The test takes place on Thursday, May 24 in class and will be 120 minutes long. It covers material from §2.1–2.4, §3.1 – 3.4, and §4.1– 4.2 of the text.

List of potential topics

- Proving statements about sets using element chasing, double inclusion and/or membership equivalence.
- Cartesian products and power sets. Indexed families, unions and intersections of indexed families.
- Understanding the definition of a relation, computing domain and range of a relation, inverse of a relation, image or preimage of a set under a relation.
- Verifying if a relation is an equivalence relation, determining equivalence classes of a relation.
- Determining if a relation is a partial order, finding minimal, maximal, minimum and maximum elements under a partial order, greatest lower bounds and least upper bounds.
- Definition of a function, proving whether a function is one-to-one or onto.

Important definitions

universal set, subset, proper subset, set equality, union, intersection, complement, relative complement ($A \backslash B$), disjoint sets, element chasing, double inclusion, membership equivalence, Venn diagrams, ordered pair, cartesian product, power set, indexed family, index set, relation, domain, range, inverse relation, image, preimage, reflexive, symmetric, transitive, equivalence relation, equivalence class, partition, irreflexive, asymmetric, antisymmetric, partial order, poset, linear order, maximal, maximum, minimal, minimum, (least) upper bound, (greatest) lower bound, product ordering, lexicographic ordering, function, one-to-one, onto, bijection.

Practice problems

The following exercises from the text are good practice problems. Solutions to some exercises are provided in the back of the book. If you are unsure about how to do some of these problems, please feel free to come to office hours or send an e-mail.
p. 54, #4–23
p. 63, # 12, 13, 16, 17
p. 67, #2 – 7
p. 78, #4, 5, 6
p. 88 #5, 6
p. 93, #2, 3
p. 100, #1,2,3,9
p. 113 #4
p. 120 #3–9
I also highly recommend reviewing Quizzes 7 – 12 and Assignments 4–6. Everything that appears on those quizzes and assignments is potentially examinable.

Reference sheet

A reference sheet identical to the one on the next page will be provided to you on the midterm.
No notes or electronic devices will be permitted.

REFERENCE

Useful tautologies

$$\sim (p \vee q) \leftrightarrow [(\sim p) \wedge (\sim q)] \tag{1}$$
$$\sim (p \wedge q) \leftrightarrow [(\sim p) \vee (\sim q)] \tag{2}$$
$$\sim (p \rightarrow q) \leftrightarrow [p \wedge (\sim q)] \tag{3}$$
$$\sim (p \leftrightarrow q) \leftrightarrow \{[p \wedge (\sim q)] \vee [q \wedge (\sim p)]\} \tag{4}$$
$$[p \vee (q \wedge r)] \leftrightarrow [(p \vee q) \wedge (p \vee r)] \tag{5}$$
$$[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)] \tag{6}$$
$$\sim [\forall x, p(x)] \leftrightarrow \exists x, \sim p(x) \tag{7}$$
$$\sim [\exists x, p(x)] \leftrightarrow \forall x, \sim p(x) \tag{8}$$

Set theory

Let U be a universal set and let A and B be subsets of U. Then,

$$A \subseteq B \text{ if } \forall x \in U, x \in A \Rightarrow x \in B \tag{9}$$
$$A = B \text{ if } \forall x \in U, x \in A \Leftrightarrow x \in B \tag{10}$$
$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\} \tag{11}$$
$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\} \tag{12}$$
$$\overline{A} = \{x \in U \mid x \notin A\} \tag{13}$$
$$A \backslash B = \{x \in U \mid x \in A \text{ and } x \notin B\} \tag{14}$$
$$\mathcal{P}(A) = \{S \mid S \subseteq A\} \tag{15}$$
$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\} \tag{16}$$

Relations

Let $R \subseteq A \times B$ be a relation from A to B. Let $S \subseteq A$ and $T \subseteq B$. Then,

$$\text{Dom}(R) = \{a \in A \mid \exists b \in B, (a, b) \in R\} \tag{17}$$
$$\text{Ran}(R) = \{b \in B \mid \exists a \in A, (a, b) \in R\} \tag{18}$$
$$R^{-1} = \{(b, a) \in B \times A \mid (a, b) \in R\} \tag{19}$$
$$R(S) = \{a \in S \mid \exists b \in B, (a, b) \in R\} \tag{20}$$
$$R^{-1}(T) = \{b \in T \mid \exists a \in A, (a, b) \in R\} \tag{21}$$

Let R be a relation on A. Then,

R is reflexive if $\forall a \in A, (a, a) \in R$.
R is symmetric if $\forall a, b \in A, (a, b) \in R \Rightarrow (b, a) \in R$.
R is transitive if $\forall a, b, c \in A, (a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.
R is irreflexive if $\forall a \in A, (a, a) \notin R$.
R is asymmetric if $\forall a, b \in A, (a, b) \in R \Rightarrow (b, a) \notin R$.
R is antisymmetric if $\forall a, b \in A, (a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$.
R is an equivalence relation if it is reflexive, symmetric and transitive.
R is a partial order if it is reflexive, antisymmetric and transitive.

Let $(A, \prec)$ be a poset, and let $B \subseteq A$. Then,

a is maximal if $\nexists b \in A \backslash \{a\}, a \prec b$.
a is a maximum if $\forall b \in A, b \prec a$.
a is minimal if $\nexists b \in A \backslash \{a\}, b \prec a$.
a is a minimum if $\forall b \in A, a \prec b$.
a is an upper bound on B if $\forall b \in B, b \prec a$. It is the least upper bound if $a \prec c$ for all upper bounds c.
a is a lower bound on B if $\forall b \in B, a \prec b$. It is the greatest lower bound if $c \prec a$ for all lower bounds c.

Let $f : A \rightarrow B$. Then,

f is one-to-one if $f(x_1) = f(x_2) \rightarrow x_1 = x_2$.
f is onto B if $\text{Ran}(f) = B$, or in other words, $\forall y \in B, \exists a \in A, f(a) = y$.