Problem (Engel Problem-Solving Strategies 1.6). *There are a white, b black, and c red chips on a table. In one step, you may choose two chips of different colors and replace them by a chip of the third color. If just one chip will remain at the end, its color will not depend on the evolution of the game. When can this final state be reached?*

This state will be reached if and only if a, b, and c don't all have the same parity. The color of the last remaining chip will be the color whose parity differs from the other two.

To see this, let us first consider the case when a, b, and c do have the same parity. In this case, a chip exchange involves changing each of a, b, and c by one, so the parities all get reversed, and they are still the same as each other after the exchange. It follows that the colors will always have the same parity, so a configuration with just one chip remaining is unreachable.

On the other hand, suppose a, b, and c don't all have the same parity. Without loss of generality, say that a is the number whose parity differs from the other two. Then, since each chip exchange reverses all the parities, a remains the "odd man out" after the exchange. It follows that a will always be the chip with different parity. Now note that each exchange reduces the total number of chips by one, so eventually it must be that only one chip remains. At this point, since a's parity always differs from the other two, we must have $(a, b, c) = (1, 0, 0)$, as desired.