

Fractional Lengths in Triangles and Prisms

Goals

- Calculate the base or the height of a triangle given its area and the other measurement.
- Determine the volume of a rectangular prism by counting how many $\frac{1}{2}$ -inch or $\frac{1}{3}$ -inch cubes it takes to build, and explain (orally and in writing) the solution method.
- Generalize that the volume of a rectangular prism with fractional edge lengths can be found by multiplying the edge lengths.

Learning Targets

- I can explain how to find the volume of a rectangular prism using cubes that have a unit fraction as their edge length.
- I can use division and multiplication to solve problems involving areas of triangles with fractional bases and heights.
- I know how to find the volume of a rectangular prism even when the edge lengths are not whole numbers.

Lesson Narrative

This lesson transitions students from thinking about the area of two-dimensional figures with fractional linear measurements to reasoning about the volume of prisms with fractional edge lengths.

Students begin by solving problems about triangles. They find the area of a triangle given a pair of base and corresponding height measurements. They also calculate an unknown base or height given the other two measurements.

Next, students explore the volume of rectangular prisms whose edge lengths are not whole numbers. In grade 5, students determined the volume of a rectangular prism by finding the number of unit cubes that can be packed into the prism without gaps or overlaps. They concluded that this number is equal to the product of the edge lengths. Here, students encounter a prism whose edge lengths are multiples of $\frac{1}{2}$ inch. They find its volume by packing it with cubes with $\frac{1}{2}$ -inch cubes, quantifying the number of cubes, and multiplying it by $\frac{1}{8}$, the volume of a single cube in cubic inches.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)
- MLR2: Collect and Display (Activity 2)

Instructional Routines

- 5 Practices

Required Materials

Materials to Gather

- $\frac{1}{2}$ -inch cubes: Activity 3
- Geometry toolkits: Activity 3

Lesson Timeline

5 min

Warm-up

10 min

Activity 1

20 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Fractional Lengths in Triangles and Prisms

Lesson Narrative (continued)

By repeating this reasoning and noticing regularity, students see that the volume of a rectangular prism with fractional edge lengths can also be found by multiplying its edge lengths directly.

Student Learning Goal

Let's explore area and volume when fractions are involved.

Warm-up

Area of Triangle

5
min

Activity Narrative

In this *Warm-up*, students review how to find the area of a triangle given a pair of base-height measurements. The reasoning here prepares students to use division of fractions to solve area problems involving triangles later in the lesson.

Launch



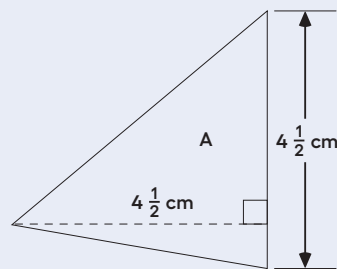
Arrange students in groups of 2.

Give students 2 minutes of quiet work time, followed by 1 minute of partner discussion.

Before students begin, review the formula for the area of a triangle. Consider displaying a drawing of a triangle with one side labeled as a base and a corresponding height shown and labeled as such.

Student Task Statement

Find the area of Triangle A in square centimeters. Show your reasoning.



$$10\frac{1}{8} \text{ cm}^2$$

Sample reasoning: The area of any triangle is $A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$ and $\frac{1}{2} \cdot \left(4\frac{1}{2}\right) \cdot \left(4\frac{1}{2}\right) = 10\frac{1}{8}$.

Activity Synthesis

Invite a student to share a solution and reasoning. Record it for all to see. Ask if others used alternative ways of reasoning, and invite them to share their approaches (as many as time permits).

If any student wrote the fraction $4\frac{1}{2}$ as 4.5 before performing any operations, consider discussing how the calculations are alike and how they are different. Tell students that they will solve more problems involving the area of triangles in this lesson.

Student Workbook

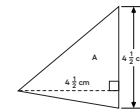
LESSON 14

Fractional Lengths in Triangles and Prisms

Let's explore area and volume when fractions are involved.

Warm-up: Area of Triangle

Find the area of Triangle A in square centimeters. Show your reasoning.



GRADE 6 • UNIT 4 • SECTION D | LESSON 14

14

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.

Invite students to verbalize their strategy for finding the unknown base or height in each triangle before they begin. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Activity 1

Bases and Heights of Triangles

10 min

Activity Narrative

In this activity, students apply their knowledge of division of fractions to find a missing length given the area of a triangle and a fractional base or height.

The formula for the area of a triangle $A = \frac{1}{2} \cdot b \cdot h$ presents a different multiplication situation than students have seen in this unit—there are three factors at play. This means that to find a missing factor, at least one additional step is needed. Students are likely to approach this in a number of ways. They may:

- Draw a duplicate of the triangle and compose a parallelogram with the same base and height, double the given area (to represent the area of the parallelogram), and then divide by the known length to find the unknown length.
- Without drawing, multiply the given area by 2 to find the value of $b \cdot h$, and then divide it by the known length.
- Perform division twice, such as divide the area by $\frac{1}{2}$ and then by the known base or length, or vice versa.
- Multiply the known length and $\frac{1}{2}$ first, so there are only two factors to work with. (For example, to find b in $\frac{1}{2} \cdot b \cdot \frac{8}{3} = 8$, they may write $\frac{1}{2} \cdot \frac{8}{3} \cdot b = 8$ and then $\frac{4}{3} \cdot b = 8$.)

Monitor for these or other approaches as students work. Select students who use different strategies, and ask them to share later.

Launch



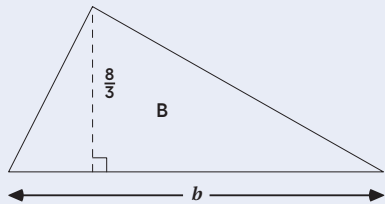
Keep students in groups of 2.

Give students 3–4 minutes of quiet work time and 2 minutes to discuss their responses and complete the activity with their partner.

Keep the formula for the area of a triangle and a labeled drawing of a triangle displayed.

Student Task Statement

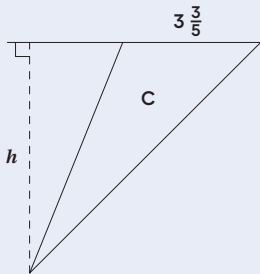
1. The area of Triangle B is 8 square units. Find the length of b . Show your reasoning.



6 units

Sample reasoning: $\frac{1}{2} \cdot \frac{8}{3} = 8$, so $\frac{8}{6} \cdot b = 8$. Dividing 8 by $\frac{8}{6}$ gives $8 \cdot \frac{6}{8}$ or $\frac{48}{8}$, which is 6.

2. The area of Triangle C is $\frac{54}{5}$ square units. What is the length of h ? Show your reasoning.



6 units

Sample reasoning: A parallelogram with a base of $3\frac{3}{5}$ (or $\frac{18}{5}$) and a height h would have an area of $2 \cdot \frac{54}{5}$ or $\frac{108}{5}$ square units. To find h , divide $\frac{108}{5}$ by $\frac{18}{5}$ (or find $\frac{108}{5} \cdot \frac{5}{18}$), which is $\frac{108}{18}$ or 6.

Activity Synthesis

Ask 2–3 previously selected students to share their solutions and reasoning. Consider starting with students who reasoned concretely (such as by duplicating the triangle to compose a parallelogram) and following with those who reasoned symbolically (such as only by manipulating expressions or equations).

Clarify that there is no single way to solve problems such as these. Point out specific points in the solving process where division of fractions enabled them to complete their reasoning. Emphasize that what we learned about fractions and operations in this unit can help us reason more effectively about problems in other areas of mathematics.

Building on Student Thinking

Some students may be unsure how to find the missing length in a triangle because finding the area of a triangle involves two operations. Encourage students to write an equation that shows the relationship between the unknown base, the known height, and the known area of the triangle. Ask students if there is any calculation they can perform first so that they are working with only one operation instead of two.

Access for Multilingual Learners
(Activity 1, Synthesis)

MLR8: Discussion Supports.
Invite students to repeat their reasoning using mathematical language:

“Can you say that again, using words such as ‘compose,’ ‘parallelogram,’ ‘base,’ ‘height,’ and ‘side length’?”

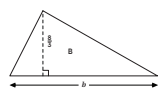
Provide all students with an opportunity to produce this language by inviting students to chorally repeat these words or phrases 1–2 times.

Advances: Speaking, Conversing

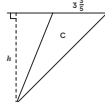
Student Workbook

1. Bases and Heights of Triangles

1 The area of Triangle B is 8 square units. Find the length of b . Show your reasoning.



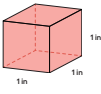
2 The area of Triangle C is $\frac{54}{5}$ square units. What is the length of h ? Show your reasoning.



2. Volumes of Cubes and Prisms

Your teacher will give you cubes that have edge lengths of $\frac{1}{2}$ inch.

1 Here is a drawing of a cube with edge lengths of 1 inch.



a. How many cubes with edge lengths of $\frac{1}{2}$ inch are needed to fill this cube?

Access for Multilingual Learners
(Activity 2, Launch)
MLR2: Collect and Display.

Circulate among the groups to listen for and collect the language that students use to explain how they reason about the volume of a $\frac{1}{2}$ -inch cube and other rectangular prisms. On a visible display, record words and phrases, such as “length,” “width,” “height,” “cubes packed into the prism,” “cubic inches,” “multiply the number of cubes by the volume of 1 cube,” and “multiply the edge lengths.” Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

Activity 2
Volumes of Cubes and Prisms
20
min

Activity Narrative
There is a digital version of this activity.

In this activity, students find the volume of a prism with fractional edge lengths. In an earlier grade, students learned that the volume of a rectangular prism with whole-number edge lengths can be found by computing the number of unit cubes that can be packed into the prism. Here, the cubes have a unit fraction (such as $\frac{1}{2}$ or $\frac{1}{4}$) for their edge lengths. Students calculate the number of such cubes that can be packed into a rectangular prism and use that number to find the volume in a standard unit of volume measurement (cubic inches, in this case).

By reasoning repeatedly with small cubes (with $\frac{1}{2}$ -inch edge lengths), students notice regularity: that the volume of a rectangular prism with fractional edge lengths can also be found by directly multiplying the edge lengths in inches.

This activity works best when each student has access to physical cubes because students will benefit from manipulating the cubes and building prisms by hand.

If physical manipulatives are not available, consider using the digital version of the activity. In the digital version, students use an applet to build rectangular prisms from cubes with edge lengths of $\frac{1}{2}$ unit. The applet allows students to measure the edge lengths of the resulting prisms.

The digital version is adapted from an applet made in GeoGebra by Susan Addington.

Launch

Display the image of the 1-inch cube for all to see. Ask students:

☞ “This cube has an edge length of 1 inch. What is its volume in cubic inches?”

1 cubic inch

☞ “How do you know?”

$1 \cdot 1 \cdot 1 = 1$

☞ “How do we find the volume of a cube with an edge length of 2 inches?”

$2 \cdot 2 \cdot 2 = 8$. We can pack the cube with eight 1-inch cubes, so its volume is $8 \cdot 1$, or 8, cubic inches.

If no students mentioned using the 1-inch cube to find the volume of the 2-inch cube, bring it up. Consider telling students that we can call a cube with an edge length of 1 inch a “1-inch cube.”

Arrange students in groups of 3–4. Give each group 20 cubes and 2 minutes to complete the first set of questions. Ask them to pause for a brief class discussion afterward.

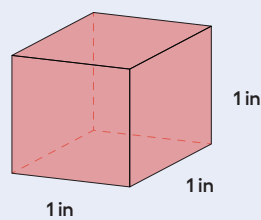
Invite students to share how they found the volume of a cube with $\frac{1}{2}$ -inch edge lengths and the prism composed of 4 stacked cubes. For the $\frac{1}{2}$ -inch cube, if students do not mention one of the two ways shown in the *Student Response*, bring it to the students’ attention. For the tower, if they don’t mention multiplying the volume of a $\frac{1}{2}$ -inch cube, which is $\frac{1}{8}$ cubic inch, ask if that is a possible way to find the volume of the prism.

🕒 Next, give students **8–10 minutes** to complete the rest of the activity.

Student Task Statement

Your teacher will give you cubes that have edge lengths of $\frac{1}{2}$ inch.

1. Here is a drawing of a cube with edge lengths of 1 inch.



a. How many cubes with edge lengths of $\frac{1}{2}$ inch are needed to fill this cube?

8 cubes

b. What is the volume, in cubic inches, of a cube with edge lengths of $\frac{1}{2}$ inch? Explain or show your reasoning.

$\frac{1}{8} \text{ in}^3$

Sample reasoning:

- Because 8 of these $\frac{1}{2}$ -inch cubes fit in 1 in^3 , each cube is $\frac{1}{8} \text{ in}^3$.
- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

2. Four cubes are piled in a single stack to make a prism. Each cube has an edge length of $\frac{1}{2}$ inch. Sketch the prism, and find its volume in cubic inches.

$\frac{4}{8} \text{ in}^3$ (or $\frac{1}{2} \text{ in}^3$)

Student Workbook

1. Bases and Heights of Triangles

1. The area of Triangle B is 8 square units. Find the length of h . Show your reasoning.

2. The area of Triangle C is $\frac{15}{2}$ square units. What is the length of h ? Show your reasoning.

2. Volumes of Cubes and Prisms

Your teacher will give you cubes that have edge lengths of $\frac{1}{2}$ inch.

1. Here is a drawing of a cube with edge lengths of 1 inch.

a. How many cubes with edge lengths of $\frac{1}{2}$ inch are needed to fill this cube?

Student Workbook

2. Volumes of Cubes and Prisms

b. What is the volume, in cubic inches, of a cube with edge lengths of $\frac{1}{2}$ inch? Explain or show your reasoning.

3. Four cubes are piled in a single stack to make a prism. Each cube has an edge length of $\frac{1}{2}$ inch. Sketch the prism, and find its volume in cubic inches.

4. Use cubes with an edge length of $\frac{1}{2}$ inch to build prisms with the lengths, widths, and heights shown in the table.

a. For each prism, record in the table how many $\frac{1}{2}$ -inch cubes can be packed into the prism and the volume of the prism.

| prism length (in) | prism width (in) | prism height (in) | number of $\frac{1}{2}$ -inch cubes in prism | volume of prism (in ³) |
|-------------------|------------------|-------------------|--|------------------------------------|
| $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | | |
| 1 | 1 | $\frac{1}{2}$ | | |
| 2 | 1 | $\frac{1}{2}$ | | |
| 2 | 2 | 1 | | |
| 4 | 2 | $\frac{1}{2}$ | | |
| 5 | 4 | 2 | | |
| 5 | 4 | $2\frac{1}{2}$ | | |

Student Workbook

Volumes of Cubes and Prisms

Are You Ready for More?

A unit fraction has a 1 in the numerator.
• These are unit fractions: $\frac{1}{3}, \frac{1}{10}, \frac{1}{100}$.
• These are not unit fractions: $\frac{2}{3}, \frac{8}{1}, 2\frac{1}{5}$.

1

Find three unit fractions whose sum is $\frac{1}{2}$. An example is: $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$.
How many examples like this can you find?

2

Find a box whose surface area in square units equals its volume in cubic units. How many like this can you find?

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3. Use cubes with an edge length of $\frac{1}{2}$ inch to build prisms with the lengths, widths, and heights shown in the table.
- a. For each prism, record in the table how many $\frac{1}{2}$ -inch cubes can be packed into the prism and the volume of the prism.

| prism length (in) | prism width (in) | prism height (in) | number of $\frac{1}{2}$ -inch cubes in prism | volume of prism (in ³) |
|-------------------|------------------|-------------------|--|------------------------------------|
| $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | $\frac{1}{8}$ |
| 1 | 1 | $\frac{1}{2}$ | 4 | $\frac{1}{2}$ |
| 2 | 1 | $\frac{1}{2}$ | 8 | 1 |
| 2 | 2 | 1 | 32 | 4 |
| 4 | 2 | $\frac{3}{2}$ | 96 | 12 |
| 5 | 4 | 2 | 320 | 40 |
| 5 | 4 | $2\frac{1}{2}$ | 400 | 50 |

Are You Ready for More?

A unit fraction has a 1 in the numerator.

- These are unit fractions: $\frac{1}{3}, \frac{1}{100}, \frac{1}{1}$.
- These are *not* unit fractions: $\frac{2}{9}, \frac{8}{1}, 2\frac{1}{5}$.

1. Find three unit fractions whose sum is $\frac{1}{2}$. An example is: $\frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$. How many examples like this can you find?

Sample response:

- $\frac{1}{3} + \frac{1}{12} + \frac{1}{12}$
- $\frac{1}{5} + \frac{1}{5} + \frac{1}{10}$
- $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

2. Find a box whose surface area in square units equals its volume in cubic units. How many like this can you find?

The denominators of the fractions that work for the first question can be used as the length, width, and height of the box.

Activity Synthesis

The purpose of the discussion is to highlight that the volume of a rectangular prism can be found by:

- Packing it with cubes with no gaps and multiplying the number of cubes by the volume of each cube.
- Multiplying the edge lengths of the prism.

Display a completed table for all to see. Give students a minute to check their responses. Invite a few students to share how they determined the volume of the prisms.

If no students mention using the number and volume of the $\frac{1}{2}$ -inch cubes, ask them to observe the relationship between the values in the last two columns of the table (the number of cubes with $\frac{1}{2}$ -inch edge lengths and the volume of prisms in cubic inches). Highlight that the volume can be found by calculating the number of cubes and multiplying it by $\frac{1}{8}$, because each small cube has a volume of $\frac{1}{8}$ cubic inch.

If no students mention finding the product of the edge lengths in inches, draw students' attention to the values in the first three columns of the table and in the last column. (Looking at the whole-number values in the second, fourth, and sixth rows of the table may be particularly helpful.) Make sure students see that the volume of each prism can also be found by multiplying its side lengths in inches.

Lesson Synthesis

To summarize ways to find the volume of a rectangular prism, display and read the following pair of problems:

- A rectangular prism measures 1 inch by 2 inches by 3 inches. What is its volume?
- A rectangular prism measures $\frac{1}{2}$ inch by 1 inch by $\frac{3}{2}$ inches. What is its volume?

Give students a minute to consider how to solve each problem, then ask questions such as:

☞ *"How are two problems different?"*

The prisms are different sizes. One prism has edge lengths that are whole numbers. The other has edge lengths that are fractions.

☞ *"Can we find the volume of both prisms by seeing how many cubes can be packed into them?"*

Yes

☞ *"Can we use 1-inch cubes for both prisms?"*

No, we'll need to use $\frac{1}{2}$ -inch cubes for the second prism.

☞ *"Is there another way to find the volume of each prism?"*

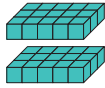
Yes, we can multiply the edge lengths.

If time allows, consider summarizing how division can be used to find an unknown base or height of a triangle. Display and read the following pair of problems:

Student Workbook

Lesson Summary

If a rectangular prism has edge lengths of 2 units, 3 units, and 5 units, we can think of it as 2 layers of unit cubes, with each layer having $(3 \cdot 5)$ unit cubes in it. So the volume, in cubic units, is $2 \cdot 3 \cdot 5$.



To find the volume of a rectangular prism with fractional edge lengths, we can think of it as being built of cubes that have a unit fraction for their edge length. For instance, if we build a prism that is $\frac{1}{2}$ -inch tall, $\frac{3}{2}$ -inch wide, and 4 inches long using cubes with a $\frac{1}{2}$ -inch edge length, we would have:

- A height of 1 cube, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
- A width of 3 cubes, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
- A length of 8 cubes, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism would be $1 \cdot 3 \cdot 8$, which is 24 cubic units.

How do we find its volume in cubic inches? We know that each cube with a $\frac{1}{2}$ -inch edge length has a volume of $\frac{1}{8}$ cubic inch, because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Since the prism is built using 24 of these cubes, its volume, in cubic inches, would then be $24 \cdot \frac{1}{8}$, which is 3 cubic inches.

The volume of the prism, in cubic inches, can also be found by multiplying the fractional edge lengths in inches: $\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$.

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- A rectangle has an area of 9 square inches and a height of $\frac{3}{2}$ inches. How long is its base?
- A triangle has an area of 9 square inches and a height of $\frac{3}{2}$ inches. How long is its base?

Give students a minute to consider how they can solve each problem, then ask questions such as:

“How would you find the base of the rectangle?”

Divide 9 by $\frac{3}{2}$

“Why does it make sense to divide?”

Because $\text{base} \cdot \frac{3}{2} = 9$

“Would dividing 9 by $\frac{3}{2}$ also give the base of the triangle? Why or why not?”

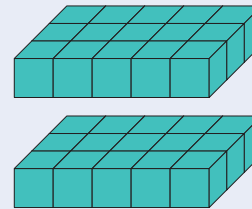
No, because the area of the triangle is $\frac{1}{2} \cdot \text{base} \cdot \frac{3}{2}$.

“How can we find the base of the triangle?”

Divide 9 by $\frac{3}{2}$ and then divide the result by $\frac{1}{2}$, or multiply it by 2.

Lesson Summary

If a rectangular prism has edge lengths of 2 units, 3 units, and 5 units, we can think of it as 2 layers of unit cubes, with each layer having $(3 \cdot 5)$ unit cubes in it. So the volume, in cubic units, is: $2 \cdot 3 \cdot 5$



To find the volume of a rectangular prism with fractional edge lengths, we can think of it as being built of cubes that have a unit fraction for their edge length. For instance, if we build a prism that is $\frac{1}{2}$ -inch tall, $\frac{3}{2}$ -inch wide, and 4 inches long using cubes with a $\frac{1}{2}$ -inch edge length, we would have:

- A height of 1 cube, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
- A width of 3 cubes, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
- A length of 8 cubes, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism would be $1 \cdot 3 \cdot 8$, which is 24 cubic units.

How do we find its volume in cubic inches? We know that each cube with a $\frac{1}{2}$ -inch edge length has a volume of $\frac{1}{8}$ cubic inch, because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

Since the prism is built using 24 of these cubes, its volume, in cubic inches, would then be $24 \cdot \frac{1}{8}$, which is 3 cubic inches.

The volume of the prism, in cubic inches, can also be found by multiplying the fractional edge lengths in inches:

$$\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$$

Cool-down

Triangles and Cubes

5
min

If time is limited, ask students to complete only the first set of questions.

Student Task Statement

1. a. How many cubes with edge lengths of $\frac{1}{3}$ inch are needed to build a cube with an edge length of 1 inch?

27 cubes

- b. What is the volume, in cubic inches, of one cube with an edge length of $\frac{1}{3}$ inch?

$\frac{1}{27} \text{ in}^3$

2. A triangle has a base of $3\frac{2}{5}$ (or $\frac{17}{5}$) inches and an area of $5\frac{1}{10}$ (or $\frac{51}{10}$) square inches. Find the height of the triangle. Show your reasoning.

3 inches

Sample reasoning: $\frac{1}{2} \cdot \frac{17}{5} \cdot h = \frac{51}{10}$, so $\frac{17}{10} \cdot h = \frac{51}{10}$. There are 3 groups of $\frac{17}{10}$ in $\frac{51}{10}$.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

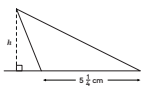
8 Problems

Student Workbook

LESSON 14
PRACTICE PROBLEMS

1. Clare is using little wooden cubes with edge lengths of $\frac{1}{2}$ inch to build a larger cube that has edge lengths of 4 inches. How many little cubes does she need? Explain your reasoning.

2. The triangle has an area of $7\frac{7}{8}$ cm² and a base of $5\frac{1}{4}$ cm.



What is the length of h ? Explain your reasoning.

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Student Workbook

Practice Problems

1. Which expression can be used to find how many cubes with edge lengths of $\frac{1}{3}$ unit fit in a prism that is 5 units by 5 units by 8 units?

- ☐ A. $(5 \cdot \frac{1}{3}) \cdot (5 \cdot \frac{1}{3}) \cdot (8 \cdot \frac{1}{3})$
☐ B. $5 \cdot 5 \cdot 8$
☐ C. $(5 \cdot 3) \cdot (5 \cdot 3) \cdot (8 \cdot 3)$
☐ D. $(5 \cdot 5 \cdot 8) \cdot (\frac{1}{3})$

Explain or show your reasoning.

2. Mai says that we can also find the answer by multiplying the edge lengths of the prism and then multiplying the result by 27. Do you agree with her? Explain your reasoning.

3. From Unit 4, Lesson 12
A builder is building a fence with $\frac{1}{2}$ -inch-wide wooden boards, arranged side-by-side with no gaps or overlaps. How many boards are needed to build a fence that is 150 inches long? Show your reasoning.

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Problem 1

Clare is using little wooden cubes with edge lengths of $\frac{1}{2}$ inch to build a larger cube that has edge lengths of 4 inches. How many little cubes does she need? Explain your reasoning.

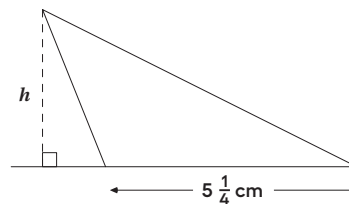
512

Sample reasoning: There are 8 half inches in 4 inches, so Clare needs $8 \cdot 8 \cdot 8$ little cubes. $8 \cdot 8 \cdot 8 = 512$

Problem 2

The triangle has an area of $7\frac{7}{8}$ cm² and a base of $5\frac{1}{4}$ cm.

What is the length of h ? Explain your reasoning.



3 cm

Sample reasoning: One half of the base ($2\frac{5}{8}$ cm) times the height is $7\frac{7}{8}$ cm². So the height in cm is $(7\frac{7}{8}) \div (2\frac{5}{8}) = 3$.

Problem 3

- a. Which expression can be used to find how many cubes with edge lengths of $\frac{1}{3}$ unit fit in a prism that is 5 units by 5 units by 8 units? Explain or show your reasoning.

- A. $(5 \cdot \frac{1}{3}) \cdot (5 \cdot \frac{1}{3}) \cdot (8 \cdot \frac{1}{3})$
 B. $5 \cdot 5 \cdot 8$
 C. $(5 \cdot 3) \cdot (5 \cdot 3) \cdot (8 \cdot 3)$
 D. $(5 \cdot 5 \cdot 8) \cdot (\frac{1}{3})$

Sample reasoning: It takes three $\frac{1}{3}$ units to make 1 unit. In terms of the edge lengths of the small cube, the prism is 15 by 15 by 24.

- b. Mai says that we can also find the answer by multiplying the edge lengths of the prism and then multiplying the result by 27. Do you agree with her? Explain your reasoning.

Mai is correct

Sample reasoning: Because it takes three $\frac{1}{3}$ units to make 1 unit, it takes $3 \cdot 3 \cdot 3$ cubes with edge lengths of $\frac{1}{3}$ unit to make one cube with edge length 1 unit.

Problem 4

from Unit 4, Lesson 12

A builder is building a fence with $6\frac{1}{4}$ -inch-wide wooden boards, arranged side-by-side with no gaps or overlaps. How many boards are needed to build a fence that is 150 inches long? Show your reasoning.

24 boards

Sample reasoning: $150 \div 6\frac{1}{4} = 150 \cdot \frac{4}{25} = 24$

Problem 5

from Unit 4, Lesson 12

Find the value of each expression. Show your reasoning and check your answer.

a. $2\frac{1}{7} \div \frac{2}{7}$
 $\frac{15}{2}$ or $7\frac{1}{2}$

b. $\frac{17}{20} \div \frac{1}{4}$
 $\frac{17}{5}$ or $3\frac{2}{5}$

Problem 6

from Unit 4, Lesson 11

A bucket contains $11\frac{2}{3}$ gallons of water and is $\frac{5}{6}$ full. How many gallons of water would be in a full bucket?

a. Write a multiplication and a division equation to represent the situation.
 $\frac{5}{6} \cdot ? = 11\frac{2}{3}$ and $11\frac{2}{3} \div \frac{5}{6} = ?$

b. Find the answer and show your reasoning.
14 gallons

Sample reasoning: $11\frac{2}{3} \div \frac{5}{6} = \frac{35}{3} \cdot \frac{6}{5}$, which equals 14.

Problem 7

from Unit 3, Lesson 12

There are 80 kids in a gym. 75% are wearing socks. How many are *not* wearing socks? If you get stuck, consider using a tape diagram.

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Sample reasoning: if 75% are wearing socks, then 25% are not wearing socks. 25% of a number is the same as $\frac{1}{4}$ of the number, and $\frac{1}{4}$ of 80 is 20.

Problem 8

from Unit 3, Lesson 11

a. The regular fare for a train ride between two cities is \$75 per person. During a promotion, train riders can save \$37.50 each. What percentage of the regular fare is the discount?

The discount is 50% of the regular fare.

Sample reasoning: 37.50 is half of 75.

b. Noah wants to save \$60. If he has saved \$45 so far, what percentage of his goal has he saved?

He has saved 75% of his goal so far.

Sample reasoning: $\frac{3}{4}$ of 60 is 45, so 45 is 75%.

Student Workbook

14 Practice Problems

from Unit 4, Lesson 12
Find the value of each expression. Show your reasoning and check your answer.

a. $2\frac{1}{7} \div \frac{2}{7}$

b. $\frac{17}{20} \div \frac{1}{4}$

from Unit 4, Lesson 11

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a. Write a multiplication and a division equation to represent the situation.

b. Find the answer and show your reasoning.

from Unit 3, Lesson 12

There are 80 kids in a gym. 75% are wearing socks. How many are not wearing socks? If you get stuck, consider using a tape diagram.

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Student Workbook

14 Practice Problems

from Unit 3, Lesson 11

a. The regular fare for a train ride between two cities is \$75 per person. During a promotion, train riders can save \$37.50 each. What percentage of the regular fare is the discount?

b. Noah wants to save \$60. If he has saved \$45 so far, what percentage of his goal has he saved?



Learning Targets

- + I can explain how to find the volume of a rectangular prism using cubes that have a unit fraction as their edge length.
- + I can use division and multiplication to solve problems involving areas of triangles with fractional bases and heights.
- + I know how to find the volume of a rectangular prism even when the edge lengths are not whole numbers.

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