

**Exercise 1.** Consider a tetrahedron. Join each vertex with the point of intersection of the medians of the opposite face. Prove that all 4 segments you constructed pass through one point. Prove that this point divides each segment in proportion 1: 3.

**Exercise 2.** Consider a 4-gon  $ABCD$ . Let  $K, L, M, N$  be the middle points of the segments  $AB, BC, CD, DA$  respectively. Let  $Q$  be the point of intersection of the lines  $LN, KM$ . Let  $P$  and  $Q$  be the middle points of the diagonals  $AC$  and  $BD$  respectively. Show that  $Q$  is the middle point of the segment  $PQ$ .

**Exercise 3.** Take a 6-gon  $A_1A_2A_3A_4A_5A_6$ . Let  $B_1, B_2, B_3, B_4, B_5, B_6$  be the middle points of the sides  $A_1A_2, A_2A_3, A_3A_4, A_4A_5, A_5A_6, A_6A_1$  respectively. Prove that the points of intersection of medians of triangles  $B_1B_3B_5$  and  $B_2B_4B_6$  coincide.

**Exercise 4.** Every vertex of a triangle was connected by two lines to the points that divide the opposite side to three equal parts. Prove that in the hexagon these six lines form the lines connecting opposite vertices are concurrent.

**Exercise 5.** Let  $O$  be the point of intersections of the medians of the triangle  $ABC$ . Let  $M, N, P$  divide the sides  $AB, BC, CA$  of the triangle  $ABC$  in equal proportions. Prove that  $O$  is also the point of intersection of the medians of triangle  $MNP$ .

**Exercise 6.** Let  $M, N, P$  be points on the sides  $AB, BC, CA$  of triangle  $ABC$ . Let  $M', N', P'$  be the reflections of the points  $M, N, P$  in the midpoints of sides  $AB, BC, CA$ . Prove that the areas of triangles  $MNP$  and  $M'N'P'$  are equal.