Lesson 10

Using Partial Quotients

Goals

- Interpret division calculations that use partial quotients, and explain (orally and in writing) what each step in the division represents.
- Use partial quotients to divide whole numbers that result in a whole-number quotient, and explain the reasoning (using words and other representations).

Learning Target

I can use partial quotients to find a quotient of two whole numbers.

Lesson Narrative

In this lesson, students continue to reason about place value and the base-ten structure in division. Instead of using base-ten representations, they use partial quotients.

Students begin by recalling that division can be done in parts, by decomposing the dividend. For instance, we can calculate $1{,}216 \div 4$ by calculating $1{,}000 \div 4$, $200 \div 4$, and $16 \div 4$. These partial quotients can be recorded as a series of equations, as students may have seen in earlier grades.

Next, students analyze a method of recording division that also involves partial quotients but is arranged vertically. Students make sense of the steps for subtracting parts of the dividend below the original number and the adding of partial quotients stacked above it. The vertical calculation foreshadows the long division algorithm.

Finally, students use this method to divide multi-digit numbers. While the dividends, divisors, and quotients are whole numbers, the challenges here are in using an unfamiliar algorithm and in looking for efficient ways to decompose each dividend. Students are likely to recognize that it helps to decompose the dividend by place value, as was the case when dividing using base-ten blocks or diagrams.

Student Learning Goal

Let's divide whole numbers.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Access for Students with Diverse Abilities

- Representation (Warm-up)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)

Instructional Routines

- 5 Practices
- MLR5: Co-Craft Questions
- Notice and Wonder

Required Materials

Materials to Gather

- Graph paper: Activity 1, Activity 2
- Base-ten blocks: Activity 2

Materials to Copy

- Squares and Rectangles Cutouts (1 copy for every 1 student): Activity 2

Required Preparation

Activity 2:
Prepare either physical base-ten blocks or paper cutouts of base-ten representations from the blackline master.

Warm-up

Notice and Wonder: Kiran's Calculations

5 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea of partial quotients, which will be useful when students learn a new way to record partial quotients in a later activity. While students may notice and wonder many things about these equations, the idea of dividing a number by decomposing the dividend into smaller and more-familiar parts are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch

Arrange students in groups of 2. Display the problem stem and four equations for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder about. Give students another minute to discuss their observations and questions.

Student Task Statement

Here are Kiran's calculations for finding 657 ÷ 3:

$$\begin{array}{rcrcr} 600 & \div & 3 & = & 200 \\ 30 & \div & 3 & = & 10 \\ 27 & \div & 3 & = & 9 \\ \hline 657 & \div & 3 & = & 219 \end{array}$$

What do you notice? What do you wonder?

Students may notice:

- The 657 is divided by 3 in parts. First, 600 is divided by 3, then 30 is divided by 3, and finally 27 is divided by 3.
- The hundreds are divided first, followed by two divisions involving tens or tens and ones.
- Each part—600, 30, and 27—can be divided by 3 without a remainder.
- The quotients from dividing each part are added at the end.

Students may wonder:

- How did Kiran know how to break up the 657?
- Can we divide other numbers this way?
- Do we always need to break up the dividend into 3 parts in order to divide?
- Why didn't Kiran decompose the 657 by place value, into 600, 50, and 7?

Instructional Routines

Notice and Wonder

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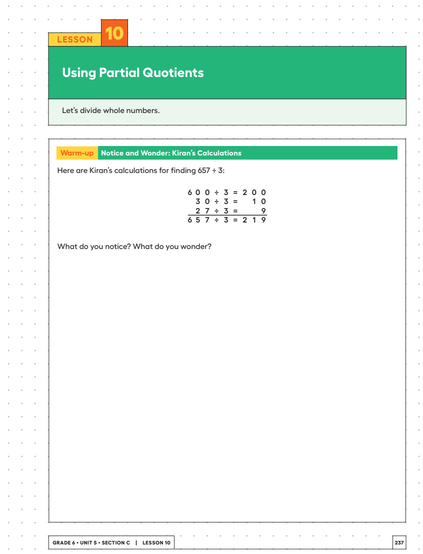
Access for Students with Diverse Abilities (Warm-up 1, Launch)

Representation: Internalize Comprehension.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Workbook


LESSON 10

Using Partial Quotients

Let's divide whole numbers.

Warm-up Notice and Wonder: Kiran's Calculations

Here are Kiran's calculations for finding 657 ÷ 3:

$$\begin{array}{rcrcr} 600 & \div & 3 & = & 200 \\ 30 & \div & 3 & = & 10 \\ 27 & \div & 3 & = & 9 \\ \hline 657 & \div & 3 & = & 219 \end{array}$$

What do you notice? What do you wonder?

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display the images and descriptions of Andre's steps without revealing the questions. Ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

> *"What do these questions have in common? How are they different?"*

Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the equations. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If no students mentioned that Kiran did not decompose the 657 strictly by place value, ask students to discuss this idea and whether it would be just as productive to decompose 657 into 600, 50, and 7.

Activity 1

Using Partial Quotients to Calculate Quotients

15 min

Activity Narrative

In this activity, students analyze another method of division in which one portion of the dividend is divided at a time. They see that there is more than one way to record this process and that different partial quotients can be used for any given division.

The first division problem, 657 ÷ 3, involves the same whole-number dividend and divisor as seen in the *Warm-up*. The calculations also show the same partial quotients of 200, 10, and 9 being added to give the quotient of 219. Unlike the strategy shown in the *Warm-up,* however:

- All the calculations are arranged vertically in a single stack rather than written as equations.
- The process involves repeatedly subtracting some amounts from the original dividend until there are no remainders.
- Each partial quotient is recorded above the line with the dividend and divisor. Partial dividends that are being subtracted and remainders are recorded below that line.

In analyzing Andre's calculations, students need to think about what calculations are performed, what each number means, why it is placed where it is or aligned as it is, and how the numbers are related. The reasoning here gives students opportunities to persevere in making sense of a problem and to attend to precision.

Monitor for students who use different partial quotients to calculate 896 ÷ 4. Select them to share later.

Launch

Arrange students in groups of 3–4. Display Andre's calculations for all to see. Read aloud (or invite a student to read) the description of Andre's steps.

Give students 5–6 minutes to make sense of Andre's method and to answer the first set of questions with their group. Pause for a brief whole-class discussion afterward.

Invite students to share their responses. As students explain, annotate Andre's calculations to illustrate the connections that students are making.

Tell students that, just like Kiran's method, Andre's method uses partial quotients. In both methods, one part of the dividend is divided at a time, which gives a part of the quotient. Both Kiran and Andre started by thinking about "how many groups of 3 are in 600?" or "what is 600 divided by 3?" but one person represented the question and the answer with a division equation (600 ÷ 3 = 200) while the other recorded the relationship between the numbers in a vertical arrangement. Consider asking:

- *"Why might it make sense to divide 30 and 27 by 3 instead of dividing 57 by 3?"*

  30 and 27 are familiar multiples of 3.

- *"Could Kiran have gotten the same quotient if he wrote 57 ÷ 3 instead?"*

  Yes

- *"What if he wrote 33 ÷ 3 and 24 ÷ 3 instead?"*

  All the partial quotients would still add up to 219.

Give students 3–4 minutes to calculate the quotient in the last question, using Andre's partial quotients method. Provide access to graph paper. Tell students that they may find a grid helpful for aligning the digits when arranging their calculations vertically as Andre has done.

Student Task Statement

1. Andre calculated 657 ÷ 3 using a method that was different from Kiran's.

He started by writing the dividend (657) and the divisor (3).	Next, he subtracted 3 groups of different amounts from 657, starting with 3 groups of 200...	...then 3 groups of 10, and then 3 groups of 9.	Andre calculated 200 + 10 + 9, and wrote 219.

```
3)6 5 7
```

```
   2 0 0
3)6 5 7
 - 6 0 0
     5 7
```

```
       9
     1 0
   2 0 0
3)6 5 7
-  6 0 0
     5 7
-    3 0
     2 7
-    2 7
       0
```

```
   [2 1 9]
       9
     1 0
   2 0 0
3)6 5 7
-  6 0 0
     5 7
-    3 0
     2 7
-    2 7
       0
```

a. Andre subtracted 600 from 657. What does the 600 represent?

The 600 represents 3 groups of 2 hundreds.

b. Andre wrote 10 above the 200, and then subtracted 30 from 57. How is the 30 related to the 10?

The 30 represents 3 tens that were distributed equally into 3 groups (1 ten in each).

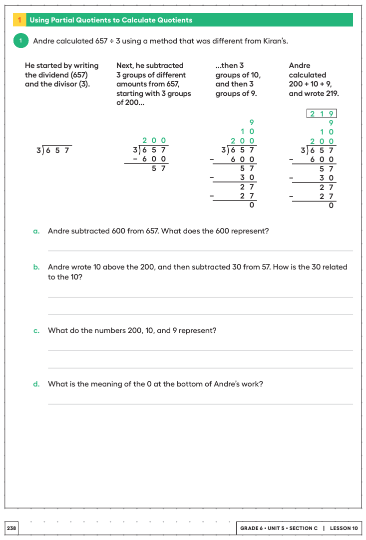
c. What do the numbers 200, 10, and 9 represent?

The 200, 10, and 9 represent the contents of each group: 2 hundreds, 1 ten, and 9 ones.

Building on Student Thinking

Students who know that 57 is 19 · 3 might wonder why Andre subtracted 10 · 3 and 9 · 3 separately rather than in one step. Encourage these students to write out the calculation that they think would be more efficient, compare it with Andre's calculation, and point out how the process is streamlined. Clarify that there are many ways to divide in portions, and that some may be quicker than others.

Student Workbook


Using Partial Quotients to Calculate Quotients

1 Andre calculated 657 ÷ 3 using a method that was different from Kiran's.

a. Andre subtracted 600 from 657. What does the 600 represent?

b. Andre wrote 10 above the 200, and then subtracted 30 from 57. How is the 30 related to the 10?

c. What do the numbers 200, 10, and 9 represent?

d. What is the meaning of the 0 at the bottom of Andre's work?

Instructional Routines

5 Practices

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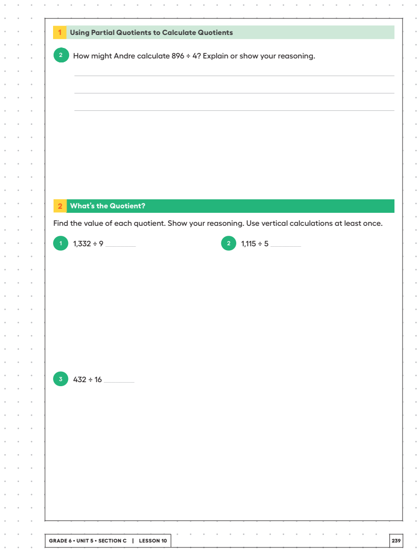
Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication. Provide continued access to base-ten blocks or paper cut-outs for students who found them helpful earlier in the lesson.

Supports accessibility for: Conceptual Processing, Organization

Student Workbook


1 Using Partial Quotients to Calculate Quotients

2 How might Andre calculate 896 ÷ 4? Explain or show your reasoning.

2 What's the Quotient?

Find the value of each quotient. Show your reasoning. Use vertical calculations at least once.

1 1,332 ÷ 9

2 1,115 ÷ 5

3 432 ÷ 16

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d. What is the meaning of the 0 at the bottom of Andre's work?

The 0 means that after subtracting 3 times 219 from 657, there is no remainder.

2. How might Andre calculate 896 ÷ 4? Explain or show your reasoning.

Andre could calculate $896 \div 4$ as shown.

```
    224
      4
     20
    200
 4)896
 - 800
    96
 -  80
    16
 -  16
     0
```

Activity Synthesis

Ask previously selected students to share their calculations for 896 ÷ 4, starting from the most involved and ending with the most streamlined.

Highlight that there are no rules about how many partial quotients we could use or what partial dividends to use. That said, it is often helpful to take out the amount in each place value and distribute it into groups. Once the entirety of the dividend is accounted for, the partial quotients can be added and the division is complete.

Activity 2

What's the Quotient?

15 min

Activity Narrative

In this activity, students choose how to find quotients of two whole numbers. As students work with larger dividends and divisors, they begin to see the merits and potential drawbacks of each method.

Here are some strategies students are likely to use, listed in the order of efficiency, from less straightforward to more straightforward:

- Using base-ten blocks or diagrams.
- Writing a series of equations to find partial quotients (similar to Kiran's equations in the *Warm-up*).
- Using vertical calculations to find partial quotients (similar to Andre's method).

Monitor for students who use these strategies, as well as for those who use other ways to divide.

Launch

Invite students to share some of the strategies for dividing two whole numbers that they have seen so far. Then, tell students that in this activity they choose a method to divide but should use partial quotients at least once. Encourage them to refer to Elena's, Kiran's, and Andre's methods of reasoning from earlier activities, if needed.

Keep students in groups of 3–4.

Give students 2–3 minutes of quiet time to work on the first division task, followed by a few minutes to discuss their response and reasoning with their group. Then give students 8–10 minutes of quiet work time on the remaining questions. Provide access to base-ten blocks or paper cutouts of base-ten representations, as well as to graph paper.

Select students who used each strategy described in the *Activity Narrative,* and ask them to share later. Aim to elicit both key mathematical ideas and a variety of student contributions, especially from students who haven't shared recently.

Student Task Statement

Find the value of each quotient. Show your reasoning. Use vertical calculations at least once.

1. 1,332 ÷ 9

   148

   Sample reasoning:

$$\begin{array}{r} 900 \div 9 = 100 \\ 270 \div 9 = 30 \\ 90 \div 9 = 10 \\ 72 \div 9 = 8 \\ \hline 1{,}332 \div 9 = 148 \end{array}$$

$$\begin{array}{rr} & \boxed{148} \\ & 8 \\ & 40 \\ & 100 \\ 9 \,\big)\!\! & 1332 \\ - & 900 \\ \hline & 432 \\ - & 360 \\ \hline & 72 \\ - & 72 \\ \hline & 0 \end{array}$$

2. 1,115 ÷ 5

   223

   Sample reasoning:

$$\begin{array}{r} 1000 \div 5 = 200 \\ 100 \div 5 = 20 \\ 15 \div 5 = 3 \\ \hline 1{,}115 \div 5 = 223 \end{array}$$

$$\begin{array}{rr} & \boxed{223} \\ & 3 \\ & 20 \\ & 200 \\ 5 \,\big)\!\! & 1115 \\ - & 1000 \\ \hline & 115 \\ - & 100 \\ \hline & 15 \\ - & 15 \\ \hline & 0 \end{array}$$

3. 432 ÷ 16

   27

   Sample reasoning:

$$\begin{array}{r} 160 \div 16 = 10 \\ 160 \div 16 = 10 \\ 80 \div 16 = 5 \\ 32 \div 16 = 2 \\ \hline 432 \div 16 = 27 \end{array}$$

$$\begin{array}{rr} & \boxed{27} \\ & 2 \\ & 5 \\ & 20 \\ 16 \,\big)\!\! & 432 \\ - & 320 \\ \hline & 112 \\ - & 80 \\ \hline & 32 \\ - & 32 \\ \hline & 0 \end{array}$$

Activity Synthesis

The purpose of this discussion is to compare the different strategies students used to divide to and draw attention to advantages and disadvantages.

Ask previously selected students to share their method for finding quotients. Sequence the discussion of the methods in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

"What are some benefits of writing equations as Kiran has done?"

It's a clear way to organize the calculations and is quicker than drawing diagrams. We're dealing with one division at a time. We can choose any number to divide.

"What might be some drawbacks of this method?"

We need to mentally keep track of how much is left of the dividend that is yet to be divided. We're writing "divide by 9" or "÷16" over and over.

"What are some benefits of using vertical calculations as Andre has done?"

It's quicker than drawing diagrams. The partial dividends and quotients are all recorded in one place. The amount that still needs to be divided can be found by subtracting and is shown as a part of the process.

"What might be some challenges with this method?"

It's not always clear what number to use each time. It involves multiplying and subtracting repeatedly, so if one calculation is wrong, the rest may also be wrong.

Lesson Synthesis

Two key ideas from this lesson are:

- Division can be done in parts and recorded in more than one way.
- We can divide efficiently by making use of place value and the structure of the base-ten system.

To highlight these ideas, ask questions such as:

"How is using partial quotients to divide similar to other division methods you have used before?"

It still involves thinking about place value and equal-size groups.

"How is it different?"

There is flexibility in how to divide and how much to divide each time. The vertical arrangement of numbers and repeated subtraction are new.

"Can we find partial quotients in any order? For example, to find 896 ÷ 4, can we subtract 16 for 4 groups of 4, then 80 for 4 groups of 20, and then 800 for 4 groups of 300?

Yes.

"Many of you started with the highest place value in the dividend. For example, when dividing 1,115 by 5, you started with 1, 000 ÷ 5. Why might that be?"

It helps to subtract the largest multiple of the divisor first, which means looking at the highest place value.

Lesson Summary

Another way to find the quotient of 345 ÷ 3 is by using partial quotients, in which we keep subtracting 3 groups of some amount from 345. We can organize the steps and record the partial quotients in a vertical calculation.

Here are two calculations for finding 345 ÷ 3:

```
   [1 1 5]
        5
      1 0
    1 0 0
 3) 3 4 5
 –  3 0 0  ◄——  3 groups of 100
      4 5
 –    3 0  ◄——  3 groups of 10
      1 5
 –    1 5  ◄——  3 groups of 5
        0
```

```
   [1 1 5]
      5 0
      5 0
      1 5
 3) 3 4 5
 –    4 5  ◄——  3 groups of 15
    3 0 0
 –  1 5 0  ◄——  3 groups of 50
    1 5 0
 –  1 5 0  ◄——  3 groups of 50
        0
```

- In the calculation on the left, first we subtract 3 groups of 100, then 3 groups of 10, and then 3 groups of 5. Adding up the partial quotients (100 + 10 + 5) gives us 115.
- The calculation on the right shows a different amount per group subtracted each time (3 groups of 15, 3 groups of 50, and 3 more groups of 50), but the total amount in each of the 3 groups is still 115.

There are other ways of calculating 345 ÷ 3 using partial quotients. We can calculate with fewer steps by removing groups of larger sizes.

Cool-down

Dividing by 11

5 min

Student Task Statement

Calculate 4,235 ÷ 11 using any method.

385

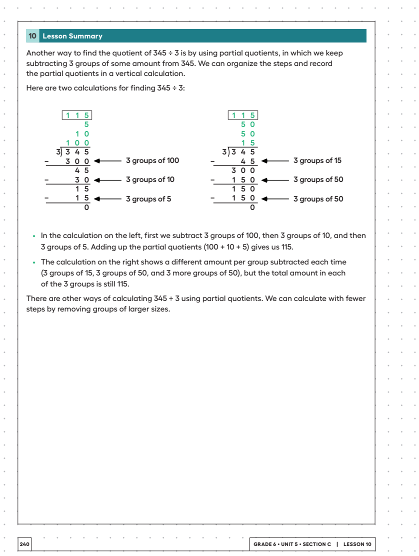
Sample reasoning:

```
     [3 8 5]
           5
         8 0
       3 0 0
 11) 4 2 3 5
 –   3 3 0 0
       9 3 5
 –     8 8 0
         5 5
 –       5 5
           0
```

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


10 Lesson Summary

Another way to find the quotient of 345 ÷ 3 is by using partial quotients, in which we keep subtracting 3 groups of some amount from 345. We can organize the steps and record the partial quotients in a vertical calculation.

Here are two calculations for finding 345 ÷ 3:

3 groups of 100
3 groups of 10
3 groups of 5
3 groups of 15
3 groups of 50
3 groups of 50

- In the calculation on the left, first we subtract 3 groups of 100, then 3 groups of 10, and then 3 groups of 5. Adding up the partial quotients (100 + 10 + 5) gives us 115.
- The calculation on the right shows a different amount per group subtracted each time (3 groups of 15, 3 groups of 50, and 3 more groups of 50), but the total amount in each of the 3 groups is still 115.

There are other ways of calculating 345 ÷ 3 using partial quotients. We can calculate with fewer steps by removing groups of larger sizes.

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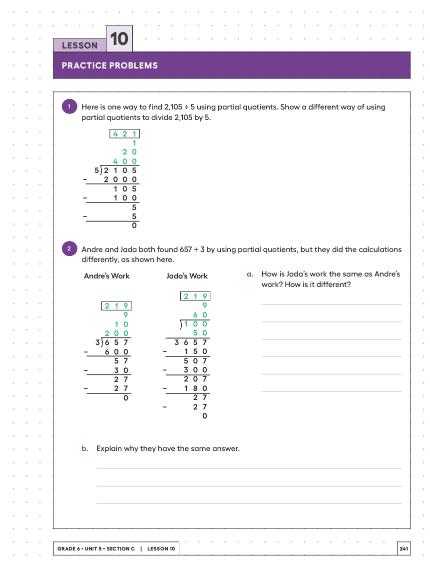
Practice Problems

7 Problems

Student Workbook


LESSON 10

PRACTICE PROBLEMS

1 Here is one way to find 2,105 ÷ 5 using partial quotients. Show a different way of using partial quotients to divide 2,105 by 5.

2 Andre and Jada both found 657 ÷ 3 by using partial quotients, but they did the calculations differently, as shown here.

Andre's Work

Jada's Work

a. How is Jada's work the same as Andre's work? How is it different?

b. Explain why they have the same answer.

Problem 1

Here is one way to find 2,105 ÷ 5 using partial quotients. Show a different way of using partial quotients to divide 2,105 by 5.

```
      [4 2 1]
            1
          2 0
        4 0 0
   5)2 1 0 5
 -   2 0 0 0
        1 0 5
 -      1 0 0
            5
 -          5
            0
```

Sample response:

```
      [4 2 1]
          2 1
        2 0 0
        2 0 0
   5)2 1 0 5
 -   1 0 0 0
     1 1 0 5
 -   1 0 0 0
        1 0 5
 -      1 0 5
            0
```

Problem 2

Andre and Jada both found 657 ÷ 3 by using partial quotients, but they did the calculations differently, as shown here.

Andre's Work

```
     [2 1 9]
          9
        1 0
      2 0 0
   3)6 5 7
 -   6 0 0
        5 7
 -      3 0
        2 7
 -      2 7
          0
```

Jada's Work

```
     [2 1 9]
          9
        6 0
      1 0 0
        5 0
   3)6 5 7
 -   1 5 0
      5 0 7
 -    3 0 0
      2 0 7
 -    1 8 0
        2 7
 -      2 7
          0
```

a. How is Jada's work the same as Andre's work? How is it different?

- Similarities: Andre and Jada both subtracted multiples of 3 several times. They both added the numbers being multiplied by 3 to find the quotient, and both ended up with 219.
- Differences: Jada subtracted multiples of 3 more times than Andre did, and the multiples of 3 that she subtracted were different.

b. Explain why they have the same answer.

Andre and Jada have the same answer because they both calculated 657 ÷ 3 by subtracting multiples of 3 until there was no remainder, and both gave the number of multiples of 3 subtracted as the answer.

Problem 3

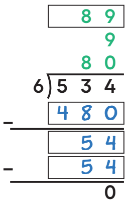
Which might be a better way to find the value of 1,150 ÷ 46: drawing base-ten diagrams or using partial quotients? Explain your reasoning.

Sample response: Using partial quotients works better. Dividing 1,150 into 46 equal groups by drawing will take too long. With partial quotients, the groups don't need to be drawn.

Problem 4

Here is an incomplete calculation of 534 ÷ 6.

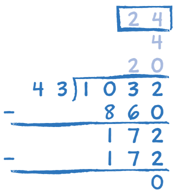
Write the missing numbers (marked with "?") that would make the calculation complete.



Problem 5

Use partial quotients to find 1,032 ÷ 43.

Sample response:



Problem 6

from Unit 5, Lesson 8

Which of the polygons has the greatest area?

A. A rectangle that is 3.25 inches wide and 6.1 inches long.

B. A square with side length of 4.6 inches.

C. A parallelogram with a base of 5.875 inches and a height of 3.5 inches.

D. A triangle with a base of 7.18 inches and a height of 5.4 inches.

Problem 7

from Unit 5, Lesson 4

One micrometer is a millionth of a meter. A certain spider web is 4 micrometers thick. A fiber in a shirt is 1 hundred-thousandth of a meter thick.

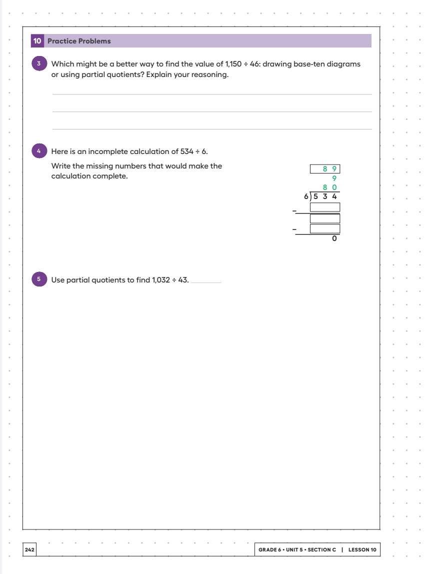
a. Which is wider, the spider web or the fiber? Explain your reasoning.

The fiber is wider. 1 hundred-thousandth is 10 millionths, and 10 millionths is more than 4 millionths.

b. How many meters wider?

6 hundred-thousandths of a meter (or equivalent)

Student Workbook



Student Workbook