Assignment 3	STMATH 300	Spring 2018

Due Thursday, April 19 before 5:00 pm

For all of the following, use an appropriate technique to prove or disprove the provided statements.

1. Prove or disprove: For all integers n, if $3 \mid (2n + 2)$, then $n \equiv_3 2$.
2. Prove or disprove: For all integers n, $n^2 \equiv_5 4$ if and only if $n \equiv_5 2$.
3. Prove or disprove: Let a, b and c be real numbers. Then, $a + b + c$ is irrational if **at least** one of a, b or c is irrational.
4. Let t, u and v be integers such that $t^2 + u^2 = v^2$. Prove that at least one of t or u is divisible by 3. (Hint: use contradiction).
5. (a) Let a and b be integers, and let d be odd. Prove that $d \mid a$ and $d \mid b$ if and only if $d \mid (a + b)$ and $d \mid (a - b)$.

   (b) Is the statement true when d is even? If so, prove it; if not, provide a counterexample.
6. Prove that $\displaystyle\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for all positive integers n.
7. Prove that $6 \mid (n^3 - n)$ for all positive integers n.
8. [**Bonus: Up to +10 points**]: The Fibonacci numbers are a famous recursive sequence defined as:

   $$f_1 = 1, \quad f_2 = 1, \quad f_n = f_{n-1} + f_{n-2} \text{ for } n > 2$$

   The first twenty Fibonacci numbers are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765.

   Use induction to prove the following: For all $n \geq 1$, f_{6n} (i.e., f_6, f_{12}, f_{18}, $\ldots$) is divisible by 8.

   Note: although this is a recursively defined sequence, the proof can be done using only the FPMI.

Note: While solutions will be provided to all of these problems, not all problems may be fully graded.