THE FACULTY OF ARTS AND SCIENCE University of Toronto

FINAL EXAMINATIONS, APRIL 13, 2010

MAT402H CLASSICAL PLANE GEOMETRIES AND THEIR TRANSFORMATIONS

Examiner: Professor A. Khovanskii

Total Marks: 100

Duration: 3 hours

NO AIDS ALLOWED.

1. [20 marks] Consider a tetrahedron ABCD. Let E be the point of intersection of the medians in the triangle ABC. Take a point F on the segment DE such that DF: FE = 6. In what proportion the plane passing through the points BCF divides the edge DA.

Hint: Put appropriate masses at points A, B, C and D.

- 2. [20 marks] Take two intersecting circles S_1, S_2 on plane. Prove that there is a Moebius transformation which maps S_1, S_2 to two equal circles.
- 5. [20 marks] Consider points O_1 , O_2 , A and a segment PQ of length R on plane. Using this data and a compass and a straightedge construct two tangent lines to the ellipse $O_1X + XO_2 = R$ passing though the point A.
- **4.** [20 marks] Take four points ABCD on an ellipse. Choose any line l tangent to the ellipse, and consider points $A' = l \cap l_A$, $B' = l \cap l_B$, $C' = l \cap l_C$, $D' = l \cap l_D$ where l_A, l_B, l_C, l_D are lines tangent to the ellipse at A, B, C, D. Prove the theorem: the cross-ratio (A', B', C', D') is independent of the choice of the tangent line l.
- **5.** [20 marks] Take a projective map $f: l \to l$ from a line l to itself. Assume that there is a point $a \in l$ such that $f(a) \neq a$ but f(f(a)) = a. Prove that f(f(x)) = x for every $x \in l$.