ST3009 Weekly Questions 4

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Question 1

a) $Y = 2$ corresponds to the event where the sum of the two rolled die is 2, in this case, there is only one case for that and it is (1, 1)

b) Same idea here, except the sum is 3 and so there are only 2 events for that case, (1, 2) and (2, 1)

c) Again, except here Y=4 and we have (2,2), (3, 1) and (1, 3)

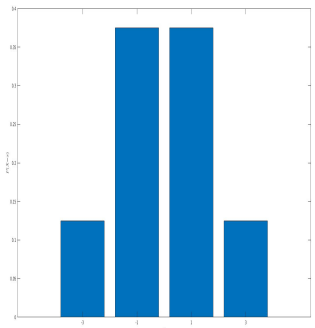
d) The event indicated by X is made out of 3 smaller event part of the sample space, (1, 1), (2, 2) and (3, 3). The sample space is made out of 36 total events, so the probability of $X = 1$ which by definition is the probability of one of the three to occur is: $\frac{3}{36} = \frac{1}{12} = 0.0833333$

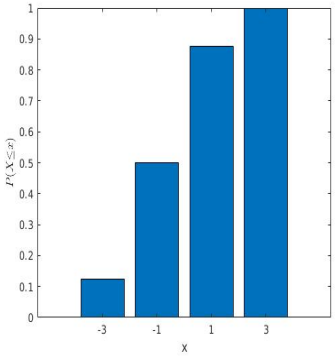
Question 2

a) X is made out of 3 coin toss and each outcome represent a $+1$ or a -1. Since X is calculated with 3 tosses, it can only take odd values and the minimum and maximum are -3 and $+3$ respectively. Other than that, it can also take the values -1 and $+1$ as these are the other odd values.

b) $X = -3$ corresponds to the event of 3 tails, which has a probability of: $\frac{1}{2^3} = 0.125$

c) $X = -1$ corresponds to the event of one extra tail, compared to the heads. With only 3 tosses, it is the probability of having 2 tails and 1 head. And there are 3 outcomes for this, namely (T, T, H), (T, H, T) and (H, T, T). $P(X = -1) = \frac{3}{8} = 0.375$





d)

Question 3

a) This asks "what is the probability for the minimum value to be greater or equal to 1". Well the dice can only a value between 1 and 6 inclusive. In the sample space, there are no outcome where the minimum value is below 1, that is just impossible. So $P(X \geq 1) = 1$

b) Same process here, except the minimum value has to be greater or equal to 2. This is only true when no 1 are rolled, so

$$P(X \geq 2) = P(No\ 1s\ rolled)$$
$$P(No\ 1s\ rolled) = \frac{5^4}{6^4} = 0.4822$$

c) We need to calculate the probability P(X≤k), ∀ k where $1 \leq k \leq 6$ For each of those, it is easier to calculate the probability's complement, and subtract that from 1. Indeed, the complement of P(X≤3), for example, is the probability of P(X>3) which is equivalent to the probability of rolling only 4s or higher. So, we get:

$$P(X \leq 1) = 1 - P(X > 1) = 1 - \frac{5^4}{6^4} = 0.51774691358$$
$$P(X \leq 2) = 1 - P(X > 2) = 1 - \frac{4^4}{6^4} = 0.8024691358$$
$$P(X \leq 3) = 1 - P(X > 3) = 1 - \frac{3^4}{6^4} = 0.9375$$
$$P(X \leq 4) = 1 - P(X > 4) = 1 - \frac{2^4}{6^4} = 0.98765432098$$
$$P(X \leq 5) = 1 - P(X > 5) = 1 - \frac{1^4}{6^4} = 0.99922839506$$
$$P(X \leq 6) = 1 - P(X > 6) = 1 - \frac{0}{6^4} = 1$$

And so, we that, we get:

$$CDF(X) = \begin{cases} 0.5177, for X \leq 1 \\ 0.802, for X \leq 2 \\ 0.9375, for X \leq 3 \\ 0.987, for X \leq 4 \\ 0.999, for X \leq 5 \\ 1, for X \leq 6 \end{cases}$$