Review of Physics for Electrotechnology

Reference: ‘Senior Physics’ by George Porter

Linear Motion

Definition of Displacement:

The displacement of a point from another point is its distance from that point in a particular direction.

Displacement is a vector quantity. The unit of displacement is the meter (m).

Example 1:

The displacement of Ballinasloe from Dublin is 129 Km due West (i.e. ‘as the crow flies’).

The distance of Ballinasloe from Dublin by road is 180Km (i.e. due to the curvature of the road).

If I drive to Ballinasloe from Dublin and back the distance I have travelled is 360Km but my displacement from Dublin is 0Km!

Example 2:

I walk 10Km due West of my home. I then walk 5Km due North. The total distance I have travelled is 15Km.

My displacement from my home is $\sqrt{125}$ Km WNW (of my home).

Definition of Velocity:

Velocity is the rate of change of displacement w.r.t. (with respect to) time.

Velocity is a vector quantity. The unit of velocity is the meter per second (ms^{-1}).

Mathematically:

Let displacement be denoted by the symbol s. Then the velocity v is:

$$v = \frac{ds}{dt}$$

Example:

The displacement of Sligo from Cork is approx. 265Km N. The distance by road is 330Km.

A motorist travels from Cork to Sligo in 5 hours.

Find:

(i) Her average speed
(ii) Her average velocity

Solution:

(i)

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time}}$$

$$= \frac{330}{5}$$

$$= 66\text{Kmph}$$

(ii)

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{265}{5}$$

$$= 53\text{Kmph N}$$

Definition of Acceleration:

Acceleration is the rate of change of velocity w.r.t. time.

Acceleration is a vector quantity.

Mathematically:

The acceleration a is:

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

The unit of acceleration is the meter per second squared $(\mathrm{ms^{-2}})$

Example:

The velocity of a car changes from $10\mathrm{ms^{-1}E}$ to $25\mathrm{ms^{-1}E}$ in $10\mathrm{s}$. Find its average acceleration.

Solution:

$$\mathrm{a} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{15}{10}$$

$$= 1.5\mathrm{ms^{-2}E}$$

Definition of Mass (1):

The mass of a body is a measure of its ability to resist changes in its velocity.

Mass is a scalar quantity. The unit of mass is the Kilogram (Kg)

Example:

It takes 10 times the force to move object A from rest as it does to move object B in the same environment. Hence the mass of object A is 10 times that of object B.

Definition of Mass (2):

The mass of a body is a measure of the quantity of matter in it.

Example:

Sample A of a particular substance is 10 times the volume of sample B of the same substance in the same environment (STP).

Hence the mass of sample A is 10 times that of sample B (and it will take 10 times the force to move sample A from rest as it does sample B).

Important Notes on Mass:

(1) The mass of a body does <u>NOT</u> depend on where it is. Hence a body of $10\mathrm{Kg}$ on Earth will have a mass of $10\mathrm{Kg}$ on the Moon, Jupiter or in outer space.
(2) The colloquial use of the term 'weight' should not be confused with mass. 'Weight' is a measure of the force of gravity on a body and its unit (being a force) is the Newton (N). The weight of a body of $10\mathrm{N}$ on earth would be approximately $1\mathrm{N}$ on the moon, $90\mathrm{N}$ on Jupiter and $0\mathrm{N}$ in outer space.

Definition of Momentum:

The momentum (p) *of a body is the product of its mass and its velocity.*

Momentum is a vector quantity. The unit of momentum is the Kgms^{-1}*.*

Mathematically:

$$p = mv$$

Example:

A truck travelling at 30Kmph hits a man and injures him. A pebble thrown at 30Kmph hits a man and it does not hurt him. A bullet travelling at 300Kmph hits a man and injures him.

Hence ‘momentum’ equals ‘impact’. Clearly the ‘impact’ of one body on another depends on its mass and its velocity. In other words:

$$p \propto m$$

$$p \propto v$$

$$\therefore p \propto mv$$

$$\Rightarrow p = cmv$$

Now define $c = 1$. Then:

$$p = mv$$

Conservation of Momentum:

The Principle of Conservation of Momentum *states that in any interaction involving a system of bodies, the total momentum before the interaction equals the total momentum after the interaction.*

Example:

A bullet of mass $0.02\mathrm{Kg}$ travelling with a velocity of $400\mathrm{ms}^{-1}\mathrm{E}$ strikes a block of wood which is at rest and which has a mass of $2.0\mathrm{Kg}$. After the collision the bullet and the block move together. Assuming there is a negligible interaction (friction) between the block and its support, calculate the velocity of the block and bullet after impact.

Solution:

Use the Principle of Conservation of Momentum which states that the total momentum before impact equals the total momentum after impact. Hence:

$$(400 \times 0.02) + (0 \times 2.0) = 2.02 \times v_{after}$$

$$\Rightarrow v_{after} = 3.96 ms^{-1}$$

Definition of Force:

A force is that which causes acceleration.

Force (F) *is a vector quantity and its unit is the Newton* (N)*.*

Newton's Laws of Motion:

Law 1:

The velocity of a body does not change (i.e. a body will not accelerate) unless a resultant external force acts on it.

Law 2:

When a resultant external force acts on a body, the rate of change of the body's momentum is proportional to the force and takes place in the direction of the force.

Law 3:

In any interaction between two bodies A and B, the force exerted by A on B is equal in magnitude and opposite in direction to the force exerted by B on A.

Law 1:

Example:

Consider a car travelling at a constant velocity of 50Kmph due east.

Fact: there is NO net force acting on the car. In other words the forward force given by the car engine is equal and opposite to the resistance to motion given by the air, friction of mechanical parts and the friction of the road surface.

Now the driver presses the accelerator and the car accelerates to a new speed of 70Kmph. During that time a resultant force is exerted on the car in the forward direction by the engine. As the speed of the car increases so too do the resistance factors given above until the new forward force equal these greater resistive forces and so the car steadies to a velocity of 70Kmph E.

Example:

A man steps from a spacecraft in deep space (i.e. far away from stars and planets (sources of gravitational pull)). If the man is unattached to the spacecraft he will continue his motion at a constant velocity away from the spacecraft until he is pulled towards some star, planet or black-hole. The reason for this continuous motion is that there is no resistance to motion in space.

Question:

Why does he not continue with a constant acceleration instead of with a constant velocity?

Note:

In the scenario described above the spacecraft will move in a direction opposite to that of the man and will continue in that direction with a constant velocity. The reason for this is given by the Principle of Conservation of Momentum.

Law 2:

Mathematically:

$$F \propto \frac{dp}{dt}$$

If the mass of the body is constant then:

$$F \propto m\frac{dv}{dt}$$

i.e.

$$F \propto ma$$

Hence:

$$F = kma$$

Defining $k = 1$ we get:

$$F = ma$$

This relationship allows us to define the unit of force.

Definition of the Unit of Force:

The unit of force is the Newton (N)*. One Newton is that force that gives a body of mass* $1\mathrm{Kg}$ *an acceleration of* $1\mathrm{ms}^{-2}$*.*

Example:

Find the acceleration produced by a force of $10\mathrm{N}\ \mathrm{SW}$ acting on a mass of $20\mathrm{Kg}$.

Solution:

$$F = ma$$
$$\Rightarrow 10\hat{f} = 20a$$
$$\Rightarrow a = 0.5ms^{-2}\ \mathrm{SW}$$

Law 3:

This law says, in effect, that forces act in pairs.

Example:

A block of wood rests on a table. The block exerts a downward force (due to its own weight) on the table. The table in turn exerts an equal and opposite force upwards on the block. The forces are equal and opposite and so the resultant force is zero. Hence there is no acceleration. If however the block is very heavy, the table is unable to sustain its weight (i.e. provide sufficient upward force). The result is that the table breaks and the block accelerates to the floor.

Question:

Why does the block accelerate to the floor?

Newton's Law of Gravitation:

The force between any two point masses is proportional to the product of the masses and inversely proportional to the square of the distance between them.

Mathematically:

$$F \propto \frac{m_1 m_2}{d^2}\hat{d}$$
$$\Rightarrow F = \frac{G m_1 m_2}{d^2}\hat{d}$$

where G is a constant called the 'Universal Constant of Gravitation'. Its value is approximately $6.7 \times 10^{-11}\,\mathrm{Nm^2Kg^2}$. The value of G is such that for everyday masses (say two billiard balls on a table) the gravitational force between them is so small that it will not overcome the friction (resistance) of the table. Hence the balls remain motionless.

For large masses such as planets and stars the effect of gravitation is very noticeable. It is because of the force of gravity that masses have associated with them a force termed 'weight'. For example, the mass of the earth is large enough to induce a noticeable force on a nearby object or an object on its surface. This object is pulled towards the centre of the earth. Hence on earth we and other masses have a certain 'weight' which is a measure of the force of gravity that binds us to the earth. If we were on the moon we would weigh less because the mass of the moon is smaller than that of earth. We would be able to jump higher, walk more easily etc. The opposite would occur on a planet larger than earth e.g. Jupiter.

Definition of Weight:

The weight of a body (on earth) is the gravitational force exerted on it by the earth.

Weight is a vector quantity. Its unit is the Newton (N)*.*

Mathematically:

$$F = W = \frac{GMm}{r^2}\hat{r}$$

where M is the mass of the earth, r is the radius of the earth and m is the mass of the object. Hence $\frac{GM}{r^2}$ is a constant for bodies on or near the earth's surface.

However:

$$F = ma$$

Hence $\frac{GM}{r^2}$ is an acceleration we term the 'acceleration due to gravity'. Its value for bodies on or near the earth's surface is $9.81\mathrm{ms}^{-2}$.

Example:

Given the mass of the earth is $6.0\times10^{24}\,\mathrm{Kg}$ and its radius is $6.4\times10^{6}\,\mathrm{m}$. Find the weight of a body whose mass is $50\mathrm{Kg}$.

Solution:

$$W = \frac{GMm}{r^2}\hat{r}$$

$$= 9.81 \times 50\text{N}$$

$$= 491\text{N}$$

Notes:

(i) Weight is a vector quantity. The direction is towards the centre of the earth (or whatever is the nearby greater mass).

(ii) The weight of a body depends on its distance from the centre of the earth, being less the further it is from the centre.

Example:

What is the acceleration due to gravity at a point 30Km above the earth's surface?

Solution:

$$g = \frac{GM}{r^2}$$

$$= \frac{6.7 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 30 \times 10^6}$$

$$= 0.3\text{ms}^{-2}$$

Energy and Work

Definition of Work:

Work is done when a force moves a body.

Work is a scalar quantity and its unit is the Joule (J) *or Newton meter* (Nm).

Mathematically:

$$W = \int_S F \cdot ds$$

In its simplest form, where a constant force F moves a body over a distance x in a straight line, then:

$$W = Fx$$

Definition of the Joule (J)***:***

$1\mathrm{J}$ *is the work done when a force of* $1\mathrm{N}$ *causes a displacement of* $1\mathrm{m}$ *in the same direction as the force.*

i.e. $1\mathrm{J}=1\mathrm{Nm}$.

Definition of Energy:

Energy is the ability to do work.

Energy is stored work and for this reason its units are also the Joule(J).

The Principle of Conservation of Energy *states that energy cannot be created or destroyed -it is only converted from one form to another.*

Forms of Energy:

Energy has many forms: sound, chemical, electrical, mass! (recall: $E = mc^2$) etc.

Two important categories of energy are:

1) Potential Energy
2) Kinetic Energy

Potential Energy:

This is the energy a body has due to its state or position.

For example: a wound up clock, a rock at the edge of a cliff etc.

The potential energy of a body is calculated by calculating the work done by the body in reaching its 'stable' condition. For example, the work done by the rock in falling to the ground. In this case it is easy to calculate the potential energy. We calculate the work done in bringing the rock from the bottom of the cliff to the top. This is work done on the rock. By the Principle of Conservation of Energy the work done on the rock (in bringing it from the bottom of the cliff to the top) equals the work done by the rock (in falling from the cliff top to the ground).

The work done to bring the rock to the top of the cliff is:

$$E = Fs$$
$$= mas$$

Hence the potential energy of the rock at the edge of a cliff is:

$$\text{Potential Energy} = mgh$$

Where h is the height of the cliff, g is the acceleration due to gravity and m is the mass of the rock.

Kinetic Energy:

- This is the energy a body has due to its motion.

Examples include a moving car, falling rain etc.

We calculate the kinetic energy of a body by calculating the amount of work it would do in coming to rest. For example, the work done in bringing a moving car to rest by applying the brakes.

Consider the scenario where a body starts with a velocity u and has a velocity v after time t. Assuming the acceleration is constant then:

$$a = \frac{v-u}{t}$$
$$\Rightarrow v = u + at$$

The displacement is:

$$s = \frac{v+u}{2} \times t$$

But:

$$v = u + at$$

$$\Rightarrow s = \frac{1}{2}(u + at + u) \times t$$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

Beginning again with $v = u + at$ then:

$$v^2 = (u + at)^2$$

$$= u^2 + 2uat + a^2t^2$$

$$= u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$$

$$\Rightarrow v^2 = u^2 + 2as$$

Consider now a car of mass m travelling with a speed u. The brakes are applied and the car comes to rest in a distance s. The work done is:

$$W = Fs$$

But:

$$F = ma$$

$$\Rightarrow W = mas$$

But:

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = u^2 - 2as$$

$$\Rightarrow as = \frac{u^2}{2}$$

$$\Rightarrow W = \frac{mu^2}{2}$$

$$\text{Kinetic Energy} = \frac{1}{2}mu^2$$

Definition of Power:

Power is the rate at which work is done.

Power is a scalar quantity and its unit is the Watt (W).

Definition of the Watt:

The power is $1\mathrm{W}$ *if work is being done at the rate of* $1\mathrm{J}$ *per second i.e.* $1\mathrm{W}=1\mathrm{Js}^{-1}$

Miscellaneous

Definition of Pressure:

The pressure at a point is the force per unit area at that point.

Pressure is a scalar quantity. Its units are the Pascal.

$$1\mathrm{Pa} = 1\mathrm{Nm}^{-2}$$

Mathematically:

$$\boxed{P = \frac{F}{A}}$$

Where F is the magnitude of the force and A is the area over which the force is applied.

Example:

Consider a rectangular tank full of fluid. What is the pressure of the water at the bottom of the tank?

Solution:

$$
\begin{aligned}
P &= \frac{F}{A} \\
&= \frac{W}{A} \\
&= \frac{mg}{A} \\
&= \frac{\rho V g}{a} \\
&= \frac{\rho h A g}{A} \\
&= \rho h g
\end{aligned}
$$

where ρ is the density of the fluid, h is the depth and g is the acceleration due to gravity.

It comes as no surprise that the pressure at the bottom of the tank is a function of these quantities since it is easy to imagine that by varying any of these the pressure will vary in like fashion. For example if we add more fluid to the tank (thereby increasing h) we would expect the pressure at the bottom of the tank to increase etc.