How Much for One?

Goals

- Calculate equivalent ratios between prices and quantities and present the solution method (using words and other representations).
- Calculate unit price and express it using the word "per" (orally and in writing).
- Understand that the phrase "at this rate" indicates that equivalent ratios are involved.

Learning Targets

- I can choose and create diagrams to help me reason about prices.
- I can explain what the phrase "at this rate" means, using prices as an example.
- If I know the price of multiple things, I can find the price per thing.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

Math Talk

Required Materials

Materials to Gather

- · Rulers: Lesson
- Tools for creating a visual display:
- · Math Community Chart: Warm-up
- Rulers: Activity 1, Activity 2

Lesson Narrative

This lesson introduces students to the idea of unit price. Students use the word "per" to refer to the cost of 1 apple, 1 pound, 1 bottle, 1 ounce, and so on, as in "\$6 per pound" or "\$1.50 per avocado." They also learn that the phrase "at this rate" is used to indicate that the ratios of price to quantity are equivalent. (For example, "Pizza costs \$1.25 per slice. At this rate, how much do 6 slices cost?") They find unit prices in different situations and notice that unit prices are useful for computing prices for other amounts.

Students choose whether to draw double number lines or other representations to support their reasoning. They continue to use precision in stating the units that go with the numbers in a ratio in both verbal statements and diagrams.

Note that students are not expected to use or understand the term "unit rate" in this lesson.

Math Community

The goal of today's exercise is to use the suggestions from the previous week's Cool-down to revise the "Norms" sections of the Math Community Chart and to invite students to reflect on one norm that will be a strength for them. Both activities begin to build shared accountability for and investment in the classroom norms.

Student Learning Goal

Let's use ratios to describe how much things cost.

Lesson Timeline

10 Warm-up 10

Activity 1

15

Activity 2

10

Lesson Synthesis

Assessment

Cool-down

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook How Much for One? Let's use ratios to describe how much things cost. | Various of Math. Talk. Dividing by 12 | Find the value of each quotient mentally. | û 4 + 12 | û 6 + 12 | û 30 = 12 | û 30 = 12

Warm-up

Math Talk: Dividing by 12



Activity Narrative

This *Math Talk* focuses on division by a two-digit number. It encourages students to think about the numbers in a computation problem and to rely on what they know about numbers in base-ten, patterns, division with remainders, and the relationship between multiplication and division to mentally find quotients. To divide larger numbers prompts students to look for and make use of structure.

Notice how students handle a remainder in a problem, which may depend on their prior experiences with division. In the next lesson, when students begin finding unit price, they will need to be able to interpret non-whole-number quotients in either decimal or fraction form.

Launch

Reveal one problem at a time. For each problem,

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each quotient mentally.

A.24 ÷ 12

2

Sample reasoning: $2 \cdot 12 = 24$

B.6 ÷ 12

 $\frac{1}{2}$ or 0.5

Sample reasoning:

- $\circ \frac{1}{2} \cdot 12 = 6$
- Two groups of 6 make 12, so one group of 6 is half of 12.

C. 30 ÷ 12

 \circ 2 $\frac{1}{2}$ or 2.5

Sample reasoning: There are 2 groups of 12 in 24, and $\frac{1}{2}$ group of 12 in 6. Thirty is 24 + 6, so there are 2 + $\frac{1}{2}$, or 2 $\frac{1}{2}$, groups of 12 in 30.

D.246 ÷ 12

 $20\frac{1}{2}$ or 20.5

Sample reasoning:

- 20 · 12 = 240 and $\frac{1}{2}$ · 12 = 6, so $20\frac{1}{2}$ · 12 = 246.
- 120 ÷ 12 is 10, so 240 ÷ 12 is 20. Adding 6 ÷ 12, which is $\frac{1}{2}$, gives 20 $\frac{1}{2}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Did anyone have the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

If students express the result of the last two divisions with "2 with a remainder of 6" and "20 with a remainder of 6," respectively, ask them if the 6 could be divided by 12, or remind them that they divided 6 by 12 in a preceding problem.

At the end of discussion, if time permits, ask a few students to share a story problem or context that $246 \div 12 = 20.5$ could represent.

After the *Warm-up*, display the Math Community Chart and a list of 2–5 revisions suggested by the class in the previous exercise for all to see. Remind students that norms are agreements that everyone in the class shares responsibility for, so everyone needs to understand and agree to work on upholding the norms. Briefly discuss any revisions and make changes to the "Norms" sections of the chart as the class agrees. Depending on the level of agreement or disagreement, it may not be possible to discuss all suggested revisions at this time. If that happens, plan to discuss the remaining suggestions over the next few lessons.

Tell students that the class now has an initial list of norms or "hopes" for how the classroom math community will work together throughout the school year. This list is just a start, and over the year it will be revised and improved as students in the class learn more about each other and about themselves and math learners.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because...." or "I noticed _____ so I...." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.

Invite students to share examples of items that they have seen sold in quantities greater than one or their experience buying such items. Ask them to reflect on possible reasons for buying a large quantity of something.

Supports accessibility for: Conceptual Processing, Memory

Building on Student Thinking

Some students may have difficulty with the answers not being integers. Either fractions or decimals are acceptable. Fractions provide the most direct route, but decimals are common for working with dollars and cents. Also, students may use the larger numbers as the dividend, simply because they are larger. Encourage students to check the reasonableness of their answers.

Activity 1

Grocery Shopping



Activity Narrative

There is a digital version of this activity.

In this activity, students continue to work with ratios that involve one unit of something. They determine the prices of grocery items and learn to use the term **unit price** to describe cost per unit. To determine unit prices, students may:

- Divide the cost by the number of items.
- · Use discrete diagrams.
- Use a double number line diagram.

As students work, monitor for students who use different methods. If students choose to draw a double number line diagram, remind them to label each number line and to circle the ratio where they find the answer.

In the digital version of the activity, students can choose to use an applet to create a double number line to find the unit price in different situations. The applet allows students to name the quantities, specify a ratio, and partition the double number line to show larger or smaller amounts that represent an equivalent ratio.

Launch

Frame the task in shopping terms. Say that when most of us go shopping, we often see prices for multiple items or units (for example, 2 bottles for \$3, or \\$1.99 for 3 pounds). Sometimes, however, we want to know how much it costs to buy a quantity different from what is posted on the price tag (for example, 5 bottles or 8 pounds of something). Tell students they will explore ways to solve such problems.

Arrange students in groups of 2-4.

Give students a few minutes of quiet think time for the first problem about avocados.

Provide access to rulers in case students choose to draw double number lines. Then, pause for a discussion.

Invite students to share their approach for each part of the problem, recording or displaying their reasoning for all to see. Focus the discussion on how they reasoned about the price of 9 avocados. If no students mentioned finding the price of a single avocado, or dollars per avocado, ask how it could be determined. Consider using a double number line diagram to illustrate students' explanations.

If any students say that the price for an avocado is \$2, ask what the cost of 8 avocados would be in that case. A double number line diagram can also help students see that 2 is the number of avocados that can be bought with 1 dollar, rather than the number of dollars for 1 avocado.

Encourage students to continue thinking about the use of "per" as they reason about the remaining problems about prices.

Student Task Statement

Answer each question and explain or show your reasoning. If you get stuck, consider drawing a double number line diagram.

- 1. Eight avocados cost \$4.
 - a. How much do 16 avocados cost?

\$8

Sample reasoning: Sixteen is twice 8, so the cost of 16 avocados is twice \$4, which is \$8.

b. How much do 20 avocados cost?

\$10

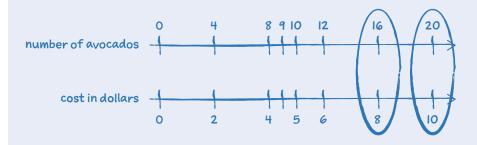
Sample reasoning: Twenty is 4 more than 16. We already know 16 avocados cost \$8. Four is half of 8, so the cost for 4 avocados is half of \$4, which is \$2. Adding 8 and 2 gives 10.

c. How much do 9 avocados cost?

\$4.50

Sample reasoning:

- If 8 avocados cost \$4, then I avocado costs \$4 ÷ 8 or \$0.50. Adding \$0.50 to the cost of 8 avocados, which is \$4, makes \$4.50.
- The cost of 9 avocados is halfway between \$4 and \$5, as shown on the double number line diagram.



- 2. Twelve large bottles of water cost \$9.
 - a. How many bottles can you buy for \$3?

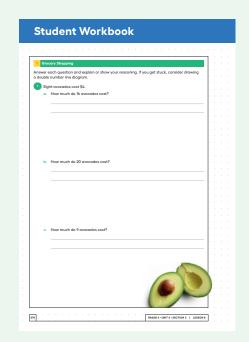
4 bottles

Sample reasoning: Three is a third of 9, so we can buy a third of the 12 bottles, which is 4 bottles, since $\frac{1}{3} \cdot 12 = 4$.

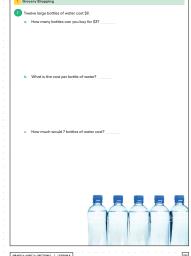
b. What is the cost per bottle of water?

\$0.75

Sample reasoning: $9 \div 12 = 0.75$







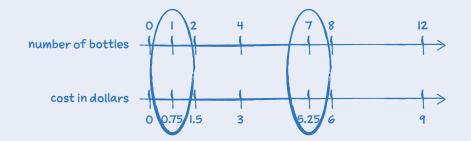


c. How much would 7 bottles of water cost?

\$5.25

Sample reasoning:

- \circ 7 · (0.75) = 5.25
- Using a double number line diagram, I can see that it's \$0.75 less than the cost of 8 bottles, which cost \$6, so it's 6 0.75 or 5.25.



- **3.** A 10-pound sack of flour costs \$8.
 - a. How much does 40 pounds of flour cost?

\$32

Sample reasoning: $10 \cdot 4 = 40$ and $8 \cdot 4 = 32$

b. What is the cost per pound of flour?

\$0.80

Sample reasoning: $8 \div 10 = 0.8$

Are You Ready for More?

It is commonly thought that buying larger packages or containers, sometimes called "buying in bulk," is a great way to save money. For example, a 6-pack of beverage might cost \$3 while a 12-pack of the same brand costs \$5.

Find 3 different cases where it is not true that buying in bulk saves money. You may use the internet or go to a local grocery store and take photographs of the cases you find. Make sure the products are the same brand. For each example that you find, give the quantity or size of each, and describe how you know that the larger size is not a better deal.

Answers vary.

Activity Synthesis

Select students who used unique methods to share their reasoning, as listed in the *Activity Narrative*. If no one used double number lines, represent one of the statements with a double number line diagram and display it for all to see. Although double number lines are not required in the task, their use in the context of problem situations helps students see their merits and illustrates how they might be used in other problems, especially as students transition from unit prices to constant speed and other contexts. Draw connections between the double number line strategy and the dividing by the numbers of items strategy.

Tell students that each "cost per one" unit being sold—avocado, pound, or bottle—is an example of a **unit price**. Ask them to name as many kinds of unit prices as they can and to think of a situation in which they might be used, starting with the list from the task:

- Cost per avocado
- Cost per pound
- · Cost per bottle

Other possibilities include cost per liter, cost per ounce, cost per jelly bean, and so on.

Activity 2

More Shopping

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students practice finding unit prices, using different reasoning strategies, and articulating their reasoning. They also learn about the term "at this rate."

For each problem, select 1–2 students or groups to share their solution and strategy later.

In the digital version of the activity, students can choose to use an applet to create a double number line to explore the problems.

Launch

Display the problem and read it aloud: Pizza costs \$1.25 per slice. At this rate, how much will 6 slices cost?

Ask students what they think "at this rate" means in the question. Ensure that they understand that "at this rate" means we know that equivalent ratios are involved:

- The ratio of cost to number of slices is 1.25 to 1 or \$1.25 per slice.
- The ratio of cost to number of slices is *something* to 6. That is, pizza costs something for 6 slices.

The *something* is the thing we are trying to figure out, and "at this rate" tells us that the two ratios in this situation are equivalent.

Building on Student Thinking

The first and third questions involve using decimals to represent cents. If the decimal point is forgotten, remind students that the cost of the bracelet is less than one dollar, and the cost of the chips is between one and two dollars.

Watch for students working in cents instead of dollars for the bracelets. They may come up with an answer of 275 cents. For these students, writing 25 cents as \$0.25 should help, or consider reminding them of the avocados from a previous activity, which had a unit price of \$0.50.

Student Workbook



Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect

After all problems have been discussed, lead a discussion comparing, contrasting, and connecting the different approaches. Ask questions such as:

"What is alike or different in how diagrams are used to solve the problems?" "The phrase 'at this rate' is used in all problems. What does it refer to in each situation?" "Where can we see the unit price in each diagram? What does it mean in each situation?" "What words or phrases did each student or group use to talk about unit price?"

Advances: Representing, Conversing

Another way to understand "at this rate" in this context is "at this price per unit" and that the price per unit is the same no matter how many items or units are purchased.

Keep students in the same groups. Explain that they will work together to solve some shopping problems and can use double number lines if they wish.

Student Task Statement

- 1. Four bags of chips cost \$6.
 - a. What is the cost per bag?

The cost per bag is \$1.50.

b. At this rate, how much will 7 bags of chips cost?

Seven bags cost \$10.50.

- 2. At a used book sale, 5 books cost \$15.
 - a. What is the cost per book?

The cost per book is \$3.

b. At this rate, how many books can you buy for \$21?

We can buy 7 books for \$21.

- 3. Neon bracelets cost \$1 for 4.
 - a. What is the cost per bracelet?

The cost per bracelet is 25 cents.

b. At this rate, how much will 11 neon bracelets cost?

Eleven bracelets cost \$2.75.

Activity Synthesis

Invite previously selected students or groups to share their work. Highlight accurate uses of the terms "at this rate" and "per," and the ways in which a double number line might have been used when working with unit price.

Lesson Synthesis

The main ideas to develop in this lesson are techniques for finding a unit price and how knowing that value can help us. Consider asking students:

"Suppose 2 bags of rice cost \$3. What are some strategies we can use to find the unit price?"

Likely strategies:

- Using division: Find 3 ÷ 2, which is 1.50.
- Using a double number line diagram: Mark \$3 for the price and 2 for the number of bags, add a tick mark for 1 bag on one number line, and find the dollar value at that same position on the other number line.
- "Why might it be helpful to find a unit price? For example, how is knowing that rice costs \$1.50 per bag useful?"

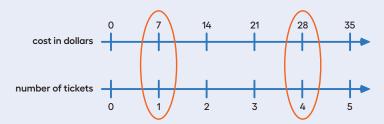
We can compute the cost of any number of items directly, by multiplying the unit price by the number of items.

Point out that when we multiply a unit price by a value to find another price, we are finding a number in an equivalent ratio. For example, the ratio "\$30 for 20 bags" is equivalent to the ratio "\$3 for 2 bags."

Lesson Summary

The **unit price** is the price of 1 thing—for example, the price of 1 ticket, 1 slice of pizza, or 1 kilogram of peaches.

If 4 movie tickets cost \$28, then the unit price would be the cost *per* ticket. We can create a double number line to find the unit price.

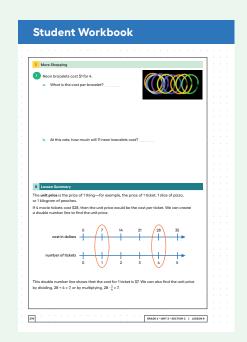


This double number line shows that the cost for 1 ticket is \$7. We can also find the unit price by dividing, $28 \div 4 = 7$, or by multiplying, $28 \cdot \frac{1}{4} = 7$.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the question "What is one of our classroom norms that is a strength for you? Why?" Tell students that as a culmination to establishing the initial list of mathematical community norms, they are now asked to share one norm they think will be a strength for them. To help students understand what the question is asking, share a personal example. For example,

"I think that 'Ask clarifying questions' is a norm that is a strength for me because I am good at asking questions when I don't think I understand how someone else is thinking about a problem. Instead of just telling you what I think you should do, I make sure to ask questions until I understand what YOU are doing."



Responding To Student Thinking

Points to Emphasize

If students struggle with calculating a unit rate, highlight unit rates in context and ways to reason about them. For example, in this activity, focus on how to find the distances traveled in 1 second:

Unit 2, Lesson 9, Activity 3 Moving for 10 Seconds

Display these prompts for all to see:

One of our classroom norms that will be a strength for me is ___.

I think this will be a strength for me because ____.

Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify which norms students feel more confident about and which norms were not listed as strengths by many students. In some cases, students may not think a norm is a strength because they are not sure what that norm looks like or sounds like. So, focus on identifying those norms in the class when they happen.

For example, during group work students ask a quiet group member which representation they prefer, and that student shares a third representation that the group had not even considered. Asking the quiet student illustrates a norm like "we invite others into the math." Pointing out that action when it happens helps students understand the norm and see how it can benefit the math thinking of the entire group. This understanding and appreciation can promote the use of that norm in the math community.

Cool-down

Unit Price of Rice

5 min

Student Task Statement

Here is a double number line showing that it costs \$3 to buy 2 bags of rice:



- 1. At this rate, how many bags of rice can you buy for \$12?
 - 8 bags
- 2. Find the cost per bag.
 - \$1.50
- 3. How much do 20 bags of rice cost?

\$30

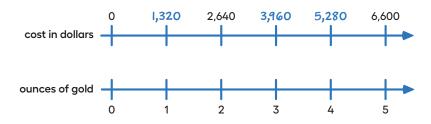
Sample reasoning: Multiply 20 by the price for one bag, or find a ratio of 20 bags to cost in dollars that is equivalent to 3 bags to 2 dollars.

Practice Problems

6 Problems

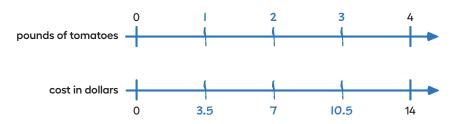
Problem 1

In 2016, the cost of 2 ounces of pure gold was \$2,640. Complete the double number line to show the cost for 1, 3, and 4 ounces of gold.



Problem 2

The double number line shows that 4 pounds of tomatoes cost \$14. Draw tick marks and write labels to show the prices of 1, 2, and 3 pounds of tomatoes.



Problem 3

4 movie tickets cost \$48. At this rate, what is the cost of:

a. 5 movie tickets?

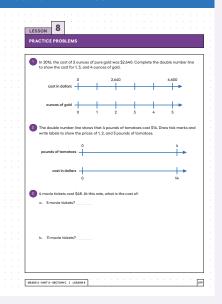
\$60

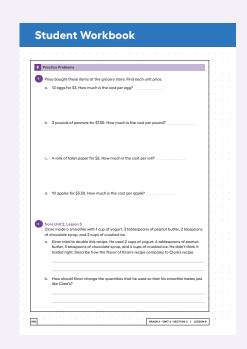
One ticket costs \$12 because $48 \div 4 = 12$. Five tickets cost \$60 because $5 \cdot 12 = 60$.

b. 11 movie tickets?

\$132, because $11 \cdot 12 = 132$

Student Workbook





Problem 4

Priya bought these items at the grocery store. Find each unit price.

- a. 12 eggs for \$3. How much is the cost per egg?25 cents or \$0.25
- **b.** 3 pounds of peanuts for \$7.50. How much is the cost per pound?
- c. 4 rolls of toilet paper for \$2. How much is the cost per roll?50 cents or \$0.50
- d. 10 apples for \$3.50. How much is the cost per apple?35 cents or \$0.35

Problem 5

from Unit 2, Lesson 3

Clare made a smoothie with 1 cup of yogurt, 3 tablespoons of peanut butter, 2 teaspoons of chocolate syrup, and 2 cups of crushed ice.

a. Kiran tried to double this recipe. He used 2 cups of yogurt, 6 tablespoons of peanut butter, 5 teaspoons of chocolate syrup, and 4 cups of crushed ice. He didn't think it tasted right. Describe how the flavor of Kiran's recipe compares to Clare's recipe.

Sample response: Kiran's smoothie would be more chocolatey than Clare's. All ingredients are doubled, but there is an extra teaspoon of chocolate syrup in his smoothie.

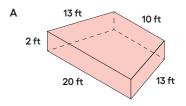
b. How should Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

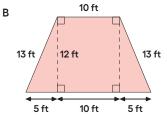
Sample response: He should use 4 teaspoons of chocolate syrup instead of 5.

Problem 6

from Unit 1, Lesson 15

A drama club is building a wooden stage in the shape of a trapezoidal prism. The height of the stage is 2 feet. Some measurements of the stage are shown here.





What is the area of all the faces of the stage, excluding the bottom? Show your reasoning. If you get stuck, consider drawing a net of the prism.

292 square feet

Sample reasoning: The trapezoidal face is I8O square feet since $(12 \cdot 10) + 2(\frac{1}{2} \cdot 12 \cdot 5) = 120 + 60 = 180$.

The side faces are $2(13 \cdot 2) + (10 \cdot 2) + (20 \cdot 2)$, or II2 square feet.

LESSON 8 • PRACTICE PROBLEMS