

- 
1. **[20 points]** Let  $S = \mathbb{R} \setminus \{0\}$ , with  $f : S \rightarrow S$ ,  $f(x) = x^2$ , and  $g : S \rightarrow \mathbb{R}$ ,  $g(x) = \ln |x|$ .
- (a) Explain why  $f$  is not onto  $S$ .
  - (b) Prove that  $g$  is onto  $\mathbb{R}$ .
  - (c) Write out an expression for  $g \circ f : S \rightarrow \mathbb{R}$ .
  - (d) Prove that  $g \circ f$  is onto  $\mathbb{R}$ .
  - (e) In class we proved the theorem: “If  $f$  is onto and  $g$  is onto, then  $g \circ f$  is onto.” In this example  $g \circ f$  is onto, but  $f$  and  $g$  are not both onto. Does this contradict the theorem? Explain why or why not.