Ryan Riley
CSCI4446
Problem Set 2
29 January 2017

Part 1 - Plotting Bifurcation Diagrams

For this part of the assignment, a program was written in python to display the bifurcation plot of the logistic map, given by the following equation.

$$x_{n+1} = Rx_n(1 - x_n)$$

This was achieved by iterating through the various R values and running the logistic map function for each R value. The following plots display the results. The first was plotted with significantly higher resolution than the second.

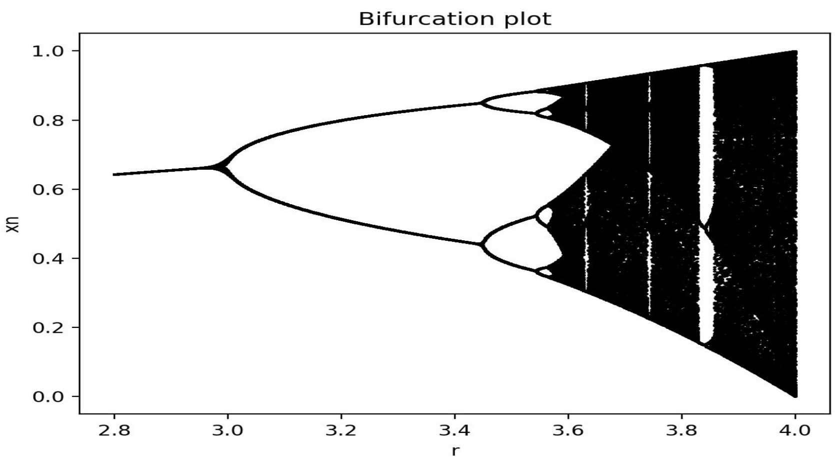

Figure 1:
Bifurcation plot with 2.8 < R < 4, 200 iterates and 100 suppressed iterates. Initial condition x_0 = .02 and intervals of 0.001 between R values

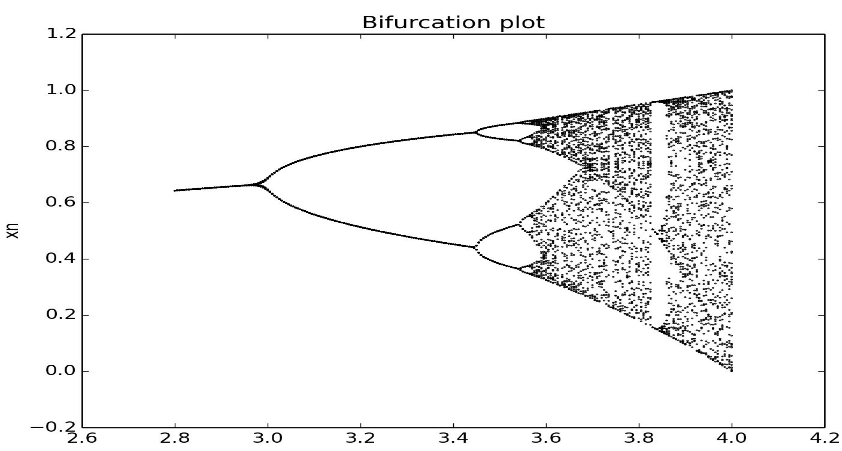

Figure 2:

Ryan Riley
CSCI4446
Problem Set 2
29 January 2017

Same plot as figure 1 but with only 50 non-suppressed iterates per R value and an interval of 0.005 between R values

Part 2 - Calculating Feigenbaum's Constant

Zooming in on the sections where the above plots bifurcate, we can calculate Feigenbaum's constant by dividing the distance between two points of bifurcation and the distance between the next. In mathematical form:

$$\delta = \lim_{n\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \approx 4.66$$

By printing the values at each R, the following were determined to be points of bifurcation. It should be noted that the resolution of the calculation was not fine grained enough to be completely accurate. As it was, the program was extremely CPU intensive.

Point	R value
1	2.97
2	3.445
3	3.54
4	3.56

Next, we can estimate the Feigenbaum number δ by using the previous points

$$\delta_1 = \frac{3.445 - 2.97}{3.54 - 3.445} = 5$$

$$\delta_2 = \frac{3.54 - 3.445}{3.56 - 3.54} = 4.75$$

As we can see, the values are trending towards 4.66.

Ryan Riley
CSCI4446
Problem Set 2
29 January 2017

Part 3 - Plotting the Henon Map

In this part of the assignment, the same process is used as part 1 but with the Henon Map equation given by:

$$x_{k+1} = y_k + 1 - ax_k^2$$
$$y_{k+1} = bx_k$$

The following plot resulted:

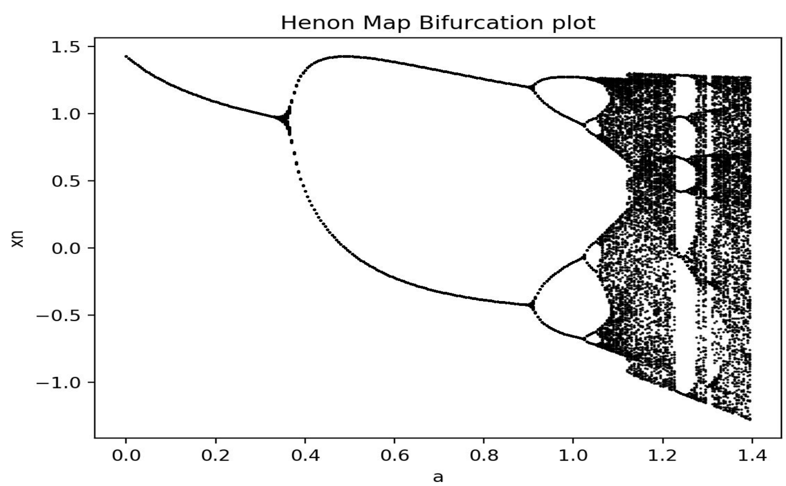

Figure 3:
Henon Map Plot with 0 < a < 1.4, b = 0.3, intervals of 0.005 between a values

The same procedure applied in part 2 to the logistic map will be applied to the Henon Map.

Again, zooming in at the values of *a* around bifurcation, the following points were estimated:

Ryan Riley
CSCI4446
Problem Set 2
29 January 2017

Point	a value
1	0.33
2	0.885
3	1.02
4	1.05

Again, we can estimate the Feigenbaum number:

$$\delta_1 = \frac{0.88 - 0.33}{1.02 - 0.88} \approx 3.92$$
$$\delta_2 = \frac{1.02 - 0.88}{1.05 - 1.02} \approx 4.66$$

This moves quickly towards the expected limit of 4.66. It does seem strange that the first value is significantly less than the limit. I attribute this to the lack of fine grain resolution when running the program and, possibly more significantly, to the fact that I was relying on my ability to view the data, point by point, and estimate the actual bifurcation point.

Part 4 - Henon Map Feigenbaum Reflections

These values are similar in part 3 (Henon Map) and part 2 (Logistic Map). This is to be expected because the Feigenbaum constant is found in any maps with a single quadratic maximum, which describes both the Henon Map and the Logistic Map. Essentially this means any of these systems will bifurcate at the same rate.