Lesson 3

Interpreting Division Situations (Optional)

Goals

- Create an equation and a diagram to represent a multiplication or division situation, and coordinate these representations (orally).
- Explain (using words and other representations) how to find the unknown quantity in a multiplication or division situation.
- Interpret a verbal description of a situation involving equal-size groups (in spoken or written language), and identify which quantity is unknown—the number of groups, the amount in one group, or the total amount.

Learning Targets

- I can create a diagram or write an equation that represents division and multiplication questions.
- I can decide whether a division question is asking "How many groups?" or "How many in each group?".

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Lesson Narrative

In this lesson, students interpret descriptions of situations involving equal-size groups. To make sense of the relationship between known and unknown quantities, students create their own diagrams or interpret given diagrams and write multiplication and division equations.

As students create representations for division situations and interpret the meaning of their answers in context, they practice reasoning abstractly and quantitatively.

Student Learning Goal

Let's explore situations that involve division.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	20 min	15 min	10 min	5 min

Assessment

Warm-up

Which Three Go Together: Situations with 4

10 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four situations involving equal-size groups. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology related to multiplication, division, and equal-size groups and how they talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the four items for all to see.

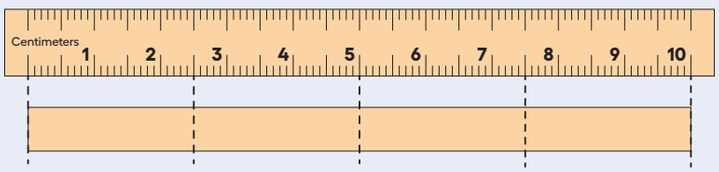
Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three items that go together and can explain why.

Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

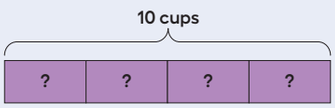
Which three go together? Why do they go together?

A.



B. Jada fills 4 jars with salsa. Each jar has 10 ounces of salsa. How many ounces of salsa are in all the jars?

C.



D. Andre is filling 4-liter jugs with water. How many jugs can he fill if he has 10 liters of water?

Inspire Math

A Fine Line video

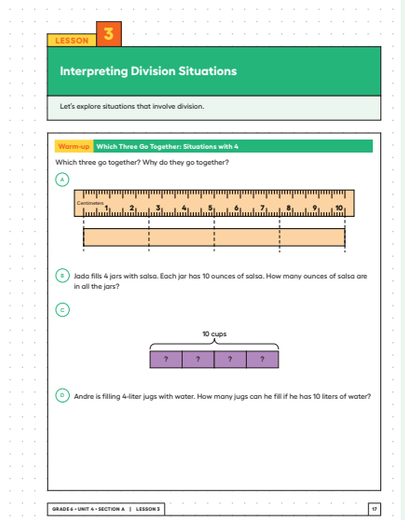
Go Online

Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614145

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 3

Interpreting Division Situations

Let's explore situations that involve division.

Warm-up Which Three Go Together: Situations with 4

Which three go together? Why do they go together?

A

Centimeters 1 2 3 4 5 6 7 8 9 10

B Jada fills 4 jars with salsa. Each jar has 10 ounces of salsa. How many ounces of salsa are in all the jars?

C

10 cups

? ? ? ?

D Andre is filling 4-liter jugs with water. How many jugs can he fill if he has 10 liters of water?

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Sample responses:

A, B, C go together because:

- There are 4 equal-size groups in each situation.

A, B, D go together because:

- We can tell what is happening in each situation and what the equal-size groups represent.

A, C, and D go together because:

- The total amount in each situation is 10 units.
- The unknown number can be found by finding $10 \div 4$, which is 2.5.
- The situations can be represented with the equations $4 \cdot ? = 10$ or $10 \div 4 = ?$.

B, C, D go together because:

- The situations are about volume.
- The situations involve putting equal amounts of something into containers.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology that they use to describe equal-size groups, such as "4 times the same amount," "divide into 4 groups," and "split into 4 equal parts." Ask students to clarify their reasoning as needed. Consider asking:

- *"How do you know ... ?"*
- *"What do you mean by ... ?"*
- *"Can you say that in another way?"*

Activity 1

Making Bubble Mixture

20 min

Activity Narrative

In this activity, students continue to investigate division problems in context, think of them in terms of equal-size groups, and represent them using diagrams and equations.

Both division problems result in quotients that are not whole numbers, but because they represent concrete quantities, students can make sense of the fractional values (number of groups and size of each group) in terms of each situation.

As students work, monitor for students who draw diagrams as they reason about the situations. Any diagram (concrete or abstract) is fine as long as it enables students to make sense of the relationship between the number of groups, the size of one group, and a total amount. Select students whose diagrams clearly show the meanings of division, and ask those students to share later. During class discussion, students will have an opportunity to look more closely at how tape diagrams can support thinking about division.

Launch

Arrange students in groups of 2. Tell students that they will look at two situations about equal-size groups of ingredients for a recipe for making bubble solution.

Ask students if they have had experience making homemade bubble mixture. Invite students to share what they know about the ingredients or the process.

Give students 5–7 minutes of quiet work time and then time to discuss their responses with a partner.

During the partner discussion, ask them to compare their equations and diagrams in the first question, and their interpretations of division in the second question.

If needed, remind students that “fl oz” refers to “fluid ounce” and is a unit to measure the volume of a liquid.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.

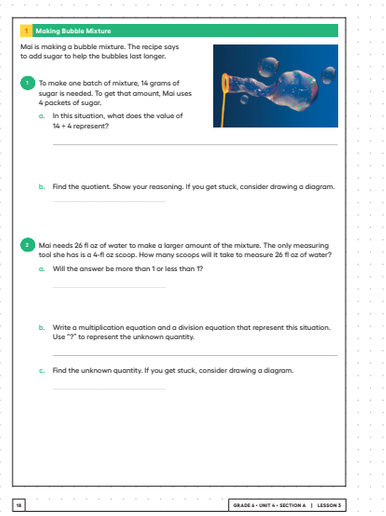
Represent the same information through different modalities using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided. Some students may benefit from support to be able to draw abstract diagrams. For example, demonstrate how 4 ounces of oats can be represented with 1 scoop in a drawing.

Supports accessibility for: Conceptual Processing; Visual-Spatial Processing

Building on Student Thinking

Some students may round their answers to the nearest whole number rather than including a fraction of a scoop or of a gram. Ask students to consider if it is possible to have a part of a scoop or a part of a unit of volume. Encourage them to think about how to show a part of a whole unit on a diagram.

Student Workbook



1 Making Bubble Mixture

Mai is making a bubble mixture. The recipe says to add sugar to help the bubbles last longer.

1. To make one batch of mixture, 14 grams of sugar is needed. To get that amount, Mai uses 4 packets of sugar.
   a. In this situation, what does the value of 14 ÷ 4 represent?
   b. Find the quotient. Show your reasoning. If you get stuck, consider drawing a diagram.
2. Mai needs 26 fl oz of water to make a larger amount of the mixture. The only measuring tool she has is a 4-fl oz scoop. How many scoops will it take to measure 26 fl oz of water?
   a. Will the answer be more than 1 or less than 1?
   b. Write a multiplication equation and a division equation that represent this situation. Use "?" to represent the unknown quantity.
   c. Find the unknown quantity. If you get stuck, consider drawing a diagram.

Student Task Statement

Mai is making a bubble mixture. The recipe says to add sugar to help the bubbles last longer.

1. To make one batch of mixture, 14 grams of sugar is needed. To get that amount, Mai uses 4 packets of sugar.
   a. In this situation, what does the value of 14 ÷ 4 represent?

   The number of grams of sugar in each packet

   b. Find the quotient. Show your reasoning. If you get stuck, consider drawing a diagram.

   $3\frac{1}{2}$

   Sample reasoning: $14 \cdot 2 = 7$, so there are 7 grams of sugar in 2 packets, which means $\frac{7}{2}$ (or $3\frac{1}{2}$) grams in 1 packet.
2. Mai needs 26 fl oz of water to make a larger amount of the mixture. The only measuring tool she has is a 4-fl oz scoop. How many scoops will it take to measure 26 fl oz of water?
   a. Will the answer be more than 1 or less than 1?

   More than 1

   b. Write a multiplication equation and a division equation that represent this situation. Use "?" to represent the unknown quantity.

   $? \cdot 4 = 26$ (or $4 \cdot ? = 26$) and $26 \div 4 = ?$

   c. Find the unknown quantity. If you get stuck, consider drawing a diagram.

   $6\frac{1}{2}$ scoops

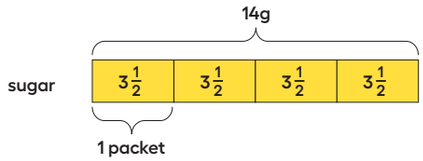
   Sample reasoning: $6 \cdot 4 = 24$ and $\frac{1}{2} \cdot 4 = 2$, so it would take $6 + \frac{1}{2}$, or $6\frac{1}{2}$, scoops to make 26 fl oz.

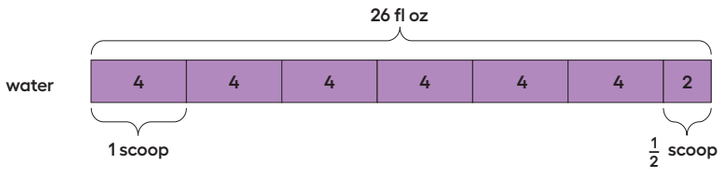
Activity Synthesis

The goals of the discussion are to solidify students' understanding of the two interpretations of division ("How many groups?" and "How much in a group?") and to clarify that division may not result in a whole-number quotient.

Invite students to share their responses and reasoning. Ask students who drew effective diagrams to display and explain them.

If no students drew tape diagrams, display the following diagrams as another way to reason about division.





Discuss what each part of the diagram and each label represent:

- The entire length of the tape represents the total amount.
- Each segment of the tape that is the same size represents one group (a packet or a scoop).
- The number inside each rectangle represents the size of one group, which can be a fraction. 14 grams in 4 packets means $3\frac{1}{2}$ grams in 1 packet.
- The number of segments tells us how many groups there are, which can also be a fraction. There are 6 full scoops and 2 extra fl oz in 26 fl oz. The 2 extra fl oz is $\frac{1}{2}$ scoop. There are $6\frac{1}{2}$ scoops in 26 fl oz.

Ask students how they can tell if the results they get from dividing 14 by 4 and 26 by 4 are correct. If no students mention multiplying each result by 4 to see if the product is 14 and 26, respectively, discuss this idea.

Activity 2: Optional

Revisiting Situations with 4

15 min

Activity Narrative

This activity offers an opportunity to practice making sense of situations involving division and representing them in different ways. Students choose one of four situations they encountered in the *Warm-up*. They create a visual or verbal representation of the situation, write multiplication and division equations to represent the relationships between the quantities, and then answer the question.

Students may use a diagram that they create or the equations that they write to reason about the answer or to find a quotient, but they may also reason in other ways. As students work, monitor for various diagrams, equations, and ways of reasoning. Select students with different representations and reasoning strategies and ask those students to share later.

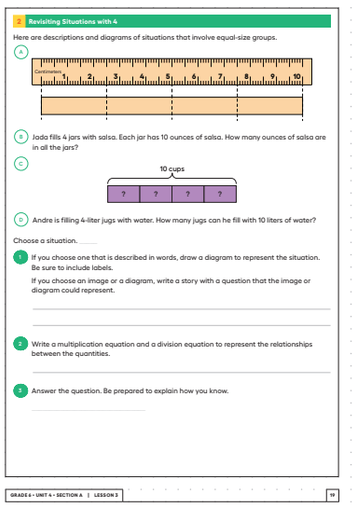
Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.

Provide students with access to one or more equations that show the general relationship between quantities in an equal-group situation, such as number of groups · size of a group = total and total ÷ size of a group = number of groups

Supports accessibility for: Language, Memory

Student Workbook


2 Revisiting Situations with 4

Here are descriptions and diagrams of situations that involve equal-size groups.

A

Centimeters 1 2 3 4 5 6 7 8 9 10

B Jada fills 4 jars with salsa. Each jar has 10 ounces of salsa. How many ounces of salsa are in all the jars?

C

10 cups

? ? ? ?

D Andre is filling 4-liter jugs with water. How many jugs can he fill with 10 liters of water?

Choose a situation.

1 If you choose one that is described in words, draw a diagram to represent the situation. Be sure to include labels.

If you choose an image or a diagram, write a story with a question that the image or diagram could represent.

2 Write a multiplication equation and a division equation to represent the relationships between the quantities.

3 Answer the question. Be prepared to explain how you know.

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Launch

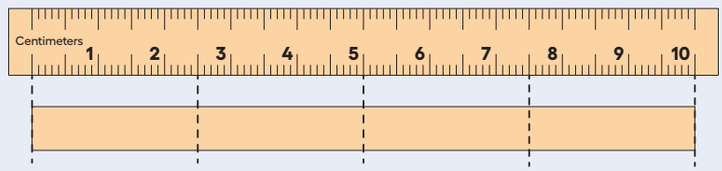
Arrange students in groups of 4. Tell students they will now practice using diagrams, words, and equations to represent four division situations that they encountered earlier. Ask each group member to choose a different situation so that all four options are selected by the group.

Give students 3–4 minutes of quiet work time and then time to share their responses with their group.

Student Task Statement

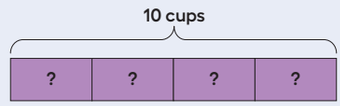
Here are descriptions and diagrams of situations that involve equal-size groups.

A.



B. Jada fills 4 jars with salsa. Each jar has 10 ounces of salsa. How many ounces of salsa are in all the jars?

C.



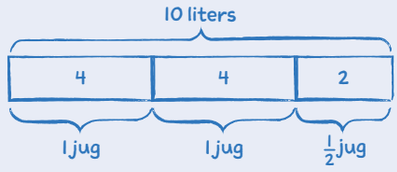
D. Andre is filling 4-liter jugs with water. How many jugs can he fill with 10 liters of water?

Choose a situation.

1. If you choose one that is described in words, draw a diagram to represent the situation. Be sure to include labels.

If you choose an image or a diagram, write a story with a question that the image or diagram could represent.

Answers vary based on the situation chosen. Sample response for situation D:



2. Write a multiplication equation and a division equation to represent the relationships between the quantities.

? · 4 = 10 and 10 ÷ 4 = ?

3. Answer the question. Be prepared to explain how you know.

Andre can fill $2\frac{1}{2}$ 4-liter jugs with 10 liters of water. $2\frac{1}{2} \cdot 4 = 10$

Activity Synthesis

The purpose of this discussion is to help students see more clearly characteristics of division situations and make connections across representations of division.

For each situation, select 1–2 students to share the responses and reasoning. Display their equations and diagrams (or descriptions) for all to see. Then, discuss questions such as:

- *"Why is it possible to think about each situation in terms of division? What is it that they all have in common?"*

  They all involve equal-size groups or some quantity put into equal groups.

- *"What is one group in each situation?"*

  In A, it is the length of each segment. In B, it is the volume of each jar. In C, it is one container represented by a segment in the diagram. In D, it is the 4-liter jug.

Lesson Synthesis

A key takeaway for this discussion is that in division situations that involve equal-size groups, we are not always looking for the same unknown. There are typically three pieces of information involved: the number of groups, the size of each group, and the total amount. Knowing what information we have and what is missing can help us answer questions.

Consider displaying and reading aloud a few more descriptions of situations and asking students to identify what information is unknown in each.

- *"Flour is sold in 3-pound bags. How many pounds are in 7 bags?"*

  The total amount is unknown.

- *"Five tickets to a play cost $38. What does each ticket cost if they all cost the same?"*

  The amount for one ticket is unknown.

- *"One quart is equal to 32 ounces. How many quarts are in 128 ounces?"*

  The number of groups is unknown.

If time permits, ask students to choose a situation and create a diagram and an equation that represent the quantities.

Access for Multilingual Learners (Activity 2, Synthesis)

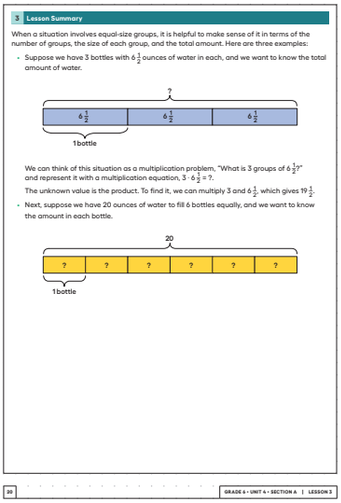
MLR7: Compare and Connect.

Lead a discussion comparing, contrasting, and connecting the different diagrams. Ask,

- *"How are the diagrams (or equations) for the four situations the same? How are they different?"*
- *"Where can we see the number of groups in each description (or diagram)?"*
- *"How do the total amount and the size of each group show up in each description (or diagram)?"*

Advances: Representing, Conversing

Student Workbook


3 Lesson Summary

When a situation involves equal-size groups, it is helpful to make sense of it in terms of the number of groups, the size of each group, and the total amount. Here are three examples:

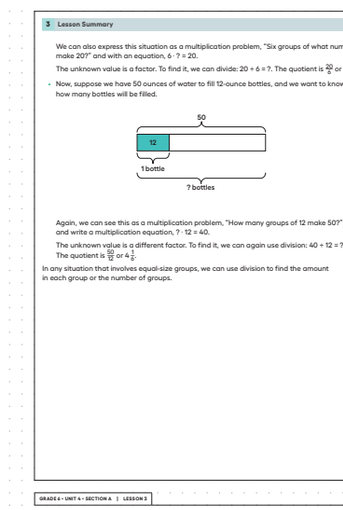
- Suppose we have 3 bottles with $6\frac{1}{2}$ ounces of water in each, and we want to know the total amount of water.

We can think of this situation as a multiplication problem, "What is 3 groups of $6\frac{1}{2}$?" and represent it with a multiplication equation, $3 \cdot 6\frac{1}{2} = ?$.

The unknown value is the product. To find it, we can multiply 3 and $6\frac{1}{2}$, which gives $19\frac{1}{2}$.

- Next, suppose we have 20 ounces of water to fill 6 bottles equally, and we want to know the amount in each bottle.

Student Workbook


3 Lesson Summary

We can also express this situation as a multiplication problem, "Six groups of what number make 20?" and with an equation, $6 \cdot ? = 20$.

The unknown value is a factor. To find it, we can divide: $20 \div 6 = ?$. The quotient is $\frac{20}{6}$ or $3\frac{1}{3}$.

- Now, suppose we have 50 ounces of water to fill 12-ounce bottles, and we want to know how many bottles will be filled.

Again, we can see this as a multiplication problem, "How many groups of 12 make 50?" and write a multiplication equation, $? \cdot 12 = 40$.

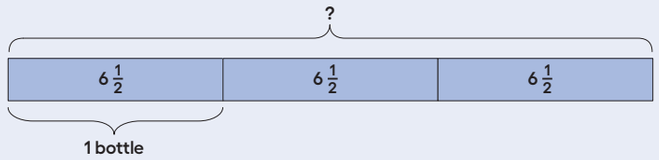
The unknown value is a different factor. To find it, we can again use division: $40 \div 12 = ?$. The quotient is $\frac{50}{12}$ or $4\frac{1}{6}$.

In any situation that involves equal-size groups, we can use division to find the amount in each group or the number of groups.

Lesson Summary

When a situation involves equal-size groups, it is helpful to make sense of it in terms of the number of groups, the size of each group, and the total amount. Here are three examples:

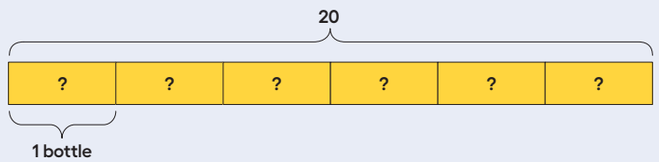
- Suppose we have 3 bottles with $6\frac{1}{2}$ ounces of water in each, and we want to know the total amount of water.



We can think of this situation as a multiplication problem, "What is 3 groups of $6\frac{1}{2}$?" and represent it with a multiplication equation, $3 \cdot 6\frac{1}{2} = ?$.

The unknown value is the product. To find it, we can multiply 3 and $6\frac{1}{2}$, which gives $19\frac{1}{2}$.

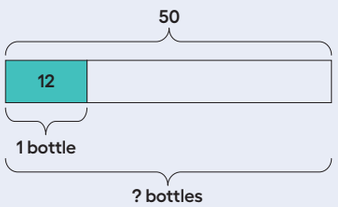
- Next, suppose we have 20 ounces of water to fill 6 bottles equally, and we want to know the amount in each bottle.



We can also express this situation as a multiplication problem, "Six groups of what number make 20?" and with an equation, $6 \cdot ? = 20$.

The unknown value is a factor. To find it, we can divide: $20 \div 6 = ?$. The quotient is $\frac{20}{6}$ or $3\frac{1}{3}$.

- Now, suppose we have 50 ounces of water to fill 12-ounce bottles, and we want to know how many bottles will be filled.



Again, we can see this as a multiplication problem, "How many groups of 12 make 50?" and write a multiplication equation, $? \cdot 12 = 40$.

The unknown value is a different factor. To find it, we can again use division: $40 \div 12 = ?$. The quotient is $\frac{50}{12}$ or $4\frac{1}{6}$.

In any situation that involves equal-size groups, we can use division to find the amount in each group or the number of groups.

Cool-down

Rice in Bags

5 min

Student Task Statement

Andre poured 27 ounces of rice into 6 bags. If all bags have the same amount of rice, how many ounces are in each bag?

1. Write an equation to represent the situation. Use a "?" to represent the unknown quantity.

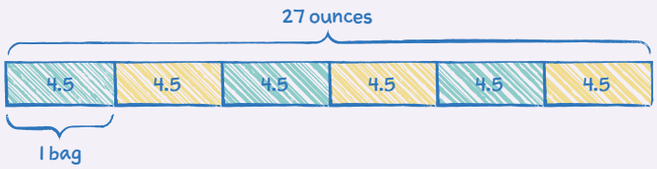
   Sample responses:

   - $6 \cdot ? = 27$
   - $? \cdot 6 = 27$
   - $27 \div 6 = ?$

2. Find the unknown quantity. Show your reasoning.

   $4\frac{1}{2}$ ounces (or equivalent)

   Sample reasoning:

   - 

   - If Andre put 4 ounces in each bag, that's 24 ounces in 6 bags. Splitting the remaining 3 ounces into 6 bags means putting $\frac{1}{2}$ ounce more in each bag.
   - If there were 3 bags, each bag would have $27 \div 3$ or 9 ounces. Splitting each 9 ounces into 2 bags gives 6 bags with 4.5 ounces in each.

Responding To Student Thinking

Points to Emphasize

If most students struggle with interpreting the situation in the *Cool-down* in terms of multiplication and division, reinforce the idea of equal-size groups in context. For example, when reviewing the practice problem referred to here, ask students how they know which quantity makes a group and how to represent its relationship to other quantities.

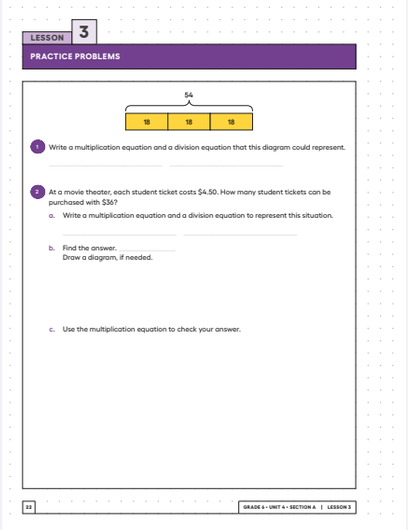
Grade 6, Unit 4, Lesson 4, Practice Problem 5

Lesson 3

Practice Problems

7 Problems

Student Workbook


LESSON 3

PRACTICE PROBLEMS

54

18 | 18 | 18

1. Write a multiplication equation and a division equation that this diagram could represent.

2. At a movie theater, each student ticket costs $4.50. How many student tickets can be purchased with $36?
   a. Write a multiplication equation and a division equation to represent this situation.
   b. Find the answer. Draw a diagram, if needed.
   c. Use the multiplication equation to check your answer.

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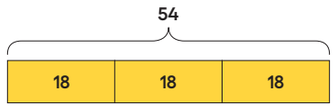
Problem 1

Write a multiplication equation and a division equation that this diagram could represent.



Multiplication: $3 \cdot 18 = 54$ (or $18 \cdot 3 = 54$), division: $54 \div 18 = 3$ (or $54 \div 3 = 18$)

Problem 2

At a movie theater, each student ticket costs $4.50. How many student tickets can be purchased with $36?

a. Write a multiplication equation and a division equation to represent this situation.

Multiplication: $? \cdot (4.50) = 36$ (or equivalent)
Division: $36 \div 4.50 = ?$ (or equivalent)

b. Find the answer. Draw a diagram, if needed.

8 student tickets

Sample reasoning:

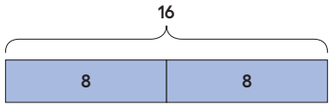
c. Use the multiplication equation to check your answer.

8 is correct because $8 \cdot (4.50) = 36$.

Problem 3

Kiran said that this diagram can show the solution to 16 ÷ 8 = ? or 16 ÷ 2 = ?, depending on how we think about the equations and the "?".

Explain or show how Kiran is correct.



The diagram can illustrate 16 ÷ 8 = ? if we interpret the equation and the "?" to mean: "How many groups of 8 are in 16?" The diagram can illustrate 16 ÷ 2 = ? if we interpret the equation and the "?" to mean: "What is in each group if 16 is divided into 2 equal groups?"

Problem 4

from Unit 4, Lesson 2

Write a sentence describing a situation that could be represented by the equation $4 \div 1\frac{1}{3} = ?$.

Sample responses:

- A group of friends met for lunch and got 4 small pizzas to share. Each person had $1\frac{1}{3}$ pizzas. How many friends went for lunch?
- A baker is filling equal-sized containers with sugar. Four pounds of sugar fill $1\frac{1}{3}$ containers. How many pounds fit in each container?

Problem 5

from Unit 4, Lesson 1

Noah said, "When you divide a number by a second number, the result will always be smaller than the first number."

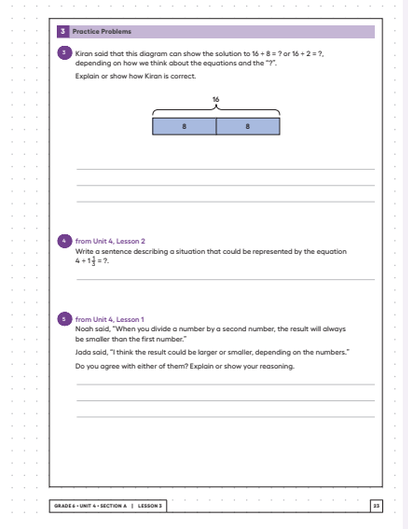
Jada said, "I think the result could be larger or smaller, depending on the numbers."

Do you agree with either of them? Explain or show your reasoning.

I agree with Jada

Sample reasoning: If a number is divided by a number that is between 0 and 1, then the result is bigger than the first number. For example, 1 ÷ 0.1 = 10, which is bigger than 1. But 1 ÷ 2 = 0.5, which is smaller than 1.

Student Workbook


3 Practice Problems

3 Kiran said that this diagram can show the solution to 16 ÷ 8 = ? or 16 ÷ 2 = ?, depending on how we think about the equations and the "?".
Explain or show how Kiran is correct.

16
8 8

4 from Unit 4, Lesson 2
Write a sentence describing a situation that could be represented by the equation $4 \div 1\frac{1}{3} = ?$.

5 from Unit 4, Lesson 1
Noah said, "When you divide a number by a second number, the result will always be smaller than the first number."
Jada said, "I think the result could be larger or smaller, depending on the numbers."
Do you agree with either of them? Explain or show your reasoning.

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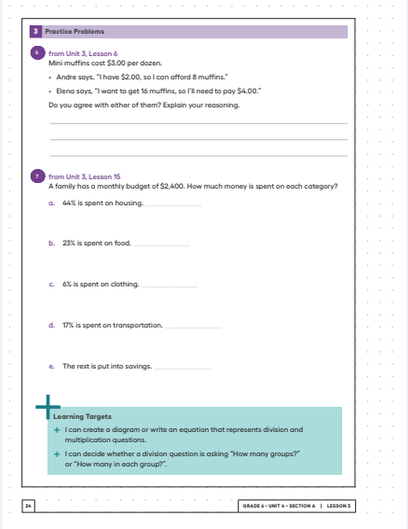
Student Workbook



3 Practice Problems

6 from Unit 3, Lesson 6
Mini muffins cost $3.00 per dozen.
- Andre says, "I have $2.00, so I can afford 8 muffins."
- Elena says, "I want to get 16 muffins, so I'll need to pay $4.00."

Do you agree with either of them? Explain your reasoning.

7 from Unit 3, Lesson 15
A family has a monthly budget of $2,400. How much money is spent on each category?

a. 44% is spent on housing.

b. 23% is spent on food.

c. 6% is spent on clothing.

d. 17% is spent on transportation.

e. The rest is put into savings.

Learning Targets
+ I can create a diagram or write an equation that represents division and multiplication questions.
+ I can decide whether a division question is asking "How many groups?" or "How many in each group?".

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Problem 6

from Unit 3, Lesson 6

Mini muffins cost $3.00 per dozen.

- Andre says, "I have $2.00, so I can afford 8 muffins."
- Elena says, "I want to get 16 muffins, so I'll need to pay $4.00."

Do you agree with either of them? Explain your reasoning.

They are both correct

Sample reasoning: Each muffin costs 25 cents because $3 \div 12 = 0.25$. Andre can afford 8 muffins because $2 \div 0.25 = 8$, and Elena will need 4 dollars because $16 \cdot 0.25 = 4$.

Problem 7

from Unit 3, Lesson 15

A family has a monthly budget of $2,400. How much money is spent on each category?

a. 44% is spent on housing.

$1,056, because $(0.44) \cdot 2,400 = 1,056$

b. 23% is spent on food.

$552, because $(0.23) \cdot 2,400 = 552$

c. 6% is spent on clothing.

$144, because $(0.06) \cdot 2,400 = 144$

d. 17% is spent on transportation.

$408, because $(0.17) \cdot 2,400 = 408$

e. The rest is put into savings.

$240, because there is 10% remaining for savings, and $(0.1) \cdot 2,400 = 240$