Lesson 14

Distances in the Coordinate Plane

Goals

- Compare and contrast (orally and in writing) the coordinates for points in different locations on the coordinate plane.
- Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.
- Generalize (orally) about the coordinates of points that are reflected across the x- or y-axis.

Learning Target

I can find horizontal and vertical distances between points on the coordinate plane.

Lesson Narrative

In this lesson, students explore the relationship between points with opposite coordinates and develop strategies for finding vertical and horizontal distances in the coordinate plane. Students begin by plotting points in one of the four quadrants. By comparing the coordinates of points in the different quadrants, students generalize patterns about their signs. Next, students explore points where the x- or y-coordinates are opposites and see that they are also reflections across the x- or y-axis. Then students develop strategies for finding the distance between two points where the coordinates may not be integers.

Student Learning Goal

Let's explore distance on the coordinate plane.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- Notice and Wonder

Required Materials

Materials to Gather

- Tracing paper: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	10 min	15 min	10 min	5 min
				Cool-down

Inspire Math

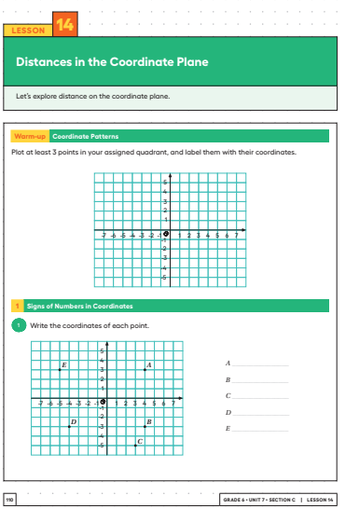
Search and Rescue video

Go Online:
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614216

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 14

Distances in the Coordinate Plane

Let's explore distance on the coordinate plane.

Warm-up Coordinate Patterns

Plot at least 3 points in your assigned quadrant, and label them with their coordinates.

1 Signs of Numbers in Coordinates

1. Write the coordinates of each point.

A _____
B _____
C _____
D _____
E _____

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Warm-up

Coordinate Patterns

10 min

Activity Narrative

The purpose of this *Warm-up* is for students to review plotting and determining the coordinates of points in the coordinate plane. Students use repeated reasoning to generalize patterns in the coordinates of points in each quadrant.

Launch

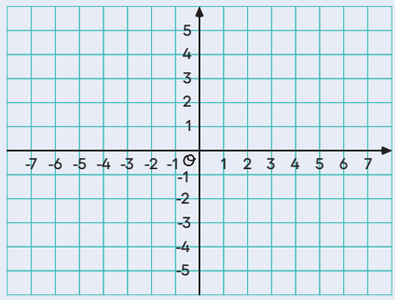
Arrange students in groups of 4.

Give students 3 minutes of quiet work time to plot and label at least three points, and up to six if they have time, in their assigned quadrant.

Follow with a whole-class discussion.

Student Task Statement

Plot at least 3 points in your assigned quadrant, and label them with their coordinates.



Sample responses:

- Quadrant I: (4, 7), (2, 3), and (6, 1)
- Quadrant II: (-5, 5), (-3, 6), and (-2, 4)
- Quadrant III: (-1, -3), (-5, -4), and (-2, -6)
- Quadrant IV: (3, -6), (7, -2), and (1, -1)

Activity Synthesis

The focus of this discussion is for students to recognize that the following patterns emerge:

- In Quadrants I and IV, the x-coordinate of a point (or the first number in an ordered pair) is positive.
- In Quadrants II and III, the x-coordinate of a point is negative.
- In Quadrants I and II, the y-coordinate of a point (or the second number in an ordered pair) is positive.
- In Quadrants III and IV, the y-coordinate of a point is negative.

Invite students to share any patterns they noticed. After each student shares, ask the rest of the class if they noticed the same pattern within their small group. Record and display these patterns for all to see. If possible, plot and label a few example points in each quadrant based on students' observations.

Activity 1
Signs of Numbers in Coordinates

10 min

Activity Narrative

In this activity, students connect opposite signs in coordinates to reflections across one or both axes. Students investigate relationships between several pairs of points in order to more generally make this connection.

The use of the word "reflection" is used informally to describe the effect of opposite signs in coordinates. In grade 8, students learn a more precise, technical definition of the word "reflection" as it pertains to rigid transformations of the plane.

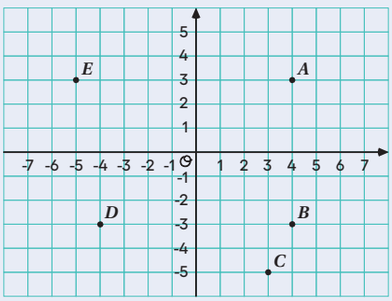
Launch

Arrange students in groups of 2. Display the first problem from the *Task Statement* for all to see. Ask students to determine the coordinates for each point, and record their responses for all to see.

Then give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. Write the coordinates of each point.



A. (4, 3)
B. (4, −3)
C. (3, −5)
D. (−4, −3)
E. (−5, 3)

2. Answer these questions for each pair of points.

- How are the coordinates the same? How are they different?
- How far away are they from the y-axis? To the left or to the right of it?
- How far away are they from the x-axis? Above or below it?

Sample responses:

a. A and B

A and B have the same x-coordinate (4) but opposite y-coordinates (3 and −3). Both points are 4 units to the right of the y-axis. Both are 3 units from the x-axis, but A is above the x-axis, and B is below it.

b. B and D

B and D have opposite x-coordinates (4 and −4) but the same y-coordinate of −3. Both points are 4 units away from the y-axis, but one is to the left of it, and the other to the right of it. Both are 3 units below the x-axis.

Access for Multilingual Learners (Activity 1, Student Task)

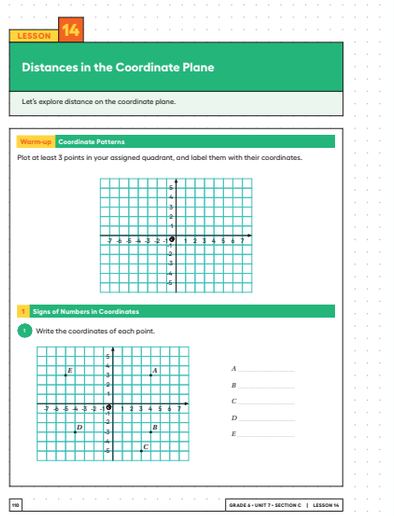
MLR2: Collect and Display
Circulate to listen for and collect the language that students use as they answer questions about pairs of points. On a visible display, record words and phrases, such as "same distance," "opposite sides," "opposite signs," and "reflection." During the *Activity Synthesis*, invite students to suggest ways to update the display:

"What are some other words or phrases we should include?"

Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student Workbook


LESSON 14
Distances in the Coordinate Plane
Let's explore distance on the coordinate plane.
Warm-up Coordinate Patterns
Plot at least 3 points in your assigned quadrant, and label them with their coordinates.
1 Signs of Numbers in Coordinates
1 Write the coordinates of each point.
A _____
B _____
C _____
D _____
E _____
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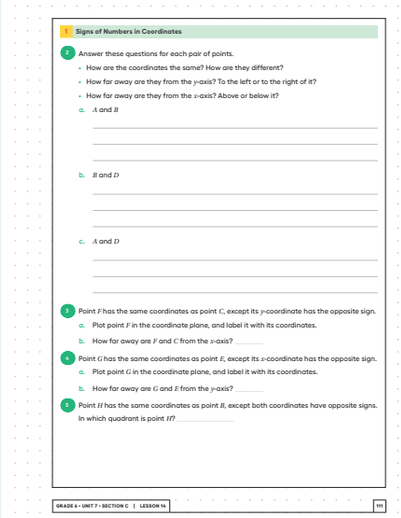
Student Workbook



1 Signs of Numbers in Coordinates

2 Answer these questions for each pair of points.

- How are the coordinates the same? How are they different?
- How far away are they from the y-axis? To the left or to the right of it?
- How far away are they from the x-axis? Above or below it?

a. A and B

b. B and D

c. A and D

3 Point F has the same coordinates as point C, except its y-coordinate has the opposite sign.

a. Plot point F in the coordinate plane, and label it with its coordinates.

b. How far away are F and C from the x-axis? _____

4 Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.

a. Plot point G in the coordinate plane, and label it with its coordinates.

b. How far away are G and E from the y-axis? _____

5 Point H has the same coordinates as point B, except both coordinates have opposite signs. In which quadrant is point H? _____

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Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to illustrate student thinking. For example, as students describe what they noticed, use color and annotations to scribe their thinking on a display of the coordinate plane that is visible for all students.

Supports accessibility for: Visual-Spatial Processing

c. A and D

A and D have opposite x-coordinates (4 and -4) and y-coordinates (3 and -3). Both are 4 units away from y-axis and 3 units away from the x-axis, but in opposite directions.

3. Point F has the same coordinates as point C, except its y-coordinate has the opposite sign.

a. Plot point F in the coordinate plane, and label it with its coordinates.

F(3, 5)

b. How far away are F and C from the x-axis?

Each point is 5 units from the x-axis.

4. Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.

a. Plot point G in the coordinate plane, and label it with its coordinates.

G(5, 3)

b. How far away are G and E from the y-axis?

Each point is 5 units from the y-axis.

5. Point H has the same coordinates as point B, except both coordinates have opposite signs. In which quadrant is point H?

Quadrant II

Activity Synthesis

The goal of this discussion is for students to see that coordinates with opposite signs correspond to reflections across the axes. Begin by asking students what patterns they noticed for pairs of points whose x-coordinates had opposite signs. Ask students to give specific examples of pairs of points and their coordinates when describing the pattern they saw. Record students' explanations for all to see. Students may use phrasing like "the point flips across the y-axis." Introduce the word "reflection," and discuss similarities between reflections across the y-axis and reflections in a mirror. Note that rigid transformations, including reflections, will be studied further in a later course, so it is not necessary for students to use this term fluently.

Repeat this discussion for pairs of points where the y-coordinates had opposite signs to see that they are reflections across the x-axis.

Close by discussing the relationship between points H and B, where both the x- and y-coordinates have opposite signs. Ask students how they might describe the relationship between H and B visually on the coordinate plane. While students may describe the relationship in terms of two reflections (once across the x-axis and again across the y-axis, or vice versa), it is not expected that students see this relationship in terms of a rotation.

Activity 2

Finding Distances in a Coordinate Plane

15 min

Activity Narrative

In this activity students develop strategies for finding the distance between two points in the coordinate plane when the coordinates might not be integers. The distances used restricted to horizontal and vertical distances—use of the general two-dimensional distance formula is not expected, nor are students expected to add or subtract negative numbers fluently. More general strategies for finding distance in the coordinate plane and rational number arithmetic is developed more completely in later courses.

Launch

Tell students to close their books or devices (or to keep them closed). Display the image from the *Task Statement* for all to see.

Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder.

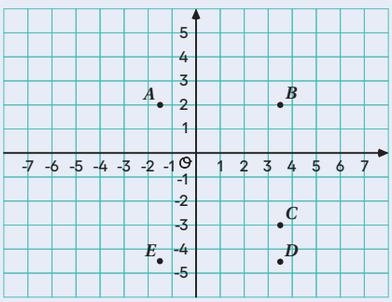
Record and display responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image.

Tell students to open their books or devices, and give them 6–7 minutes of quiet work time, and follow with a whole-class discussion.

Provide access to tracing paper.

Student Task Statement

1. Label each point with its coordinates.



A(-1.5, 2)
B(3.5, 2),
C(3.5, -3)
D(3.5, -4.5)
E(-1.5, -4.5)

2. Find the distance between each pair of points.

a. Points B and C

5 units

b. Points D and B

6.5 units

c. Points D and E

5 units

3. Which of the points are 5 units from (-1.5, -3)?

Points A and C

Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

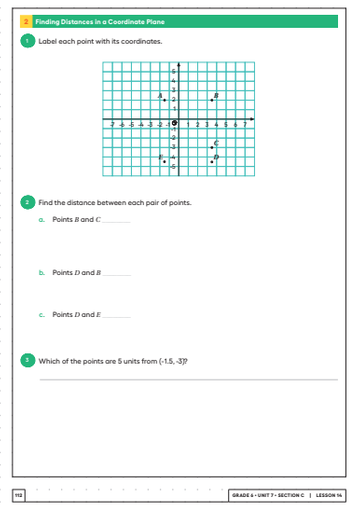
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If some students claim that a diagonal distance across a certain number of squares is equal to a vertical or horizontal distance across the same number of squares, consider asking:

"How did you determine the distances between both sets of points?"
"Does a ruler (or tracing paper) confirm that those distances are the same?"

Student Workbook


2 Finding Distances in a Coordinate Plane

1 Label each point with its coordinates.

2 Find the distance between each pair of points.

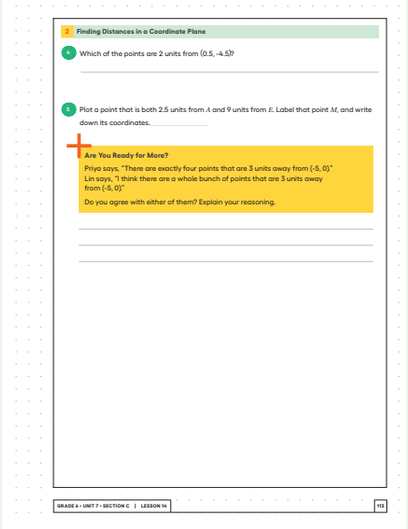
a. Points B and C

b. Points D and B

c. Points D and E

3 Which of the points are 5 units from (-1.5, -3)?

Student Workbook


2 Finding Distances in a Coordinate Plane

A Which of the points are 2 units from (0.5, -4.5)?

5 Plot a point that is both 2.5 units from A and 9 units from E. Label that point M, and write down its coordinates.

Are You Ready for More?

Priya says, "There are exactly four points that are 3 units away from (-5, 0)." Lin says, "I think there are a whole bunch of points that are 3 units away from (-5, 0)."

Do you agree with either of them? Explain your reasoning.

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4. Which of the points are 2 units from (0.5, -4.5)?

Point *E*

5. Plot a point that is both 2.5 units from A and 9 units from E. Label that point M, and write down its coordinates.

$M(-1.5, 4.5)$

Are You Ready for More?

Priya says, "There are exactly four points that are 3 units away from (-5, 0)." Lin says, "I think there are a whole bunch of points that are 3 units away from (-5, 0)."

Do you agree with either of them? Explain your reasoning.

Sample response: I agree with Lin, because I can measure a length of 3 units from the point -5,0 in any direction. I don't have to just go up, down, left, or right.

(Students will learn in grade 7 that a circle is the (infinite) set of points that are equidistant from a center point. For grade 6 we focus on vertical and horizontal distances, where naming the coordinates of points at a given distance is clear.)

Activity Synthesis

The purpose of this discussion is for students to share strategies for determining the distance between two points on the same horizontal or vertical line. Begin by inviting students to share the distances they found between pairs of points and their reasoning. Record and display their responses for all to see. Some common strategies include:

- Counting grid spaces in a horizontal or vertical direction.
- Determining the distance between each point and the x- or y-axis and adding the distances.
- Reasoning using each point's coordinates.
- Using tracing paper.

Here are some questions for discussion:

- *"How are these strategies the same or different?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Would these strategies still work if both points were located in the same quadrant?"* Yes

Lesson Synthesis

The purpose of this discussion is to summarize the effect of replacing coordinates with their opposites and how to find horizontal and vertical distances in the coordinate plane. Here are some questions for discussion:

- *"Without graphing, what can you say about the points (5, -3) and (5, 3) in the coordinate plane?"*

  The first point is in Quadrant IV, and the second point is in Quadrant I. They are both 3 units from the x-axis. They are reflections of each other across the x-axis. They are 6 units apart. They are both on the same vertical line.

"Without graphing, what can you say about (-6, 4)and (6, 4) in the coordinate plane?"

The first point is in Quadrant II and the second point is in Quadrant I. They are both 6 units from the y-axis. They are reflections across the y-axis. They are 12 units apart. They are both on the same horizontal line.

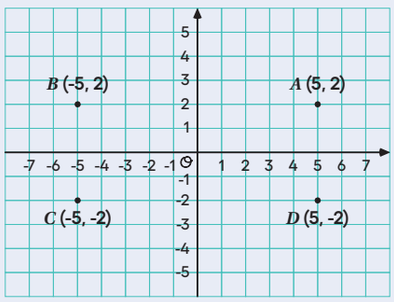
"Without graphing, what can you say about (2.5, 1) and (6, 1) in the coordinate plane?"

They are both in Quadrant I. They are both on the same horizontal line because they have the same y-value. The second point is 3.5 units to the right of the first point.

If time allows, challenge students to draw a rectangle with given side lengths and identify its vertices. This will be useful in a future lesson where students explore shapes in the coordinate plane.

Lesson Summary

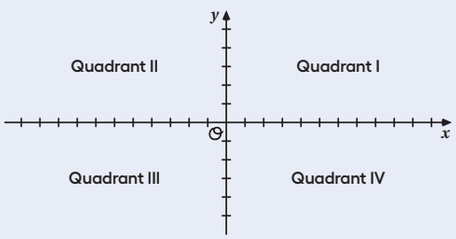
The points $A(5, 2)$, $B(-5, 2)$, $C(-5, -2)$, and $D(5, -2)$ are shown in the coordinate plane. Notice that they all have almost the same coordinates, except the signs are different. They are all the same distance from each axis but are in different quadrants.



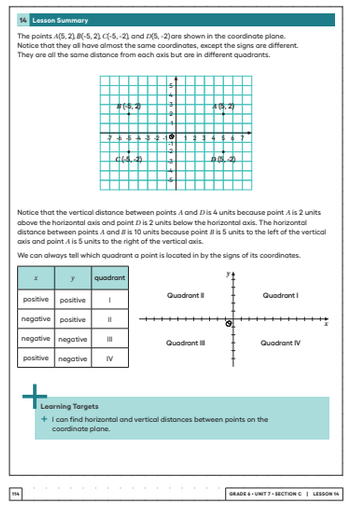
Notice that the vertical distance between points A and D is 4 units because point A is 2 units above the horizontal axis and point D is 2 units below the horizontal axis. The horizontal distance between points A and B is 10 units because point B is 5 units to the left of the vertical axis and point A is 5 units to the right of the vertical axis.

We can always tell which quadrant a point is located in by the signs of its coordinates.

x	y	quadrant
positive	positive	I
negative	positive	II
negative	negative	III
positive	negative	IV



Student Workbook


14 Lesson Summary

The points $A(5, 2)$, $B(-5, 2)$, $C(-5, -2)$, and $D(5, -2)$ are shown in the coordinate plane. Notice that they all have almost the same coordinates, except the signs are different. They are all the same distance from each axis but are in different quadrants.

Notice that the vertical distance between points A and D is 4 units because point A is 2 units above the horizontal axis and point D is 2 units below the horizontal axis. The horizontal distance between points A and B is 10 units because point B is 5 units to the left of the vertical axis and point A is 5 units to the right of the vertical axis.

We can always tell which quadrant a point is located in by the signs of its coordinates.

x	y	quadrant
positive	positive	I
negative	positive	II
negative	negative	III
positive	negative	IV

Quadrant II Quadrant I Quadrant III Quadrant IV

Learning Targets

+ I can find horizontal and vertical distances between points on the coordinate plane.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

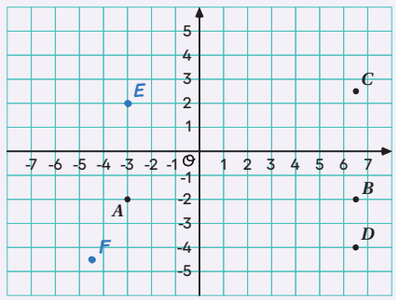
Cool-down

Points and Distances

5 min

Student Task Statement

Here are four points in a coordinate plane.



1. What is the distance between points A and B?

   About 9.5 units

2. What is the distance between points C and D?

   About 6.5 units

3. Plot the point (-3, 2). Label it E.

   See image.

4. Plot the point (-4.5, -4.5). Label it F.

   See image.

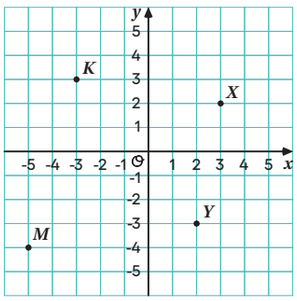
Lesson 14

Practice Problems

5 Problems

Problem 1

Here are 4 points on a coordinate plane.



a. Label each point with its coordinates.

- $X(3, 2)$
- $Y(2, -3)$
- $K(-3, 3)$
- $M(-5, -4)$

b. Plot a point that is 3 units from point K. Label it P.

Sample responses: $(0, 3)$ or $(-3, 6)$

c. Plot a point that is 2 units from point M. Label it W.

Sample responses: $(-3, -4)$ or $(-5, -2)$

Problem 2

Each set of points are connected to form a line segment. What is the length of each?

a. A = (3, 5) and B = (3, 6)

1 unit

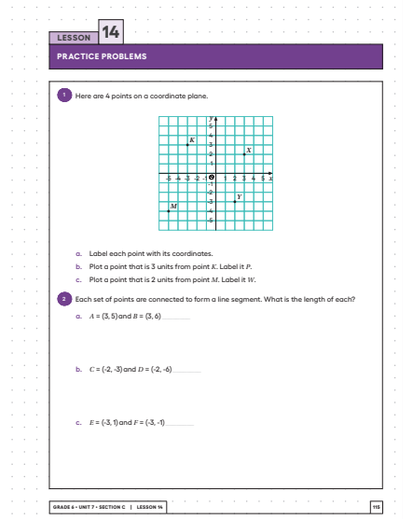
b. C = (-2, -3) and D = (-2, -6)

3 units

c. E = (-3, 1) and F = (-3, -1)

2 units

Student Workbook


LESSON 14

PRACTICE PROBLEMS

1. Here are 4 points on a coordinate plane.

a. Label each point with its coordinates.
b. Plot a point that is 3 units from point K. Label it P.
c. Plot a point that is 2 units from point M. Label it W.

2. Each set of points are connected to form a line segment. What is the length of each?

a. A = (3, 5) and B = (3, 6) _____

b. C = (-2, -3) and D = (-2, -6) _____

c. E = (-3, 1) and F = (-3, -1) _____

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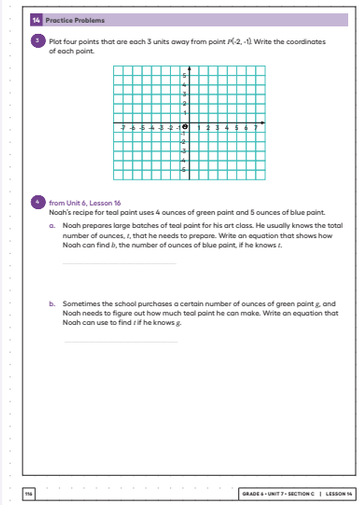
Student Workbook


14 Practice Problems

3 Plot four points that are each 3 units away from point $P(-2, -1)$. Write the coordinates of each point.

4 from Unit 6, Lesson 16
Noah's recipe for teal paint uses 4 ounces of green paint and 5 ounces of blue paint.

a. Noah prepares large batches of teal paint for his art class. He usually knows the total number of ounces, t, that he needs to prepare. Write an equation that shows how Noah can find b, the number of ounces of blue paint, if he knows t.

b. Sometimes the school purchases a certain number of ounces of green paint g, and Noah needs to figure out how much teal paint he can make. Write an equation that Noah can use to find t if he knows g.

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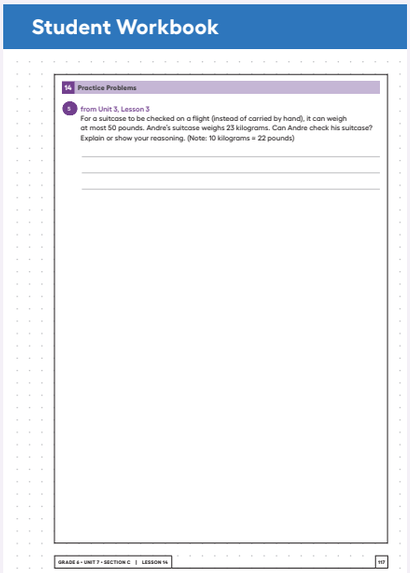
Student Workbook


14 Practice Problems

5 from Unit 3, Lesson 3
For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 pounds. Andre's suitcase weighs 23 kilograms. Can Andre check his suitcase? Explain or show your reasoning. (Note: 10 kilograms ≈ 22 pounds)

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Problem 3

Plot four points that are each 3 units away from point $P(-2, -1)$. Write the coordinates of each point.

Sample response

- $A(-5, -1)$
- $B(1, -1)$
- $C(-2, 2)$
- $D(-2, -4)$

(Students are unlikely to come up with other possible solutions at this stage.)

Problem 4

from Unit 6, Lesson 16

Noah's recipe for teal paint uses 4 ounces of green paint and 5 ounces of blue paint.

a. Noah prepares large batches of teal paint for his art class. He usually knows the total number of ounces, t, that he needs to prepare. Write an equation that shows how Noah can find b, the number of ounces of blue paint, if he knows t.

$b = \frac{5}{4}t$

b. Sometimes the school purchases a certain number of ounces of green paint g, and Noah needs to figure out how much teal paint he can make. Write an equation that Noah can use to find t if he knows g.

$t = \frac{4}{5}g$

Problem 5

from Unit 3, Lesson 3

For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 pounds. Andre's suitcase weighs 23 kilograms. Can Andre check his suitcase? Explain or show your reasoning.
(Note: 10 kilograms ≈ 22 pounds)

No, Andre will not be able to check his suitcase if they are strict about following the rule.

Sample reasoning: 1 kilogram weighs 2.2 pounds, so 23 kilograms weighs $(2.2) \cdot 23 = 50.6$ pounds.