

**Due Thursday, May 3 by 5:00 pm**

For Problems #1 – 5, let  $A$ ,  $B$ ,  $C$  and  $D$  be subsets of a universal set  $U$ . Prove the following statements.

*REMEMBER: Venn diagrams are not proofs! They can help with reasoning, but a proof requires making an element chasing, double inclusion or membership equivalence argument.*

1.  $A \setminus (A \setminus B) = A \cap B$
2.  $A \setminus (B \setminus C) = A \cap (\overline{B} \cup C)$
3.  $B \subseteq A$  if and only if  $A \cup \overline{B} = U$ .
4.  $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
5.  $(A \times C) \setminus (B \times C) = (A \setminus B) \times C$

For Problems #6 and 7, find expressions for  $\bigcup_{i \in I} A_i$  and  $\bigcap_{i \in I} A_i$ , and prove that the expressions are equal to the original union/intersection.

6.  $I = \mathbb{N}$ ,  $A_i = \{x \in \mathbb{R} \mid i < x < i + 1\}$
7.  $I = \mathbb{N}$ ,  $A_i = \{j \cdot i \mid j \in \mathbb{Z}\}$