Lecture 2

- Theory
  - Unification
  - Unification in Prolog
  - Proof search

- Exercises
  - Exercises of LPN chapter 2
  - Practical work

Aim of this lecture

- Discuss **unification** in Prolog
  - Show how Prolog unification differs from standard unification
- Explain the search strategy that Prolog uses when it tries to deduce new information from old, using modus ponens

Unification

- Recall previous example, where we said that Prolog unifies

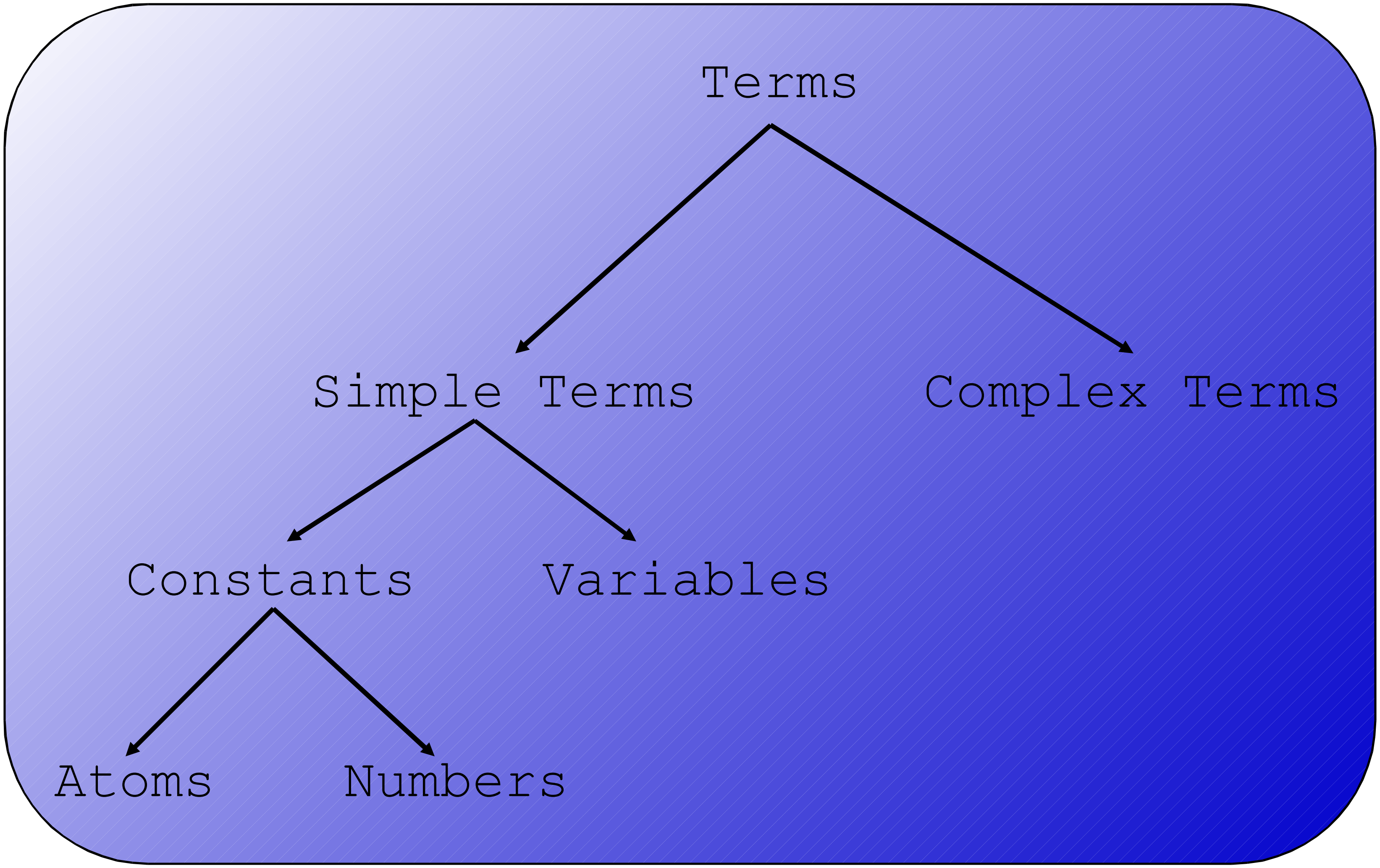
  woman(X)

  with

  woman(mia)

  thereby instantiating the variable **X** with the atom **mia**.

Recall Prolog Terms



Unification

- Working definition:
  - Two terms unify if they are the same term or if they contain variables that can be uniformly instantiated with terms in such a way that the resulting terms are equal

Unification

- This means that:
  - **mia** and **mia** unify
  - **42** and **42** unify
  - **woman(mia)** and **woman(mia)** unify

- This also means that:
  - **vincent** and **mia** do not unify
  - **woman(mia)** and **woman(jody)** do not unify

Unification

- What about the terms:
  - **mia** and **X**

Unification

- What about the terms:
  - **mia** and **X**
  - **woman(Z)** and **woman(mia)**

Unification

- What about the terms:
  - **mia** and **X**
  - **woman(Z)** and **woman(mia)**
  - **loves(mia,X)** and **loves(X,vincent)**

Instantiations

- When Prolog unifies two terms it performs all the necessary instantiations, so that the terms are equal afterwards
- This makes unification a powerful programming mechanism

Revised Definition 1/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.

Revised Definition 2/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.
2. If T_1 is a variable and T_2 is any type of term, then T_1 and T_2 unify, and T_1 is instantiated to T_2. (and vice versa)

Revised Definition 3/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.
2. If T_1 is a variable and T_2 is any type of term, then T_1 and T_2 unify, and T_1 is instantiated to T_2. (and vice versa)
3. If T_1 and T_2 are complex terms then they unify if:
   a) They have the same functor and arity, and
   b) all their corresponding arguments unify, and
   c) the variable instantiations are compatible.

Prolog unification: =/2

```
?- mia = mia.
yes
?-
```

Prolog unification: =/2

```
?- mia = mia.
yes
?- mia = vincent.
no
?-
```

Prolog unification: =/2

```
?- mia = X.
X=mia
yes
?-
```

How will Prolog respond?

?- X=mia, X=vincent.

How will Prolog respond?

?- X=mia, X=vincent.

no

?-

Why? After working through the first goal, Prolog has instantiated X with **mia**, so that it cannot unify it with **vincent** anymore. Hence the second goal fails.

Example with complex terms

```
?- k(s(g),Y) = k(X,t(k)).
```

Example with complex terms

```
?- k(s(g),Y) = k(X,t(k)).
X=s(g)
Y=t(k)
yes
?-
```

Example with complex terms

?- k(s(g),t(k)) = k(X,t(Y)).

Example with complex terms

```
?- k(s(g),t(k)) = k(X,t(Y)).
X=s(g)
Y=k
yes
?-
```

One last example

?- loves(X,X) = loves(marsellus,mia).

Prolog and unification

- Prolog does not use a standard unification algorithm
- Consider the following query:

```
?- father(X) = X.
```

- Do these terms unify or not?

Infinite terms

```
?- father(X) = X.
X=father(father(father(father(father(father
   (father(father(father(father(father(father
   (father(father(father(father(father(father
   (father(father(father(father(father(father
   (father(father(father(father(father(father
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   (father(father(father(father(father(father
   (father(father(father(father(father(father
   (
```

Infinite terms

```
?- father(X) = X.
X=father(father(father(…))))
yes
?-
```

Occurs Check

- A standard unification algorithm carries out an occurs check
- If it is asked to unify a variable with another term it checks whether the variable occurs in the term
- In Prolog:

```
?- unify_with_occurs_check(father(X), X).
no
```

Programming with Unification

```
vertical( line(point(X,Y),
               point(X,Z))).

horizontal( line(point(X,Y),
                 point(Z,Y))).
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- vertical(line(point(1,1),point(1,3))).
yes
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- vertical(line(point(1,1),point(1,3))).
yes
?- vertical(line(point(1,1),point(3,2))).
no
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- horizontal(line(point(1,1),point(1,Y))).
Y = 1;
no
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- horizontal(line(point(2,3),Point)).
Point = point(_554,3);
no
?-
```

Exercise: unification

Proof Search

- Now that we know about unification, we are in a position to learn how Prolog searches a knowledge base to see if a query is satisfied.
- In other words: we are ready to learn about proof search

Example

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

```
?- k(Y).
```

Example: search tree

f(a).

f(b).

g(a).

g(b).

h(b).

k(X):- f(X), g(X), h(X).

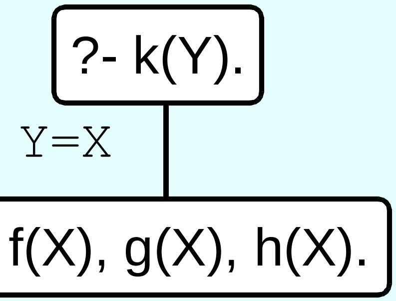
?- k(Y).

?- k(Y).

Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

```
?- k(Y).
```



Example: search tree

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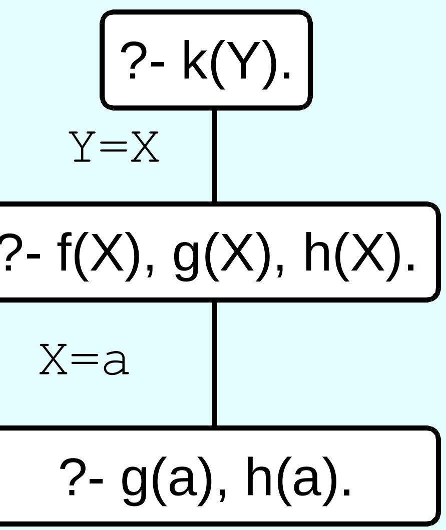
```

```
?- k(Y).
```



Example: search tree

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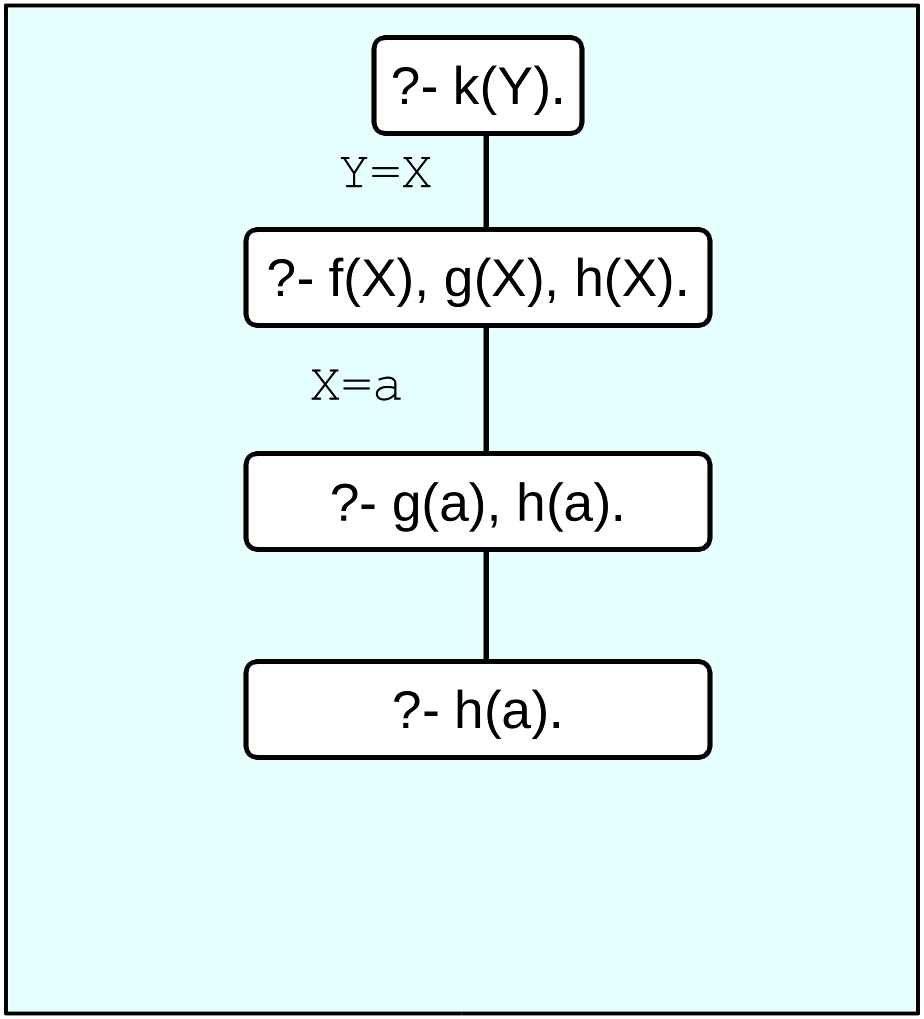
k(X):- f(X), g(X), h(X).
```

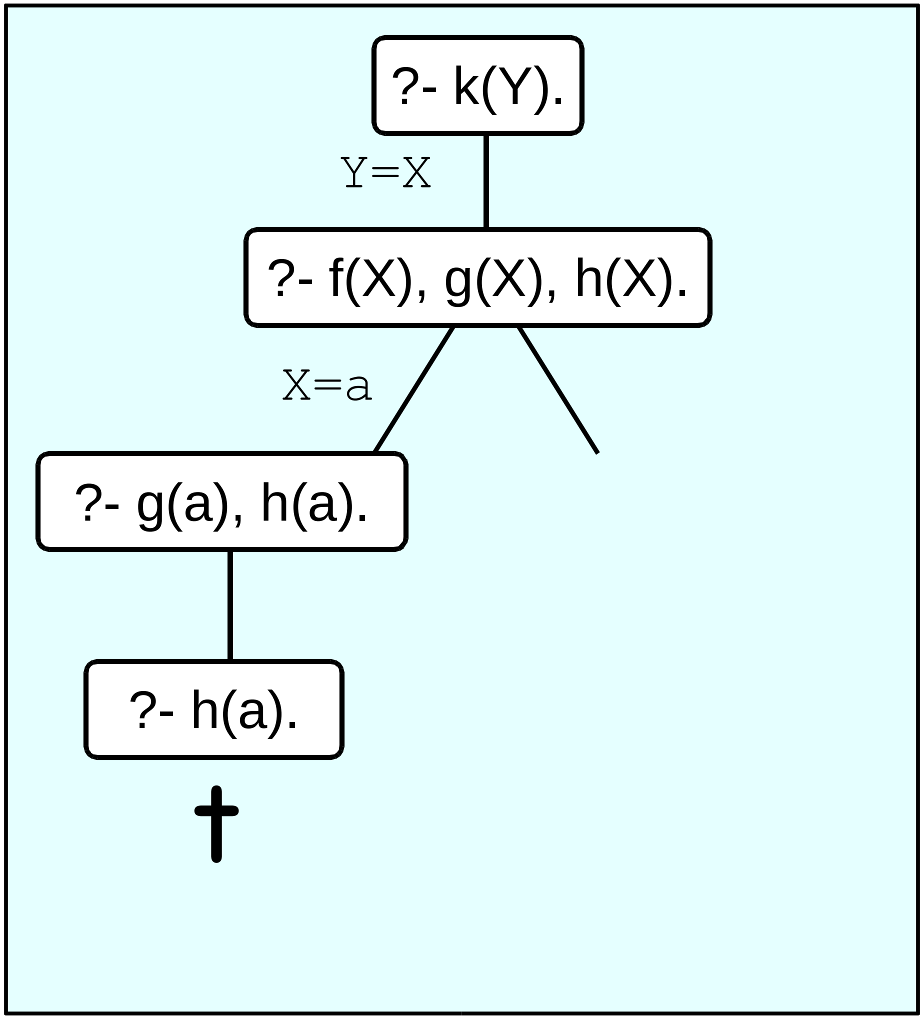
```
?- k(Y).
```



Example: search tree

f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).

?- k(Y).



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```
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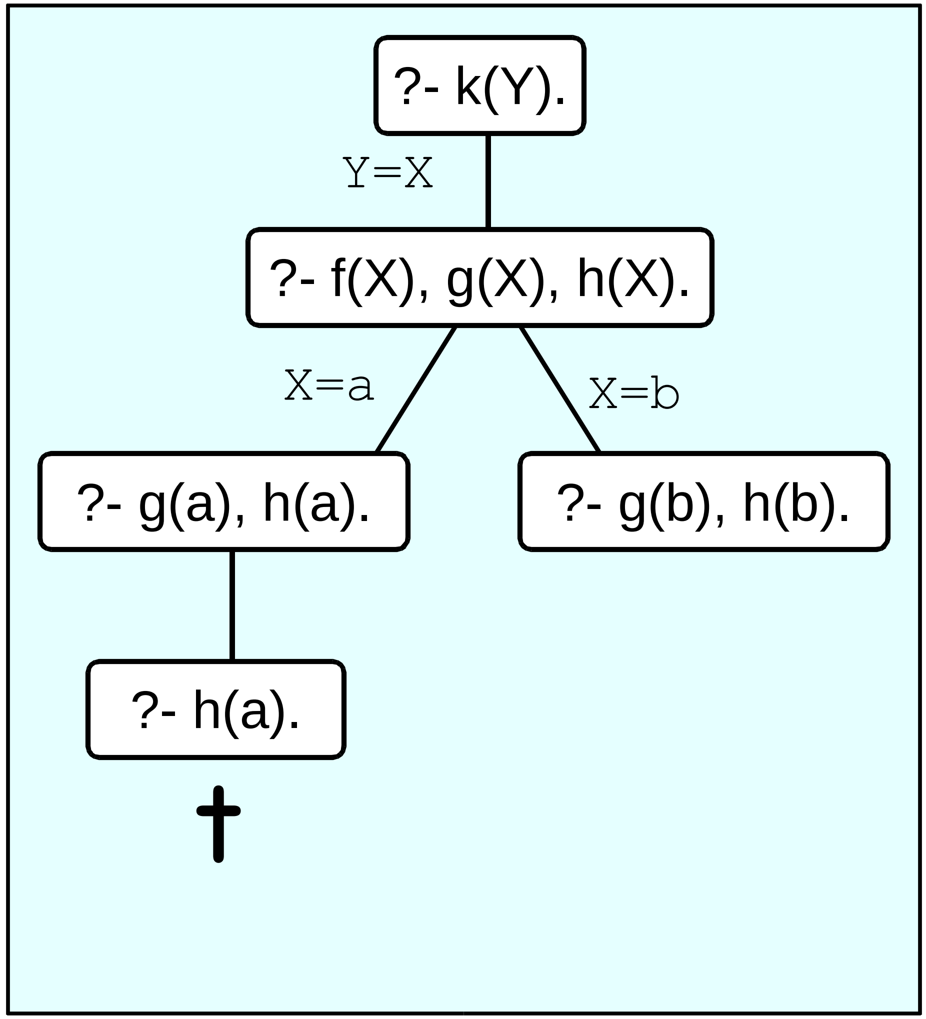

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f(a).
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```

```
?- k(Y).
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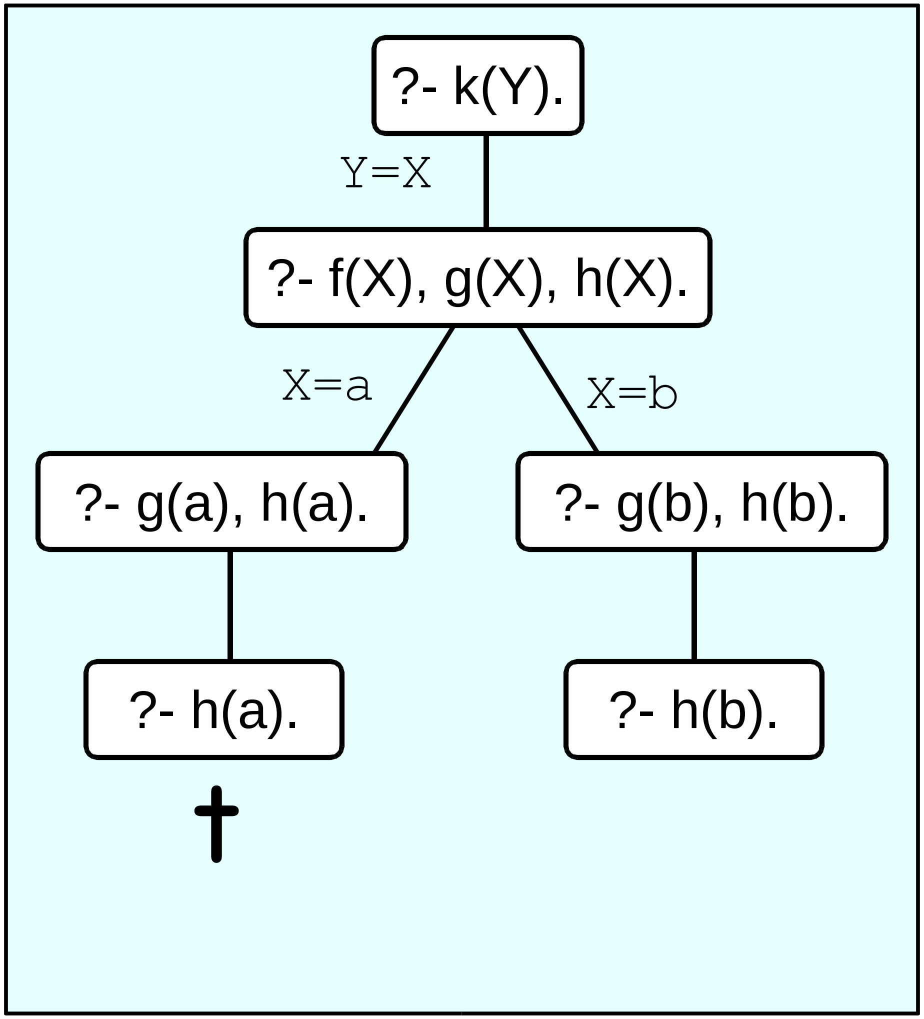
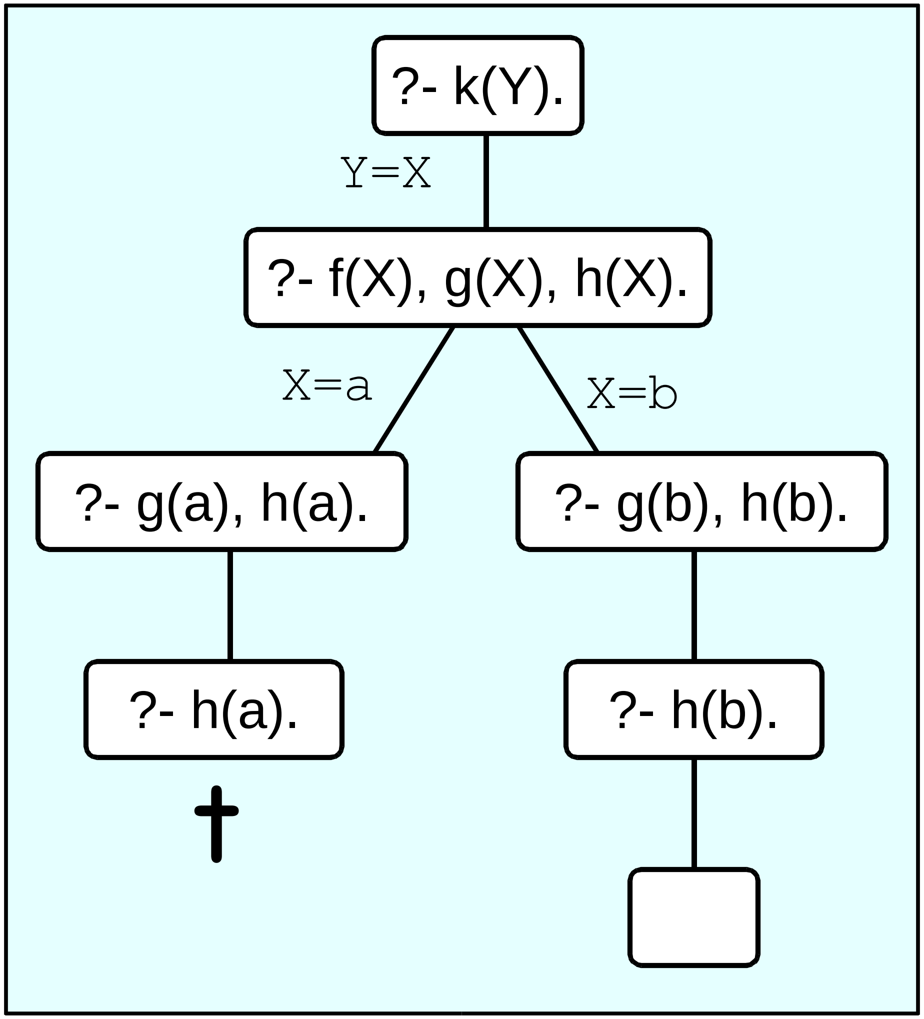


Example: search tree

f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).

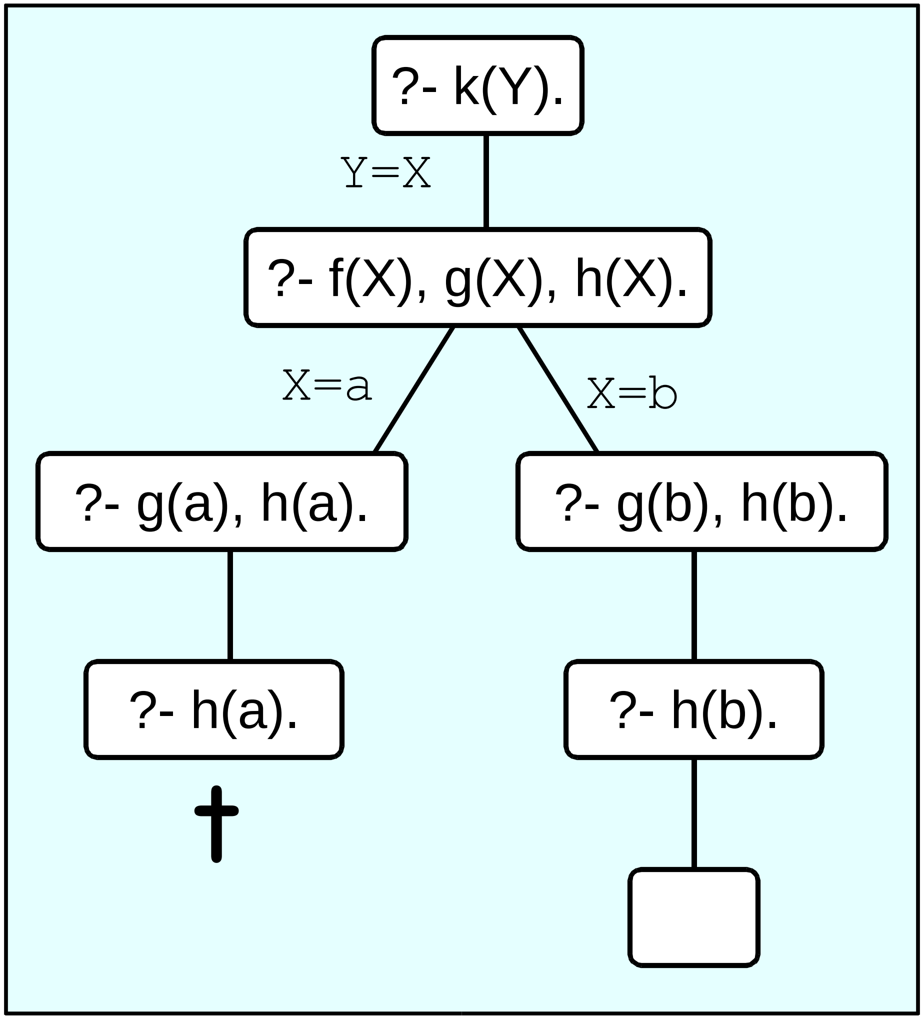
?- k(Y).
Y=b



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

```
?- k(Y).
Y=b;
no
?-
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
?- jealous(X,Y).
```

Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
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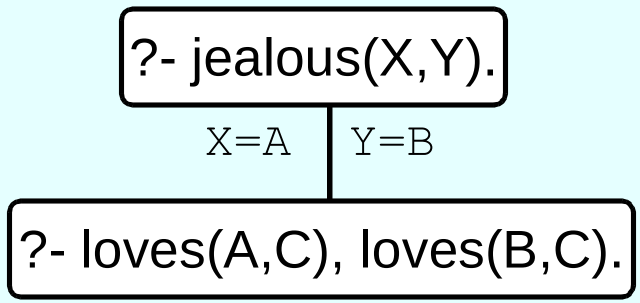

Another example

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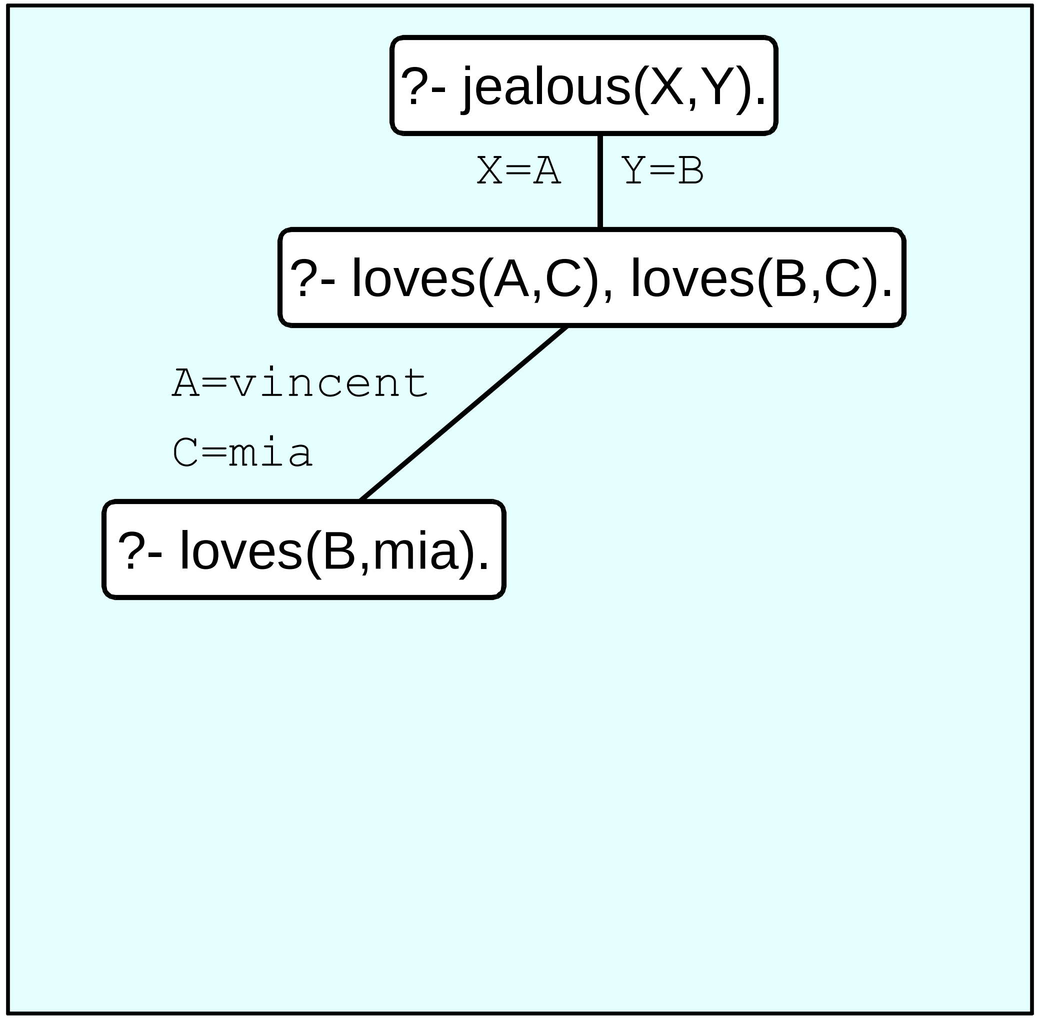
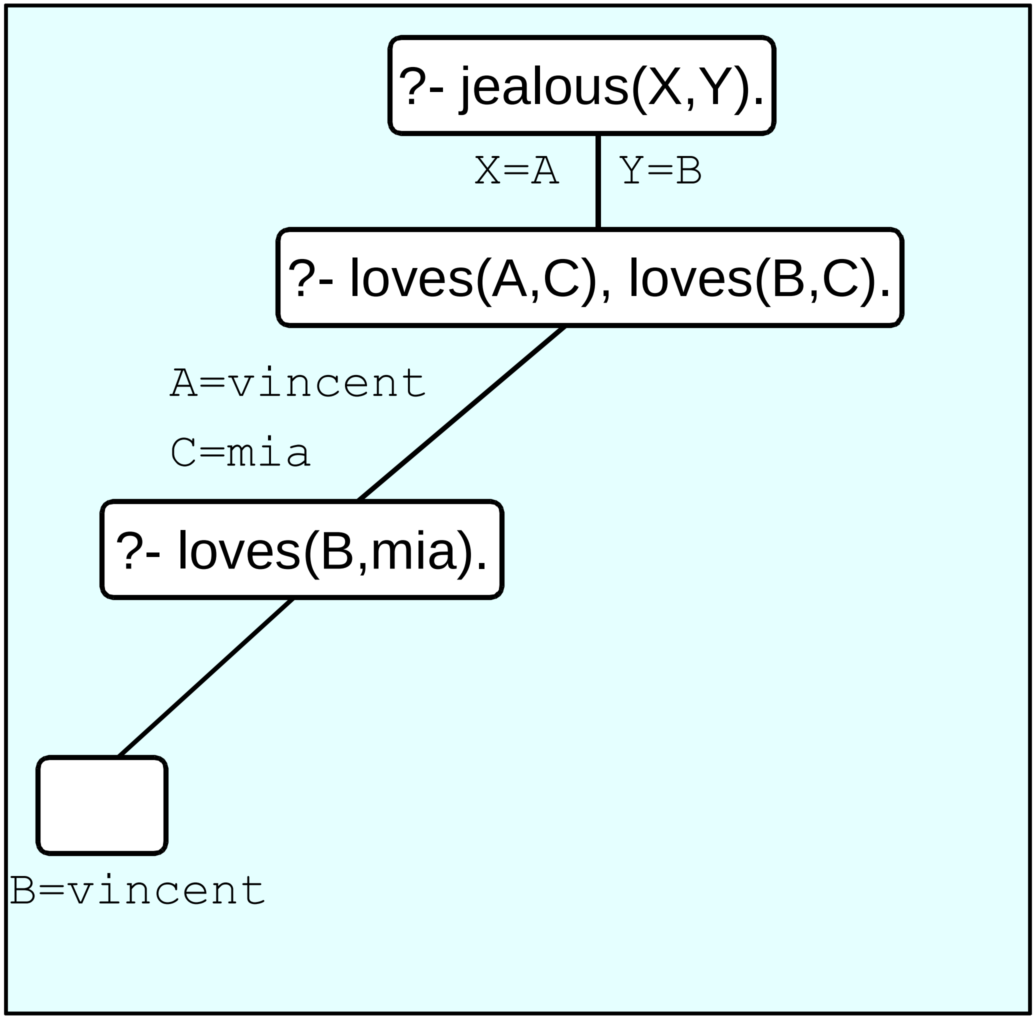


Another example

loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).

?- jealous(X,Y).
X=vincent
Y=vincent

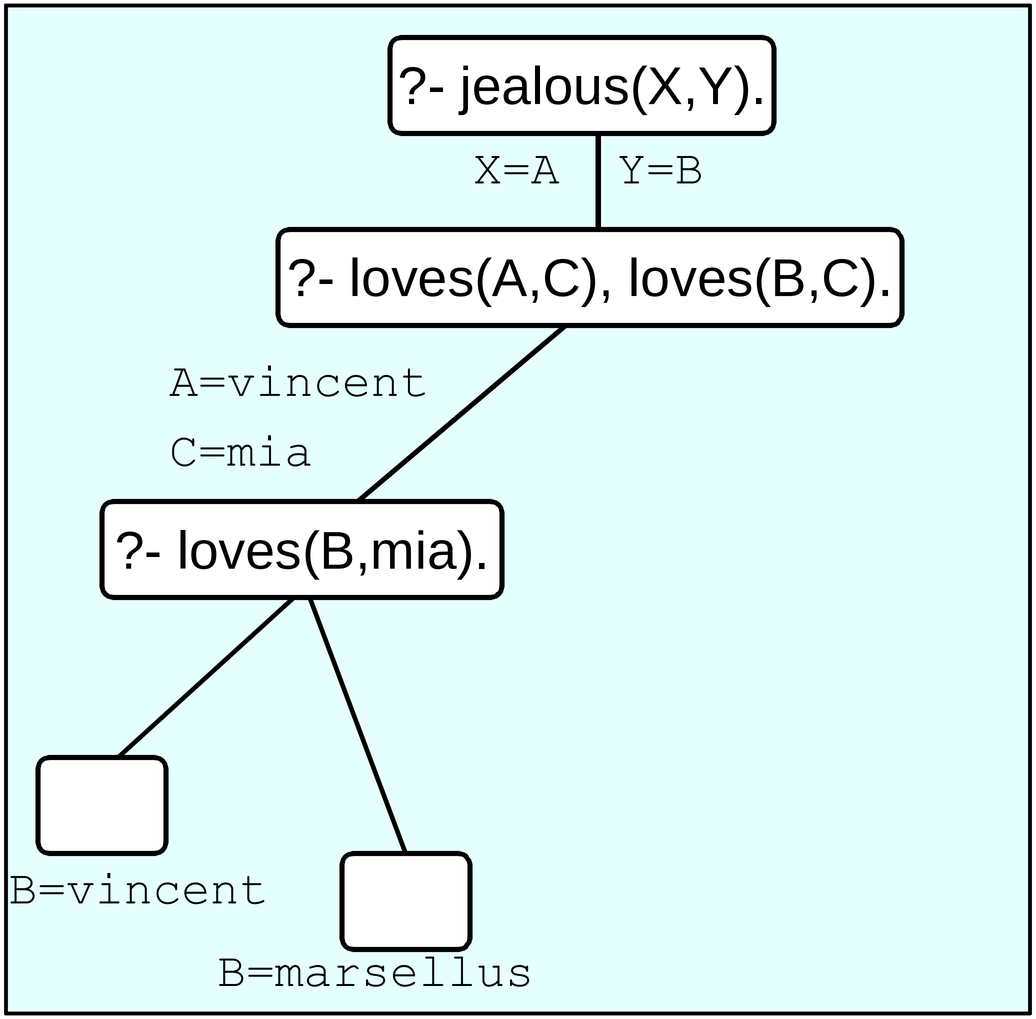


Another example


```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
?- jealous(X,Y).
X=vincent
Y=vincent;
X=vincent
Y=marsellus
```



Another example

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loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
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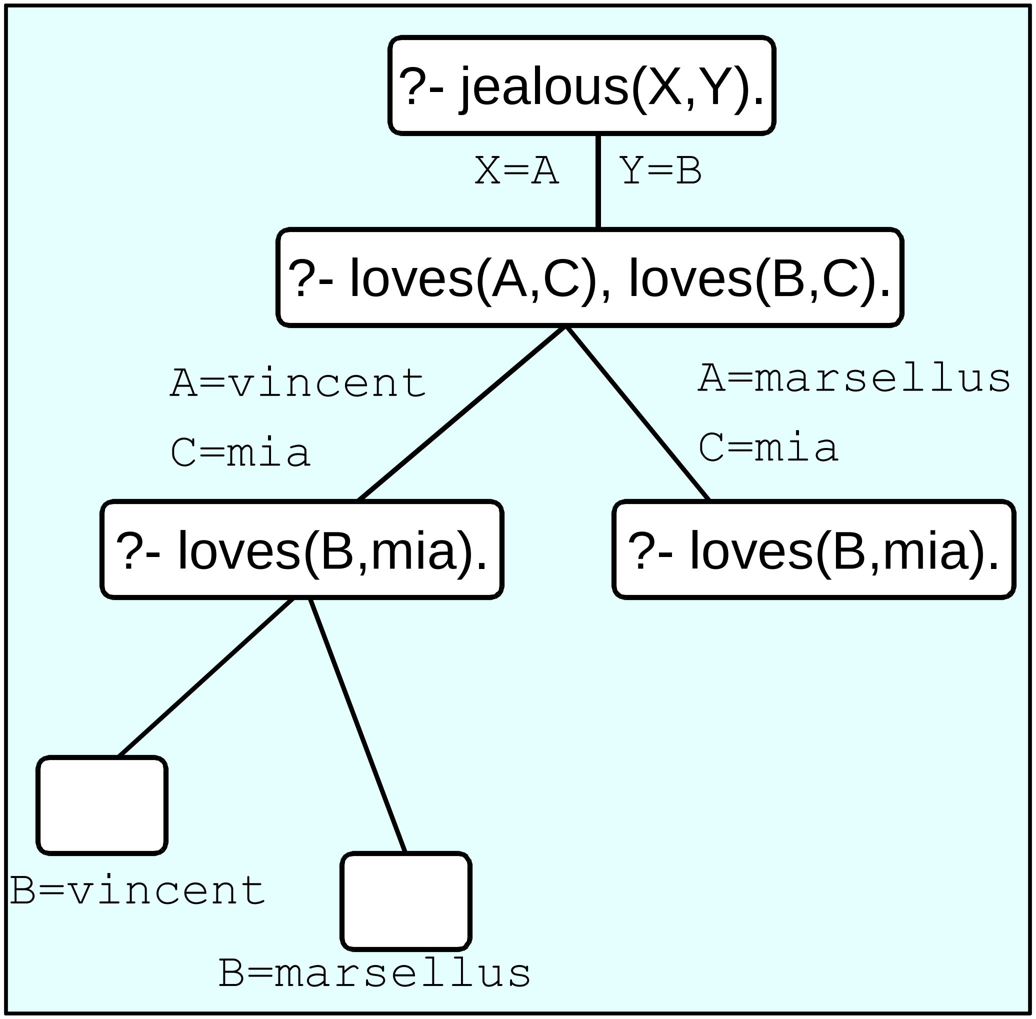
```

```
?- jealous(X,Y).
X=vincent
Y=vincent;
X=vincent
Y=marsellus;
```



Another example

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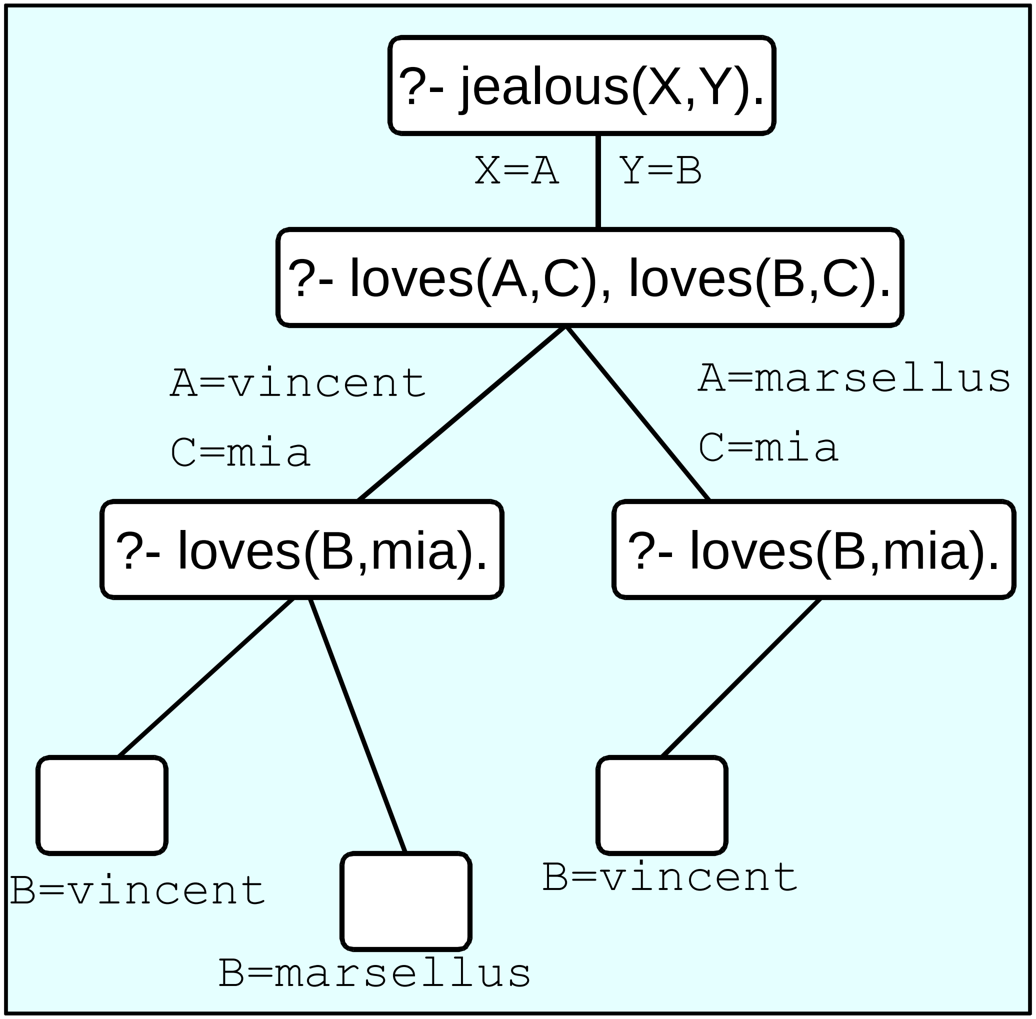
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
….
X=vincent
Y=marsellus;
X=marsellus
Y=vincent
```



Another example

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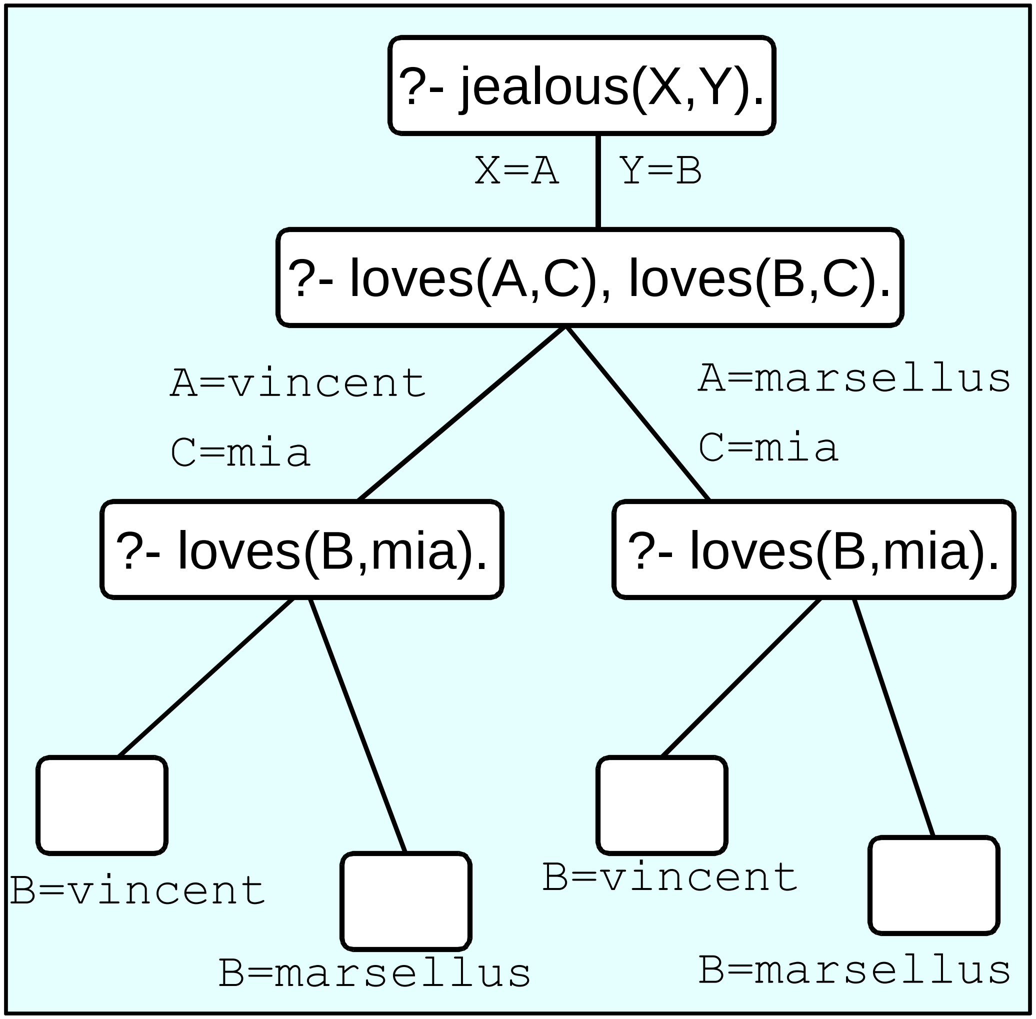
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
….
X=marsellus
Y=vincent;
X=marsellus
Y=marsellus
```



Another example

```
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loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
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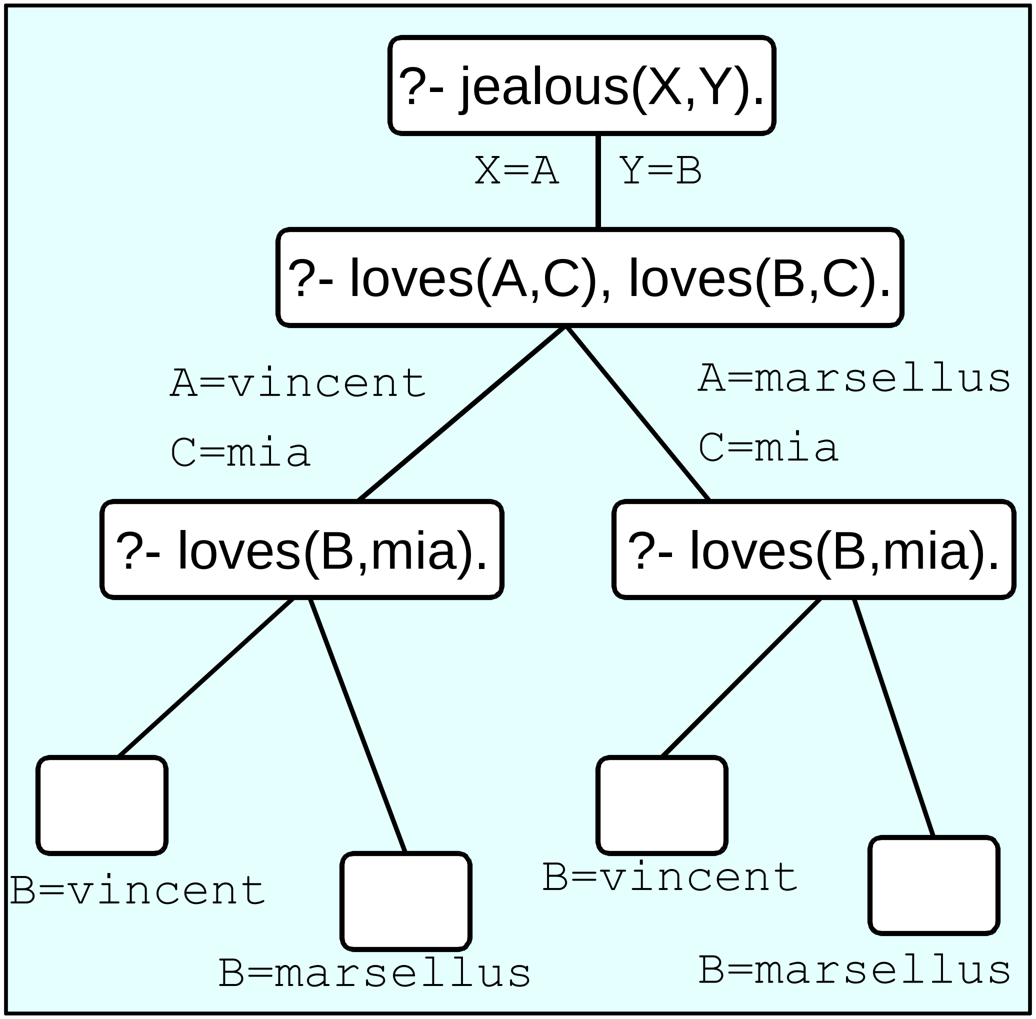
```

```
….
X=marsellus
Y=vincent;
X=marsellus
Y=marsellus;
no
```



Exercises

Summary of this lecture

- In this lecture we have
  - defined unification
  - looked at the difference between standard unification and Prolog unification
  - introduced search trees

Next lecture

- Discuss **recursion** in Prolog
  - Introduce recursive definitions in Prolog
  - Show that there can be mismatches between the declarative meaning of a Prolog program, and its procedural meaning.