V. Real number axioms

In these axioms, x, y, and z denote real numbers. Axioms 1–12 of this group are called the **field axioms**, while axioms 1–17 are called the **ordered field axioms.**

(1) Additive closure: $\forall x, y \, \exists z \, (x + y = z)$

(2) Multiplicative closure: $\forall x, y \, \exists z \, (x \cdot y = z)$

(3) Additive associativity: $x + (y + z) = (x + y) + z$

(4) Multiplicative associativity: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

(5) Additive commutativity: $x + y = y + x$

(6) Multiplicative commutativity: $x \cdot y = y \cdot x$

(7) Distributivity: $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ and $(y + z) \cdot x = (y \cdot x) + (z \cdot x)$[7]

(8) Additive identity: There is a number, denoted 0, such that for all x, $x + 0 = x$.[8]

(9) Multiplicative identity: There is a number, denoted 1, such that for all x, $x \cdot 1 = 1 \cdot x = x$.[7,8]

(10) Additive inverses: For every x there is a number, denoted $-x$, such that $x + (-x) = 0$.[8]

(11) Multiplicative inverses: For every nonzero x there is a number, denoted x^{-1}, such that $x \cdot x^{-1} = x^{-1} \cdot x = 1$.[7,8]

(12) $0 \neq 1$

(13) Irreflexivity of $<$: $\sim(x < x)$

(14) Transitivity of $<$: If $x < y$ and $y < z$, then $x < z$

(15) Trichotomy: Either $x < y$, $y < x$, or $x = y$

(16) If $x < y$, then $x + z < y + z$

(17) If $x < y$ and $0 < z$, then $x \cdot z < y \cdot z$ and $z \cdot x < z \cdot y$[7]

(18) Completeness: If a nonempty set of real numbers has an upper bound, then it has a *least* upper bound.