Problem (Putnam 2009 - A1)**.** *Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points in the plane?*

The answer is yes. To see this, let (x, y) be an arbitrary point in the plane. Then, we have

$$\begin{aligned}
f(x,y) + f(x+1,y) + f(x,y+1) + f(x+1,y+1) &= 0 \\
f(x,y) + f(x-1,y) + f(x-1,y+1) + f(x,y+1) &= 0 \\
f(x,y) + f(x-1,y) + f(x-1,y-1) + f(x,y-1) &= 0 \\
f(x,y) + f(x+1,y) + f(x+1,y-1) + f(x,y-1) &= 0 \\
-\left[f(x-1,y-1) + f(x+1,y+1) + f(x+1,y-1) + f(x-1,y-1)\right] &= -(0) = 0 \\
-2\left[f(x,y+1) + f(x+1,y) + f(x,y-1) + f(x-1,y)\right] &= -2(0) = 0.
\end{aligned}$$

Adding the equations and simplifying, we get

$$4f(x,y) = 0,$$

which means

$$f(x,y) = 0.$$