Lesson 1

Tape Diagrams and Equations

Goals

- Draw tape diagrams to represent equations of the forms $x + p = q$ and $px = q$.
- Interpret (orally and in writing) tape diagrams that represent equations of the form $p + x = q$ or $px = q$.

Learning Targets

- I can tell whether or not an equation could represent a tape diagram.
- I can use a tape diagram to represent an equation.

Lesson Narrative

The purpose of this lesson is to remind students how tape diagrams and equations can be used to represent operations. There are two roles that tape diagrams (or any diagrams) can play: helping to visualize a relationship and helping to solve a problem. The focus of this lesson is using tape diagrams to represent relationships between values in equations. Students both interpret tape diagrams and create their own. This prepares students to use both diagrams and equations to solve problems in later lessons.

Note that the terms "solution" and "variable" aren't defined until the next lesson, nor should any solution methods be generalized yet. Students should engage with the activities and reason about quantities in ways that make sense to them.

Student Learning Goal

Let's recall how tape diagrams and equations can show relationships between amounts.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1, Activity 2)

Instructional Routines

- MLR2: Collect and Display

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	15 min	10 min	10 min

Warm-up

Which Diagram Is Which?

5 min

Activity Narrative

Students recall tape diagram representations of addition and multiplication relationships.

In these materials, when multiplication is used to represent equal groups, such as “5 groups of 2,” the factors are written in the same order as when described verbally: the first factor is the number of groups and the second is the number in each group (or size of each group). But students do not have to follow that convention. They may use their understanding of the commutative property of multiplication to represent relationships in ways that make sense to them.

Launch

Give students 2 minutes of quiet think time, followed by a whole-class discussion.

Student Task Statement

1. Here are two diagrams. One represents $2 + 5 = 7$. The other represents $5 \cdot 2 = 10$. Which is which? Label each diagram with the value that represents the total.

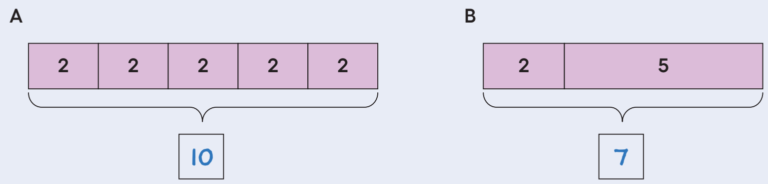


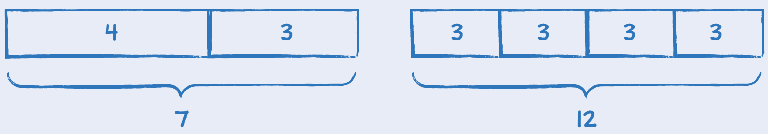
Diagram A: $5 \cdot 2 = 10$, Diagram B: $2 + 5 = 7$

2. Draw a diagram that represents each equation.

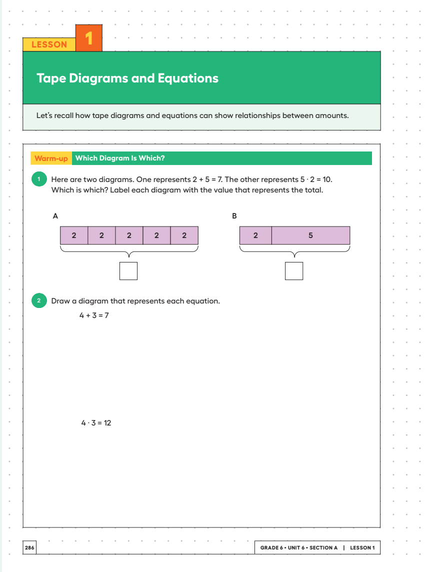
   $4 + 3 = 7$

   $4 \cdot 3 = 12$

   Sample response:



Student Workbook


LESSON 1

Tape Diagrams and Equations

Let's recall how tape diagrams and equations can show relationships between amounts.

Warm-up Which Diagram Is Which?

1 Here are two diagrams. One represents $2 + 5 = 7$. The other represents $5 \cdot 2 = 10$. Which is which? Label each diagram with the value that represents the total.

A 2 2 2 2 2

B 2 5

2 Draw a diagram that represents each equation.

$4 + 3 = 7$

$4 \cdot 3 = 12$

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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Activity Synthesis

Invite students to share their responses, diagrams, and rationales. The purpose of the discussion is to give students an opportunity to articulate how operations can be represented by tape diagrams. Consider asking:

- *"Where do you see the 5 in the first diagram?"*

  There are 5 equal parts represented by 5 same-size boxes.

- *"Why is there only one 2 in the second diagram?"*

  2 is only added once in the equation $2 + 5 = 7$.

- *"How did you find the value of the total to write in the first diagram?"*

  Compute $2 + 2 + 2 + 2 + 2$ or $5 \cdot 2$ or find the total value in the multiplication equation that the diagram represents.

- *"Explain how you knew what the diagrams for $4 + 3 = 7$ and $4 \cdot 3 = 12$ should look like."*

  "How are the representations of $4 \cdot 3$ alike? How are they different?"

  They all show the same total, 12. Some represent $4 \cdot 3$ as 4 groups of size 3, while others represent it as 3 groups of size 4.

Activity 1
Match Equations and Tape Diagrams

10 min

Activity Narrative

In this activity, students use what they know about operations to identify multiple equations with variables that match a given diagram. For example, the relationship between the quantities 2, 5, and 7 expressed by the equation $2 + 5 = 7$ can also be written as $2 = 7 - 5$, $5 = 7 - 2$, $7 = 2 + 5$, and $7 - 2 = 5$. Ask students to explain how the parts of these equations match the parts of the tape diagram.

Note that the word "variable" is not defined until the next lesson. It is not necessary to use that term with students yet. Also, this activity includes the equivalent expressions $x + x + x + x$ and $4 \cdot x$ because these equivalent ways of writing it should be familiar from earlier grades, but the term *equivalent expressions* is defined more carefully later in this unit. Even though this familiar example appears, the general idea of equivalent expressions is not a focus of this lesson.

Throughout the activity, students have opportunities to use both informal and more formal language to describe equations.

Launch

Arrange students in groups of 2.

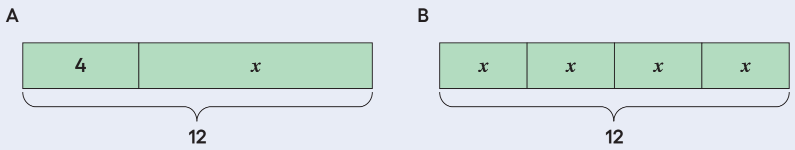
Give students 2 minutes of quiet work time. Then ask them to share their responses with their partner and follow with whole-class discussion.

If necessary, explain that the x in each diagram is just standing in for a number, like the box did in the previous activity. Encourage students to refer to x as representing the unknown value in the relationship.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe the diagrams and equations. Display words and phrases, such as "equal parts," "total," "letter x," and "unknown," "multiplied by," "added to," and "expression."

Student Task Statement

Here are two tape diagrams. Match each equation to one of the tape diagrams.



- $4 + x = 12$ A
- $12 \div 4 = x$ B
- $4 \cdot x = 12$ B
- $12 = 4 + x$ A
- $12 - x = 4$ A
- $12 = 4 \cdot x$ B
- $12 - 4 = x$ A
- $x = 12 - 4$ A
- $x + x + x + x = 12$ B

Activity Synthesis

Focus the discussion on why the same relationship can be expressed in more than one way. Elicit ideas such as:

- A multiplicative relationship can also be expressed using division.
- An additive relationship can also be expressed using subtraction.
- It does not matter how expressions are arranged around an equal sign. For example, $4 + x = 12$ and $12 = 4 + x$ mean the same thing.
- Repeated addition (or expressions with equal addends) can also be represented with multiplication. For example, $4 \cdot x$ is another way to express $x + x + x + x$.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they matched each equation to a diagram. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. (For example, the display may have "4 added to x is 12" already on it and can be updated with the more precise phrase "4 added to the unknown value x is equal to 12." In the next lesson, students will learn even more precise language to describe the relationships represented by the equations. They will revisit this display to update that language.)

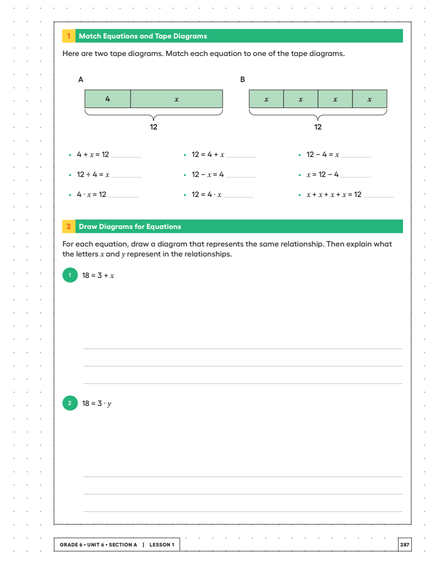
Consider asking questions such as:

"How can you tell if a diagram represents addition or multiplication?"

"Once you were sure about one equation, how did you find others that matched the same diagram?"

"What is the same about the equations that represent the same diagram? What is different?"

Student Workbook



Activity 2

Draw Diagrams for Equations

15 min

Activity Narrative

In this activity, students draw tape diagrams to match given equations. Then they reason about the unknown value and what it represents in each diagram and equation. Although some students may be tempted to identify the value of the variable, that should not be the focus of this activity. Students will focus on what it means to solve an equation in the next lesson.

Launch

Give students 5 minutes quiet work time followed by a whole-class discussion.

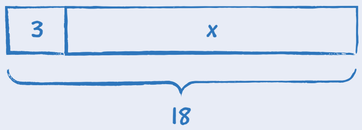
Use *Collect and Display* to direct attention to words collected and displayed from the previous activity. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Student Task Statement

For each equation, draw a diagram that represents the same relationship. Then explain what the letters x and y represent in the relationships.

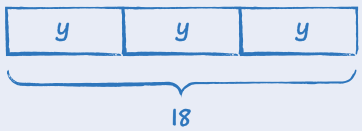
1. $18 = 3 + x$

Sample response: The letter x represents the unknown number that can be added to 3 to get a total of 18.



2. $18 = 3 \cdot y$

Sample response: The letter y represents the unknown size of each part when 3 equal parts make a total of 18.



Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
For each equation, provide a blank template of a tape diagram for students to complete.

Supports accessibility for: Visual-spatial processing; Organization

Are You Ready for More?

Which diagram can be represented by more equations? Explain or show your reasoning.

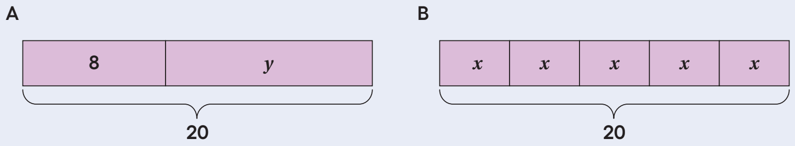


Diagram B

Sample reasoning: Diagram B can be represented by 3 operations: multiplication, division, and addition. Diagram A can only be represented by 2 operations: addition and subtraction.

Examples for Diagram A:

- $8 + x = 20$
- $20 - 8 = x$

Examples for Diagram B:

- $5 \cdot x = 20$ (or equivalent)
- $20 \div 5 = x$ (or equivalent)
- $x + x + x + x + x = 20$ (or equivalent)

Activity Synthesis

Invite students to share their diagrams and explanations of the meanings of x and y. Emphasize explanations that use the terms "unknown number," "unknown value," or "unknown quantity."

To highlight how a tape diagram represents the quantities and relationship shown in each equation, ask questions such as:

"Where do you see the addition (or multiplication) in this diagram?"

"Where do you see the 3 in the tape diagram representing the multiplication equation?"

"How is the equal sign represented in the diagram?"

The parts have the same value as the whole or total. The numbers and letters in the boxes add up to the total shown for the whole rectangle.

Building on Student Thinking

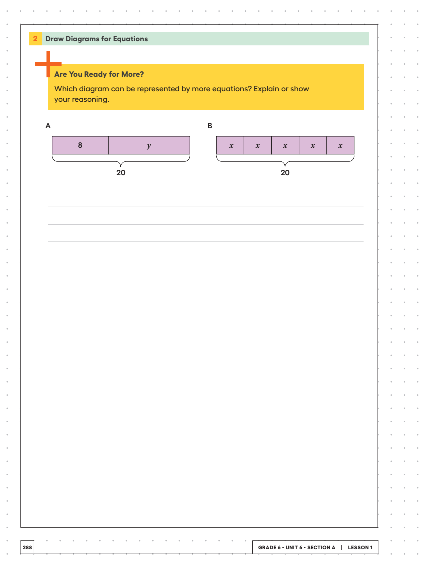
If students draw a tape diagram with two parts—labeled "3" and "y"—for the equation $18 = 3 \cdot y$, consider asking:

"What does $3 \cdot y$ mean?"

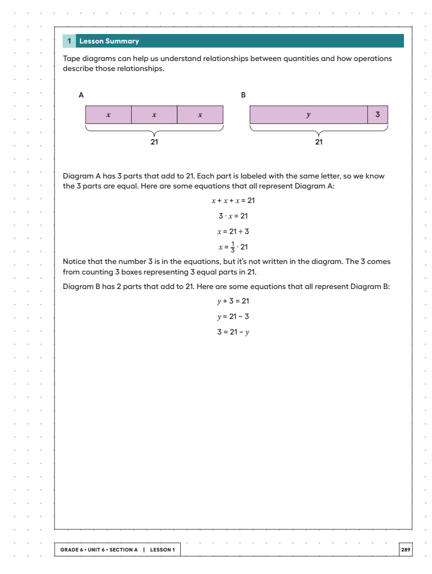
"How could you represent 3 equal groups with unknown size y?"

Students might think they need to show an unknown number (y) of equal groups of 3. While this is possible, showing 3 equal groups with unknown size y is simpler to represent.

Student Workbook



Student Workbook


1 Lesson Summary

Tape diagrams can help us understand relationships between quantities and how operations describe those relationships.

Diagram A has 3 parts that add to 21. Each part is labeled with the same letter, so we know the 3 parts are equal. Here are some equations that all represent Diagram A:

Notice that the number 3 is in the equations, but it's not written in the diagram. The 3 comes from counting 3 boxes representing 3 equal parts in 21.

Diagram B has 2 parts that add to 21. Here are some equations that all represent Diagram B:

Lesson Synthesis

To ensure that students understand the use and usefulness of tape diagrams and equations to represent relationships between values and to strengthen students' mathematical language use, consider asking:

"How can we use a tape diagram to represent two numbers being added?"

"How can we use a tape diagram to represent two numbers being multiplied?"

Lesson Summary

Tape diagrams can help us understand relationships between quantities and how operations describe those relationships.

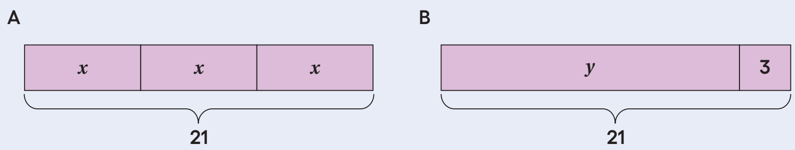


Diagram A has 3 parts that add to 21. Each part is labeled with the same letter, so we know the 3 parts are equal. Here are some equations that all represent Diagram A:

$$x + x + x = 21$$
$$3 \cdot x = 21$$
$$x = 21 \div 3$$
$$x = \frac{1}{3} \cdot 21$$

Notice that the number 3 is in the equations, but it's not written in the diagram. The 3 comes from counting 3 boxes representing 3 equal parts in 21.

Diagram B has 2 parts that add to 21. Here are some equations that all represent Diagram B:

$$y + 3 = 21$$
$$y = 21 - 3$$
$$3 = 21 - y$$

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

“What norm(s) should stay the way they are?”

“What norm(s) do you think should be made more clear? How?”

“What norms are missing that you would add?”

“What norm(s) should be removed?”

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

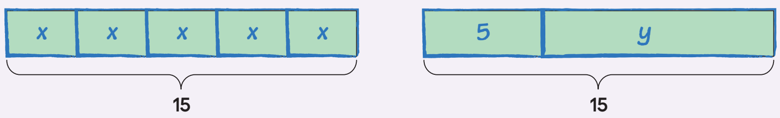
Complete the Diagrams

10 min

Student Task Statement

Complete the first diagram so it represents $5 \cdot x = 15$. Complete the second diagram so it represents $5 + y = 15$.

Sample response:



Responding To Student Thinking

Points to Emphasize

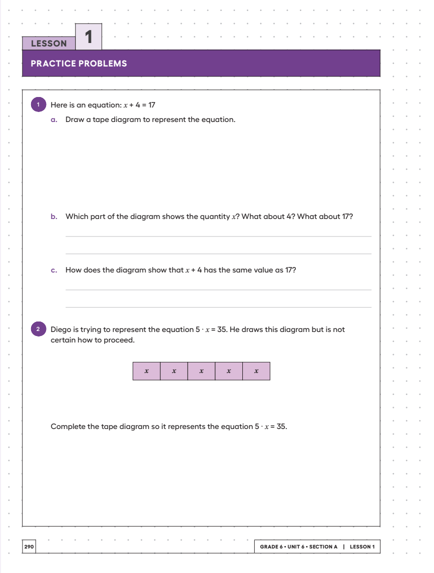
If students struggle with representing given equations with a tape diagram, focus on tape diagrams when opportunities arise over the next several lessons. For example, encourage students to draw a tape diagram for each equation in:

Grade 6, Unit 6, Lesson 2, Activity 1
Three Letters

Practice Problems

7 Problems

Student Workbook


LESSON 1

PRACTICE PROBLEMS

1 Here is an equation: $x + 4 = 17$

a. Draw a tape diagram to represent the equation.

b. Which part of the diagram shows the quantity x? What about 4? What about 17?

c. How does the diagram show that $x + 4$ has the same value as 17?

2 Diego is trying to represent the equation $5 \cdot x = 35$. He draws this diagram but is not certain how to proceed.

Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

Problem 1

Here is an equation: $x + 4 = 17$

a. Draw a tape diagram to represent the equation.

Students draw a tape diagram showing one part labeled x and another labeled 4 and a total of 17.

b. Which part of the diagram shows the quantity x? What about 4? What about 17?

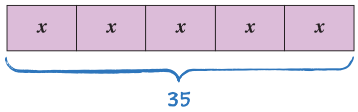
The rectangle labeled x represents the quantity x, and the rectangle labeled 4 represents the quantity 4. The big rectangle (the combination of the two smaller ones) represents 17.

c. How does the diagram show that $x + 4$ has the same value as 17?

The large rectangle is labeled 17, but it is also obtained by joining the two smaller rectangles labeled x and 4.

Problem 2

Diego is trying to represent the equation $5 \cdot x = 35$. He draws this diagram but is not certain how to proceed.

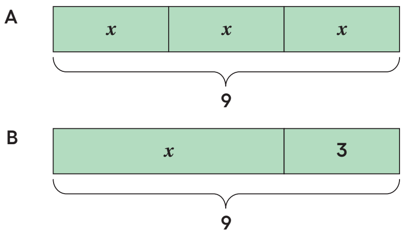


Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

Problem 3

Match each equation to one of the two tape diagrams.

- $x + 3 = 9$ B
- $3 \cdot x = 9$ A
- $9 = 3 \cdot x$ A
- $3 + x = 9$ B
- $x = 9 - 3$ B
- $x = 9 \div 3$ A
- $x + x + x = 9$ A



Problem 4

For each equation, draw a tape diagram that represents the same relationship. Then explain what the letter x represents in each relationship.

a. $x + 9 = 16$

Sample response: Students draw a tape diagram showing one part labeled 9 and another labeled x and a total of 16. The letter x represents the number that when added to 9 makes a total of 16.

b. $4 \cdot x = 28$

Sample response: Students draw a tape diagram showing 4 groups labeled x and a total of 28. The letter x represents the number that when multiplied by 4 makes a total of 28.

Problem 5

from Unit 5, Lesson 13

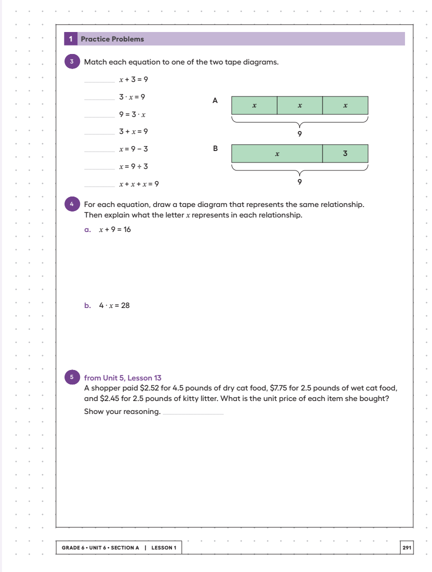
A shopper paid \$2.52 for 4.5 pounds of dry cat food, \$7.75 for 2.5 pounds of wet cat food, and \$2.45 for 2.5 pounds of kitty litter. What is the unit price of each item she bought? Show your reasoning.

Dry cat food costs \$0.56 per pound, wet cat food costs \$3.10 per pound, and kitty litter costs \$0.98 per pound.

Sample reasoning:

- $2.52 \div 4.5 = 252 \div 450$, which equals 0.56.
- $7.75 \div 2.5 = 775 \div 250$, which equals 3.1 or 3.10.
- $2.45 \div 2.5 = 245 \div 250$, which equals 0.98.

Student Workbook


1 Practice Problems

3 Match each equation to one of the two tape diagrams.

$x + 3 = 9$
$3 \cdot x = 9$
$9 = 3 \cdot x$
$3 + x = 9$
$x = 9 - 3$
$x = 9 \div 3$
$x + x + x = 9$

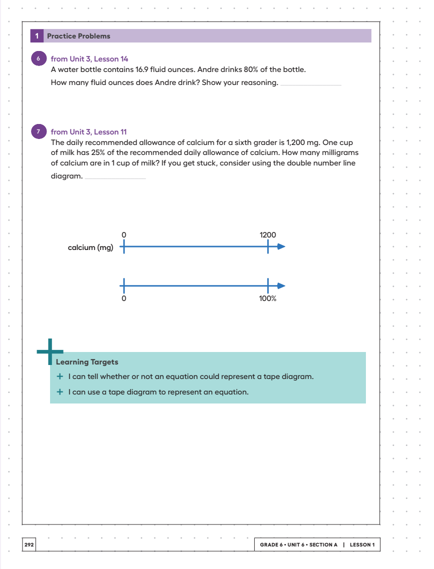
4 For each equation, draw a tape diagram that represents the same relationship. Then explain what the letter x represents in each relationship.

a. $x + 9 = 16$

b. $4 \cdot x = 28$

5 from Unit 5, Lesson 13
A shopper paid \$2.52 for 4.5 pounds of dry cat food, \$7.75 for 2.5 pounds of wet cat food, and \$2.45 for 2.5 pounds of kitty litter. What is the unit price of each item she bought? Show your reasoning.

Student Workbook


1 Practice Problems

6 from Unit 3, Lesson 14
A water bottle contains 16.9 fluid ounces. Andre drinks 80% of the bottle. How many fluid ounces does Andre drink? Show your reasoning.

7 from Unit 3, Lesson 11
The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in 1 cup of milk? If you get stuck, consider using the double number line diagram.

calcium (mg) 0 1200
0 100%

Learning Targets
+ I can tell whether or not an equation could represent a tape diagram.
+ I can use a tape diagram to represent an equation.

GRADE 6 • UNIT 6 • SECTION A | LESSON 1

Problem 6

from Unit 3, Lesson 14

A water bottle contains 16.9 fluid ounces. Andre drinks 80% of the bottle. How many fluid ounces does Andre drink? Show your reasoning.

13.52 fluid ounces

Sample reasoning: $(0.8) \cdot (16.9) = 13.52$

Problem 7

from Unit 3, Lesson 11

The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in 1 cup of milk? If you get stuck, consider using the double number line diagram.

300 mg

Sample reasoning: