Representing Percentages with Double Number Line Diagrams

Goals

- Comprehend a phrase like "A% of B" (in written and spoken language) to refer to the value that makes a ratio with B that is equivalent to A: 100.
- Explain (orally) how to use a double number line diagram to solve problems such as A of B is ? and A of ? is C.
- Interpret statements about percentages and identify the quantity that corresponds to 100%.

Learning Target

I can use double number line diagrams to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?"

Access for Students with Diverse Abilities

• Engagement (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR6: Three Reads (Activity 3)

Instructional Routines

- 5 Practices
- MLR5: Co-Craft Questions
- MLR6: Three Reads

Required Materials

Materials to Gather

 Tools for creating a visual display: Activity 2

Lesson Narrative

In this lesson, students explore percentages of quantities other than 100 and 1 and in a variety of contexts. In all activities, the percentages describe multiplicative comparisons of two quantities, rather than part-to-whole relationships.

Students reason about percentages when the value for 100% is known (such as finding 75% of \$10) and about the value of 100% when a percentage is known (such as finding a weight given that 10% of that weight is 9 pounds). The percentages in all problems are limited to multiples of 10% and 25%.

The optional activity (about fundraising goals) gives students another opportunity to find percentages in a money context before reasoning about values that correspond to 100% in other contexts.

Blank or partially labeled double number line diagrams are provided to support students in their reasoning. This is done for several reasons:

- Using a double number line diagram involves identifying the value that corresponds to 100%, reinforcing the idea of a percentage as a rate per 100.
- The equal intervals on the two number lines encourage students to reason about equivalent ratios.



10 min

Warm-up

10 min

Activity 1

15 min

Activity 2

10 min

Activity 3

10

Lesson Synthesis

Assessment

5 min

Cool-down

Representing Percentages with Double Number Line Diagrams

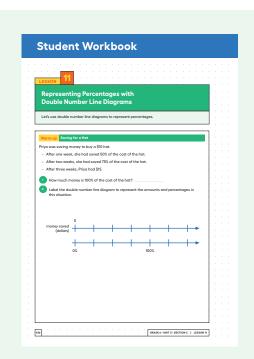
Lesson Narrative (continued)

• Thinking about how to extend or further partition the given intervals to represent a situation also prompts students to look for and make use of structure.

It is perfectly acceptable, however, for students to choose other representations or use other strategies for solving percentage problems.

Student Learning Goal

Let's use double number line diagrams to represent percentages.



Warm-up

Saving for a Hat



Activity Narrative

In this *Warm-up*, students make sense of percentages when 100% corresponds to 10. Students can likely reason mentally about 50% of 10, 75% of 10, and relate 15 to 10, but are prompted to use a double number line diagram to represent the situation. Doing so encourages them to see percentages as rates and in terms of equivalent ratios. It also promotes the use of a double number line diagram as a tool for reasoning about percentages, which will be useful in subsequent activities and lessons.

The percentages are limited to multiples of 25 and 50, and the number lines are partitioned but only partially labeled. As students represent quantities and identify percentages on the diagram, they practice reasoning quantitatively and abstractly.

Launch 22

Remind students that in the previous lesson, we found percentages of 100 and of 1 using double number line diagrams. Explain that in this lesson we will find percentages of other numbers.

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time, followed by 1–2 minutes to discuss their responses with a partner.

Student Task Statement

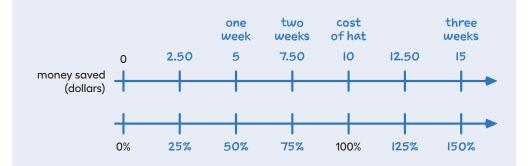
Priya was saving money to buy a \$10 hat.

- After one week, she had saved 50% of the cost of the hat.
- After two weeks, she had saved 75% of the cost of the hat.
- After three weeks, Priya had \$15.
- 1. How much money is 100% of the cost of the hat?

\$10

2. Label the double number line diagram to represent the amounts and percentages in this situation.

Sample response:



Activity Synthesis

This discussion is to highlight two ideas: that 100% can correspond to an amount other than 100 or 1, and that percentage as a rate per 100 still applies just the same.

Display the blank diagram for all to see. Invite students to share how they found the percentages and how they went about labeling the diagram to represent the situation. Annotate the diagram based on students' explanations. Make sure students see that in this case 100% corresponds to \$10, which is the cost of the hat and the target amount that Priya wanted to have.

Then, discuss how the definition of "rate per 100" is still in play when comparing an amount to 10 units. If 100% of the cost of the hat is \$10, then:

- 50% of the cost is \$5 because the ratio of \$5 to \$10 is equivalent to 50 to 100.
- 75% of the cost is \$7.50 because the ratio of \$7.50 to \$10 is equivalent to 75 to 100.
- 150% of the cost is \$15 because the ratio of \$15 to \$10 is equivalent to 150 to 100.

Consider referring to the completed double number line diagram to illustrate these equivalent ratios.

Activity 1: Optional

Fundraising Goal

10 min

Activity Narrative

In this activity, students continue to reason about percentages where 100% corresponds to a value other than 100 or 1. Blank double number line diagrams are given to support students' thinking, but because the percentages are multiples of 50, students may find it intuitive to reason without diagrams. For instance, to find the amount that Tyler raised (150% of his goal), they may:

- Add the values of 50% of \$40 and 100% of \$40.
- Reason that 150 is $3 \cdot 50$ and triple the value of 50% of \$40, or find $3 \cdot 20$.
- Reason that 150 is (1.5) · 100 and find (1.5) · 40.

Monitor for students who use these and other ways of reasoning and consider inviting them to share during discussion.

Launch



Arrange students in groups of 2. Introduce the context of fundraising and goal setting. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Display only the stem of the first problem (Lin, Diego, and Tyler's goals and what percent each person raised), without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 1)

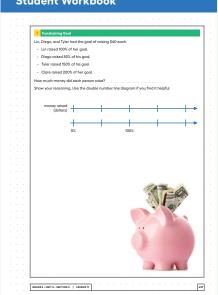
MLR5: Co-Craft Questions.

This activity uses *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Building on Student Thinking

If students are unsure how to interpret percentages greater than 100% in this situation, ask them to compare the dollar amounts that Tyler and Clare each raised to the goal. Clarify that percentages such as 150% and 200% in this case tell us that the amounts raised exceeded the goal.

Student Workbook



☐ "What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "finding percentages of \$40" or "finding 50% (or 100%, 150%, or 200%) of the goal."

Reveal the question "How much money did each person raise?" and give students 1 minute to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking,

"Which of your questions is most similar to or different from the ones provided? Why?"

Give students 2–3 minutes to complete the activity.

Student Task Statement

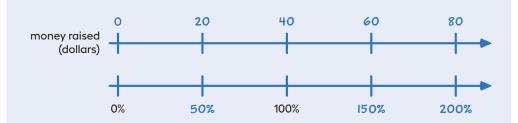
Lin, Diego, and Tyler had the goal of raising \$40 each.

- Lin raised 100% of her goal.
- Diego raised 50% of his goal.
- Tyler raised 150% of his goal.
- Clare raised 200% of her goal.

How much money did each person raise?

Lin raised \$40, Diego raised \$20, Tyler raised \$60, Clare raised \$80

Show your reasoning. Use the double number line diagram if you find it helpful.



Activity Synthesis

Focus the discussion on how students found 150% and 200% of \$40. Invite students who reasoned with and without double number line diagrams to share.

If not mentioned in students' explanations, emphasize that we are finding percentages of \$40, since this number—not 100 cents, 1 dollar, or 10 dollars—is the fundraising goal for the three friends. Since 100% of a goal of \$40 is \$40, the 100% and \$40 are lined up on the double number line diagram. To highlight these correspondences, display and annotate them on the double number line diagrams (as shown in the Student Response).

Activity 2

Three-Day Biking Trip



Activity Narrative

In this activity, students find percentages of a value in the context of distance. They begin by relating a given value to 100% and reasoning about other percentages. Then students reason in the other direction: given a percentage and a corresponding distance, students determine the number of miles that corresponds to 100%. The context does not explicitly state that the value being sought (the distance that Han's uncle biked) is the value for 100%, so students need to first make that connection.

Double number line diagrams are provided to support students' thinking and to encourage them to reason in terms of equivalent ratios, but students may also use other representations and strategies.

Monitor for the different approaches students take for each set of questions. Here are some ways in which students may answer the first set of questions, from less direct to more direct:

- Align 8 miles to 100% on the number line, partition the bottom number line into intervals of 25% (100 ÷ 4), find the corresponding intervals of 2 (8 ÷ 4) on the top line, and skip count by that value to find distances for 75% and 125%.
- Reason about 75% and 125% of the distance as 25% less and 25% more than 100% of the distance, which means subtracting $\frac{1}{4} \cdot 8$ from 8 and adding $\frac{1}{4} \cdot 8$ to 8, respectively.
- Reason about 75% of 8 directly by multiplying 8 by $\frac{3}{4}$ and 125% of 8 by multiplying 8 by $\frac{5}{4}$

Here are some ways in which they may answer the second question:

- Label the top number line with multiples of 3 and the bottom number line with multiples of 50, and see what distance aligns with 100%.
- Double the 3 to find the distance Han's uncle biked because 100 is $2 \cdot 50$.

The diagrams are partially marked and labeled. As students partition and label the diagrams to represent the quantities and percentages in the situation, they practice reasoning abstractly and quantitatively and attending to precision.

Launch



Keep students in groups of 2. Ask students if they have traveled a long distance on foot or on wheels (such as on a bicycle, scooter, skateboard, wheelchair, or roller skates) before. Ask them to estimate the longest distance they have traveled in one day—not in a car, bus, or train—and to tell their partner about it. Invite a few students to share. Tell students they will now look at situations about distances biked in one day.

Give students 3–4 minutes of quiet work time and time to discuss their responses with their partner. Encourage students to use the double number line diagram to help them answer the questions if needed.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially those of students who haven't shared recently.

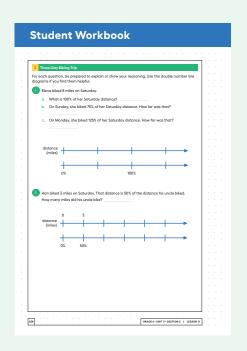
Instructional Routines

5 Practices

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Student Workbook 2 Three-Day Biking Trip Let's You Ready for More? In July 2012, Austrian cycles Christoph Stosser broke the record for the distance bleed in one day for distances was ready if 200% of Servick Soutubry distance. Bleed in one day for distances was ready if 200% of Servick Soutubry distance. About how many misss did for size if Septem or show flow you brown. 3 Puppins Grew Up 1 Dade has a new puppy that weight 9 pounds. The vest soys that the puppy is now at about 20% of its adde was lower pupping. What will be the adult weight of the puppy? Weight [pounds] OX 20%

Student Task Statement

For each question, be prepared to explain or show your reasoning. Use the double number line diagrams if you find them helpful.

- 1. Elena biked 8 miles on Saturday.
 - a. What is 100% of her Saturday distance?

8 miles

Sample reasoning: That's how far she biked in total.

- **b.** On Sunday, she biked 75% of her Saturday distance. How far was that?
 - 6 miles

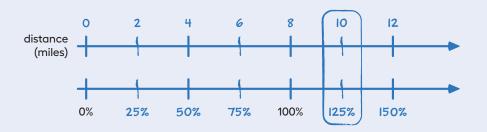
Sample reasoning: 75% of 8 is 6.

c. On Monday, she biked 125% of her Saturday distance. How far was that?

10 miles

Sample reasoning:

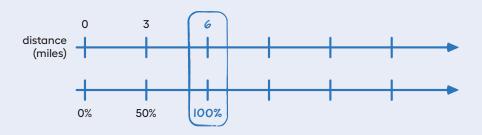
o On the diagram, 125% lines up with 10 miles.



- Every 25% is 2 miles, so 125%, which is 25% more than 100%, is 2 miles more than 8 miles, or 8 + 2, which is 10.
- **2.** Han biked 3 miles on Saturday. That distance is 50% of the distance his uncle biked. How many miles did his uncle bike?

6 miles

Sample reasoning: Han's distance is half of his uncle's distance, so his uncle's distance is twice 3 miles, which is 6 miles.



Are You Ready for More?

In July 2021, Austrian cyclist Christoph Strasser broke the record for the distance biked in one day. That distance was nearly 8,000% of Elena's Saturday distance. About how many miles did he ride? Explain or show how you know.

About 640 miles

Sample reasoning: 8,000 is $80 \cdot 100$. If 100% of Elena's distance is 8 miles, then 8,000% of Elena's distance is $80 \cdot 8$ or 640 miles.

Activity Synthesis

One key idea to highlight is that the value assigned to 100% can vary depending on the comparisons being made and the questions being asked in a situation.

Invite previously selected students to share their strategies for finding 75% and 125% of Elena's biking distance on Saturday and for finding Han's uncle biking distance. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

☐ "In Elena's case, how did you know what distance corresponds to 100%?"

"In Han's case, how did you know that to find the distance that Han's uncle biked you were looking for a value that corresponds to 100%?"

"We saw several ways to find 125% of Elena's Saturday distance. What is the same about the strategies? What is different?"

Activity 3

Puppies Grow Up

10 min

Activity Narrative

This activity offers another opportunity for students to find 100% of quantities given other percentages. As in an earlier activity, students are not told that the values being sought correspond to 100%, so they will need to interpret each situation and make that connection.

Double number line diagrams are provided as a reasoning tool, but students may also use a table of equivalent ratios or other methods. Those who use double number line diagrams are likely to find them effective for the first question (find 100% of a quantity given 20%) but less straightforward for the second question (find 100% of a quantity given 30%). Since 100 is not a multiple of 30, students may partition the double number line into intervals of 10% and scale up from there to find the value of 100%. They may also divide 9 by 3 and then multiply by 10, without using the diagram. (Some may multiply 9 by $\frac{10}{3}$ directly, though this is less likely.)

The last question prompts students to make a quick comparison about the two situations. Students' responses don't need to be fully formed for class discussions to begin.

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Internalize Self-Regulation.

Provide students with an opportunity to self-assess and reflect on their own sense-making process. Ask them how they interpreted the given information and figured out what 100% represents in each situation.

Supports accessibility for: Organization, Conceptual Processing

Instructional Routines

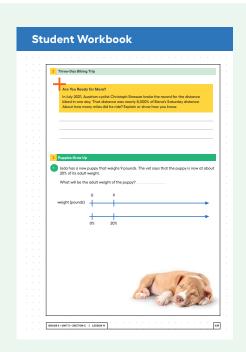
MLR6: Three Reads ilclass.com/r/10695568



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Access for Multilingual Learners (Activity 3)

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.



Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the diagram about Jada's puppy, without revealing the questions.

- For the first read, read the problem aloud then ask,
- "What is this situation about?"

Jada's puppy's visit to the vet, the puppy's weight

Listen for and clarify any questions about the context.

 After the second read, ask students to list any quantities that can be counted or measured

its weight now, how its weight now compares to its adult weight.

- After the third read, reveal the question:
- "What will be the adult weight of the puppy?"
 and ask,
- ☐ "What are some ways we might get started on this?"

Invite students to name some possible starting points, referencing quantities from the second read.

mark and label the double number lines to find the weight that lines up with 100%, skip count by 9 five times since there are five 20% in 100%, multiply 9 by 5.

If not mentioned by students, highlight that in this situation, the puppy's weight is expressed as a percentage of the adult weight, so 100% refers to the adult weight.

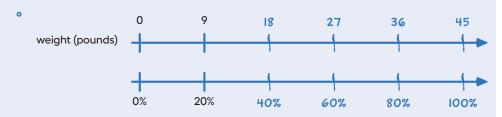
Arrange students in groups of 2. Give students 3–4 minutes of quiet think time and then time to share their responses with their partner.

Student Task Statement

- **1.** Jada has a new puppy that weighs 9 pounds. The vet says that the puppy is now at about 20% of its adult weight.
 - What will be the adult weight of the puppy?

45 pounds

Sample reasoning:



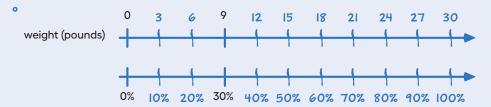
- Since 100 is 5 times 20, multiply 9 by 5 to get 45.
- If 20% is 9 pounds, then 10% is 4.5 pounds and 100% is 10 times 4.5, which is 45 pounds.

2. Andre also has a puppy that weighs 9 pounds. The vet says that this puppy is now at about 30% of its adult weight.

What will be the adult weight of Andre's puppy?

30 pounds

Sample reasoning:



- If 30% of the adult weight is 9 pounds, then 10% is 3 pounds ($9 \div 3 = 3$) and 100% is 10 times that 10%, so it is 10 · 3, which is 30 pounds.
- **3.** What is the same about Jada's and Andre's puppies? What is different? Sample responses:
 - Both puppies weigh the same right now. The puppies will weigh different amounts when they are adults.
 - The puppies are both 9 pounds now, but this weight represents different percentages of their adult weight.

Activity Synthesis

One key idea to emphasize here is that reasoning about percentages is like reasoning about equivalent ratios.

Invite students to share their responses and reasoning. For each question, start with strategies that involve double number lines and follow with strategies that might be more efficient. Keep the diagrams displayed for all to see and to refer to throughout discussion.

Highlight that the reasoning involved here is comparable to finding equivalent ratios in an earlier unit. For Jada's puppy, we are looking for a ratio of weight in pounds to percentage, _____:100 that is equivalent to 9:20. For Andre's puppy, we are looking for _____:100 that is equivalent to 9:30.

Point out that the same amount (9 pounds, in this case) can represent different percentages depending on the amount to which it is compared or the value for 100%.

- If the value for 100% is 45, then 9 is 20% of that value.
- If the value for 100% is 30, then 9 is 30% of that value.

Building on Student Thinking

Students may see that the adult weight of Andre's puppy corresponds to 100% but may be unsure how to find its value since 100 is not a multiple of 30. Consider asking:

"If you know that 30% of the adult weight corresponds to 9 pounds, what are some other percentages you can find?"

15% or half of 30%, 10% or a third of 30%, and 60% or twice 30%

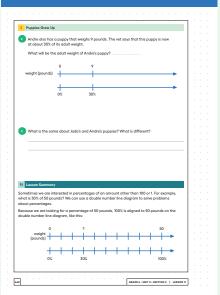
"How can you show those on the double number line diagram?"

partition the interval between 0 and 30% and 0 and 9 into halves for 15% or thirds for 10%, double the interval for 60%

"Which of those percentages can help you find 100%?"

10%

Student Workbook



Lesson Synthesis

Some important ideas to highlight are:

- We can talk about percentages of a quantity as "rates per 100" even when the value of that quantity is not 100.
- To find percentages of a number, n, is to assign 100% to n and look for ratios that are equivalent to n to 100.
- Double number line diagrams can help us see these equivalent ratios and reason about percentages of a quantity.

Display a partially labeled double number line diagram as shown.



Consider asking students:

- "What information does the diagram give us?"
 100% of 80 pounds is 80 pounds. 0% of 80 pounds is 0 pounds.
- O "How can we use the diagram to help us find 10% of 80?"

Partition the interval between 0 and 80 and between 0% and 100% into 10 equal parts. Label each top tick mark with increments of 8, which is $80 \div 10$, and label the bottom tick mark with increments of 10%, which is $100 \div 10$. Find the weight in pounds that aligns vertically with 10%.

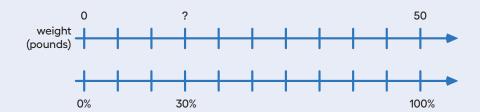
- \bigcirc "How can we find 40% of 80 pounds?"
 - Look for the weight in pounds that aligns vertically with 40%
- Can it help us find 130% of 80 pounds? If so, how?"

Yes. We can continue to add increments of 10% on the bottom line, past 100%, and increments of 8 on the top line, past 80 pounds. Then, we look for the weight in pounds that aligns to 130%.

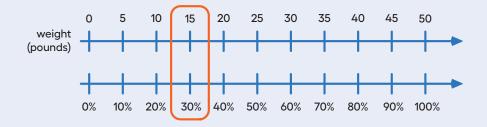
Lesson Summary

Sometimes we are interested in percentages of an amount other than 100 or 1. For example, what is 30% of 50 pounds? We can use a double number line diagram to solve problems about percentages.

Because we are looking for a percentage of 50 pounds, 100% is aligned to 50 pounds on the double number line diagram, like this:



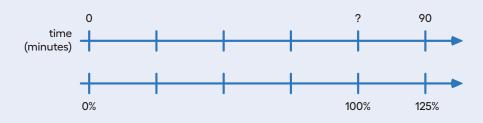
We divide the distance between 0% and 100% and that between 0 and 50 pounds into ten equal parts. The tick marks on the top line can be labeled by counting by 5s (50 \div 10 = 5). Those on the bottom line can be labeled by counting by 10% (100 \div 10 = 10). We can see that 30% of 50 pounds is 15 pounds.



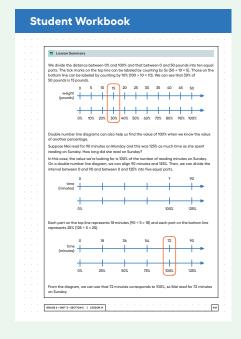
Double number line diagrams can also help us find the value of 100% when we know the value of another percentage.

Suppose Mai read for 90 minutes on Monday and this was 125% as much time as she spent reading on Sunday. How long did she read on Sunday?

In this case, the value we're looking for is 100% of the number of reading minutes on Sunday. On a double number line diagram, we can align 90 minutes and 125%. Then, we can divide the interval between 0 and 90 and between 0 and 125% into five equal parts.



Student Workbook 1 Pupples Graw Up 2 Andre do he has puppy that weight 9 pounds. The vet says that this puppy is now and a subset of the sub

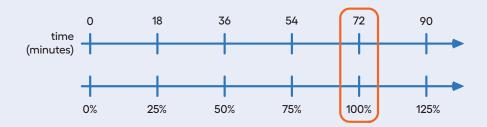


Responding To Student Thinking

Points to Emphasize

If students struggle with using a double number line diagram to find a percentage greater than 100%, plan to highlight the meaning of "percentages over 100%" when opportunities arise over the next several lessons. For example, before students label and reason with the given double number line diagrams, discuss with students how they interpret a weight that is "150% as much as Jada's puppy" in this activity: Unit 3, Lesson 12, Activity 1 Revisiting Jada's Puppy

Each part on the top line represents 18 minutes (90 \div 5 = 18) and each part on the bottom line represents 25% (125 \div 5 = 25).



From the diagram, we can see that 72 minutes corresponds to 100%, so Mai read for 72 minutes on Sunday.

Cool-down

Recycling Goal

5 min

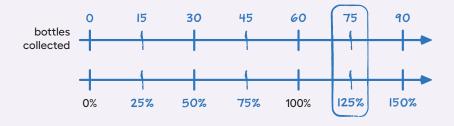
Student Task Statement

Noah set a goal of collecting 60 plastic bottles for recycling each week. He has reached 125% of his goal for this week. How many bottles has he collected?

75 bottles

Use the double number line diagram if you find it helpful.

Sample reasoning:



Practice Problems

5 Problems

Problem 1

Solve each problem. If you get stuck, consider using the double number line diagrams.

a. During a basketball practice, Mai attempted 40 free throws and was successful on 25% of them. How many successful free throws did she make?

b. Yesterday, Priya successfully made 12 free throws. Today, she made 150% as many. How many successful free throws did Priya make today?

18 free throws

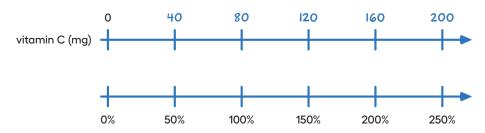
10 free throws

Problem 2

A bottle of orange juice contains 200 milligrams of vitamin C, which is 250% of the daily intake recommended for adults.

What is 100% of the recommended daily intake of vitamin C for adults?

80 mg



Explain or show your reasoning.

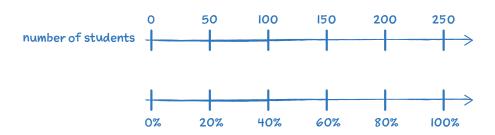
Sample reasoning: Align 200 mg to 250% on the double number line diagram and divide both 200 and 250% by 5 to get the values for 50% increments. IOO% is aligned to 80 mg.

Problem 3

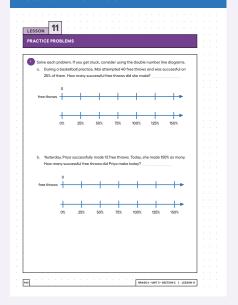
At a school, 40% of the sixth-grade students said that hip-hop is their favorite kind of music. If 100 sixth-grade students prefer hip hop music, how many sixth-grade students are at the school? Explain or show your reasoning.

250

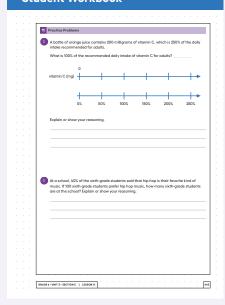
Sample reasoning:

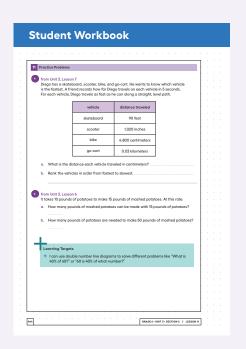


Student Workbook



Student Workbook





Problem 4

from Unit 3, Lesson 7

Diego has a skateboard, scooter, bike, and go-cart. He wants to know which vehicle is the fastest. A friend records how far Diego travels on each vehicle in 5 seconds. For each vehicle, Diego travels as fast as he can along a straight, level path.

vehicle	distance traveled
skateboard	90 feet
scooter	1,020 inches
bike	4,800 centimeters
go-cart	0.03 kilometers

- **a.** What is the distance each vehicle traveled in centimeters?
 - o skateboard: 2,743.2 cm
 - scooter: 2,590.8 cm
 - o bike: 4,800 cm
 - o go-cart: 3,000 cm
- **b.** Rank the vehicles in order from fastest to slowest.

Bike, go-cart, skateboard, scooter

Problem 5

from Unit 3, Lesson 6

It takes 10 pounds of potatoes to make 15 pounds of mashed potatoes. At this rate:

- **a.** How many pounds of mashed potatoes can be made with 15 pounds of potatoes?
 - $22\frac{1}{2}$ pounds of mashed potatoes (or equivalent)
- **b.** How many pounds of potatoes are needed to make 50 pounds of mashed potatoes?
 - 33 pounds of potatoes (or equivalent)