CS2010: Data Structures and Algorithms II

Substring Search

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Outline of Substring search algorithms

› Brute force

› KMP (Knuth-Morris-Pratt)

› Boyer-Moore

› Rabin-Karp

› Many many many others

› Suffix arrays

› LCP (longest common prefix) arrays

String matching algorithms

› [Handbook of Exact String Matching Algorithms](#)
› [http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.133.4896&rep=rep1&type=pdf](#)
› [http://www-igm.univ-mlv.fr/~lecroq/string/](#)

Java String implementation

Java library. The `indexOf()` method in Java's string library returns the index of the first occurrence of a given string, starting at a given offset.

› Which algorithm does String.IndexOf(String) use?
  – Naïve loop (brute force)
  – Why?

› String.contains()

Common interview questions

› Implement a needle-in-a-haystack
  – public int Search(String haystack, String needle)

› Implement strstr()
  – Find the first instance of a string in another string

› Longest common substring between 2 files

› Longest substring that's a palindrome

› Longest repeated substring

› Etc etc

Different to Pattern Matching

› Find a pattern, i.e. one of the specified set of substrings in a text

› Regular expression – notation to specify a set of strings

› For more info see 5.4 in Sedgewick and Wayne

Substring search - definition

Goal. Find pattern of length M in a text of length N.

typically N >> M

pattern → N E E D L E

text → I N A H A Y S T A C K <u>N E E D L E</u> I N A

match

Substring search – brute force

Check for pattern starting at each text position.

i	j	i+j	0	1	2	3	4	5	6	7	8	9	10
		txt →	A	B	A	C	A	D	A	B	R	A	C
0	2	2	A	B	R	A ← pat							
1	0	1		A	B	R	A						
2	1	3			A	B	R	A					
3	0	3				A	B	R	A				
4	1	5					A	B	R	A			
5	0	5						A	B	R	A		
6	4	10								A	B	R	A

entries in red are mismatches

entries in gray are for reference only

entries in black match the text

return i when j is M

A B R A

match

Substring search – brute force

Check for pattern starting at each text position.

i	j	i + j	0	1	2	3	4	5	6	7	8	9	10
			A	B	A	C	A	D	A	B	R	A	C
4	3	7					A	D	A	C	R		
5	0	5						A	D	A	C	R	

```
public static int search(String pat, String txt)
{
    int M = pat.length();
    int N = txt.length();
    for (int i = 0; i <= N - M; i++)
    {
        int j;
        for (j = 0; j < M; j++)
            if (txt.charAt(i+j) != pat.charAt(j))
                break;
        if (j == M) return i;        ← index in text where
    }                                  pattern starts
    return N;     ← not found
}
```

Substring search – brute force

Brute-force substring search: worst case

Brute-force algorithm can be slow if text and pattern are repetitive.

i	j	i+j	0	1	2	3	4	5	6	7	8	9
		txt→	A	A	A	A	A	A	A	A	A	B
0	4	4	A	A	A	A	B	← pat				
1	4	5		A	A	A	A	B				
2	4	6			A	A	A	A	B			
3	4	7				A	A	A	A	B		
4	4	8					A	A	A	A	B	
5	5	10						A	A	A	A	B

match

Worst case. $\sim MN$ char compares.

Substring search – backup

Substring search – explicit backup

Same sequence of char compares as previous implementation.
- i points to end of sequence of already-matched chars in text.
- j stores # of already-matched chars (end of sequence in pattern).

```
i   j    0  1  2  3  4  5  6  7  8  9  10
         A  B  A  C  A  D  A  B  R  A  C
7   3                 A  D  A  C  R
5   0                    A  D  A  C  R
```

```
public static int search(String pat, String txt)
{
   int i, N = txt.length();
   int j, M = pat.length();
   for (i = 0, j = 0; i < N && j < M; i++)
   {
      if (txt.charAt(i) == pat.charAt(j)) j++;
      else { i -= j; j = 0;  }                    ←——— explicit backup
   }
   if (j == M) return i - M;
   else              return N;
}
```

Algorithmic challenges in substring search

Brute-force is not always good enough.

Theoretical challenge. Linear-time guarantee. ⟵ fundamental algorithmic problem

Practical challenge. Avoid backup in text stream. ⟵ often no room or time to save text

Now is the time for all people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for a lot of good people to come to the aid of their party. Now is the time for all of the good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for each good person to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Republicans to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many or all good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Democrats to come to the aid of their party. Now is the time for all people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for a lot of good people to come to the aid of their party. Now is the time for all of the good people to come to the aid of their party. Now is the time for all good people to come to the aid of their attack at dawn party. Now is the time for each person to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Republicans to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many or all good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Democrats to come to the aid of their party.

Knuth-Morris-Pratt (KMP)

KMP

› 1970 by Donald Knuth and Vaughan Pratt

› + Independently by James H. Morris.

› (Donald Knuth - The Art of Computer Programming - comprehensive monograph that covers many kinds of programming algorithms and their analysis – 4 volumes and counting)

KMP

Knuth-Morris-Pratt substring search

Intuition.   Suppose we are searching in text for pattern BAAAAAAAAA.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.
- We know previous 6 chars in text are BAAAAB.
- Don't need to back up text pointer!

assuming { A, B } alphabet

```
                                    i
                                    ↓
text ——       A  B  A  A  A  A  B  A  A  A  A  A  A  A  A  A
after mismatch
on sixth char ——  B  A  A  A  A  A  A  A  A  A  ←— pattern
brute-force backs ——  B  A  A  A  A  A  A  A  A  A  A
up to try this
                         B  A  A  A  A  A  A  A  A  A
and this ——               B  A  A  A  A  A  A  A  A  A  A
and this ——                  B  A  A  A  A  A  A  A  A  A
and this ——                     B  A  A  A  A  A  A  A  A  A  A
and this ——
but no backup ——                    B  A  A  A  A  A  A  A  A  A
is needed
```

Knuth-Morris-Pratt algorithm.   Clever method to always avoid backup. (!)

KMP – avoid back up how? DFA

› DFA – Deterministic Final State Automaton
› Finite State Automaton/Finite State Machine
  – mathematical model of computation
  – an abstract machine that can be in exactly one of a finite number of states at any given time.
  – can change from one state to another in response to some external inputs
  – the change from one state to another is called a transition
  – defined by a list of its states, its initial state, and the conditions for each transition.
› Deterministic - produces a unique computation (or run) of the automaton for each input string
› DFA - finite-state machine that accepts and rejects strings of symbols

FSA – an example

State diagram for a turnstile

A turnstile
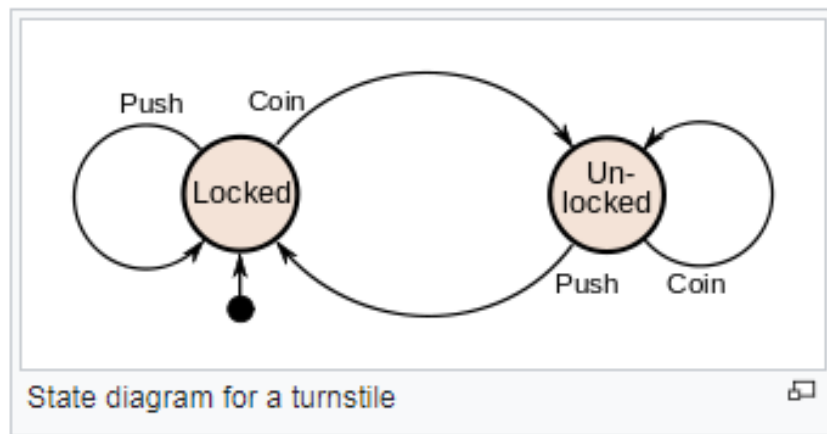
Finite State Machine – more formally

A **finite state automaton** is a quintuple (Q, Σ, E, S, F) with

- Q a finite set of states

- Σ a finite set of symbols, the alphabet

- $S \subseteq Q$ the set of start states

- $F \subseteq Q$ the set of final states

- E a set of edges $Q \times (\Sigma \cup \{\epsilon\}) \times Q$

  The **transition function** d can be defined as

  $$d(q, a) = \{q' \in Q | \exists (q, a, q') \in E\}$$

- Deterministic Finite Automata are always *complete*: they define a transition for each state and each input symbol.

DFA

DFA is abstract string-searching machine.

- Finite number of states (including start and halt).
- Exactly one transition for each char in alphabet.
- Accept if sequence of transitions leads to halt state.

internal representation

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

If in state j reading char c:
   if j is 6 halt and accept
   else move to state dfa[c][j]

graphical representation
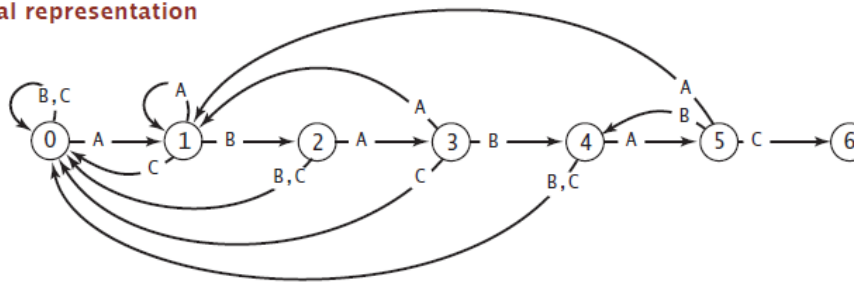
DFA simulation

DFA simulation

B   C   B   A   A   B   A   C   A   A   B   A   B   A   C   A   A

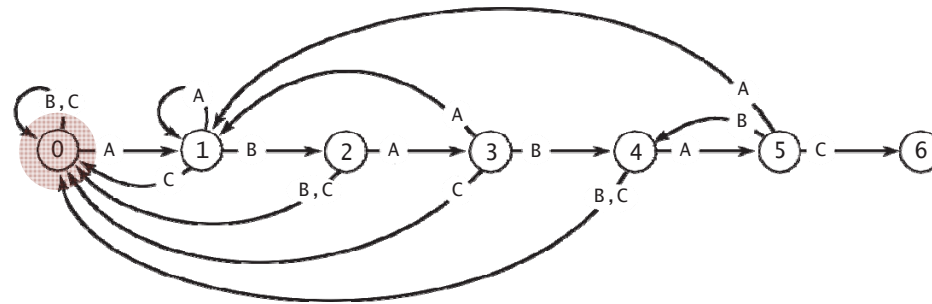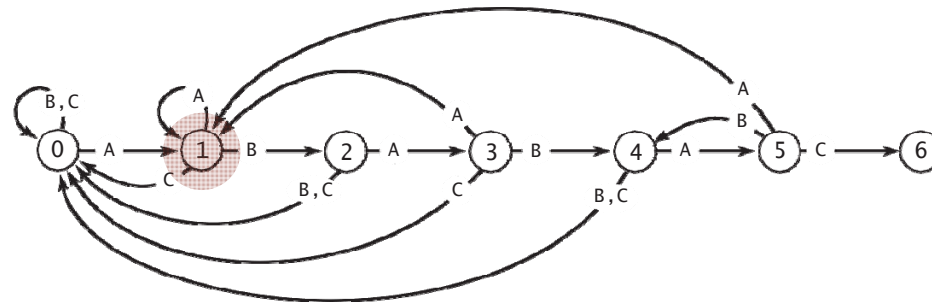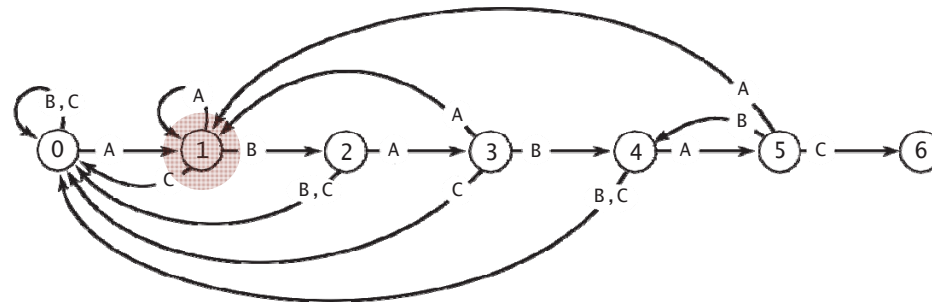| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
dfa[][j] A	1	1	3	1	5	1
B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

DFA simulation

B C B A A B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
dfa[][j] A	1	1	3	1	5	1
B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

B C B A A B A C A A B A B A C A A

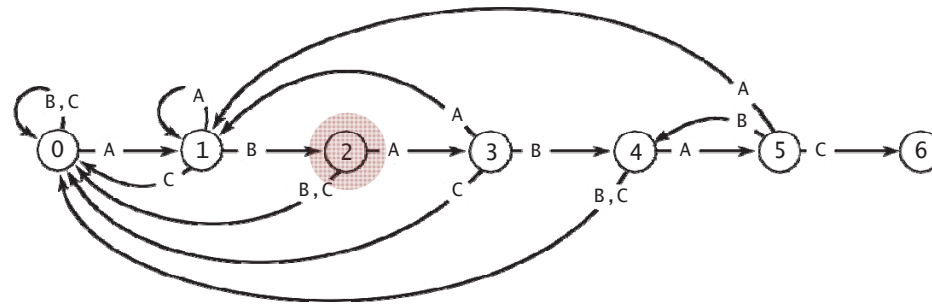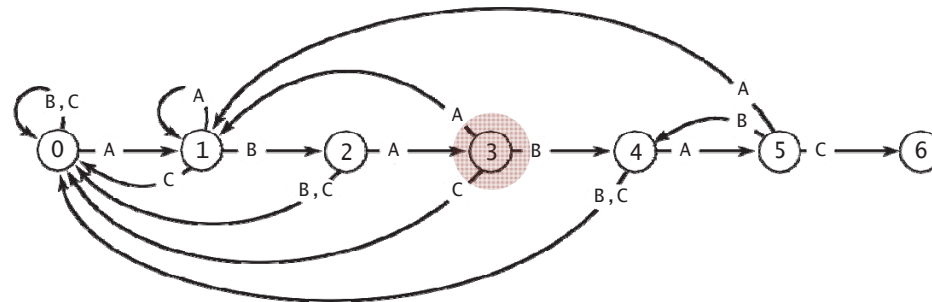| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

DFA simulation

B C B A A B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

B C B A A B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

B C B A **A** B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

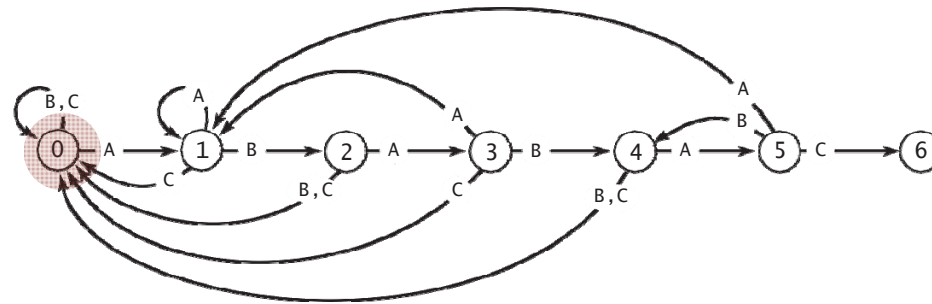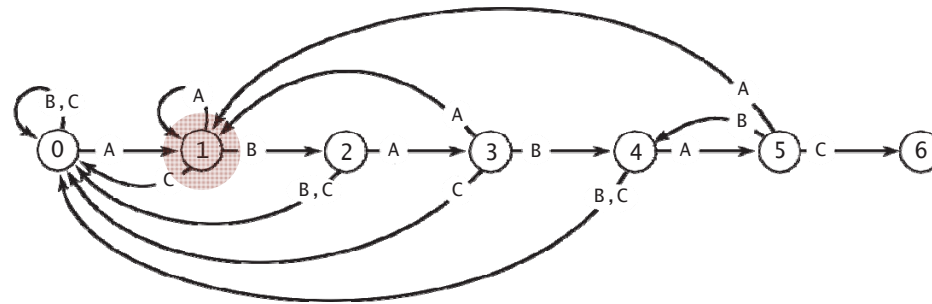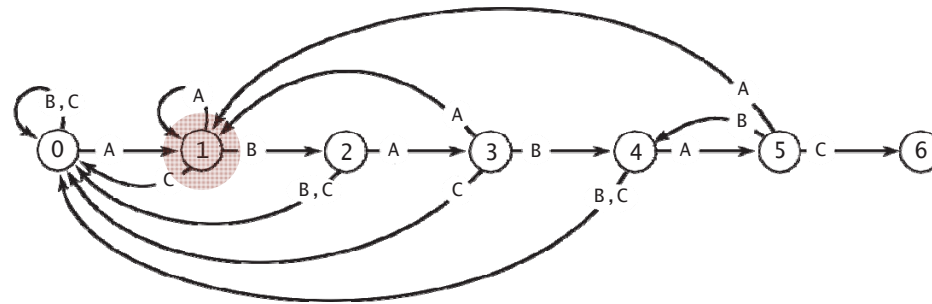TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A B A B A C

DFA simulation

B C B A A B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

B C B A **A B A** C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

B C B A A B A C A A B A B A C A A

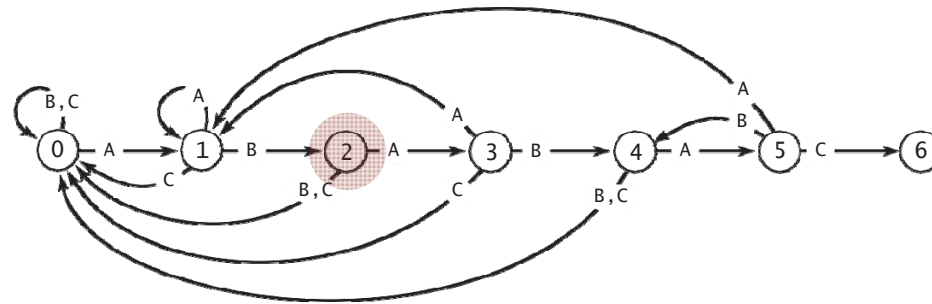| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

10

DFA simulation

B C B A A B A C **A** A B A B A C A A

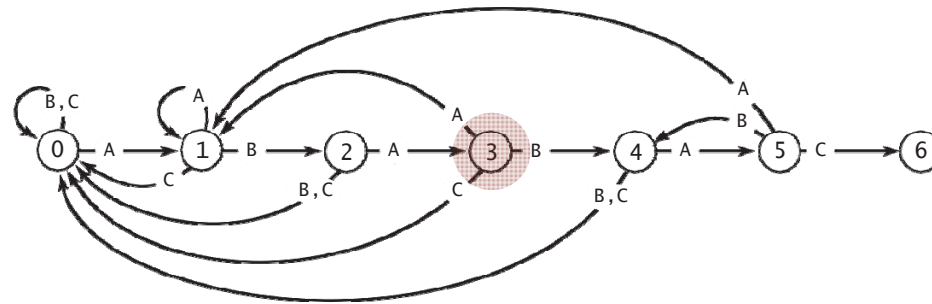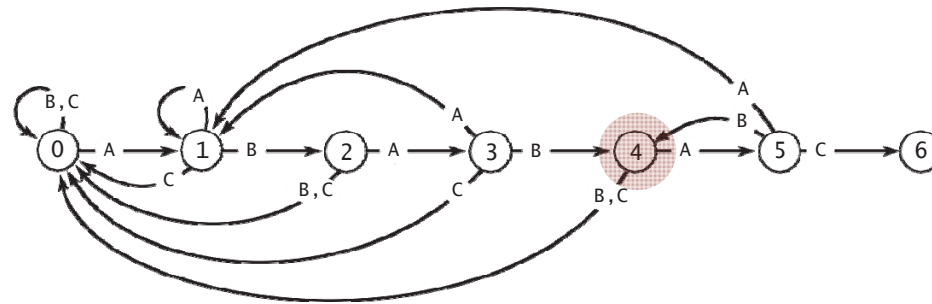| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

DFA simulation

B C B A A B A C A **A** B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

B C B A A B A C A **A B** A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

B C B A A B A C A **A B A** B A C A A

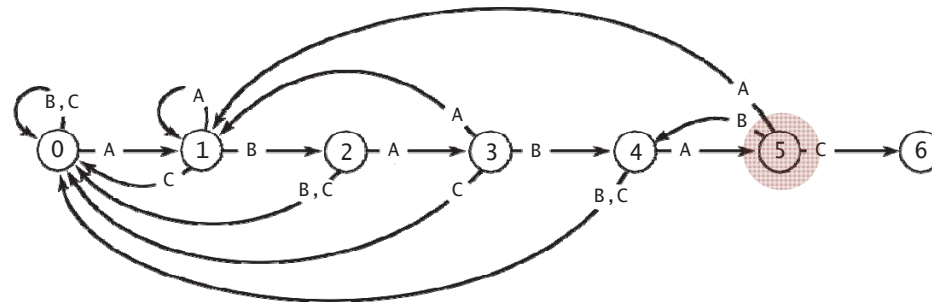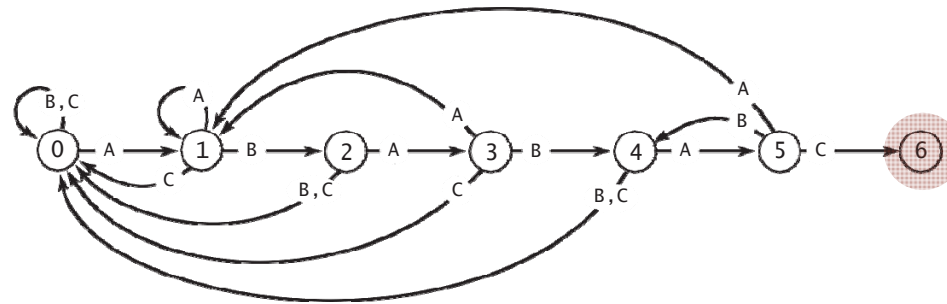| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

DFA simulation

B C B A A B A C A A B A B A C A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

B C B A A B A C A **A B A B A** C A A
↑

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

DFA simulation

B C B A A B A C A **A B A B A C** A A

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

TRACE OF KMP SUBSTRING SEARCH (DFA SIMULATION) FOR A
B A B A C

DFA States – number of characters matched
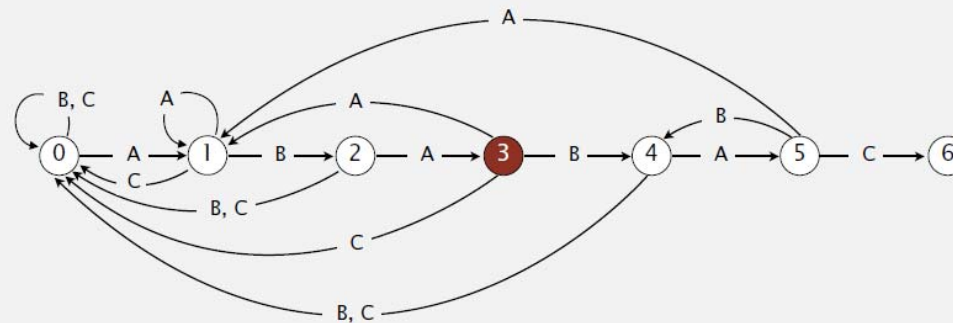
DFA simulation exercise

› Consider the following DFA for searching for a string "IVANA"

› For simplicity, we assume the alphabet contains only letters A, I, N, V

› DFA is therefore as follows

j	0	1	2	3	4
char?	I	V	A	N	A
A	0	0	3	0	0
I	1	1	1	1	1
N	0	0	0	4	0
V	0	2	0	0	5

Exercise 1 Simulating DFA:

› 1. Construct graphical representation of the DFA table

› 2. Write the trace of states when searching for a string "IVANA" in input "ANVAIVAAIVANAAN"

› 3. Vote on turning point for the correct trace

Exercise 1 Simulating DFA:

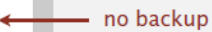| j	0	1	2	3	4
char?	I	V	A	N	A
A	0	0	3	0	0
I	1	1	1	1	1
N	0	0	0	4	0
V	0	2	0	0	5

Text ANVAIVAAIVANAAN

Search string IVANA

KMP Java Implementation

Key differences from brute-force implementation.

- Need to precompute dfa[][] from pattern.
- Text pointer i never decrements.

```java
public int search(String txt)
{
    int i, j, N = txt.length();
    for (i = 0, j = 0; i < N && j < M; i++)
        j = dfa[txt.charAt(i)][j];                    ←——— no backup
    if (j == M) return i - M;
    else        return N;
}
```

Running time.

- Simulate DFA on text: at most N character accesses.
- Build DFA: how to do efficiently? [warning: tricky algorithm ahead]

Key differences from brute-force implementation.

- Need to precompute dfa[][] from pattern.
- Text pointer i never decrements.
- Could use input stream.

```java
public int search(In in)
{
    int i, j;
    for (i = 0, j = 0; !in.isEmpty() && j < M; i++)
        j = dfa[in.readChar()][j];                              ← no backup
    if (j == M) return i - M;
    else        return NOT_FOUND;
}
```
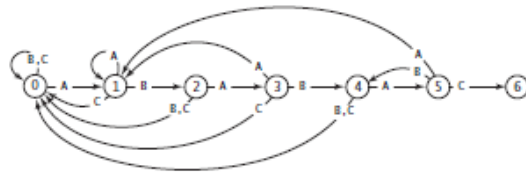
DFA construction

Knuth-Morris-Pratt construction

Include one state for each character in pattern (plus accept state).

```
            j    0   1   2   3   4   5
   pat.charAt(j) A   B   A   B   A   C
               | A
      dfa[][j] | B
               | C
```

(0)   (1)   (2)   (3)   (4)   (5)   (6)

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Match transition: advance to next state if `c == pat.charAt(j)`.

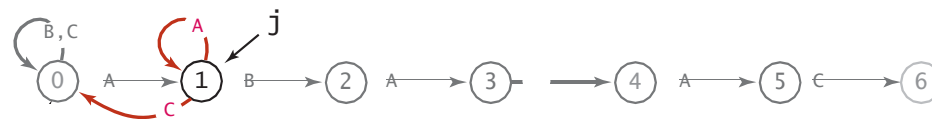| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1		3		5	
dfa[][j] B		2		4		
C						6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition: back up if `c != pat.charAt(j)`.

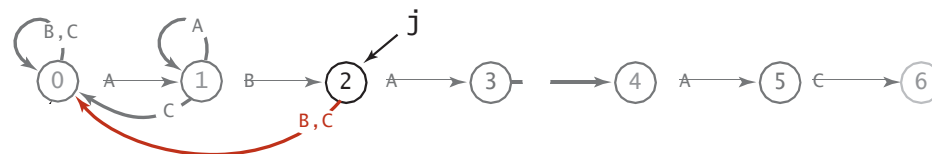|	j	0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
	A	1		3		5	
dfa[][j]	B	0	2		4		
	C	0					6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition: back up if `c != pat.charAt(j)`.

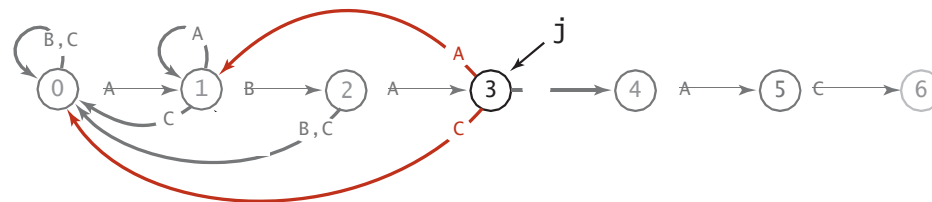|	j	0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
	A	1	1	3		5	
dfa[][j]	B	0	2			4	
	C	0	0				6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition: back up if `c != pat.charAt(j)`.

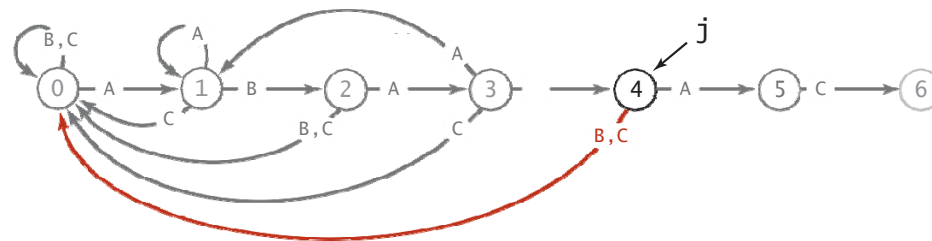| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3		5	
dfa[][j] B	0	2	0	4		
C	0	0	0			6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition: back up if `c != pat.charAt(j)`.

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	**B**	A	C
A	1	1	3	1	5	
dfa[][j] B	0	2	0	4		
C	0	0	0	0		6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction
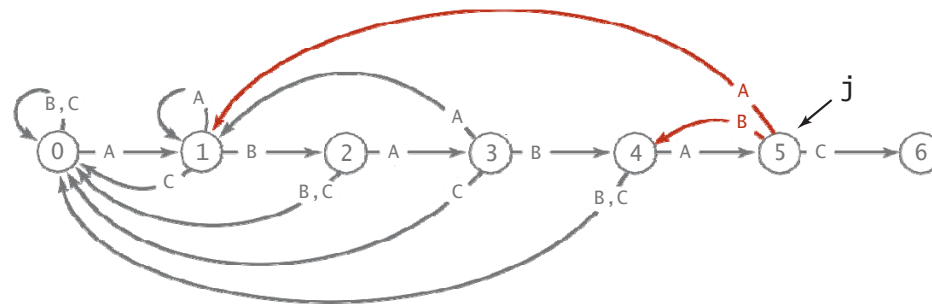
Mismatch transition: back up if `c != pat.charAt(j)`.

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	
dfa[][j] B	0	2	0	4	0	
C	0	0	0	0	0	6

Knuth-Morris-Pratt construction

Mismatch transition: back up if `c != pat.charAt(j)`.

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6

Knuth-Morris-Pratt construction

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	1
dfa[][j] B	0	2	0	4	0	4
C	0	0	0	0	0	6
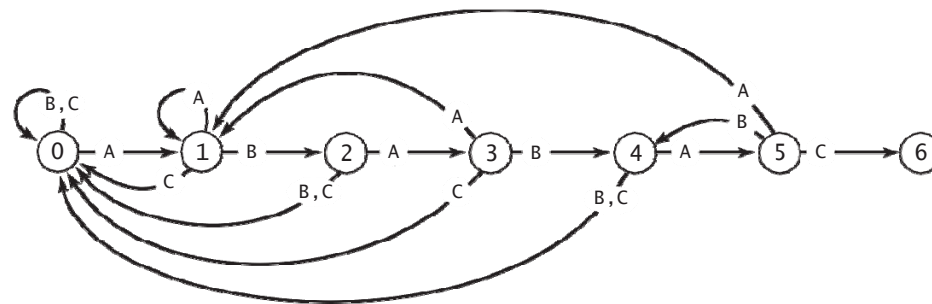
Exercise 2 constructing DFA:

› Search for your first name in a 15-character string you make up (you can decide whether you want the string to contain your name or not)

› Assume the alphabet contains only the letters that are present in your name (in DFA table list them in alphabetical order)

› Construct DFA table and graphical representation

› Write out the trace of search for your name in the madeup string

› Hand up the exercise for my Friday night entertainment

DFA construction – Java code

Include one state for each character in pattern (plus accept state).

```
         j   0   1   2   3   4   5
pat.charAt(j) A   B   A   B   A   C
            | A
 dfa[][j]   | B
            | C
```

⓪   ①   ②   ③   ④   ⑤   ⑥

Constructing the DFA for KMP substring search for A B A B A C

DFA Construction – Java code

Match transition. If in state j and next char c == pat.charAt(j), go to j+1.

first j characters of pattern have already been matched

next char matches

now first j +1 characters of pattern have been matched

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1		3		5	
	B		2		4		
	C						6

0 — A → 1 — B → 2 — A → 3 — B → 4 — A → 5 — C → 6

DFA construction – Java code

Match transition. For each state `j`, `dfa[pat.charAt(j)][j] = j+1.`

first j characters of pattern have already been matched

now first j+1 characters of pattern have been matched

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1		3		5	
dfa[][j] B		2		4		
C						6

Constructing the DFA for KMP substring search for A B A B A C

DFA Construction – Java code

Mismatch transition. If in state j and next char c != pat.charAt(j), then the last j-1 characters of input are pat[1..j-1], followed by c.

To compute dfa[c][j]: Simulate pat[1..j-1] on DFA and take transition c.
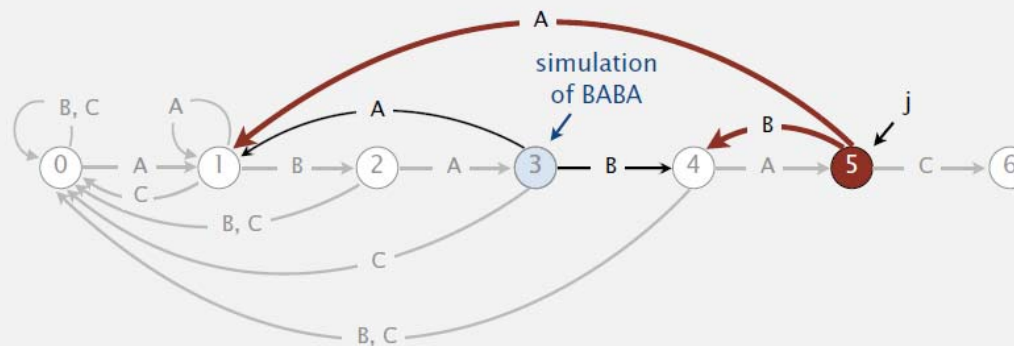Running time. Seems to require j steps.   still under construction (!)

Ex. dfa['A'][5] = 1;   dfa['B'][5] = 4

simulate BABA;	simulate BABA;
take transition 'A'	take transition 'B'
= dfa['A'][3]	= dfa['B'][3]

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C

DFA Construction – Java code

Mismatch transition. If in state j and next char c != pat.charAt(j), then the last j-1 characters of input are pat[1..j-1], followed by c.

state X

To compute dfa[c][j]: Simulate pat[1..j-1] on DFA and take transition c.
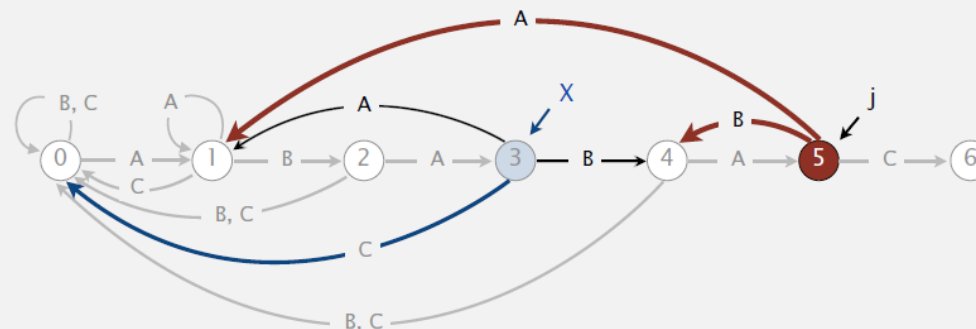Running time. Takes only constant time if we maintain state X.

Ex. dfa['A'][5] = 1;   dfa['B'][5] = 4   X' = 0

from state X,
take transition 'A'
= dfa['A'][X]

from state X,
take transition 'B'
= dfa['B'][X]

from state X,
take transition 'C'
= dfa['C'][X]

0	1	2	3	4	5
A	B	A	B	A	C

DFA construction

› So lets do this again, while maintaining state x

Knuth-Morris-Pratt construction

Include one state for each character in pattern (plus accept state).

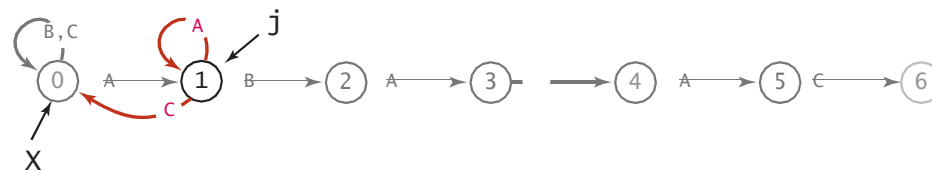| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
dfa[][j] A						
B						
C						

(0)   (1)   (2)   (3)   (4)   (5)   (6)

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Match transition. For each state `j`, `dfa[pat.charAt(j)][j] = j+1.`

first j characters of pattern
have already been matched

now first j+1 characters of
pattern have been matched

	j	0	1	2	3	4	5
	pat.charAt(j)	A	B	A	B	A	C
	A	1		3		5	
dfa[][j]	B		2		4		
	C						6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition.

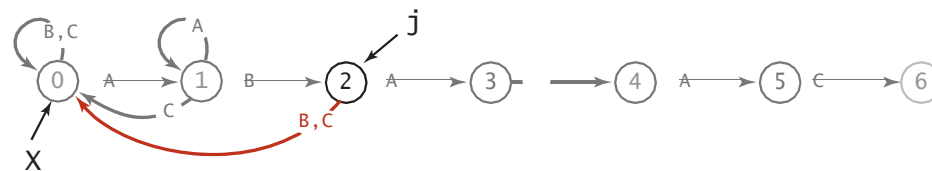| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
dfa[][j] A	1		3		5	
dfa[][j] B	0	2		4		
dfa[][j] C	0					6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`,
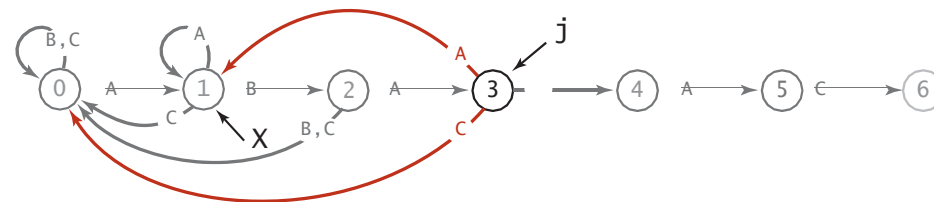`dfa[c][j] = dfa[c][X];` then update `x = dfa[pat.charAt(j)][X].`

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`,
`dfa[c][j] = dfa[c][X];` then update `x = dfa[pat.charAt(j)][X]`.

```
                         X
                         |
                         v
         j       0   1   2   3   4   5
   pat.charAt(j) A   B   A   B   A   C
              A (1)  1   3       5
   dfa[][j] B  0   2   0   4
              C  0   0   0           6
```
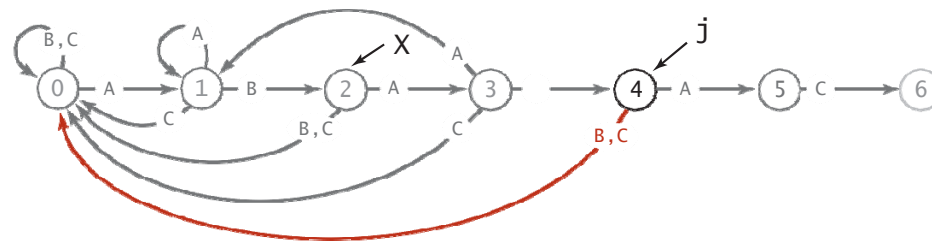

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`,
`dfa[c][j] = dfa[c][X]`; then update `x = dfa[pat.charAt(j)][X]`.

	X					
j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
A	1	1	3	1	5	
dfa[][j] B	0	②	0	4		
C	0	0	0	0		6

Constructing the DFA for KMP substring search for A B A B A C

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`,
`dfa[c][j] = dfa[c][X]`; then update `x = dfa[pat.charAt(j)][X]`.

Knuth-Morris-Pratt construction

Mismatch transition. For each state `j` and char `c != pat.charAt(j)`,
`dfa[c][j] = dfa[c][X]`; then update `x = dfa[pat.charAt(j)][X]`.
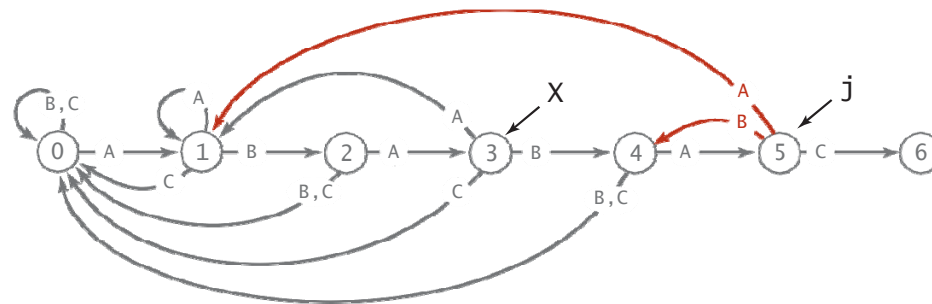
Knuth-Morris-Pratt construction

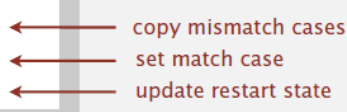| j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C
dfa[][j] A	1	1	3	1	5	1
B	0	2	0	4	0	4
C	0	0	0	0	0	6

DFA Construction – Java code

For each state j:

- Copy `dfa[][X]` to `dfa[][j]` for mismatch case.
- Set `dfa[pat.charAt(j)][j]` to j+1 for match case.
- Update X.

```java
public KMP(String pat)
{
    this.pat = pat;
    M = pat.length();
    dfa = new int[R][M];
    dfa[pat.charAt(0)][0] = 1;
    for (int X = 0, j = 1; j < M; j++)
    {
        for (int c = 0; c < R; c++)
            dfa[c][j] = dfa[c][X];          ← copy mismatch cases
        dfa[pat.charAt(j)][j] = j+1;        ← set match case
        X = dfa[pat.charAt(j)][X];          ← update restart state
    }
}
```

Running time. M character accesses (but space/time proportional to $R\,M$).

KMP search – Java code

```java
for (i = 0, j = 0; i < n && j < m; i++) {
    j = dfa[txt.charAt(i)][j];
}
if (j == m) return i - m;      // found
return n;                      // not found
```

KMP search performance

KMP substring search analysis

Proposition. KMP substring search accesses no more than $M+N$ chars to search for a pattern of length M in a text of length N.

Pf. Each pattern char accessed once when constructing the DFA; each text char accessed once (in the worst case) when simulating the DFA.

Proposition. KMP constructs `dfa[][]` in time and space proportional to RM.

Larger alphabets. Improved version of KMP constructs `nfa[]` in time and space proportional to M.

KMP NFA for ABABAC