

## Function Application/Types

- ▶ Function application is denoted by juxtaposition, and is left associative
- ▶ `f x y z` parses as `((f x) y) z`
- ▶ If we want `f` applied to both `x`, and the result of the application of `g` to `y`, we must write `f x (g y)`
- ▶ In types, the function arrow is right associative  
`Int -> Char -> Bool` parses as `Int -> (Char -> Bool)`
- ▶ The type of a function whose first argument is itself a function,  
has to be written as `(a -> b) -> c`
- ▶ Note the following types are identical:  
`(a -> b) -> (c -> d)`  
`(a -> b) -> c -> d`

## Sections [H2010 3.5]

- ▶ A “section” is an operator, with possibly one argument surrounded by parentheses, which can be treated as a prefix function name.
- ▶ `(+)` is a prefix function adding its arguments  
(e.g. `(+) 2 3 = 5`)
- ▶ `(/)` is a prefix function dividing its arguments  
(e.g. `(/) 2.0 4.0 = 0.5`)
- ▶ `(/4.0)` is a prefix function dividing its single argument by 4  
(e.g. `(/4.0) 10.0 = 2.5`)
- ▶ `(10.0/)` is a prefix function dividing 10 by its single argument  
(e.g. `(10/) 4.0 = 2.5`)
- ▶ `(- e)` is not a section, use `subtract e` instead.  
(e.g. `(subtract 1) 4 = 3`)

## Writing Functions (I) — using other functions

(Examples from Chp 4, Programming in Haskell, 2nd Ed., Graham Hutton 2016)

- ▶ Function `even` returns true if its integer argument is even  
`even n = n `mod` 2 == 0`  
We use the modulo function `mod` from the Prelude
- ▶ Function `recip` calculates the reciprocal of its argument  
`recip n = 1/n`  
We use the division function `/` from the Prelude
- ▶ Function call `splitAt n xs` returns two lists, the first with the first `n` elements of `xs`, the second with the rest of the elements  
`splitAt n xs = (take n xs, drop n xs)`  
We use the list functions `take` and `drop` from the Prelude

## Writing Functions (II) — using recursion

- ▶ We shall show how to write the functions `take` and `drop` using recursion.
- ▶ We shall consider what this means for the execution efficiency of `splitAt`.
- ▶ We then do a direct recursive implementation of `splitAt` and compare.

## Implementing take

- ▶ `take :: Int -> [a] -> [a]`  
Let `xs1 = take n xs` below.  
Then `xs1` is the first `n` elements of `xs`.  
If `n <= 0` then `xs1 = []`.  
If `n >= length xs` then `xs1 = xs`.
- ▶ `take n _ | n <= 0 = []`  
`take _ [] = []`  
`take n (x:xs) = x : take (n-1) xs`
- ▶ How long does `take n xs` take to run?  
(we count function calls as a proxy for execution time)
- ▶ It takes time proportional to `n` or `length xs`, whichever is shorter.

## Implementing drop

- ▶ `drop :: Int -> [a] -> [a]`  
Let `xs2 = drop n xs` below.  
Then `xs2` is `xs` with the first `n` elements removed.  
If `n <= 0` then `xs2 = xs`.  
If `n >= length xs` then `xs2 = []`.
- ▶ `drop n xs | n <= 0 = xs`  
`drop _ [] = []`  
`drop n (x:xs) = drop (n-1) xs`
- ▶ How long does `drop n xs` take to run?
- ▶ It takes time proportional to `n` or `length xs`, whichever is shorter.

## Implementing splitAt recursively

- ▶ `splitAt :: Int -> [a] -> ([a],[a])`  
Let `(xs1,xs2) = splitAt n xs` below.  
Then `xs1` is the first `n` elements of `xs`.  
Then `xs2` is `xs` with the first `n` elements removed.  
If `n >= length xs` then `(xs1,xs2) = (xs,[])`.  
If `n <= 0` then `(xs1,xs2) = ([],xs)`.
- ▶ `splitAt n xs | n <= 0 = ([],xs)`  
`splitAt _ [] = ([],[])`  
`splitAt n (x:xs)`  
    `= let (xs1,xs2) = splitAt (n-1) xs`  
       `in (x:xs1,xs2)`
- ▶ How long does `splitAt n xs` take to run?
- ▶ It takes time proportional to `n` or `length xs`, whichever is shorter, which is twice as fast as the version using `take` and `drop` explicitly!

## Switcheroo!

- ▶ Can we implement `take` and `drop` in terms of `splitAt`?
- ▶ Hint: the Prelude provides the following:  
`fst :: (a,b) -> a`  
`snd :: (a,b) -> b`
- ▶ Solution:  
`take n xs = fst (splitAt n xs)`  
`drop n xs = snd (splitAt n xs)`
- ▶ How does the runtime of these definitions compare to the direct recursive ones?