Lesson 17

Fitting Boxes into Boxes

Goals

- Compare and contrast (orally and using other representations) different ways jewelry boxes could be packed inside larger shipping boxes.
- Determine which size shipping box is least expensive, and present (orally and in writing) a justification.
- Make simplifying assumptions and determine what information is needed to solve a problem about shipping costs.

Learning Target

I can use multiplication and division of fractions to reason about real-world volume problems.

Lesson Narrative

In this lesson, students use what they learned about volume and about operations with fractions to solve an optimization problem. Given several shipping boxes with fractional edge lengths, students determine the most efficient and economical way to pack and ship 270 necklaces, each placed in a smaller gift box.

In the first activity, students make sense of the context and problem, outline what they will need to know and do to answer the question, and map out a plan.

Next, students use available measurements to determine the number of gift boxes that can fit each shipping box and the number of shipping boxes needed. This investigation involves experimenting with different orientations of the gift boxes to optimize the space in a shipping box. (The quantities in this lesson are based on the flat-rate shipping options of the United States Postal Service (USPS), but other sizes and rates can be used as well.)

Lesson Timeline

15 min	30 min	25 min	10 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Instructional Routines

- MLR6: Three Reads

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2
- Four-function calculators: Activity 3

Required Preparation

Lesson:
Bring in samples of United States Postal Service flat-rate boxes, or have images of these boxes available.

Lesson 17

Fitting Boxes into Boxes

Lesson Narrative (continued)

Finally, students use what they find about the capacity of each shipping box and the necessary number of boxes to calculate costs, compare them, and identify the least expensive option.

Throughout the lesson, students have opportunities to explain their thinking, listen to feedback from their peers, and critique the reasoning of others. To model a situation mathematically, they also need to make assumptions about relevant conditions or constraints.

To complete all activities would likely take more than a typical class meeting. The "How Many Shipping Boxes?" activity allows students to consolidate key concepts from this unit and is therefore prioritized. The last activity, which prompts students to calculate shipping costs, summarize their findings, and make a recommendation, is marked as optional.

Student Learning Goal

Let's use what we learned about fractions to find shipping costs.

Instructional Routines

MLR6: Three Reads

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR6: Three Reads

This activity uses *Three Reads* to to support reading comprehension and sense-making about this problem

Activity 1

What Do We Need to Know and Do?

15 min

Activity Narrative

This activity introduces the context and constraints of a shipping problem. It invites students to make sense of the situation (packing 270 necklaces in boxes) and determine the information they need to solve the problem (finding the least expensive way to ship them).

To find the most economical shipping box combination, students will need to:

- Find out the measurements of the gift boxes and shipping boxes, as well as the costs for mailing a shipping box of each size.
- Decide on an orientation for the gift boxes inside each shipping box and calculate how many gift boxes will fit with that particular orientation.
- Test out different orientations and how they affect the number of gift boxes to be fitted and the cost.

As they think about necessary information and steps to solve a real-world problem, students engage in aspects of modeling.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the two paragraphs, without revealing the last sentence ("She wants to know ...") or the questions.

- For the first read, read the problem aloud then ask, "What is this situation about?" (An artist is packing necklaces in individual boxes to ship to a store.) Listen for and clarify any questions about the context, such as "flat rate" or "shipping rates."
- After the second read, ask students to list any quantities that can be counted or measured. (The number of necklaces, the edge lengths of each gift box)
- After the third read, reveal the statement: "She wants to know which boxes to use to minimize her shipping cost" and ask, "What are some ways we might get started on this?" Invite students to name some possible starting points, referring to quantities from the second read. (Find out the size and cost of each shipping box. Figure out how many gift boxes can fit in it. Think about different ways to arrange the gift boxes.)

Arrange students in groups of 4.

Give students 2 minutes of quiet think time to brainstorm the information needed to solve this problem.

Then invite students to share their ideas, and record them for all to see.

Consider displaying some flat-rate boxes from the United States Postal Service (USPS) or an image of each of the boxes. Demonstrate the idea of the task by putting a small box inside a larger box in different orientations. Tell students that the postal service offers shipping boxes in a few standard sizes and charges a fix rated for shipping a box of each size.

Next, give students 4–5 minutes to work with their group and to think about how to effectively find the best shipping option.

Student Task Statement

An artist makes necklaces. She packs each necklace in a small gift box that is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch.

A department store ordered 270 necklaces. The artist plans to ship the necklaces to the store using flat-rate shipping boxes from the post office. She wants to know which boxes to use to minimize her shipping cost.

1. What information would she need to find out?

   Sample response:

   Information needed:

   - The size and cost of each shipping box
   - The measurements of each shipping box
   - Whether there are any rules about how heavy each box can be or how the gift boxes need to be packed in a shipping box
   - Whether the artist has a budget for shipping

2. How would you use this information to find the most inexpensive way to ship the necklaces? With your group, make a plan and write down the main steps.

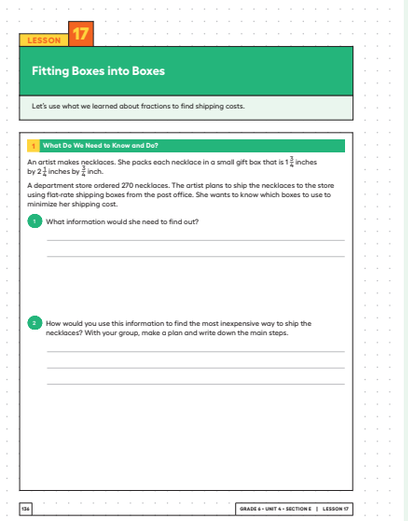
   A plan for solving the problem:

   - Decide which group member or members will work with which shipping box.
   - Find out how many gift boxes fit into each shipping box.
   - Find out how many and what combination of shipping boxes we need.
   - Calculate the total cost for shipping the gift boxes.
   - Compare the costs.

Activity Synthesis

The purpose of this discussion is to elicit ideas students have for managing the problem-solving process. Ask each group to share a couple of specific steps that they could take to answer the question. If not mentioned by students, suggest that each group split up the calculations to be done so each person is responsible for finding the cost associated with one shipping box.

Student Workbook


LESSON 17

Fitting Boxes into Boxes

Let's use what we learned about fractions to find shipping costs.

1 What Do We Need to Know and Do?

An artist makes necklaces. She packs each necklace in a small gift box that is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch.

A department store ordered 270 necklaces. The artist plans to ship the necklaces to the store using flat-rate shipping boxes from the post office. She wants to know which boxes to use to minimize her shipping cost.

1 What information would she need to find out?

2 How would you use this information to find the most inexpensive way to ship the necklaces? With your group, make a plan and write down the main steps.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: Students can share their learning using a video or demonstration of how they would pack the boxes, or by drawing a labeled diagram.

Supports accessibility for: Language, Fine Motor Skills

Activity 2

How Many Shipping Boxes?

30 min

Activity Narrative

In this activity, each student determines the number of gift boxes that can fit in a particular shipping box and finds how many shipping boxes are needed to contain 270 necklaces in gift boxes.

Along the way, students consider how the arrangement of the gift boxes inside each shipping box affects the number of gift boxes that can fit. Encourage students to create drawings of the boxes to show the calculations that they need to make and to visualize various constraints.

Students also have an opportunity to make assumptions as they model with mathematics. For instance, they may assume that only the same type or size of shipping boxes could be used, that some room for protective padding is needed, and so on.

In the *Activity Synthesis*, students are prompted to explain their reasoning and listen to the reasoning of others. They discuss their decisions, evaluate the accuracy of their calculations, and revise their work based on feedback.

If additional modeling demand is desired, consider asking students to research information on sizes and costs of shipping boxes or to measure the shipping boxes rather than providing this information for them.

Launch

Display the following information for all to see and keep it displayed throughout the activity.

Gift box: $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch

box size	measurements	cost
small box	$5\frac{3}{8}$ inches by $8\frac{5}{8}$ inches by $1\frac{5}{8}$ inches	$6.80
medium box 1	11 inches by $8\frac{1}{2}$ inches by $5\frac{1}{2}$ inches	$13.45
medium box 2	$11\frac{7}{8}$ inches by $3\frac{3}{8}$ inches by $13\frac{5}{8}$ inches	$13.45
large box	12 inches by 12 inches by $5\frac{1}{2}$ inches	$18.75

Keep students in the same groups of 4. Ask each group member to select a different size of shipping box so that all boxes are studied in each group. Provide access to geometry toolkits, especially paper and rulers.

Give students quiet time to investigate the shipping box that they selected. Allow at least 10 minutes for discussions afterward.

Student Task Statement

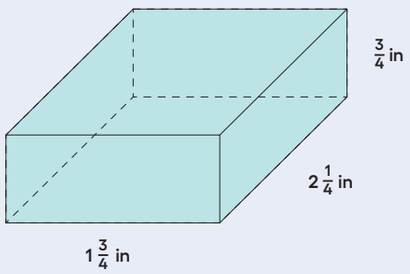
Work with your group to find out how many shipping boxes are needed to ship 270 necklaces. Each group member should select a different type of flat-rate shipping box to investigate.

For each type of shipping box:

- Find how many gift boxes can be packed in it. Explain or show your reasoning. Draw a sketch, if needed.
- Find out how many shipping boxes are needed to fit 270 gift boxes.

Show your reasoning and organize it so that it can be followed by others.

Record the size, measurements, and cost of the shipping box that you are investigating.



box size	measurements	cost

For each box, different orientations and results are possible. Answers vary based on the orientation chosen and whether any gaps are filled with additional boxes in a different orientation. Sample responses for the USPS flat-rate boxes:

- Small box: If we place the gift boxes on their largest face and with the $2\frac{1}{4}$-inch side along the $8\frac{5}{8}$-inch side of the shipping box, we can fit $3 \cdot 3 \cdot 2$ or 18, plus 2 more, making 20 gift boxes. (Other orientations would lead to fewer boxes).
- Medium box 1: If we place the gift boxes on their largest face and with the $1\frac{3}{4}$-inch side along the 11-inch side of the shipping box, we can fit $6 \cdot 3 \cdot 7$ or 126 boxes. There would be a gap for an additional $1 \cdot 4 \cdot 7$ (or 28 boxes), for a total of 154 boxes.
- Medium box 2: If we place the gift boxes on their largest face and with the $2\frac{1}{4}$-inch side along the $13\frac{5}{8}$-inch side of the shipping box, we can fit $6 \cdot 6 \cdot 4$ or 144 gift boxes. This will leave a gap for additional $1 \cdot 1 \cdot 7$ or (7 boxes), for a total of 151 boxes.
- Large box: If we place the gift boxes on their longest and narrowest face, we can fit 255 boxes: $16 \cdot 5 \cdot 3$ or 240 oriented one way, and additional $5 \cdot 1 \cdot 3$ (or 15) boxes oriented differently, squeezed in the gap left by the first set of boxes.

Building on Student Thinking

It may not occur to students to try more than one way to arrange the gift boxes in a shipping box. Consider asking students:

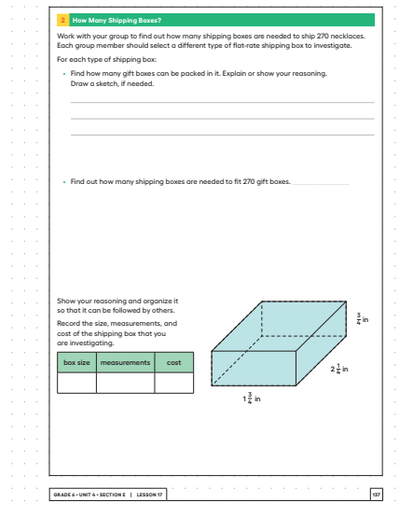
"Does it matter which way the gift boxes are placed in a shipping box? Does the orientation affect the number of boxes that can fit in the box?"

"What are some possible ways to arrange the gift boxes?"

"How much empty space would be in the shipping box if the gift boxes are arranged this way?"

If needed, consider demonstrating how to use drawings to help visualize the different arrangement of the boxes.

Student Workbook


2 How Many Shipping Boxes?

Work with your group to find out how many shipping boxes are needed to ship 270 necklaces. Each group member should select a different type of flat-rate shipping box to investigate.

For each type of shipping box:

- Find how many gift boxes can be packed in it. Explain or show your reasoning. Draw a sketch, if needed.
- Find out how many shipping boxes are needed to fit 270 gift boxes.

Show your reasoning and organize it so that it can be followed by others.

Record the size, measurements, and cost of the shipping box that you are investigating.

box size	measurements	cost

$\frac{3}{4}$ in
$2\frac{1}{4}$ in
$1\frac{3}{4}$ in

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Access for Students with Diverse Abilities (Activity 3, Launch)

Engagement: Develop Effort and Persistence.
Students may benefit from feedback that emphasizes effort and time on task. For example, check in with each group and highlight how much work they have already completed toward solving a difficult problem. Encourage them to continue, and if necessary remind them of the tools and resources available to them, such as calculators and the given information about the boxes.

Supports accessibility for: Social-Emotional Functioning, Organization

Activity Synthesis

Focus this time on small-group discussions. After students have made their first round of calculations, ask them to partner with a fellow group member and take turns explaining their work. Explain that their job is to check each other's reasoning, give feedback to one another, correct any errors, and revise their work based on the feedback.

Display questions such as the following. Ask students to use them to guide their discussion.

- *"How many different ways can the gift boxes fit into this shipping box?"*
- *"For this box, does one orientation create less or more wasted space than others? Why?"*
- *"Can you use diagrams to show and compare the unused spaces in different arrangements?"*
- *"Is there a way to fit more gift boxes into this shipping box? How?"*
- *"Is it possible to fit 270 gift boxes in fewer shipping boxes of this size? How?"*

If time permits, facilitate a second round of small-group discussions. This time, ask students to discuss their work with a classmate from another group who worked on the same shipping box as they did and to revise their thinking as needed.

Activity 3: Optional

How Much Would It Cost to Ship?

25 min

Activity Narrative

In this activity, students calculate the costs of shipping the 270 necklaces in different-size boxes, decide on the least expensive option, and discuss their conclusions. Students also reflect on their reasoning and problem-solving process.

As students work, monitor for groups who arrive at different conclusions on the number of shipping boxes needed or the most economical options. Consider asking them to share during class discussion.

Launch

Keep students in the same groups of 4. If needed, give them time to finish revising their calculations on the number of shipping boxes needed to ship all the necklaces.

Explain to students that now that they know how many gift boxes can fit into each shipping box and how many shipping boxes of each size are needed to ship all 270 necklaces, their next step is to calculate and compare the costs.

Instruct students to first share their individual findings with their group, record them in the table, and then work together to calculate the costs. Explain that the last row in the table can be used to calculate the cost of using a combination of sizes, if students choose to explore that option.

Give students 10–15 minutes of group work time. Provide access to calculators.

Student Task Statement

Work with your group to find the best plan for shipping the boxes of necklaces.

- For each size of shipping box and based on your work earlier, record the number of boxes needed to ship all 270 necklaces. You can also include an option that involves using shipping boxes in two different sizes and record that information in the last row.
- Calculate the total cost of shipping for each box size or a combination of sizes. Show your reasoning.
- Decide which shipping boxes the artist should use. Be prepared to explain your reasoning.

Sample response:

- The numbers of USPS flat-rate boxes and costs:

box size	gift boxes that can fit	shipping boxes needed	cost per box	total cost
small box	20	14	$6.80	$95.20
medium box 1	154	2	$13.45	$26.90
medium box 2	151	2	$13.45	$26.90
large box	255	2	$18.75	$37.50
large box and small box	255 and 20	1 of each	$18.75 and $6.80	$25.55

- The two cheapest options: using 1 large box and 1 small box ($25.55), or using 2 medium boxes of either type ($26.90).

Activity Synthesis

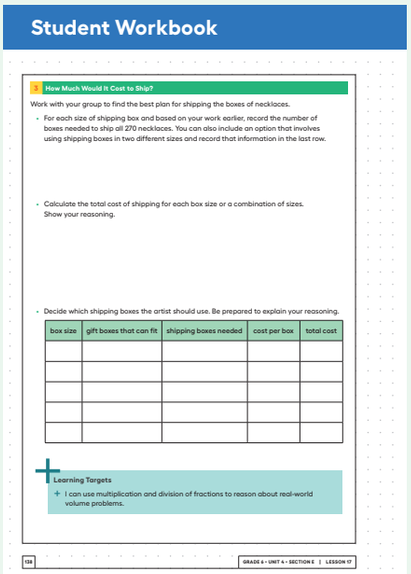
If time allows, select 4–5 groups to present their findings about the 4–5 shipping boxes (with each group presenting one option).

Otherwise, invite students to share their conclusions on the most economical shipping option for the artist. Display or record their reasoning for all to see.

To involve more students in the conversation, consider asking:

- *"Did anyone arrive at the same conclusion but for a different reason?"*
- *"Did anyone arrive at a different conclusion?"*
- *"Do you agree or disagree? Why?"*
- *"Is any group rethinking their original conclusion after hearing another group's reasoning? Which parts of their reasoning led you to reconsider your answer?"*

Student Workbook



Lesson Synthesis

Consider wrapping up by discussing the ways that students applied multiplication and division of fractions in solving the problem. Ask questions such as:

- *"How did you calculate how many gift boxes would fit in a box? Did you multiply the lengths of the gift boxes or divide the lengths of the shipping boxes?"*
- *"Did the size of fractions affect how you performed division? What methods did you use to divide?"*
- *"How did you confirm or check your calculations?"*

If time permits, invite students to reflect on how they engaged in mathematical modeling. Ask questions such as:

- *"Did you make any assumptions in order to solve the problem? What assumptions did you make?"*
- *"Were there times when you had to change course because your initial approach was not fruitful? When was that?"*
- *"If you had a chance to solve a similar problem, what might you do differently to improve the efficiency or accuracy of your work?"*