Week #9 Exercises

Qs. #1

1. Simplify (without a calculator)

$$\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1}$$

2. $(+k : low \leq k \leq high \wedge P(k) : f(k)) =$
   $g(low) + g(low + 1) + \cdots + g(high)$
   where $g(k) = if\ P(k)\ then\ f(k)\ else\ 0$
   Example: ($\mathbb{N}$ is the set of Natural numbers)
   $(+k \in \mathbb{N} : 1 \leq k \leq 9 \wedge odd(k) : k) = 1 + 3 + 5 + 7 + 9 = 25$
   Calculate:
   $(+k \in \mathbb{N} : 0 \leq k \leq 20 \wedge prime(k) : k)$
   where
   $prime(k)$ iff k has exactly two divisors, itself and 1.

Qs. #2

The functions, (*a div b*) and (*a mod b*) are defined so that, for $b \neq 0$,

$$a = b * (a\ div\ b) + (a\ mod\ b)\ \wedge\ 0 \leq (a\ mod\ b) < |b|$$

Determine

- 123 *div* 10 and 123 *mod* 10
  i.e. determine 123 *div* 10 and 123 *mod* 10 so that
  $123 = 10 * (123\ div\ 10) + (123\ mod\ 10)\ \wedge$
  $0 \leq (123\ mod\ 10) < 10$
  is satisfied.

- (−123) *div* 10 and (−123) *mod* 10
  i.e. determine (−123) *div* 10 and (−123) *mod* 10 so that
  $123 = 10 * ((-123)\ div\ 10) + ((-123)\ mod\ 10)\ \wedge$
  $0 \leq ((-123)\ mod\ 10) < 10$
  is satisfied.

Qs. 2 Cont'd

- 123 *div* (−10) and 123 *mod* (−10)
  i.e. determine 123 *div* (−10) and 123 *mod* (−10) so that
  $123 = 10 * (123 \, div \, (−10)) + (123 \, mod \, (−10)) \land$
  $0 \leq (123 \, mod \, (−10)) < 10$
  is satisfied.

- (−123) *div* (−10) and (−123) *mod* (−10)
  i.e. determine (−123) *div* (−10) and (−123) *mod* (−10) so
  that
  $−123 = (−10) * ((−123) \, div \, (−10)) + ((−123) \, mod \, (−10)) \land$
  $0 \leq ((−123) \, mod \, (−10)) < 10$
  is satisfied.