INSTRUCTIONS: Find the determinant of the following matrices. Please show all work (minimal credit will be given to correct answers stated without supporting evidence).

(1) [16 points] Let $A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$. Find the characteristic equation, eigenvalues, and a basis for each eigenvalue.

Solution. Since

$$\begin{vmatrix} 5-\lambda & -6 \\ 3 & -4-\lambda \end{vmatrix} = (5-\lambda)(-4-\lambda) - (-18) = \lambda^2 - \lambda - 20 + 18 = \lambda^2 - \lambda - 2 = (\lambda-2)(\lambda+1), \qquad \text{(6 points)}$$

it follows that the characteristic equation is

$$\lambda^2 - \lambda - 2 = (\lambda-2)(\lambda+1) = 0. \qquad \text{(2 points)}$$

Thus, $\sigma(A) = \{-1, 2\}$.

Since

$$A + I = \begin{bmatrix} 6 & -6 \\ 3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix},$$

it follows that a basis for E_{-1} is

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \qquad \text{(4 points)}$$

and since

$$A - 2I = \begin{bmatrix} 3 & -6 \\ 3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix},$$

it follows that a basis for E_2 is

$$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}. \qquad \text{(4 points)}$$

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