Lesson 6

Area of Parallelograms

Goals

- Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Apply the formula for the area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.

Learning Target

I can use the area formula to find the area of any parallelogram.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Take Turns
- Which Three Go Together?

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Narrative

This lesson allows students to practice using the formula for the area of parallelograms, and to choose the measurements to use as a base and its corresponding height.

The parallelograms are shown both on and off a grid. As students work to find the areas, they begin to see that some measurements are more helpful than others. For example, if a parallelogram on a grid has a vertical or horizontal side, and one of those sides is used as a base, then both the base and the height can be relatively easily determined. As students think about which measurements to use, they practice looking for and making use of structure.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
5 min	25 min	10 min	5 min

Lesson 6

Area of Parallelograms

Lesson Narrative (continued)

Students also see that parallelograms with the same base and the same height have the same area because the products of those two numbers are equal, even if the parallelograms look very different.

Student Learning Goal

Let’s practice finding the area of parallelograms.

Warm-up

Which Three Go Together: Squares

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four figures with shaded regions. The figures are similar to the ones for which students found the area earlier in the unit. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear how students talk about attributes of figures and two-dimensional regions.

While students may or may not think to compare the four areas, they are likely to notice regions that can be decomposed, rearranged, and subtracted. They may also notice that figures that look different can have the same area. These observations will support students in reasoning about the areas of parallelograms and other polygons later.

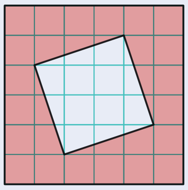
Launch

Arrange students in groups of 2–4. Display the four figures for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three figures that go together and can explain why. Next, tell each student to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A



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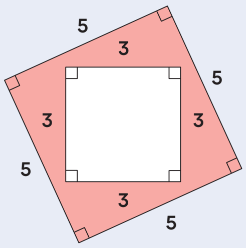

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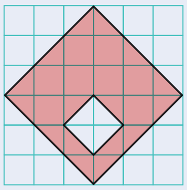

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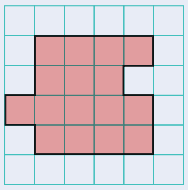

Instructional Routines

Which Three Go Together?

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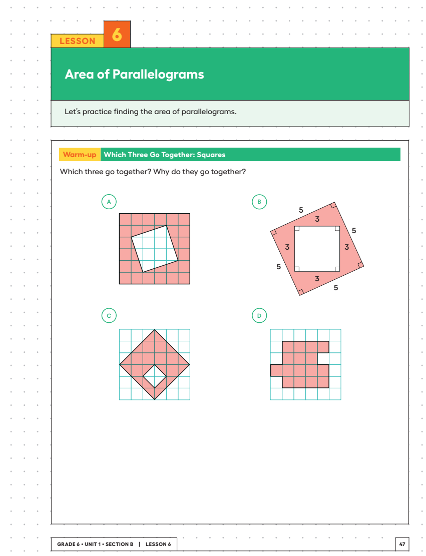
Student Workbook



Instructional Routines

Take Turns

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Sample responses:

A, B, and C go together because:

- They all show two squares of different sizes.
- There is a small square inside a larger square. The smaller square is not shaded.
- There is at least one square that is tilted (or that has no horizontal or vertical sides).

A, B, and D go together because:

- They have shapes with vertical and horizontal sides.
- We can tell the side lengths of the outer shape.

A, C, and D go together because:

- The shapes are on a grid.
- The sides are not labeled with their length.

B, C, and D go together because:

- The area of the shaded region is 16 square units.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, ask students to clarify their reasoning as needed. For example, a student may claim that each of the Figures A, B, and C has a smaller square removed from a larger square. Ask how they know that the smaller unshaded rectangles in Figures A and C are squares.

If no students mentioned the areas of the shaded regions, ask them if and how the areas could be compared. As needed, reiterate strategies for reasoning about area and the idea that different shapes can have the same area.

Activity 1

More Areas of Parallelograms

25 min

Activity Narrative

There is a digital version of this activity.

This activity allows students to practice finding and reasoning about the area of various parallelograms—on and off a grid. Students make sense of the measurements and relationships in the given figures and identify an appropriate pair of base-height measurements to use (the length of a side and that of a segment that is perpendicular to that side). Students learn to recognize that two parallelograms with the same base-height measurements (or with different base-height measurements but the same product) have the same area.

As they work individually, notice how students determine base-height pairs to use. As they work in groups, listen to their discussions and identify those who can explain how they found the area of each of the parallelograms.

In the digital version of the activity, students use an applet to create two parallelograms with the same area.

Launch

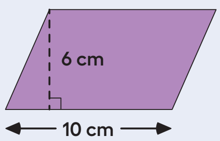
Arrange students in groups of 4. Give each student access to their geometry toolkit and 5 minutes of quiet time to find the areas of the parallelograms in the first question. Then, assign each student one parallelogram (A, B, C or D). Ask the students to take turns explaining to the group how they found the area of their assigned parallelogram. Explain that while one group member explains, the others should listen and make sure they agree. If they don't agree, they should discuss their thinking and work to reach an agreement before moving to the next parallelogram.

Afterward, give students another 5–7 minutes of quiet work time to complete the rest of the activity.

Student Task Statement

1. Find the area of each parallelogram. Show your reasoning.

A



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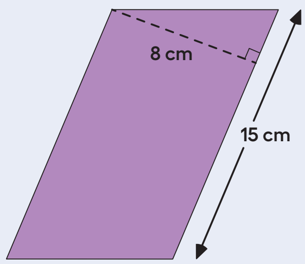

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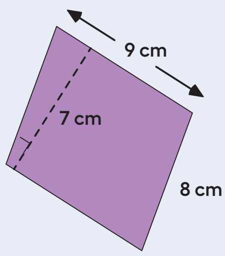

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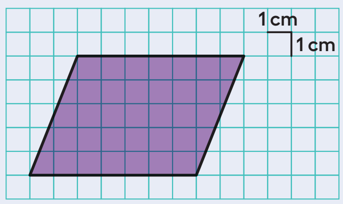


A: $10 \cdot 6 = 60$ square centimeters

B: $15 \cdot 8 = 120$ square centimeters

C: $9 \cdot 7 = 63$ square centimeters

D: $7 \cdot 5 = 35$ square centimeters

2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

12 centimeters. Sample reasoning: We found the area of the parallelogram to be 120 square centimeters. If the side that is 10 centimeters is the base, then $10 \cdot h$ must equal 120, so the height must be $120 \div 10$ or 12 centimeters.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example:

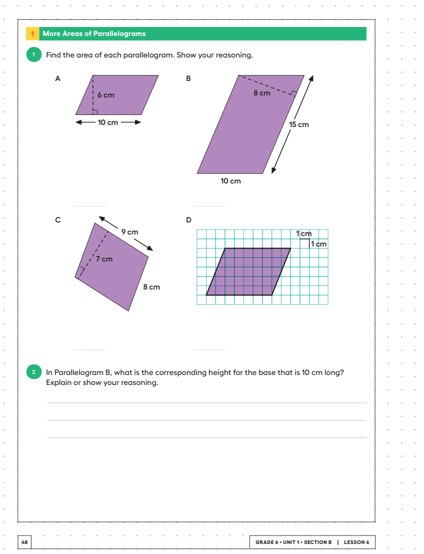
- "First, I ______ because …"
- "Then, I …"
- "I noticed ______ so I …"
- "How did you get …?"

Supports accessibility for: Language, Social-Emotional Functioning

Building on Student Thinking

Some students may continue to use visual reasoning strategies (decomposition, rearranging, enclosing, and subtracting) to find the area of parallelograms. This is fine at this stage, but to help them gradually transition toward abstract reasoning, encourage them to try solving one problem both ways—using visual reasoning and using their generalization about bases and heights from an earlier lesson. They can start with one method and use the other to check their work.

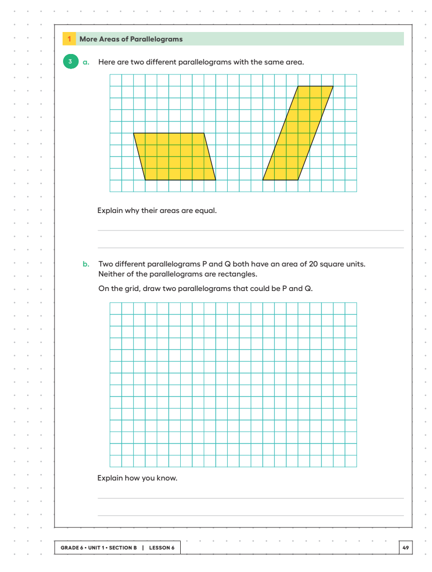
Student Workbook


1 More Areas of Parallelograms

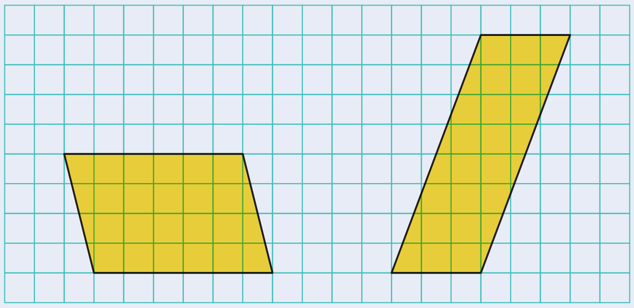
1 Find the area of each parallelogram. Show your reasoning.

2 In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

Student Workbook



3. **a.** Here are two different parallelograms with the same area. Explain why their areas are equal.



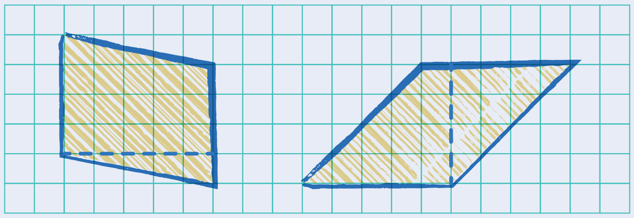
Sample response: They both have an area of 24 square units. The first parallelogram can be decomposed and rearranged into a rectangle that is 6 units by 4 units. The second one can be rearranged into a rectangle that is 3 units by 8 units.

b. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

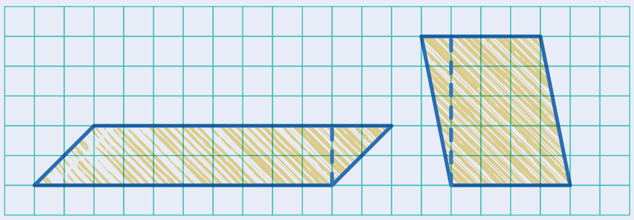
On the grid, draw two parallelograms that could be P and Q. Explain how you know.

Sample responses:

- One parallelogram has a base of 5 units and a height of 4 units. The other has a base that is 4 units and a height that is 5 units. They can both be decomposed and rearranged into a rectangle that is 4 units by 5 units, which has an area of 20 square units.



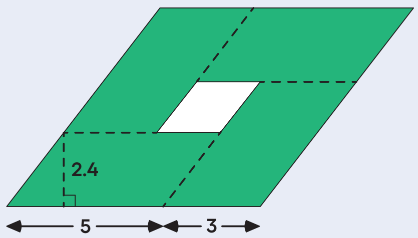
- One parallelogram has a base that is 10 units and a height that is 2 units. The other has a base that is 4 units and a height that is 5 units. They could be P and Q because $10 \cdot 2$ and $4 \cdot 5$ both give 20, which is the area.



- Two parallelograms with equal base and equal height but with different orientations or with the pair of parallel bases positioned differently. They could be P and Q because multiplying the base and height of each parallelogram gives 20, which is the area.

Are You Ready for More?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.



What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

3.2 square inches

Sample reasoning:

- The area of one shaded parallelogram is 12 square inches, because one base is 5 inches and its corresponding height is 2.4 inches $(5 \cdot (2.4) = 12)$. This means the corresponding height for the side that is 3 inches is 4 inches $(3 \cdot 4 = 12)$. The height of the small parallelogram is the difference between 4 inches and 2.4 inches, which is 1.6 inches. The horizontal side of the unshaded parallelogram, which can be a base, is 2 inches $(5 - 3 = 2)$. The area of the unshaded parallelogram is therefore $2 \cdot (1.6)$ or 3.2 square inches.
- The base of the overall parallelogram is 8 inches $(5 + 3 = 8)$. Its height is 6.4 inches $(4 + 2.4 = 6.4)$. Its area is therefore $8 \cdot (6.4)$ or 51.2 square inches. The area of the four shaded parallelograms is $4 \cdot 12$ or 48 square inches. The area of the unshaded region is therefore $51.2 - 48$ or 3.2 square inches.

Activity Synthesis

Use whole-class discussion to highlight three important points:

- We need base and height information to calculate the area of a parallelogram, so we generally look for the length of one side and the length of a perpendicular segment that connects that side to the opposite side. Other measurements may not be as useful.
- A parallelogram has two pairs of base and corresponding height. Both pairs produce the same area, so the product of one pair of numbers should equal the product of the other pair.
- Two parallelograms with different pairs of base and corresponding height can have the same area, as long as their products are equal. A 3-by-6 rectangle and a parallelogram with a base of 1 and a height of 18 will have the same area because $3 \cdot 6 = 1 \cdot 18$.

To highlight the first point, consider asking:

"The parallelograms in the first question show multiple measurements. How did you know which ones would help you find the area?"

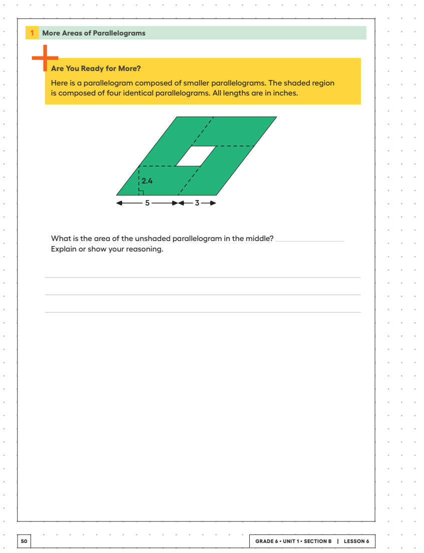
"Which pieces of information in Parallelograms B and C were not needed? Why not?"

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each explanation that is shared about creating two parallelograms of equal area, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

Student Workbook



To highlight the second point, select 1 or 2 students to share how they found the missing height in the second question. Emphasize that the product of 815 and that of 10 and the unknown h must be equal because both give us the area of the same parallelogram.

To highlight the last point, invite a few students to share their pair of parallelograms and how they know that the areas are equal. If not made explicit in students' explanations, stress that the base-height pairs must have the same product. Consider displaying the applet for all to see and using it to facilitate this discussion.

Lesson Synthesis

In this lesson, students used the formula for area to practice finding the area of various parallelograms. Discuss with students:

- *"When a parallelogram is shown on a grid, how do we know which side to choose for a base? Can we use any side?"*

  It is helpful to use a horizontal or a vertical side as a base because it would be easier to tell the length of that side and of its corresponding height.

- *"Off a grid, how do we know which measurements can help us find the area of a parallelogram?"*

  We need the length of one side of the parallelogram and of a perpendicular segment that connects that side to the opposite side.

- *"Do parallelograms that have the same area always look the same?"*

  No.

- *"Can you show an example?"*

  "Do parallelograms that have the same base and height always look the same?"

  No.

- *"Can you show an example?"*

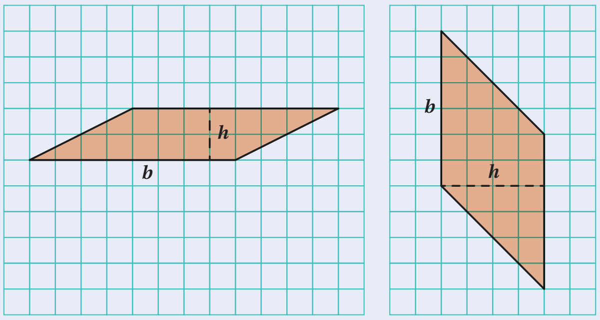
  "How can we draw two different parallelograms with the same area?"

  We can find any two pairs of base-height lengths that have the same product. We can also use the same pair of numbers but draw the parallelograms differently.

Lesson Summary

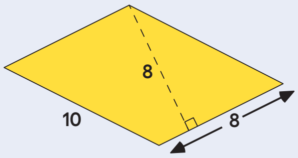
Any pair of a base and a corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily found than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

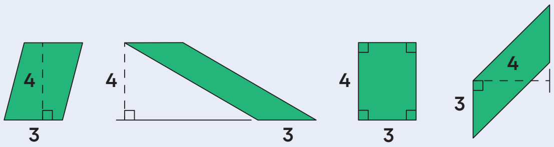


When a parallelogram is *not* drawn on a grid, we can still find its area if we know a base and a corresponding height.

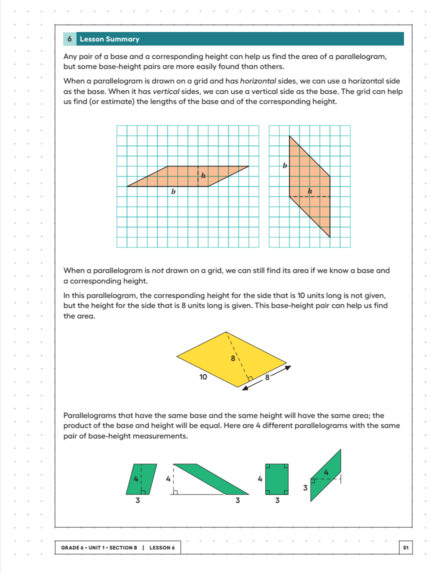
In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.



Parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are 4 different parallelograms with the same pair of base-height measurements.



Student Workbook


Lesson Summary
Any pair of a base and a corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily found than others.
When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.
When a parallelogram is *not* drawn on a grid, we can still find its area if we know a base and a corresponding height.
In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.
Parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are 4 different parallelograms with the same pair of base-height measurements.

Responding To Student Thinking

Points to Emphasize
If students struggle with finding the area of a parallelogram that is not on a grid, integrate discussions about how to find the area of a parallelogram more generally.
For example, when students find the areas of parallelograms in the practice problems of the lesson referred to here, encourage them to decompose and rearrange the parallelograms into rectangles, reason about the bases and heights, and notice regularity in the process.

Unit 1, Lesson 7 From Parallelograms to Triangles

Cool-down

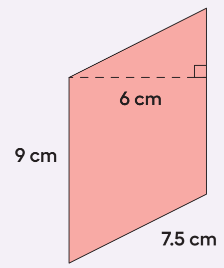
One More Parallelogram

5 min

Launch

Access to geometry toolkits.

Student Task Statement



1. Find the area of the parallelogram. Explain or show your reasoning.

   54 sq cm

   Sample reasoning: A base is 9 cm and its corresponding height is 6 cm. $9 \cdot 6 = 54$.

2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

   The 7.5 cm length was not used

   Sample reasoning:

   - If the side that is 7.5 cm was used to find area, we would need the length of a perpendicular segment between that side and the opposite side as its corresponding height. We don't have that information.
   - The parallelogram can be decomposed and rearranged into a rectangle by cutting it along the horizontal line and moving the right triangle to the bottom side. Doing this means the side that is 7.5 cm is no longer relevant. The rectangle is 6 cm by 9 cm; we can use those side lengths to find area.

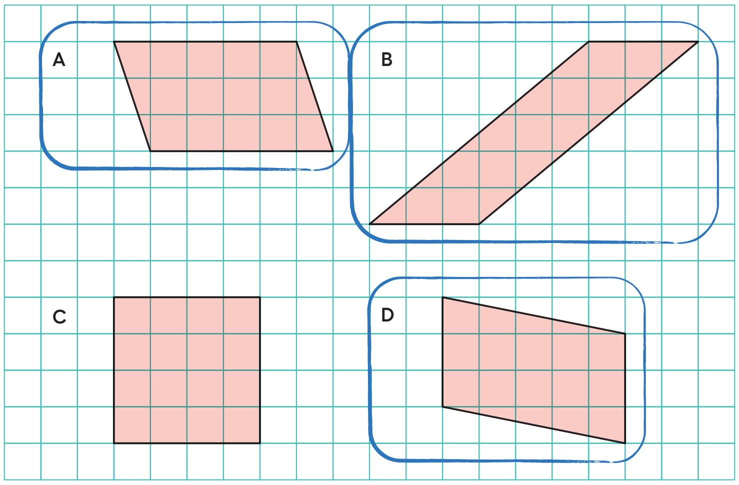
Lesson 6

Practice Problems

6 Problems

Problem 1

Which three of these parallelograms have the same area as each other?



Problem 2

Which pair of base and height produces the greatest area? All measurements are in centimeters.

A. $b = 4$, $h = 3.5$

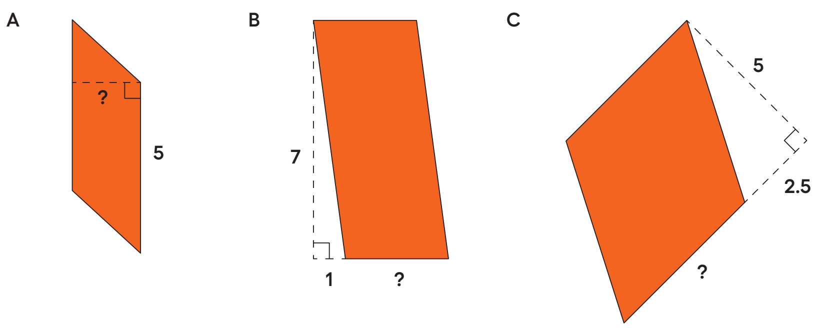
B. $b = 0.8$, $h = 20$

C. $b = 6$, $h = 2.25$

D. $b = 10$, $h = 1.4$

Problem 3

Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.



A. 10 square units

2 units

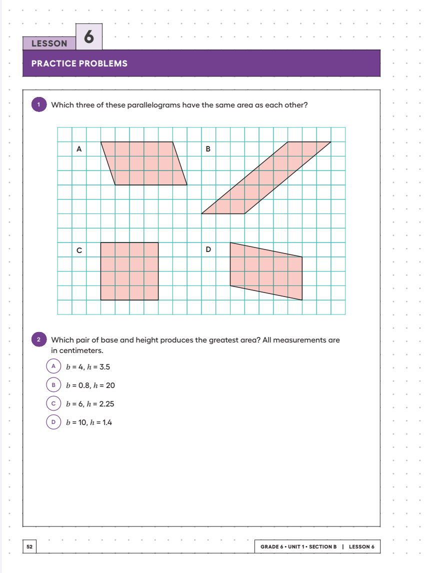
B. 21 square units

3 units

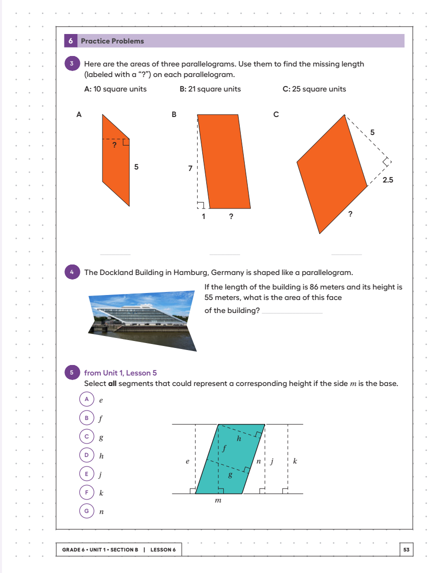
C. 25 square units

5 units

Student Workbook



Student Workbook



Problem 4

The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

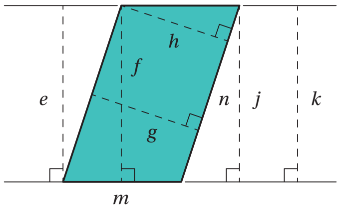
4,730 square meters

Sample reasoning: $86 \cdot 55 = 4{,}730$

Problem 5

from Unit 1, Lesson 5

Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

E. j

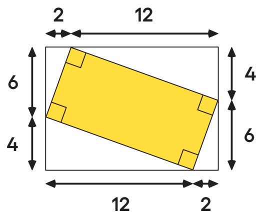
F. k

G. n

Problem 6

from Unit 1, Lesson 3

Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



80 square centimeters

Sample reasoning: The area of the large rectangle is 140 square centimeters, because $14 \cdot 10 = 140$. The areas of the small, unshaded right triangles are each 6 square centimeters, because $6 \cdot 2 \div 2 = 6$. The areas of the larger, unshaded right triangles are each 24 square centimeters, because $4 \cdot 12 \div 2 = 24$. Subtracting the areas of the four unshaded right triangles from the area of the large rectangle gives 80: $140 - 6 - 6 - 24 - 24 = 80$.

Student Workbook