

Due Thursday, May 10 by 5:00 pm

1. Let $A = \{2, 3, \dots, 20\}$, and define $R = \{(a, b) \in A \times A \text{ such that } a \mid b\}$.
 - (a) What are the domain and range of R ? Explain.
 - (b) List three ordered pairs that are in the inverse relation, R^{-1} .
 - (c) Let $S = T = \{6, 8\}$. List all the elements of $R(S)$ and $R^{-1}(T)$.
2. Let $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + 4y^2 = 4\}$
 - (a) What are the domain and range of R ? Explain.
 - (b) Let $S = T = \{0\}$. List all the elements of $R(S)$ and $R^{-1}(T)$.
3. Let $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = \frac{1}{x} + 1\}$.
 - (a) What are the domain and range of R ? Explain.
 - (b) Express the inverse relation, R^{-1} , in the form $R^{-1} = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = f(x)\}$ for some function $f(x)$.
 - (c) Let $S = T = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$. What is $R(S)$? What is $R^{-1}(T)$?

For Problems 4 – 7:

- i) Determine which of the three properties (reflexive, symmetric, transitive) R satisfies. Be sure to briefly justify your answers.
 - ii) If R is an equivalence relation, determine the equivalence classes of R .
4. $R = \{(a, a), (a, c), (b, b), (b, c), (c, a), (c, b)\}$, where R is a relation on the set $A = \{a, b, c\}$.
 5. $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \text{ such that } a^2 \equiv_3 b^2\}$
 6. $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \text{ such that } a \mid b \text{ or } b \mid a\}$
 7. $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \text{ such that } 3 \mid (a + 2b)\}$