Lesson 2

Meanings of Division

Goals

- Identify or generate a multiplication equation that represents the same relationship as a division expression, and explain (orally) the reasoning.
- Interpret a division expression in two ways: as an answer to a "How many groups?" question or to a "How many in each group?" question.
- Interpret and create tape diagrams that represent situations involving equal-size groups.

Learning Targets

- I can explain how multiplication and division are related.
- I can explain two ways of interpreting a division expression such as 27 ÷ 3.
- When given a division equation, I can write a multiplication equation that represents the same situation.

Lesson Narrative

In this lesson, students revisit the relationship between multiplication and division to solidify their understanding of the meanings of division. Specifically, students recall that:

- We can think of multiplication as a way to quantify equal-size groups, and that we can find a product if we know the number of groups and the size of each group.
- We can interpret division as a way of finding a missing factor, which can either be the number of groups, or the size of one group.

Students explore these ideas in the context of concrete situations and by using diagrams and equations to support their reasoning. The given quantities are small whole-numbers that are easy to work with mentally, allowing students to focus on the structure that relates the quantities. Understanding this structure will support students in reasoning about division of fractions.

As students represent division situations with diagrams and equations and interpret division equations in context, they practice reasoning abstractly and quantitatively.

Student Learning Goal

Let's explore ways to think about division.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
5 min	25 min	10 min	5 min

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Warm-up)

Instructional Routines

- MLR2: Collect and Display

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

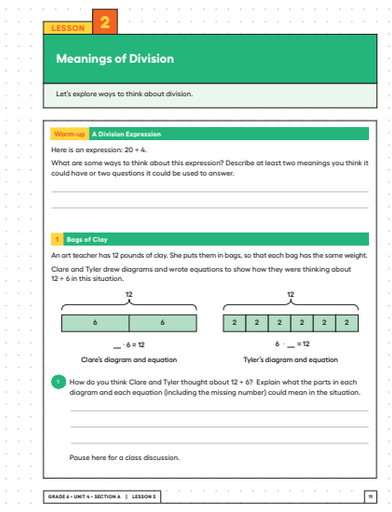
Access for Multilingual Learners (Warm-up)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Building on Student Thinking

Some students may simply write the value of the expression because they are not sure how to describe how they think about it. Encourage them to think of a story with a question, in which the expression could be used to answer the question, or to draw a diagram that illustrates what the numbers could mean.

Student Workbook


LESSON 2

Meanings of Division

Let's explore ways to think about division.

Warm-up A Division Expression

Here is an expression: 20 ÷ 4.

What are some ways to think about this expression? Describe at least two meanings you think it could have or two questions it could be used to answer.

1 Bags of Clay

An art teacher has 12 pounds of clay. She puts them in bags, so that each bag has the same weight.

Clare and Tyler drew diagrams and wrote equations to show how they were thinking about 12 ÷ 6 in this situation.

12

6 6

_____ · 6 = 12

Clare's diagram and equation

12

2 2 2 2 2 2

6 · _____ = 12

Tyler's diagram and equation

1 How do you think Clare and Tyler thought about 12 ÷ 6? Explain what the parts in each diagram and each equation (including the missing number) could mean in the situation.

Pause here for a class discussion.

GRADE 6 • UNIT 4 • SECTION A | LESSON 2 11

Warm-up

A Division Expression

10 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the ways in which students understand and interpret a division expression with whole numbers, preparing them to explore the meanings of division in the lesson.

Launch

Arrange students in groups of 2. Ask students to write at least two different ways in which they think about 20 ÷ 4. Explain that they can write what the expression means to them, how they think about it when finding the value of the expression, or a situation that the expression could represent.

Give students 1–2 minutes of quiet think time, followed by 1 minute of partner discussion.

During the discussion, ask students to share their responses and notice what they have in common.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language that students use to describe their interpretations of 20 ÷ 4. Display words and phrases such as "dividend," "divisor," "equal groups," "split equally into 4 groups (or parts)," and "put into groups of 4."

Student Task Statement

Here is an expression: 20 ÷ 4.

What are some ways to think about this expression? Describe at least two meanings you think it could have or two questions it could be used to answer.

Sample responses:

- It could mean:
  - 20 things divided equally among 4 people or into 4 groups
  - 20 things divided equally so that each person or each group has 4 things
- It could answer questions such as:
  - How many groups of 4 are in 20?
  - How many are in each group if we split 20 into 4 groups?
  - How many 4s are in 20?
  - What times 4 equals 20?
  - What is the length of a rectangle with a width of 4 units and an area of 20 square units?

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask partners to share the interpretations of 20 ÷ 4 that they had in common. Invite students to borrow language from the display as needed.

Record and display students' responses for all to see. Ask students to notice any themes or trends in the range of responses.

Highlight the two ways students will be thinking about division in this unit:

- Division means partitioning a number or a quantity into equal groups and finding out *how many groups can be made*.
- Division means partitioning a number or a quantity into equal groups, and finding out *how much is in each group*.

Activity 1

Bags of Clay

20 min

Activity Narrative

This activity makes explicit the two ways of interpreting division in terms of equal-size groups: as a way to find the size of one group and a way to find the number of groups. Students explore these two ways of seeing division by connecting it to multiplication, making sense of it in the context of a situation, and drawing visual representations.

While tape diagrams are used to help students see what 12 ÷ 6 could mean, students may choose to draw other kinds of diagrams to represent other division expressions in the activity.

As students interpret equations and diagrams that represent the quantities in a situation and create their own representations, they practice reasoning abstractly and quantitatively.

Launch

Keep students in groups of 2. Ask students to keep their materials closed. Display the following question for all to see:

> *"An art teacher has 12 pounds of clay. She puts them in bags, so that each bag has the same weight."*

Ask students:

> *"In terms of pounds and bags of clay, what could 12 ÷ 6 mean?"*

Give students a minute of quiet think time and another minute to explain their thinking to their partner. Ask a few students to share their interpretations with the class. If students do not bring up one of the two ways to interpret the 6, ask them about it:

> *"What else could the 6 mean in this situation?"*

Once students see that the divisor could be interpreted in two ways, give them 5 minutes to complete the first question. Ask students to pause for a brief discussion before moving on.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Develop Expression and Communication.

Develop fluency with tape diagrams. Provide access to blank or partially completed tape diagrams for students to use in answering the questions.

Supports accessibility for: Visual-Spatial Processing, Attention

Building on Student Thinking

Students may interpret $12 \div \frac{1}{2}$ to mean "divide 12 in half," say that its value is 6, and draw a diagram showing 12 partitioned into 2 equal parts. Urge students to revisit their reasoning by comparing $12 \div \frac{1}{2}$ and $12 \div 2$. Discuss questions such as:

"Would $12 \div 2$ and $12 \div \frac{1}{2}$ have the same value? Why or why not?"

No, because the 12 is divided by different numbers, so the quotients will be different.

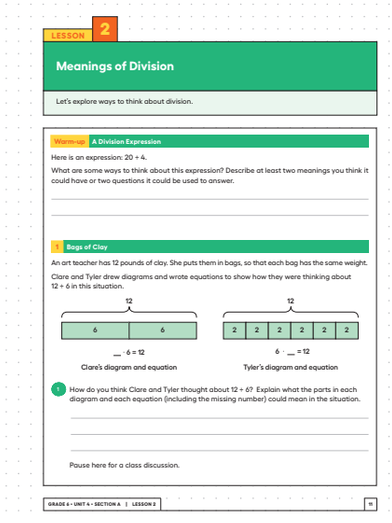
"If $12 \div 2$ can mean '12 divided into groups of 2,' what can $12 \div \frac{1}{2}$ mean?"

12 divided into groups of $\frac{1}{2}$s

"To check if $12 \div 2$ is 6, we can multiply $6 \cdot 2$ to see if it is 12. To see if $12 \div \frac{1}{2}$ equals 6, we can find $6 \cdot \frac{1}{2}$. Does it give 12?"

No, it gives 3, which means $12 \div \frac{1}{2}$ is not 6.

Student Workbook


LESSON 2

Meanings of Division

Let's explore ways to think about division.

Warm-up A Division Expression

Here is an expression: $20 \div 4$.

What are some ways to think about this expression? Describe at least two meanings you think it could have or two questions it could be used to answer.

1 Bags of Clay

An art teacher has 12 pounds of clay. She puts them in bags, so that each bag has the same weight.

Clare and Tyler drew diagrams and wrote equations to show how they were thinking about $12 \div 6$ in this situation.

12 | 6 | 6

$\_ \cdot 6 = 12$

Clare's diagram and equation

12 | 2 | 2 | 2 | 2 | 2 | 2

$6 \cdot \_ = 12$

Tyler's diagram and equation

1 How do you think Clare and Tyler thought about $12 \div 6$? Explain what the parts in each diagram and each equation (including the missing number) could mean in the situation.

Pause here for a class discussion.

GRADE 6 • UNIT 4 • SECTION A | LESSON 2

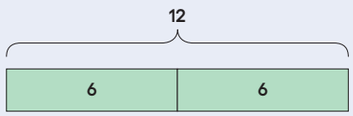
Select students to explain Clare and Tyler's diagrams and equations. Highlight that, in this context, $12 \div 6$ could mean 12 pounds of clay being divided equally into 6 bags, or 12 pounds of clay being divided so that each bag has 6 pounds.

Give students quiet time to complete the rest of the activity.

Student Task Statement

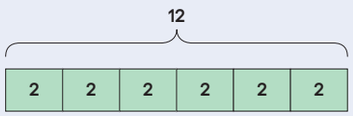
An art teacher has 12 pounds of clay. She puts them in bags, so that each bag has the same weight.

Clare and Tyler drew diagrams and wrote equations to show how they were thinking about $12 \div 6$ in this situation.



$\_ \cdot 6 = 12$

Clare's diagram and equation



$6 \cdot \_ = 12$

Tyler's diagram and equation

1. How do you think Clare and Tyler thought about $12 \div 6$? Explain what the parts in each diagram and each equation (including the missing number) could mean in the situation.

Sample response: Clare might have seen 6 as the number of pounds in a bag, so the missing factor would mean how many bags the teacher has. Tyler might have seen 6 as the number of bags, so the missing factor would mean how many pounds of clay are in each bag.

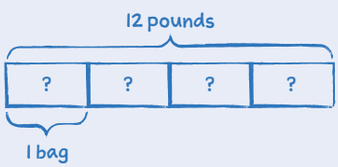
Pause here for a class discussion.

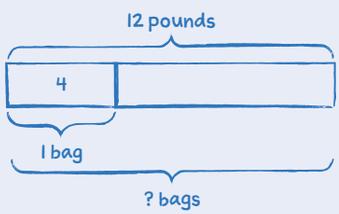
2. What could each division expression mean in the situation with the bags of clay? Draw a diagram and write a multiplication equation to show how you are thinking about each of these expressions.

Any diagram that represents one of the two interpretations of division is acceptable.

a. $12 \div 4$

12 ÷ 4 could mean, "How many pounds in 1 bag if there were 12 pounds in 4 bags?" ($4 \cdot \_\_\_\_ = 12$) or "How many bags can we get if we put 4 pounds in each bag?" ($\_\_\_\_ \cdot 4 = 12$).





b. $12 \div 2$

12 ÷ 2 could mean, "How many pounds in 1 bag if there were 12 pounds in 2 bags?" ($2 \cdot \_\_\_\_ = 12$) or "How many bags can we get if we put 2 pounds in each bag?" ($\_\_\_\_ \cdot 2 = 12$).

c. $12 \div \frac{1}{2}$

$12 \div \frac{1}{2}$ could mean, "How many pounds in 1 bag if there were 12 pounds in $\frac{1}{2}$ bag?" ($\frac{1}{2} \cdot \_\_\_\_ = 12$) or "How many bags can we get if we put $\frac{1}{2}$ pound in each bag?" ($\_\_\_\_ \cdot \frac{1}{2} = 12$).

Are You Ready for More?

A rectangular block of clay is cut into slices.

1. If each slice is $\frac{1}{2}$ of a block, how many slices are there?

   2

2. If each slice is $\frac{1}{5}$ of a block, how many slices are there?

   5

3. What happens to the number of slices as each slice gets smaller?

   The number of slices grows as the slices get smaller. This process is limited by how small the clay can get physically.

4. What would dividing by 0 mean in this situation about slicing a block of clay?

   It would mean dividing the clay into slices of length 0, which doesn't make sense.

Activity Synthesis

The goal of this discussion is to make sure students recognize that a division expression such as 12 ÷ 4 can be interpreted as the answer to the question "4 times what number equals 12?" or the question, "What number times 4 equals 12?"

Select a few students to share their diagrams and equations for the problems in the last question. After each explanation, highlight the connections between the expression, the diagram, and the context.

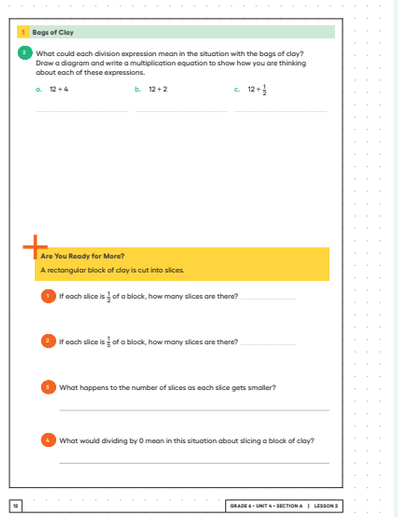
Note that students may write either ___ · 4 = 12 or 4 · ___ = 12 for each interpretation as long as they understand what each factor represents. Because in these materials we tend to say "___ groups of ___," we follow that order in writing the multiplication:

(number of groups) · (size of each group) = total amount

Point out that, in general, division can be interpreted as a way to find two values:

- The size of each group when we know the number of groups and a total amount
- How many groups are in a total amount given the size of one group

Student Workbook



1 Bags of Clay

2 What could each division expression mean in the situation with the bags of clay? Draw a diagram and write a multiplication equation to show how you are thinking about each of these expressions.

a. $12 \div 4$ b. $12 \div 2$ c. $12 \div \frac{1}{3}$

Are You Ready for More?

A rectangular block of clay is cut into slices.

1. If each slice is $\frac{1}{3}$ of a block, how many slices are there?
2. If each slice is $\frac{1}{5}$ of a block, how many slices are there?
3. What happens to the number of slices as each slice gets smaller?
4. What would dividing by 0 mean in this situation about slicing a block of clay?

Lesson Synthesis

The goal of this discussion is to highlight that both multiplication and division can be reasoned in terms of equal-size groups. Just as we can interpret $3 \cdot 5 = 15$ to mean "3 groups of 5 make 15," we can think of division equations such as $15 \div 3 = 5$ in terms of the number of groups and how much is in each group. Diagrams and equations can help us make sense of the numbers and what they mean.

Ask students questions such as:

- *"What are two ways to interpret $20 \div 8$?"*

  "How many groups of 8 are in 20?" or "How much is in each group if there are 20 in 8 groups?"

- *"Suppose we interpret it as 'How much is in each group if there are 20 in 8 groups?' How might we draw a diagram to show this?"*

  A tape that represents 20 divided into 8 equal parts.

- *"What multiplication equation can we write?"*

  Either $? \cdot 8 = 20$ or $8 \cdot ? = 20$ can work, as long as we know what each factor represents.

- *"If we think of it as 'How many groups of 8 are in 20?', how would the diagram be different?"*

  A tape that represents 20 divided into equal parts of 8.

- *"What multiplication equation can we write?"*

  Either $? \cdot 8 = 20$ or $8 \cdot ? = 20$ can work, as long as we are clear what each factor represents.

A note about the term "group:"

Students may be most familiar with the idea of a group as a collection of people or objects. Clarify that the term is used more broadly here. A "group" can refer to a part, a batch, a bag, a section, or another quantity with a particular value. So "equal-size groups" can refer to collections with the same number of items or people in each, as well as parts with the same value, sections of equal length, bags of the same weight, and so on. As students reason about various multiplication and division situations, the meaning of "group" in each situation will become more intuitive.

Lesson Summary

Suppose 24 books are being distributed into boxes. The expression 24 ÷ 3 can be understood in two ways:

- 24 books are distributed equally into 3 boxes, as represented by this diagram:

In this case, the quotient, 8, is the number of books in each box.

- 24 books are distributed into boxes, 3 books in each box, as represented by this diagram:

In this case, the quotient, 8, is the number of boxes used.

These two ways of seeing division are connected to the way 3, 8, and 24 are related by multiplication.

- $3 \cdot 8 = 24$ can be read as "3 groups of 8 make 24."
- $8 \cdot 3 = 24$ can be read as "8 groups of 3 make 24."

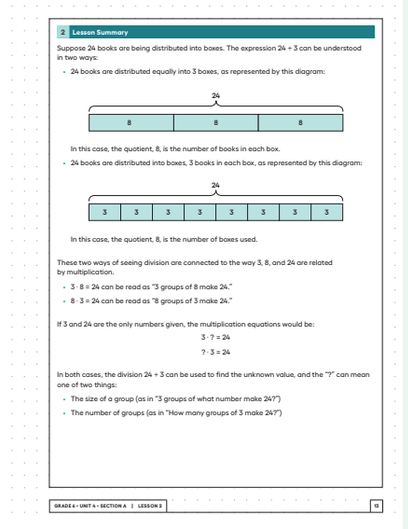
If 3 and 24 are the only numbers given, the multiplication equations would be:

$$3 \cdot ? = 24$$
$$? \cdot 3 = 24$$

In both cases, the division 24 ÷ 3 can be used to find the unknown value, and the "?" can mean one of two things:

- The size of a group (as in "3 groups of what number make 24?")
- The number of groups (as in "How many groups of 3 make 24?")

Student Workbook



2 Lesson Summary

Suppose 24 books are being distributed into boxes. The expression 24 ÷ 3 can be understood in two ways:

- 24 books are distributed equally into 3 boxes, as represented by this diagram:

In this case, the quotient, 8, is the number of books in each box.

- 24 books are distributed into boxes, 3 books in each box, as represented by this diagram:

In this case, the quotient, 8, is the number of boxes used.

These two ways of seeing division are connected to the way 3, 8, and 24 are related by multiplication.

- $3 \cdot 8 = 24$ can be read as "3 groups of 8 make 24."
- $8 \cdot 3 = 24$ can be read as "8 groups of 3 make 24."

If 3 and 24 are the only numbers given, the multiplication equations would be:

$3 \cdot ? = 24$
$? \cdot 3 = 24$

In both cases, the division 24 ÷ 3 can be used to find the unknown value, and the "?" can mean one of two things:

- The size of a group (as in "3 groups of what number make 24?")
- The number of groups (as in "How many groups of 3 make 24?")

GRADE 6 • UNIT 4 • SECTION A | LESSON 2 19

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

Groups on A Field Trip

10 min

Student Task Statement

During a field trip, 60 students are put into equal-size groups.

1. Describe two ways to interpret 60 ÷ 5 in this situation.

   60 ÷ 5 could represent:

   - "How many students are in each group if there are 5 groups?"
   - "How many groups can be formed if there are 5 students per group?"

2. Find the value of the expression. Explain what it could mean in this situation.

   12

   It could mean that there are 12 students in each of the 5 groups, or that there are 12 groups with 5 students in each group.

3. Write a multiplication equation that can describe the same situation

   Any of the following equations are acceptable:

   - $5 \cdot 12 = 60$
   - $5 \cdot 12 = 60$
   - $5 \cdot ? = 60$
   - $? \cdot 5 = 60$

Lesson 2

Practice Problems

7 Problems

Problem 1

Twenty pounds of strawberries are being shared equally by a group of friends. The equation 20 ÷ 5 = 4 represents the division of strawberries.

a. If the 5 represents the number of people, what does the 4 represent?

The number of pounds of strawberry each person got

b. If the 5 represents the pounds of strawberries per person, what does the 4 represent?

The number of friends who were sharing the strawberries

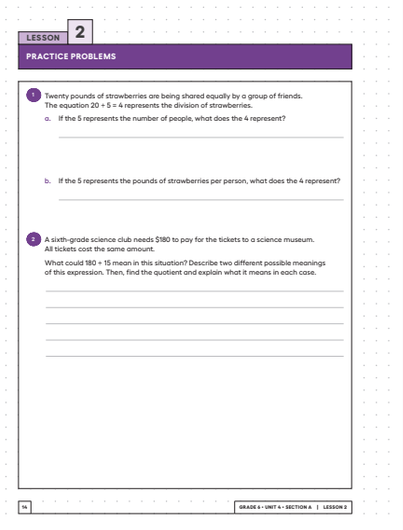
Problem 2

A sixth-grade science club needs $180 to pay for the tickets to a science museum. All tickets cost the same amount.

What could 180 ÷ 15 mean in this situation? Describe two different possible meanings of this expression. Then, find the quotient and explain what it means in each case.

180 ÷ 15 could mean: "How many tickets could the club buy with $180 if each ticket costs $15?" or "How much does each ticket cost if $180 buys 15 tickets?" The answer is 180 ÷ 15 = 12. In the first case, it means the club could buy 12 tickets. In the second, it means each ticket costs $12.

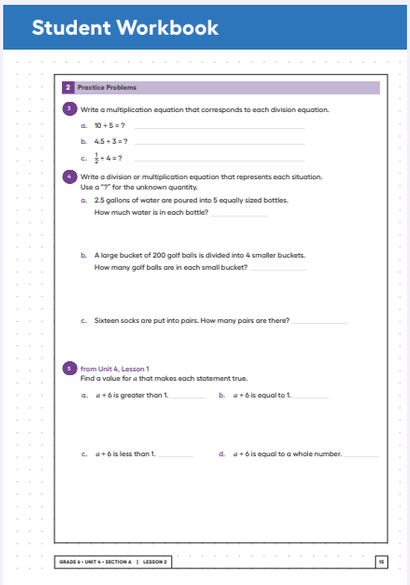
Student Workbook


LESSON 2

PRACTICE PROBLEMS

1. Twenty pounds of strawberries are being shared equally by a group of friends. The equation 20 ÷ 5 = 4 represents the division of strawberries.
   a. If the 5 represents the number of people, what does the 4 represent?
   b. If the 5 represents the pounds of strawberries per person, what does the 4 represent?
2. A sixth-grade science club needs $180 to pay for the tickets to a science museum. All tickets cost the same amount.
   What could 180 ÷ 15 mean in this situation? Describe two different possible meanings of this expression. Then, find the quotient and explain what it means in each case.

Student Workbook



2 Practice Problems

3 Write a multiplication equation that corresponds to each division equation.

a. $10 \div 5 = ?$ _____

b. $4.5 \div 3 = ?$ _____

c. $\frac{1}{2} \div 4 = ?$ _____

4 Write a division or multiplication equation that represents each situation. Use a "?" for the unknown quantity.

a. 2.5 gallons of water are poured into 5 equally sized bottles. How much water is in each bottle? _____

b. A large bucket of 200 golf balls is divided into 4 smaller buckets. How many golf balls are in each small bucket? _____

c. Sixteen socks are put into pairs. How many pairs are there? _____

5 from Unit 4, Lesson 1

Find a value for a that makes each statement true.

a. $a + 6$ is greater than 1. _____ b. $a + 6$ is equal to 1. _____

c. $a + 6$ is less than 1. _____ d. $a + 6$ is equal to a whole number. _____

GRADE 6 • UNIT 4 • SECTION A | LESSON 2 15

Problem 3

Write a multiplication equation that corresponds to each division equation.

a. $10 \div 5 = ?$

$? \cdot 5 = 10$ or $5 \cdot ? = 10$

b. $4.5 \div 3 = ?$

$? \cdot 3 = 4.5$ or $3 \cdot ? = 4.5$

c. $\frac{1}{2} \div 4 = ?$

$? \cdot 4 = \frac{1}{2}$ or $4 \cdot ? = \frac{1}{2}$

Problem 4

Write a division or multiplication equation that represents each situation. Use a "?" for the unknown quantity.

a. 2.5 gallons of water are poured into 5 equally sized bottles. How much water is in each bottle?

$2.5 \div 5 = ?$ or $5 \cdot ? = 2.5$

b. A large bucket of 200 golf balls is divided into 4 smaller buckets. How many golf balls are in each small bucket?

$200 \div 4 = ?$ or $4 \cdot ? = 200$

c. Sixteen socks are put into pairs. How many pairs are there?

$16 \div 2 = ?$ or $? \cdot 2 = 16$

Problem 5

from Unit 4, Lesson 1

Find a value for a that makes each statement true.

a. $a \div 6$ is greater than 1

Any number greater than 6

b. $a \div 6$ is equal to 1

6

c. $a \div 6$ is less than 1

Any number less than 6

d. $a \div 6$ is equal to a whole number

Any positive number that is a multiple of 6

Problem 6

from Unit 3, Lesson 14

Complete the table with percentages of 1. You can write your answer in fraction or decimal form.

value	percentage
1	100%
$\frac{1}{4}$ or 0.25	25%
0.1	10%
$\frac{3}{4}$ or 0.75 (or equivalent)	75%
$\frac{1}{5}$	20%
1.5	150%
$1\frac{4}{10}$ or 1.4 (or equivalent)	140%

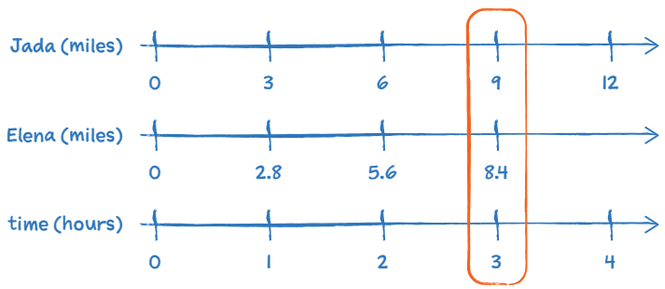
Problem 7

from Unit 3, Lesson 9

Jada walks at a speed of 3 miles per hour. Elena walks at a speed of 2.8 miles per hour. If they both begin walking along a walking trail at the same time, how much farther will Jada walk after 3 hours? Explain your reasoning.

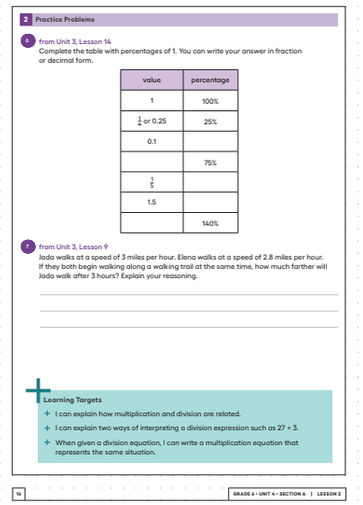
Jada will have walked 0.6 miles farther.

Sample reasoning:



After 3 hours Jada will have walked 9 miles, and Elena will have walked 8.4 miles. 9 − 8.4 = 0.6.

Student Workbook


2 Practice Problems

6 from Unit 3, Lesson 14
Complete the table with percentages of 1. You can write your answer in fraction or decimal form.

value	percentage
1	100%
$\frac{1}{4}$ or 0.25	25%
0.1	
	75%
$\frac{1}{5}$	
1.5	
	140%

7 from Unit 3, Lesson 9
Jada walks at a speed of 3 miles per hour. Elena walks at a speed of 2.8 miles per hour. If they both begin walking along a walking trail at the same time, how much farther will Jada walk after 3 hours? Explain your reasoning.

Learning Targets
+ I can explain how multiplication and division are related.
+ I can explain two ways of interpreting a division expression such as 27 ÷ 3.
+ When given a division equation, I can write a multiplication equation that represents the same situation.

14 GRADE 6 • UNIT 4 • SECTION A | LESSON 2