Positive and Negative Numbers

Goals

- Comprehend the words "positive" and "negative" (in spoken and written language) and the symbol "-" (in written language). Say "negative" when reading numbers written with the "-" symbol.
- Interpret positive and negative numbers that represent temperature or elevation, and understand the convention of what "below zero" typically means in each of these contexts.
- Recognize that the number line can be extended to represent negative numbers.

Learning Targets

- I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation.
- I can use positive and negative numbers to describe temperature and elevation.
- I know what positive and negative numbers are.

Lesson Narrative

In this lesson, students extend their understanding of the number system from just **positive numbers**, numbers greater than 0, to also include **negative numbers**, numbers less than 0. By exploring temperature and elevation, students see a need to describe numbers less than 0. In these two contexts, zero represents a physical situation (the freezing point of water and sea level), and numbers less than zero describe a physical state in the real world. Students reason quantitatively and abstractly as they represent temperatures and elevations with positive and negative numbers on a number line.

Student Learning Goal

Let's explore how we represent temperatures and elevations.

Lesson Timeline

5_{min}

Warm-up

10 min

Activity 1

20 min

Activity 2

10 min

Lesson Synthesis

Access for Students with Diverse Abilities

• Engagement (Activity 2)

Access for Multilingual Learners

• MLR5: Co-Craft Questions (Activity 2)

Instructional Routines

· Notice and Wonder

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Assessment

5_{min}

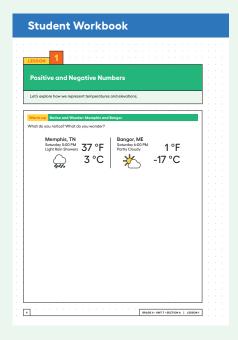
Cool-down

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.





Warm-up

Notice and Wonder: Memphis and Bangor



Activity Narrative

The purpose of this *Warm-up* is to introduce students to temperatures measured in degrees Celsius, which will be useful when students read thermometers using the Celsius scale in a later activity. Many students have an intuitive understanding of temperature ranges in degrees Fahrenheit that are typical of the city or town in which they live, but many are unfamiliar with the Celsius scale.

While students may notice and wonder many things about these images, the idea that there are different scales for measuring temperature is an important discussion point.

Launch



Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?

Memphis, TN Saturday 5:00 PM

Light Rain Showers 3

Bangor, ME Saturday 6:00 PM Partly Cloudy



1°F 7°⊂

Students may notice:

- · The weather is different in the two cities.
- There are two temperatures for each city.
- The temperatures have different letters.
- · The times are different.
- There is a minus sign on one of the temperatures.

Students may wonder:

- · Where are these two cities located on a map?
- How far are either of these cities from us?
- · What do the two temperatures mean?
- Why isn't it snowing in the colder city?
- · What is the minus sign doing there?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Explain to students that temperatures are usually measured in either degrees Fahrenheit, which is what they are probably most familiar with, or degrees Celsius, which may be new for them. Tell them that many other countries measure temperature in degrees Celsius and that scientists often use this temperature scale. One thing that is special about the Celsius scale is that at sea level, water freezes at 0 degrees and boils at 100 degrees.

Activity 1

Above and Below Zero

10 min

Activity Narrative

There is a digital version of this activity.

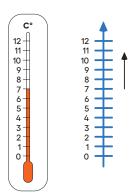
In this task, students are introduced to certain contexts for which it would be difficult to answer mathematical questions if restricted to only positive numbers. The need for negative numbers leads to the natural representation of them on the number line.

This task is not about operations with signed numbers, but rather why we extend our number system beyond positive numbers. Students reason abstractly and quantitatively when they represent the change in temperature on a number line.

In the digital version of the activity, students use an applet to represent changes in temperature. The applet allows students to quickly represent an increase or decrease in temperature.



Display this image for all to see.



Tell students,

"The thermometer showed a temperature of 7 degrees Celsius one morning. Later, the temperature increased 4 degrees. We can use a vertical number line to represent this change in temperature."

Arrange students in groups of 2.

Give students 2 minutes of quiet work time for the first question. Then give students 2 minutes to discuss the second question with their partner, and follow with a whole-class discussion.

Student Task Statement

- **1.** Here are three number lines and three situations involving changes in temperature. Represent the change in temperature for each situation on a number line, and then answer the question.
 - **a.** At noon, the temperature was 5 degrees Celsius. By late afternoon, it had risen 6 degrees Celsius. What was the temperature late in the afternoon?

II degrees Celsius

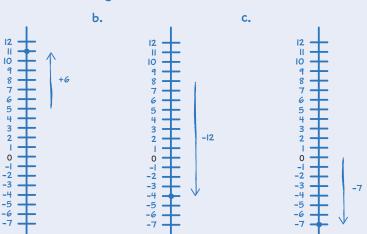
b. The temperature was 8 degrees Celsius at midnight. By dawn, it had dropped 12 degrees Celsius. What was the temperature at dawn?

4 degrees below 0 or -4 degrees Celsius

c. Water freezes at 0 degrees Celsius, but the freezing temperature can be lowered by adding salt to the water. A student discovered that adding half a cup of salt to a gallon of water lowers its freezing temperature by 7 degrees Celsius. What is the freezing temperature of the gallon of salt water?

7 degrees below 0 or -7 degrees Celsius

a.



2. Discuss with a partner:

- a. How did each of you name the resulting temperature in each situation?
 Sample response: II degrees Celsius, 4 degrees below zero or -4 degrees
 Celsius, 7 degrees below zero or -7 degrees Celsius
- **b.** How do temperatures above 0 compare to temperatures below 0?

 Sample response: Temperatures below 0 are colder than temperatures above 0.
- **c.** Come up with an example other than temperature where numbers below 0 make sense.

Sample response: It's possible to go underground, like in a cave or a basement. If ground level is 0, underground would be below 0.

d. Come up with an example where numbers below 0 do not make sense.

Sample response: When counting things like students or pencils, numbers below 0 don't make sense.

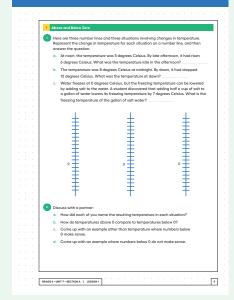
Building on Student Thinking

If students count tick marks rather than the space between tick marks, and they include the starting tick mark in their count, consider asking:

"How did you represent the change in temperature on your number line?"

"What would a temperature change of 1 degree Celsius look like on a number line?"

Student Workbook



Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions

Keep books or devices closed. Display only the table of elevations from the *Task Statement*, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Activity Synthesis

The purpose of this discussion is to introduce a **negative number** as a number that is less than zero, in contrast to a **positive number** as a number greater than zero. Some students will have a pre-existing understanding of positive and negative numbers.

Begin by inviting several students to share the different ways they described the final temperature in the second situation (4 degrees below zero, -4 degrees Celsius). Tell students that this is an example of a negative number because it describes a number less than zero, and we use the - symbol to show that.

Ask students how they would describe a number that is greater than zero (a positive number). Explain that a + symbol is used to indicate a positive number, though it is not always written out. For example, +7 and 7 both represent positive 7. Negative 7 is represented as -7.

Activity 2

High Places, Low Places



Activity Narrative

There is a digital version of this activity.

This activity presents a second, natural context for negative numbers, and students start comparing positive and negative numbers in preparation for ordering them in a following activity. Students may use the structure of a vertical number line in order to compare the relative location of each elevation.

In the digital version of the activity, students use an applet to represent the elevations of the highest points on land and lowest points in the ocean on a vertical number line. The applet allows students to drag points to a vertical number line to mark different mountains or trenches and quickly check their answers.

Launch



Display the table of elevations for all to see. Ask students to think of a way to explain in their own words what the numbers mean, and invite 2–3 students to share their ideas.

If not mentioned in students' explanations, tell students that the term "elevation" is commonly used to describe the height of a place (such as a city) or an object (such as an aircraft) compared to sea level. Denver, CO, is called "The Mile High City" because its elevation is 1 mile, or 5,280 feet, above sea level.

Arrange students in groups of 2, and give students 5 minutes of quiet work time to answer the first set of questions.

Ask students to be prepared to explain their thinking in a wholeclass discussion.

Students using the digital activity are provided with an interactive map in addition to the questions about elevation. Pause for a whole-class discussion to ensure that students understand what is meant by an elevation that is below sea level.

Then give students 5 more minutes of quiet work time to finish the remaining questions.

Student Task Statement

1. The table shows the elevations of various cities.

city	elevation (feet)
Harrisburg, PA	320
Bethell, IN	1,211
Denver, CO	5,280
New Orleans, LA	-8
Death Valley, CA	-282
New York City, NY	33
Miami, FL	0

a. Which city on the list has the second highest elevation?

Bethell, IN

b. How would you describe the elevation of Harrisburg, PA in relation to sea level?

320 feet above sea level

c. How would you describe the elevation of Death Valley, CA in relation to sea level?

282 feet below sea level

d. How would you describe the elevation of Miami, FL?

Miami, FL, is at sea level.

e. A different city not on this list has a higher elevation than New Orleans,
 LA. Select all numbers that could represent the new city's elevation.
 Be prepared to explain your reasoning.

A.-11 feet

B. 3 feet

C.-4 feet

D.-9 feet

E. 0 feet

Sample reasoning: 3 feet is above sea level, so it is higher in elevation than anything below sea level. O feet is exactly at sea level, so it is also higher in elevation than anything below sea level. -4 feet is below sea level, but is closer to the surface than -8 feet, so it also has a higher elevation.

f. Pause here for a whole-class discussion.

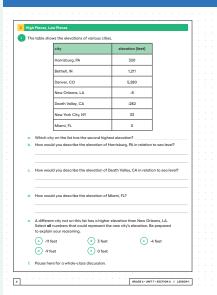
Access for Students with Diverse Abilities (Activity 1, Student Task)

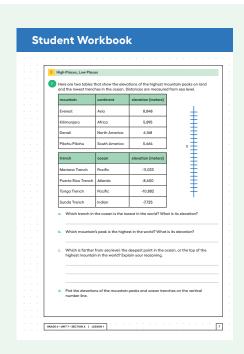
Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Have students complete the first problem, then plot the elevations of high points, and then plot the elevations of low points. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook





2. Here are two tables that show the elevations of the highest mountain peaks on land and the lowest trenches in the ocean. Distances are measured from sea level.

mountain	continent	elevation (meters)
Everest	Asia	8,848
Kilimanjaro	Africa	5,895
Denali	North America	6,168
Pikchu Pikchu	South America	5,664

trench	ocean	elevation (meters)
Mariana Trench	Pacific	-11,033
Puerto Rico Trench	Atlantic	-8,600
Tonga Trench	Pacific	-10,882
Sunda Trench	Indian	-7,725

a. Which trench in the ocean is the lowest in the world? What is its elevation?

Mariana Trench in the Pacific Ocean; -II,033 meters

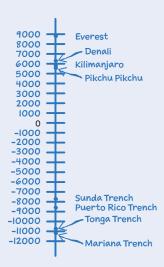
b. Which mountain's peak is the highest in the world? What is its elevation?

Mt. Everest in Asia; 8,848 meters

c. Which is farther from sea level: the deepest point in the ocean, or the top of the highest mountain in the world? Explain your reasoning.

The deepest point in the ocean is farther from sea level. Sample reasoning: -II,033 meters is farther from 0 on a number line than 8,848 meters is.

d. Plot the elevations of the mountain peaks and ocean trenches on the vertical number line.



Are You Ready for More?

A spider spins a web in the following way:

- It starts at sea level.
- It moves up one inch in the first minute.
- It moves down two inches in the second minute.
- It moves up three inches in the third minute.
- It moves down four inches in the fourth minute.

Assuming that the pattern continues, what will the spider's elevation be after an hour has passed?

30 inches below sea level.

The elevation after each minute is +1, -1, +2, -2, etc.

Activity Synthesis

above sea level.

below sea level.

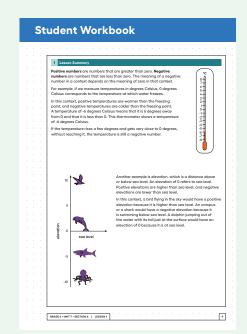
The goal of this discussion is for students to compare negative elevations. Begin by displaying a blank vertical number line for all to see.



Ask students where the elevation of New Orleans, LA could be on this number line, and plot and label the point for all to see. Continue adding points to the number line for the 5 elevations given for the new city not on the list (-11 feet, 3 feet, -4 feet, -9 feet, 0 feet). Discuss the following questions:

- "What does it mean when a point is above 0 on a vertical number line?"
 The number is positive, and the elevation represented by that point is
- "What does it mean when a point is below 0 on a vertical number line?"
 The number is negative, and the elevation represented by that point is
- "Is -11 feet higher or lower in elevation than -8 feet?"
 - -II is lower in elevation because it is lower than -8 on the vertical number line.
- "Is -4 feet higher or lower in elevation than -8 feet?"
 - -4 is higher in elevation because it is higher than -8 on the vertical number line.





Lesson Synthesis

The goal of this discussion is for students to explain what zero represents in certain contexts and how that affects the meaning of positive and negative numbers.

Ask students to recall the situations from earlier activities (temperature in degrees Celsius and elevation). Ask students:

"What does zero represent in each situation?"
freezing point of water, sea level

"What does a positive number represent in each situation?"
temperatures above freezing, elevations above sea level

"Where would positive numbers be located on a vertical number line?"
above zero, on the same side as I

"What does a negative number represent in each situation?"
temperatures below freezing, elevations below sea level

"Where would negative numbers be located on a vertical number line?"
below zero, on the same side as -I

Lesson Summary

Positive numbers are numbers that are greater than zero. **Negative numbers** are numbers that are less than zero. The meaning of a negative number in a context depends on the meaning of zero in that context.

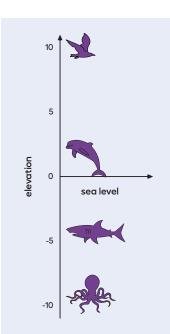
For example, if we measure temperatures in degrees Celsius, 0 degrees Celsius corresponds to the temperature at which water freezes.

In this context, positive temperatures are warmer than the freezing point, and negative temperatures are colder than the freezing point.

A temperature of -6 degrees Celsius means that it is 6 degrees awa from 0 and that it is less than 0. This thermometer shows a temperature of -6 degrees Celsius.

If the temperature rises a few degrees and gets very close to 0 degrees, without reaching it, the temperature is still a negative number.





Another example is elevation, which is a distance above or below sea level. An elevation of 0 refers to sea level. Positive elevations are higher than sea level, and negative elevations are lower than sea level.

In this context, a bird flying in the sky would have a positive elevation because it is higher than sea level. An octopus or a shark would have a negative elevation because it is swimming below sea level. A dolphin jumping out of the water with its tail just at the surface would have an elevation of 0 because it is at sea level.

Cool-down

Agree or Disagree?

5 min

Student Task Statement

State whether you agree with each of the following statements. Explain your reasoning.

1. A temperature of 35 degrees Fahrenheit is as cold as a temperature of -35 degrees Fahrenheit.

Disagree

Sample reasoning: 35 degrees Fahrenheit is above 0 degrees Fahrenheit, and -35 degrees Fahrenheit is below 0 degrees Fahrenheit. -35 degrees is 70 degrees colder than 35 degrees.

2. A city that has an elevation of 15 meters is closer to sea level than a city that has an elevation of -10 meters.

Disagree

Sample reasoning: -10 meters is 10 meters from sea level. 15 meters is 15 meters from sea level. -10 meters is closer to sea level.

3. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.

Agree

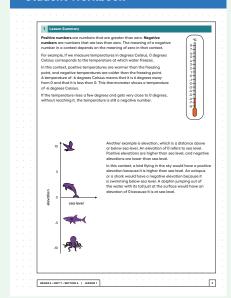
Sample reasoning: -17 meters is 17 meters from sea level. -40 meters is 40 meters from sea level. -17 meters is closer to sea level.

Responding To Student Thinking

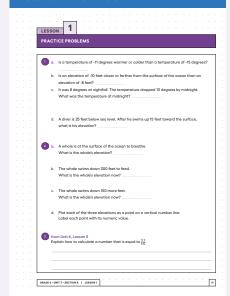
More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Student Workbook



Practice Problems

5 Problems

Problem 1

a. Is a temperature of -11 degrees warmer or colder than a temperature of -15 degrees?

warmer

b. Is an elevation of -10 feet closer or farther from the surface of the ocean than an elevation of -8 feet?

farther

- **c.** It was 8 degrees at nightfall. The temperature dropped 10 degrees by midnight. What was the temperature at midnight?
 - -2 degrees
- **d.** A diver is 25 feet below sea level. After he swims up 15 feet toward the surface, what is his elevation?
 - -10 feet, or 10 feet below sea level

Problem 2

a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?

0 feet

Sample reasoning: Sea level is 0 feet above or below sea level.

b. The whale swims down 300 feet to feed. What is the whale's elevation now?

-300 feet

Sample reasoning: The whale is 300 feet below sea level.

c. The whale swims down 150 more feet. What is the whale's elevation now?

-450 feet

Sample reasoning: The whale was 300 feet below sea level, and now it is an additional 150 feet below sea level.

d. Plot each of the three elevations as a point on a vertical number line. Label each point with its numeric value.

A number line with 0, -300, and -450 marked.

Problem 3

from Unit 6, Lesson 5

Explain how to calculate a number that is equal to $\frac{2.1}{1.5}$.

Sample response: $\frac{2.1}{1.5}$ means $2.1 \div 1.5$. This can be done by long division.

(The question doesn't require it, but the quotient is 1.4.)

Problem 4

from Unit 6, Lesson 4

Write an equation to represent each situation, and then solve the equation.

Accept all equivalent forms of each response.

a. Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x for the number of ounces of water the bottle holds.

$$\frac{3}{5}x = 15$$

Solution: 25

b. A bottle holds 15 ounces of water. Jada drank 8.5 ounces of water. How many ounces of water are left in the bottle? Use y for the number of ounces of water left in the bottle.

$$y + 8.5 = 15$$

Solution: 6.5

c. A bottle holds z ounces of water. A second bottle holds 16 ounces, which is $\frac{8}{5}$ times as much water. How much does the first bottle hold?

$$\frac{8}{5}z = 16$$

Solution: 10

Problem 5

from Unit 4, Lesson 13

A rectangle has an area of 24 square units and a side length of $2\frac{3}{4}$ units. Find the other side length of the rectangle. Show your reasoning.

$$8\frac{8}{11}$$
 (or equivalent)

Sample reasoning: $24 \div \frac{||}{4} = 24 \cdot \frac{4}{||} = \frac{96}{||} = 8\frac{8}{||}$

