Lesson 2

Truth and Equations

Goals

- Comprehend the word "variable" to refer to a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (in spoken and written language).
- Generate values that make an equation true or false and justify (orally and in writing) whether they are "solutions" to the equation.
- Use substitution to determine whether a given number makes an equation true.

Learning Target

I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a "solution" to the equation.

Lesson Narrative

Students begin the lesson by analyzing the role variables play in equations. They then dig into what it means for an equation to be true or not true. Since grade 1, students have been identifying whether numerical equations are true or false. Now, students extend that understanding to algebraic equations. Students learn that, for an equation with a variable, a value of the variable that makes the equation true is called a "solution" of the equation. They then use substitution to determine whether a given number in a set is a solution of the equation.

This lesson is where "next to" notation is introduced for multiplication. (For example, $10m$ means $10 \cdot m$.)

Student Learning Goal

Let's see what it means to find a solution for an equation with a variable.

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

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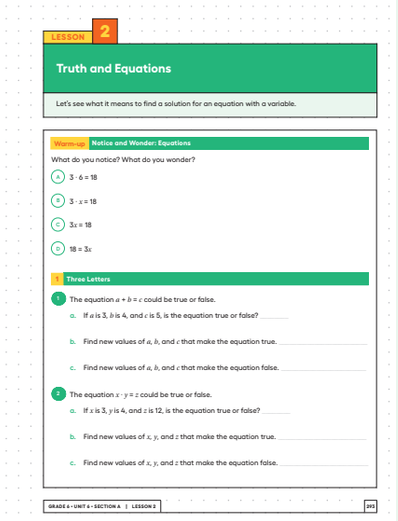
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 2

Truth and Equations

Let's see what it means to find a solution for an equation with a variable.

Warm-up Notice and Wonder: Equations

What do you notice? What do you wonder?

A. $3 \cdot 6 = 18$

B. $3 \cdot x = 18$

C. $3x = 18$

D. $18 = 3x$

1 Three Letters

1. The equation $a + b = c$ could be true or false.
   a. If a is 3, b is 4, and c is 5, is the equation true or false? _____
   b. Find new values of a, b, and c that make the equation true. _____
   c. Find new values of a, b, and c that make the equation false. _____
2. The equation $x \cdot y = z$ could be true or false.
   a. If x is 3, y is 4, and z is 12, is the equation true or false? _____
   b. Find new values of x, y, and z that make the equation true. _____
   c. Find new values of x, y, and z that make the equation false. _____

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Warm-up

Notice and Wonder: Equations

10 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that a **variable** represents an unknown number in an equation or diagram, which will be useful when students identify solutions for equations in a later activity. While students may notice and wonder many things about these equations, the fact that the variable x represents the number 6 is the most important discussion point.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is that the number 6 in the first equation is replaced by the letter x in all the other equations. They may also notice in the last three equations that replacing each x with 6 makes the equations true.

Launch

Arrange students in groups of 2. Display the equations for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?

A. $3 \cdot 6 = 18$

B. $3 \cdot x = 18$

C. $3x = 18$

D. $18 = 3x$

Students may notice:

- The numbers 3 and 18 are in all the equations.
- Three equations have the letter x.
- There is no operation symbol in the last two equations.

Students may wonder:

- What does the letter x represent?
- Do all of these equations show multiplication?
- Do all of these equations represent the same relationship?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the equations. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the fact that the number 6 is replaced by the letter x does not come up during the conversation, ask students to discuss this idea. Explain that a letter used to stand in for a number is called a **variable**. Throughout this unit, students will have many chances to understand and use this term.

Students may wonder about the $3x$ in the last two equations since it's likely that this is their first experience with "next to" notation. Explain that $3x$ means the same thing as $3 \cdot x$, and we will frequently use this notation from now on. Explain that in the expression $3x$, the number 3 is called the **coefficient**. Use this term throughout the lesson when the need naturally arises to name it.

Activity 1

Three Letters

10 min

Activity Narrative

In this activity, students consider what it means for an equation to be true or false. They also practice substituting values for a variable and evaluating expressions with addition and multiplication.

Launch

Allow students 2 minutes of quiet work time on the first part of the first question and ask them to pause their work.

Ask a student to explain how they decided whether the equation was true or false given the values of a, b, and c. As the student explains, demonstrate this process by writing:

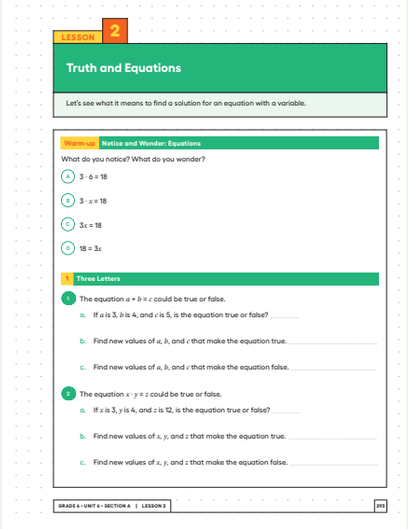
$a + b = c$

$3 + 4 = 5$

Elicit from students that since the sum of 3 and 4 is not 5, the equation is false for these values for the variables.

Give students 2 minutes to complete the rest of the task.

Student Workbook


LESSON 2

Truth and Equations

Let's see what it means to find a solution for an equation with a variable.

Warm-up Notice and Wonder: Equations

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C. $3x = 18$

D. $18 = 3x$

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   a. If a is 3, b is 4, and c is 5, is the equation true or false?
   b. Find new values of a, b, and c that make the equation true.
   c. Find new values of a, b, and c that make the equation false.
2. The equation $x \cdot y = z$ could be true or false.
   a. If x is 3, y is 4, and z is 12, is the equation true or false?
   b. Find new values of x, y, and z that make the equation true.
   c. Find new values of x, y, and z that make the equation false.

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Student Task Statement

1. The equation $a + b = c$ could be true or false.
   a. If a is 3, b is 4, and c is 5, is the equation true or false?

      False
   b. Find new values of a, b, and c that make the equation true.

      Sample response: a is 1, b is 2, c is 3
   c. Find new values of a, b, and c that make the equation false.

      Sample response: a is 4, b is 5, c is 10
2. The equation $x \cdot y = z$ could be true or false.
   a. If x is 3, y is 4, and z is 12, is the equation true or false?

      True
   b. Find new values of x, y, and z that make the equation true.

      Sample response: x is 3, y is 5, z is 15
   c. Find new values of x, y, and z that make the equation false.

      Sample response: x is 1, y is 2, z is 3

Activity Synthesis

This discussion should focus on the idea that an equation can be either true or false, and that the truth of an equation with variables depends on the values of those variables. If needed, remind students that the equal sign shows that the two expressions on its two sides are equal to each other.

Invite students to share their values for true and false equations. To support students in expressing their ideas about equations, consider asking:

- *"What makes an equation true?"*

  The expressions on both sides of the equal sign have the same value.
- *"What makes an equation false?"*

  The expressions on each side have different values.
- *"How can we tell if an equation without variables is true or false?"*

  Find the value of the expression on each side and see whether or not those two values are equal.
- *"How can we tell if an equation with a variable is true or false when we are given a value for the variable?"*

  Replace the variable with the given value, find the value of each side, and see whether or not those two values are equal.

Emphasize that the equation 2+3=7 can be understood as a mathematical statement that is not true instead of something that is "wrong" or "a mistake."

Activity 2
Find a Solution

10 min

Activity Narrative

In this activity, students look for solutions to equations and determine which value from a given set makes an equation with a variable true. Since so many values are given, students may realize that using only substitution is not the most efficient method for finding a solution. As a result, students may use the structure of the equations and given values to eliminate some of the values as possible solutions.

Launch

Remind students that a letter used to represent an unknown value is called a "variable," and an equation with a variable can be true or false. Explain that a value that can be used in place of the variable and that makes the equation true is called a **solution** to the equation. Emphasize that this means students should look for a value that can be substituted for, or used in place of, a variable in an equation that makes the equation true.

Arrange students in groups of 2. Allow students 5 minutes to work quietly and share their responses with a partner, followed by a whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Ask students to suggest ways to update the display:

- *"Are there any new words or phrases that you would like to add? Is there any language you would like to revise or remove?"*

For example, the display may have "4 added to the unknown value x is equal to 12" already on it and can be updated with the more precise "4 added to the variable x is equal to 12."

Encourage students to use the display as a reference.

Student Task Statement

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, consider drawing a diagram. Be prepared to explain your reasoning.

List:	$\frac{1}{8}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{7}{3}$	0.01	0.1	0.5
	1	2	8.5	9.5	16.7	20	400	600	1400

1. $1000 - a = 400$

a = 600

2. $8c = 8$

c = 1

3. $10w = 1$

w = 0.1

4. $10 = \frac{1}{2}f$

f = 20

5. $0.99 = 1 - g$

g = 0.01

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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Please log in to the site before using the QR code or URL.

Building on Student Thinking

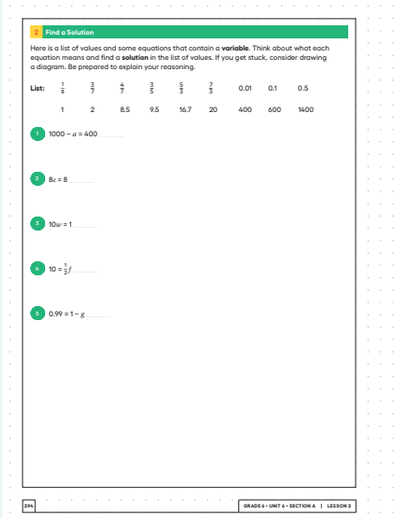
If students only use substitution to identify the solution for each equation, consider asking:

"How did you find a solution for this equation?"

"How could you use the structure of the equation to eliminate some of the possible solutions?"

Encourage students to use words to express the relationship represented by the equation. For example, the equation $1000 - a = 400$ can be stated as "When a number is subtracted from 1,000, the difference is 400." This process will help students know that the value of a must be less than 1,000. Some might also recognize that it means a must be a whole number.

Student Workbook



2 Find a Solution

Here is a list of values and some equations that contain a **variable**. Think about what each equation means and find a **solution** in the list of values. If you get stuck, consider drawing a diagram. Be prepared to explain your reasoning.

List: $\frac{1}{8}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{3}{5}$ $\frac{5}{3}$ $\frac{7}{3}$ 0.01 0.1 0.5

1 2 8.5 9.5 16.7 20 400 600 1400

1. $1000 - a = 400$
2. $8c = 8$
3. $10w = 1$
4. $10 = \frac{1}{2}f$
5. $0.99 = 1 - g$

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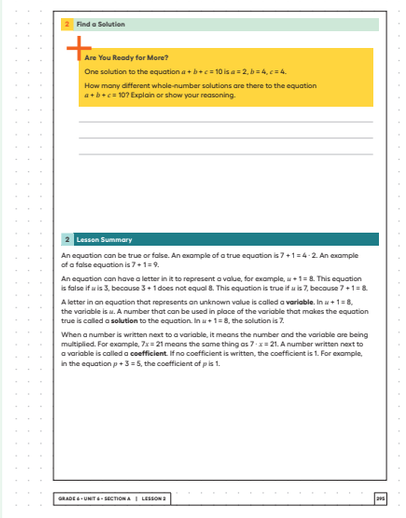
Student Workbook



2 Find a Solution

Are You Ready for More?

One solution to the equation $a + b + c = 10$ is $a = 2$, $b = 4$, $c = 4$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

2 Lesson Summary

An equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$. An example of a false equation is $7 + 1 = 9$.

An equation can have a letter in it to represent a value, for example, $w + 1 = 8$. This equation is false if w is 3, because $3 + 1$ does not equal 8. This equation is true if w is 7, because $7 + 1 = 8$.

A letter in an equation that represents an unknown value is called a **variable**. In $w + 1 = 8$, the variable is w. A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In $w + 1 = 8$, the solution is 7.

When a number is written next to a variable, it means the number and the variable are being multiplied. For example, $7x = 21$ means the same thing as $7 \cdot x = 21$. A number written next to a variable is called a **coefficient**. If no coefficient is written, the coefficient is 1. For example, in the equation $p + 3 = 5$, the coefficient of p is 1.

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Are You Ready for More?

One solution to the equation $a + b + c = 10$ is $a = 2$, $b = 4$, $c = 4$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

There are 66 solutions. Sample reasoning:

If $a = 0$, there are 11 combinations of b and c:

- $b = 0$, $c = 10$
- $b = 1$, $c = 9$
- $b = 2$, $c = 8$
- $b = 3$, $c = 7$
- $b = 4$, $c = 6$
- $b = 5$, $c = 5$
- $b = 6$, $c = 4$
- $b = 7$, $c = 3$
- $b = 8$, $c = 2$
- $b = 9$, $c = 1$
- $b = 10$, $c = 0$

If $a = 1$, there are 10 combinations of b and c:

- $b = 0$, $c = 9$
- $b = 1$, $c = 8$
- $b = 2$, $c = 7$
- $b = 3$, $c = 6$
- $b = 4$, $c = 5$
- $b = 5$, $c = 4$
- $b = 6$, $c = 3$
- $b = 7$, $c = 2$
- $b = 8$, $c = 1$
- $b = 9$, $c = 0$

If $a = 2$, there are 9 combinations of b and c:

- $b = 0$, $c = 8$
- $b = 1$, $c = 7$
- $b = 2$, $c = 6$
- $b = 3$, $c = 5$
- $b = 4$, $c = 4$
- $b = 5$, $c = 3$
- $b = 6$, $c = 2$
- $b = 7$, $c = 1$
- $b = 8$, $c = 0$

This pattern continues: as a increases by 1, the number of combinations decreases by 1, until there is only 1 combination when $a = 10$: $b = 0$ and $c = 0$.

So, the total number of solutions (for a value of 0 through 10) is $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, which is 66.

Activity Synthesis

The goal of the discussion is to ensure that students understand what "a solution to an equation" means. To guide the discussion, consider doing the following:

- Solicit or display correct solutions.
- Choose a few equations and ask students to explain how they know a solution is correct.
- Highlight correct uses of the new terms "variable," "solution," and "coefficient."
- Demonstrate substituting an identified solution in the equation to verify that it makes the equation true. Emphasize that this substitution method can always be used to check a solution to an equation.

Lesson Synthesis

The purpose of the discussion is to review appropriate use and understanding of new vocabulary and the concepts they represent. Consider asking some of the following questions:

- *"What does it mean for an equation to be true (or false)?"*

  "What does the equal sign have to do with whether the equation is true or false?"

  A true equation has expressions with equal value on each side of the equal sign.

- *"Is an equation with a variable always true?"*

  It is only true for values of the variable that make the values of the expressions on the two sides equal.

Display the equation $3x = 18$ from the *Warm-up*.

- *"Which part of this equation is a variable?"*

  x

- *"Which part is a coefficient?"*

  3

- *"What operation does the expression 3x represent?"*

  multiplication

- *"Is 6 a solution for this equation? How do you know?"*

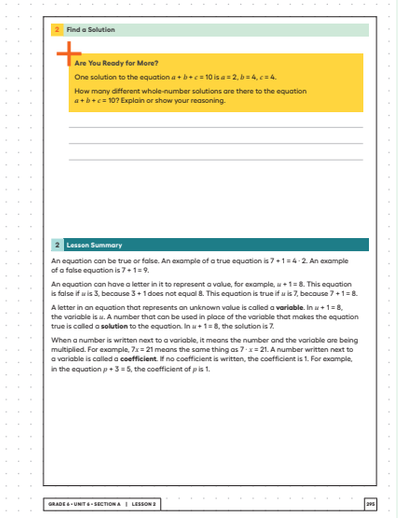
  Yes. When I replace x with 6, the equation is true. Both sides have the same value.

Emphasize that finding the solution to an equation with a variable is answering the question "Which value for the variable makes the equation true?"

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



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Cool-down

5 min

How Do You Know a Solution Is a Solution?

Student Task Statement

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing the sentences:

The word "solution" means … a value for the variable that makes the equation true.

88 is a solution to $\frac{1}{8}x = 11$ because … if x is 88, the equation is $\frac{1}{8} \cdot 88 = 11$, which is true.

Lesson 2

Practice Problems

7 Problems

Problem 1

Select **all** the true equations.

A. $5 + 0 = 0$

B. $15 \cdot 0 = 0$

C. $1.4 + 2.7 = 4.1$

D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$

E. $4\frac{2}{3} = 5 - \frac{1}{3}$

Problem 2

Select **all** the equations that are true when the value of the variable is 4.

A. $9 \cdot x = 32$

B. $31 - x = 17$

C. $1 = \frac{1}{4}x$

D. $x + 18 = 22$

E. $10x = 14$

Problem 3

Which equation shows that $\frac{1}{5}$ is a solution to the equation $30x = 6$?

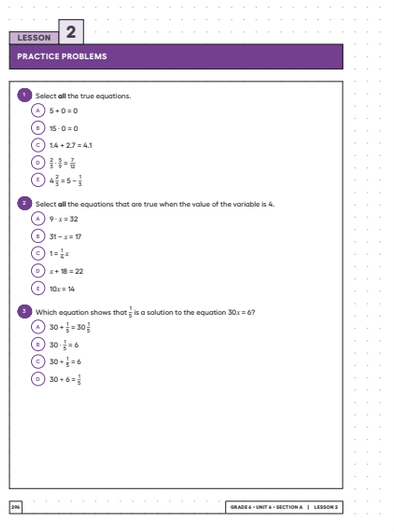
A. $30 + \frac{1}{5} = 30\frac{1}{5}$

B. $30 \cdot \frac{1}{5} = 6$

C. $30 \div \frac{1}{5} = 6$

D. $30 \div 6 = \frac{1}{5}$

Student Workbook


LESSON 2

PRACTICE PROBLEMS

1. Select **all** the true equations.
   - A. $5 + 0 = 0$
   - B. $15 \cdot 0 = 0$
   - C. $1.4 + 2.7 = 4.1$
   - D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$
   - E. $4\frac{2}{3} = 5 - \frac{1}{3}$
2. Select **all** the equations that are true when the value of the variable is 4.
   - A. $9 \cdot x = 32$
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   - C. $1 = \frac{1}{4}x$
   - D. $x + 18 = 22$
   - E. $10x = 14$
3. Which equation shows that $\frac{1}{5}$ is a solution to the equation $30x = 6$?
   - A. $30 + \frac{1}{5} = 30\frac{1}{5}$
   - B. $30 \cdot \frac{1}{5} = 6$
   - C. $30 \div \frac{1}{5} = 6$
   - D. $30 \div 6 = \frac{1}{5}$

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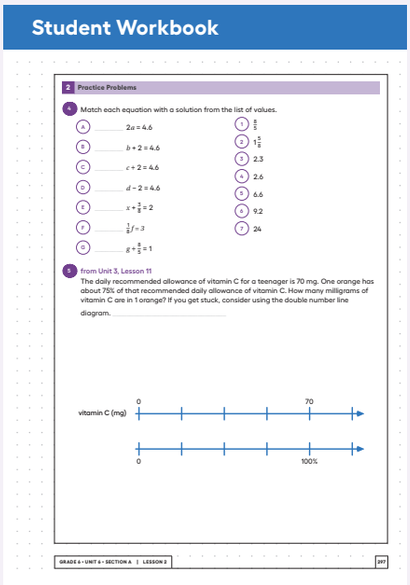
Student Workbook


2 Practice Problems

4 Match each equation with a solution from the list of values.

A. _____ $2a = 4.6$
B. _____ $b + 2 = 4.6$
C. _____ $c \div 2 = 4.6$
D. _____ $d - 2 = 4.6$
E. _____ $x + \frac{3}{8} = 2$
F. _____ $\frac{1}{8}f = 3$
G. _____ $g \div \frac{8}{5} = 1$

1. $\frac{8}{5}$
2. $1\frac{5}{8}$
3. 2.3
4. 2.6
5. 6.6
6. 9.2
7. 24

5 from Unit 3, Lesson 11
The daily recommended allowance of vitamin C for a teenager is 70 mg. One orange has about 75% of that recommended daily allowance of vitamin C. How many milligrams of vitamin C are in 1 orange? If you get stuck, consider using the double number line diagram.

vitamin C (mg) 0 70
0 100%

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Problem 4

Match each equation with a solution from the list of values.

A. 3 $2a = 4.6$

B. 4 $b + 2 = 4.6$

C. 6 $c \div 2 = 4.6$

D. 5 $d - 2 = 4.6$

E. 2 $x + \frac{3}{8} = 2$

F. 7 $\frac{1}{8}f = 3$

G. 1 $g \div \frac{8}{5} = 1$

1. $\frac{8}{5}$

2. $1\frac{5}{8}$

3. 2.3

4. 2.6

5. 6.6

6. 9.2

7. 24

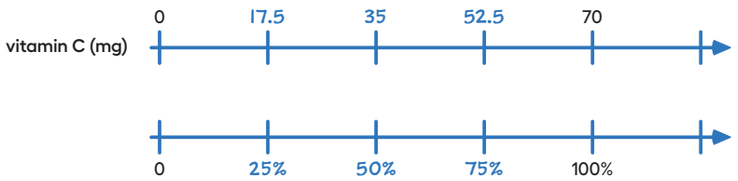
Problem 5

from Unit 3, Lesson 11

The daily recommended allowance of vitamin C for a teenager is 70 mg. One orange has about 75% of that recommended daily allowance of vitamin C. How many milligrams of vitamin C are in 1 orange? If you get stuck, consider using the double number line diagram.

52.5 mg

Sample reasoning:



Problem 6

from Unit 3, Lesson 12

There are 90 kids in the band. 20% of the kids own their own instruments, and the rest rent them.

a. How many kids own their own instruments?

18 kids (90 · 0.2 = 18)

b. How many kids rent instruments?

72 kids (90 – 18 = 72)

c. What percentage of kids rent their instruments?

80% (100 – 20 = 80)

Problem 7

from Unit 5, Lesson 8

Find each product.

a. (0.25) · (1.4)

0.35

b. (0.061) · (0.43)

0.02623

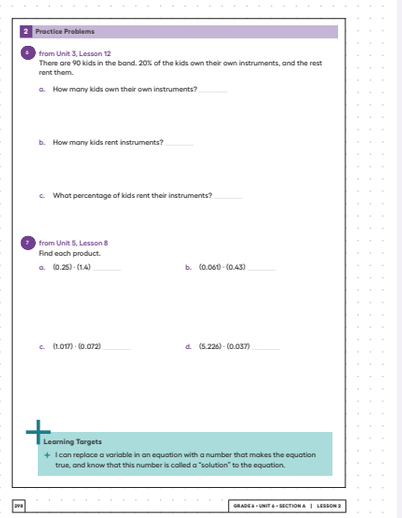
c. (1.017) · (0.072)

0.073224

d. (5.226) · (0.037)

0.193362

Student Workbook


2 Practice Problems

6 from Unit 3, Lesson 12
There are 90 kids in the band. 20% of the kids own their own instruments, and the rest rent them.
a. How many kids own their own instruments? _____
b. How many kids rent instruments? _____
c. What percentage of kids rent their instruments? _____

7 from Unit 5, Lesson 8
Find each product.
a. (0.25) · (1.4) _____
b. (0.061) · (0.43) _____
c. (1.017) · (0.072) _____
d. (5.226) · (0.037) _____

Learning Targets
+ I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a "solution" to the equation.

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