Due Thursday, May 17 by 5:00 pm

For problems #1 – 4, prove or disprove the statement.

1. Let R be a relation on A with $\text{Dom}(R) = A$. If R is symmetric and transitive, then R is also reflexive.
2. Let R be a relation that is antisymmetric. Then R is not symmetric.
3. Let R be a relation that is asymmetric. Then R is antisymmetric.
4. Let $(A, \prec)$ be a poset where $\prec$ is a **linear (total)** order. Then any maximal element of $(A, \prec)$ must be a maximum.
5. Let U be a nonempty universal set. As discussed in class, $(\mathcal{P}(U), \subseteq)$ is a partially ordered set. Suppose as well that $\mathcal{B} \subseteq \mathcal{P}(U)$, that is, $\mathcal{B}$ is a set containing some (but not necessarily all) subsets of U.
   (a) Give an example demonstrating why $\subseteq$ is not a total order on $\mathcal{P}(U)$.
   (b) Give an example demonstrating why $\mathcal{B}$ might not have a maximum element.
   (c) Prove that $\displaystyle\bigcup_{B \in \mathcal{B}} B$ is an upper bound on $\mathcal{B}$.
   (d) Prove that $\displaystyle\bigcap_{B \in \mathcal{B}} B$ is a lower bound on $\mathcal{B}$.
   (e) Prove that $\displaystyle\bigcup_{B \in \mathcal{B}} B$ and $\displaystyle\bigcap_{B \in \mathcal{B}} B$ are in fact the least upper bound and greatest lower bound, respectively, on $\mathcal{B}$.

   Hint: Parts (c)-(e) require using element chasing arguments. The proofs should not be too long, but be careful with notation!

For problems # 6 – 8, determine whether the specified function is

i) one-to-one

ii) onto the specified set B.

6. $g : \mathbb{R}\backslash\{1\} \to \mathbb{R}\backslash\{1\}$, $g(x) = \dfrac{x}{x-1}$ (Graphing is not an acceptable proof).
7. $\mathcal{I} : F \to \mathbb{R}$, $\mathcal{I}(f) = \displaystyle\int_{-1}^{1} f(x)\,dx$, where F is the set of all functions with a finite integral on $[-1, 1]$.
8. $\varphi : \mathbb{N} \to \mathbb{N}$, where $\varphi(n)$ is the sum of the divisors of n.
   So for example $\varphi(6) = 1 + 2 + 3 + 6 = 12$, $\varphi(12) = 1 + 2 + 3 + 4 + 6 + 12 = 28$, etc.
9. **Bonus: [up to 10 additional points]**. Let $(A, \prec_A)$ and $(B, \prec_B)$ be posets.
   (a) Prove that the lexicographic ordering $\prec_L$ (defined on p. 99 of the text) is a partial ordering of $A \times B$.
   (b) Prove that if $\prec_A$ and $\prec_B$ are total orderings, then $\prec_L$ is also a total ordering.

Note: While solutions will be provided to all of these problems, not all problems may be fully graded.