## What Fraction of a Group?

#### Goals

- Comprehend the phrase "What fraction of a group?" (in spoken and written language) as a variation of the question "How many groups?" that is used when the number of groups is less than 1.
- Create a tape diagram to represent and solve a problem asking "How many groups?" in which the number of groups is a fraction less than 1.
- Write multiplication and division equations to represent a problem asking "How many times as long?"

#### **Learning Targets**

- I can tell when a question is asking for the number of groups and that number is less than 1.
- I can use diagrams and multiplication and division equations to represent and answer "What fraction of a group?" questions.

In this lesson, students extend their exploration of "How many groups?" questions to include cases where the number of groups is a fraction less than 1. In such situations, the total amount is smaller than the size of 1 group, so the question becomes "What fraction of a group?"

Students notice that they can use the same reasoning strategies as before because each situation here still has the same structure of (number of groups)  $\cdot$  (size of a group) = total amount. They continue to write multiplication equations of this form and the corresponding division equations.

Students first reason about "What fraction of a group?" questions in the context of fractional batches of a recipe. Students can make sense of fractions of a group in the optional activity, which provides another concrete situation involving lengths of ropes before answering questions without a context in the last activity.

#### **Access for Students with Diverse Abilities**

• Engagement (Activity 1)

#### **Access for Multilingual Learners**

- MLR5: Co-Craft Questions (Activity 2)
- MLR6: Three Reads (Activity 1)

#### **Required Materials**

#### **Materials to Gather**

- · Colored pencils: Activity 2
- · Colored pencils: Activity 4

#### **Activity 3:**

For the digital version of the activity, acquire devices that can run the applet.

#### **Lesson Timeline**



Warm-up

10

**Activity 1** 

20

**Activity 2** 

10

**Lesson Synthesis** 

Assessment

Cool-down

## What Fraction of a Group?

## **Lesson Narrative (continued)**

Throughout the lesson, students need to pay close attention to how their diagram, description, and equations represent the relationship between a given total amount and the size of 1 group. In doing so, they practice reasoning abstractly and quantitatively.

#### **Student Learning Goal**

Let's think about dividing things into groups when we can't even make one whole group.

#### Warm-up

#### **Notice and Wonder: Cups and Days**



#### **Activity Narrative**

The purpose of this *Warm-up* is twofold: to familiarize students with the quantities and representations they will see later in the lesson, and to elicit observations about the size of a quantity relative to the size of 1 group. The insights will be useful later when students interpret division situations involving quotients that are either greater than 1 or less than 1.

While students may notice and wonder many things about these diagrams, observations about whether the amount shown for each day represents more or less than 1 batch are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

#### Launch

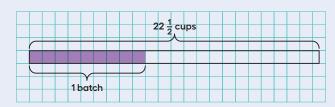
Display the two tape diagrams for all to see.

Give students **1 minute** of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

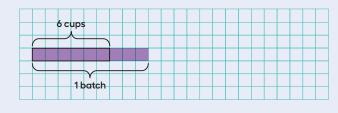
#### **Student Task Statement**

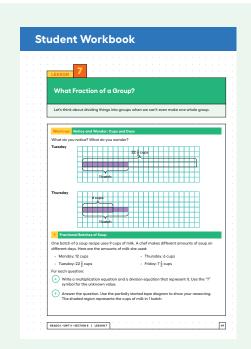
What do you notice? What do you wonder?

#### **Tuesday**



#### **Thursday**





**Lesson 7** Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

#### Students may notice:

- The tape diagrams show different numbers of cups of something on two days of the week.
- In each diagram, the size of I batch is 9 cups.
- · On Tuesday, the number of cups is more than one batch.
- On Thursday, the number of cups is less than one batch. It is  $\frac{6}{9}$  of a batch.

#### Students may wonder:

- What is the situation about?
- · Why is the amount for Thursday less than one batch?
- How many batches do 22 \frac{1}{2} cups each make?
- · On Thursday, what fraction of a batch is being made?
- Why might someone want to make  $\frac{6}{9}$  of a batch?

### **Activity Synthesis**

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the diagrams. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the number of batches represented in each diagram does not come up during the conversation, ask students to discuss this idea.

#### **Activity 1**

#### **Fractional Batches of Soup**

20 min

#### **Activity Narrative**

This activity helps to transition students from thinking about "How many groups?" to "What fraction of a group?" Students make sense of division situations in which the number of groups could be greater than 1 or less than 1.

Given the amount of milk required for 1 batch of soup (or the size of 1 group in this situation), students find out how many batches or what fraction of a batch can be made with different amounts of milk. They write multiplication and division equations to represent the questions. To support their reasoning, students may also choose to use the partially started tape diagrams they previewed in the *Warm-up*.

As students work, monitor for various ways of reasoning and of marking up the diagrams. Select students who reason effectively and in different ways, and ask them to share later.

#### Launch



Arrange students in groups of 2.

Give students 6-8 minutes of quiet work time, followed by 2-3 minutes to discuss their responses with their partner.

Provide access to colored pencils, as some students may find it helpful to identify whole groups and partial groups on a tape diagram by coloring.

#### **Student Task Statement**

One batch of a soup recipe uses 9 cups of milk. A chef makes different amounts of soup on different days. Here are the amounts of milk she used:

- Monday: 12 cups
- Tuesday: 22 ½ cups
- · Thursday: 6 cups
- Friday:  $7\frac{1}{2}$  cups

For each question:

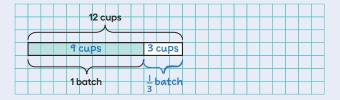
- **A.** Write a multiplication equation and a division equation that represent it. Use the "?" symbol for the unknown value.
- B. Answer the question. Use the partially started tape diagram to show your reasoning. The shaded region represents the cups of milk in 1 batch.
- 1. How many batches of soup did she make on Monday?
  - a. Multiplication equation: Division equation:

Monday:

 $? \cdot 9 = 12 \text{ and } 12 \div 9 = ?$ 

**b.** Answer:

 $1\frac{1}{3}$  batches (or equivalent)



#### **Access for Multilingual Learners** (Activity 1, Launch)

#### MLR6: Three Reads.

Keep student workbooks or devices closed. Display only the problem stem (including the amounts of milk used on different days), without revealing the questions. Tell the students,

"We are going to read this problem 3 times."

After the 1st read, say,

"Tell your partner what this situation is about." After the 2nd read, say,

"List the quantities. What can be counted or measured?" For the 3rd read: Reveal and read the first question. Ask,

"What are some ways we might get started on this?" Advances: Reading, Representing

#### **Access for Students with Diverse Abilities (Activity 1, Launch)**

#### **Engagement: Provide Access by** Recruiting Interest.

Invite students to share their experiences making multiple batches of a food item at home.

Supports accessibility for: Conceptual Processing, Memory

#### **Building on Student Thinking**

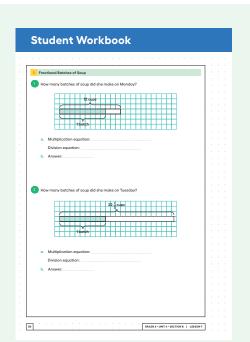
If students are unsure how to write equations for each situation, urge them to start with a multiplication equation. Remind them that a question such as

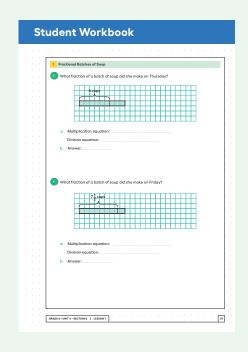
"How many groups of 5 are in 20?" can be thought of as

"What number times 5 is 20?" or ?  $\cdot$  5 = 20. Ask them what the equal-size groups are in this situation. Once students see the questions as

"How many groups of 9 are in some number?" and write multiplication equations, ask them:

"How would you write a division equation that represents the same question?"





2. How many batches of soup did she make on Tuesday?

Activity 3

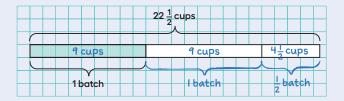
**a.** Multiplication equation: Division equation:

Tuesday:

$$? \cdot 9 = 22\frac{1}{2}$$
 and  $22\frac{1}{2} \div 9 = ?$ 

**b.** Answer:

2 1/2 batches (or equivalent)



- 3. What fraction of a batch of soup did she make on Thursday?
  - a. Multiplication equation:

Division equation:

Thursday:

$$? \cdot 9 = 6 \text{ and } 6 \div 9 = ?$$

**b.** Answer:

 $\frac{2}{3}$  of a batch (or equivalent)

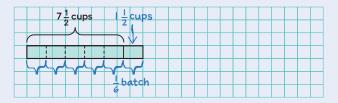


- 4. What fraction of a batch of soup did she make on Friday?
  - a. Multiplication equation: Division equation:

? • 9 = 
$$7\frac{1}{2}$$
 and  $7\frac{1}{2} \div 9 = ?$ 

**b.** Answer:

 $\frac{5}{6}$  of a batch (or equivalent)



#### **Activity Synthesis**

Ask previously identified students to share their responses (including diagrams) and to explain their reasoning. Display the diagrams for all to see.

To help students see the structure in the diagrams, ask:

(How are the diagrams for Monday and Tuesday like the ones for Thursday and Friday? How are they different?"

If not mentioned by students, point out that:

- The size of 1 group (or the amount of milk in 1 batch) is the same in every diagram, but the amount we are comparing to 1 batch varies. On Monday and Tuesday, that amount is each greater than 1 batch (9 cups). On Thursday and Friday, it is less than 1 batch (9 cups).
- Comparison to the size of 1 group is also reflected in the language used. The questions start with "How many batches?" for the first two days and "What fraction of a batch?" for the other two.

To help students notice the structure in the equations, display the four multiplication and division equations (showing the quotients). Ask students:

? · 9 = 12	12 ÷ 9 = ?
$? \cdot 9 = 22\frac{1}{2}$	$22\frac{1}{2} \div 9 = ?$
? · 9 = 6	6 ÷ 9 = ?
$? \cdot 9 = 7\frac{1}{2}$	$7\frac{1}{2} \div 9 = ?$

"What do the multiplication equations have in common? What about the division equations?"

Highlight that:

- The four questions can also be phrased as "How many groups of 9 are in each amount of milk?" Represented as multiplication, they all have the structure of "What number times 9 equals a given amount of milk?" or ? · 9 = amount of milk.
- Whether the questions are asking "How many batches (of 9 cups)" or "What fraction of a batch (of 9 cups)," they can be expressed with a division by 9 or amount of milk ÷ 9 = number of batches.

**Lesson 7** Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

## Access for Multilingual Learners (Activity 2, Launch)

#### MLR5: Co-Craft Questions.

Keep student workbooks or devices closed. Display only the problem stem and the diagram, without revealing the questions, and ask students to write a list of mathematical questions that could be asked about the four ropes. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?" Reveal the incomplete sentences in the table that students are to complete, and invite additional connections.

#### **Activity 2: Optional**

#### **Fractions of Ropes**



#### **Activity Narrative**

#### There is a digital version of this activity.

This activity offers students another opportunity to make sense of division situations in which the quotients can be either greater than 1 or less than 1. Students examine different lengths of ropes and compare their lengths relative to one another in multiplicative terms. For example, Ropes B and C are 5 and  $2\frac{1}{2}$  times as long as Rope A, respectively. Rope D, which is shorter than Rope A, is  $\frac{3}{4}$  times as long as Rope A.

To make such comparisons, students need to recognize that equal-size groups are also at play in this context, and that the size of 1 group is the length of one rope to which another rope is compared. As students work, monitor for students who make these connections.

In the digital version of the activity, students use an applet to compare the lengths of rope to one another. The applet allows students to move the ropes around and mark off increments. This activity works best when each student has access to the applet because students will benefit from being able to move the shorter ropes around as they compare them to the longer ones. If students don't have individual access, displaying the applet for all to see would be helpful during the synthesis.

## Launch 2

Arrange students in groups of 2. Tell students that they compare lengths of four ropes in this activity and use multiplication and division to make the comparisons.

Give students 4–5 minutes of quiet think time and then a couple of minutes to compare their responses with a partner and discuss any disagreements.

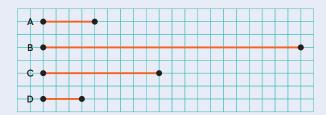
Clarify that rope A is 4 units long.

Activity 1

#### **Student Task Statement**

Here is a diagram that shows four ropes of different lengths.

Warm-up



Complete each sentence comparing the ropes' lengths. Then write a multiplication equation and a division equation for each comparison.

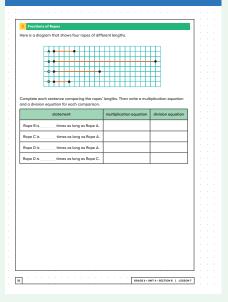
statement	multiplication equation	division equation
Rope B is 5 times as long as Rope A.	5 · 4 = 20	20 ÷ 4 = 5
Rope C is 2 <sup>1</sup> / <sub>4</sub> times as long as Rope A.	2 <del> </del> · 4 = 9	9 ÷ 4 = 2 <sup>1</sup> / <sub>4</sub>
Rope D is $\frac{3}{4}$ times as long as Rope A.	$\frac{3}{4} \cdot 4 = 3$	$3 \div 4 = \frac{3}{4}$
Rope D is $\frac{1}{3}$ times as long as Rope C.	$\frac{1}{3} \cdot 9 = 3$	$3 \div 9 = \frac{1}{3}$

Equations that are equivalent to the ones shown are acceptable.

#### **Building on Student Thinking**

Some students might associate the wrong lengths with the ropes or confuse the order of comparison (such as comparing A to C instead of C to A). Encourage students to record the length of each rope next to the diagram and to attend more closely to the ropes being compared.

#### **Student Workbook**



**Lesson 7** Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

#### **Activity Synthesis**

Display the completed table for all to see. Give students a minute to check their answers and ask questions. Consider displaying the diagram and marking off the lengths of Rope A on each of the other ropes.

Next, discuss the connections between "How many times as long?" questions and "How many groups?" questions. Ask students:

"How are these two types of questions alike?"

They both involve finding how many groups of one amount are in another amount. The number of groups may or may not be a whole number.

"How are they different?"

The equal-size groups here are lengths. The questions involve the phrase "times as long as."

Then, discuss the values that complete the comparison statements. Ask students:

"How are the values in the last two statements different from the first two?"

In the former two, each value is greater than I. In the latter two, each value is less than I.

"In this situation, what do the values that are less than 1 mean about Rope D?"

Rope D is shorter than the other ropes, so its length is a fraction of that of the other ropes.

If time permits, discuss how the multiplication and division equations that students wrote represent comparisons between Rope D and Ropes A and C.

#### **Activity 3**

#### **Not Quite One Group**

10 min

#### **Activity Narrative**

In this activity, students apply their insights from earlier activities to make sense of division questions without a context. While the questions begin with the phrase "what fraction of"— suggesting that the situations deal with an amount that is less than the size of 1 group, students still need to determine which of the two given values in each question represents the size of 1 group.

As students make sense of quantities described in words, represent them with diagrams and equations, and interpret numerical answers in terms of the questions, they reason abstractly and quantitatively.

## Launch

Keep students in groups of 2.

Give students 4–5 minutes of quiet work time, followed by a minute to discuss their responses with their partner.

Provide continued access to colored pencils.

#### **Student Task Statement**

For each question, write a multiplication equation and a division equation. Then answer the question. You can draw a tape diagram if you find it helpful.

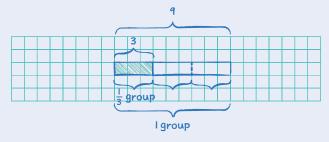
Warm-up

- 1. What fraction of 9 is 3?
  - **a.** Multiplication equation: Division equation:

$$? \cdot 9 = 3 \text{ and } 3 \div 9 = ?$$

**b.** Answer:

 $\frac{1}{3}$  (or equivalent)

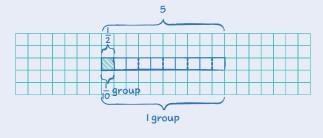


- **2.** What fraction of 5 is  $\frac{1}{2}$ ?
  - **a.** Multiplication equation: Division equation:

$$? \cdot 5 = \frac{1}{2}$$
 and  $\frac{1}{2} \div 5 = ?$ 

**b.** Answer:

 $\frac{1}{10}$  (or equivalent)



#### **Activity Synthesis**

Invite students to share their responses. Display their equations and diagrams for all to see. Discuss how the two given values and the answer to the question can be seen in each representation. Ask questions such as:

- What do the quotients  $\frac{1}{3}$  and  $\frac{1}{10}$  mean in terms of the questions?" There is  $\frac{1}{3}$  of 9 in 3, or 3 is  $\frac{1}{3}$  of 9. There is  $\frac{1}{10}$  of 5 in  $\frac{1}{2}$ , or  $\frac{1}{2}$  is  $\frac{1}{10}$  of 5.
- "How can we check if the answers are correct?"

We can use multiplication:  $\frac{1}{3} \cdot 9 = 3$  and  $\frac{1}{10} \cdot 5 = \frac{1}{2}$ 

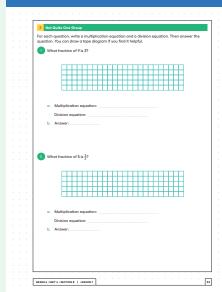
#### **Building on Student Thinking**

In earlier tape diagrams, students saw that each square represented 1 unit. That scale worked well then and would also work fine for representing the first question here. If students default to using the same scale for the last question, however, the parts showing  $\frac{1}{2}$  would be very small, making the diagram difficult to use. Urge students to think strategically when deciding the overall length or the scale to use so that the diagram can be helpful. Ask questions such as:

"What is the smallest value you need to represent? What is the largest value?"

"What should each grid square represent to best show those values? How many squares should represent 1 whole?"

#### **Student Workbook**



# 

#### **Lesson Synthesis**

To highlight the structure of division situations involving a fractional group and its connections to other situations, ask questions such as:

- "How are the problems in this lesson like those in earlier lessons?"
  They are all about finding the number of groups.
- "How are they different?"

The total amount is less than the size of I group. In tape diagrams, the length representing the given total is shorter than the length representing I group.

"How can we tell if a division situation involves less than one whole group?"

The question asks "What fraction of ...?" The total amount given is less than the size of I group. In a recipe situation, an amount less than what is in I batch means a fractional group less than I.

"How do we figure out what fraction of a group a number is?"

We can draw a tape diagram to help us compare the amount to the size of I group and express it as a fraction. We can also write a multiplication equation with a missing factor and think about what number makes the equation true.

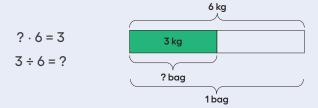
#### **Lesson Summary**

It is natural to think about groups when we have more than one group, but we can also have a fraction of a group.

Sometimes an amount is less than the size of 1 group, and we want to know what fraction of a group that amount is.

Suppose a full bag of flour weighs 6 kg. A chef used 3 kg of flour. What fraction of a full bag was used? In other words, what fraction of 6 kg is 3 kg?

We can still write equations and draw a diagram to represent the situation.



We can see from the diagram that 3 is  $\frac{1}{2}$  of 6, so  $3 \div 6 = \frac{1}{2}$ . We can check this quotient by multiplying:  $\frac{1}{2} \cdot 6 = 3$ .

In *any* situation where we want to know what fraction one number is of another number, we can write a multiplication equation and a division equation to help us find the answer.

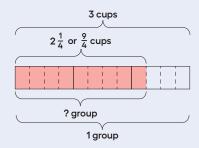
For example, "What fraction of 3 is  $2\frac{1}{4}$ ?" can be expressed as:

Warm-up

$$? \cdot 3 = 2\frac{1}{4}$$

$$2\frac{1}{4} \div 3 = ?$$

The value of  $2\frac{1}{4} \div 3$  is also the answer to the original question.



We can use a diagram to reason that there are 12 fourths in 3 and 9 fourths in  $2\frac{1}{4}$ , so  $2\frac{1}{4}$  is  $\frac{9}{12}$ , or  $\frac{3}{4}$ , of 3. If we multiply  $\frac{3}{4}$  and 3, we get  $2\frac{1}{4}$ .

#### Cool-down

## A Partially Filled Fish Tank

5 min

#### Launch

Provide continued access to colored pencils.

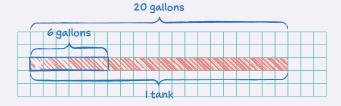
#### **Student Task Statement**

There are 6 gallons of water in a 20-gallon fish tank. What fraction of the tank is filled?

**1.** Write a multiplication equation and a division equation to represent the situation.

$$? \cdot 20 = 6$$
 and  $6 \div 20 = ?$ 

**2.** Answer the question. You can draw a tape diagram if you find it helpful.  $\frac{6}{20}$  or  $\frac{3}{10}$  of the tank



#### **Responding To Student Thinking**

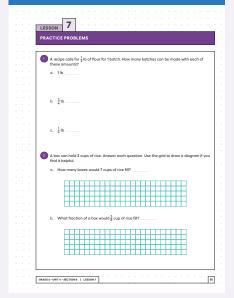
#### Points to Emphasize

**Lesson Synthesis** 

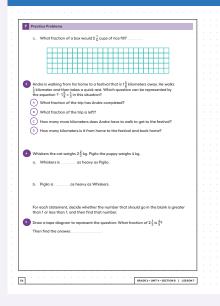
If students struggle with writing equations to represent the situation, focus on interpreting known and unknown quantities in terms of equal-size groups. For example, for each situation in the activity referred to here, ask students whether the given values represent the number of groups, the size of 1 group, or the total amount.

Grade 6, Unit 4, Lesson 8, Activity 2 How Much in One Batch?

## Student Workbook



#### Student Workbook



#### **Problem 1**

A recipe calls for  $\frac{1}{2}$  lb of flour for 1 batch. How many batches can be made with each of these amounts?

- **a.** 1 lb

#### **Problem 2**

A box can hold 3 cups of rice. Answer each question. Use the grid to draw a diagram if you find it helpful.

- a. How many boxes would 7 cups of rice fill?
  - $2\frac{1}{3}$  boxes
- **b.** What fraction of a box would  $\frac{3}{8}$  cup of rice fill?
  - $\frac{1}{8}$  of a box
- **c.** What fraction of a box would  $2\frac{1}{4}$  cups of rice fill?
  - $\frac{3}{4}$  of a box

#### **Problem 3**

Andre is walking from his home to a festival that is  $1\frac{5}{8}$  kilometers away. He walks  $\frac{1}{3}$  kilometer and then takes a quick rest. Which question can be represented by the equation ?  $\cdot 1\frac{5}{8} = \frac{1}{3}$  in this situation?

- **A.** What fraction of the trip has Andre completed?
- B. What fraction of the trip is left?
- C. How many more kilometers does Andre have to walk to get to the festival?
- **D.** How many kilometers is it from home to the festival and back home?

#### **Problem 4**

Whiskers the cat weighs  $2\frac{2}{3}$  kg. Piglio the puppy weighs 4 kg.

**a.** Whiskers is \_\_\_\_\_ as heavy as Piglio.

Less than  $I_{\frac{8}{12}}$  or  $\frac{2}{3}$  (or equivalent)

**b.** Piglio is \_\_\_\_\_\_ as heavy as Whiskers.

More than I.  $\frac{12}{8}$  or  $I_{\frac{1}{2}}$  (or equivalent)

For each statement, decide whether the number that should go in the blank is greater than 1 or less than 1, and then find that number.

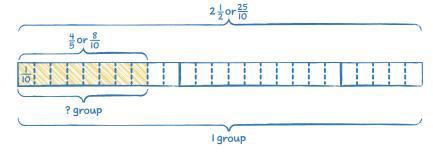
#### **Problem 5**

Draw a tape diagram to represent the question: What fraction of  $2\frac{1}{2}$  is  $\frac{4}{5}$ ?

Then find the answer.

8 25

## Sample reasoning:



#### **Problem 6**

from Unit 4, Lesson 6

How many groups of  $\frac{3}{4}$  are in each of these quantities?

**a.**  $\frac{11}{4}$ 

 $3\frac{2}{3}$ 

Sample reasoning: Create a tape diagram showing  $\frac{11}{4}$  divided into groups of  $\frac{3}{4}$  each.

**b.**  $6\frac{1}{2}$ 

8=

Sample reasoning: Create a tape diagram showing  $6\frac{1}{4}$  divided into groups of  $\frac{3}{4}$  each.

#### **Problem 7**

from Unit 4, Lesson 4

Which question can be represented by the equation  $4 \div \frac{2}{7} = ?$ 

- **A.** What is 4 groups of  $\frac{2}{7}$ ?
- **B.** How many  $\frac{2}{7}$ s are in 4?
- **C.** What is  $\frac{2}{7}$  of 4?
- **D.** How many 4s are in  $\frac{2}{7}$ ?

