Rectangles with Fractional Side Lengths

Goals

- Calculate one side length of a rectangle given its area and the other side length.
- Coordinate (orally)
 diagrams and equations
 that represent the area of
 a rectangle with fractional
 side lengths.
- Draw and label a diagram to justify the area of a rectangle with fractional side lengths.

Learning Target

I can use division and multiplication to solve problems involving areas of rectangles with fractional side lengths.

Access for Students with Diverse Abilities

• Representation (Activity 1)

Access for Multilingual Learners

• MLR6: Three Reads (Activity 2)

Instructional Routines

- 5 Practices
- · Notice and Wonder

Required Materials

Materials to Gather

- Straightedges: Activity 2, Activity 4
- Four-function calculators: Activity 4

Activity 4:

Consider preparing $\frac{3}{4}$ -inch square stickers, $\frac{5}{8}$ -binder clips, and $1\frac{3}{4}$ -inch paper clips available for students to verify their answers.

Lesson Narrative

In this lesson, students solve problems about the side lengths and area of rectangles.

The lesson begins with two optional activities that activate geometric ideas from grade 5. Students revisit different ways to find the area of a rectangle with fractional side lengths. They verify that the area can be found multiplying the side lengths regardless of whether they are or are not whole numbers.

Next, students solve a contextual problem about finding the length of a rectangular tray whose area and width are known. They are also asked to find the number of rectangular tiles—which have fractional side lengths—that can cover the tray completely. Solving these problems prompts students to apply their understanding of division of fractions.

Student Learning Goal

Let's explore rectangles that have fractional measurements.

Lesson Timeline

5_{min}

Warm-up

15 min

Activity 1

10 min

Activity 2

20 min

Activity 3

10 min

Lesson Synthesis



5_{min}

Cool-down

Warm-up

Notice and Wonder: Areas of Squares



Activity Narrative

The purpose of this *Warm-up* is to elicit observations about the areas of squares with whole-number and fractional side lengths, which will be useful when students reason about rectangles with fractional side lengths in a later activity. While students may notice and wonder many things about these images, ideas about finding areas by tiling are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch



Arrange students in groups of 2. Display the image of squares for all to see.

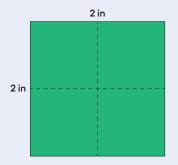
Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement

What do you notice? What do you wonder?







Students may notice:

- There are 3 squares.
- The side lengths of the squares are $\frac{1}{2}$ inch, I inch, and 2 inches.
- · Each side length is twice the side length of the square before it.
- Two of the squares are partitioned into 4 smaller squares.
- Four $\frac{1}{2}$ -inch squares can fit in the I-inch square. Four I-inch squares can fit in the 2-inch square.

Students may wonder:

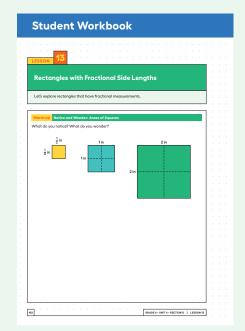
- · Are the squares part of a pattern?
- · Will the side length of the next square be 4 inches?
- · What is the area of each square?

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.





Lesson 13 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

Instructional Routines

5 Practices

ilclass.com/r/10690701

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Activity Synthesis

Consider telling students that we can call a square with 1-inch side length "a 1-inch square."

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If reasoning about the area of a region by covering or tiling it with squares of known area does not come up during the conversation, ask students to discuss this idea.

Highlight the following points:

- A square with a side length of 1 inch (a 1-inch square) has an area of 1 in².
- A 2-inch square has an area of 4 in², because 4 squares with 1-inch side length are needed to cover it.
- A ¹/₂-inch square has an area of ¹/₄ in² because 4 of them are needed to completely cover a 1-inch square.

Activity 1: Optional

Areas of Squares and Rectangles



Activity Narrative

The purpose of this activity is to review and illustrate the idea that the area of a rectangle with fractional side lengths can be found by multiplying the two side lengths, just as in the case of whole numbers.

Students are to justify that the area of a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches is $7\frac{7}{8}$ square inches. As they make their case and listen to others' cases, students practice constructing a logical argument and critiquing the reasoning of others.

Monitor for students who use different approaches to reason about the area of the rectangle in the last question. Here are some likely approaches, from less direct to more direct:

- Find the number of $\frac{1}{4}$ -inch squares in the rectangle, and multiply it by $\frac{1}{16}$, which is the area of one $\frac{1}{4}$ -inch square.
- Tile the rectangle with 1-inch squares and parts of 1-inch squares, find the area of each part, and add the areas.
- Decompose each side length into a whole number and a fraction (3 and $\frac{1}{2}$ on one side, 2 and $\frac{1}{4}$ on the other), partition the rectangle accordingly, find the area of each part, and add the areas.
- Multiply the side lengths of the rectangle.

Launch



Tell students that they will now think about the areas of other squares and rectangles with fractional side lengths.

Keep students in groups of 2.

Give students 5–6 minutes of quiet work time and 2–3 minutes to share their responses and reasoning with their partner.

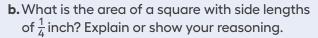
Provide access to straightedges.

Select students who used each strategy described in the *Activity Narrative* and ask them to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

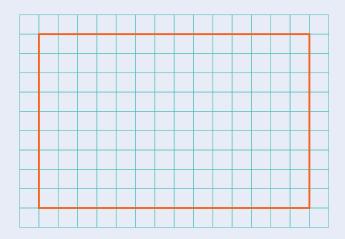
- 1. Here is a square with side lengths of 1 inch.
 - **a.** How many squares with side lengths of $\frac{1}{4}$ inch can fit in a square with side lengths of 1 inch?







2. Here is a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches. The side length of each grid square is $\frac{1}{4}$ inch.



Show that a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches has an area of $7\frac{7}{8}$ square inches. You can use the drawing if you find it helpful.

Access for Students with Diverse Abilities (Activity 1)

Representation: Internalize Comprehension.

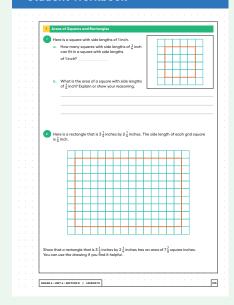
Begin by asking,

"Does this problem remind anyone of something we have done before?"

Cool-down

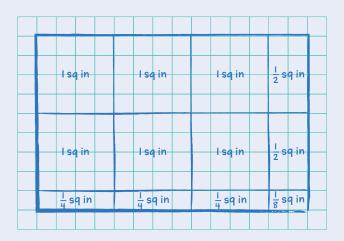
Supports accessibility for: Memory, Attention

Student Workbook

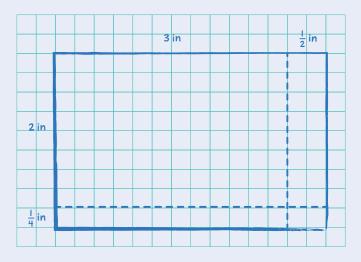


Sample responses:

• The rectangle has $14 \cdot 9$, or 126, $\frac{1}{4}$ -inch squares., which is $\frac{63}{8}$ or $7\frac{7}{8}$ square inches.



•
$$(6 \cdot 1) + (2 \cdot \frac{1}{2}) + (3 \cdot \frac{1}{4}) + (1 \cdot \frac{1}{6}) = 6 + 1 + \frac{3}{4} + \frac{1}{6} = 7\frac{7}{6}$$



•
$$(3 \cdot 2) + (3 \cdot \frac{1}{4}) + (\frac{1}{2} \cdot 2) + (\frac{1}{2} \cdot \frac{1}{4}) = 6 + \frac{3}{4} + 1 + \frac{1}{8} = 7\frac{7}{8}$$

Activity Synthesis

The goal of the discussion is to make explicit that even when the side lengths of a rectangle are not whole numbers, its area is still the product of the two numbers.

Invite previously selected students to share their responses to the last question. Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see

If no students reasoned about the area by decomposing the rectangle and finding the sum of partial areas, display one or both diagrams that are shown in the *Student Response*. Ask students to explain how each diagram allows us to find the area. If no students calculated the product of $3\frac{1}{2}$ and $2\frac{1}{4}$, ask students to find that value and compare it to the area found using their way.

Connect the different responses to the learning goals by asking questions such as:

"Which strategies are more elaborate or involve more steps? Which ones are quicker or more straightforward?"

"What are some benefits of reasoning about the areas of smaller squares or rectangles in the $3\frac{1}{2}$ -by- $2\frac{1}{4}$ rectangle? What are some disadvantages?"

"What are some benefits of multiplying the side lengths? What are some disadvantages?"

Activity 2

Areas of Rectangles: Optional

10 min

Activity Narrative

This activity gives students another opportunity to reason about multiplication and the area of a rectangle with fractional side lengths. Specifically, it helps students see how the product of two mixed numbers (or two fractions that are greater than 1) can be found by decomposing the factors into whole numbers and fractions and finding partial products.

Students are first presented with multiplication expressions and diagrams of rectangles with a shaded region. They match each diagram with an expression that can represent the area of the rectangle. Then students use their understanding of the connections across representations to show that the value of one expression they matched, $2\frac{1}{2}\cdot 4\frac{3}{4}$, is $11\frac{7}{8}$.

Because the diagrams are unlabeled, students need to look for and make use of the structure in the expressions and diagrams to make a match. To justify that a multiplication has a certain value, they need to construct a logical argument.

Launch

Give students 3-4 minutes of guiet work time.

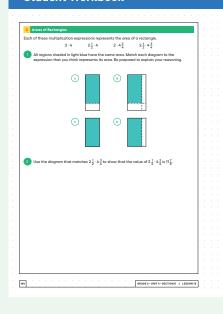
Emphasize the direction that states, "All regions shaded in light blue have the same area" before students begin working.

Building on Student Thinking

In justifying $2\frac{1}{2} \cdot 4\frac{3}{4} = 11\frac{7}{8}$, some students may simply multiply the whole numbers in the side lengths (the 2 and 4), multiply the fractions (the $\frac{1}{2}$ and $\frac{3}{4}$), and then add them. When they recognize that their sum of areas is less than $11\frac{7}{8}$, refer them to the diagram. Ask them to identify the rectangles whose areas they have calculated and those they have not accounted for, and to think about how they could find the area of the entire rectangle.

When adding partial products with fractions in different denominators, some students may simply add the numerators and denominators. Remind them to attend to the size of the fractional parts when adding or subtracting fractions.

Student Workbook



Student Task Statement

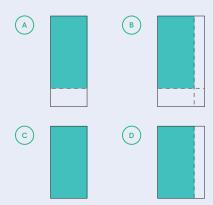
Each of these multiplication expressions represents the area of a rectangle.

$$2\frac{1}{2} \cdot 4$$

$$2\cdot 4\frac{3}{4}$$

$$2\frac{1}{2} \cdot 4$$
 $2 \cdot 4\frac{3}{4}$ $2\frac{1}{2} \cdot 4\frac{3}{4}$

1. All regions shaded in light blue have the same area. Match each diagram to the expression that you think represents its area. Be prepared to explain your reasoning.



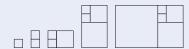
- 2 · 4 represents the area of Diagram C.
- $2\frac{1}{2}$ · 4 represents the area of Diagram D.
- $2 \cdot 4\frac{3}{4}$ represents the area of Diagram A.
- $2\frac{1}{2} \cdot 4\frac{3}{4}$ represents the area of Diagram B.
- **2.** Use the di agram that matches $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$

Sample response: (2 · 4) + $(\frac{1}{2} \cdot 4)$ + $(\frac{3}{4} \cdot 2)$ + $(\frac{3}{4} \cdot \frac{1}{2})$ = 8 + 2 + $\frac{3}{2}$ + $\frac{3}{8}$ = $11\frac{7}{8}$

Are You Ready for More?

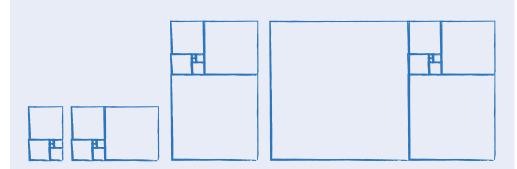
The following rectangles are composed of squares, and each rectangle is constructed using the rectangle before it. The side length of the first square is 1 unit.

Activity 1



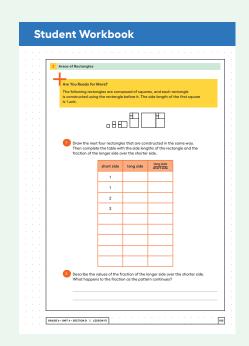
1. Draw the next four rectangles that are constructed in the same way. Then complete the table with the side lengths of the rectangle and the fraction of the longer side over the shorter side.

short side	long side	long side short side
1	1	2
1	2	2
2	3	l <u>1</u>
3	5	l ² / ₃
5	8	1 3 5
8	13	1 <u>5</u>
13	21	1 8
21	34	l <u>13</u>
34	55	1 21 34



2. Describe the values of the fraction of the longer side over the shorter side. What happens to the fraction as the pattern continues?

Sample response: The fractions go up and down around a value that is near $1\frac{2}{3}$.



Lesson 13 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

Access for Multilingual Learners (Activity 3, Launch)

MLR6: Three Reads.

Keep student workbooks or devices closed. Display only the first sentence, without revealing the questions. Tell students,

"We are going to read this statement 3 times."

After the 1st read: Say to students,

"Tell your partner what this situation is about."

After the 2nd read: Say to students

"List the quantities. What can be counted or measured?"
For the 3rd read: Reveal and read the first question. Ask,

"What are some ways we might get started on this?" Advances: Reading, Representing

Activity Synthesis

Display the expressions and diagrams for all to see. Invite one or more students to match each expression to a rectangle whose area the expression can represent. Ask students to explain their reasoning. If possible, record students' reasoning on or near the representation referred to.

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Does anyone want to add on to ____'s reasoning?"

"Do you agree or disagree? Why?"

Next, invite other students to share how they used Diagram B to show that $2\frac{1}{2}\cdot 4\frac{3}{4}=11\frac{7}{8}$, starting with how they knew the side lengths of each subrectangle. As students explain, label each sub-rectangle with its side lengths and its area.

If not articulated by students, highlight that combining all the partial areas gives us a sum of $11\frac{7}{8}$, which is the area of the entire rectangle.

Activity 3

How Many Would it Take?



Activity Narrative

In this activity, students determine how many tiles with fractional side lengths are needed to completely cover another rectangular region that also has fractional side lengths. To do so, students need to apply their understanding of the area of rectangles and of division of fractions. They also need to plan their approach, think about how the orientation of the tiles affects their calculation and solution, and attend carefully to the different measurements and steps in their calculation. The task engages students in aspects of mathematical modeling and prompts them to attend to precision.

As students work, monitor for those whose diagrams or solutions show different tile orientations.

Launch



Keep students in groups of 2.

Give students 7–8 minutes of quiet work time and 2–3 minutes to share their responses with their partner, or give 10 minutes for them to complete the activity in groups.

Provide access to straightedges and four-function calculators (for computing products or quotients of large whole numbers).

Student Task Statement

Noah would like to cover a rectangular tray with rectangular tiles. The tray has a width of $11\frac{1}{4}$ inches and an area of $50\frac{5}{8}$ square inches.

Warm-up

1. Find the length of the tray in inches. Show your reasoning. $4\frac{1}{2}$ inches

Sample reasoning: $50\frac{5}{8} \div 11\frac{1}{4} = \frac{405}{8} \div \frac{45}{4} = \frac{405}{8} \cdot \frac{4}{45} = \frac{405}{40} = 4\frac{45}{40}$

2. The tiles are $\frac{3}{4}$ inch by $\frac{9}{16}$ inch. Draw a diagram to show one way Noah could lay the tiles. Your diagram does not need to show every tile but should show known measurements.

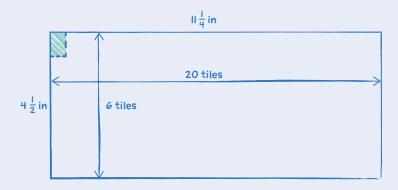
Diagram should show a rectangle with $II\frac{1}{4}$ inch and $4\frac{1}{2}$ inch side lengths and one or more tiles oriented vertically or horizontally in the rectangle.

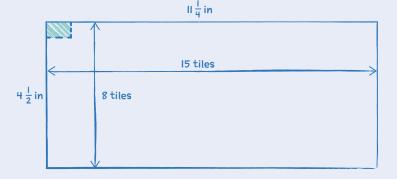
3. Draw a diagram to show how Noah could lay the tiles. Your diagram should show how many tiles would be needed to cover the length and width of the tray, but does not need to show every tile.

120 tiles

Sample reasoning:

- If the $\frac{3}{4}$ -inch side of the tiles are placed along $4\frac{1}{2}$ -inch side of the tray, 6 tiles are needed for that shorter side $(4\frac{1}{2} \div \frac{3}{4} = 6)$ and 20 tiles needed for the longer side $(11\frac{1}{4} \div \frac{9}{16} = 20)$. So, $6 \cdot 20 = 120$
- If the $\frac{9}{16}$ -inch side of the tiles are placed along $4\frac{1}{2}$ -inch side of the tray, 8 tiles are needed for that shorter side $(4\frac{1}{2} \div \frac{9}{16} = 8)$ and 15 tiles are needed for the longer side $(11\frac{1}{4} \div \frac{3}{4} = 15)$. So, $8 \cdot 15 = 120$



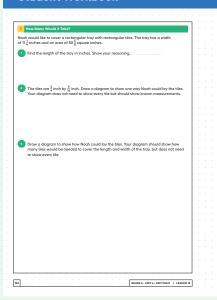


Building on Student Thinking

Students might find only the number of tiles needed to line the four sides of the tray. Suggest that they refer to their drawing of the tray and check whether their calculations include tiles that cover the entire tray.

Students might try to reason about a division such as $11\frac{1}{4} \div \frac{9}{16}$ by drawing a diagram or another likely timeconsuming way (given the numbers involved). Remind them, as needed, that they have one or more efficient strategies at their disposal.

Student Workbook



Lesson 13 Warm-up Activity 1 Activity 2 **Activity 3 Lesson Synthesis** Cool-down

Activity Synthesis

Invite students who chose different tile orientations to show their diagrams and explain their reasoning. Display the two diagrams shown in the *Student Response*, if needed.

If time permits, point out how in this problem, the two different tile orientations do not matter because the length and the width of the tiles are factors of both the length and the width of the tray. This means that we can fit a whole number of tiles in either direction, and we can use the same number of tiles to cover the tray regardless of orientation.

But if the side lengths of the tiles do not both fit into $11\frac{1}{4}$ and $4\frac{1}{2}$ evenly, then the orientation of the tiles does matter. In that case, we may need more or fewer tiles, or we may not be able to tile the entire tray without gaps if the tiles are oriented a certain way.

Use the opportunity to point out that a diagram does not have to show all the details (such as every single tile) to be useful.

Lesson Synthesis

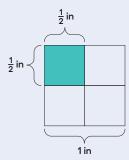
Invite students to summarize different ways to reason about the area of a rectangle with fractional side lengths, such as a rectangle that is $5\frac{1}{2}$ cm by $3\frac{1}{2}$ cm. If time permits, consider asking students to draw a diagram and annotate it to show each reasoning strategy.

Emphasize that because we can multiply the side lengths of a rectangle to find its area, if we know the area of a rectangle and one side length, then we can find the length of the other side by dividing the former by the latter. Afterward, we can check the quotient by multiplying it by the given side length.

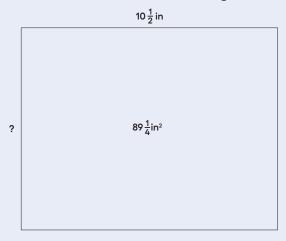
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Lesson Summary

If a rectangle has side lengths a units and b units, the area is $a \cdot b$ square units. For example, if we have a rectangle with $\frac{1}{2}$ -inch side lengths, its area is $\frac{1}{2} \cdot \frac{1}{2}$ (or $\frac{1}{4}$) square inches.



This means that if we know the *area* and *one side length* of a rectangle, we can divide to find the *other* side length.



If one side length of a rectangle is $10\frac{1}{2}$ in and its area is $89\frac{1}{4}$ in², we can write this equation to show their relationship:

$$10\frac{1}{2}$$
 ? = $89\frac{1}{4}$

Then, we can find the other side length, in inches, using division:

$$89\frac{1}{4} \div 10\frac{1}{2} = ?$$

Cool-down

Two Frames

Student Task Statement

Two rectangular picture frames have the same area of 45 square inches but have different side lengths. Frame A has a length of $6\frac{3}{4}$ inches, and Frame B has a length of $7\frac{1}{2}$ inches.

1. Without calculating, predict which frame has the shorter width. Explain your reasoning.

Frame B has a longer length, so its width is shorter if the two pairs of side lengths produce the same product of 45.

2. Find the width that you predicted to be shorter. Show your reasoning.

6 inches

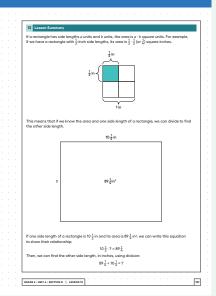
Sample reasoning: $45 \div 7\frac{1}{2} = 45 \div \frac{15}{2} = 45 \cdot \frac{2}{15} = 6$

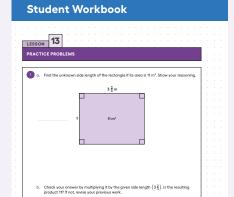
Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

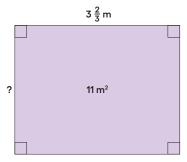
Student Workbook





Problem 1

a. Find the unknown side length of the rectangle if its area is 11 m². Show your reasoning.



3 m, because II ÷
$$\left(3\frac{2}{3}\right)$$
 = 3

b. Check your answer by multiplying it by the given side length $(3\frac{2}{3})$. Is the resulting product 11? If not, revise your previous work.

$$3\frac{2}{3} \cdot 3 = 11$$

Problem 2

A worker is tiling the floor of a rectangular room that is 12 feet by 15 feet. The tiles are square with side lengths $1\frac{1}{3}$ feet. How many tiles are needed to cover the entire floor? Show your reasoning.

$$101\frac{1}{4}$$
 or 102 tiles

Sample reasoning: I2 ÷ $\frac{4}{3}$ = 9, so 9 tiles are needed to cover the I2 feet of length. I5 ÷ $\frac{4}{3}$ = $\frac{45}{4}$, so II $\frac{1}{4}$ tiles are needed to cover the I5 feet of length. To find the number of tiles, we multiply: 9 · $\frac{45}{4}$ = $\frac{405}{4}$ or IOI $\frac{1}{4}$ tiles, which can be rounded to IO2

Problem 3

A television screen has length $16\frac{1}{2}$ inches, width w inches, and area 462 square inches. Select **all** the equations that represent the relationship of the side lengths and area of the television.

A.
$$w \cdot 462 = 16\frac{1}{2}$$

B.
$$16\frac{1}{2} \cdot w = 462$$

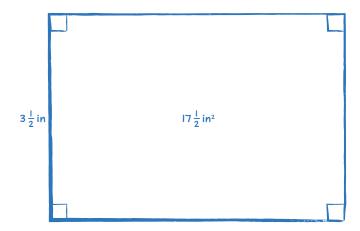
C. 462 ÷ 16
$$\frac{1}{2}$$
 = w

D. 462 ÷
$$w = 16\frac{1}{2}$$

E.
$$16\frac{1}{2} \cdot 462 = w$$

Problem 4

The area of a rectangle is $17\frac{1}{2}$ in² and its shorter side is $3\frac{1}{2}$ in. Draw a diagram that shows this information. What is the length of the longer side?



5 in. (The sides perpendicular to the $3\frac{1}{2}$ -inch side each have length in inches of $\left(17\frac{1}{2}\right) \div \left(3\frac{1}{2}\right) = \frac{35}{2} \cdot \frac{2}{7} = 5$.)

Problem 5

from Unit 4, Lesson 12

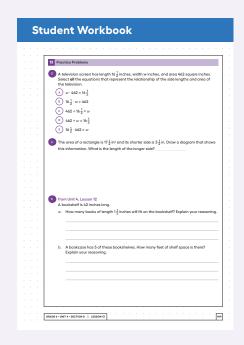
A bookshelf is 42 inches long.

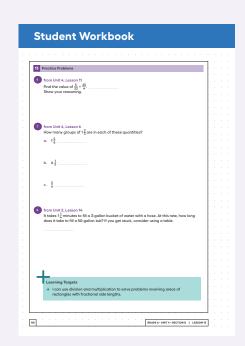
a. How many books of length $1\frac{1}{2}$ inches will fit on the bookshelf? Explain your reasoning.

28 books.
$$42 \div 1\frac{1}{2} = 42 \div \frac{3}{2} = \frac{84}{3} = 28$$

b. A bookcase has 5 of these bookshelves. How many feet of shelf space is there? Explain your reasoning.

$$17\frac{1}{2}$$
 feet. $5 \cdot 42 = 210$ and 210 inches is $17\frac{1}{2}$ feet, since $210 \div 12 = 17\frac{1}{2}$.





Problem 6

from Unit 4, Lesson 11

Find the value of $\frac{5}{32} \div \frac{25}{4}$. Show your reasoning.

$$\frac{1}{40}$$
 $\frac{5}{32} \div \frac{25}{4} = \frac{5}{32} \cdot \frac{4}{25}$, which is equal to $\frac{1}{40}$

Problem 7

from Unit 4, Lesson 6

How many groups of $1\frac{2}{3}$ are in each of these quantities?

- **a.** $1\frac{5}{6}$
- **b.** $4\frac{1}{3}$
- **c.** $\frac{5}{6}$

Problem 8

from Unit 2, Lesson 14

It takes $1\frac{1}{4}$ minutes to fill a 3-gallon bucket of water with a hose. At this rate, how long does it take to fill a 50-gallon tub? If you get stuck, consider using a table.

 $\frac{125}{6}$ minutes (or equivalent)

Sample response:

gallons of water	time in minutes
3	5 4
300	I25 (or equivalent)
50	$\frac{125}{6}$ (or equivalent)