Lesson 14

# Nets and Surface Area

## Goals

- Match polyhedra with their nets and justify (orally) that they match.
- Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.
- Visualize and identify the polyhedron that can be assembled from a given net.

## Learning Targets

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

## Lesson Narrative

This lesson extends students' understanding of polyhedra, their nets, and their surface area.

Students begin by matching polyhedra and their nets. Then, they work in groups to assemble two prisms and a pyramid from given nets and use nets to find the surface area of each polyhedron. To support students in reasoning about area, the nets are presented with a grid.

As students calculate surface area and consider how to account for all the faces of a polyhedron, they have opportunities to look for and make use of structure, in both the arrangement and measurements of the polygons that compose the polyhedron.

## Student Learning Goal

Let's use nets to find the surface area of polyhedra.

### Access for Students with Diverse Abilities

- Representation (Activity 1)

### Access for Multilingual Learners

- MLR2: Collect and Display (Warm-up)
- MLR8: Discussion Supports (Activity 1)

### Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

### Required Materials

**Materials to Gather**

- Nets of polyhedra: Warm-up, Activity 1
- Scissors: Warm-up, Activity 1
- Geometry toolkits: Activity 1
- Glue or glue sticks: Activity 1
- Tape: Activity 1

### Required Preparation

**Warm-up:**
Prepare physical copies of the nets in the *Warm-up*, in case needed to support students with visualization. The blackline master contains a larger version of these nets.

**Activity 1:**
Every student workbook contains a copy of the nets for this activity. Each group of 3 students will need one set of nets (A, B, and C) and some glue or tape.

## Lesson Timeline

| Warm-up | Activity 1 | Lesson Synthesis | Assessment: Cool-down |
|---|---|---|---|
| 10 min | 25 min | 10 min | 5 min |

### Access for Multilingual Learners (Warm-up, Student Task)

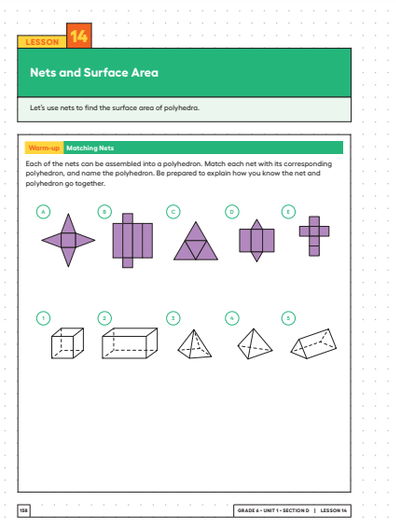
**MLR2: Collect and Display.**
Direct attention to words collected and displayed from the previous lesson. Invite students to borrow language from the display as needed when explaining how they know a net and a polyhedron go together. Update the display throughout the lesson.

*Advances: Conversing, Reading*

### Building on Student Thinking

If students have trouble distinguishing between Figures A, C, and D, remind them that prisms and pyramids can both contain faces that are triangles. In a pyramid, all triangular faces that are not the base meet at a one vertex and have shared edges. In a prism, there can be a triangular base, but the other faces are quadrilaterals.

### Student Workbook


LESSON 14

Nets and Surface Area

Let's use nets to find the surface area of polyhedra.

Warm-up Matching Nets

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.

## Warm-up

## Matching Nets

10 min

### Activity Narrative

This *Warm-up* prompts students to match nets to polyhedra. It invites them to think about the polygons that make up a polyhedron and to mentally manipulate nets, which helps develop their visualization skills.

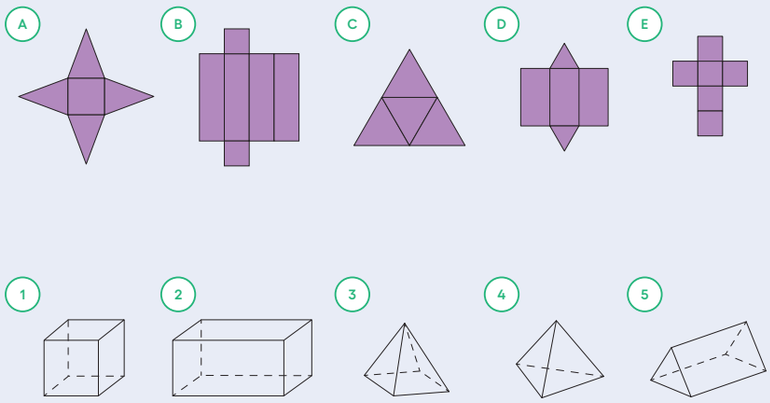
### Launch

Give students 3 minutes of quiet think time to match nets to polyhedra and then another 2 minutes to discuss their response and reasoning with a partner. Encourage students to use the terminology that they learned in prior lessons.

To support students who need more time or help in visualization, prepare physical models of the polyhedra. Have scissors available so that students can cut out and assemble the nets in the student workbook and test their ideas.

### Student Task Statement

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.




Students are likely to give similar reasoning for each figure as in the given sample reasoning.

Net A is a square pyramid (3). Sample reasoning: It has five faces: one square and four triangles, just like the square pyramid.

Net B is a rectangular prism (2).

Net C is a triangular pyramid (4).

Net D is a triangular prism (5).

Net E is a cube or square prism (1).

### Activity Synthesis

Invite a few students to share their matching decisions and reasoning with the class. Ask students:

- *"What clues did you use to help you match? How did you check if you were right?"*

If there is not unanimous agreement on any of the nets, ask students with differing opinions to explain their reasoning. Discuss to come to an agreement.

## Activity 1

## Using Nets to Find Surface Area

25 min

### Activity Narrative

In this activity, students cut and assemble nets into polyhedra. They use nets to find surface area, applying what they learned earlier about areas of triangles and parallelograms. The presence of a grid supports students in their calculations. It also reinforces the idea of area as the number of unit squares in a region and the connection between area and surface area.

As they find surface area, students have an opportunity to look for and make use of structure. For instance, they may identify multiple copies of the same polygon and find the combined area with a single calculation. They may also group rectangles with a common side length into a larger rectangle and find the area of the latter.

Monitor for the different approaches that students take. Here are some likely approaches, starting from less systematic to more systematic:

- Find the area of each face separately and add the areas.
- Identify unique shapes, find their areas, multiply the area of each unique shape by how many there are, and add the areas.
- Rearrange and group two or more faces (for instance, rearrange a pair of triangles into a rectangle or a parallelogram) before finding and adding the areas.

Also monitor for how students organize their work, such as whether they record the measurements of each face, label the shapes in the net and the associated calculations, and account for all the faces of their polyhedron. Encourage students with disorganized or scattered work to take a more methodical approach.

### Launch

Arrange students in groups of 3. Each group will need one of each net (A, B, and C), tape, and access to their geometry toolkits (especially scissors). Explain to students that they will cut some nets, assemble them into polyhedra, and calculate their surface areas. Remind students that the surface area of a three-dimensional figure is the sum of the areas of all of its faces. Ask students to complete the first question before cutting anything.

Point out that the net has shaded and unshaded polygons. Explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are "flaps" to make it easier to glue or tape the polygons together. They will get tucked behind the shaded polygons and are

**Instructional Routines**

**5 Practices**

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**Instructional Routines**

**MLR8: Discussion Supports**

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**Access for Students with Diverse Abilities (Activity 1, Student Task)**

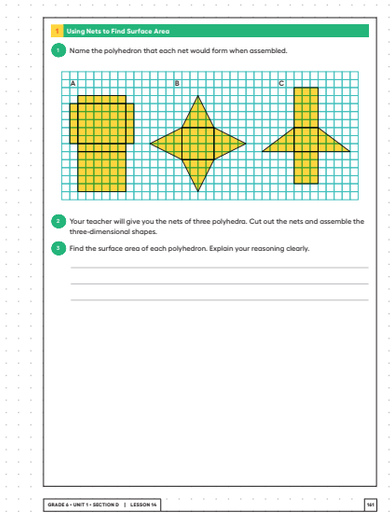
**Representation: Access for Perception.**
Provide access to pre-cut nets and the polyhedra. Ask students to identify correspondences between the faces of the polyhedra and regions of each net.

*Supports accessibility for: Visual-Spatial Processing, Organization*

**Building on Student Thinking**

If students do not identify the specific type of prism or pyramid, remind them that they should also name each figure by the shape of its base.

**Student Workbook**




not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold up the net and attach the various polygons together. A straightedge can be very helpful for making the creases.

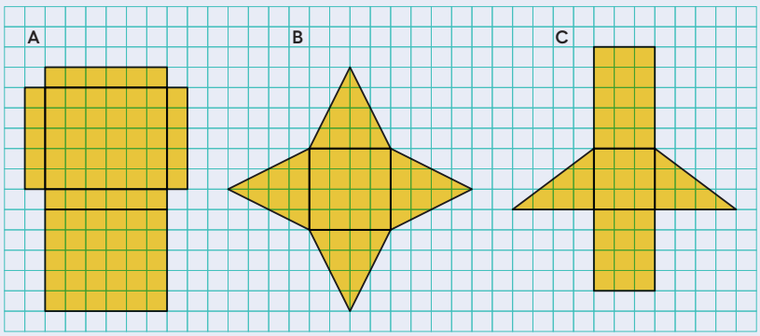
Tell students that it is easy to miss or double-count the area of a face when finding surface area. Ask them to think carefully about how to record their calculations to ensure that all faces are accounted for, correct measurements are used, and errors are minimized.

When students have completed their calculations, ask them to compare and discuss their work with another student who has the same polyhedron.

For whole-class discussion, select 1 or 2 polyhedra whose surface area is found using different approaches, including different ways to keep track of calculations. Select students who use different strategies to share later.

### Student Task Statement

**1.** Name the polyhedron that each net would form when assembled.




- A: rectangular prism
- B: square pyramid
- C: triangular prism

**2.** Your workbook contains the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.

No answer required.

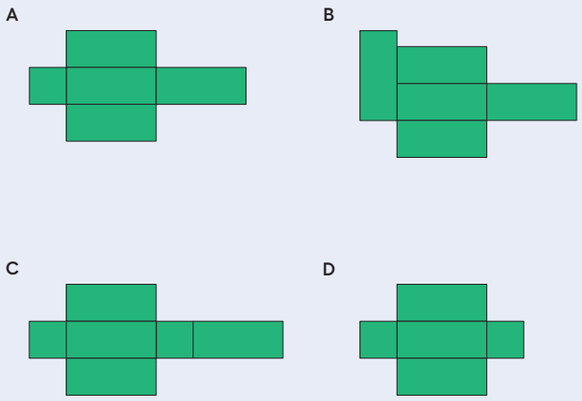
**3.** Find the surface area of each polyhedron. Explain your reasoning clearly.

Sample responses:

- A: The surface area is 82 square units
  $2(6 \cdot 1) + 2(5 \cdot 1) + 2(6 \cdot 5) = 82$
- B: The surface area is 48 square units
  $(4 \cdot 4) + 4\left(\frac{1}{2} \cdot 4 \cdot 4\right) = 48$
- C: The surface area is 48 square units
  $(3 \cdot 5) + (3 \cdot 3) + (3 \cdot 4) + 2\left(\frac{1}{2} \cdot 3 \cdot 4\right) = 48$
- C: The combined area of the three rectangular faces is 36 square units. $3 \cdot 12 = 36$. The combined area of the two right triangles is 12 square units. $2\left(\frac{1}{2} \cdot 3 \cdot 4\right) = 12$. The surface area is 48 square units because $36 + 12 = 48$

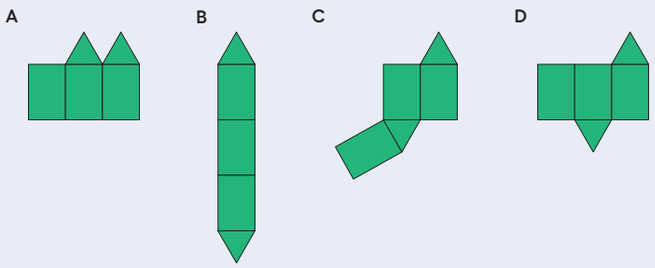
## Are You Ready for More?

**1.** For each net, decide if it can be assembled into a rectangular prism.




Only C can be folded into a rectangular prism.

**2.** For each net, decide if it can be folded into a triangular prism.




C and D can be folded into triangular prisms.

## Activity Synthesis

Invite previously selected students to share their responses and reasoning. For each polyhedron, sequence the discussion of the strategies in the order listed in the activity narrative. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"Before cutting out and assembling your net, how did you know what polyhedron it would create?"*
- *"We saw more than one strategy for finding the surface area of a polyhedron [letter]. How are the strategies alike? How are they different?"*
- *"How can we make sure that we've included the areas of all the faces of the polyhedron?"*

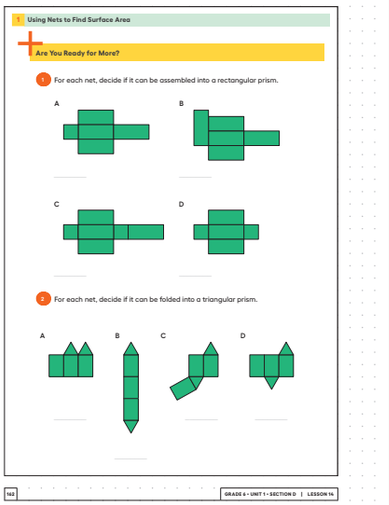
If methods for organizing the work—such as listing measurements and labeling polygons and computations—are not mentioned, bring them up. If needed, demonstrate these organizational methods and discuss their benefits.

### Access for Multilingual Learners (Activity 1, Synthesis)

**MLR8: Discussion Supports.** Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Advances: Speaking, Representing*

### Student Workbook



Connect the approaches used here to those used earlier in the unit by asking students:

- *"How is finding the surface area of a polyhedron like finding the area of a polygon? How is it different?"*

Highlight that even though polyhedra are three-dimensional figures, surface area is a two-dimensional measure, so the same strategies we used to find area—such as decomposing and rearranging—still apply here.

## Lesson Synthesis

In this lesson, students matched nets to polyhedra, assembled polyhedra from nets, and used nets to find surface area. Discuss with students:

- *"How do we use a net to find surface area?"*

  We calculate the area of each polygon on the net and add all the areas.

- *"How is finding surface area using a net different from finding surface area by looking at a picture of a polyhedron—as we had done with the filing cabinet, or by studying the actual object—as we had done with the snap cubes?"*

  A net allows us to see all the faces of a polyhedron at once. When working from a picture or drawing, we need to visualize the hidden faces. Working with an actual polyhedron could help, but again we are not looking at all the faces at once; we have to rotate the object and might miss or double-count a face.

- *"When using a net, how do we keep track of the calculations or make sure that all faces are accounted for?"*

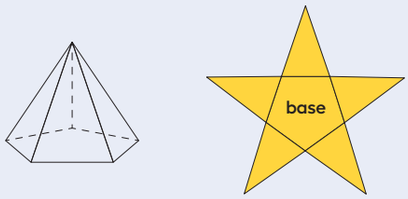
  We can label all the polygons and the calculations.

- *"Are there ways to simplify the calculations? Or is it necessary to find the area of each polygon one at a time?"*

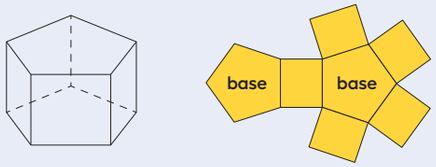
  Sometimes we can simplify the process by combining polygons and finding the area of the combined region—for example, finding the area of a group of rectangles with the same side length. If there are several polygons that are identical, we can find the area of one polygon and multiply it by the number of identical polygons in the net.

## Lesson Summary

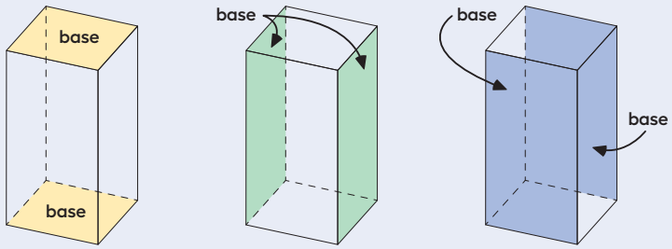
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



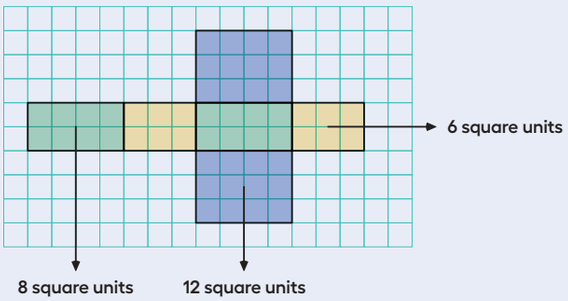

A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.




In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.

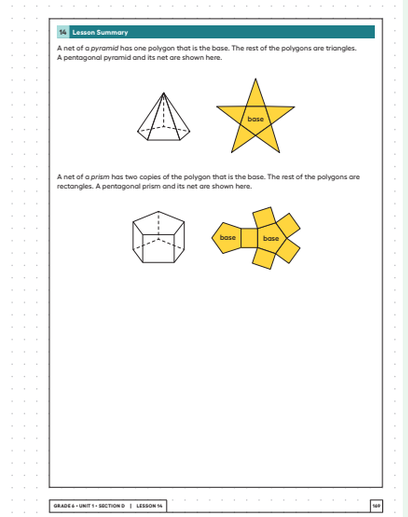



Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.

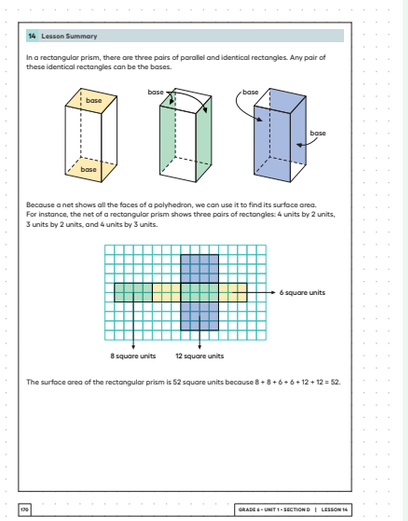



The surface area of the rectangular prism is 52 square units because $8 + 8 + 6 + 6 + 12 + 12 = 52$.

### Student Workbook




### Student Workbook



**Responding To Student Thinking**

**More Chances**
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

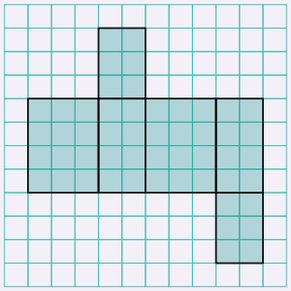
## Cool-down

5 min

### Unfolded

#### Student Task Statement

1. What kind of polyhedron can be assembled from this net?



A rectangular prism

2. Find the surface area (in square units) of the polyhedron. Show your reasoning.

52 square units

Sample reasoning: $2(3 \cdot 4) + 2(2 \cdot 4) + 2(2 \cdot 3) = 52$

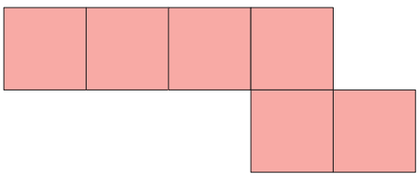
Lesson 14

## Practice Problems

6 Problems

### Problem 1

Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.

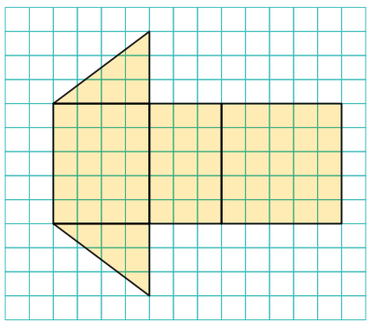


No

Sample reasoning: The four squares placed side by side can only be folded in one way to meet up with one another, making a cube without a top and bottom. One of the remaining two squares can be folded to make the top or bottom, but the other one cannot be used.

### Problem 2

**a.** What polyhedron can be assembled from this net? Explain how you know.



A triangular prism

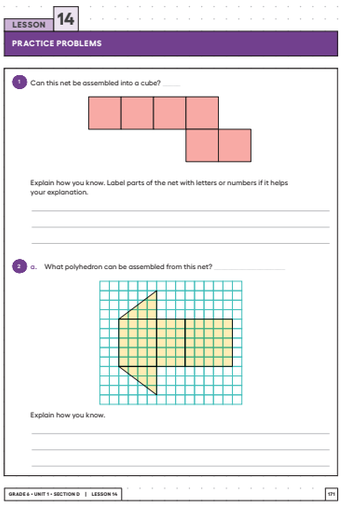
Sample reasoning: There are two identical triangles that are the bases. The rest of the faces are rectangles.

**b.** Find the surface area of this polyhedron. Show your reasoning.

72 square units

Sample reasoning: The area of the three rectangles are 20, 15, and 25 square units. The area of the two triangles are $2\left(\frac{1}{2} \cdot 4 \cdot 3\right)$ or 12 square units. $20 + 15 + 25 + 2(6) = 72$

### Student Workbook


LESSON 14

PRACTICE PROBLEMS

1. Can this net be assembled into a cube?

Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.

2. a. What polyhedron can be assembled from this net?

Explain how you know.

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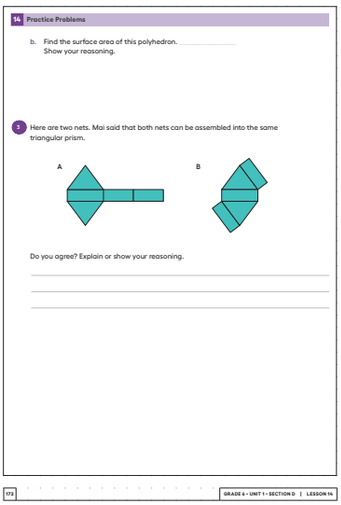
**Student Workbook**


14 Practice Problems

b. Find the surface area of this polyhedron. _____
Show your reasoning.

3 Here are two nets. Mai said that both nets can be assembled into the same triangular prism.

A B

Do you agree? Explain or show your reasoning.

_____

_____

_____

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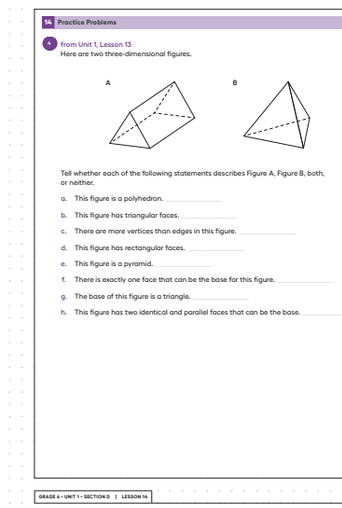
**Student Workbook**


14 Practice Problems

4 from Unit 1, Lesson 13
Here are two three-dimensional figures.

A B

Tell whether each of the following statements describes Figure A, Figure B, both, or neither.

a. This figure is a polyhedron. _____
b. This figure has triangular faces. _____
c. There are more vertices than edges in this figure. _____
d. This figure has rectangular faces. _____
e. This figure is a pyramid. _____
f. There is exactly one face that can be the base for this figure. _____
g. The base of this figure is a triangle. _____
h. This figure has two identical and parallel faces that can be the base. _____

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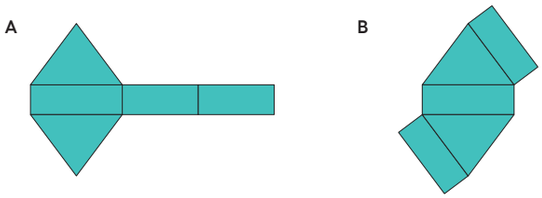
## Problem 3

Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.




Agree

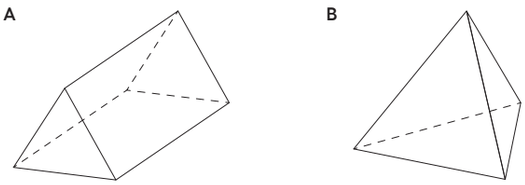
Sample reasoning: Both nets are composed of the same set of polygons. The positions of the one rectangular face are different, but when assembled, that face will meet the same edge of three other polygons.

## Problem 4

from Unit 1, Lesson 13

Here are two three-dimensional figures.

Tell whether each of the following statements describes Figure A, Figure B, both, or neither.




**a.** This figure is a polyhedron.

Both

**b.** This figure has triangular faces.

Both

**c.** There are more vertices than edges in this figure.

Neither

**d.** This figure has rectangular faces.

Figure A

**e.** This figure is a pyramid.

Figure B

**f.** There is exactly one face that can be the base for this figure.

Neither

**g.** The base of this figure is a triangle.

Both

**h.** This figure has two identical and parallel faces that can be the base.

Figure A

## Problem 5

from Unit 1, Lesson 12

Select **all** units that can be used for surface area.

**A.** square meters

**B.** feet

**C.** centimeters

**D.** cubic inches

**E.** square inches

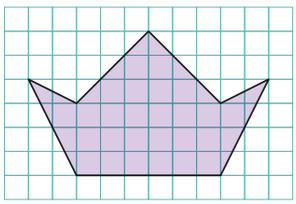
**F.** square feet

## Problem 6

from Unit 1, Lesson 11

Find the area of this polygon.

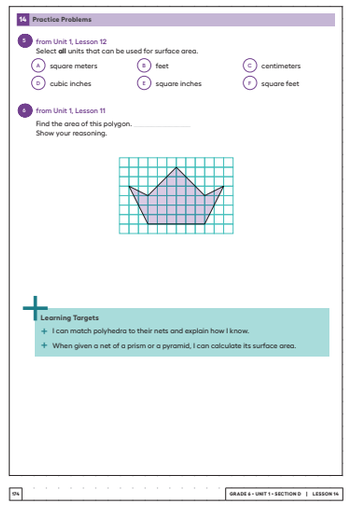
33 square units



Show your reasoning.

Sample reasoning: The figure can be decomposed into two triangles with a base of 3 units and a height of 2 units (area: $2 \cdot \left(\frac{1}{2} \cdot 3 \cdot 2\right) = 6$), a triangle with a base of 6 units and a height of 3 units (area: $\frac{1}{2} \cdot 6 \cdot 3 = 9$), and a rectangle that is 6 units by 3 units (area: $6 \cdot 3 = 18$). The total area is $6 + 9 + 18$, which is 33.

### Student Workbook



14 Practice Problems

5 from Unit 1, Lesson 12
Select **all** units that can be used for surface area.
A square meters B feet C centimeters
D cubic inches E square inches F square feet

6 from Unit 1, Lesson 11
Find the area of this polygon.
Show your reasoning.

Learning Targets
+ I can match polyhedra to their nets and explain how I know.
+ When given a net of a prism or a pyramid, I can calculate its surface area.