Ryan Riley

CSCI4446 - Chaotic Dynamics

Problem Set 3

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# Part 0 - Newton's Fractal

In this part of the assignment, the plot of Newton's Method on $x^3 - 1 = 0$ is explored. Zooming in shows many interesting plots. Figure 1, see below, iis an example:

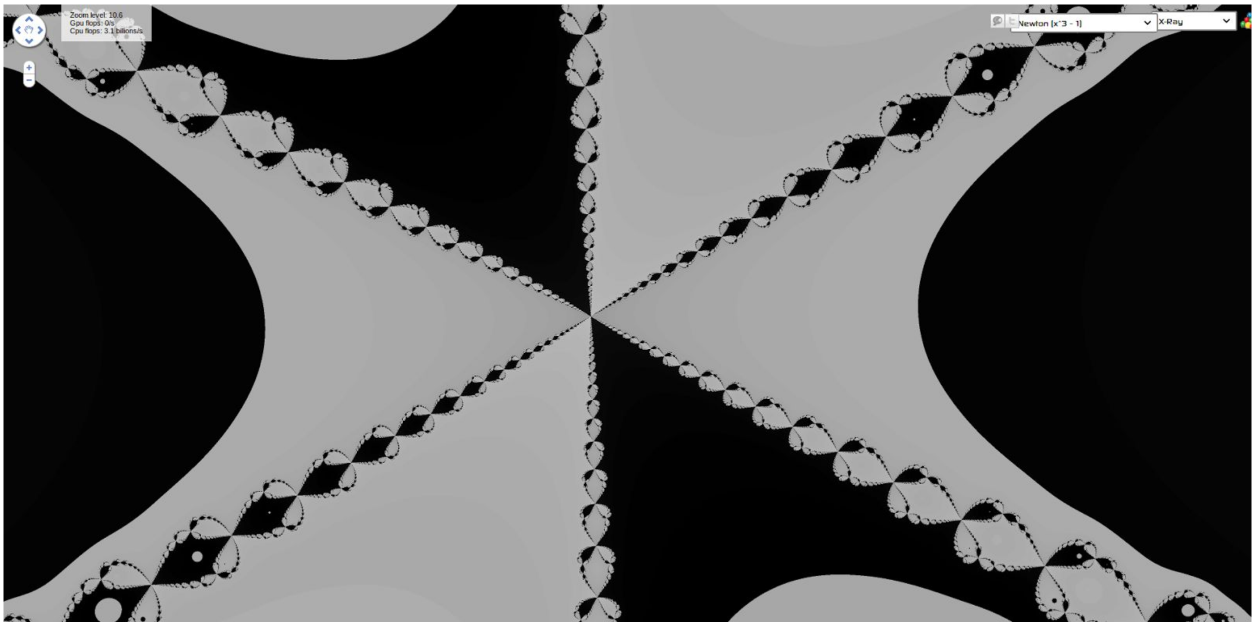

Figure 1

I found this particular image segment interesting because it displays the fractal nature of the image. This particular view is zoomed in quite far and yet it is apparent that there are innumerable copies of the general form within itself

# Part 1 - Cantor Set

Reference: http://www.wahl.org/fe/HTML_version/link/FE4W/c4.htm

To calculate the capacity dimension $d_{cap}$ of the middle sixth removed cantor set, we observe that during the first iteration, we have two segments, each of length $\frac{5}{12}$. After k iterations, there will be $2^k$ segments, each of length $(\frac{5}{12})^k$. In other words, $N(\varepsilon) = 2^k$ and $\varepsilon = (\frac{5}{12})^k$.
Therefore,

$$d_{cap} = \lim_{k \to \infty} \frac{log(2^k)}{log((\frac{12}{5})^k)} \approx 0.792$$

Note that L'Hospital's rule was used to eliminate the k in the numerator and denomator

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# Part 2 - ODEs

2a.

$$x' = y$$
$$y' = z$$
$$z' = \frac{1}{2}(3tan(\frac{1}{2}z) - 16log(y) + x)$$

2b.

$$\frac{d^3}{dt}x(t) - \frac{d^2}{dt}x(t)\frac{d}{dt}x(t) - log(\frac{d}{dt}x(t)) = 0$$

2c.

Both of the previous systems are nonlinear. The system in part a is nonlinear because of the trigonometric function and the system in part b is nonlinear because it contains a product of terms. Both contain a logarithmic function.

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# Part 3 - Fractal Trees

3a.

In this part, a simple python program was written to draw fractal trees by starting with a line segment and drawing line segments of some fraction of the previous at specific angles to the end of the previous to an arbitrary depth. Figure 2, shown below, displays this behavior with a right angle and a segments 0.6 length of the previous to a depth of 13.

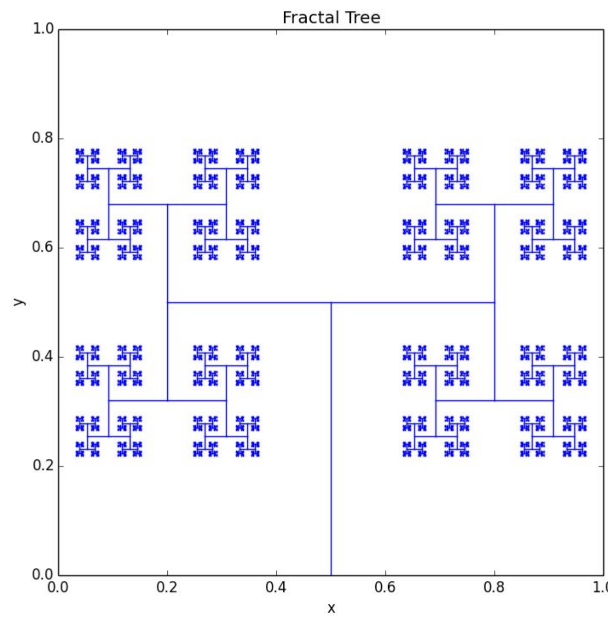


Figure 2

I could not distinguish new line segments at depths greater than 13. It should be noted that the values of the axes are unitless; they are simply there to show proportions

3b.

If each segment is exactly half as long as the previous, each new branch is exactly half the size of the original tree. This gives the appearance that the tree grows more slowly and branches less. Of course, it branches just as frequently, but it is more difficult to see from a fully zoomed-out perspective. This trend continues as the ratio of segments to the previous decreases. The more the ratio is decreased, the less branches that are readily visible. It is almost as if the tree converges more quickly.

On the other hand, if the ratio is greater than $\sqrt{2}$, the branches grow so quickly, the image no longer appears to be a tree. The tree seems to be growing unbounded.

3c.

In this section, we investigate the consequences of allowing the ratio of the segment to the previous and the angle between them vary between left and right branches. Figure 3, shown below, displays the findings:

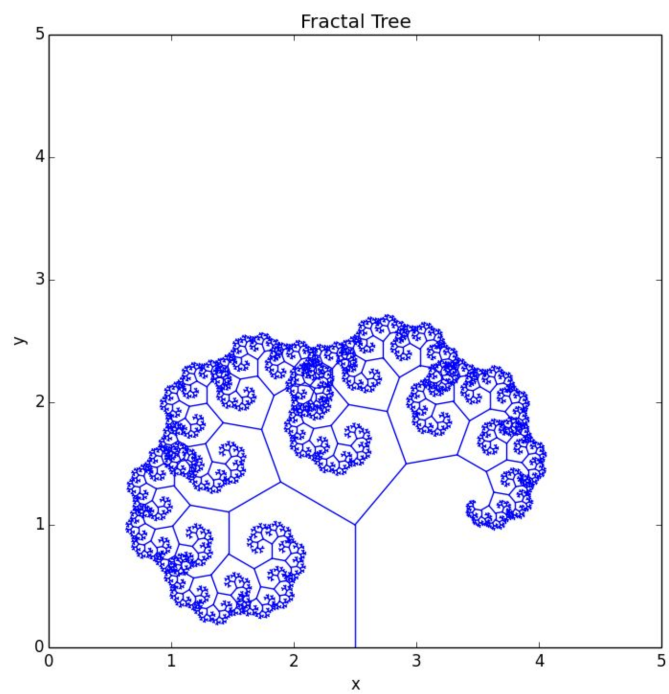


Figure 3

Figure 3 is a plot with the left ratio set to 0.7, the right set to 0.65, the left angle set to $60$ degrees and the right angle set to 40 degrees

3d.

Finally, various angles and ratios are explored to find interesting plots. The following plots are representative of the findings:

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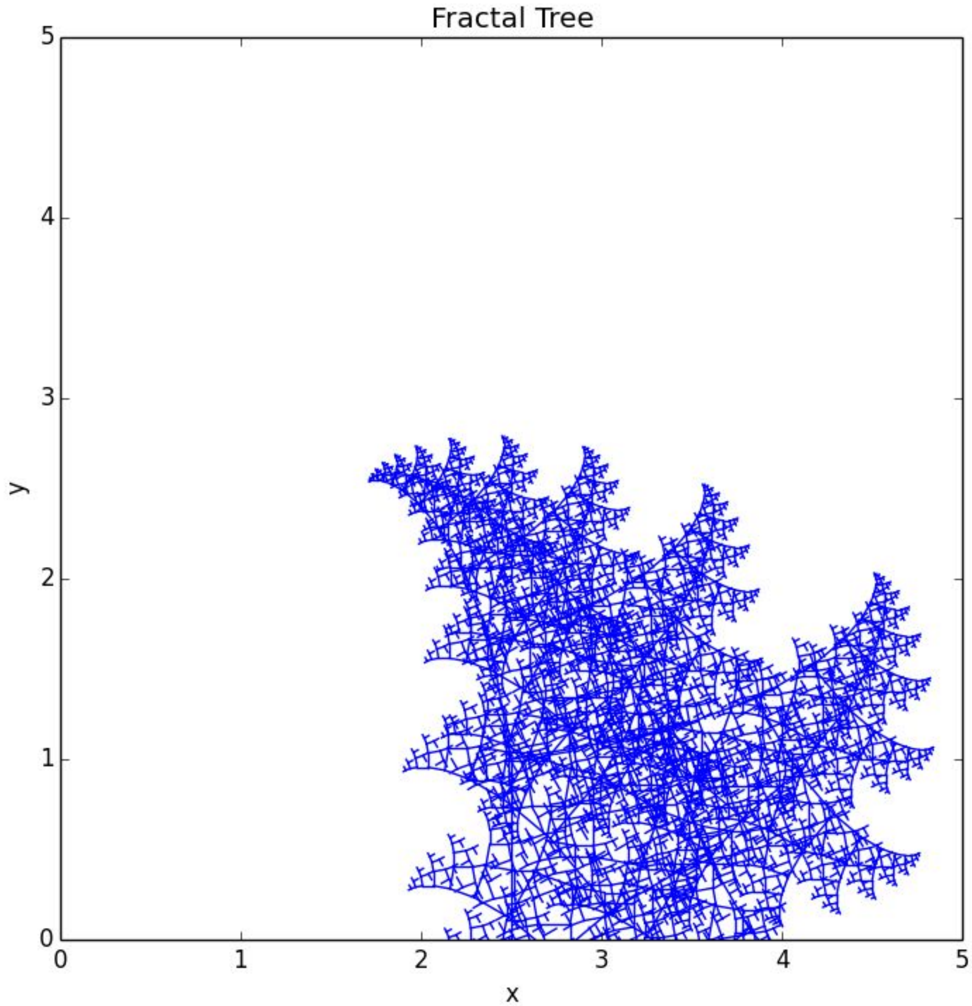


Figure 4

Parameters for Figure 4:
Left ratio: 0.65
Right ratio 0.85
Left angle: 10 degrees
Right angle: 80 degrees

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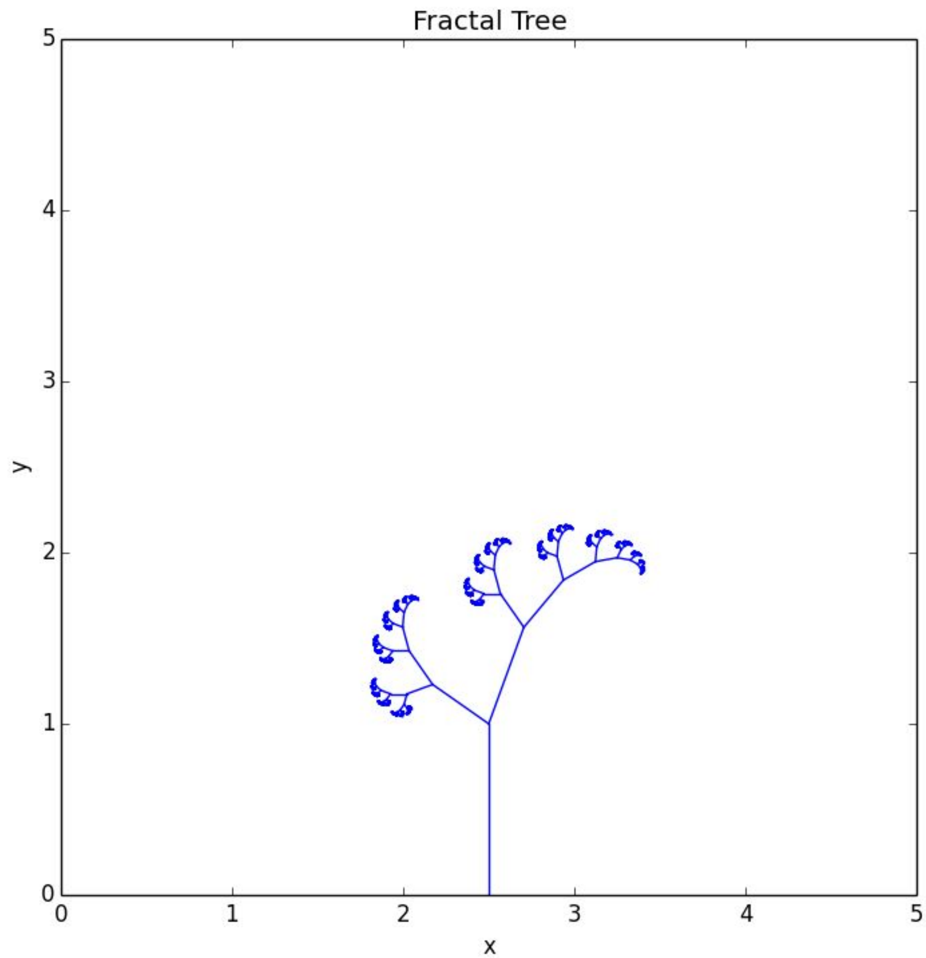


Figure 5

Parameters for Figure 5:
Left ratio: 0.4
Right ratio 0.6
Left angle: 55 degrees
Right angle: 20 degrees

I particularly like this image because it reminds me of my Bonzai tree

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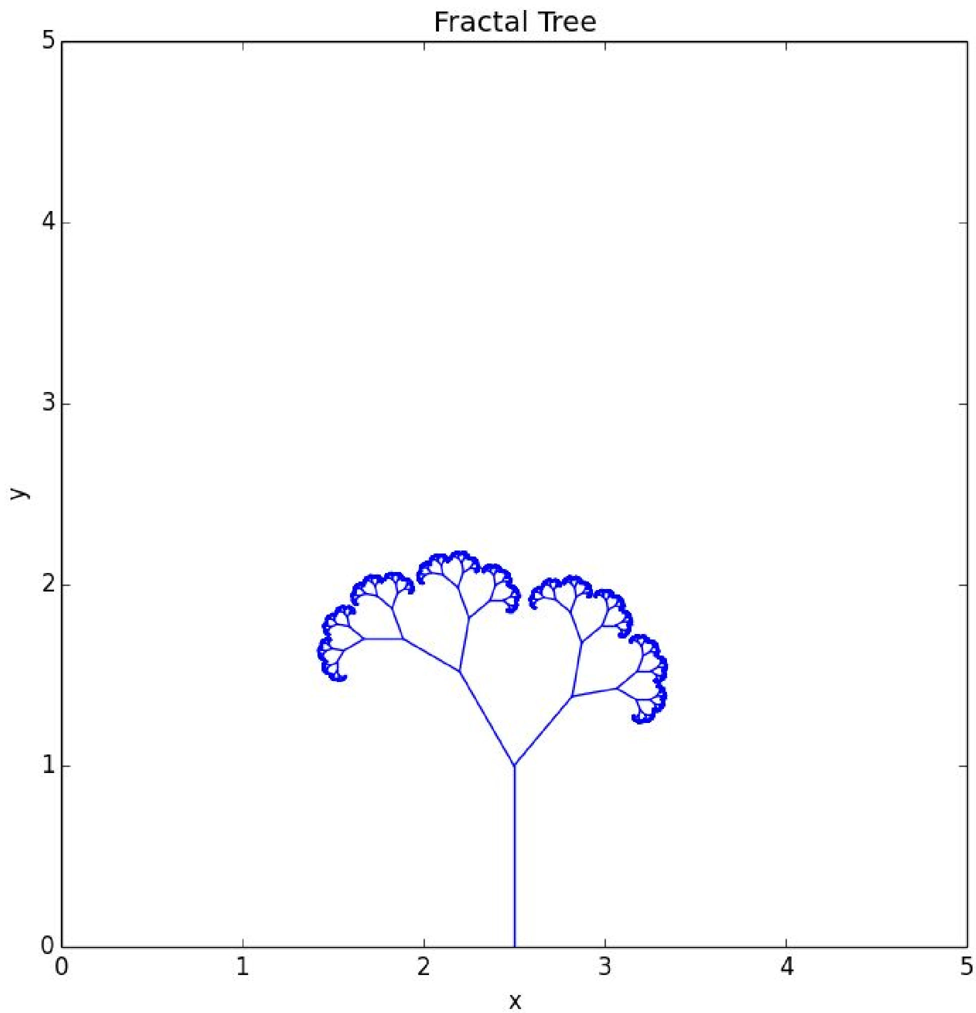


Figure 6

Parameters for Figure 6:
Left ratio: 0.6
Right ratio 0.55
Left angle: 30 degrees
Right angle: 40 degrees

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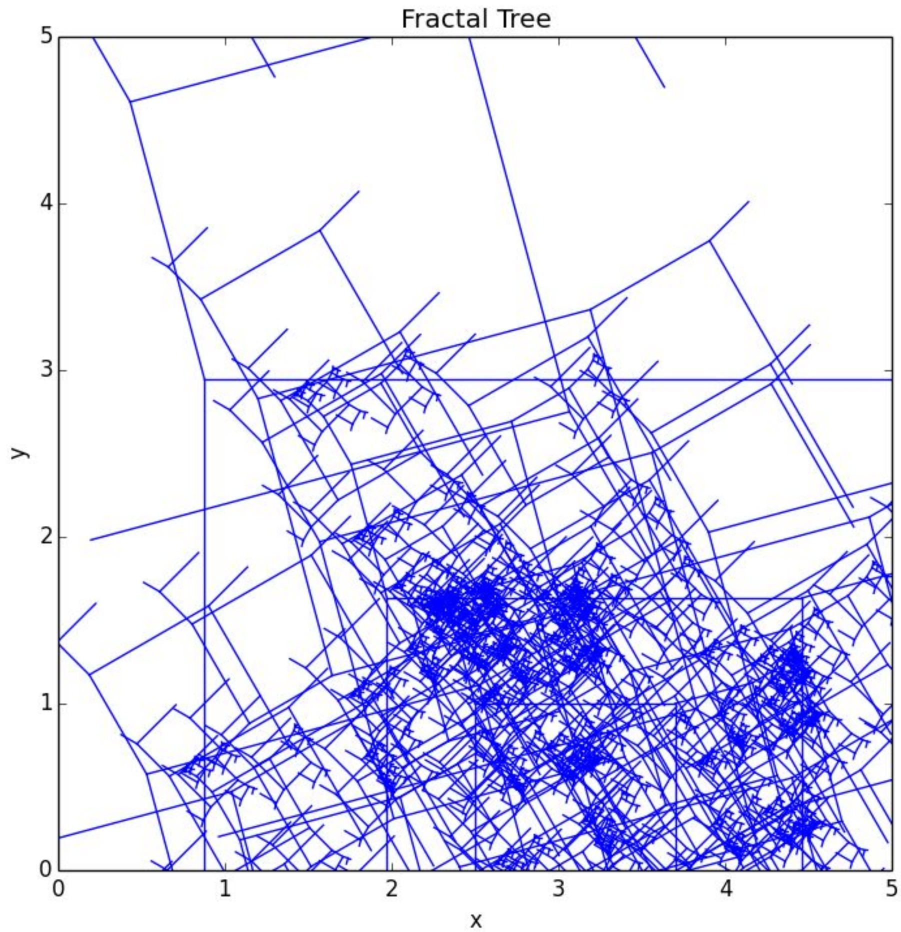


Figure 7

Parameters for Figure 7:
Left ratio: 0.4
Right ratio 1.2
Left angle: 15 degrees
Right angle: 90 degrees

This image reminds of me of abstract expressionism