Analysis of Steady State Flow in Systems

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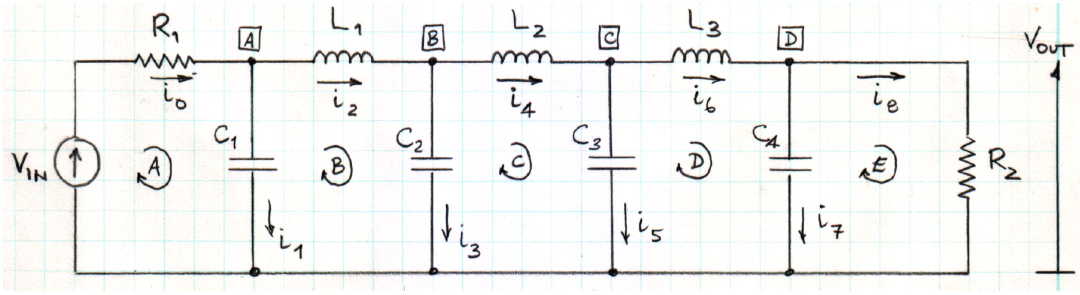

Figure 1: Chebyshev Filter Circuit

To analyse the circuit provided in Figure 1, Mesh and Nodal Analysis techniques are used. Mesh Analysis is derived from Kirchoff's Voltage Law which states that 'the directed sum of the electrical potential differences (voltage) around any closed network is zero'. Nodal Analysis is derived form Kirchoff's Current Law which states that 'at any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node'. Both tools are required to derive the nine equations in order to solve for the unknowns, ω and V_{IN}, in the system. The five Mesh equations (equations 1 to 5) and four Node equations (equations 6 to 9) were found to be

$$\text{A} \quad V_{IN} - Z_{R1}i_0 - Z_{C1}i_1 = 0 \tag{1}$$

$$V_{IN} = Z_{R1}i_0 + Z_{C1}i_1$$

$$\text{B} \quad Z_{C1}i_1 - Z_{L1}i_2 - Z_{C2}i_3 = 0 \tag{2}$$

$$\text{C} \quad Z_{C2}i_3 - Z_{L2}i_4 - Z_{C3}i_5 = 0 \tag{3}$$

$$\text{D} \quad Z_{C3}i_5 - Z_{L3}i_6 - Z_{C4}i_7 = 0 \tag{4}$$

$$\text{E} \quad Z_{C4}i_7 - Z_{R2}i_8 = 0 \tag{5}$$

$$\boxed{A} \quad i_0 - i_1 - i_2 = 0 \tag{6}$$

$$\boxed{B} \quad i_2 - i_3 - i_4 = 0 \tag{7}$$

$$\boxed{C} \quad i_4 - i_5 - i_6 = 0 \tag{8}$$

$$\boxed{D} \quad i_6 - i_7 - i_8 = 0 \tag{9}$$

These equations can then be compiled into matrix form in order to make use of software tools.

$$
\begin{bmatrix}
Z_{R1} & Z_{C1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z_{C1} & -Z_{L1} & -Z_{C2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z_{C2} & -Z_{L2} & -Z_{C3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Z_{C3} & -Z_{L3} & -Z_{C4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{C4} & -Z_{R2} \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1
\end{bmatrix}
*
\begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}
=
\begin{bmatrix} V_{IN} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Which becomes

$$
\begin{bmatrix}
R_1 & \frac{-j}{\omega C_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-j}{\omega C_1} & -j\omega L_1 & \frac{j}{\omega C_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-j}{\omega C_2} & -j\omega L_2 & \frac{j}{\omega C_3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-j}{\omega C_3} & -j\omega L_3 & \frac{j}{\omega C_4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-j}{\omega C_4} & -R_2 \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1
\end{bmatrix}
*
\begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}
=
\begin{bmatrix} V_{IN} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

3 A Water-Supply Network