Lecture 5

Tuesday, April 10, 2018 8:45 AM

* Announcements

* Office-hours: 11am-12pm, T, W, Th or by appt. (15-20mins).
* Lowest quiz score will be dropped (does not apply for unexcused absence).
* Tutoring available through QSC
  - math.stackexchange.com

§1.3 Q/A

#11 $\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \vec{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

Sol'n: Solve:

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 3 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

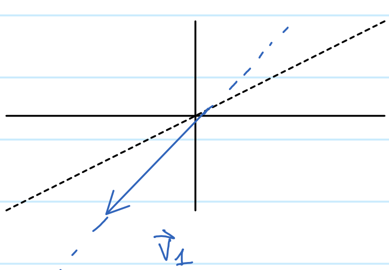
Yes; all solutions are of the form

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5x_3 + 2 \\ -4x_3 + 3 \\ x_3 + 0 \end{bmatrix} = \begin{bmatrix} -5x_3 \\ -4x_3 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -5 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad x_3 \in \mathbb{R}.$$

#19. Give a geometric description of Span(v_1, v_2) with $\vec{v}_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$.

Sol'n:



Notice that $\vec{v}_2 = \frac{3}{2}\vec{v}_1$. If $y \in \text{Span}(v_1, v_2)$, then there are scalars $x_1, x_2 \in \mathbb{R}$ s.t

$$y = x_1\vec{v}_1 + x_2\vec{v}_2$$
$$= x_1\vec{v}_1 + x_2\left(\tfrac{3}{2}\vec{v}_1\right)$$
$$= \left(x_1 + \frac{3x_2}{2}\right)\vec{v}_1 \in \text{Span}(\vec{v}_1).$$

Section 1.3 Q/A

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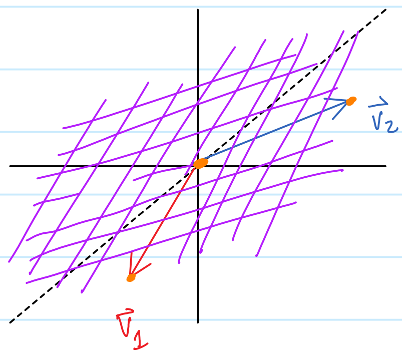
#15. List 5 vectors in Span(v_1, v_2), $\vec{v}_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$.



Sol'n:

$\vec{v}_1$, $(1,0)$

$\begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$, $(1,1)$

$\vec{v}_2$, $(0,1)$

$\vec{0}$, $(0,0)$

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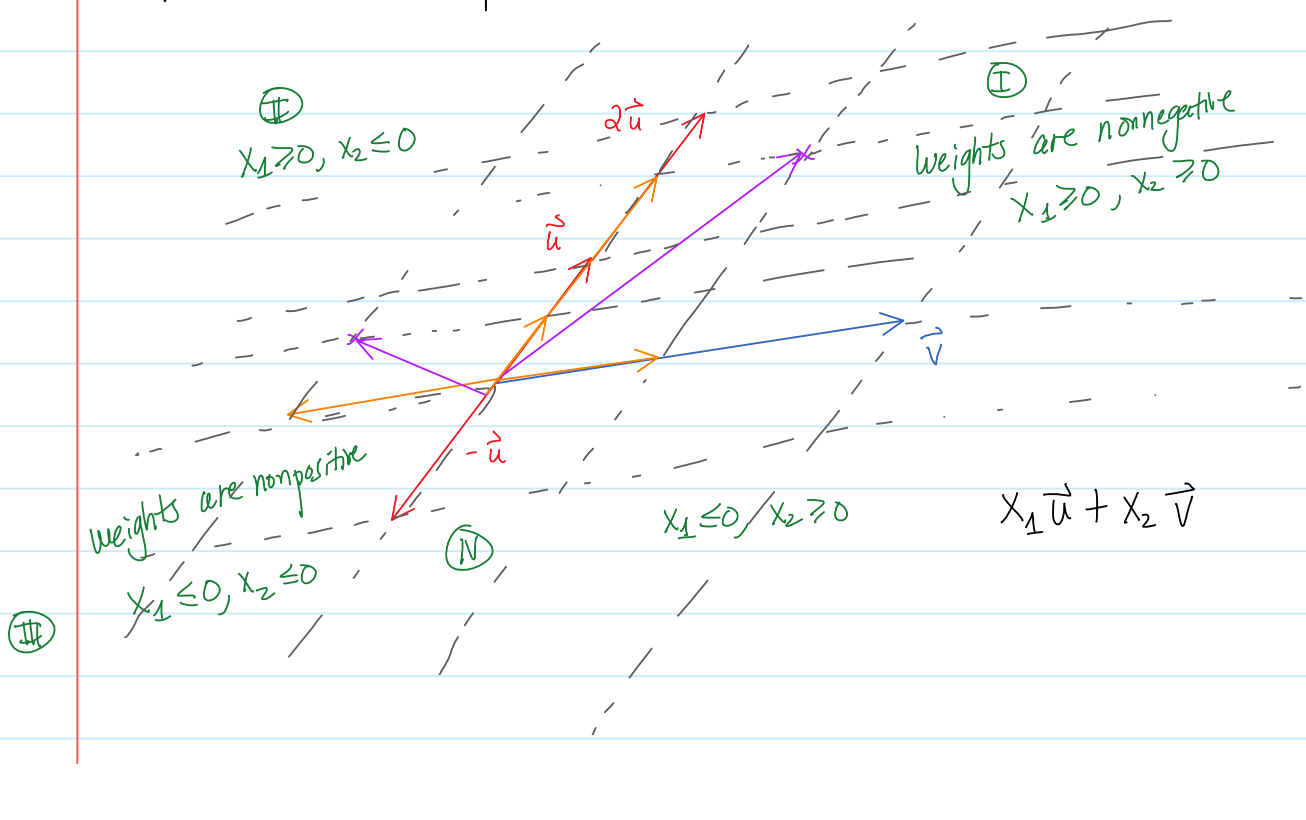
§1.3 (continued)

Recall: If $\vec{v}_1, \dots, \vec{v}_p \in \mathbb{R}^n$, then the span of $\vec{v}_1, \dots, \vec{v}_p$, denoted by Span$(\vec{v}_1, \dots, \vec{v}_p)$, is the set of all possible linear combinations (i.e, weighted sums) of the vectors $\vec{v}_1, \dots, \vec{v}_p$.

Eg. Suppose $\vec{u}, \vec{v} \in \mathbb{R}^2$ in which $\vec{u}$ and $\vec{v}$ are nonzero and $\vec{v}$ is not a scalar multiple of $\vec{u}$. What does span$(\vec{u}, \vec{v})$ look like?



Properties of Span

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Properties:

(i) $\vec{v}_i \in \text{span}(\vec{v}_1, \ldots, \vec{v}_i, \ldots, \vec{v}_p)$.

Pf: If we use an ordered n-tuple, the corresponding weight is $(0, \ldots, \underset{i^{th}\text{-position.}}{1}, \ldots, 0)$.

(ii) $\vec{0} \in \text{span}(\vec{v}_1, \ldots, \vec{v}_p)$

Pf: Select all weights to be zero. In particular,

$$\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + \cdots + 0\vec{v}_p.$$

§1.4 The Matrix Equation $A\vec{x} = \vec{b}$.

Def'n: If $A = [\vec{a}_1 | \vec{a}_2 | \cdots | \vec{a}_n]$ is an $m \times n$ matrix and $\vec{x} \in \mathbb{R}^n$, then the product of A and $\vec{x}$, denoted by $A\vec{x}$, is defined by

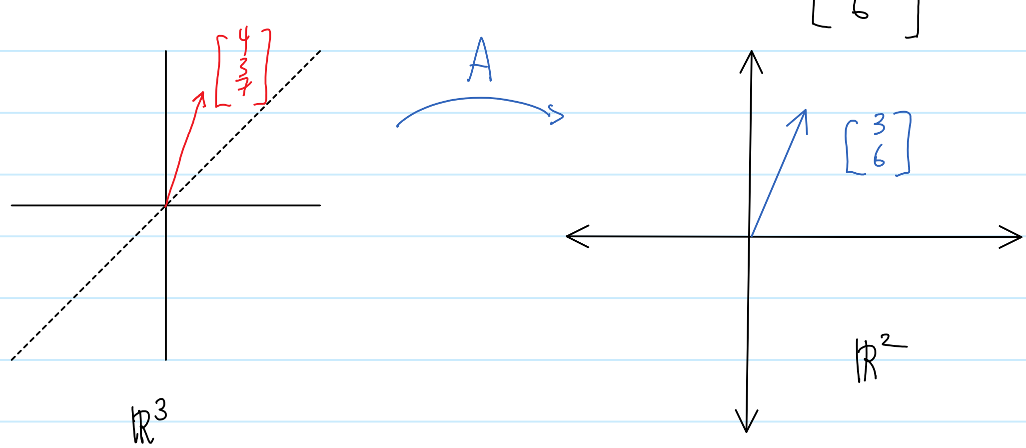
$$A\vec{x} = [\vec{a}_1 | \vec{a}_2 | \cdots | \vec{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n = \sum_{i=1}^{n} x_i \vec{a}_i.$$

Remark: $A\vec{x}$ is defined only if the number of columns of A equals the number of entries of $\vec{x}$.

E.g. Compute

(a)
$$\begin{array}{c} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix} \\ \vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3 \end{array} \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 2 \\ -5 \end{bmatrix} + 7\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ -15 \end{bmatrix} + \begin{bmatrix} -7 \\ 21 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \end{bmatrix}.$$



Matrix-vector "product"

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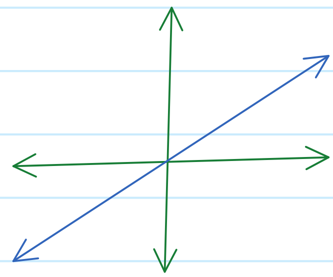
(b) $\begin{bmatrix} 2 & -3 \\ 8 & 0 \\ -5 & 2 \end{bmatrix} \overset{\mathbb{R}^2}{\begin{bmatrix} 4 \\ 7 \end{bmatrix}} = 4\begin{bmatrix} 2 \\ 8 \\ -5 \end{bmatrix} + 7\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 32 \\ -20 \end{bmatrix} + \begin{bmatrix} -21 \\ 0 \\ 14 \end{bmatrix} = \overset{\mathbb{R}^3}{\begin{bmatrix} -13 \\ 32 \\ -6 \end{bmatrix}}.$

$3 \times 2 \quad 2 \times 1$

(c) $\begin{bmatrix} 2 & -3 \\ 8 & 0 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ undefined

$y = ax$



$\vec{y} = A\vec{x}$