Due Thursday, April 5, by 5:00 pm

1. Use direct arguments to prove the following statements.
   (a) Let $n \in \mathbb{Z}$. Prove that $n^2 + n + 1$ is odd.
   (b) Let a and b be integers such that $a \mid b$. Prove that $a^2 \mid b^2$.
   (c) Let a, b, and c be integers such that $a \mid b$ and $a \mid c$. Prove that $a \mid (bx + cy)$ for all $x, y \in \mathbb{Z}$.
2. Find the quotient q and remainder r when a is divided by b in each of the following cases:
   (a) $a = 142$, $b = 15$
   (b) $a = 15$, $b = 142$
   (c) $a = -76$, $b = 8$
3. Use modular arithmetic to determine whether $8^{13} + 13^8$ is divisible by:
   (a) 3
   (b) 7
4. A well-known rule for divisibility states that a is divisible by 11 if the **alternating** sum of a's digits is also divisible by 11. For example, $198,836$ is divisible by 11 since $1 - 9 + 8 - 8 + 3 - 6 = -11$, and $11 \mid -11$. (In this case, $198,836 = 11 \times 18,076$).

   Use modular arithmetic to explain why this rule works.

Note: While solutions will be provided to all of these problems, not all problems may be fully graded.