Using Fractions to Multiply Decimals

Goals

Identify and generate multiplication expressions that use whole numbers and decimals such as 0.1, 0.01, and 0.001 to represent the product of two decimals, and find its value.

Use decimal fractions to represent and justify (orally and in writing) how to find the product of two decimals.

Learning Target

I can use place value and fractions to reason about the multiplication of decimals.

Lesson Narrative

In this lesson, students use what they know about fractions and place value to reason about products of decimals beyond the hundredths. They first recall that the fractions $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ are equivalent to the decimals 0.1, 0.01, and 0.001, respectively, and that multiplying a number by these values is equivalent to dividing the number by 10, 100, and 1,000. Students observe how such operations affect the number of decimal places in the resulting products or quotients.

Next, students express each decimal factor as a product of a whole number and a fraction, and then use the commutative and associative properties to compute the product. For example, they see that (0.6) \cdot (0.5) can be viewed as 6 \cdot (0.1) \cdot 5 \cdot (0.1) and thus as $\left(6 \cdot \frac{1}{10}\right) \cdot \left(5 \cdot \frac{1}{10}\right)$. Multiplying the whole numbers and the fractions gives 30 $\cdot \frac{1}{100}$, which is 0.3.

As they reason about the relationship between decimals and fractions, and about its effects on the number of decimal places in the product, students practice making use of the structure of the base-ten system.

Student Learning Goal

Let's look at products that are decimals.

Lesson Timeline

5_{min}

Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Lesson Synthesis

Access for Students with Diverse Abilities

• Engagement (Activity 2)

Access for Multilingual Learners

• MLR8: Discussion Supports (Activity 1)

Assessment

5 min

Cool-down

Lesson 5 Warm-up Activity 1 Activity 2 Lesson Synthesis Cool-down

Warm-up

Multiplying by 10



Activity Narrative

In this *Warm-up*, students encounter four multiplication equations, each with a variable *x*. Students are likely to notice that:

- The equations all have x and 10 as their factors.
- The four products are related by powers of 10, or that each product is onetenth of the one before it.
- The products all have the digits 8 and 1, and those digits move to the right, to smaller place values.

They make use of the structure of the equations to determine where the value of x is the greatest and how the values of the variables are related.

Launch

Display the four equations for all to see.

Give students 1–2 minutes of quiet time to analyze the equations and to answer the questions. Ask them to do so without writing anything and to be prepared to explain their reasoning. Follow that with a whole-class discussion.

Student Task Statement

1. In which equation is the value of x the largest? Explain your reasoning.

 $x \cdot 10 = 810$ $x \cdot 10 = 81$ $x \cdot 10 = 8.1$ $x \cdot 10 = 0.81$

The x has the largest value in the first equation.

Sample reasoning:

- When multiplied by 10, the x in the first equation has the largest product.
- Each x is one tenth of the product, and the largest product is 810.
- 2. How many times the size of 0.81 is 810?

810 is 1,000 times the size of 0.81.

Sample reasoning:

- Multiplying 0.81 by 10 moves the digits one place to the left, so multiplying 0.81 by 1,000 moves the digits 8 and 1 three places to the left.
- 810 is 10 times 81, and 81 is 100 times 0.81, so 810 must be 1,000 times 0.81.

Inspire Math Gold Mine video

Go Online

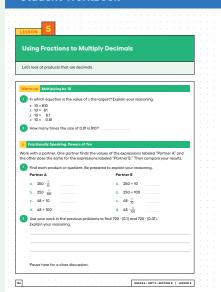
Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614152

Please log in to the site before using the QR code or URL.



Student Workbook



Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.

Invite students to begin partner interactions by repeating the question.

"What is the value of your expression?"

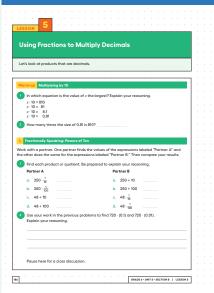
This gives both students an opportunity to produce language.

Advances: Conversing

Building on Student Thinking

Students may readily see that $36\cdot(0.1)=3.6$ but be unsure about what to do when the factor being multiplied by the decimal 0.1 and 0.01 is also a decimal (for instance, $(24.5)\cdot(0.1)$). Ask students to think about what happens when 24.5 is multiplied by 10, 100, and 1,000, and then think about what might make sense when it is multiplied by 0.1 and 0.01.

Student Workbook



Activity Synthesis

Before discussing the responses to the questions, invite students to share what they noticed about the four equations. Record observations about the structure of the equations, as noted in the *Activity Narrative*.

Then ask students to share their responses and reasoning. Highlight responses that clarify that multiplying a number by 10 moves the digits one place to the left. So if a number times 10 is 8.1, that number must be 0.81. Discuss how this understanding can help find how many times the size of 0.81 is 810.

Activity 1

Fractionally Speaking: Powers of Ten



Activity Narrative

In this activity, students recognize and use the fact that multiplying by 0.1, 0.01, and 0.001 is equivalent to multiplying by $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$, respectively. In all cases, the essential point to understand is that in the base-ten system, the value of each place is $\frac{1}{10}$ the value of the place immediately to its left. Writing the decimals 0.1, 0.01, and 0.001 in fraction form will help students recognize how the number of decimal places in the factors affect the number of decimal places in the product. Though students are not expected to make a generalization at this point, the reasoning here prompts students to look for and make use of structure.

Through repeated reasoning, students see how the number of decimal places in the factors can help them place the decimal point in the product.

Launch



Arrange students in groups of 2. Ask one student in each group to find the values of the expressions for Partner A and the other to do the same for Partner B. Then ask them to discuss their responses, answer the second question together, and pause for a brief class discussion before completing the last set of questions.

Student Task Statement

Work with a partner. One partner finds the values of the expressions labeled "Partner A" and the other does the same for the expressions labeled "Partner B." Then compare your results.

1. Find each product or quotient. Be prepared to explain your reasoning.

Partner A	Partner B
a. 250 $\cdot \frac{1}{10}$	a. 250 ÷ 10
b. 250 $\cdot \frac{1}{100}$	b. 250 ÷ 100
c. 48 ÷ 10	c. 48 · $\frac{1}{10}$
d. 48 ÷ 100	d. 48 $\cdot \frac{1}{100}$
For both Partner A and Partner B:	
a. 25	b. 2. 5
c. 4.8	d. 0.48

2. Use your work in the previous problems to find $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your reasoning.

Pause here for a class discussion.

 $720 \cdot (0.1) = 72$ and $720 \cdot (0.01) = 7.2$

Sample reasoning:

- 0.1 is equal to $\frac{1}{10}$, so 720 · (0.1) = 720 · $\frac{1}{10}$, which is equal to 720 ÷ 10, or 72.
- 0.01 is equal to $\frac{1}{100}$, so 720 · (0.01) = 720 · $\frac{1}{100}$, which is equal to 720 ÷ 100, or 72
- 3. Find each product. Show your reasoning.
 - a.36 · (0.1) 3.6

Sample reasoning: 36 \cdot (0.1) means 36 groups of I tenth, or 36 $\cdot \frac{1}{10}$, which equals 3.6.

b. (24.5) · (0.1) **2.45**

Sample reasoning: $(24.5) \cdot \frac{1}{10} = 24.5 \div 10$, which is 2.45.

c. (1.8) · (0.1) O.18

Sample reasoning: (1.8) $\cdot \frac{1}{10}$ = 1.8 ÷ 10, which is 0.18.

d.54 · (0.01) 0.54

Sample reasoning: 54 · (0.01) means 54 groups of I hundredth, or 54 hundredths.

e. (9.2) · (0.01) 0.092

Sample reasoning: $9.2 \div 100 = 0.092$.

Activity Synthesis

The purpose of this discussion is to highlight the number of decimal places in a product of two numbers in base-ten. Consider asking some of the following questions:

"How does the size of a product compare to the size of the factor when the factor is multiplied by 0.1?"

The product is one tenth the size of the factor.

"What does that mean in terms of the number of decimal places?"

All the digits move one place to the right, so the product has one more decimal place compared to the factor.

"How does the size of a product compare to the size of the factor when the factor is multiplied by 0.01?"

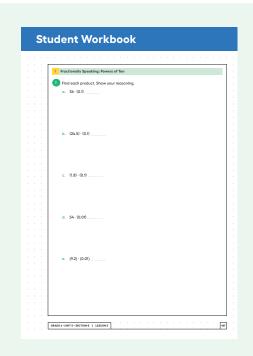
The product is one hundredth the size of the factor.

"What does that mean in terms of the number of decimal places?"

All the digits move two places to the right, so the product has two more decimal places compared to the factor.

"Can you predict the outcome of multiplying 750 by 0.1 or 0.01 without calculating?"

Multiplying 750 by 0.1 would produce 75. Multiplying 750 by 0.01 would produce 7.5.



Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Differentiate the degree of difficulty or complexity. For example, limit the answer choices for the first question to A-D.

Supports accessibility for: Conceptual Processing, Memory

Building on Student Thinking

If students try to use vertical calculation to find the products, ask them to instead reason by writing equivalent expressions and thinking of the decimals as fractions (if helpful).

Student Workbook

	uivalent to (0.6) · (0.5). Be prepared to explain
your reasoning.	© 6 · (0.01) · 5 · (0.1)
A 6 · (0.1) · 5 · (0.1)	~
C 6 · 10 · 5 · 10	0 6 - 1 1000 - 5 - 1
E 6 · (0.001) · 5 · (0.01)	6 · 5 · 1/10 · 10
© \$ · 5 \$\overline{\pi}\$	
Find the value of (0.6) · (0.5) Show your reasoning.	
Show your recooning.	
Find the value of each product by	writing equivalent expressions. Show your reasoning.
a. (0.3) · (0.02)	
b. (0.7) · (0.05)	
b. (0.7) · (0.05)	

Activity 2

Fractionally Speaking: Multiples of Powers of Ten

15

Activity Narrative

In this activity, students continue to think about products of decimals. They use what they know about the fractions $\frac{1}{10}$ and $\frac{1}{100}$, as well as the commutative and associative properties, to identify and write multiplication expressions that could help them find the product of two decimals.

While students may be able to start by calculating the value of each decimal product, the goal is for them to look for and use the structure of equivalent expressions, and later generalize the process to multiply any two decimals.

As students work, listen for the different ways that students decide on which expressions are equivalent to $(0.6) \cdot (0.5)$. Identify a few students or groups with differing approaches, and ask them to share later.

Launch



Arrange students in groups of 2.

Give groups 3–4 minutes to work on the first two questions, and then pause for a whole-class discussion. Ask auestions such as:

 \bigcirc "Why is (0.6) · (0.5) equivalent to 6 · $\frac{1}{10}$ · 5 · $\frac{1}{10}$?"

0.6 is 6 tenths, which is the same as $6 \cdot \frac{1}{10}$, and 0.5 is 5 tenths, or $5 \cdot \frac{1}{10}$

 \bigcirc "Why is the expression $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$ equivalent to $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$?"

We can 'switch' the places of 5 and $\frac{1}{10}$ in the multiplication and not change the product. This follows the commutative property of operations.

 \bigcirc "How did you find the value of 30 $\cdot \frac{1}{100}$?"

Multiplying by $\frac{1}{100}$ means dividing by 100, which moves the digits 2 places to the right, so the result is 0.30, or 0.3.

Give students 1–2 minutes to complete the last question, leaving time for discussion.

Student Task Statement

1. Select all expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your reasoning.

B.6
$$\cdot$$
 (0.01) \cdot 5 \cdot (0.1)

C.
$$6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$$

D.6
$$\cdot \frac{1}{1,000} \cdot 5 \cdot \frac{1}{100}$$

F.
$$6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$$
G. $\frac{6}{10} \cdot \frac{5}{10}$

G.
$$\frac{6}{10} \cdot \frac{5}{10}$$

A, because $6 \cdot (0.1) = 0.6$ and $5 \cdot (0.1) = 0.5$.

C, because
$$6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10} = \frac{6}{10} \cdot \frac{5}{10} = (0.6) \cdot (0.5)$$

C, because $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10} = \frac{6}{10} \cdot \frac{5}{10} = (0.6) \cdot (0.5)$. F, because $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10} = 6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$ which is $6 \cdot (0.1) \cdot 5 \cdot (0.1)$, or $(0.6) \cdot (0.5)$.

G, because $\frac{6}{10}$ is equivalent to 0.6 and $\frac{5}{10}$ is equivalent to 0.5.

2. Find the value of $(0.6) \cdot (0.5)$. Show your reasoning.

0.3

Sample reasoning: (0.6) • (0.5) = $\frac{6}{10} \cdot \frac{5}{10}$, which is $\frac{30}{100}$, or 0.3.

- **3.** Find the value of each product by writing equivalent expressions. Show your reasoning.
 - a.(0.3) · (0.02) 0.006

Sample reasoning: $\frac{3}{10} \cdot \frac{2}{100} = \frac{6}{1,000}$, which is 0.006.

b. (0.7) · (0.05) **0.035**

Sample reasoning: (0.7) • (0.05) = $\frac{7}{10} \cdot \frac{5}{100}$, which is $\frac{35}{1,000}$ or 0.035.

Are You Ready for More?

Ancient Romans used letters to represent numbers, as shown in the table.

1.

Roman numeral	number
I	1
V	5
X	10
L	50
С	100
D	500
М	1,000

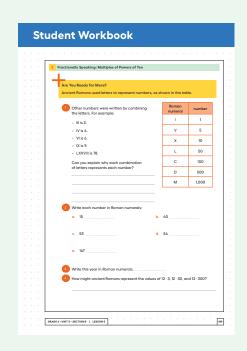
Other numbers were written by combining the letters. For example:

- III is 3.
- IV is 4.
- VI is 6.
- IX is 9.
- LXXVIII is 78

Can you explain why each combination of letters represents each number?

Sample response:

- 3 is written as 3 ones or 3 l's.
- 4 is written as 5 I and 9 is written as IO I. The number being used to subtract is put before the number being subtracted.
- 6 is written as 5 + 1 and 78 is written as 50 + 10 + 10 + 5 + 1 + 1 + 1. The number being added is placed after the number being added to.



2. Write each number in Roman numerals:

a. 15 XV

b. 40 XL

c. 53 LIII

d. 54 LIV

e. 167 CLXVII

3. Write this year in Roman numerals.

Sample response: MMXXIV for 2024

4. How might ancient Romans represent the values of 12 \cdot 3, 12 \cdot 30, and 12 \cdot 300?

XXXVI, CCCLX, MMMDC

Activity Synthesis

Invite students to share their responses and reasoning for the last question. Ask questions such as:

- "Why might it be helpful to write $\frac{3}{10} \cdot \frac{2}{100}$ or (0.3) \cdot (0.02) as $3 \cdot 2 \cdot \frac{1}{1,000}$?"

 Using whole numbers or fractions can help make the multiplication simpler. Even if there is division, it is division by a power of 10.
- \bigcirc "We can write (0.3) \cdot (0.02) as $3 \cdot$ (0.1) \cdot 2 \cdot (0.01), and as $3 \cdot 2 \cdot$ (0.1) \cdot (0.01). Why is it okay to rearrange the factors?"

It doesn't change the product.

 \bigcirc "What do you notice about the number of decimal places in the product of (0.3) \cdot (0.02)?"

The number is in the thousandths. It has 3 decimal places.

Lesson Synthesis

The key takeaway here is that we can use our understanding of fractions and place value in calculating the product of two decimals. Writing decimals in fraction form can help us determine the number of decimal places that the product will have and help us place the decimal point in the product.

Ask students questions such as:

Write 0.2 as $\frac{2}{10}$, and 0.0009 as $\frac{9}{10,000}$, multiply the fractions to get $\frac{18}{100,000}$, and write the product as the decimal 0.00018. Or write 0.2 as $2 \cdot \frac{1}{10}$, and 0.0009 as $9 \cdot \frac{1}{10,000}$, multiply the whole numbers and the fractions, and convert the fractional product into a decimal.

 \bigcirc "Which product would have more decimal places: (0.03) \cdot (0.001) or (0.3) \cdot (0.0001)? How can we tell?"

The products have the same number of decimal places. Written as fractions, both products equal $\frac{3}{100,000}$ or 0.00003.

Tell students that in future lessons, they will apply this reasoning to find products of more elaborate decimals, such as $(0.24) \cdot (0.011)$.

Lesson Summary

We can use fractions like $\frac{1}{10}$ and $\frac{1}{100}$ to reason about the location of the decimal point in a product of two decimals.

Let's take $24 \cdot (0.1)$ as an example. There are several ways to find the product:

- We can interpret it as 24 groups of 1 tenth (or 24 tenths), which is 2.4.
- We can think of it as $24 \cdot \frac{1}{10}$, which is equal to $\frac{24}{10}$ (and also equal to 2.4).
- Because multiplying by $\frac{1}{10}$ has the same result as dividing by 10, we can also think of it as $24 \div 10$, which is equal to 2.4.

Similarly, we can think of (0.7) \cdot (0.09) as 7 tenths times 9 hundredths, and write:

$$\left(7 \cdot \frac{1}{10}\right) \cdot \left(9 \cdot \frac{1}{100}\right)$$

We can rearrange the whole numbers and fractions:

$$(7\cdot 9)\cdot \left(\frac{1}{10}\cdot \frac{1}{100}\right)$$

This tells us that $(0.7) \cdot (0.09) = 0.063$.

$$63 \cdot \frac{1}{1,000} = \frac{63}{1,000}$$

Here is another example: To find $(1.5) \cdot (0.43)$, we can think of 1.5 as 15 tenths and 0.43 as 43 hundredths. We can write the tenths and hundredths as fractions and rearrange the factors.

$$\left(15 \cdot \frac{1}{10}\right) \cdot \left(43 \cdot \frac{1}{100}\right) = 15 \cdot 43 \cdot \frac{1}{1,000}$$

Multiplying 15 and 43 gives us 645, and multiplying $\frac{1}{10}$ and $\frac{1}{100}$ gives us $\frac{1}{1,000}$. So (1.5) \cdot (0.43) is 645 \cdot $\frac{1}{1,000}$, which is 0.645.

Cool-down

Explaining and Calculating Products

5 min

Student Task Statement

1. Use what you know about decimals or fractions to explain why $(0.2) \cdot (0.002) = 0.0004$.

Sample response: 0.2 is $\frac{2}{10}$, and 0.002 is $\frac{2}{1,000}$. Multiplying the two we have: $\frac{2}{10} \cdot \frac{2}{1,000} = \frac{4}{10,000}$, which is 0.0004.

2. A rectangular plot of land is 0.4 kilometer long and 0.07 kilometer wide. What is its area in square kilometers?

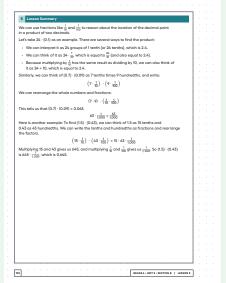
0.028 square kilometers, because $(0.4) \cdot (0.07) = 0.028$

Responding To Student Thinking

More Chances

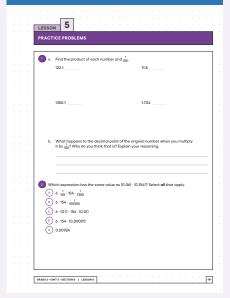
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook

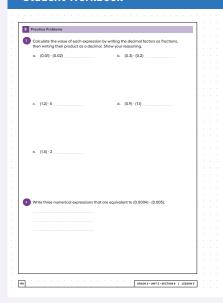


5

Student Workbook



Student Workbook



Problem 1

a. Find the product of each number and $\frac{1}{100}$.

122.1 I.22I

11.8 0.118

1350.1 13.501

1.704 0.01704

b. What happens to the decimal point of the original number when you multiply it by $\frac{1}{100}$? Why do you think that is? Explain your reasoning.

Sample response: The digits move 2 places to the right. Multiplying a decimal number by $\frac{1}{100}$ means dividing by 100, which gives a number that is one-hundredth of the original number.

Problem 2

Which expression has the same value as $(0.06) \cdot (0.154)$? Select **all** that apply.

A.
$$6 \cdot \frac{1}{100} \cdot 154 \cdot \frac{1}{1,000}$$

B.
$$6 \cdot 154 \cdot \frac{1}{100,000}$$

E. 0.00924

Problem 3

Calculate the value of each expression by writing the decimal factors as fractions, then writing their product as a decimal. Show your reasoning.

a. $(0.01) \cdot (0.02)$

0.0002, because 0.01 = $\frac{1}{100}$, and 0.02 = $\frac{2}{100}$, so the product is $\frac{2}{10,000}$.

b. (0.3) · (0.2)

0.06, because 0.3 = $\frac{3}{10}$, and 0.2 = $\frac{2}{10}$, so the product is $\frac{6}{100}$.

c. (1.2) · 5

6, because 1.2 is $\frac{12}{10}$, and 5 is $\frac{5}{1}$ so the product is $\frac{60}{10}$, or 6.

0.99, because 0.9 = $\frac{9}{10}$, and I.I = $\frac{11}{10}$, so the product is $\frac{99}{100}$.

e. (1.5) · 2

3, because 1.5 = $\frac{3}{2}$, and twice this is 3.

Problem 4

Write three numerical expressions that are equivalent to $(0.0004) \cdot (0.005)$.

Sample responses:

- 4 · (0.000I) · 5 · (0.00I)
- 4.5.(0.000I).(0.00I)
- $\frac{4}{10,000} \cdot \frac{5}{1,000}$
- \frac{1}{10,000} \cdot 4 \cdot \frac{1}{1,000} \cdot 5

Problem 5

from Unit 5, Lesson 3

Calculate each sum.

Problem 6 from Unit 5, Lesson 4

Calculate each difference. Show your reasoning.

b. 23.11 – 0.376 **22.73**4

c. 0.9 - 0.245 0.655

Problem 7

from Unit 1, Lesson 3

On the grid, draw a quadrilateral *that is not a rectangle* that has an area of 18 square units. Show how you know the area is 18 square units.

Sample response: Multiplying the base and height of each parallelogram gives 18: $6 \cdot 3 = 18$ and $2 \cdot 9 = 18$.

