ST3009 Weekly Questions 2

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Question 1

a) Each die can roll 6 different number and we roll 3 die, so the total number of outcomes is 6^3 which is equal to 216.

b) The set of outcomes with at least one 2 rolled is the same as the total set of outcomes minus the all the outcomes with no 2 in them. In terms of probability, it means that the probability of at least one 2, is 1 minus the probability of no 2 rolled: $1 - \frac{5^3}{6^3} = 0.421$

c) The MATLAB simulation gives a similar result

d) There are 3 possibilities for the dice rolls to sum up to 17, there is (6,6,5), (6,5,6) and (5,6,6). So the probability for that is $\frac{3}{216} = 0.013888$

e) If the first roll was a 1, it means that the two other rolls need to add up to 11, and so that gives us (5,6) and (6,5) but this time, out of only 6^2 possibilities, so the probability for that is $\frac{2}{36} = 0.0555$

Question 2

a) The probability for the second roll to be a five is sum of the probability of first rolling a 1 and then a 5 using the 6 sided die, and the probability of rolling anything but a 1 and finally rolling a 5 using the 20 sided die. That is because these two cases are mutually exclusive because of the first roll. So we get:

$$P(Second\, throw\, is\, 5) = \frac{1}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{20}$$
$$P(Second\, throw\, is\, 5) = 0.0694$$

b) Same process here but since we want the second roll to be equal to 15, only the 20 sided die can achieve such a roll, so the probability to roll 15 if the first roll is 1, is equal to 0 and we can just ignore that probability, so we get

$$P(Second\ throw\ is\ 15) = 0 + \frac{5}{6} \times \frac{1}{20}$$
$$P(Second\ throw\ is\ 15) = 0.0416$$

Question 3

We want to know the probability of the suspect being guilty, knowing he has the characteristic, we shall write as P(G—C). We'll use Bayes rule for that, and so we need to know the probability of P(G), that is 60%, the probability of him having the characteristic, knowing he is guilty. Well for that we know the actual criminal has it, so if he is guilty, he has a 100% of having the characteristic. Finally we need to calculate the probability P(C), for that we'll use marginalisation. So we get:

$$P(G|C) = \frac{P(C|G)P(G)}{P(C)}, s.t P(C) = P(C|G)P(G) + P(C|\neg G)P(\neg G)$$
$$P(G|C) = \frac{1 * 0.6}{1 * 0.6 + 0.2 * 0.4} = 0.88$$

Question 4

$$\begin{bmatrix} 0.0744 & 0.1885 & 0.0744 & 0.005 \\ 0.005 & 0.1488 & 0.0942 & 0.0744 \\ 0.001 & 0.005 & 0.1488 & 0.0942 \\ 0.001 & 0.001 & 0.0099 & 0.0744 \end{bmatrix}$$

I get this grid matrix for the probability of having two bar signal, knowing the location. To get this, I used Bayes rule for each of the grid cell. If $\mathrm{L}_{\mathrm{i,\ j}}$ corresponds to the j^{th} cell of the i^{th} row for the location grid and $\mathrm{S}_{\mathrm{i,\ j}}|\mathrm{L}_{\mathrm{i,\ j}}$ correspond to the i, j cell on the other grid, we get the following equation:

$$P(L_{\mathrm{i,\ j}}|S_{\mathrm{i,\ j}}) = \frac{P(S_{\mathrm{i,\ j}}|L_{\mathrm{i,\ j}})P(L_{\mathrm{i,\ j}})}{P(S)}, \forall i, j\ s.t\ 1 \leq i, j \leq 4$$

Where P(S) is the probability of having signal, at all. This is calculated by using marginilisation and summing up all the probabilitiies of having two bar of signal in a given location, for all the location. All the probabilities from the left grid a mutually exclusive, someone can not be in two locations at the same time. So for P(S), we get:

$$P(S) = \sum_{i,j=1}^{4} P(S_{\mathrm{i,\ j}}|L_{\mathrm{i,\ j}}) \times P(L_{\mathrm{i,\ j}}) = 0.504$$