Lesson 10

Rectangle Madness

Goals

- Coordinate diagrams and expressions involving equivalent fractions.
- Interpret and create diagrams involving a rectangle decomposed into squares.

Lesson Narrative

This lesson is optional. In this exploration in pure mathematics, students tackle a series of activities that explore the relationship between the greatest common factor of two numbers and related fractions using a geometric representation. The activities in this lesson build on each other, providing students an opportunity to express the relationship between the greatest common factor of two numbers and related fractions through repeated reasoning. Thus, the activities should be done in order. It is not necessary to do the entire set of problems to get some benefit from the activities in this lesson, although more connections are made the farther one gets.

Student Learning Goal

Let's cut up rectangles.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR2 : Collect and Display (Activity 1)
- MLR8 : Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Materials to Gather

- Graph paper: Activity 2

Lesson Timeline

20 min	30 min
Activity 1	Activity 2

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display.
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

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Activity 1

Squares in Rectangles

20 min

Activity Narrative

This first activity helps students understand the geometric process that they use in later activities to connect the greatest common factor with related fractions. The first question helps students focus on the effect on side lengths of decomposing a rectangle into smaller rectangles. The second question has students analyze a rectangle that has been decomposed into squares. The third question has students themselves decompose a rectangle into squares. As students work with each rectangle, they make use of the structure to approach the problems. In the next activity, students relate this process to greatest common factors and fractions.

Launch

Tell students to close their books or devices (or to keep them closed). Display rectangle $ABCD$ for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses, without editing or commentary, for all to see. If possible, record the relevant reasoning on or near the image.

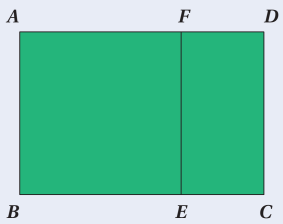
If naming shapes by vertices does not come up during the conversation, ask students to discuss this idea.

Tell students to open their books or devices, and arrange students in groups of 2. Give 7–10 minutes for students to complete the problems, and follow that with a whole-class discussion.

As students work, use Collect and Display to create a shared reference that captures students' developing mathematical language. Collect the language that students use to decompose rectangles. Display words and phrases such as "rectangle," "square," "split," "divide," "segment," and "pieces."

Student Task Statement

1. Rectangle $ABCD$ is not a square. Rectangle $ABEF$ is a square. Use the possible segment lengths to find the missing segment length.



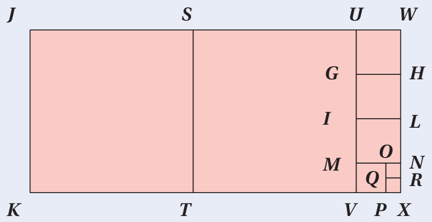
a. If segment AF is 5 units long and segment FD is 2 units long, how long would segment AD be? 7 units

b. If segment BC is 10 units long and segment BE is 6 units long, how long would segment EC be? 4 units

c. If segment AF is 12 units long and segment FD is 5 units long, how long would segment FE be? 12 units

d. If segment AD is 9 units long and segment AB is 5 units long, how long would segment FD be? 4 units

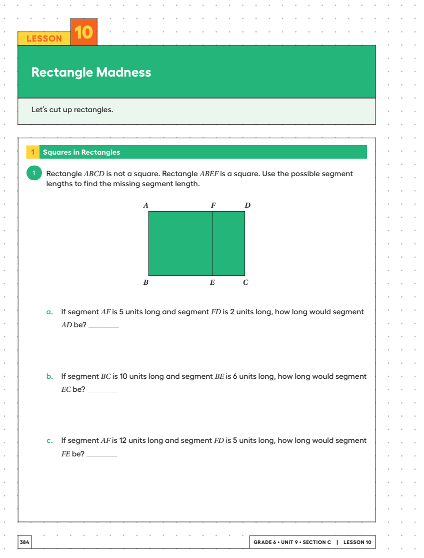
2. Rectangle $JKXW$ has been decomposed into squares.



Segment JK is 33 units long and segment JW is 75 units long. Find the areas of all of the squares in the diagram.

1,089 square units, 81 square units, 36 square units, and 9 square units

Student Workbook


LESSON 10

Rectangle Madness

Let's cut up rectangles.

1 Squares in Rectangles

1 Rectangle $ABCD$ is not a square. Rectangle $ABEF$ is a square. Use the possible segment lengths to find the missing segment length.

a. If segment AF is 5 units long and segment FD is 2 units long, how long would segment AD be? ______

b. If segment BC is 10 units long and segment BE is 6 units long, how long would segment EC be? ______

c. If segment AF is 12 units long and segment FD is 5 units long, how long would segment FE be? ______

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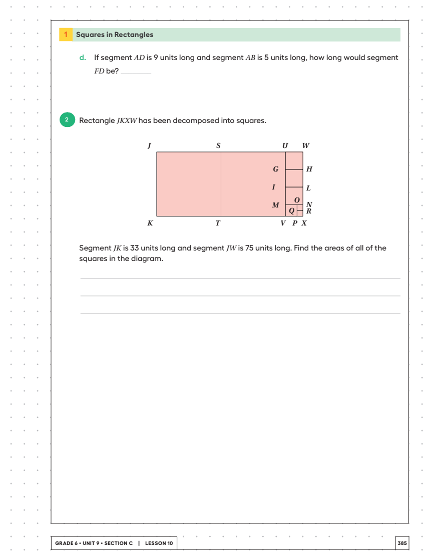
Student Workbook


1 Squares in Rectangles

d. If segment AD is 9 units long and segment AB is 5 units long, how long would segment FD be? ______

2 Rectangle $JKXW$ has been decomposed into squares.

Segment JK is 33 units long and segment JW is 75 units long. Find the areas of all of the squares in the diagram.

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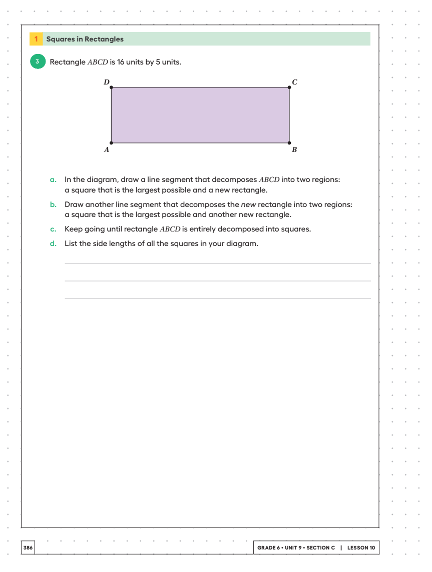
Student Workbook



1 Squares in Rectangles

3 Rectangle $ABCD$ is 16 units by 5 units.

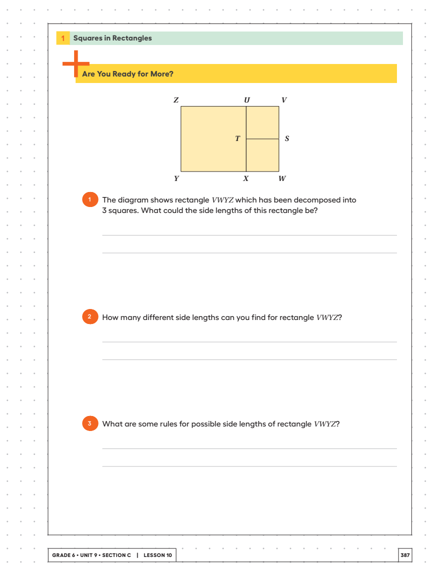
a. In the diagram, draw a line segment that decomposes $ABCD$ into two regions: a square that is the largest possible and a new rectangle.

b. Draw another line segment that decomposes the new rectangle into two regions: a square that is the largest possible and another new rectangle.

c. Keep going until rectangle $ABCD$ is entirely decomposed into squares.

d. List the side lengths of all the squares in your diagram.

Student Workbook



1 Squares in Rectangles

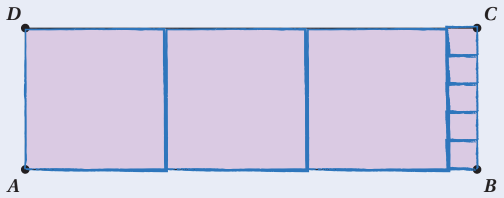
Are You Ready for More?

1 The diagram shows rectangle $VWYZ$ which has been decomposed into 3 squares. What could the side lengths of this rectangle be?

2 How many different side lengths can you find for rectangle $VWYZ$?

3 What are some rules for possible side lengths of rectangle $VWYZ$?

3. Rectangle $ABCD$ is 16 units by 5 units.



a. In the diagram, draw a line segment that decomposes $ABCD$ into two regions: a square that is the largest possible and a new rectangle.

b. Draw another line segment that decomposes the *new* rectangle into two regions: a square that is the largest possible and another new rectangle.

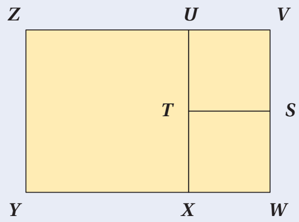
c. Keep going until rectangle $ABCD$ is entirely decomposed into squares.

d. List the side lengths of all the squares in your diagram.

Sample response:

There are three 5-by-5 squares and five 1-by-1 squares.

Are You Ready for More?



1. The diagram shows rectangle $VWYZ$ which has been decomposed into 3 squares. What could the side lengths of this rectangle be?

Sample response: Height is 2 units, width is 3 units.

2. How many different side lengths can you find for rectangle $VWYZ$?

Sample response: There are an infinite number of solutions.

3. What are some rules for possible side lengths of rectangle $VWYZ$?

Sample response: The length of line segment XW can be any number. Then the width of $VWYZ$ is 3 times the length of line segment XW, and the height of $VWYZ$ is 2 times the length of line segment XW.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they decomposed rectangle $ABCD$ in the last problem. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.
The goal of this activity is to familiarize students with decomposing rectangles into squares before using this tool with fractions. Here are some questions for discussion:

- *"Do you think every rectangle can be decomposed into squares?"*

  Yes

- *"What challenges did you run into while decomposing rectangles? How did you resolve them?"*

  "If a line segment of length x units is decomposed into two segments of lengths y and z it can be represented with the equation $x = y + z$. How else can this relationship be represented?"

  $x - z = y$

Activity 2

More Rectangles, More Squares

30 min

Activity Narrative

In this activity, students apply the geometric process that they saw in the last activity and are asked to make connections between the result and the greatest common factor of the side lengths of the original rectangle. They work on a sequence of similar problems, allowing them to see and begin to articulate a pattern. Then they make connections between this pattern and fractions that are equivalent to the fraction made up of the side lengths of the original rectangle.

Launch

Arrange students in groups of 2. Students work on problems alone and check work with a partner.

If needed, briefly review rewriting fractions before students begin. Consider drawing a diagram to support students' thinking about the different forms representing the same value.

1. Write a fraction that is equal to the mixed number $3\frac{4}{5}$.

   We can think of $3\frac{4}{5}$ as $\frac{15}{5} + \frac{4}{5}$, so it is equal to $\frac{19}{5}$.

2. Write a mixed number that is equal to this fraction: $\frac{11}{4}$.

   This is equal to $\frac{8}{4} + \frac{3}{4}$, so it is equal to $2\frac{3}{4}$.

It is quicker to sketch the rectangles on blank paper, but some students may benefit from using graph paper to support entry into these problems.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Begin with a physical demonstration of drawing line segments to decompose rectangles to support connections between new situations and prior understandings. Consider using these prompts:

> *"Should we start with a square on the left or the right?"*
> *"Where should the next square be placed?"*

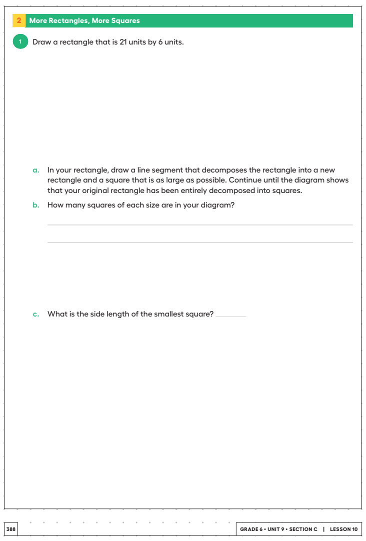
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Building on Student Thinking

If students do not see connections between the decomposition of the rectangles and the fraction problems, consider asking:

> *"Tell me more about the squares you decomposed the rectangle into."*
> *"Where do you see the parts of the mixed number in the squares?"*

Student Workbook


2 More Rectangles, More Squares

1 Draw a rectangle that is 21 units by 6 units.

a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been entirely decomposed into squares.

b. How many squares of each size are in your diagram?

c. What is the side length of the smallest square?

If students did not do the previous activity the same day as this activity, remind them of the earlier work:

- Draw a rectangle that is 16 units by 5 units.
  - **a.** In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been decomposed into squares.
  - **b.** How many squares of each size are there?

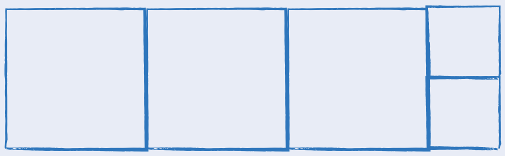
    Three 5-by-5 squares and five 1-by-1 squares.
  - **c.** What is the side length of the smallest square?

    1

Student Task Statement

1. Draw a rectangle that is 21 units by 6 units.

Sample diagram:



a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been entirely decomposed into squares.

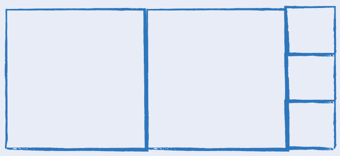
b. How many squares of each size are in your diagram?

Three 6-by-6 squares and two 3-by-3 squares

c. What is the side length of the smallest square? 3

2. Draw a rectangle that is 28 units by 12 units.

Sample diagram:



a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been decomposed into squares.

b. How many squares of each size are in your diagram?

Two 12-by-12 squares and three 4-by-4 squares

c. What is the side length of the smallest square? 4

3. Write each of these fractions as a mixed number with the smallest possible numerator and denominator:

   a. $\frac{16}{5}$ $3\frac{1}{5}$

   b. $\frac{21}{6}$ $3\frac{3}{6} = 3\frac{1}{2}$

   c. $\frac{28}{12}$ $2\frac{4}{12} = 2\frac{1}{3}$

4. What do the fraction problems have to do with the earlier rectangle decomposition problems?

   Sample responses: The given fractions had the same numbers as the side lengths of the given rectangles. The mixed numbers included the numbers of squares. For example, the 28-by-12 rectangle was partitioned into 2 large squares and 3 small squares, and the associated mixed number was $2\frac{1}{3}$. The size of the smallest square is related to the fraction used to make the mixed number. For example, in the 28-by-12 rectangle, the smallest square had sides of length 4. To make the mixed number, $\frac{4}{12}$ is rewritten as $\frac{1}{3}$.

Activity Synthesis

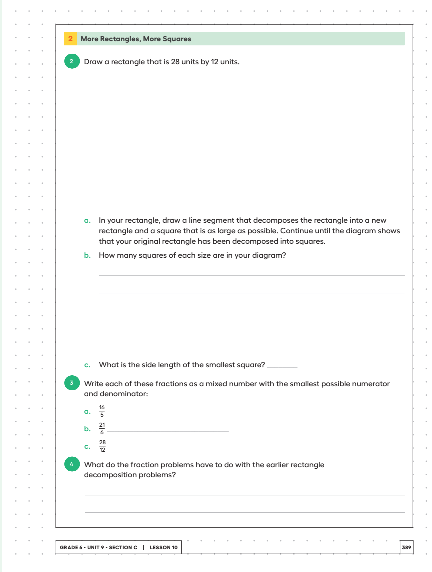
Invite students to share some of their observations. Ask,

"What connections do you see between the rectangle drawings and the fractions?"

At this point, it is sufficient for students to notice some connections between a partitioned rectangle and its associated fraction. They will have more opportunities to explore, so there's no need to make sure they notice all of the connections right now. Here are examples of things that it is possible to notice using the rectangles in this activity but might not be noticed until students see more examples:

- The given fractions had the same numbers as the side lengths of the given rectangles.
- The mixed numbers included the numbers of squares. For example, the 28-by-12 rectangle was partitioned into 2 large squares and 3 small squares, and the associated mixed number was $2\frac{1}{3}$.
- The size of the smallest square is related to the fraction used to make the mixed number. For example, in the 28-by-12 rectangle, the smallest square had sides of length 4. To make the mixed number, $\frac{4}{12}$ is rewritten as $\frac{1}{3}$.

Student Workbook


2 More Rectangles, More Squares

2 Draw a rectangle that is 28 units by 12 units.

a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been decomposed into squares.

b. How many squares of each size are in your diagram?

c. What is the side length of the smallest square?

3 Write each of these fractions as a mixed number with the smallest possible numerator and denominator:

a. $\frac{16}{5}$

b. $\frac{21}{6}$

c. $\frac{28}{12}$

4 What do the fraction problems have to do with the earlier rectangle decomposition problems?

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. For each observation that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.

Advances: Listening, Speaking