Lesson 17

A Fermi Problem

Goals

- Apply reasoning developed throughout this unit to an unfamiliar problem.
- Decide what information is needed to solve a real-world problem.
- Make simplifying assumptions about a real-world situation.

Learning Targets

- I can apply what I have learned about ratios and rates to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem about ratios and rates.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- Poll the Class

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Narrative

In this lesson, students apply the strategies for reasoning about situations involving equivalent ratios to solve an unfamiliar, Fermi-type problem. Students must take a problem that is not well-posed and simplify the problem so that it can be solved.

To understand the problem, students need to break down a larger question into more-manageable sub-questions, which requires sense making and perseverance. To solve the problem, they need to make assumptions and approximations, plan an approach, and reason with the mathematics they know. Engineers, computer scientists, physicists, and economists often make simplifying assumptions as they use mathematics to tackle complex problems. This process is central to mathematical modeling.

Later in the course, students will have more opportunities to explore and solve Fermi problems and engage in mathematical modeling.

Student Learning Goal

Let's solve a Fermi problem.

Lesson Timeline

5 min	40 min	40 min	10 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis

Inspire Math

Alligators video

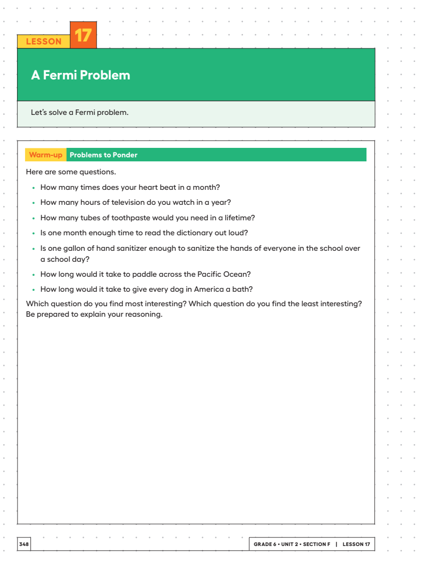
Go Online

Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614227

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 17

A Fermi Problem

Let's solve a Fermi problem.

Warm-up Problems to Ponder

Here are some questions.

- How many times does your heart beat in a month?
- How many hours of television do you watch in a year?
- How many tubes of toothpaste would you need in a lifetime?
- Is one month enough time to read the dictionary out loud?
- Is one gallon of hand sanitizer enough to sanitize the hands of everyone in the school over a school day?
- How long would it take to paddle across the Pacific Ocean?
- How long would it take to give every dog in America a bath?

Which question do you find most interesting? Which question do you find the least interesting? Be prepared to explain your reasoning.

Warm-up

Problems to Ponder

5 min

Activity Narrative

This *Warm-up* allows students to preview examples of Fermi problems, two of which they will explore further in the next activity. The questions are presented with a series of open-ended questions. As they identify questions that they find the most and the least intriguing, students are likely to notice that answering the questions requires finding some missing information and making some assumptions.

Launch

Arrange students in groups of 2–4.

Give students a minute of quiet think time and another minute to share their responses with their group.

Student Task Statement

Here are some questions.

- How many times does your heart beat in a month?
- How many hours of television do you watch in a year?
- How many tubes of toothpaste would you need in a lifetime?
- Is one month enough time to read the dictionary out loud?
- Is one gallon of hand sanitizer enough to sanitize the hands of everyone in the school over a school day?
- How long would it take to paddle across the Pacific Ocean?
- How long would it take to give every dog in America a bath?

Which question do you find most interesting? Which question do you find the least interesting? Be prepared to explain your reasoning.

Answers vary.

Activity Synthesis

Invite 1–2 students to share the questions they find most interesting and why. For each question shared, ask if others in the class also selected that question and whether they did so for a different reason. Repeat the steps with the questions that students find the least interesting.

Explain that these questions are called "Fermi problems," named after Enrico Fermi, an Italian physicist. Fermi loved to think up and discuss problems that are impossible to measure directly but can be roughly estimated using known facts and calculations. Tell students that they will now explore a Fermi problem together.

Activity 1
Solving a Fermi Problem

40 min

Activity Narrative

In this activity, students are introduced to the type of thinking useful for solving Fermi problems. Students see different ways to break a Fermi problem down into smaller questions that can be measured, estimated, or calculated. Then, students work on answering those questions to solve the problem.

As students work, notice the range of their estimates and the sub-questions they formulate to help them solve the larger problem. Some examples of productive sub-questions might be:

- What information do we *know*?
- What information can be *measured*?
- What information cannot be measured but can be *calculated*?
- What *assumptions* should we make?

Part of the appeal of Fermi problems is in making estimates for some things that in modern times we could easily look up. Challenge students to work without performing any internet searches.

Launch

Arrange students in groups of 3–4. As a class, decide on a Fermi problem to explore. Display it for all to see and prompt students to record it in the first graphic organizer in their workbook.

To orient students to the process of solving a Fermi problem, guide them through a series of discussions:

- Ask students:

"What are some pieces of information you would need to know to solve this problem?"

Give students 2–3 minutes of quiet think time and then time to discuss with their group.

Invite groups to share their responses with the class. If their responses are not in the form of questions, reframe them as such and record them for all to see. (For example, if a group says, "we need to know the number of students in the school," reframe it as "how many students are in the school?")

Explain that asking and answering smaller questions or sub-questions is a necessary process in solving Fermi problems. Instruct students to record relevant sub-questions in the first graphic organizer.

- Next, ask students:

"Which sub-question would you answer first? Which would you answer next?"

Give groups 1–2 minutes to discuss how the sub-questions should be organized or ordered and then time to share with the class.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.
Ask students to read the directions for the activity aloud to their group. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language, Attention

Instruct students to label the sub-questions in the order in which they think the questions should be answered.

- Ask students:

"Are there gaps in your sub-questions? Are there additional pieces of information you would need to solve the Fermi problem?"

Instruct students to write new sub-questions to fill any gaps they notice.

- Select 2–3 sub-questions that can be answered in different ways, such as by researching a fact, measuring, estimating, calculating, making an assumption, or a combination of these. Then, ask students:

"How would you go about answering these questions? What are some ways to get the information you need?"

Give groups 1–2 minutes to brainstorm and time to share their ideas with the class.

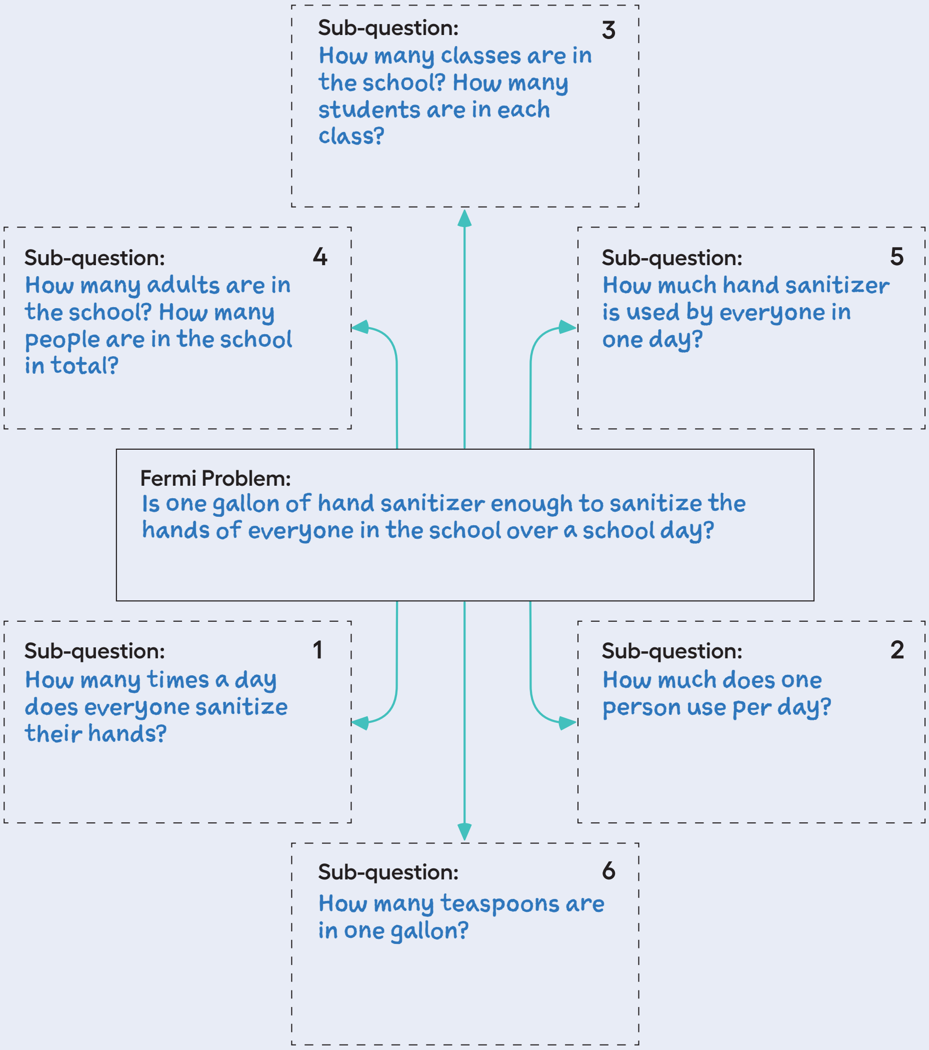
Solicit various ways to attain information or perform research besides using the internet.

If the ideas of estimating and making assumptions are not mentioned, ask students about them. If needed, give an example of an assumption and an estimate that might be needed to solve the Fermi problem.

Tell students to work with their group to complete the rest of the activity. Consider preparing resources to support students with their investigation. For instance, if the problem pursued is about paddling across the Pacific Ocean, a globe or a world map would be helpful. Students may also need access to tools such as stopwatches, tape measures, calculators, and measurement conversion charts.

Student Task Statement

1. What are some smaller questions or sub-questions to figure out before solving the chosen Fermi problem? Record the Fermi problem and your sub-questions here.



Answers vary depending on the Fermi problem chosen.

Sample response:

See graphic organizer.

2. Think about how the sub-questions should be organized. Label each sub-question to show the order in which they should be answered.

   If you notice a gap in your sub-questions (or that some information is needed before the next sub-question could be answered), write a new sub-question to fill the gap.

3. Let's start answering the sub-questions! Use the given organizer.

   Write your sub-questions in order.

- Find the information you need to answer each sub-question. Research, measure, estimate, and perform any necessary calculations.
- Record any fact you find and any assumption you make.

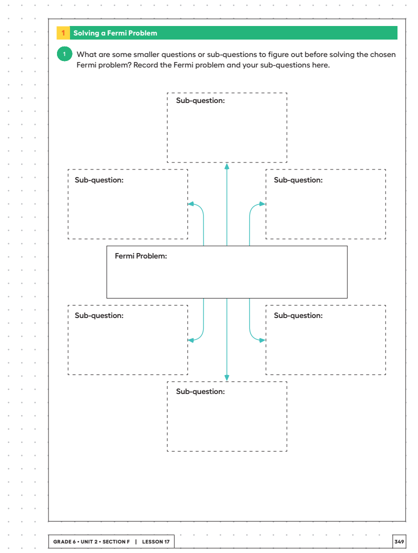
Building on Student Thinking

If students are unsure how to break down the chosen Fermi problem into smaller sub-questions, consider demonstrating with a different example. Take this problem, for instance: "If you have a locker full of oranges, could you squeeze enough juice for everyone at your school?" Tell students that one thing we might want to know is the size of the locker: How big is it? Urge students to think of other questions that might be helpful to answer.

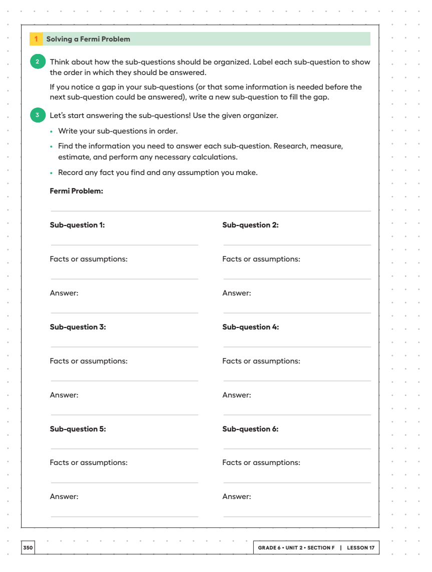
Here are some ideas:

- How many people are in the school?
- How many oranges are needed to get a cup of juice?
- How large are the oranges?
- How much juice does each person get?
- Do adults and children get the same amount of juice?

Student Workbook



Student Workbook



1 Solving a Fermi Problem

2 Think about how the sub-questions should be organized. Label each sub-question to show the order in which they should be answered.

If you notice a gap in your sub-questions (or that some information is needed before the next sub-question could be answered), write a new sub-question to fill the gap.

3 Let's start answering the sub-questions! Use the given organizer.

- Write your sub-questions in order.
- Find the information you need to answer each sub-question. Research, measure, estimate, and perform any necessary calculations.
- Record any fact you find and any assumption you make.

Fermi Problem:

Sub-question 1:

Facts or assumptions:

Answer:

Sub-question 2:

Facts or assumptions:

Answer:

Sub-question 3:

Facts or assumptions:

Answer:

Sub-question 4:

Facts or assumptions:

Answer:

Sub-question 5:

Facts or assumptions:

Answer:

Sub-question 6:

Facts or assumptions:

Answer:

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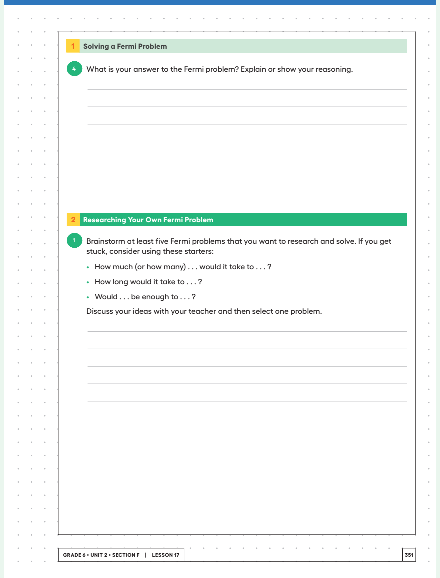
Student Workbook



1 Solving a Fermi Problem

4 What is your answer to the Fermi problem? Explain or show your reasoning.

2 Researching Your Own Fermi Problem

1 Brainstorm at least five Fermi problems that you want to research and solve. If you get stuck, consider using these starters:

- How much (or how many) . . . would it take to . . . ?
- How long would it take to . . . ?
- Would . . . be enough to . . . ?

Discuss your ideas with your teacher and then select one problem.

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Fermi Problem:

Is one gallon of hand sanitizer enough to sanitize the hands of everyone in the school over a school day?

Sub-question 1:

How many times a day does everyone sanitize their hands?

Facts or assumptions:

Everyone sanitizes their hands when they arrive at school, after recess or break time, before lunch, at the end of the day.

Answer:

4 times

Sub-question 2:

How much does one person use per day?

Facts or assumptions:

Each person gets a squirt of hand sanitizer. Each squirt is teaspoon. Adults and children use the same amount.

Answer:

2 teaspoons per day, because .

Sub-question 3:

How many classes are in the school and how many students are in each class?

Facts or assumptions:

Answer:

There are 20 classes with 25 students in each class.

Sub-question 4:

How many adults are in the school? How many people are in the school in total?

Facts or assumptions:

Answer:

There are 30 teachers and 12 adults who are not teachers, so there are 42 adults.

There are 542 people in total, because $(20 \cdot 25) + 42 = 500 + 42 = 542$.

Sub-question 5:

How much hand sanitizer is used by everyone in one day?

Facts or assumptions:

Answer:

1,084 teaspoons. If each person uses 2 teaspoons, then 542 people use or 1,084 teaspoons.

Sub-question 6:

How many teaspoons are in a gallon?

Facts or assumptions:

There are 16 cups in a gallon, 8 ounces in a cup, and 6 teaspoons in an ounce.

Answer:

768 teaspoons

cups	ounces
1	8
8	16
16	128

There are 128 ounces in 16 cups. There are 6 teaspoons in an ounce, so in 128 ounces there are or 768 teaspoons.

4. What is your answer to the Fermi problem? Explain or show your reasoning.

One gallon of hand sanitizer is not enough for sanitizing everyone's hands over a school day. There are only 768 teaspoons in a gallon and almost 1,100 teaspoons are needed per day.

Activity Synthesis

Poll the class on their solution to the Fermi problem. Record and display the responses for all to see. It is highly unlikely that any two groups would make the exact same assumptions or use the exact same estimates, so different answers are to be expected. Discuss the variations in the solutions and likely reasons for them. If possible, display each group's sub-questions and corresponding answers for all to see.

A few key points to emphasize:

- There are multiple ways to go about solving a Fermi problem, all of which require making assumptions, estimating, measuring, calculating, relying on known facts, or a combination of these.
- The assumptions and estimates that we make about the situation in the problem affect how we solve the problem and the solution we get.
- While the solutions to the same Fermi problem may vary a little or a lot depending on the solving process and the assumptions made, the range of solutions can help us see what answers might be realistic or reasonable.

Activity 2: Optional

Researching Your Own Fermi Problem

40 min

Activity Narrative

This optional activity gives students an opportunity to apply the reasoning and tools developed in this unit to solve another Fermi problem. Students work with a partner to brainstorm a few new problems and select one to solve together.

To encourage ratio reasoning, look for problems that involve two or more quantities and that are related by several rates. The rates may be familiar (such as conversion across two units of measurement) or easy to find (such as the number of heartbeats in a unit of time). They may also be less straightforward and require estimation (such as the amount of time needed to paddle a unit of distance).

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.
If students are brainstorming new problems, use this routine to support students' understanding of Fermi problems. Present non-examples such as "How many students are in our classroom right now?" and "How tall is a stack of 20 pennies?" Ask groups of 2–3 students to select and critique one of the questions and then collaborate to revise it into a Fermi-type question. Invite students to share their new questions and ask the class to identify the changes that made them Fermi questions.

Advances: Conversing

Access for Students with Diverse Abilities (Activity 2, Student Task)

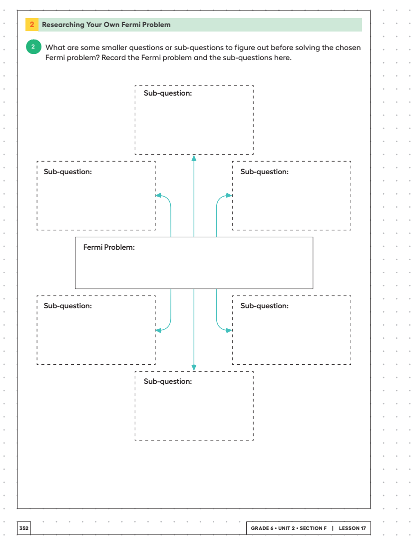
Engagement: Develop Effort and Persistence.
Differentiate the degree of difficulty or complexity. Begin with the Fermi questions from the *Warm-up* and ask students to identify a question or a situation of interest. Then, invite students to adjust the question for the same situation or to think of a similar question but for a different situation. For example, start with the question "How many tubes of toothpaste would you need in a lifetime?" from the *Warm-up* and ask what other amounts in a lifetime might be interesting to investigate.

Supports accessibility for: Conceptual Processing, Memory

Building on Student Thinking

When brainstorming, students may think of problems that involve a lot of steps but can be calculated with certainty. For instance, "How many minutes are in a decade with two leap years?" can be answered with multi-step multiplication. The question "How long would it take to show all the episodes of your favorite television series non-stop?" can be answered by adding up the durations of the episodes, information that is likely available online. Clarify that Fermi problems generally involve quantities that are unknown or impossible to measure directly, so it is necessary to make estimates and assumptions. This leads to a range of possible answers. Consider referring to a couple of Fermi problems in the *Warm-up* and discussing what needs to be estimated and assumed in each problem.

Student Workbook



Launch

Explain to students that they will now brainstorm some Fermi problems they are interested in answering and select one to solve. Encourage students to revisit the examples given in the *Warm-up* activity to jumpstart their thinking and to use the given question starters.

If time is limited or if desired, consider allowing students to select a Fermi problem from the list in the *Warm-up* activity. They may choose to modify the problem or solve it as given.

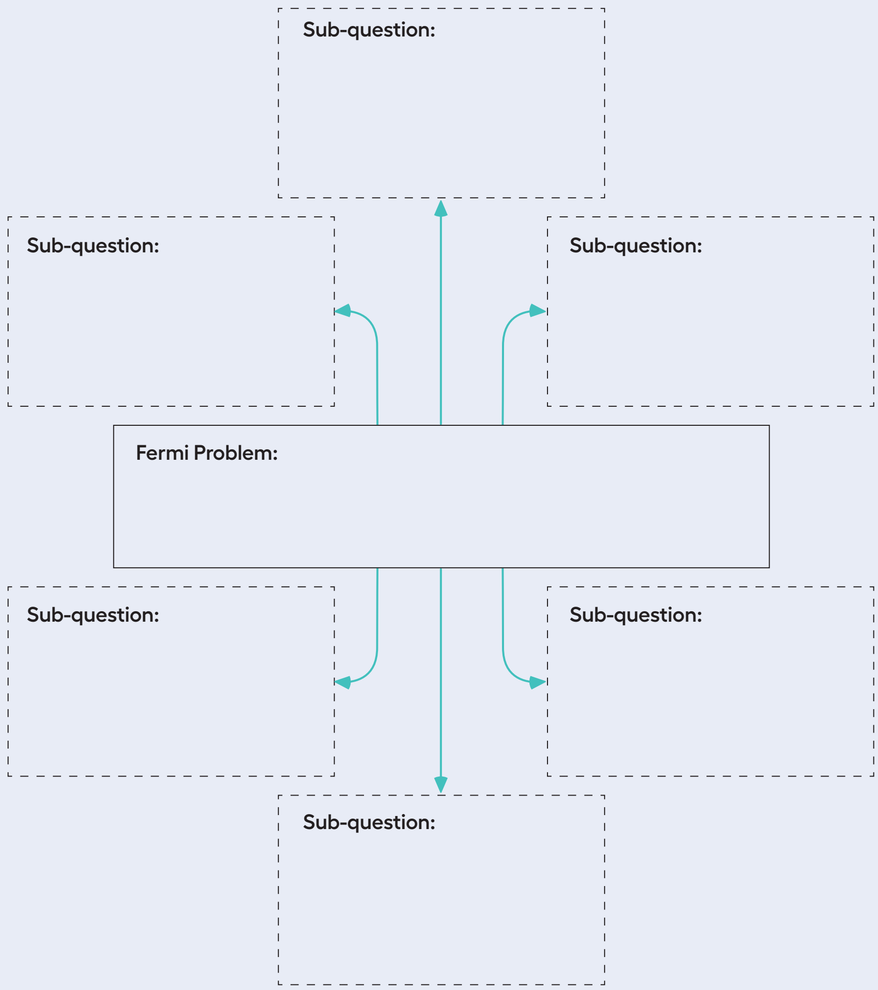
Arrange students in groups of 2. Provide tools for creating a visual display.

Student Task Statement

1. Brainstorm at least five Fermi problems that you want to research and solve. If you get stuck, consider using these starters:
   - How much (or how many) … would it take to …?
   - How long would it take to …?
   - Would … be enough to …?

   Discuss your ideas with your teacher and then select one problem.
2. What are some smaller questions or sub-questions to figure out before solving the chosen Fermi problem? Record the Fermi problem and the sub-questions here.

   Answer vary.



3. Let's start answering the sub-questions! Use the given organizer.
   - Write your sub-questions in order.
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   Facts or assumptions:

   Answer:

   Sub-question 5:

   Facts or assumptions:

   Answer:

   Sub-question 6:

   Facts or assumptions:

   Answer:

4. What is your answer to the Fermi problem? Explain or show your reasoning.

   Answers vary.

Activity Synthesis

Ask some students (or all, if time permits) to present their problems and solutions to the class.

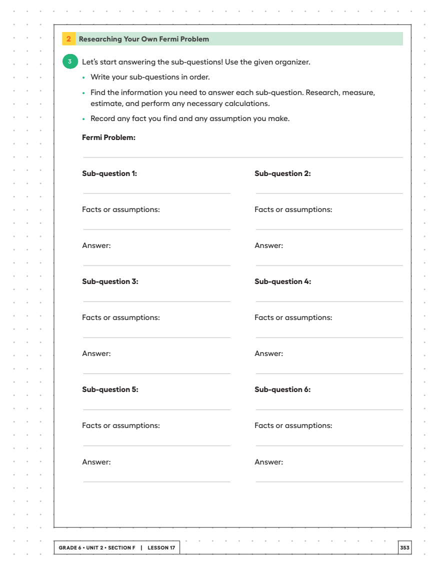
If time allows, consider asking students to create a visual display of their work and then ask them to display them throughout the classroom. Invite students to quietly circulate and read at least 2 of the posters or visual presentations in the room. Ask students to consider what is the same and what is different about the sub-questions and solution methods for the different Fermi problems.

Discuss any similarities and differences in the ways the Fermi problems were broken down and the smaller questions were answered. Highlight instances of ratio and rate reasoning, particularly productive use of double number lines or tables.

Lesson Synthesis

Find opportunities to summarize key ideas from this unit when discussing students' work and presentations. Point out the specific ways in which reasoning about ratio and use of representations helped students tackle difficult problems. Also draw attention to aspects of mathematical modeling that were at play as students solved problems. Highlight instances in which students made estimates in order to proceed, figured out additional information that was needed to make progress, or made simplifying assumptions.

Student Workbook


2 Researching Your Own Fermi Problem

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Answer:

Sub-question 6:

Facts or assumptions:

Answer:

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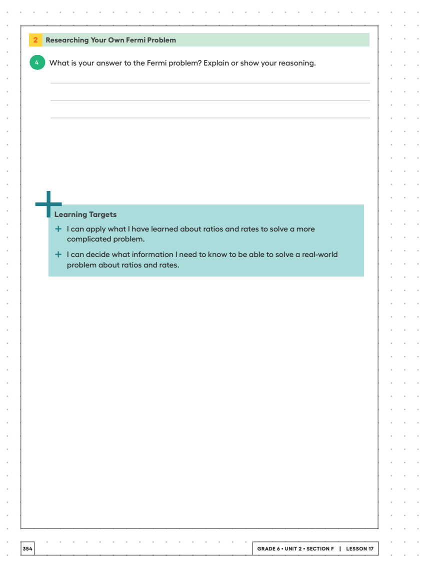
Student Workbook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Learning Targets
- I can apply what I have learned about ratios and rates to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem about ratios and rates.

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