Week #8 Exercise

Ex: 1

Show by KE Deduction or otherwise

1. $(P \rightarrow Q), (\neg P \rightarrow Q) \vdash Q$
2. $P \rightarrow Q \rightarrow R \vdash (P \rightarrow Q) \rightarrow P \rightarrow R$
3. $P \vee Q, P \rightarrow R, Q \rightarrow S \vdash R \vee S$

Ex: 2

Determine by KE Deduction or otherwise whether the following argument is valid.

The only people in the mansion were the butler and the maid.
If the only people in the mansion were the butler and the maid, then the butler or the maid did it.
If the maid did it, then she had a motive.
The maid did not have a motive,
therefore
the butler did it.

P: *The only people in the mansion were the butler and the maid.*
Q: *The maid did it.*
R: *The maid had a motive*
S: *The butler did it.*

$$\frac{P,\ P \rightarrow Q \vee S,\ Q \rightarrow R,\ \neg R}{S}$$

KE Deduction Rules

For convenience and reference, the KE Deduction rules are divided into categories.

α (alpha) Rules

$$\frac{P \wedge Q}{\begin{array}{c} P \\ Q \end{array}} \qquad \frac{\neg(P \vee Q)}{\begin{array}{c} \neg P \\ \neg Q \end{array}} \qquad \frac{\neg(P \rightarrow Q)}{\begin{array}{c} P \\ \neg Q \end{array}} \qquad \frac{\neg\neg P}{P}$$

KE Deduction Rules (Cont'd)

β (beta) Rules

$$\frac{P \vee Q \quad \neg P}{Q} \qquad \frac{\neg(P \wedge Q) \quad P}{\neg Q} \qquad \frac{P \rightarrow Q \quad P}{Q} \qquad \frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

Branching Rule, B :