The test takes place on Thursday, April 26 in class and will be 120 minutes long. It covers Chapter 0, Chapter 1, and Section 6.2 of the textbook (induction)

Review Session

Instead of my regular office hours on **Wednesday, April 25**, I have booked **DISC-464 from 3:30 – 5:30** for a drop-in review session. Please feel free to bring in any questions, problems, etc. to work over.

List of potential topics

- Basic number theory: divisibility, division algorithm, modular arithmetic.
- Propositional logic: basic operations ($\sim$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$), truth tables, tautologies and contradictions.
- Working with statements involving universal and existential quantifiers, negating statements of this type.
- Proofs using direct arguments, contrapositive, and contradiction. Proofs involving biconditional statements and cases.
- Proofs using the first and second principles of mathematical induction.

Important definitions

divisibility ($a \mid b$), even and odd integers, quotient, remainder, congruency modulo n ($a \equiv_n b$), statement (proposition), negation ($\sim$), conjunction ($\wedge$, and), disjunction ($\vee$, or), conditional statement ($\rightarrow$), sufficient, necessary, biconditional statement ($\leftrightarrow$, if and only if), truth table, converse, inverse and contrapositive statement, tautology, contradiction, logical equivalence, existential and universal quantifier, universe of discourse, direct proof, proof by contraposition, proof by contradiction, first and second principles of mathematical induction, base case, induction hypothesis, induction step.

Practice problems

The following exercises from the text are good practice problems. Solutions to some exercises are provided in the back of the book. If you are unsure about how to do some of these problems, please feel free to come to office hours or send an e-mail.
p. 11 #3,4,5
pp. 22–23 # 3, 4, 6, 7
p. 29 #1–6
p. 32–33 #1, 4
p. 40 #5 – 7
p. 161 #1, 2
I also highly recommend reviewing Quizzes 1–6 and Assignments 1–3. Everything that appears on those quizzes and assignments is potentially examinable.

Reference sheet

A reference sheet identical to the one on the next page will be provided to you on the midterm.
No notes or electronic devices will be permitted.

REFERENCE

Division algorithm

Let a and b be integers with $b > 0$.
Then there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$.

Converse, inverse, and contrapositive

Given a conditional statement of the form $p \to q$, the *converse* statement is $q \to p$, the *inverse* statement is $\sim p \to \sim q$, and the contrapositive statement is $\sim q \to \sim p$.

Useful tautologies

$$\sim (p \vee q) \leftrightarrow [(\sim p) \wedge (\sim q)] \tag{1}$$
$$\sim (p \wedge q) \leftrightarrow [(\sim p) \vee (\sim q)] \tag{2}$$
$$\sim (p \to q) \leftrightarrow [p \wedge (\sim q)] \tag{3}$$
$$\sim (p \leftrightarrow q) \leftrightarrow \{[p \wedge (\sim q)] \vee [q \wedge (\sim p)]\} \tag{4}$$
$$[p \vee (q \wedge r)] \leftrightarrow [(p \vee q) \wedge (p \vee r)] \tag{5}$$
$$[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)] \tag{6}$$
$$\sim [\forall x, p(x)] \leftrightarrow \exists x, \sim p(x) \tag{7}$$
$$\sim [\exists x, p(x)] \leftrightarrow \forall x, \sim p(x) \tag{8}$$