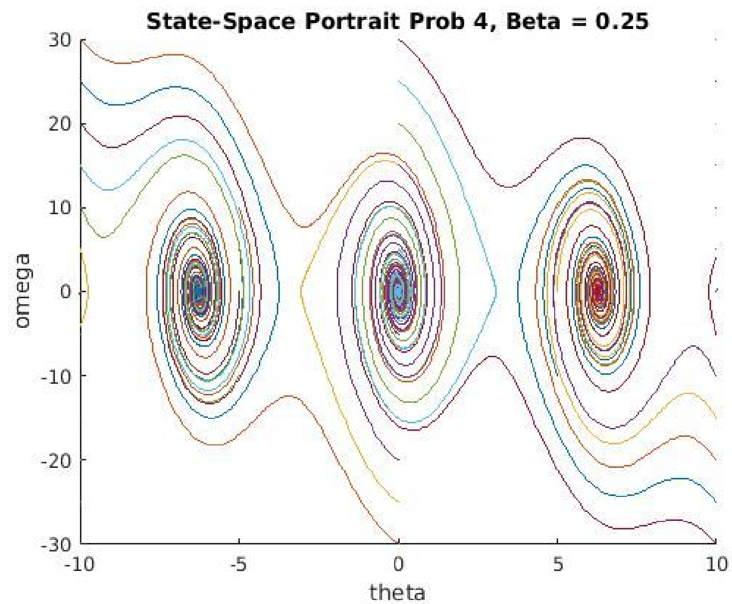

Figure 4

β is the damping factor. A positive β means the system is dissipative. The system converges to fixed points at a rate proportional to β. For larger β, the convergence is quick and, conversely small β converges more slowly.

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Problem 5 - $\theta \ mod \ 2\pi$

Figure 5 displays the same plot as Figure 4 but with $\theta \ mod \ 2\pi$. This effectively wraps the plot to fit in the domain of 0 to 2π

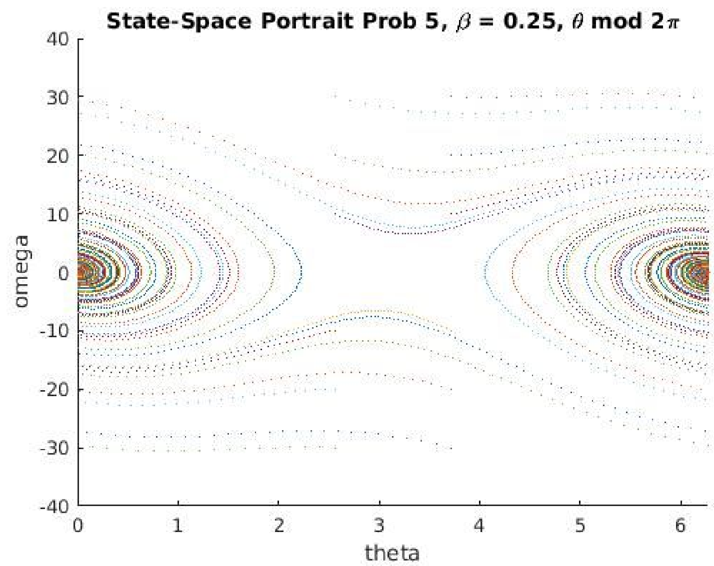

Figure 5

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Problem 6 - Chaos with $\theta \; mod \; 2\pi$

The natural frequency of the system was determined by using the formula $\alpha = \sqrt{\frac{g}{l}} \approx 9.8995$ The drive amplitude was slowly increased from 0 by intervals of 0.1, with the drive frequency at $\frac{3}{4}$ of the natural frequency. The periodic orbits bifurcate, resulting in chaotic orbits. Figure 6 displays this chaos.

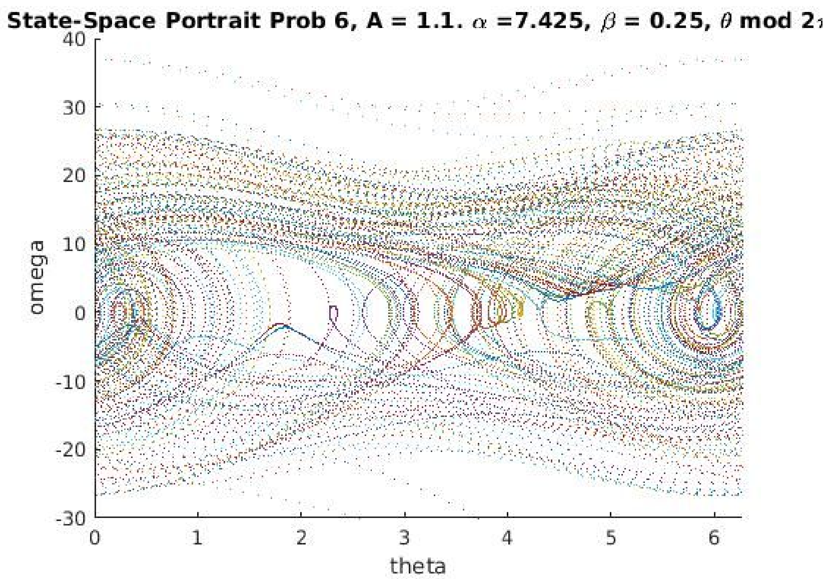

Figure 6

Problem 7 - Timestep manipulation

Next, the drive frequency, amplitude and damping factors were all set to 0. The timestep was then altered to obtain novel results. The initial results are as expected with a small enough timestep as shown in Figure 7. As the timestep grows larger, the plot looks more scattered (figure 8). Eventually, no trajectories are displayed (figure 9). I believe this is because the RK4 solver is missing the actual point because the timestep is too large. Essentially, it overshoots the point and, in the case of Figure 9, catastrophically so.

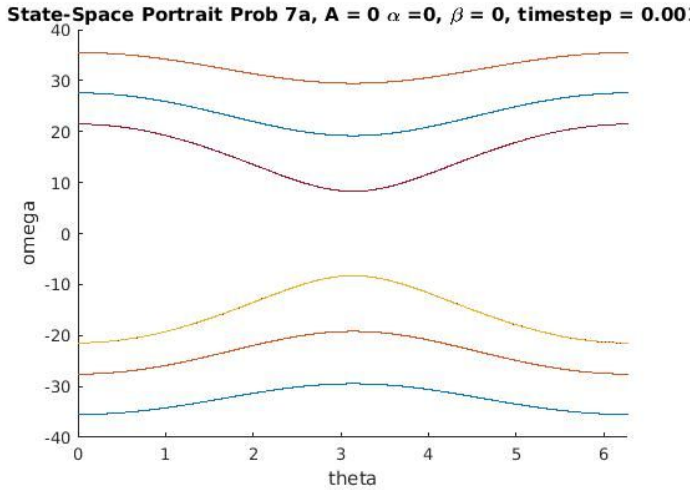

Figure 7

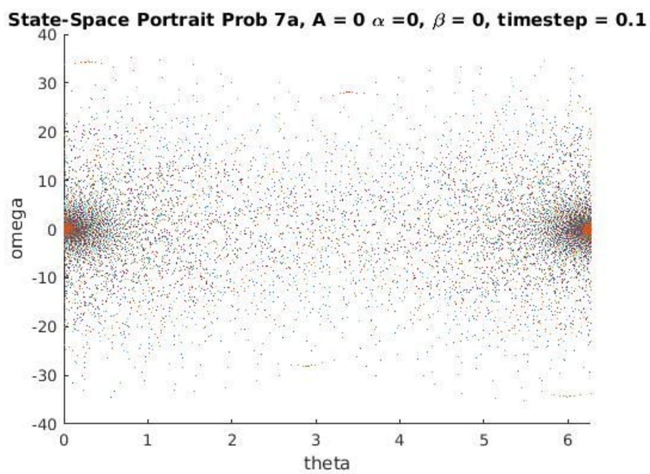

Figure 8

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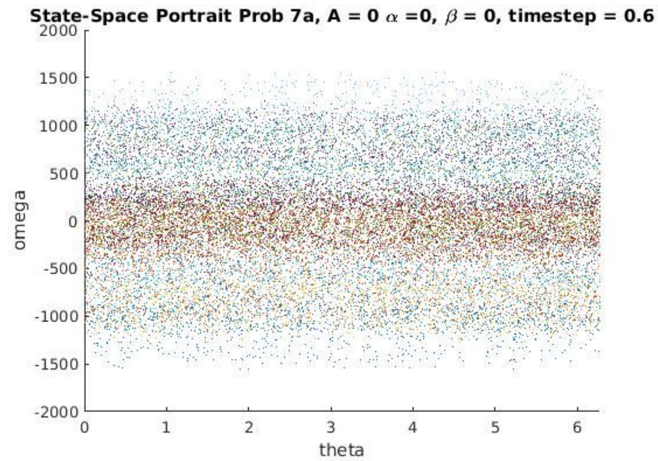

Figure 9