MECH2700 Engineering Analysis I, 2015
Exercise Sheet 5

Data processing and solution of linear systems of equations

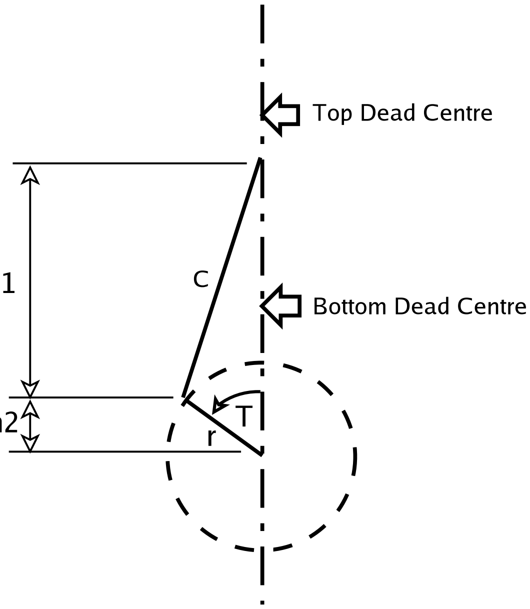
1. As part of the traditional Thermodynamics course, there is an experiment on "The Performance of a Diesel Engine". Pressure and crank angle are recorded for a single cycle of the engine and are written to a file with three values to a line. The first value is the sample number and is unimportant. The second value is the crank-angle signal and the third value is the voltage output from the pressure transducer.

   Write a Python program that takes the LabView output file (output.lab) containing sampled values for the crank angle and pressure transducer signals and computes (i) the crank angle, θ, in radians, (ii) the cylinder volume, V, in m^3 and (iii) the pressure, P, in kPa. You can get this file from the course Blackboard page as the text file `output.lab`

   Write the transformed data to a file called "pv.dat" and plot both the pressure versus crank angle and a P-V diagram. A second phase of your program should integrate the PV data to obtain the work done by the gas during the compression stroke ($\pi < \theta \leq 2\pi$) and the expansion (power) stroke ($2\pi < \theta \leq 3\pi$). To integrate the sampled data, use

   $$W = \int P \, dV \approx \sum \left[\frac{1}{2}(P_i + P_{i-1})(V_i - V_{i-1}) \right]$$

   where subscript i refers to the current sample point and $i-1$ refers to the previous sample point.

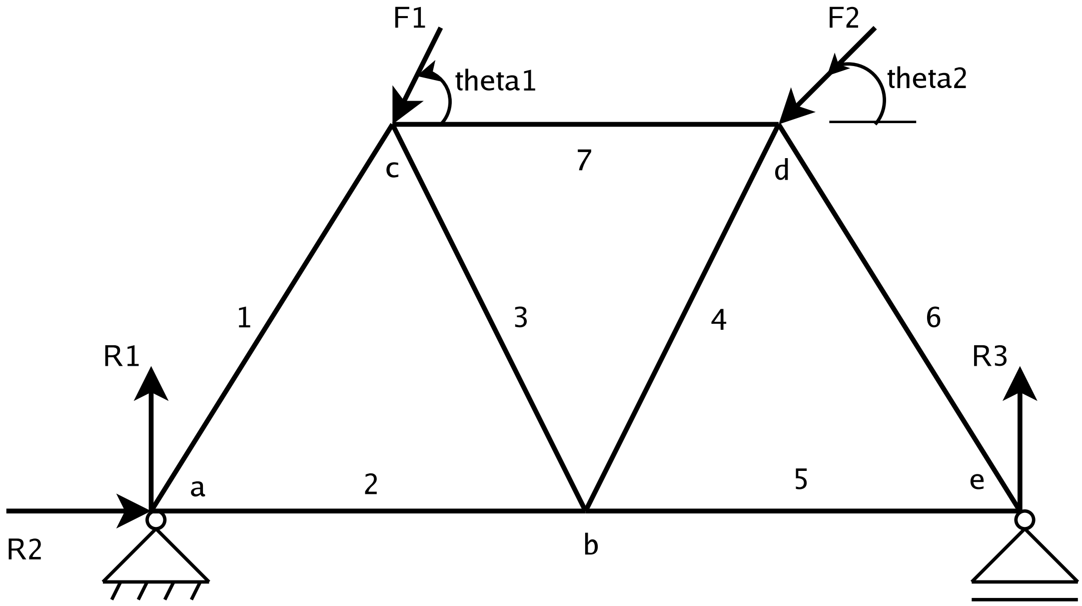


The pressure transducer responds linearly with a sensitivity of 1185 kPa/Volt. The crank-angle signal maps linearly from the range $-5 \leq s \leq 15$ to crank angle in radians $-\pi \leq \theta \leq 3\pi$. The volume inside the cylinder is given as

$$V = V_1 + A\left[r + C - h_1 - h_2\right]$$

where
T is crank angle
$h_1 = r \cos T$
$h_2 = \sqrt{c^2 - (r \sin T)^2}$
L is the stroke, 0.075 m
D is the cylinder bore, 0.080 m
C is the connecting-rod length, 0.133 m
A is the area of the cylinder cross-section, $\pi D^2/4$
r is the crank radius, $L/2$
$V_2 - V_1$ is the swept volume, $A \times L$
V_2/V_1 is the compression ratio, 21.2:1

2. The planar truss shown below has 7 equal-length members (labelled 1 through 7) and 5 pin joints (labelled a through e). The reaction forces at the supports are shown as R_1, R_2 and R_3.



With the external loads F_1 and F_2, write the equations of static equilibrium of the truss by considering vertical and horizontal forces at each pin. Denote the tension in the members as T_1 through T_7 (with a negative value indicating that the member is in compression).

3. Write your set of linear equations for the equilibrium of the truss in standard (matrix) form $\mathbf{A}\vec{x} = \vec{y}$ with the vector of unknowns being

$$\vec{x} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{bmatrix}$$

4. Write a Python program that solves your system of linear equations using a direct solver (such as Gauss-Jordan elimination) for the external loads $|F_1| = 10\,\mathrm{kN}$, $|F_2| = 7\,\mathrm{kN}$, $\theta_1 = 75^o$ and $\theta_2 = 45^o$. You may find that it is convenient to use the "numpy" module and its "matrix" object to solve the matrix equation. For this exercise, you are to write your own linear system solver, as shown in lectures.

5. Write a Python program to solve the equations from question 2 using Gauss-Seidel iteration.