Problem 1 (Putnam 2009 - A1). *Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points in the plane?*

The answer is yes. To see this, let (x, y) be an arbitrary point in the plane. Then, we have

$$f(x, y) + f(x + 1, y) + f(x, y + 1) + f(x + 1, y + 1) = 0$$
$$f(x, y) + f(x - 1, y) + f(x - 1, y + 1) + f(x, y + 1) = 0$$
$$f(x, y) + f(x - 1, y) + f(x - 1, y - 1) + f(x, y - 1) = 0$$
$$f(x, y) + f(x + 1, y) + f(x + 1, y - 1) + f(x, y - 1) = 0$$
$$- [f(x - 1, y - 1) + f(x + 1, y + 1) + f(x + 1, y - 1) + f(x - 1, y - 1)] = -(0) = 0$$
$$-2 [f(x, y + 1) + f(x + 1, y) + f(x, y - 1) + f(x - 1, y)] = -2(0) = 0.$$

Adding the equations and simplifying, we get

$$4f(x, y) = 0,$$

which means

$$f(x, y) = 0.$$