

Last day we saw that given two posets, (A, \prec_A) and (B, \prec_B) , we can define the product ordering on $A \times B$ as:

$(a, b) \prec_{prod} (c, d)$ if and only if $a \prec_A c$ and $b \prec_B d$.

1. **[8 points]** Prove that \prec_{prod} is transitive. (Hint: we know that both \prec_A and \prec_B are transitive because they are partial orders).
2. **[6 points]** Let $A = B = \{2, 4, 6\}$, and let $\prec_A = \prec_B = \leq$ when defining \prec_{prod} .
 - (a) Is $(4, 6)$ a maximal element under \prec_{prod} ? Explain.
 - (b) Is $(4, 6)$ a maximum element? Explain.
3. **[6 points]** Let $A = B = \{2, 4, 6\}$, and let $\prec_A = \prec_B = \ll$ (divisibility ordering) when defining \prec_{prod} .
 - (a) Is $(4, 6)$ a maximal element under \prec_{prod} ? Explain.
 - (b) Is $(4, 6)$ a maximum element? Explain.