(**1**) Let

$$A = \begin{array}{c} \begin{matrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{a}_4 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ -2 & 1 & 1 & -2 \\ -4 & 1 & 1 & -4 \end{bmatrix} \end{array} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

(**i**) Find an explicit description for any general solution of $[\ A \mid \vec{b}\ ]$.

(**ii**) Find the *reduced echelon form* of A.

(iii) Does the linear system $[\ A \mid \vec{b}\ ]$ have a solution for every $\vec{b} \in \mathbb{R}^4$?

(iv) Find a basis for $\mathrm{Col}(A)$ and determine $\dim(\mathrm{Col}(A))$.

(v) Find a basis for $\mathrm{Nul}(A)$ and determine $\dim(\mathrm{Nul}(A))$.

(**vi**) Without using a cofactor expansion or row-reduction, explain why $\det(A) = 0$.

(**2**) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ -1 & 3 & 4 \end{bmatrix}$.

(**i**) Determine whether A is invertible and, if so, find A^{-1}.

(ii) Find the characteristic polynomial of A and the eigenvalues of A. *Hint*: Expand $\det(A - \lambda I)$ across the first row.

(iii) Find a basis for the eigenspace corresponding to each of the eigenvalues of A.

(**3**) Find $\det(A)$ if

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & -1 & 0 \\ 1 & 0 & -1 & -1 \\ 0 & -1 & 3 & -1 \end{bmatrix}.$$

(**4**) Show that

$$H = \{\vec{x} \in \mathbb{R}^3 : x_1 - 2x_2 - x_3 = 0\}.$$

is a subspace of $\mathbb{R}^3$.

(**5**) Let K be the subset of $\mathbb{R}^3$ containing points of the form

$$\begin{bmatrix} -1 \\ x_2 \\ 0 \end{bmatrix}.$$

Show that K is <u>not</u> a subspace of $\mathbb{R}^3$.