(1) Solve the following linear systems:

(a) $$\left\{\begin{array}{rcrcr} x_1 & - & 4x_2 & = & 6 \\ 2x_1 & + & 5x_2 & = & -1 \end{array}\right.$$

(b) $$\left\{\begin{array}{rcrcrcr} 2x_1 & + & x_2 & - & x_3 & = & 0 \\ -4x_1 & - & x_2 & - & 5x_3 & = & 0 \end{array}\right.$$

(2) Determine, by solving a linear system, if the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are linearly independent or linearly dependent.

(3) <u>Without solving a linear system</u>, determine whether the following sets are linearly independent or dependent (please provide adequate justification):

(a) $\left\{ \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

(4) Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} -20 \\ 25 \\ 24 \end{bmatrix}$.

(a) Is $\vec{b}$ a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$? If so, find scalars x_1, x_2, and x_3 such that

$$x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}.$$

(b) Let $A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix}$. If $\vec{b} \in \mathbb{R}^3$, is $A\vec{x} = \vec{b}$ consistent?

(5) Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ satisfying $T(\vec{e}_1) = \vec{e}_1 - 2\vec{e}_2$, $T(\vec{e}_2) = 4\vec{e}_3 - 2\vec{e}_2$, and $T(\vec{e}_3) = 5\vec{e}_3$, where $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ are the columns of the 3×3 identity matrix.