Lesson 14

Solving Percentage Problems

Goals

- Choose and create a tape diagram, double number line diagram, or table to solve problems involving percentages and explain (orally) the solution method.
- Determine what information is needed to solve a problem involving percentages. Ask questions to elicit that information.

Learning Target

I can choose and create diagrams to help me solve problems about percentages.

Lesson Narrative

In this lesson, students solve percentage problems with less scaffolding and engage with all three kinds of questions that are commonly asked in situations where $A\%$ of B is C:

- What is $A\%$ of B?
- What is B if $A\%$ of B is C?
- What percentage of B is C?

In the *Warm-up*, students reason about what percentage one whole number is of another whole number. (Most problems involve benchmark percentages.) In the main activity, an *Info Gap*, students practice identifying information that they need to solve percentage problems, asking for the information and solving the problems.

Unlike in earlier lessons, no blank representations or intermediate prompts are given, so students need to choose approaches that seem appropriate. They may opt to use a previously introduced representation or to reason more directly, by applying insights about the relationship between percentages and quantities from earlier work.

Student Learning Goal

Let's solve more percentage problems.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Cool-down
10 min	25 min	10 min	5 min

Assessment

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap (Activity 1)

Instructional Routines

- MLR4: Information Gap

Required Materials

Materials to Gather

- Math Community Chart: Activity 1

Materials to Copy

- Music Devices Cards (1 copy for every 2 students): Activity 1

Warm-up

Matching the Percentage

10 min

Activity Narrative

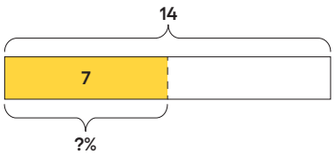
In this activity, students determine what percentage one number is of another number. They answer questions of the form "B is what percentage of C?" and match each question with an answer in a list of percentages.

Each pair of values is related by a factor that is a benchmark fraction, so students are likely to identify the matches quickly. They should spend some time discussing how they reason about each question.

Although students are likely to make the matches mentally, some may find it helpful to draw a tape diagram to visualize the relationship between the numbers.

Launch

Display this tape diagram for all to see.



Ask students to think about what question it could represent and to give a signal if they have a response. Select a couple of students to share their responses. Record questions along the lines of "7 is what percentage of 14?" and display the question(s) for all to see. Invite students to share the answer to the question and to explain how they know.

Tell students that they will now answer similar questions and think about what percentage one number is of another number.

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and another minute to share their responses and reasoning with a partner.

Student Task Statement

Match each question in the left column with a percentage in the right column. One percentage will be left over. Be prepared to explain your reasoning.

1. 5 is what percentage of 20?

C. 25%

Sample reasoning: 5 times 4 is 20, so 5 is $\frac{1}{4}$ (or 25%) of 20.

2. 3 is what percentage of 30?

B. 10%

Sample reasoning: 3 times 10 is 30, so 3 is $\frac{1}{10}$ (or 10%) of 30.

A. 4%

B. 10%

C. 25%

D. 75%

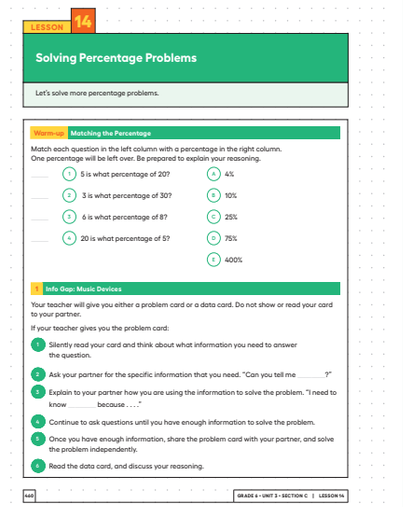
E. 400%

Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Provide Access for Physical Action. Activate or supply background knowledge. Provide students with access to blank tape diagrams to support information processing.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook


LESSON 14

Solving Percentage Problems

Let's solve more percentage problems.

Warm-up Matching the Percentage

Match each question in the left column with a percentage in the right column. One percentage will be left over. Be prepared to explain your reasoning.

1. 5 is what percentage of 20?
2. 3 is what percentage of 30?
3. 6 is what percentage of 8?
4. 20 is what percentage of 5?

A. 4%
B. 10%
C. 25%
D. 75%
E. 400%

1 Info Gap: Music Devices

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

Building on Student Thinking

Because 5 goes into 20 four times, students might rush to say that 5 is 4% of 20. If this happens, encourage students to check their thinking by asking questions such as:

"10 is what percentage of 20?"
"If 10 is 50% of 20, what percentage of 20 is 5?"
"If 5 is 4% of 20, then does that mean 10 is 8% of 20?"

3. 6 is what percentage of 8?

D. 75%

Sample reasoning: 2 is $\frac{1}{4}$ of 8, so 6 is $\frac{3}{4}$ (or 75%) of 8.

4. 20 is what percentage of 5?

E. 400%

Sample reasoning: 5 is 100% of 5, and 20 is 4 times that, so 20 is 400% of 5.

Activity Synthesis

The goal of the discussion is to help students see that a number can be expressed as a percentage of another number by comparing the numbers multiplicatively and using fractions.

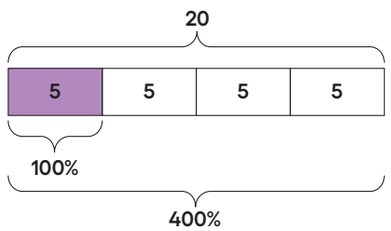
Invite students to briefly share their responses and reasoning. Highlight that when finding what percentage one number is of another number, it can be helpful to think about what fraction one number is of the other number, for instance:

- 5 is what fraction of 20?
- 3 is what fraction of 30?
- 6 is what fraction of 8?

Once we know that 5 is $\frac{1}{4}$ of 20, 3 is $\frac{1}{10}$ of 30, and 6 is $\frac{6}{8}$ (or $\frac{3}{4}$) of 8, we can relate the fractions to the percentages 25% of 20, 10% of 30, and 75% of 8, respectively.

Then, discuss how students reasoned about the last question. If not mentioned by students, emphasize that unlike in the first question, the 5 in this question is the value that corresponds to 100%, so the percentage of 20 of 5 must be greater than 100. Because 20 is 4 · 5, the percentages must also be related by a factor of 4. Consider using a table or a tape diagram to illustrate these relationships.

value	percentage
5	100
20	400



Activity 1

Info Gap: Music Devices

25 min

Activity Narrative

In this activity, students find both A and C where $A\%$ of B is C but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

In each question on the problem cards, the value that corresponds to 100% is different. This means that students who opt to use a diagram or a table need to create a separate representation for each question. Monitor for this awareness.

Also monitor for the strategies that students use when reasoning about \$24 as a percentage of \$25. For instance, students may:

- Create a double number line diagram, draw and label the first few tick marks, see that each tick mark corresponds to \$1 and 4%, bypass drawing the remaining tick marks, and multiply 24 by 4.
- Notice that \$24 is \$1 away from \$25 and subtract 4% from 100%.
- Use a table to find the percentage corresponding to \$1 and then multiply that percentage by 24.

Consider selecting students who use efficient strategies to share during discussion.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Tell students that they will continue to solve problems involving percentages. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it. There is an extra set of cards available for demonstration purposes.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Instructional Routines

MLR4: Information Gap

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL

Access for Multilingual Learners (Activity 1)

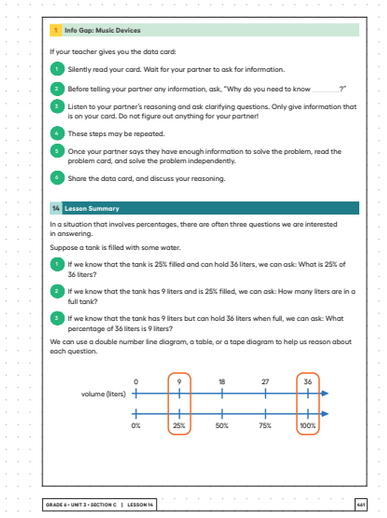
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Workbook



1 Info Gap: Music Devices

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

14 Lesson Summary

In a situation that involves percentages, there are often three questions we are interested in answering.

Suppose a tank is filled with some water.

1. If we know that the tank is 25% filled and can hold 36 liters, we can ask: What is 25% of 36 liters?
2. If we know that the tank has 9 liters and is 25% filled, we can ask: How many liters are in a full tank?
3. If we know that the tank has 9 liters but can hold 36 liters when full, we can ask: What percentage of 36 liters is 9 liters?

We can use a double number line diagram, a table, or a tape diagram to help us reason about each question.

GRADE 6 • UNIT 3 • SECTION C | LESSON 14 441

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

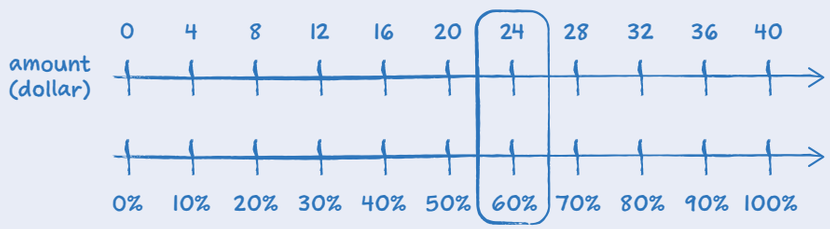
1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

Problem Card 0: $36.90

Sample reasoning: 10% of 41 is $4.10. Tyler has 90% of $41, which is $4.10 less than $41. 41 − 4.10 = 36.90

Problem Card 1:

- Jada can afford only Device A. Sample reasoning: 60% of $40 is $24. Both Device B and Device C cost more than $24.



- Jada has 96% of the money needed for Device B.

Sample reasoning:

- Use a table to find the percentage for 1 dollar and then multiply it by 24.

amount (dollars)	percentage
25	100
1	4
24	96

- Reason that since 100 is 25 · 4, we also multiply 24 by 4.

Problem Card 2: $60

Sample reasoning: If $24 is 40% of the cost, then $6 is 10% of the cost and $60 is 100% of the cost.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Consider inviting previously selected students to share their strategies, ending with methods that are more efficient.

Here are some questions for discussion:

- *"To answer the first question on Problem Card 1, we needed to find 60% of the cost of Device C. In that case, what amount corresponds to 100%?"*

  $40, the cost of Device C

- *"In the second question, what amount corresponds to 100%?"*

  $25, the cost of Device B

- *"In the question on Problem Card 2, what amount corresponds to 100%?"*

  $60, the cost of Device D

Then, display the following questions for all to see. Explain that they represent the questions on the cards but are phrased differently:

- *"What is 60% of $40?"*

  "$24 is what percentage of $25?"

  "If $24 is 40% of a number, what is that number?"

Highlight for students that in each question, we were looking for different pieces of information. In the first question, it was the dollar amount for 60% that was unknown. In the second question, it was the percentage. In the last one, it was the dollar amount for 100%. We can reason about the answer to each question using known strategies and representations.

Building on Student Thinking

If students draw a complete double number line diagram with all 25 tick marks on each line and take the time to label each tick mark by counting by 4s, ask them to explain their strategy. Then, urge them to look for a pattern in their process to help them answer the question more efficiently. For example, consider asking:

"How many times would you need to count by 4s?"
"What might be a quicker way to find the percentage for $24 other than adding 4 twenty-four times?"

Math Community

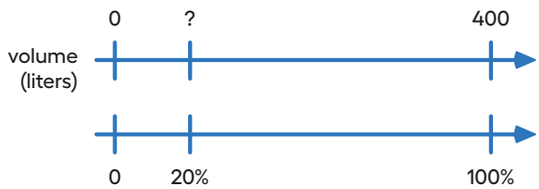
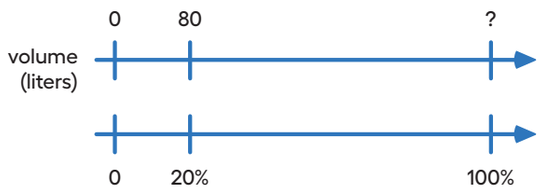
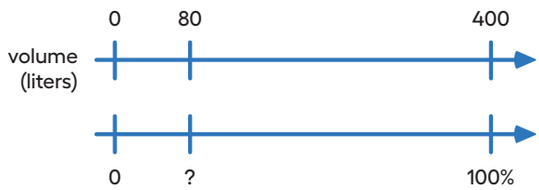
Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- "I noticed our norm '_____' in action today and it really helped me/my group because _____."

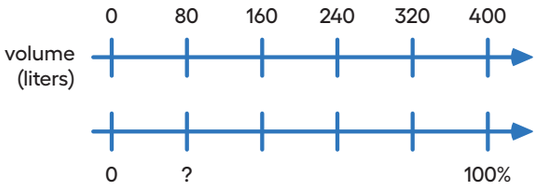
Lesson Synthesis

In situations that can be described with percentages, there are generally three different questions we can ask. To illustrate this point, present a scenario and display a partially labeled double number line diagram. Annotate the diagram with the known and unknown values as each question is discussed.

Suppose a rain barrel has collected some rainwater.

If we know ...	we can ask ...
The barrel is 20% filled and can hold 400 liters when full.	What is 20% of 400 liters? 
The barrel is 20% filled and has 80 liters of water.	How many liters are in a full barrel if 20% of a full barrel is 80 liters? 
The barrel has 80 liters of water and can hold 400 liters when full.	80 liters is what percentage of 400 liters? 

In earlier lessons, we solved the first two kinds of problems in different ways. In this lesson, we also solved the last kind of problem. We can start by asking: "What fraction of 400 is 80?" $80 \div 400 = \frac{1}{5}$, so we can partition into five equal parts the intervals between 0 and 400 and between 0% and 100% on a double number line diagram, as shown here:



Since $\frac{1}{5}$ of 100 is 20, that percentage is 20. We can also use a table:

volume (liters)	percentage
400	100
80	20

Highlight that regardless of the question asked, it is important to think about which quantity corresponds to 100%.

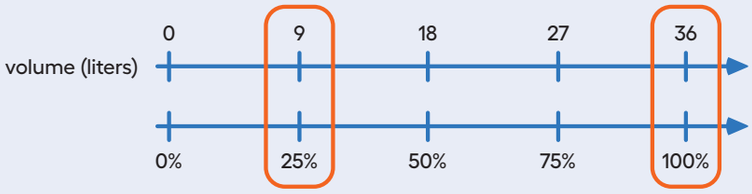
Lesson Summary

In a situation that involves percentages, there are often three questions we are interested in answering.

Suppose a tank is filled with some water.

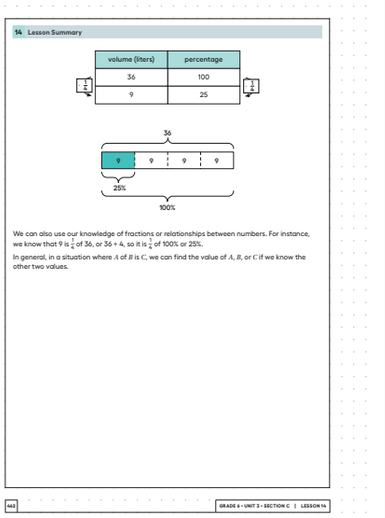
1. If we know that the tank is 25% filled and can hold 36 liters, we can ask: What is 25% of 36 liters?
2. If we know that the tank has 9 liters and is 25% filled, we can ask: How many liters are in a full tank?
3. If we know that the tank has 9 liters but can hold 36 liters when full, we can ask: What percentage of 36 liters is 9 liters?

We can use a double number line diagram, a table, or a tape diagram to help us reason about each question.



volume (liters)	percentage
36	100
9	25

Student Workbook


14 Lesson Summary

volume (liters)	percentage
36	100
9	25

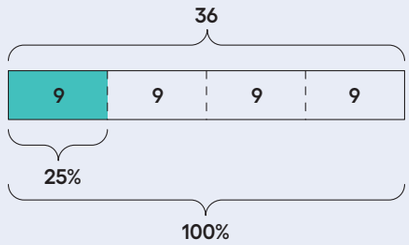
We can also use our knowledge of fractions or relationships between numbers. For instance, we know that 9 is $\frac{1}{4}$ of 36, or 36 ÷ 4, so it is $\frac{1}{4}$ of 100% or 25%.

In general, in a situation where A of B is C, we can find the value of A, B, or C if we know the other two values.

Responding To Student Thinking

Points to Emphasize
If most students struggle with finding a percentage of a given number, elicit strategies for solving percentage problems in which different pieces of information are unknown. For instance, when completing the practice problems in this lesson, ask students to discuss with a partner their understanding of each situation and their reasoning:

Unit 3, Lesson 15 Finding This Percent of That



We can also use our knowledge of fractions or relationships between numbers. For instance, we know that 9 is $\frac{1}{4}$ of 36, or 36 ÷ 4, so it is $\frac{1}{4}$ of 100% or 25%.

In general, in a situation where A of B is C, we can find the value of A, B, or C if we know the other two values.

Cool-down

Walking to School

5 min

Student Task Statement

It takes Jada 20 minutes to walk to school.

1. It takes Andre 80% as long to walk to school. How long does it take Andre to walk to school?

   16 minutes

   Sample reasoning:

   - 10% of 20 minutes is 2 minutes. 8 · 2 = 16, so it takes 16 minutes for Andre to walk to school.
   -

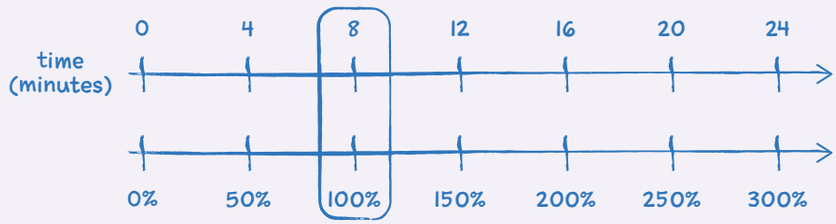
time (minutes)	percentage
20	100
2	10
16	80

2. Jada's walk to school takes 250% as long as Tyler's walk. How long does it take Tyler to walk to school?

   8 minutes

   Sample reasoning:

   - If 20 minutes is 250% of Tyler's walk, then 4 minutes is 50% of Tyler's walk and 8 minutes is 100% of Tyler's walk.
   -



Lesson 14

Practice Problems

7 Problems

Problem 1

For each problem, explain or show your reasoning.

a. 160 is what percentage of 40?

400%

Sample reasoning: 400% of a number is 4 times that number and $4 \cdot 40 = 160$

b. 40 is 160% of what number?

25

Sample reasoning:

value	percentage
40	160
10	40
5	20
25	100

c. What number is 40% of 160?

64

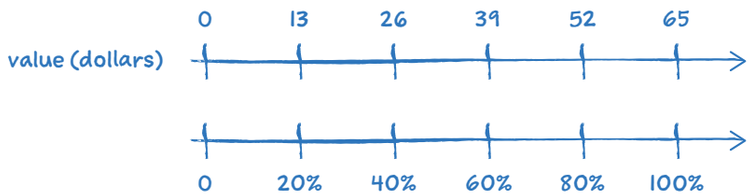
Sample reasoning: 10% of 160 is 16, so 40% of 160 is $4 \cdot 16$, which is 64.

Problem 2

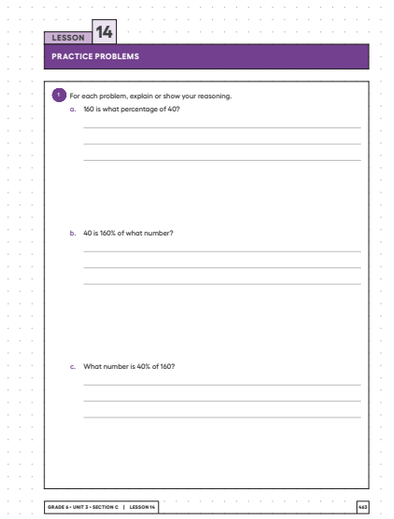
A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $13, what was the original price of her purchase? Explain or show your reasoning.

$65

Sample reasoning: On a double number line diagram, line up 13 and 20%, and then multiply both by 5, which gives 65 and 100%.



Student Workbook


LESSON 14

PRACTICE PROBLEMS

1 For each problem, explain or show your reasoning.

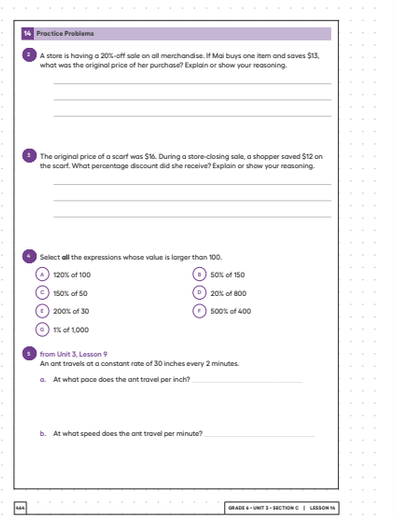
a. 160 is what percentage of 40?

b. 40 is 160% of what number?

c. What number is 40% of 160?

GRADE 6 • UNIT 3 • SECTION C | LESSON 14

Student Workbook


14 Practice Problems

2 A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $13, what was the original price of her purchase? Explain or show your reasoning.

3 The original price of a scarf was $16. During a store-closing sale, a shopper saved $12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.

4 Select all the expressions whose value is larger than 100.

A 120% of 100
B 50% of 150
C 150% of 50
D 20% of 800
E 200% of 30
F 500% of 400
G 1% of 1,000

5 from Unit 3, Lesson 9
An ant travels at a constant rate of 30 inches every 2 minutes.

a. At what pace does the ant travel per inch?

b. At what speed does the ant travel per minute?

Problem 3

The original price of a scarf was $16. During a store-closing sale, a shopper saved $12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.

75%

Sample reasoning:

- 12 is $\frac{3}{4}$ of 16, so 12 is 75% of 16.
- Using a table:

value (dollars)	percentage
16	100
12	75

Problem 4

Select **all** the expressions whose value is larger than 100.

A. 120% of 100

B. 50% of 150

C. 150% of 50

D. 20% of 800

E. 200% of 30

F. 500% of 400

G. 1% of 1,000

Problem 5

from Unit 3, Lesson 9

An ant travels at a constant rate of 30 inches every 2 minutes.

a. At what pace does the ant travel per inch?

The pace is $\frac{1}{15}$ of a minute per inch.

b. At what speed does the ant travel per minute?

The speed is 15 inches per minute.

Problem 6

from Unit 3, Lesson 3

Is 3.5 cups more or less than 1 liter? Explain or show your reasoning.
(Note: 1 cup is approximately 236.6 milliliters.)

Less

Sample reasoning:

cups	milliliters
1	236.6
0.5	118.3
3	709.8
3.5	828.1

Problem 7

from Unit 3, Lesson 1

Name a unit of measurement that is about the same size as each object.

a. The distance of a doorknob from the floor is about 1 yard or meter.

b. The thickness of a fingernail is about 1 millimeter.

c. The volume of a drop of honey is about 1 milliliter.

d. The weight or mass of a pineapple is about kilogram or pound.

e. The thickness of a picture book is about 1 centimeter or inch.

f. The weight or mass of a buffalo is about 1 ton.

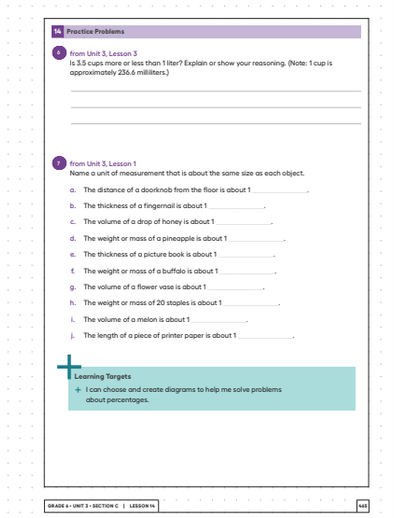
g. The volume of a flower vase is about 1 cup, quart, or liter.

h. The weight or mass of 20 staples is about 1 gram.

i. The volume of a melon is about 1 gallon.

j. The length of a piece of printer paper is about 1 foot.

Student Workbook


14 Practice Problems

6 from Unit 3, Lesson 3
Is 3.5 cups more or less than 1 liter? Explain or show your reasoning. (Note: 1 cup is approximately 236.6 milliliters.)

7 from Unit 3, Lesson 1
Name a unit of measurement that is about the same size as each object.
a. The distance of a doorknob from the floor is about 1 _____.
b. The thickness of a fingernail is about 1 _____.
c. The volume of a drop of honey is about 1 _____.
d. The weight or mass of a pineapple is about 1 _____.
e. The thickness of a picture book is about 1 _____.
f. The weight or mass of a buffalo is about 1 _____.
g. The volume of a flower vase is about 1 _____.
h. The weight or mass of 20 staples is about 1 _____.
i. The volume of a melon is about 1 _____.
j. The length of a piece of printer paper is about 1 _____.

Learning Targets
+ I can choose and create diagrams to help me solve problems about percentages.

GRADE 6 • UNIT 3 • SECTION C | LESSON 14