Lesson 16

Two Related Quantities, Part 1

Goals

- Comprehend the terms "independent variable" and "dependent variable" (in spoken and written language).
- Create a table, graph, and equation with two variables to represent the relationship between quantities.

Learning Targets

- I can create tables and graphs that show the relationship between two amounts.
- I can write an equation with two variables that shows the relationship between two amounts.

Lesson Narrative

This lesson is the first of two that apply new understanding of algebraic expressions and equations to represent relationships between two quantities. Students use tables, graphs, and equations that represent these relationships and make connections across these representations.

In the first activity, students analyze a relationship that can be defined by addition or subtraction. The context, about the difference of two measurements, is one that students encountered earlier in the unit. At that point, students wrote expressions in one variable that can be used to find a quantity when the relationship to the other quantity is known. Here, they write equations that relate two quantities and examine the graphs that can represent the relationship.

In the second activity, students revisit and extend their understanding of equivalent ratios. A familiar scenario of mixing paints in a given ratio provides the context for writing equations that represent a multiplicative relationship between two quantities. Students then create a table of values that shows how changes in one quantity affect changes in the other, and graph the points from the table on a coordinate grid.

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Cool-down (Assessment)
10 min	25 min	10 min	5 min

Lesson 16

Two Related Quantities, Part 1

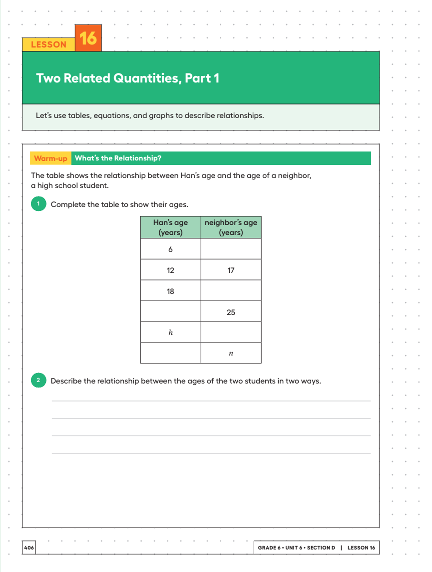
Lesson Narrative (continued)

Students learn that relationships between the two quantities can be described by two different but related equations with one quantity, the **dependent variable**, affected by changes in the other quantity, the **independent variable**. In mathematical modeling, which variable is considered independent and which is considered dependent is often the choice of the modeler, although sometimes the situation suggests choosing one way over the other. The contexts in this lesson were chosen because they do not suggest which quantity should be selected as the independent variable.

Student Learning Goal

Let's use tables, equations, and graphs to describe relationships.

Student Workbook


LESSON 16

Two Related Quantities, Part 1

Let's use tables, equations, and graphs to describe relationships.

Warm-up What's the Relationship?

The table shows the relationship between Han's age and the age of a neighbor, a high school student.

1 Complete the table to show their ages.

Han's age (years)	neighbor's age (years)
6	
12	17
18	
	25
h	
	n

2 Describe the relationship between the ages of the two students in two ways.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Warm-up

What's the Relationship?

10 min

Activity Narrative

This *Warm-up* invites students to describe an additive relationship between two quantities, first using specific numbers, and then more generally, using descriptions. During the *Activity Synthesis*, students learn that they can write equations with two variables to represent the relationship between the quantities.

The familiar context of age difference allows students to see that two equations are possible, because we can describe the age of a younger person based on the age of the older person (such as, "A is some years younger than B") and the other way around (such as, "B is some years older than A"). Students have an opportunity to attend to precision as they use words and equations to represent relationships.

Launch

Arrange students in groups of 2. Give students 2–3 min of partner work time. Follow with a whole-class discussion.

Student Task Statement

The table shows the relationship between Han's age and the age of a neighbor, a high school student.

1. Complete the table to show their ages.

Han's age (years)	neighbor's age (years)
6	11
12	17
18	23
20	25
h	$h+5$
$n-5$	n

2. Describe the relationship between the ages of the two students in two ways.

   Sample response:

   - Han's neighbor is 5 years older than Han.
   - Han is 5 years younger than his neighbor.

Activity Synthesis

Invite students to share their descriptions of the relationship between Han's and his neighbor's ages, and to explain how those relationships can be seen in the table.

To highlight that two equations can be written to represent the same relationship, ask students:

- *"Let's say h represents Han's age in years and n represents his neighbor's age. What equations can we write to represent the relationship between Han's and his neighbor's ages?"*

  $n = h + 5$ and $h = n - 5$

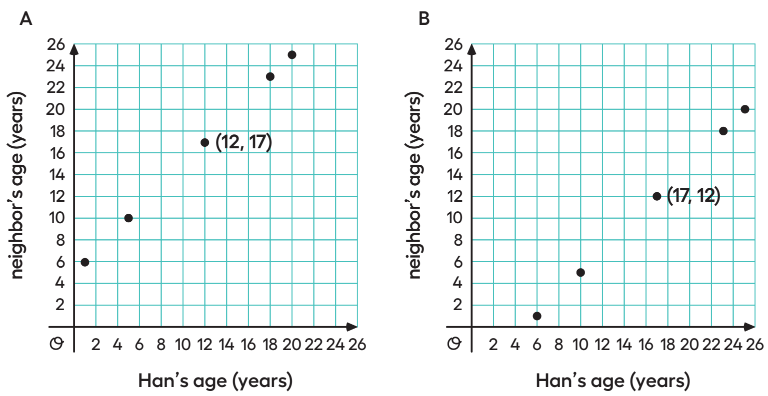
- *"If we know Han's age, which equation would help us find his neighbor's age?"*

  $n = h + 5$

- *"If we know Han's neighbor's age, which equation would help us find Han's age?"*

  $h = n - 5$

Next, display the following two graphs for all to see. Give students a minute to observe the graphs.



Ask students:

- *"Which graph represents the relationship between Han's age and his neighbor's age? How do you know?"*

  Both graphs

  The coordinates of the points match the values in the table but the coordinate values are switched.

- *"Which graph corresponds to which equation? How can we tell?"*

  Graph A corresponds to $n = h + 5$.

  Adding 5 to a horizontal coordinate value gives the vertical value. Graph B corresponds to $h = n - 5$ because subtracting the first coordinate value gives the second value.

Tell students that they will explore other relationships in which one quantity affects the other and describe them using words, tables, equations, and graphs.

Activity 1
Painting the Set

25 min

Activity Narrative

There is a digital version of this activity.

In this activity, students analyze the relationship between two quantities that form a ratio, and think about how tables, graphs, and equations represent the relationship. The work here activates students' prior knowledge of ratios, ways to reason about them, and the language used to describe ratios.

The familiar context of mixing paint is used to elicit language related to ratios and rates, but students may also use language that is less formal to describe one quantity in terms of the other. This will serve as the basis for thinking about dependent and independent variables and for writing equations with two variables.

Students begin by completing a table of equivalent ratios and plotting the values on two coordinate grids, creating two graphs that represent the same situation. Then, students describe the relationship between the two quantities in as many ways as they can. Thinking about how one quantity relates to the other in turns allows students to interpret and write equations with variables. Along the way, students reason quantitatively and abstractly.

In the *Activity Synthesis*, the terms "dependent variable" and "independent variable" are introduced to describe the two variables.

In the digital version of the activity, students can choose to use an applet to plot points on a graph to show the relationship between cups of red paint and cups of yellow paint in the mixture. The applet allows students to place a point directly on to the graph, using the axis labels to identify the correct location. Then they can double- click the point to check or change the point's coordinates.
If students don't have individual access, consider displaying the applet for all to see after students create their own graphs on paper.

Launch

Invite students to share what they know about plays and the set of a play. As needed, explain that the set describes the scenery and other props and objects that are used on stage during a play or production.

Keep students in groups of 2. Give 3–4 minutes of quiet work time on the first three questions, followed by 2–3 minutes to discuss their responses with a partner. Ask students to pause for a brief whole-class discussion after they describe the relationship between the amounts of red paint and yellow paint in multiple ways.

Invite students to share their descriptions and to point out how the described relationship can be seen in the table and the graphs. Record the responses for all to see, and highlight those that describe the amount of one color of paint in terms of the other.

If no students express the relationship using rate or ratio language, provide the following sentence frames for students to complete:

- *"As the number of cups of red paint increases, the number of yellow cups of paint __________."*

  "For every __________ cups of red paint, you need __________ cups of yellow paint."

  "For each cup of red paint, you need __________ cups of yellow paint."

Give students time to complete the remaining questions, leaving at least 5 minutes for discussion.

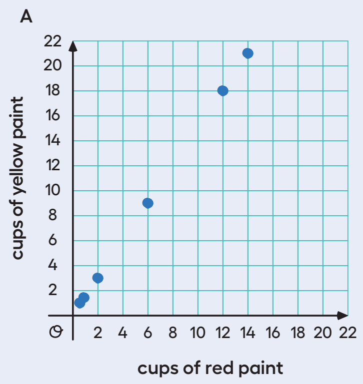
Student Task Statement

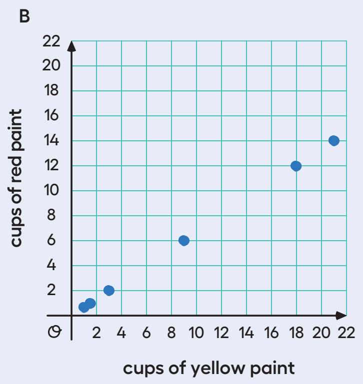
Lin needs to mix a specific shade of orange paint for the set of the school play. The color is a mixture of red and yellow paint.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

cups of red paint	cups of yellow paint
2	3
6	9
8	12
12	18
14	21
1	$\frac{3}{2}$ (or equivalent)
$\frac{2}{3}$ (or equivalent)	1

2. Use the values in the table to create two graphs that can represent the relationship between cups of red paint and cups of yellow paint.

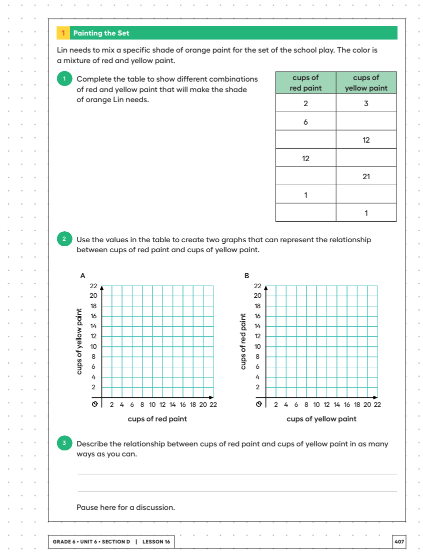




Building on Student Thinking

Students may think that, like the equation Lin wrote, their equation for the last question also needs to start with "$y =$ __________." Encourage students to start by thinking about how much red paint is needed for different amounts of yellow paint, starting with 3 cups and 1 cup (the values in the table), and then other values such as 4 cups and 10 cups. Prompt students to apply the same thinking to find r (cups of red paint) for y (cups of yellow paint).

Student Workbook


1 Painting the Set

Lin needs to mix a specific shade of orange paint for the set of the school play. The color is a mixture of red and yellow paint.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

cups of red paint	cups of yellow paint
2	3
6	
	12
12	
	21
1	
	1

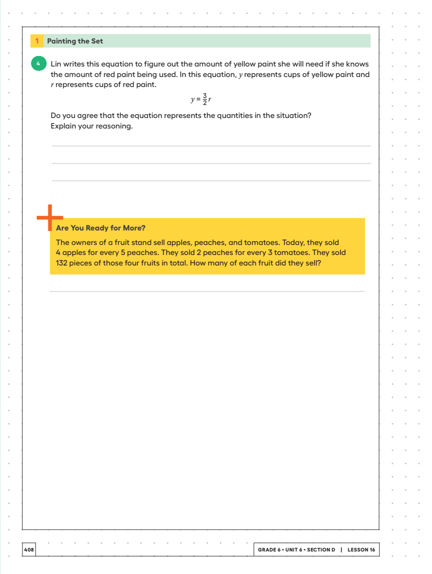
2. Use the values in the table to create two graphs that can represent the relationship between cups of red paint and cups of yellow paint.

3. Describe the relationship between cups of red paint and cups of yellow paint in as many ways as you can.

Pause here for a discussion.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Student Workbook



1 Painting the Set

4 Lin writes this equation to figure out the amount of yellow paint she will need if she knows the amount of red paint being used. In this equation, y represents cups of yellow paint and r represents cups of red paint.

$y = \frac{3}{2}r$

Do you agree that the equation represents the quantities in the situation? Explain your reasoning.

Are You Ready for More?

The owners of a fruit stand sell apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of those four fruits in total. How many of each fruit did they sell?

3. Describe the relationship between cups of red paint and cups of yellow paint in as many ways as you can.

Sample responses:

- The cups of yellow paint increase as the cups of red paint increase.
- For every 2 cups of red paint, 3 cups of yellow paint are needed.
- The ratio of red paint to yellow paint is 2:3.
- For each cup of red paint, $\frac{3}{2}$ cups of yellow paint are needed.
- For each cup of yellow paint, $\frac{3}{2}$ cup of red paint is needed.

Pause here for a discussion.

4. Lin writes this equation to figure out the amount of yellow paint she will need if she knows the amount of red paint being used. In this equation, y represents cups of yellow paint and r represents cups of red paint.

$$y = \frac{3}{2}r$$

Do you agree that the equation represents the quantities in the situation? Explain your reasoning.

Agree

Sample reasoning:

- For every cup of red paint, $\frac{3}{2}$ cups of yellow paint is needed, so it makes sense that y is equal to r times $\frac{3}{2}$.
- Substituting every pair of values in the table into the equation makes the equation true. For example: if r is 8, then $y = \frac{3}{2} \cdot 8$, which is $\frac{24}{2}$ or 12.
- $r = \frac{2}{3} \cdot y$. Sample reasoning: For every cup of yellow paint used, $\frac{2}{3}$ of red paint is needed, so multiplying y by $\frac{2}{3}$ gives r.

Are You Ready for More?

The owners of a fruit stand sell apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of those four fruits in total. How many of each fruit did they sell?

32 apples, 40 peaches, 60 tomatoes

Activity Synthesis

Invite students to share whether they agree that the equation Lin wrote relates the amount of yellow paint to red paint and explain their reasoning. Then discuss the equation that students wrote for the last question.

If no students mentioned using the values in the table to check Lin's equation or to help write a second equation, ask them to discuss this idea.

Explain that in a situation that has two quantities that change in relation to each other, we can use the words "dependent" and "independent" to describe the two variables:

- If we know the amount of red paint, we can use it to calculate the amount of yellow paint, as Lin has done by writing $y = \frac{3}{2}r$.
- In this case, we say that the amount of yellow paint depends on the amount of red paint, so y is the **dependent variable** and r is the **independent variable**.

- If we know the amount of yellow paint, we can use it to calculate the amount of red paint, as students have done by writing $r = \frac{2}{3}y$.
- In this case, we say that the amount of red paint depends on the amount of yellow paint so, r is the dependent variable and y is the independent variable

Draw students' attention to the two graphs they created. Ask:

"Which equation is represented by the first graph?"

$y = \frac{3}{2}r$

"How can we tell?"

Multiplying the horizontal or first value of the ordered pairs by $\frac{3}{2}$ gives the second or vertical value.

Lesson Synthesis

To summarize the major ideas in this lesson, prompt students to refer to their work in the two activities. Discuss questions such as:

"How are the relationships between the quantities in the two situations—about ages and about paint mixture—alike?"

The value of one quantity depends on the other. The quantities can be represented using a table and graphs. We can write two equations and create two graphs for each relationship.

"How are they different?"

In the situation about ages, we added or subtracted the value of one quantity by a number to get the other. In the situation about paint mixture, we multiplied one quantity by a number to get the other.

"How do we know what equations to write to describe the relationship between the quantities?"

We can think about how to find the second quantity if we know the first quantity, and the other way around. We can use a table and think about how to complete the value in one column if we have the value in the other column.

Consider annotating the relationship between the pair of quantities in the table for each situation, as shown.

Han's age (years)		neighbors' age (years)
6	+ 5	11
12	+ 5	17
h	+ 5	$h + 5$

$h + 5 = n$ or $n = h + 5$

cups of red paint		cups of yellow paint
2	$\cdot \frac{3}{2}$	3
1	$\cdot \frac{3}{2}$	$\frac{3}{2}$
r	$\cdot \frac{3}{2}$	$\frac{3}{2} \cdot r$

$r \cdot \frac{3}{2} = y$ or $y = \frac{3}{2}r$

Han's age (years)		neighbors' age (years)
6	− 5	11
12	− 5	17
$n - 5$	− 5	n

$n - 5 = h$ or $h = n - 5$

cups of red paint		cups of yellow paint
2	$\cdot \frac{2}{3}$	3
$\frac{2}{3}$	$\cdot \frac{2}{3}$	1
$\frac{2}{3} \cdot y$	$\cdot \frac{2}{3}$	y

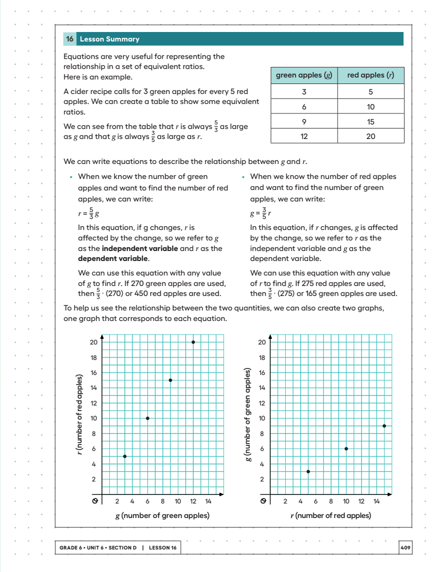
$y \cdot \frac{2}{3} = r$ or $r = \frac{2}{3}y$

Access for Multilingual Learners Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students in explaining how they know an equation represents the relationship in the situation. Examples: "I agree (or disagree) that $y = \frac{3}{2}r$ represents the quantities of paint because ..." "I know that the equation ________ can be used to find the amount of red paint because ..."

Advances: Speaking, Conversing

Student Workbook



Lesson Summary

Equations are very useful for representing the relationship in a set of equivalent ratios. Here is an example.

A cider recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r.

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can write equations to describe the relationship between g and r.

- When we know the number of green apples and want to find the number of red apples, we can write:

  $r = \frac{5}{3}g$

  In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

  We can use this equation with any value of g to find r. If 270 green apples are used, then $\frac{5}{3} \cdot (270)$ or 450 red apples are used.

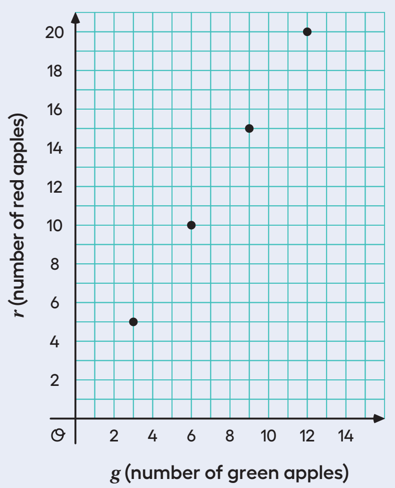
- When we know the number of red apples and want to find the number of green apples, we can write:

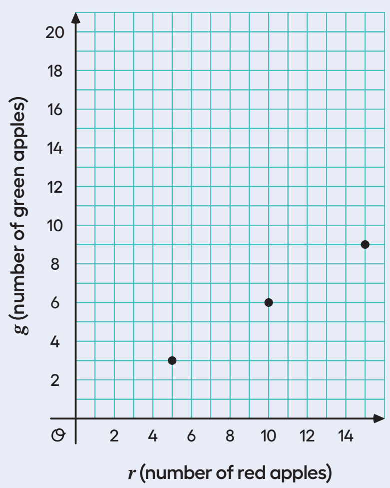
  $g = \frac{3}{5}r$

  In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

  We can use this equation with any value of r to find g. If 275 red apples are used, then $\frac{3}{5} \cdot (275)$ or 165 green apples are used.

To help us see the relationship between the two quantities, we can also create two graphs, one graph that corresponds to each equation.





Cool-down
Kitchen Cleaner

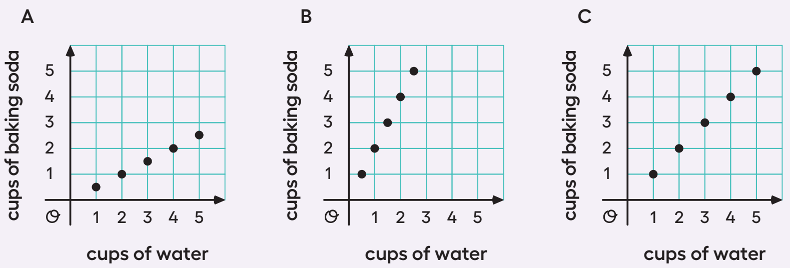
5 min

Student Task Statement

To remove grease from kitchen surfaces, a recipe says to use 1 cup of baking soda for every $\frac{1}{2}$ cup of water.

cups of baking soda	cups of water
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$

1. Which graph represents the relationship between cups of baking soda and cups of water? Explain how you know.



Graph B

Sample reasoning:

- In all graphs, the first value of the coordinates represents the amount of water. The amount of baking soda is twice the amount of water, so the coordinates of the points should be $\left(\frac{1}{2},1\right)$, $(1,2)$, $\left(\frac{3}{2},3\right)$, and so on.
- I matched the coordinates of the points to the values in the table: $\frac{1}{2}$ cup of water goes with 1 cup of baking soda, 1 cup of water goes with 2 cups of baking soda, and so on.
- The ratio of cups of water to cups of baking soda is 2 to 1, so I looked at the coordinate points that show the same ratio.

2. Select all equations that can represent the relationship between b, cups of baking soda, and w, cups of water, in this situation.

A. $w = \frac{1}{2}b$

B. $b = \frac{1}{2}w$

C. $b = w$

D. $b = 2w$

E. $w = 2b$

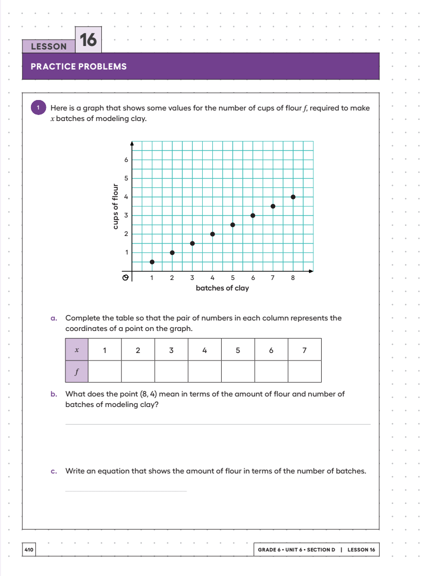
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

6 Problems

Student Workbook


LESSON 16
PRACTICE PROBLEMS

1 Here is a graph that shows some values for the number of cups of flour f, required to make x batches of modeling clay.

a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

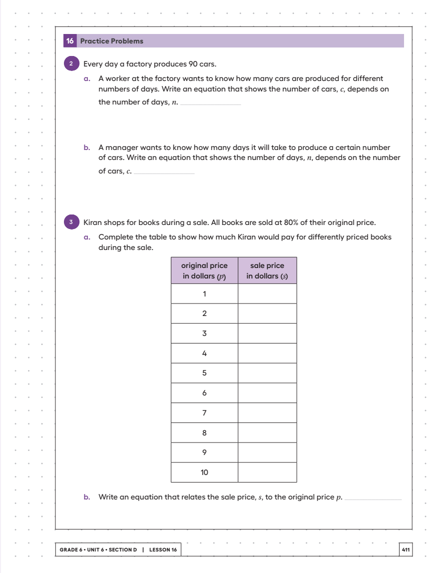
x	1	2	3	4	5	6	7
f							

b. What does the point (8, 4) mean in terms of the amount of flour and number of batches of modeling clay?

c. Write an equation that shows the amount of flour in terms of the number of batches.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Student Workbook


16 Practice Problems

2 Every day a factory produces 90 cars.

a. A worker at the factory wants to know how many cars are produced for different numbers of days. Write an equation that shows the number of cars, c, depends on the number of days, n.

b. A manager wants to know how many days it will take to produce a certain number of cars. Write an equation that shows the number of days, n, depends on the number of cars, c.

3 Kiran shops for books during a sale. All books are sold at 80% of their original price.

a. Complete the table to show how much Kiran would pay for differently priced books during the sale.

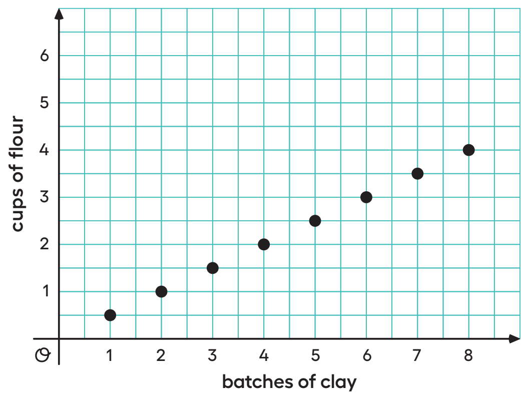
original price in dollars (p)	sale price in dollars (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

b. Write an equation that relates the sale price, s, to the original price p.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Problem 1

Here is a graph that shows some values for the number of cups of flour f, required to make x batches of modeling clay.



a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

x	1	2	3	4	5	6	7
f	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$

b. What does the point (8, 4) mean in terms of the amount of flour and number of batches of modeling clay?

To make 8 batches of modeling clay, 4 cups of flour are needed.

c. Write an equation that shows the amount of flour in terms of the number of batches.

$f = \frac{1}{2}x$ (or equivalent)

Problem 2

Every day a factory produces 90 cars.

a. A worker at the factory wants to know how many cars are produced for different numbers of days. Write an equation that shows the number of cars, c, depends on the number of days, n.

$c = 90n$

b. A manager wants to know how many days it will take to produce a certain number of cars. Write an equation that shows the number of days, n, depends on the number of cars, c.

$n = \frac{c}{90}$ or $n = c \div 90$

Problem 3

Kiran shops for books during a sale. All books are sold at 80% of their original price.

a. Complete the table to show how much Kiran would pay for differently priced books during the sale.

original price in dollars (p)	sale price in dollars (s)
1	0.80
2	1.60
3	2.40
4	3.20
5	4.00
6	4.80
7	5.60
8	6.40
9	7.20
10	8.00

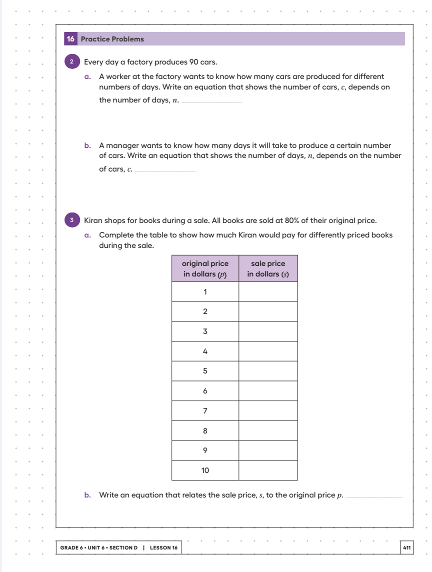
b. Write an equation that relates the sale price, s, to the original price p.

$s = 0.8p$ (or equivalent)

c. Complete the graph showing the relationship between the sale price and the original price by plotting the points from the table.

Sample response:

Student Workbook


16 Practice Problems

2 Every day a factory produces 90 cars.

a. A worker at the factory wants to know how many cars are produced for different numbers of days. Write an equation that shows the number of cars, c, depends on the number of days, n.

b. A manager wants to know how many days it will take to produce a certain number of cars. Write an equation that shows the number of days, n, depends on the number of cars, c.

3 Kiran shops for books during a sale. All books are sold at 80% of their original price.

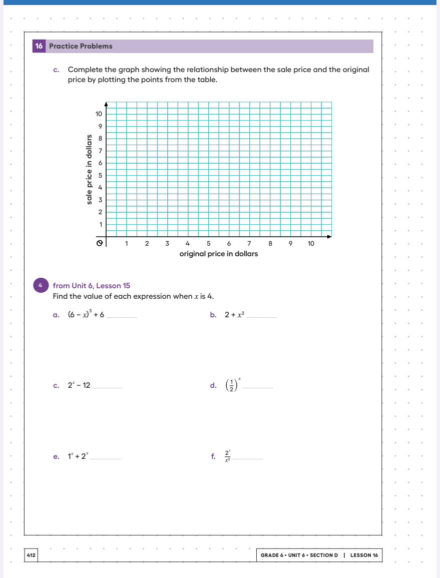
a. Complete the table to show how much Kiran would pay for differently priced books during the sale.

original price in dollars (p)	sale price in dollars (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

b. Write an equation that relates the sale price, s, to the original price p.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Student Workbook


16 Practice Problems

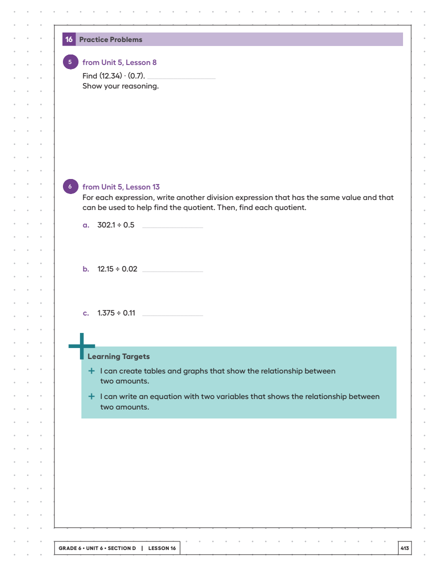
c. Complete the graph showing the relationship between the sale price and the original price by plotting the points from the table.

4 from Unit 6, Lesson 15

Find the value of each expression when x is 4.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Student Workbook


16 Practice Problems

5 from Unit 5, Lesson 8

Find (12.34) · (0.7).

Show your reasoning.

6 from Unit 5, Lesson 13

For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

a. 302.1 ÷ 0.5

b. 12.15 ÷ 0.02

c. 1.375 ÷ 0.11

Learning Targets

+ I can create tables and graphs that show the relationship between two amounts.

+ I can write an equation with two variables that shows the relationship between two amounts.

GRADE 6 • UNIT 6 • SECTION D | LESSON 16

Problem 4

from Unit 6, Lesson 15

Find the value of each expression when x is 4.

a. $(6 - x)^3 + 6$

14

b. $2 + x^3$

66

c. $2^x - 12$

4

d. $\left(\frac{1}{2}\right)^x$

$\frac{1}{16}$

e. $1^x + 2^x$

17

f. $\frac{2^x}{x^2}$

1

Problem 5

from Unit 5, Lesson 8

Find (12.34) · (0.7). Show your reasoning.

8.638

Sample reasoning: $1{,}234 \cdot 7 = 8{,}638$. Because 12.34 is $\frac{1}{100}$ of 1,234 and 0.7 is $\frac{1}{10}$ of 7, the product 8,638 needs to be multiplied by $\left(\frac{1}{100} \cdot \frac{1}{10}\right)$ or $\frac{1}{1{,}000}$.

Problem 6

from Unit 5, Lesson 13

For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

a. 302.1 ÷ 0.5

Sample response:

3,021 ÷ 5

The quotient is 604.2

b. 12.15 ÷ 0.02

Sample response:

1,215 ÷ 2

The quotient is 607.5

c. 1.375 ÷ 0.11

Sample response:

137.5 ÷ 11

The quotient is 12.5