Lecture 5-B

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* Announcements

(i) Office-hours: 11am - 12pm, T, W, Th or by appt (15-20 mins).
(ii) Lowest quiz-score will be dropped (does not apply for unexcused absence).
(iii) Tutoring available through QSC.
- math.stackexchange.com
- Youtube: 3 blue, 1 brown (Linear Algebra unit).

§1.3 Q/A

#17. $\vec{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \vec{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. Q: For what value(s) of h is $\vec{b} \in \text{Span}(\vec{a}_1, \vec{a}_2)$?

Span: all possible linear combos = weighted sum $x_1\vec{a}_1 + x_2\vec{a}_2$

Sol'n: Solve:

$$\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{array}\right]$$

If $h \neq -17$, then the LS is inconsistent; thus it is consistent if $h = -17$.

#15. $\vec{v}_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$

Exercise: find five vectors that belong to span$(\vec{v}_1, \vec{v}_2)$.

Sol'n:

(1) $\vec{v}_1$; weights/scalars: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$1\vec{v}_1 + 0\vec{v}_2 = \vec{v}_1 \in \text{span}(\vec{v}_1, \vec{v}_2)$

(2) $\vec{v}_2$; $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$\vec{v}_2 - \vec{v}_1 = (-1)\vec{v}_1 + (1)\vec{v}_2$

(3) $\vec{0}$; $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\pi\vec{v}_1 - \sqrt{2}\vec{v}_2 \in \text{span}(\vec{v}_1, \vec{v}_2)$

(4) $\vec{v}_1 + \vec{v}_2 = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$; $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(5) $\vec{v}_1 - \vec{v}_2 = \begin{bmatrix} 12 \\ -2 \\ -6 \end{bmatrix}$; $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Span (§ 1.3)

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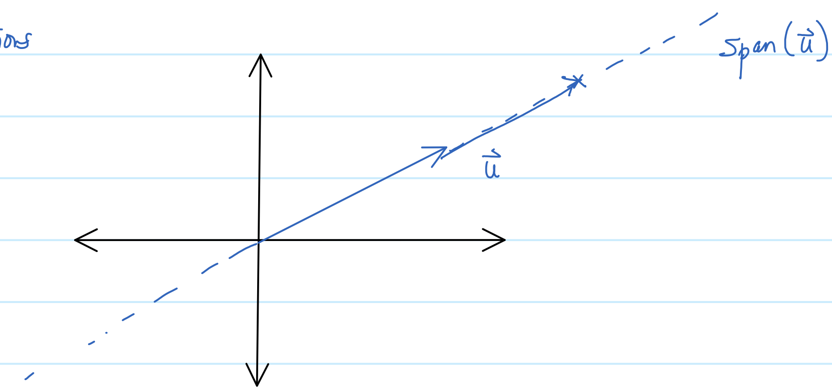
§1.3 (continued)

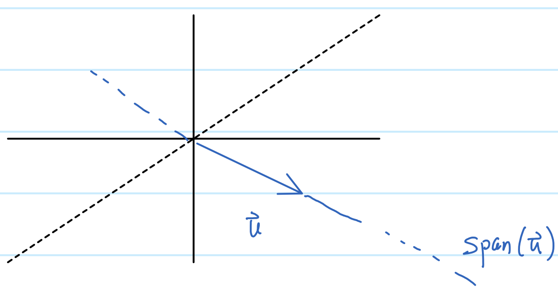
Recall: If $\vec{v}_1, .., \vec{v}_p \in \mathbb{R}^n$, then the span of $\vec{v}_1, .., \vec{v}_p$, denoted by Span$(\vec{v}_1, ..., \vec{v}_p)$ (Span$\{\vec{v}_1, .., \vec{v}_p\}$), is the set of all possible linear combinations (or weighted sums) of the vectors $\vec{v}_1, .., \vec{v}_p$.

* Geometric considerations

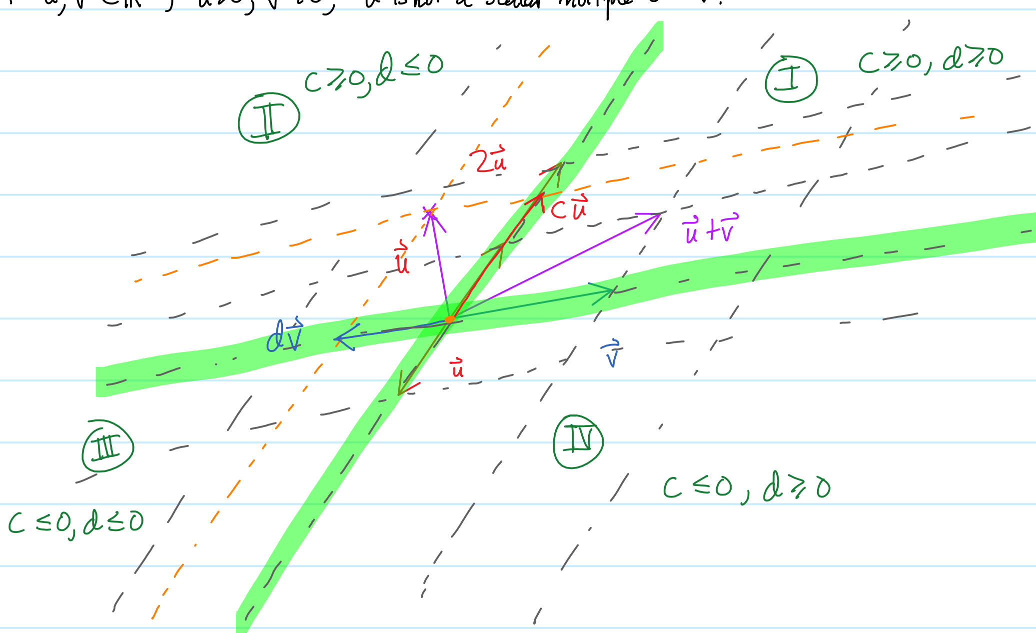
* $\vec{u} \in \mathbb{R}^2$, $\vec{u} \neq \vec{0}$



* $\vec{u} \in \mathbb{R}^3$, $\vec{u} \neq \vec{0}$



* $\vec{u}, \vec{v} \in \mathbb{R}^2$, $\vec{u} \neq 0$, $\vec{v} \neq 0$, $\vec{u}$ is not a scalar multiple of $\vec{v}$.



Span (continued)

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Properties: Let $\vec{v}_1, \dots, \vec{v}_p \in \mathbb{R}^n$.

(i) $\vec{v}_i \in \text{Span}(\vec{v}_1, \dots, \vec{v}_p)$, for every $i = 1, \dots, p$.

Pf: $\vec{v}_i = \underline{0}\,\vec{v}_1 + \dots + \underline{1}\,\vec{v}_i + \dots + \underline{0}\,\vec{v}_p \in \text{Span}(\vec{v}_1, \dots, \vec{v}_p)$.

(ii) $\vec{0} \in \text{Span}(\vec{v}_1, \dots, \vec{v}_p)$.

Pf: $\vec{0} = 0\,\vec{v}_1 + \dots + 0\,\vec{v}_i + \dots + 0\,\vec{v}_p \in \text{Span}(\vec{v}_1, \dots, \vec{v}_p)$.

§1.4 The Matrix Equation $A\vec{x} = \vec{b}$.

Defn: If $A = [\vec{a}_1\ \vec{a}_2\ \cdots\ \vec{a}_n]$ is an $m \times n$ matrix and $\vec{x} \in \mathbb{R}^n$, then the **product of A and $\vec{x}$**, denoted by $A\vec{x}$, is defined by

misnomer

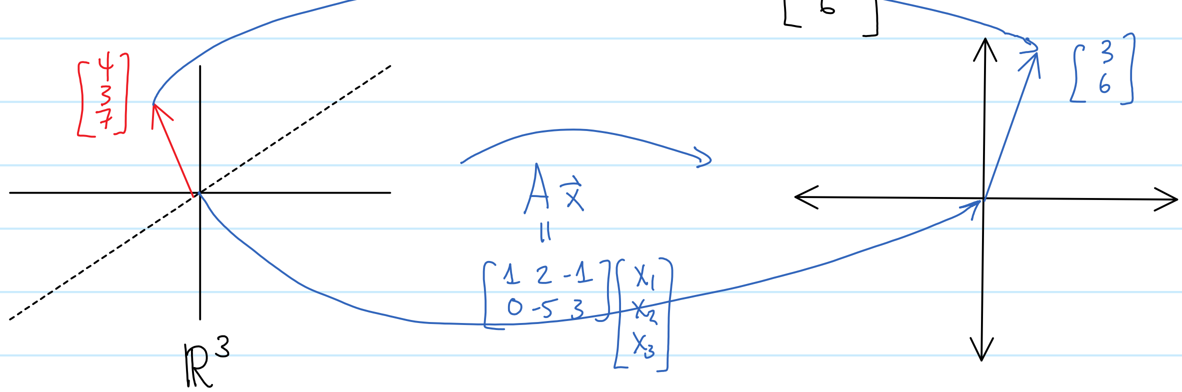
$$A\vec{x} = [\vec{a}_1\ \vec{a}_2\ \cdots\ \vec{a}_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1\vec{a}_1 + x_2\vec{a}_2 + \dots + x_n\vec{a}_n = \sum_{i=1}^{n} x_i\vec{a}_i \in \text{Span}(\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n)$$

Remark: $A\vec{x}$ is defined only if the number of columns of A equals the number of entries of $\vec{x}$.

E.g. Compute

(a)
$$\underset{2\times 3}{\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix}}\underset{3\times 1}{\begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}} = 4\underset{\vec{a}_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + 3\underset{\vec{a}_2}{\begin{bmatrix} 2 \\ -5 \end{bmatrix}} + 7\underset{\vec{a}_3}{\begin{bmatrix} -1 \\ 3 \end{bmatrix}} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ -15 \end{bmatrix} + \begin{bmatrix} -7 \\ 21 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \end{bmatrix}.$$



Matrix-vector product

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(b) $$\underset{3\times 2}{\begin{bmatrix} 2 & -3 \\ 8 & 0 \\ -5 & 2 \end{bmatrix}} \underset{2\times 1}{\overset{\text{input}: \mathbb{R}^2}{\begin{bmatrix} 4 \\ 7 \end{bmatrix}}} = 4\begin{bmatrix} 2 \\ 8 \\ -5 \end{bmatrix} + 7\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 32 \\ -20 \end{bmatrix} + \begin{bmatrix} -21 \\ 0 \\ 14 \end{bmatrix} = \overset{\text{output}: \mathbb{R}^3}{\begin{bmatrix} -13 \\ 32 \\ -6 \end{bmatrix}}$$

(c) $$\begin{bmatrix} 2 & -3 \\ 8 & 0 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{undefined}$$

The function $\vec{y} = A\vec{x}$ (A is a fixed $m\times n$ matrix, $\vec{x} \in \mathbb{R}^n$, $\vec{y} \in \mathbb{R}^m$) can be viewed as a generalization $y = ax$, $a \in \mathbb{R}$.