Size of Divisor and Size of Quotient

Goals

- Comprehend the terms "dividend" and "divisor" (in spoken language) to refer to the numbers in a division problem.
- Explain (orally) how to estimate quotients, by comparing the size of the dividend and divisor.
- Generalize about the size of a quotient, i.e., predicting whether it is a very large number, a very small number, or close to 1.

Learning Target

When dividing, I know how the size of a divisor affects the quotient.

Lesson Narrative

This lesson invites students to investigate the relationships between the numbers in a division situation. Students learn that reasoning about the relative sizes of the divisor and the dividend can tell us about the size of the quotient.

Students begin exploring these relationships in concrete situations. They estimate how many thinner and thicker objects are needed to make a stack of a given height, and they then describe these relationships in terms of division. For example, height of stack ÷ thickness of object = number of objects in the stack. As they interpret division situations and equations that represent them, students practice reasoning abstractly and quantitatively.

Then students think about the relationships more generally. They reason about the values of division expressions with the same dividend but different-size divisors (such as $1 \div 80$, $1 \div 800$, and $1 \div 80,000$), as well as those with different dividends but the same divisor (such as 1,000 \div 25, 75 \div 25, and 24 \div 25). Along the way, students have opportunities to look for structure.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR2: Collect and Display (Activity 2)

Instructional Routines

- Math Talk
- MLR2: Collect and Display

Required Materials

Materials to Gather

 Pre-printed slips, cut from copies of the blackline master: Activity 2

Materials to Copy

 All in Order Cards (1 copy for every 3 students): Activity 2

Activity 2:

Consider copying Set 1 and Set 2 of the blackline master on paper of different colors.

Lesson Timeline



Warm-up

15 min

Activity 1

20 min

Activity 2

10 min

Lesson Synthesis

Assessment



Cool-down

Size of Divisor and Size of Quotient

Lesson Narrative (continued)

Students observe that dividing by a number that is much smaller than the dividend results in a quotient that is larger than 1, that dividing by a number that is much larger than the dividend gives a quotient that is close to 0, and that dividing by a number that is close to the dividend results in a quotient that is close to 1.

Student Learning Goal

Let's explore quotients of different sizes.

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Building on Student Thinking

If students say that they (mentally) cross out the zeros to divide, consider asking the class during discussion to explain what they believe is happening mathematically when zeros are crossed out. Clarify any confusion accordingly.

Students may say that 5,000 ÷ 10,000 is 2 because they automatically assign the larger number to be the dividend. Urge them to check their reasoning by referring to the preceding expression or to related division expressions with smaller numbers:

"Can 5,000 ÷ 2,500 and 5,000 ÷ 10,000 both have the same value of 2?" "What is 10 ÷ 5? What is 5 ÷ 10?"

Warm-up

Math Talk: Size of Dividend and Divisor



Activity Narrative

This *Math Talk* focuses on division of whole numbers. It encourages students to think about how the size of the divisor affects the quotient. It also prompts them to rely on what they know about properties of operations and the relationship between multiplication and division, to mentally solve problems. The reasoning elicited here will be helpful later when students further explore meanings of division and the relationship between the dividend, divisor, and quotient.

To divide large numbers mentally, students need to look for and make use of structure. In explaining their reasoning strategies, students need to be precise in their word choice and use of language.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation, before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

 $A.5,000 \div 5$

1,000

Sample reasoning: 5 thousands divided by 5 is I thousand.

 $B.5,000 \div 2,500$

2

Sample reasoning: There are 2 groups of 2,500 in 5,000.

C. 5,000 ÷ 10,000

 $\frac{1}{2}$ (or 0.5)

Sample reasoning: 5,000 is half of 10,000, and 5,000 divided into 10,000 groups means 0.5 in each group.

D.5,000 ÷ 500,000

 $\frac{1}{100}$ (or 0.01)

Sample reasoning: 5,000 \div 1,000 = 5, and 500,000 \div 1,000 = 500, so 5 \div

 $500 = \frac{5}{500}$, which is $\frac{1}{100}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

After evaluating all four expressions, ask students:

"What do you notice about the value of each expression as the number we use to divide gets larger?"

It gets smaller.

"Why do you think that is?"

Highlight explanations that support two ways of thinking about division, though at this point it is not important to discuss both if one of them is not mentioned.

- **1.** Dividing means breaking a number into a certain number of equal parts, and when there are more parts, the size of each part gets smaller.
- **2.** Dividing means breaking a number into parts of a particular size, and when the size of each part gets larger, the number of parts gets smaller.

Activity 1

All Stacked Up

15 min

Activity Narrative

This activity aims to give students a concrete context for thinking about division. Students estimate how many of each given object are needed to make a stack of a particular height. To do so, they use what they know about the sizes of familiar objects (boxes, egg cartons, notebooks, and coins) and their intuition that it takes more of a thinner object and fewer of a thicker object to reach the same height. Later, they will use this idea to think about division more generally.

We often refer to certain objects (such as coins and books) as having a thickness rather than a height. Clarify that in these examples "thickness" and "height" refer to the same dimension.

As students discuss in groups, monitor for those who can:

- Explain clearly the relationship between the height of the object being stacked and the height of a stack.
- Explain clearly why the situation can be represented with a division expression.

Students engage in abstract and quantitative reasoning as they represent a situation mathematically and interpret mathematical expressions in context.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _______ because ..." or "I noticed ______ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.

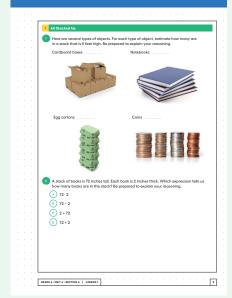
Use cardboard boxes, egg cartons, notebooks, or coins to demonstrate the stacking of objects. If possible, allow students to explore, but not measure the objects.

Supports accessibility for: Conceptual Processing, Language, Memory

Building on Student Thinking

If students are unsure how to make estimates for the first set of questions, ask them which object we would need the most of and the least of to reach 1 foot. Next, prompt them to reason about how many of each object it would take to reach 1 foot before doing the same for a height of 5 feet.

Student Workbook



Launch

Arrange students in groups of 3-4.

Give students 4–5 minutes of quiet think time and 2 minutes to discuss their solutions with their group. Follow with a whole-class discussion.

If needed, remind students to consider units of measurement when reasoning about the first set of questions. The target height of a stack is given in feet, but height or thickness of each object is more commonly measured in smaller units of length. Provide unit conversion rates as requested.

Student Task Statement

1. Here are several types of objects. For each type of object, estimate how many are in a stack that is 5 feet high. Be prepared to explain your reasoning.

Cardboard boxes

Sample response: About 6 boxes. Assuming that each box is about 10 inches tall, it would take 6 of them to reach 60 inches or 5 feet.

Egg cartons

Sample response: About 20 egg cartons. An egg carton is about 3 inches thick, so it would take about 4 to reach I foot, or 5.4 to reach 5 feet.

Notebooks

Sample response: About 60 books, assuming the notebooks are about I inch thick. $60 \cdot I = 60$.

Coins

Sample response: About 960 pennies. Assuming that each penny is about $\frac{1}{16}$ inch thick, it would take 16 pennies to make 1 inch, and $60 \cdot 16$, or 960, pennies to make 60 inches, or 5 feet.

- **2.** A stack of books is 72 inches tall. Each book is 2 inches thick. Which expression tells us how many books are in the stack? Be prepared to explain your reasoning.
 - 72 · 2
 - 72 2
 - 2 ÷ 72

• 72 ÷ 2

Sample reasoning: To find out how many books are in the stack is to find out how many 2 inches are in 72 inches, which can be done by dividing 72 by 2.

Activity Synthesis

The purpose of the discussion is to highlight the relationship between the quantities in a situation that involves division.

Ask a few students to share their estimates and explanations for the first set of questions. After all four objects are discussed, ask students:

"How did your estimates for the number of objects change as the object got thinner?"

The estimates got larger.

○ Why might that be?"

As the thickness decreased, more objects were needed to make a height of 5 feet.

Select previously identified students to share their responses and reasoning to the last two questions. Emphasize that we can make sense of all situations in the activity in terms of division.

In the case of the 5-foot or 60-inch stacks, the relationship between quantities can be represented as: $60 \div \text{height of an object in inches} = \text{number of objects.}$

Remind students that the result of division is called the "quotient." In this division, the number of objects is the quotient. The greater the height or thickness of the object, the smaller the quotient, and vice versa.

If time permits, consider using this applet to further illustrate the relationship between the size of the object in the stack and the size of the quotient.

The Geogebra applet 'All Stacked Up' is available here: https://www.geogebra.org/m/RJQyS6av.

Activity 2

All in Order

15 min

Activity Narrative

In this activity, students continue to explore the relationship between dividends, divisors, and quotients.

First, they study two sets of division expressions and arrange them in the order of their values—from the largest to smallest. The first set of expressions has the same dividend (800). The second set has the same divisor (25).

Next, students estimate the values of another set of expressions and classify the quotients as being close to 0, close to 1, or much greater than 1.

As students work, monitor how they think about placing the expressions. Select students or groups to explain their rationale later.

Also notice the expressions that students find difficult to put in order. Expressions with fractional or decimal divisors, or expressions in which the dividends and divisors are very close to each other (such as $800 \div 800.1$ and $800 \div 799.5$), may be particularly challenging. This is an opportunity for students to make use of the structure that relates the three quantities in a division: to see that as the divisor gets larger, even if only by a very small amount, the quotient necessarily gets smaller.

In Are You Ready for More, students have the option to work in groups of 2. Instead of writing a list of expressions, partners may take turns writing expressions with values that are increasingly closer to 1 without equaling 1.

Instructional Routines

MLR2: Collect and Display

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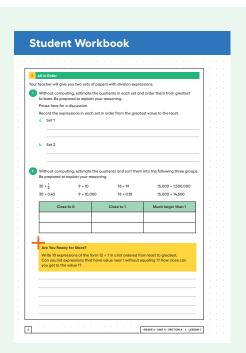
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Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.



Launch

Arrange students in groups of 2. Give each group one set of pre-cut slips—either Set 1 or Set 2—from the blackline master.

Give students 2–3 minutes to order the expressions by value—from greatest to least.

Next, ask them to discuss their ordered list with another group who sorted a different set, check each other's work, and record both ordered lists.

Then, ask students to work with their original partner to complete the last question, leaving a few minutes for discussion.

Use Collect and Display to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe the numbers in a division expression, their size, and their relationships. Display words and phrases such as "the number being divided," "the number we are using to divide," "the result of division," "the quotient," and "when the second number in a division gets larger, the answer gets smaller."

Student Task Statement

Your teacher will give you two sets of papers with division expressions.

1. Without computing, estimate the quotients in each set and order them from greatest to least. Be prepared to explain your reasoning.

Pause here for a discussion.

Record the expressions in each set in order from the greatest value to the least.

a.Set 1

$$800 \div 0.0001, 800 \div \frac{1}{10}, 800 \div 2.5, 800 \div 250, 800 \div 799.5, 800 \div 801, 800 \div 1,250, 800 \div 10,000$$

b. Set 2

2. Without computing, estimate the quotients and sort them into the following three groups. Be prepared to explain your reasoning.

$$30 \div \frac{1}{2}$$
 9 ÷ 10 18 ÷ 19 15,000 ÷ 1,500,000
 $30 \div 0.45$ 9 ÷ 10,000 18 ÷ 0.18 15,000 ÷ 14,500

Close to 0	Close to 1	Much larger than 1
15,000 ÷ 1,500,000	9 ÷ 10	30 ÷ 0.45
	18 ÷ 19	18 ÷ 0.18
9 ÷ 10,000	15,000 ÷ 14,500	30 ÷ ½

Are You Ready for More?

Write 10 expressions of the form $12 \div ?$ in a list ordered from least to greatest. Can you list expressions that have value near 1 without equaling 1? How close can you get to the value 1?

Sample response: $12 \div 13$, $12 \div 12.5$, $12 \div 12.1$, $12 \div 12.05$, $12 \div 12.01$, $12 \div 12.001$. I can get closer and closer to the value of I by dividing 12 by a number that is closer and closer to 12 but is not exactly 12.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students or groups to share how they ordered the two sets of expressions in the first question. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

Discuss division expressions that students found hard to place (such as those involving decimal, fractional, or very large divisors), and select other students to share how they made their decisions.

At this point, students may find it helpful to have more concise terms to describe the numbers in a division expression.

Introduce (or remind students of) the terms "dividend" for the number being divided and "divisor" for the number used to divide. Display a division equation labeled with these terms so students can refer to it as needed:

dividend ÷ divisor = quotient

Next, discuss ways to reason about the size of quotients relative to 0 and 1. Encourage students to use the new terms as they respond to the following questions:

"How did you decide that a quotient is close to 0?"

The divisor is much larger than the dividend.

"How did you decide that a quotient is close to 1?"

If the divisor is close to the dividend, then it would be closer to I.

"Is there a way to tell if a quotient is less than 1 or more than 1?"

If the divisor is smaller than the dividend, then the quotient is more than I. Otherwise, it is less than I.

"Suppose a divisor is less than the dividend. How can we tell if the quotient is just a little larger than 1 or much larger than 1?"

If the dividend and divisor are very far apart in size, then the quotient is much larger than I.

Lesson Synthesis

A key takeaway from this lesson is that in a division, the size of the dividend (the number that we are dividing) and the size of the divisor (the number we use to divide) both affect the quotient (the result of dividing).

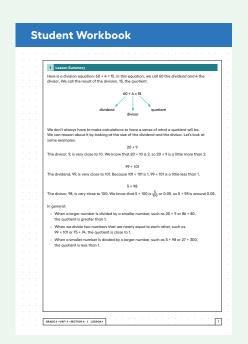
To highlight what the relative sizes of the dividend and the divisor can tell us about the size of the quotient, ask students:

Building on Student Thinking

Students may try to compute the value of each expression because they are unsure how to begin otherwise. Suggest that they compare two quotients at a time, starting with those that have very different divisors. Ask, for instance,

"Which is greater, 800 ÷ 250 or 800 ÷ 2.5?"

Urge them to use the patterns they saw earlier about how the size of a divisor affects the quotient.



"What happens to the quotient when we divide by smaller and smaller numbers?"

The quotient gets larger and larger.

- "Which would result in a smaller quotient: dividing a number by 0.5 or dividing it by 5? Why?"
 - 5, because we would need fewer 5s than 0.5s to reach the size of the dividend.
- "What can we say about the quotient when the divisor and dividend are about the same size?"

The quotient will be close to I.

"What can we say about the quotient when the divisor is much smaller than the dividend?"

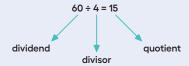
The quotient will be much greater than I.

"What about when we divide a number by another number that is much larger?"

The quotient will be close to 0.

Lesson Summary

Here is a division equation: $60 \div 4 = 15$. In this equation, we call 60 the dividend and 4 the divisor. We call the result of the division, 15, the quotient.



We don't always have to make calculations to have a sense of what a quotient will be. We can reason about it by looking at the size of the dividend and the divisor. Let's look at some examples.

The divisor, 9, is very close to 10. We know that $20 \div 10$ is 2, so $20 \div 9$ is a little more than 2.

The dividend, 99, is very close to 101. Because 101 \div 101 is 1, 99 \div 101 is a little less than 1.

The divisor, 98, is very close to 100. We know that $5 \div 100$ is $\frac{5}{100}$ or 0.05 so $5 \div 98$ is ground 0.05.

In general:

- When a larger number is divided by a smaller number, such as $20 \div 9$ or $86 \div 80$, the quotient is greater than 1.
- When we divide two numbers that are nearly equal to each other, such as $99 \div 101$ or $75 \div 74$, the quotient is close to 1.
- When a smaller number is divided by a larger number, such as $5 \div 98$ or $27 \div 300$, the quotient is less than 1.

Cool-down

Result of Division

5 min

Student Task Statement

Without computing, decide whether the value of each expression is much smaller than 1, close to 1, or much larger than 1.

1. 1,000,001 ÷ 99

Much larger than I

2. 3.7 ÷ 4.2

Close to I

3.1 ÷ 835

Much smaller than I

4. 100 ÷ $\frac{1}{100}$

Much larger than I

5. 0.006 ÷ 6,000

Much smaller than I

6. 50 ÷ 50 $\frac{1}{4}$

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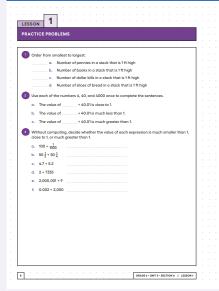
Close to I

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Problem 1

Order from smallest to largest:

- a. Number of pennies in a stack that is 1 ft high
- **b.** Number of books in a stack that is 1 ft high
- c. Number of dollar bills in a stack that is 1ft high
- d. Number of slices of bread in a stack that is 1 ft high

number of books, number of slices of bread, number of pennies, number of dollar bills

Problem 2

Use each of the numbers 4, 40, and 4000 once to complete the sentences.

- **a.** The value of $\frac{40}{2}$ ÷ 40.01 is close to 1.
- **b.** The value of $\frac{4}{3}$ ÷ 40.01 is much less than 1.
- **c.** The value of $\frac{4000}{2}$ ÷ 40.01 is much greater than 1.

Problem 3

Without computing, decide whether the value of each expression is much smaller than 1, close to 1, or much greater than 1.

- **a.** $100 \div \frac{1}{1000}$ much greater than I
- **b.** $50\frac{1}{3} \div 50\frac{1}{4}$ close to I
- c. 4.7 ÷ 5.2 close to I
- d. 2 ÷ 7335 much smaller than I
- e. 2,000,001 ÷ 9 much greater than I
- f. 0.002 ÷ 2,000 much smaller than I

Problem 4

from Unit 3, Lesson 16

A wheelbarrow has a weight limit of 120 pounds.

- a. What percentage of the weight limit is 66 pounds? 55%
- b. What percentage of the weight limit is 228 pounds? 190%
- c. What weight is 95% of the limit? II4 pounds

Problem 5

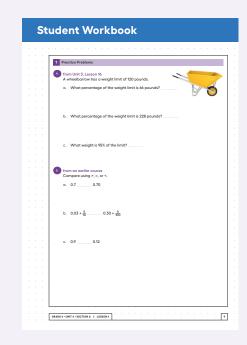
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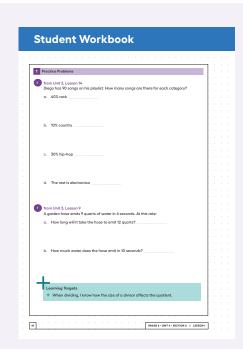
Compare using >, =, or <.

a.
$$0.7 = 0.70$$
 because $\frac{7}{10} = \frac{70}{100}$

b.
$$0.03 + \frac{6}{10} > 0.30 + \frac{6}{100}$$
 because $0.63 > 0.36$

c.
$$0.9 \ge 0.12$$
 because $\frac{9}{10} = \frac{90}{100}$





Problem 6

from Unit 3, Lesson 14

Diego has 90 songs on his playlist. How many songs are there for each category?

- **a.** 40% rock
 - 36, because $(0.4) \cdot 90 = 36$
- **b.** 10% country
 - 9, because $(0.1) \cdot 90 = 9$
- c. 30% hip-hop
 - 27, because $(0.3) \cdot 90 = 27$
- d. The rest is electronica
 - 18, because (0.2) · 90 = 18

Problem 7

from Unit 3, Lesson 9

A garden hose emits 9 quarts of water in 6 seconds. At this rate:

- **a.** How long will it take the hose to emit 12 quarts?
 - 8 seconds
- **b.** How much water does the hose emit in 10 seconds?
 - 15 quarts