IV. Set axioms

In these axioms, the variables A, B, C, and D denote sets but w, x, y, and z can be any sort of objects, not necessarily real numbers. This group of axioms is included here primarily for completeness; most of them are not discussed in the text.

(1) Extensionality: $A = B \leftrightarrow \forall x\,(x \in A \leftrightarrow x \in B)$

(2) Pairing: $\forall x, y\, \exists A\, \forall z\, [z \in A \leftrightarrow (z = x \vee z = y)]$. (Less formally, this says: for every x and y, the set $\{x, y\}$ exists.)

(3) There is a set $\mathbb{R}$ of all real numbers.[6]

*(4) For every x and y, the ordered pair (x, y) exists.

*(5) $(w, x) = (y, z)$ iff $w = y$ and $x = z$.

(6) Power set axiom: $\forall A\, \exists B\, \forall C\, (C \in B \leftrightarrow C \subseteq A)$. (Less formally, this says: for every set A, $\wp(A)$ exists.)

(7) Union axiom: $\forall \mathscr{A}\, \exists B\, \forall x\, [x \in B \leftrightarrow \exists C\, (C \in \mathscr{A} \wedge x \in C)]$. (Less formally, this says: for every set of sets $\mathscr{A}$, the union of all the sets in $\mathscr{A}$ ($\bigcup \mathscr{A}$) exists.)

*(8) Separation axiom: For every proposition $P(x)$ and every set A, the set $\{x \in A \mid P(x)\}$ exists.

(9) Replacement axiom: For every proposition $P(x, y)$ and every set A,

$$[\forall x \in A\, \exists! y\, P(x, y)] \rightarrow \exists B\, \forall y\, [y \in B \leftrightarrow \exists x \in A\, P(x, y)]$$

(Less formally, this says: if $P(x, y)$ defines a function whose domain is the set A, then its range is also a set.)

(10) Foundation axiom: $\forall A\, [A \neq \varnothing \rightarrow \exists B \in A\, (B \cap A = \varnothing)]$

(11) Axiom of choice (AC): For every collection $\mathscr{A}$ of nonempty sets, there is a function f such that, for every B in $\mathscr{A}$, $f(B) \in B$.