Lesson 16

Solving Problems Involving Fractions (Optional)

Goals

- Generate an equation to represent a situation involving fractions, and justify (orally) the operation chosen.
- Use operations with fractions to solve problems in a variety of situations, and explain (orally and in writing) the reasoning.

Learning Target

I can use mathematical expressions to represent and solve word problems that involve fractions.

Lesson Narrative

In this lesson, students apply their understanding of all four operations to represent and solve problems involving fractions. In all activities, students are prompted to interpret situations or expressions, represent situations and questions mathematically, and estimate answers that would make sense in terms of the situations. Along the way, students practice reasoning abstractly and quantitatively.

The last activity requires students to think about how different constraints limit possible answers to a question, prompting students to make sense of a problem and persevere in solving it.

Student Learning Goal

Let's add, subtract, multiply, and divide fractions.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)
- MLR5: Co-Craft Questions (Activity 3)

Instructional Routines

- Take Turns

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2
- Math Community Chart: Activity 3

Required Preparation

Activity 3:
Before class, plan an efficient way to assign at least 1 division problem and 1 problem involving another operation to each student (or group).

Lesson:
For the second activity ("Pairs of Problems"), plan an efficient way to assign at least 1 division problem and 1 problem involving another operation to each student (or group).

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	15 min	25 min	15 min	10 min	5 min
					Cool-down

Inspire Math

A Fine Line video

Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614234

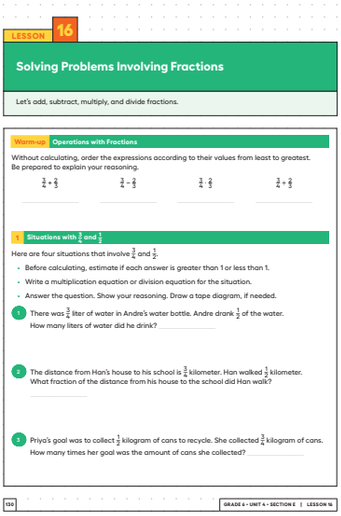
Please log in to the site before using the QR code or URL.

Building on Student Thinking

Some students may assign the division expression to be the one with the lowest value because they still assume that the quotient will always be less than the dividend. Prompt them to test their assumption with a counterexample, such as

$2 \div \frac{1}{3}$ or $\frac{1}{2} \div \frac{1}{4}$. If the assumption is common, consider addressing it during a whole-class discussion.

Student Workbook



LESSON 16

Solving Problems Involving Fractions

Let's add, subtract, multiply, and divide fractions.

Warm-up Operations with Fractions

Without calculating, order the expressions according to their values from least to greatest. Be prepared to explain your reasoning.

$\frac{3}{4}+\frac{2}{3}$ $\frac{3}{4}-\frac{2}{3}$ $\frac{3}{4}\cdot\frac{2}{3}$ $\frac{3}{4}\div\frac{2}{3}$

1 Situations with $\frac{3}{4}$ and $\frac{1}{2}$

Here are four situations that involve $\frac{3}{4}$ and $\frac{1}{2}$.

- Before calculating, estimate if each answer is greater than 1 or less than 1.
- Write a multiplication equation or division equation for the situation.
- Answer the question. Show your reasoning. Draw a tape diagram, if needed.

1. There was $\frac{3}{4}$ liter of water in Andre's water bottle. Andre drank $\frac{1}{2}$ of the water. How many liters of water did he drink?
2. The distance from Han's house to his school is $\frac{3}{4}$ kilometer. Han walked $\frac{1}{2}$ kilometer. What fraction of the distance from his house to the school did Han walk?
3. Priya's goal was to collect $\frac{1}{2}$ kilogram of cans to recycle. She collected $\frac{3}{4}$ kilogram of cans. How many times her goal was the amount of cans she collected?

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Warm-up

Operations with Fractions

5 min

Activity Narrative

This *Warm-up* reinforces students' understanding of what each of the four operations (addition, subtraction, multiplication, and division) does when performed on fractions. The same pair of fractions are used in each problem so that students can focus on the meaning of each operation. Because students are not to calculate exact values, to order the expressions they need to rely on what they know about the size of the fractions, as well as to look for and make use of structure.

Launch

Arrange students in groups of 2. Display problems for all to see. Ask students to put the expressions in order based on their value, from least to greatest, but without calculating the exact values. Instead, they should estimate the value of each expression by reasoning about the operation and the fractions. Ask students to give a signal as soon as they have determined an order and can support it with an explanation.

Give students 1–2 minutes of quiet think time and another minute to discuss their reasoning with a partner and come to an agreement.

Student Task Statement

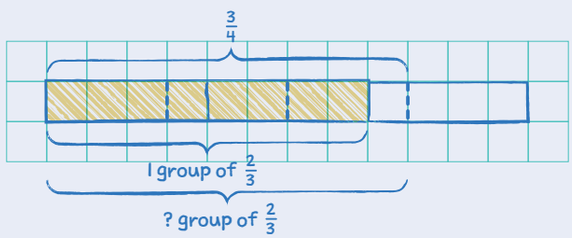
Without calculating, order the expressions according to their values from least to greatest. Be prepared to explain your reasoning.

$\frac{3}{4}+\frac{2}{3}$ $\frac{3}{4}-\frac{2}{3}$ $\frac{3}{4}\cdot\frac{2}{3}$ $\frac{3}{4}\div\frac{2}{3}$

$\frac{3}{4}-\frac{2}{3}$ $\frac{3}{4}\cdot\frac{2}{3}$ $\frac{3}{4}\div\frac{2}{3}$ $\frac{3}{4}+\frac{2}{3}$

Sample reasoning:

- $\frac{3}{4}+\frac{2}{3}$ and $\frac{3}{4}\div\frac{2}{3}$ are both greater than 1.
- $\frac{3}{4}-\frac{2}{3}$ and $\frac{3}{4}\cdot\frac{2}{3}$ are both less than 1.
- $\frac{3}{4}$ is greater than $\frac{1}{2}$ and $\frac{2}{3}$ is also greater than $\frac{1}{2}$, so $\frac{3}{4}+\frac{2}{3}$ is greater than 1.
- $\frac{3}{4}$ is 0.75 and $\frac{2}{3}$ is about 0.67, so their sum is a little less than 1.5.
- $\frac{3}{4}\div\frac{2}{3}$ can be viewed as "How many $\frac{2}{3}$s are in $\frac{3}{4}$?". Since $\frac{3}{4}$ is just a little over $\frac{2}{3}$, the quotient is a little more than 1. If we were to draw a tape diagram, we can see it is just a little bit more than 1.



- $\frac{3}{4}\cdot\frac{2}{3}$ can be viewed as $\frac{3}{4}$ of $\frac{2}{3}$, so the product is less than $\frac{2}{3}$ but more than $\frac{1}{3}$.
- $\frac{3}{4}$ is $\frac{1}{4}$ more than $\frac{1}{2}$, and $\frac{2}{3}$ is greater than $\frac{1}{2}$ by an even smaller amount, so $\frac{3}{4}-\frac{2}{3}$ is less than $\frac{1}{4}$.

Activity Synthesis

Invite 1–2 groups to share how they ordered their expressions from least to greatest. Record it for all to see.

To involve more students in the conversation, consider asking:

- *"Does anyone want to add on to ___'s strategy?"*
- *"Did anyone compare the expressions in a different way?"*
- *"Do you agree or disagree? Why?"*

If there are disagreements, ask students with opposing views to explain their reasoning, and discuss it to reach an agreement on a correct order.

Activity 1: Optional

Situations with $\frac{3}{4}$ and $\frac{1}{2}$

15 min

Activity Narrative

This activity offers an additional opportunity for students to make sense of word problems, set up an appropriate representation, use that representation for reasoning, and estimate before solving. Students are presented with four situations that involve only fractions. Two of them require multiplication to solve, and the other two require division. Students decide which operation is needed to answer each question, and before solving, make an estimate based on the given context.

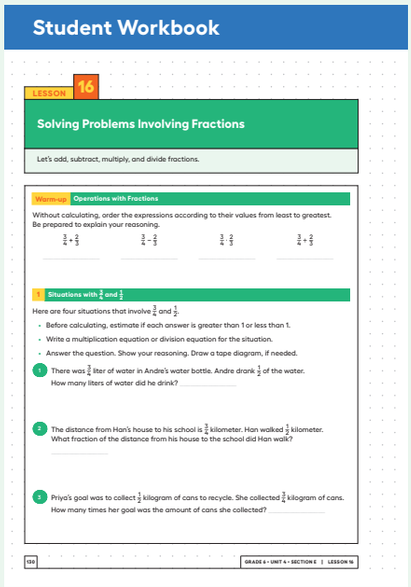
As students work, monitor for how they determine appropriate operations to use. Note any common challenges so they can be discussed later.

Launch

Keep students in groups of 2. Explain to students that the situations in this activity all involve the same two fractions, but they do not all require the same operation to solve. Encourage them to make sense of each situation carefully before calculating or reasoning about the answer. Provide access to geometry toolkits (especially graph paper and colored pencils).

Give students 8–10 minutes to work on the activity either individually or with their partner, and then some time to discuss or check their responses.

If time is limited, consider asking students to answer either the first two or the last two questions.


Student Workbook

LESSON 16

Solving Problems Involving Fractions

Let's add, subtract, multiply, and divide fractions.

Warm-up Operations with Fractions

Without calculating, order the expressions according to their values from least to greatest. Be prepared to explain your reasoning.

$\frac{3}{4} + \frac{2}{3}$ $\frac{3}{4} - \frac{2}{3}$ $\frac{3}{4} \cdot \frac{2}{3}$ $\frac{3}{4} \div \frac{2}{3}$

1 Situations with $\frac{3}{4}$ and $\frac{1}{2}$

Here are four situations that involve $\frac{3}{4}$ and $\frac{1}{2}$.

- Before calculating, estimate if each answer is greater than 1 or less than 1.
- Write a multiplication equation or division equation for the situation.
- Answer the question. Show your reasoning. Draw a tape diagram, if needed.

1. There was $\frac{3}{4}$ liter of water in Andre's water bottle. Andre drank $\frac{1}{2}$ of the water. How many liters of water did he drink?

2. The distance from Han's house to his school is $\frac{3}{4}$ kilometer. Han walked $\frac{1}{2}$ kilometer. What fraction of the distance from his house to the school did Han walk?

3. Priya's goal was to collect $\frac{1}{2}$ kilogram of cans to recycle. She collected $\frac{3}{4}$ kilogram of cans. How many times her goal was the amount of cans she collected?

Student Task Statement

Here are four situations that involve $\frac{3}{4}$ and $\frac{1}{2}$.

- Before calculating, estimate if each answer is greater than 1 or less than 1.
- Write a multiplication equation or division equation for the situation.
- Answer the question. Show your reasoning. Draw a tape diagram, if needed.

1. There was $\frac{3}{4}$ liter of water in Andre's water bottle. Andre drank $\frac{1}{2}$ of the water. How many liters of water did he drink?

Water that Andre drank:

- Estimate: Less than 1
- Equation: $\frac{1}{2} \cdot \frac{3}{4} = ?$
- Answer: $\frac{3}{8}$

  Sample reasoning: $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

2. The distance from Han's house to his school is $\frac{3}{4}$ kilometer. Han walked $\frac{1}{2}$ kilometer. What fraction of the distance from his house to the school did Han walk?

Fraction of distance from Han's house:

- Estimate: Less than 1
- Equation: $\frac{1}{2} \div \frac{3}{4} = ?$ or $? \cdot \frac{3}{4} = \frac{1}{2}$
- Answer: $\frac{2}{3}$ (or equivalent)

  Sample reasoning: $\frac{1}{2} \cdot \frac{4}{3} = \frac{4}{6}$

3. Priya's goal was to collect $\frac{1}{2}$ kilogram of cans to recycle. She collected $\frac{3}{4}$ kilogram of cans. How many times her goal was the amount of cans she collected?

Collected cans compared to Priya's goal:

- Estimate: Greater than 1
- Equation: $\frac{3}{4} \div \frac{1}{2} = ?$ or $? \cdot \frac{1}{2} = \frac{3}{4}$
- Answer: $1\frac{1}{2}$ (or equivalent)

  Sample reasoning: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot 2 = \frac{6}{4} = \frac{3}{2}$

4. Mai's class volunteered to clean a park with an area of $\frac{1}{2}$ square mile. Before they took a lunch break, the class had cleaned $\frac{3}{4}$ of the park. How many square miles had they cleaned before lunch?

Area Mai's class cleaned before lunch:

- Estimate: Less than 1
- Equation: $\frac{3}{4} \cdot \frac{1}{2} = ?$
- Answer: $\frac{3}{8}$

  Sample reasoning: $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$

Activity Synthesis

Display the solutions for all to see and give students time to check their work. If time permits, discuss students' reasoning. Ask:

- *"How did you estimate the answers?"*
- *"How did you know what operation you needed to perform to find the answer?"*
- *"For which problems was it difficult to tell what operation to use?"*
- *"For which problems was it helpful to draw a diagram?"*

Some students may notice that the second and third questions involve the phrases "how many times?" and "what fraction of?" which suggests that division might be involved. Ask them to identify the size of 1 group in those cases.

Activity 2

Pairs of Problems

25 min

Activity Narrative

In this partner activity, students take turns making sense of and writing equations for a variety of situations involving fractions and all four operations. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

After writing equations, students are assigned two problems to solve, at least one of which is a division problem. Before calculating, students first estimate their answer. Doing so helps them to attend to the meaning of the operation and the reasonableness of their calculated answer in the context of the situation. The work here offers students opportunities to practice reasoning quantitatively and abstractly.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms, or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Keep students in groups of 2. Tell students that they will take turns writing equations to represent situations. Share the following steps:

- Both partners read the first problem in the first column.
- One partner writes an equation to represent the situation and explains why it is correct.
- The other partner listens to the writer's reasoning and decides if it makes sense.
- If there is a disagreement, the partners discuss the issues until coming to an agreement.

Instructional Routines

Take Turns

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Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. Prior to writing equations, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

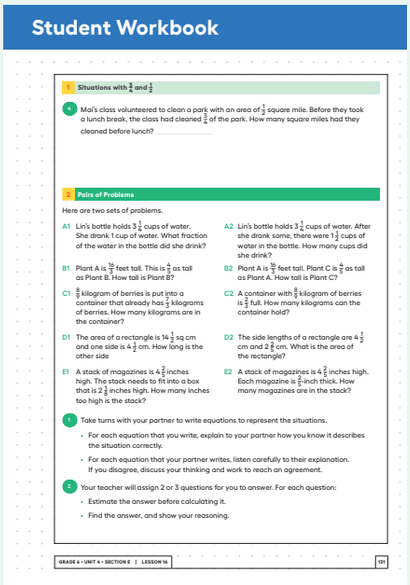
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, check in with groups after they complete both parts of Problem A. Monitor for correctness of equations and for equal sharing of the work.

Supports accessibility for: Organization, Attention

Student Workbook



1 Situations with $\frac{3}{4}$ and $\frac{1}{2}$

A Mai's class volunteered to clean a park with an area of $\frac{1}{2}$ square mile. Before they took a lunch break, the class had cleaned $\frac{3}{4}$ of the park. How many square miles had they cleaned before lunch?

2 Pairs of Problems

Here are two sets of problems.

A1 Lin's bottle holds $3\frac{1}{4}$ cups of water. She drank 1 cup of water. What fraction of the water in the bottle did she drink?

A2 Lin's bottle holds $3\frac{1}{4}$ cups of water. After she drank some, there were $1\frac{1}{2}$ cups of water in the bottle. How many cups did she drink?

B1 Plant A is $\frac{16}{3}$ feet tall. This is $\frac{4}{5}$ as tall as Plant B. How tall is Plant B?

B2 Plant A is $\frac{16}{3}$ feet tall. Plant C is $\frac{4}{5}$ as tall as Plant A. How tall is Plant C?

C1 $\frac{8}{9}$ kilogram of berries is put into a container that already has $\frac{7}{3}$ kilograms of berries. How many kilograms are in the container?

C2 A container with $\frac{8}{9}$ kilogram of berries is $\frac{2}{3}$ full. How many kilograms can the container hold?

D1 The area of a rectangle is $14\frac{1}{2}$ sq cm and one side is $4\frac{1}{2}$ cm. How long is the other side

D2 The side lengths of a rectangle are $4\frac{1}{2}$ cm and $2\frac{2}{5}$ cm. What is the area of the rectangle?

E1 A stack of magazines is $4\frac{2}{5}$ inches high. The stack needs to fit into a box that is $2\frac{1}{8}$ inches high. How many inches too high is the stack?

E2 A stack of magazines is $4\frac{2}{5}$ inches high. Each magazine is $\frac{2}{5}$-inch thick. How many magazines are in the stack?

1 Take turns with your partner to write equations to represent the situations.
- For each equation that you write, explain to your partner how you know it describes the situation correctly.
- For each equation that your partner writes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

2 Your teacher will assign 2 or 3 questions for you to answer. For each question:
- Estimate the answer before calculating it.
- Find the answer, and show your reasoning.

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- Next, both partners read the first problem in the second column and swap roles.
- Repeat until all situations are represented with equations.

Next, assign at least 1 division problem and 1 problem involving another operation for each student (or group) to solve. Consider preparing the assignments in advance or deciding ahead of time how to efficiently assign the problems.

Give students 4–5 minutes of quiet work time or partner work time to complete the activity.

Student Task Statement

Here are two sets of problems.

A1. Lin's bottle holds $3\frac{1}{4}$ cups of water. She drank 1 cup of water. What fraction of the water in the bottle did she drink?

A2. Lin's bottle holds $3\frac{1}{4}$ cups of water. After she drank some, there were $1\frac{1}{2}$ cups of water in the bottle. How many cups did she drink?

B1. Plant A is $\frac{16}{3}$ feet tall. This is $\frac{4}{5}$ as tall as Plant B. How tall is Plant B?

B2. Plant A is $\frac{16}{3}$ feet tall. Plant C is $\frac{4}{5}$ as tall as Plant A. How tall is Plant C?

C1. $\frac{8}{9}$ kilogram of berries is put into a container that already has $\frac{7}{3}$ kilograms of berries. How many kilograms are in the container?

C2. A container with $\frac{8}{9}$ kilogram of berries is $\frac{2}{3}$ full. How many kilograms can the container hold?

D1. The area of a rectangle is $14\frac{1}{2}$ sq cm and one side is $4\frac{1}{2}$ cm. How long is the other side?

D2. The side lengths of a rectangle are $4\frac{1}{2}$ cm and $2\frac{2}{5}$ cm. What is the area of the rectangle?

E1. A stack of magazines is $4\frac{2}{5}$ inches high. The stack needs to fit into a box that is $2\frac{1}{8}$ inches high. How many inches too hiah is the stack

E2. A stack of magazines is $4\frac{2}{5}$ inches high. Each magazine is $\frac{2}{5}$-inch thick. How many magazines are in the stack?

1. Take turns with your partner to write equations to represent the situations.
- For each equation that you write, explain to your partner how you know it describes the situation correctly.
- For each equation that your partner writes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

2. Your teacher will assign 2 or 3 questions for you to answer. For each question:

- Estimate the answer before calculating it.
- Find the answer, and show your reasoning.

Sample response:

A1. Equation:
$1 \div \left(3\frac{1}{4}\right) = ?$
- Estimate:
  Less than $\frac{1}{3}$ of the water in the bottle.
- $\frac{4}{13}$ of the bottle.
  $1 \div \left(3\frac{1}{4}\right) = 1 \div \frac{13}{4} = 1 \cdot \frac{4}{13} = \frac{4}{13}$

A2. Equation:
$3\frac{1}{4} - ? = 1\frac{1}{2}$
- Estimate:
  A little less than 2 cups.
- $1\frac{3}{4}$ cups.
  $3\frac{1}{4} - 1\frac{1}{2} = 1\frac{3}{4}$

B1. Equation:
$\frac{16}{3} = \frac{4}{5} \cdot ?$
- Estimate:
  Around 7 feet.
- $6\frac{2}{3}$ feet.
  $\frac{16}{3} \div \frac{4}{5} = \frac{16}{3} \cdot \frac{5}{4} = \frac{20}{3} = 6\frac{2}{3}$

B2. Equation:
$\frac{4}{5} \cdot \left(5\frac{1}{3}\right) = ?$
- Estimate:
  A little bit less than 5 feet.
- $4\frac{4}{15}$ feet. $\frac{4}{5} \cdot \frac{16}{3} = \frac{64}{15} = 4\frac{4}{15}$

C1. Equation:
$\frac{8}{9} + \frac{7}{3} = ?$
- Estimate: Around 3 kg.
- $3\frac{2}{9}$ kg. $\frac{8}{9} + \frac{7}{3} = \frac{29}{9} = 3\frac{2}{9}$

C2. Equation:
$\frac{8}{9} = \frac{2}{3} \cdot ?$
- Estimate:
  Between 1 and 2 kg.
- $\frac{4}{3}$ kg. $\frac{8}{9} \div \frac{2}{3} = \frac{4}{3}$

D1. Equation:
$\left(14\frac{1}{2}\right) \div \left(4\frac{1}{2}\right) = ?$
- Estimate: Between 3 and 4 cm.
- $3\frac{2}{9}$ cm.
  $\left(14\frac{1}{2}\right) \div \left(4\frac{1}{2}\right) = \frac{29}{2} \div \frac{9}{2} = \frac{29}{9}$

D2. Equation:
$4\frac{1}{2} \cdot 2\frac{2}{5} = ?$
- Estimate:
  Around 10 square centimeters.
- $10\frac{4}{5}$ square centimeters.
  $\left(4\frac{1}{2}\right) \cdot \left(2\frac{2}{5}\right) = \frac{9}{2} \cdot \frac{12}{5} = \frac{54}{5} = 10\frac{4}{5}$

E1. Equation:
$4\frac{2}{5} - 2\frac{1}{8} = ?$
- Estimate:
  A little more than 2 inches.
- $2\frac{11}{40}$ inches.
  $4\frac{2}{5} - 2\frac{1}{8} = 4\frac{16}{40} - 2\frac{5}{40} = 2\frac{11}{40}$.

E2. Equation:
$\left(4\frac{2}{5}\right) \div \frac{2}{5} = ?$
- Estimate:
  A little more than 10 magazines.
- 11 magazines.
  $\left(4\frac{2}{5}\right) \div \frac{2}{5} = 11$.

Access for Multilingual Learners (Activity 3, Student Task)

MLR5: Co-Craft Questions. Keep student workbooks or devices closed. Display only the first paragraph and the list of ingredients that Mai, Kiran, and Clare brought, without revealing the question. Ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Activity Synthesis

Much of the discussion takes place between partners. Use this time to address common issues or misconceptions. Consider having the solutions accessible for students to check their answers.

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- *"I noticed our norm '___' in action today, and it really helped me/my group because ___."*

Activity 3: Optional

Making Ornaments

15 min

Activity Narrative

This activity gives students another opportunity to use what they have learned about operations with fractions to model and solve a problem in a recipe context. The question asks for the whole-number of batches that can be made with available ingredients. Because two known quantities don't translate to the same number of batches, students need to consider how to find the number of batches that can be made and which ingredient limits what is possible. The reasoning here prompts students to make sense of the problem and persevere in solving it.

Students may approach the problem in different ways (such as by drawing diagrams, making computations, or reasoning verbally). Students may also choose different operations to obtain the information they need. For instance, instead of dividing the available amount of an ingredient by the amount in a batch, they may perform repeated subtraction. Monitor for the different methods that students use, and select strategies or explanations that should be shared with the class.

Launch

Ask students if they have experience making homemade ornaments. Invite students to briefly share the materials and the process involved. If any students are familiar with dough ornaments, invite them to share what they know. Explain that this activity is about making ornaments from dough made with flour and salt as the main ingredients. The dough can be cut into interesting shapes, baked, and decorated.

Keep students in groups of 2.

Give students 5–7 minutes of quiet work time and then 1–2 minutes to discuss their response with their partner.

Ask students to be prepared to explain their reasoning.

Student Task Statement

Mai, Kiran, and Clare are making dough ornaments together. To make one batch of the dough, they need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of salt. They each brought the ingredients they had at home.

- Mai brought 2 cups of flour and $\frac{1}{4}$ cup of salt.
- Kiran brought 1 cup of flour and $\frac{1}{2}$ cup of salt.
- Clare brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cup of salt.

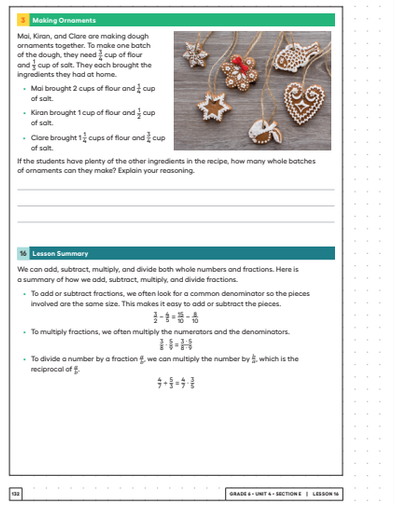
If the students have plenty of the other ingredients in the recipe, how many whole batches of ornaments can they make? Explain your reasoning.

They have enough for 4 batches of dough.

Sample reasoning:

- The students brought a total of $4\frac{1}{4}$ cups of flour ($2 + 1 + 1\frac{1}{4} = 4\frac{1}{4}$) and $1\frac{1}{2}$ cups of salt ($\frac{1}{4} + \frac{1}{2} + \frac{3}{4} = 1\frac{1}{2}$).
- The amount of flour is enough for $5\frac{2}{3}$ batches, because $4\frac{1}{4} \div \frac{3}{4} = \frac{17}{4} \cdot \frac{4}{3} = \frac{17}{3} = 5\frac{2}{3}$.
- The amount of salt is enough for $4\frac{1}{2}$ batches, because $1\frac{1}{2} \div \frac{1}{3} = \frac{3}{2} \cdot \frac{3}{1} = \frac{9}{2} = 4\frac{1}{2}$. Given the amount of flour, they can make 5 whole batches of cookies.
- Given the amount of salt, the whole number of batches they can make is only 4.

Student Workbook



3 Making Ornaments

Mai, Kiran, and Clare are making dough ornaments together. To make one batch of the dough, they need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of salt. They each brought the ingredients they had at home.

- Mai brought 2 cups of flour and $\frac{1}{4}$ cup of salt.
- Kiran brought 1 cup of flour and $\frac{1}{2}$ cup of salt.
- Clare brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cup of salt.

If the students have plenty of the other ingredients in the recipe, how many whole batches of ornaments can they make? Explain your reasoning.

16 Lesson Summary

We can add, subtract, multiply, and divide both whole numbers and fractions. Here is a summary of how we add, subtract, multiply, and divide fractions.

- To add or subtract fractions, we often look for a common denominator so the pieces involved are the same size. This makes it easy to add or subtract the pieces.
$\frac{3}{2} - \frac{3}{5} = \frac{15}{10} - \frac{6}{10}$
- To multiply fractions, we often multiply the numerators and the denominators.
$\frac{3}{8} \cdot \frac{5}{3} = \frac{3 \cdot 5}{8 \cdot 3}$
- To divide a number by a fraction $\frac{a}{b}$, we can multiply the number by $\frac{b}{a}$, which is the reciprocal of $\frac{a}{b}$.
$\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \cdot \frac{3}{5}$

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Activity Synthesis

Consider combining every group of 2–3 students and having students discuss their responses and reasoning in larger groups of 4–6.

If time permits, reconvene for a whole-class discussion. Highlight a couple of strategies, and invite students to reflect on the effectiveness and efficiency of the strategies. For example, if some students performed repeated addition instead of multiplying (or repeated subtraction instead of dividing), ask students to discuss the efficiency of each operation and consider when one method might be preferred over the other.

Lesson Synthesis

Invite students to reflect on their problem-solving process, how they made sense of given situations, and how they decided on which steps to take. Ask questions such as:

- *"How did you know which operations to use for each situation?"*
- *"How did you know if you've chosen the right operation?"*
- *"How did you know if your answer makes sense and is correct?"*

Responding To Student Thinking

Press Pause
If students struggle with interpreting the division problem or finding the quotient, make time to revisit related work in earlier lessons. For example, ask students to interpret each situation in the activity referred to here, and discuss ways to reason about the answer to each question.

Grade 6, Unit 4, Lesson 9, Activity 3
Amount in One Group

Lesson Summary

We can add, subtract, multiply, and divide both whole numbers and fractions. Here is a summary of how we add, subtract, multiply, and divide fractions.

- To add or subtract fractions, we often look for a common denominator so the pieces involved are the same size. This makes it easy to add or subtract the pieces.

$$\frac{3}{2} - \frac{4}{5} = \frac{15}{10} - \frac{8}{10}$$

- To multiply fractions, we often multiply the numerators and the denominators.

$$\frac{3}{8} \cdot \frac{5}{9} = \frac{3 \cdot 5}{8 \cdot 9}$$

- To divide a number by a fraction $\frac{a}{b}$, we can multiply the number by $\frac{b}{a}$, which is the reciprocal of $\frac{a}{b}$.

$$\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \cdot \frac{3}{5}$$

Cool-down

A Box of Pencils

5 min

Student Task Statement

A box of pencils is $5\frac{1}{4}$ inches wide. Seven pencils, laid side by side, take up $2\frac{5}{8}$ inches of the width.

1. How many inches of the width of the box is *not* taken up by the pencils? Explain or show your reasoning.

   $2\frac{5}{8}$ inches, because $5\frac{1}{4} - 2\frac{5}{8} = 2\frac{5}{8}$

2. All 7 pencils have the same width. How wide is each pencil? Explain or show your reasoning.

   $\frac{3}{8}$ inch, because $2\frac{5}{8} \div 7 = \frac{21}{8} \cdot \frac{1}{7} = \frac{3}{8}$

Lesson 16

Practice Problems

7 Problems

Problem 1

An orange has about $\frac{1}{4}$ cup of juice. How many oranges are needed to make $2\frac{1}{2}$ cups of juice?

Select **all** the equations that represent this question.

A. $? \cdot \frac{1}{4} = 2\frac{1}{2}$

B. $\frac{1}{4} \div 2\frac{1}{2} = ?$

C. $? \cdot 2\frac{1}{2} = \frac{1}{4}$

D. $2\frac{1}{2} \div \frac{1}{4} = ?$

Problem 2

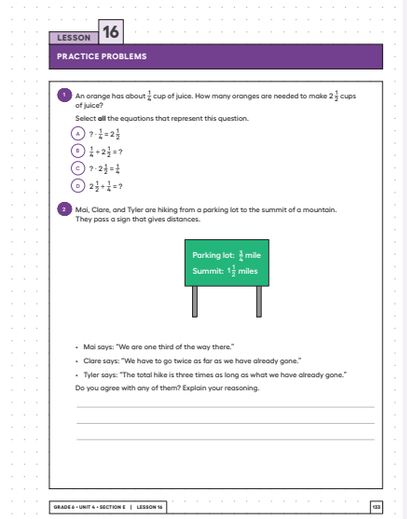
Mai, Clare, and Tyler are hiking from a parking lot to the summit of a mountain. They pass a sign that gives distances.

- Mai says: "We are one third of the way there."
- Clare says: "We have to go twice as far as we have already gone."
- Tyler says: "The total hike is three times as long as what we have already gone."

Do you agree with any of them? Explain your reasoning.

Yes, they are all correct. The total distance in miles from the parking lot to the summit is $\frac{3}{4} + 1\frac{1}{2}$, which is $2\frac{1}{4}$ miles. Mai computed: $\frac{3}{4} = \frac{1}{3} \cdot 2\frac{1}{4}$ (or $\frac{3}{4} \div 2\frac{1}{4} = \frac{1}{3}$). Clare computed: $1\frac{1}{2} = 2 \cdot \frac{3}{4}$. Tyler computed: $2\frac{1}{4} \div \frac{3}{4} = 3$.

Student Workbook


LESSON 16

PRACTICE PROBLEMS

1. An orange has about $\frac{1}{4}$ cup of juice. How many oranges are needed to make $2\frac{1}{2}$ cups of juice?
Select **all** the equations that represent this question.
A. $? \cdot \frac{1}{4} = 2\frac{1}{2}$
B. $\frac{1}{4} \div 2\frac{1}{2} = ?$
C. $? \cdot 2\frac{1}{2} = \frac{1}{4}$
D. $2\frac{1}{2} \div \frac{1}{4} = ?$

2. Mai, Clare, and Tyler are hiking from a parking lot to the summit of a mountain. They pass a sign that gives distances.
Parking lot: $\frac{3}{4}$ mile
Summit: $1\frac{1}{2}$ miles
- Mai says: "We are one third of the way there."
- Clare says: "We have to go twice as far as we have already gone."
- Tyler says: "The total hike is three times as long as what we have already gone."

Do you agree with any of them? Explain your reasoning.

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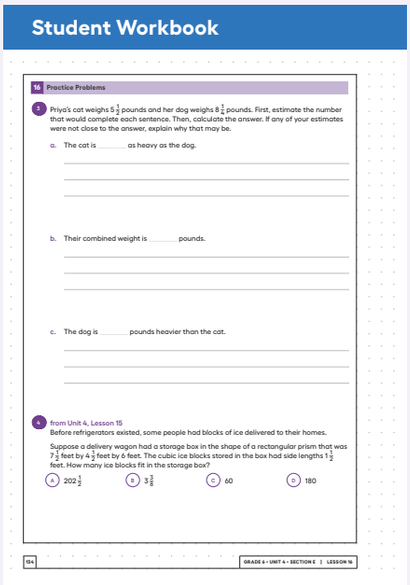
Student Workbook



Practice Problems

3 Priya's cat weighs $5\frac{1}{2}$ pounds and her dog weighs $8\frac{1}{4}$ pounds. First, estimate the number that would complete each sentence. Then, calculate the answer. If any of your estimates were not close to the answer, explain why that may be.

a. The cat is _____ as heavy as the dog.

b. Their combined weight is _____ pounds.

c. The dog is _____ pounds heavier than the cat.

4 from Unit 4, Lesson 15
Before refrigerators existed, some people had blocks of ice delivered to their homes.

Suppose a delivery wagon had a storage box in the shape of a rectangular prism that was $7\frac{1}{2}$ feet by $4\frac{1}{2}$ feet by 6 feet. The cubic ice blocks stored in the box had side lengths $1\frac{1}{2}$ feet. How many ice blocks fit in the storage box?

A $202\frac{1}{2}$ B $3\frac{3}{8}$ C 60 D 180

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Problem 3

Priya's cat weighs $5\frac{1}{2}$ pounds and her dog weighs $8\frac{1}{4}$ pounds. First, estimate the number that would complete each sentence. Then, calculate the answer. If any of your estimates were not close to the answer, explain why that may be.

a. The cat is $\frac{2}{3}$ as heavy as the dog.

Sample response: Estimate: The cat weighs less than the dog but more than half as much, so somewhere between $\frac{1}{2}$ and 1. Calculation: $\left(5\frac{1}{2}\right) \div \left(8\frac{1}{4}\right) = \frac{2}{3}$. This matches the estimate.

b. Their combined weight is $13\frac{3}{4}$ pounds.

Sample response: Estimate: Combined, they weigh more than 13 pounds, almost 14 pounds. Calculation: $5\frac{1}{2} + 8\frac{1}{4} = 13\frac{3}{4}$.

c. The dog is $2\frac{3}{4}$ pounds heavier than the cat.

Sample response: Estimate: The dog weighs about 3 pounds more than the cat—a little less than 3 pounds. Calculation: $8\frac{1}{4} - 5\frac{1}{2} = 2\frac{3}{4}$.

Problem 4

from Unit 4, Lesson 15

Before refrigerators existed, some people had blocks of ice delivered to their homes.

Suppose a delivery wagon had a storage box in the shape of a rectangular prism that was $7\frac{1}{2}$ feet by $4\frac{1}{2}$ feet by 6 feet. The cubic ice blocks stored in the box had side lengths $1\frac{1}{2}$ feet. How many ice blocks fit in the storage box?

A. $202\frac{1}{2}$

B. $3\frac{3}{8}$

C. 60

D. 18

Problem 5

from Unit 4, Lesson 1

Fill in the blanks with 0.001, 0.1, 10, or 1000 so that the value of each quotient is in the correct column.

Close to $\frac{1}{100}$

- 0.1 ÷ 9
- 12 ÷ 1,000

Close to 1

- 0.1 ÷ 0.12
- $\frac{1}{8}$ ÷ 0.1

Greater than 100

- 1,000 ÷ $\frac{1}{3}$
- 700.7 ÷ 0.001 or 0.1

Problem 6

from Unit 3, Lesson 15

A school club sold 300 shirts. 31% were sold to fifth graders, 52% were sold to sixth graders, and the rest were sold to teachers. How many shirts were sold to each group—fifth graders, sixth graders, and teachers? Explain or show your reasoning.

- 93 shirts were sold to fifth graders, because (0.31) · 300 = 93.
- 156 shirts were sold to sixth graders, because (0.52) · 300 = 156.
- 51 shirts were sold to teachers, because 300 − 93 − 156 = 51.

Problem 7

from Unit 2, Lesson 15

Jada has some pennies and dimes. The ratio of Jada's pennies to dimes is 2 to 3.

a. From the information given, can you determine how many coins Jada has?

No, there is not enough information. We only know that for every 2 pennies, there are 3 dimes.

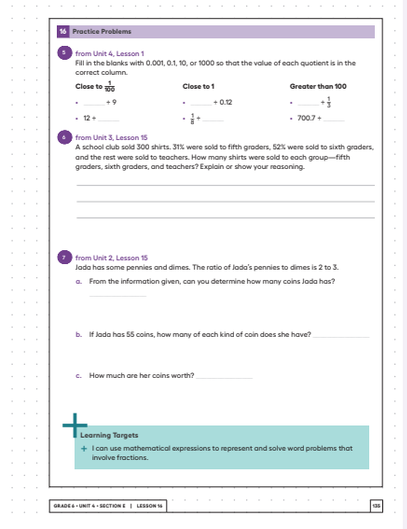
b. If Jada has 55 coins, how many of each kind of coin does she have?

22 pennies and 33 dimes. There are 5 coins total in each group of 2 pennies and 3 dimes. If Jada has 55 coins, that means there are 11 groups, because 55 ÷ 5 = 11. There are 22 pennies (11 · 2 = 22) and 33 dimes (11 · 3 = 33) in total.

c. How much are her coins worth?

$3.52. The 22 pennies are worth $0.22, and the 33 dimes are worth $3.30. 0.22 + 3.30 = 3.52.

Student Workbook


16 Practice Problems

5 from Unit 4, Lesson 1
Fill in the blanks with 0.001, 0.1, 10, or 1000 so that the value of each quotient is in the correct column.

Close to $\frac{1}{100}$: • ____ + 9 • 12 + ____
Close to 1: • ____ + 0.12 • $\frac{1}{8}$ + ____
Greater than 100: • ____ + $\frac{1}{3}$ • 700.7 + ____

6 from Unit 3, Lesson 15
A school club sold 300 shirts. 31% were sold to fifth graders, 52% were sold to sixth graders, and the rest were sold to teachers. How many shirts were sold to each group—fifth graders, sixth graders, and teachers? Explain or show your reasoning.

7 from Unit 2, Lesson 15
Jada has some pennies and dimes. The ratio of Jada's pennies to dimes is 2 to 3.
a. From the information given, can you determine how many coins Jada has?
b. If Jada has 55 coins, how many of each kind of coin does she have?
c. How much are her coins worth?

Learning Targets
+ I can use mathematical expressions to represent and solve word problems that involve fractions.

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