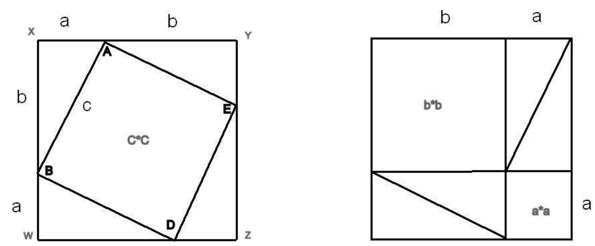
Pythagoras' Theorem

The following diagrams are equal as both are equal to $(a+b)^2 = a^2 + 2*a*b + b^2$.

i.e. Left $= c^2 + 4*\frac{1}{2}*a*b$ and Right $= a^2 + b^2 + 4*\frac{1}{2}*a*b$,

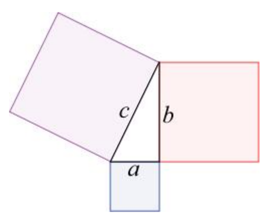
i.e. $c^2 + 4*\frac{1}{2}*a*b = a^2 + b^2 + 4*\frac{1}{2}*a*b$

i.e. $c^2 = a^2 + b^2$



Pythagoras Theorem (Standard Diagram)

Standard Diagram for Pythagoras' Theorem



Trigonometry version of Pythagoras' Theorem:

Let θ be the angle between lines c and a then

$cos\,\theta = \frac{a}{c}$ and $sin\,\theta = \frac{b}{c}$

From Pythagoras' Theorem $a^2 + b^2 = c^2$

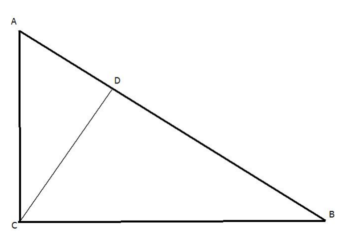
tf.

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

i.e.

$$cos^2\theta + sin^2\theta = 1$$

Similar Triangles

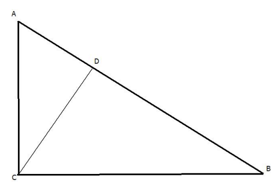


Consider the right angled triangle $\triangle ACB$ with a perpendicular constructed from C to D.

The angles $\angle ACB$ and $\angle ADC$ are right angles and so are so are equal. The angles $\angle CAB$ and $\angle CAD$ are the same. The remaining angles $\angle CBA$ and $\angle DCA$ are thus equal.

Since the angles are equal, the triangles $\triangle ACB$ and $\triangle ADC$ are similar and therefore corresponding sides are proportional.

Similar Triangles Cont'd



Similarly, The angles$\angle ACB$ and $\angle ADB$ are right angles and so are equal. The angles $\angle CBA$ and $\angle CBD$ are the same. The remaining angles $\angle CAB$ and $\angle DCB$ are thus equal.

Since the angles are equal, the triangles $\triangle ACB$ and $\triangle CDB$ are similar and therefore corresponding sides are proportional.

[Euclid's Elements: http://aleph0.clarku.edu/~djoyce/java/elements/elements.html]

Similar Triangles Cont'd

Using $\cong$ for 'similar',

we have $\triangle ADC \cong \triangle ACB$ and $\triangle ACB \cong \triangle CDB$.

In similar triangles the corresponding sides are proportional.

$$\begin{array}{ccccc} \triangle ADC & \cong & \triangle ACB & \cong & \triangle CDB \\ \frac{AD}{AC} & = & \frac{AC}{AB} & & \\ & & & & \\ & & \frac{CB}{AB} & = & \frac{DB}{CB} \end{array}$$

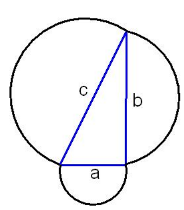
i.e. $AD * AB = AC^2$ and $CB^2 = AB * DB$.

tf. $AC^2 + CB^2 = AB * (AD + DB)$

i.e. $AC^2 + CB^2 = AB^2$ since $AB = AD + DB$

Pythagoras and Circles

Is Pythagoras' Thm true if 'squares' are replaced by 'semi-circles'?



Area of semi-circle on c = $\frac{\pi c^2}{8}$;
Area of semi-circle on b =$\frac{\pi b^2}{8}$; Area of semi-circle on a = $\frac{\pi a^2}{8}$;
From Pythagoras' Thm:

$$c^2 = b^2 + a^2$$

tf.

$$\frac{\pi c^2}{8} = \frac{\pi b^2}{8} + \frac{\pi a^2}{8}$$

Pythagorean Triples

The triple of natural numbers, a, b and c are Pythagorean when

$$a^2 + b^2 = c^2$$

e.g. 3, 4 and 5 as $3^2 + 4^2 = 5^2$.

More generally, if u and v are natural numbers with $u > v$, then the 3 numbers

$a = 2 * u * v$,
$b = u^2 - v^2$, and
$c = u^2 + v^2$
form a Pythagorean Triple.
$a^2 + b^2$
$= (2 * u * v)^2 + (u^2 - v^2)^2$
$= 4 * u^2 * v^2 + u^4 - 2 * u^2 * v^2 + v^4$
$= u^4 + 2 * u^2 * v^2 + v^4$
$= (u^2 + v^2)^2$
$= c^2$

Example

Example 1. Let $u = 5$ and $v = 3$ then
$30 = 2 * 5 * 3$, $16 = 5^2 - 3^2$ and $34 = 5^2 + 3^2$
tf. $30^2 + 16^2 = 34^2$
i.e. $900 + 256 = 1156$.