

Row--Vector Rule for Computing *A***x**

If the product A**x** is defined

Then the i’th entry in A**x** is the sum of the products of corresponding entries from row i of A and from the vector **x**.

*Let A be an m \_ n matrix. Then the following statements are logically equivalent.*

*That is, for a particular A, either they are all true statements, or they are all false.*

a. For each **b** in Rm, the equation A**x** D **b** has a solution.

b. Each **b** in Rm is a linear combination of the columns of A.

c. The columns of A span Rm.

d. A has a pivot position in every row.

Existence of Solutions

The definition of A**x** leads directly to the following useful fact:

The equation A**x** D **b** has a solution if and only if **b** is a linear combination of the columns of A.

Existence and Uniqueness Theorem

*A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column—that is, if and only if an echelon form of the augmented matrix has no row of the form:*

[0 … 0 b ] with b nonzero

If a linear system is consistent, then the solution set contains either (i) a unique solution, when there are no free variables





The homogeneous equation A**x** = **0** has a nontrivial solution if and only if the

equation has at least one free variable.





















