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Week 1

John Machin

John Machin was a professor of astronomy at Gresham College, London and is best known for developing a quickly converging series for Pi in 1706 and using it to compute Pi to 100 decimal places. The formula is:

$$\frac{\pi}{4} = 4arctan\frac{1}{5} - arctan\frac{1}{239}$$

The benefit of the new formula, a variation on the Gregory/Leibniz series (Pi/4 = arctan 1), was that it had a significantly increased rate of convergence, which made it a much more practical method of calculation. (https://en.wikipedia.org/wiki/John_Machin)

Taylor Series

The Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. The concept of a Taylor series was formulated by the Scottish mathematician James Gregory and formally introduced by the English mathematician Brook Taylor in 1715. If the Taylor series is centered at zero, then that series is also called a Maclaurin series, named after the Scottish mathematician Colin Maclaurin, who made extensive use of this special case of Taylor series in the 18th century. (https://en.wikipedia.org/wiki/Taylor_series)

$$arctanx = \sum_{n=0}^{\infty} \frac{(-1)^{-n}}{2n+1} x^{2n+1} \qquad \text{for} |x| \leq 1, x \pm i$$

Approximation for π

Following the Taylor Series from 0 to 2 for Machin's Formula the following is obtained:

$$\frac{\pi}{4} = (4\frac{1}{5} - \frac{1}{239} - 4\frac{1}{375} + \frac{1}{40955757} + 4\frac{1}{15625} - 2.56 \times 10^{-13})$$

The series is dominated by the first term but converges quickly.

Machin1.py

<u>Code from the shell:</u>
Begin program Machin1...
('Truncated series estimate is ', 3.1416210293250346)
('Math library estimate is ', 3.141592653589793)
('Difference is ', 2.837573524150372e-05)
End of program Machin1.

The approximation of π differs to the Math Library estimate, on the fifth significant figure which is not accurate due as the computer can calculate this value to a much higher precision. This occurs as the approximation is only calculated to the first 3 terms of the series. Higher accuracy can be achieved by increasing the number of terms.

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Machin2.py

$$arctanx = \sum_{n=0}^{k} \frac{(-1)^{-n}}{2n+1} x^{2n+1} \qquad \text{for} |x| \leq 1, x \pm i$$

Begin program Machin2...
Enter a value for the number of terms: 10
('Truncated series estimate is ', 3.1415926535897922, ' for n=', 10)
('Math library estimate is ', 3.141592653589793)
('Difference is ', -8.881784197001252e-16)
End of program Machin2.

Questions

Question 1

help(machin)
Help on function machin in module __main__:

machin(n)
    Computes pi using Machin's formula.

Question 2

>>> import math
>>> help(math.fabs)
Help on built-in function fabs in module math:

fabs(...)
    fabs(x)

    Return the absolute value of the float x.

Question 3

math.fabs(math.pi - machin(10))
8.881784197001252e-16

Question 4

Table 1: Approximation of π using Machin's formula

No. of Terms	3	5	7	9	11	13	15
Error	$2.838e^{-05}$	$2.881e^{-08}$	$3.376e^{-11}$	$4.308e^{-14}$	$8.882e^{-16}$	$8.882e^{-16}$	$8.882e^{-16}$

Question 5

Table 2: Approximation of π using an alternative formula

No. of Terms	3	5	7	9	11	13	15
Error	1.564	1.564	1.564	1.564	1.564	1.564	1.564

Week 2

Iterative Algorithms

1

Box Number	Decision / input / output	N	g
1	Start…	?	?
2	Input N as 27	27.0	
3			1.0
4	$\lvert g^3 - N \rvert > 0.01$ is true	27	
5	g <— 1/3 (N/g^2 + 2g)		9.6667
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		15.444
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		19.296
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		21.864
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		23.576
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		24.717
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		25.478
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		25.986
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		26.324
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		26.549
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		26.699
4	$\lvert g3 - N \rvert > 0.01$ is true		
5	g <— 1/3 (N/g^2 + 2g)		26.8

This trend continues until g converges to 26.9999. This is due to block 4. A more appropriate block 4 would be “while abs(g - N) >= 0.01” and this would then terminate after 20 loops.

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Week 3

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Week 4

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Week 5

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Week 6

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Week 7

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Week 8

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Week 9

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Week 10

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Week 11

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Week 12