Tutorial Exercises
Week 12
Maclaurin/Taylor Series, Binomial Series

Q1 Taylor Series

Taylor Series

For small x, we can approximate, $f(a+x)$

$$f(a+x) = f(a) + x * f'(a) + \frac{x^2}{2!} * f''(a) + \frac{x^3}{3!} * f'''(a) + \frac{x^4}{4!} * f^{(4)}(a) + \ldots$$

1. Let $f(x) = \frac{1}{x}$, find a Taylor series for $\frac{1}{1+x}$ i.e. $f(1+x)$ (find the series to the first 4 terms.)

Q2 Binomial Series

Binomial Series

$$(1+x)^n = 1 + n*x + \frac{n(n-1)}{2!}*x^2 + \frac{n(n-1)(n-2)}{3!}*x^3 + \dots$$

Find a Binomial series for $(1+x)^{\frac{1}{2}}$ (up to x^3 term).
Hence find an approximation for $\sqrt{2}$ using the following:
$2 = \frac{16}{8} = \frac{16}{9} * \frac{9}{8} = \frac{16}{9} * \left(1+\frac{1}{8}\right) \therefore$
$\sqrt{2} = \sqrt{\frac{16}{9} * \left(1+\frac{1}{8}\right)} = \frac{4}{3}\sqrt{1+\frac{1}{8}} = \frac{4}{3} * \left(1+\frac{1}{8}\right)^{\frac{1}{2}}$.

Q3 Maclaurin Series

Maclaurin Series

$$f(x) = f(0) + x * f'(0) + \frac{x^2}{2!} * f''(0) + \frac{x^3}{3!} * f'''(0) + \dots$$

Find a Maclaurin series for $tan\,x$
(find the series to the term involving x^3)

Note:
$\frac{d}{dx} tan\,x = sec^2 x$
$\frac{d}{dx} sec\,x = (sec\,x) * (tan\,x)$
$tan^2 x + 1 = sec^2 x$