What Are Percentages?

Goals

- Comprehend the word "percentage" (in written and spoken language) and the symbol "%" (in written language) to mean a rate per 100.
- Draw and label a double number line diagram to represent percentages of a dollar and to find corresponding monetary values or percentages.

Learning Targets

- I can create a double number line diagram with percentages on one line and dollar amounts on the other line.
- I can explain the meaning of percentages using dollars and cents as an example.

Lesson Narrative

This lesson introduces students to **percent** and **percentages**. They learn that percent means "per hundred" and that it describes the ratio of a quantity to 100.

Students begin with gridded 10-by-10 squares that each represent an area of 100 square units. They use "percent of the large square" to describe the shaded region in each diagram and are prompted to articulate their initial interpretation of the term "percent."

Next, students explore percentages as rates per 100 with money as the main context. They use their knowledge that there are 100 cents in a dollar to express the values of coins as percentages of a dollar. For example:

- The value of a nickel is 5 percent of a dollar because there are 5 cents in a nickel to 100 cents in a dollar.
- The value of 6 quarters is 150 percent of a dollar because there are 150 cents in 6 quarters to 100 cents in a dollar.

Access for Students with Diverse Abilities

• Engagement (Activity 1)

Access for Multilingual Learners

 MLR2: Collect and Display (Warm-up, Activity 1)

Instructional Routines

• MLR2: Collect and Display

Lesson Timeline



Warm-up



Activity 1



Activity 2



Lesson Synthesis

Assessment

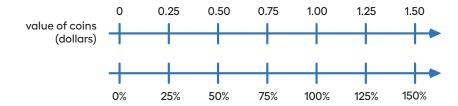


Cool-down

What Are Percentages?

Lesson Narrative (continued)

Double number line diagrams are used to develop the idea of a percentage as a rate per 100 and to make the connections to equivalent ratios. A double number line diagram can help students see, for instance, that when we refer to a percentage "of a dollar," \$1 is "100 per 100" or 100 percent of a dollar. Students can then reason that \$0.50 is "50 per 100" or 50 percent of a dollar, \$1.25 is 125 percent of a dollar, and so on.



In this lesson, 100% corresponds only to 100 units of a quantity (100 square units of area) and to 1 dollar (understood as equivalent to 100 cents). In upcoming lessons, students will explore percentages in which 100% corresponds to other values.

Student Learning Goal

Let's learn about percentages.

Inspire Math

Climbing Mount Everest video



First Optional Viewing: Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614204

Please log in to the site before using the QR code or URL.



Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754





Access for Multilingual Learners (Warm-up)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Warm-up

A Large Square of 100 Square Units



Activity Narrative

This Warm-up introduces the term "percent" in a geometric context and elicits students' initial ideas about its meaning. The work here enables students to approach the idea of "per 100" visually before they reason about it more abstractly later.

Students first quantify the area of a shaded region in several large gridded squares. Then, they describe each area relative to 100 square units, the area of a single large square, using the phrase "percent of the large square." The quantity being compared to 100 includes values less than, equal to, and greater than 100.

During class discussion, students offer their interpretations of the term "percent," which they will revisit in subsequent work.

Launch



Arrange students in groups of 2. Give students 2 minutes of quiet time to work on the first question. Ask them to pause for a discussion before completing the last question.

Display Diagrams A–E for all to see. Ask students to share the area of each shaded region and record it near the diagram. Tell students:

"We can say that in Diagram A, 1 percent of the large square is shaded, and in Diagram B, 15 percent of the large square is shaded."

Ask students:

○ "What percent of the large square is shaded in Diagram C?"

30 percent

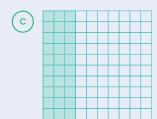
Repeat the question for Diagrams D and E (100 percent and 108 percent of the large square is shaded, respectively). Then, ask students to complete the last question.

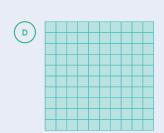
Student Task Statement

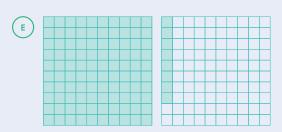
1. A large square represents an area of 100 square units. How many square units are shaded in each diagram?











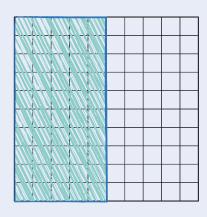
A: I square units, B: 15 square units, C: 30 square units, D: 100 square units, E: 108 square units

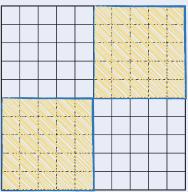
Pause here so your teacher can review your work.

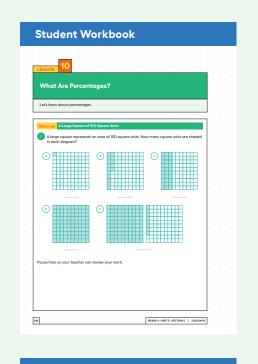
2. We can say that "1 percent of the large square" is shaded in diagram A and "15 percent of the large square" is shaded in diagram B.

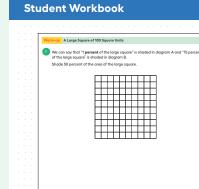
Shade 50 percent of the area of the large square.

Sample responses:









Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL



Access for Multilingual Learners (Activity 1)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Activity Synthesis

Invite students to share their depictions of 50 percent of the area of the large square and how they knew how much to shade.

Then, ask partners to discuss what they think the term "percent" means and invite them to share their initial ideas with the class.

Because the percent values in the sample statements match the number of shaded square units in the diagrams, some students may infer that "percent" is another name for square units. Others may think in terms of the ratio of the area of the shaded region to the area of a large square, relating the former to 100 square units.

Use Collect and Display to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe what "percent" means in the context of area. Display words and phrases such as "compared to 100," "out of 100 square units," "the ratio of shaded squares to 100 squares," or "how many shaded squares in a larger square of 100 squares."

Tell students that they will check and refine these interpretations in upcoming activities and lessons.

Activity 1

Coins



Activity Narrative

This activity builds students' understanding of "percent of ______" in a different context, prompting them to revisit their previous idea. Students begin to see the term "percent" as describing a comparison to 100 more broadly. The activity also introduces the percent symbol (%).

First, students see examples of the phrase "percent of a dollar" being used to describe the values of some U.S. coins. They interpret "percent" in terms of the amount of money in cents relative to the 100 cents in a dollar. Then, students write a definition for the term, accounting for the usage seen in this activity.

Launch 4



Ask students:

 \bigcirc "A pen costs \$1.50. How many cents is that?"

150 cents

(a) "A sticker costs 25 cents. How many dollars is that?"

 $\frac{1}{4}$ or 0.25 of a dollar

☐ "How many cents are in a dollar?"

100 cents

"How many dollars are in one cent?"

 $\frac{1}{100}$ or 0.01 of a dollar

Display the image of U.S. coins. Ask students which coins they have seen or used. Students may not be familiar with the half-dollar and dollar coins. Clarify that these coins exist but are not as commonly used. (They are sometimes used as change in vending machines.)

Lesson 10 **Activity 1** Activity 2 Lesson Synthesis Cool-down Warm-up

Tell students that they will look at examples of "percent" being used to talk about money. Arrange students in groups of 2. Give students 3–4 minutes of quiet work time and then a minute to discuss their responses with their partner.

Use Collect and Display to direct attention to words collected and displayed from an earlier activity. Collect the language that students use to define "percent" in the context of coin values. Display words and phrases such as "out of 100 cents," "relative to 100 cents," or "per 100 of something."

Student Task Statement

Here are the names and values of some U.S. coins.



penny



nickel



dime



quarter





dollar

10 cents

25 cents

50 cents

1 dollar

Here are some ways to describe the coins' values using percent (%):

- The value of a dime is 10 percent of a dollar.
- The value of a guarter is 25 percent of a dollar.
- The value of 5 dimes is 50 percent of a dollar.
- 1. Describe the value of each coin using "percent of a dollar."

a. A penny

b. A nickel

I percent of a dollar

5 percent of a dollar

c. A half dollar

d. A dollar

50 percent of a dollar

100 percent of a dollar

- 2. Jada has two coin collections. Each collection has the same type of coins.
 - a. Her first collection of coins is worth 105 percent of a dollar. What is the value of this collection?

105 cents or \$1.05

b. Her second collection of coins is worth \$2.00. What percent of a dollar is that?

200 percent of a dollar

3. Write a definition for "percent" based on your understanding now.

Sample responses:

- · Percent is a way to compare to 100.
- · Percent means per 100 of something.

Are You Ready for More?

Find two different sets of coins that each make 120% of a dollar, where no type of coin is in both sets.

Sample response: A dollar and two dimes, four quarters and four nickels

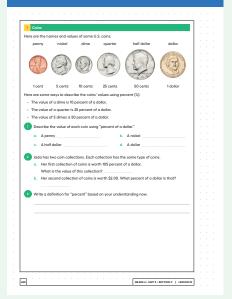
Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.

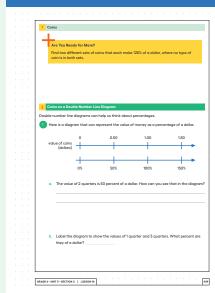
Invite students to share their experiences with coins of different denominations. For example, invite students to share about a coin collection they have or a family member has, or times when certain kinds of coins are of particular interest to them.

Supports accessibility for: Conceptual Processing, Memory

Student Workbook



Student Workbook



Activity Synthesis

The goals of the discussion are to elicit new insights on the idea of percentage, clarify the meaning of "percent," and introduce the symbol for percent.

Invite students to share their responses to the first two sets of questions. Then, direct their attention to the reference created using *Collect and Display*. Ask students to share their definitions of "percent," borrowing language from the display as needed. As they respond, update the reference to include additional phrases.

Discuss questions such as:

"We used 'percent' to talk about the value of coins and the area of a shaded region. What is the same about the use of 'percent' in the two situations? What is different?"

"How would you describe the meaning of '25 percent of a dollar' to someone else? What about '25 percent of a large square'?"

"What does '100 percent of a dollar' mean? What about '100 percent of a large square'?"

"How would you define the word 'percent' now?"

Clarify that **percent** means "per 100" or "for every 100." It is a way to describe a comparison to 100 or the ratio of a quantity to 100. Use examples to illustrate this idea:

- The value of a nickel is 5 percent of a dollar because there are 5 cents in a nickel to 100 cents in a dollar. (The values of a nickel and a dollar are in the ratio of 5 to 100.)
- The value of a dime is 10 percent of a dollar because there are 10 cents in a dime to 100 cents in a dollar. (The values of a dime and a dollar are in the ratio of 10 to 100.)
- Two dollars is 200 percent of a dollar because there are 200 cents in 2 dollars to 100 cents in a dollar. (The ratio of their values is 200 to 100.)
- In Diagram E of the *Warm-up*, the area of the shaded regions is 108 percent of the area of the large square because their areas in square units are in the ratio of 108 to 100.

Display the symbol "%" for all to see. Tell students that it is used to express "percent."

Activity 2

Coins on a Double Number Line Diagram



Activity Narrative

In this activity, students develop their understanding of percentages as rates by reasoning with double number line diagrams and relating 100% to 1 dollar.

Prior to this point, students related "100% of a quantity" to 100 units of that quantity (square units of area, or cents in a dollar). The work here makes it more explicit to students that 100% can correspond to other amounts. Students see "1.00" (dollar)—rather than "100" (cents)—being aligned to 100% on a double number line diagram. They use the diagram to find amounts that correspond to some percentages of a dollar as well as to identify given amounts of money as percentages of a dollar.

The reasoning here encourages students to attend to precision. In working with partially labeled number lines, students need to attend to the units being used and the meaning of the intervals before locating and labeling other relevant values. In expressing a value as a percentage of another value, they need to include "percent" or the % symbol.

Launch 22

Keep students in groups of 2. Draw students' attention to the first double number line diagram. Give partners 2 minutes to make sense of the diagram and answer the first set of questions. Ask students to pause for a brief class discussion before moving on to the second set of questions.

Invite students to share their explanations for the first question. Reiterate that "percent" means "per 100" or "for every 100," and explain that **percentage** is a rate per 100. Because we are interested in percentages of a dollar, \$1 is 100% of a dollar. This means:

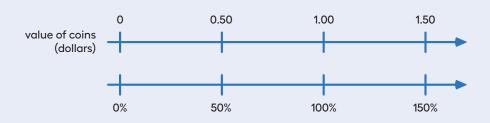
- The value of 2 quarters is 50% of a dollar because the ratio of \$0.50 to \$1.00 is 50 to 100.
- The value of 3 quarters is 75% of a dollar because the ratio of \$0.75 to \$1.00 is 75 to 100.

Give students 3-4 minutes to complete the second half of the activity.

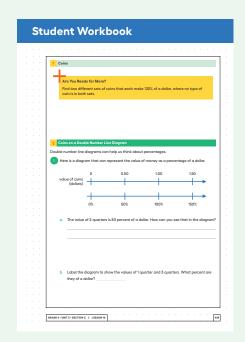
Student Task Statement

Double number line diagrams can help us think about percentages.

1. Here is a diagram that can represent the value of money as a percentage of a dollar.



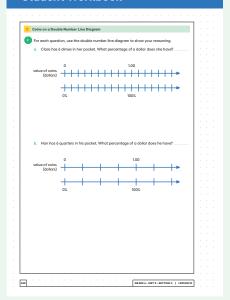
a. The value of 2 quarters is 50 percent of a dollar. How can you see that in the diagram?



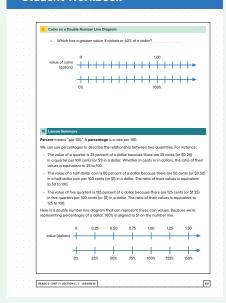
Building on Student Thinking

Students may more readily see coin values as percentages of 100 cents than as percentages of 1 dollar. To support this transition, ask students: "What percentage of 100 cents is 6 dimes?" Then, ask how this percentage (60%) can be shown on a double number line diagram such as the one given in the first question. If the idea of expressing the coin values in cents instead of dollars doesn't come up, ask them about it. Consider inviting students to write "100 cents" above "\$1.00" and "50 cents" above "\$0.50," and so on.

Student Workbook



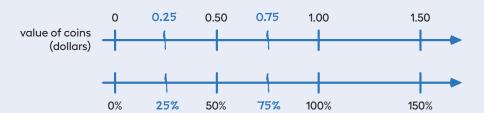
Student Workbook



Sample responses:

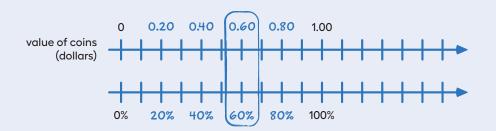
- Two quarters are worth \$0.50, which lines up with 50% on the diagram.
- On the top number line, \$0.50 is halfway between \$0 and \$1.00. On the bottom number line, 50% is halfway between 0% and 100%.
- **b.** Label the diagram to show the values of 1 quarter and 3 quarters. What percent are they of a dollar?

25% and 75% of a dollar



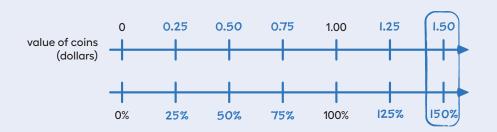
- **2.** For each question, use the double number line diagram to show your reasoning.
 - **a.** Clare has 6 dimes in her pocket. What percentage of a dollar does she have?

60% of a dollar



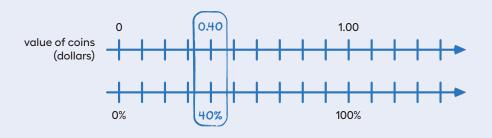
b. Han has 6 quarters in his pocket. What percentage of a dollar does he have?

150% of a dollar



c. Which has a greater value: 8 nickels or 40% of a dollar?

They have the same value. The value of 8 nickels is 40 cents, and 40% of a dollar is also 40 cents.



Activity Synthesis

The purpose of this discussion is to reinforce the idea of percentage as a rate and highlight that double number lines are helpful for reasoning about percentages.

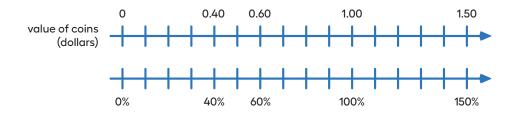
Invite students to explain how the diagrams support or show their reasoning about 6 dimes, 6 quarters, and 8 nickels as percentages. If no students mention the ratios of coin values to 1 dollar, consider asking:

"What do we know about the ratios of the coins' values to 1 dollar? How can these ratios help us find percentages of a dollar?"

Emphasize that a percentage is a rate relating a quantity to 100. One dollar is 100% of a dollar (or 100 per 100), so:

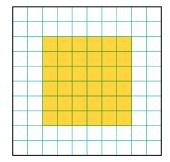
- 6 dimes is 60% of a dollar because the ratio of \$0.60 to \$1.00 is 60 to 100.
- 6 quarters is 150% of a dollar because the ratio of \$1.50 to \$1.00 is 150 to 100.
- 8 nickels is 40% of a dollar because the ratio of \$0.40 to \$1.00 is 40 to 100

On a double number line diagram, these coin values and percentages—\$0.60 and 60%, \$1.50 and 150%, and \$0.40 and 40%—are aligned vertically, just as \$1.00 and 100% are.



Lesson Synthesis

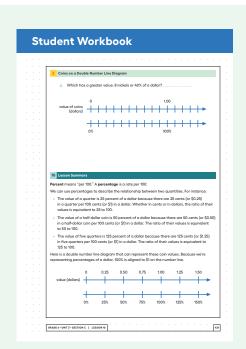
To highlight the idea of "percent" and "percentage" as relating a quantity to 100, revisit the two main contexts used in the lesson. Consider asking students: Display this diagram.



"The large square is 10 units by 10 units. Can we say that 36% of the large square is shaded? Why or why not?"

Yes, because 36% means 36 per 100. There are 100 square units in the square and 36 of them are shaded.

"How can we show 100% of the large square being shaded?"
Shade the entire large square or all 100 square units.



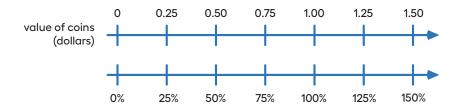
○ "What is 100% of a dollar?"

\$1 or 100 cents

"What is 50% of a dollar?"

\$0.50 or 50 cents

Display this double number line diagram.



○ "How does this double number line diagram show those percentages?"

The \$1 is lined up with 100% and the \$0.50 is lined up with 50%.

○ "How much money is 20% of a dollar?"

\$0.20 or 20 cents

○ "How do you know?"

20% means 20 per 100. We can see from the double number line diagram that if \$1 is 100% then \$0.20 is 20%.

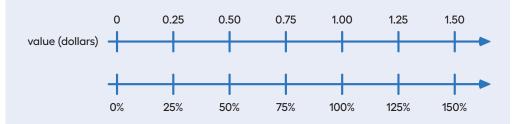
Lesson Summary

Percent means "per 100." A percentage is a rate per 100.

We can use percentages to describe the relationship between two quantities. For instance:

- The value of a quarter is 25 percent of a dollar because there are 25 cents (or \$0.25) in a quarter per 100 cents (or \$1) in a dollar. Whether in cents or in dollars, the ratio of their values is equivalent to 25 to 100.
- The value of a half-dollar coin is 50 percent of a dollar because there are 50 cents (or \$0.50) in a half-dollar coin per 100 cents (or \$1) in a dollar. The ratio of their values is equivalent to 50 to 100.
- The value of five quarters is 125 percent of a dollar because there are 125 cents (or \$1.25) in five quarters per 100 cents (or \$1) in a dollar. The ratio of their values is equivalent to 125 to 100.

Here is a double number line diagram that can represent these coin values. Because we're representing percentages of a dollar, 100% is aligned to \$1 on the number line.



Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

Kiran & Mai's Coins

5 min

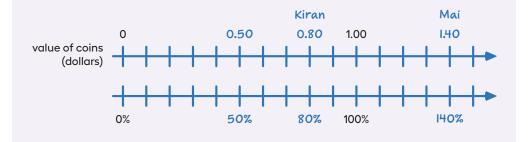
Student Task Statement

Kiran and Mai each have some coins.

- Kiran has 80 cents. What percent of a dollar does he have?
 percent of a dollar
- 2. Mai has 140% of a dollar. How much money does she have?

 140 cents or \$1.40

Use the double number line diagram to show your reasoning.

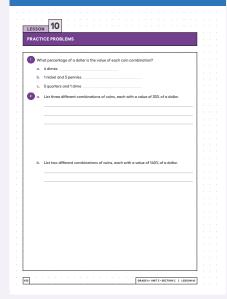


Responding To Student Thinking

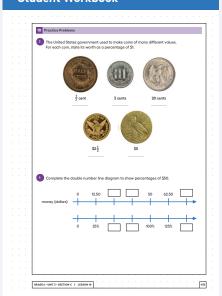
More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.





Student Workbook



Problem 1

What percentage of a dollar is the value of each coin combination?

a. 4 dimes

40%

b. 1 nickel and 3 pennies

8%

c. 5 quarters and 1 dime

135%

Problem 2

a. List three different combinations of coins, each with a value of 30% of a dollar.

30 pennies, 6 nickels, or 3 dimes

b. List two different combinations of coins, each with a value of 140% of a dollar.

140 pennies, 14 dimes, or 5 quarters and 3 nickels

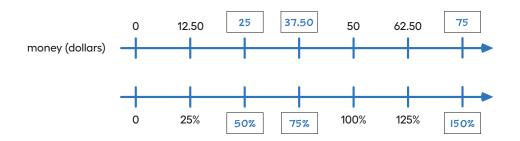
Problem 3

The United States government used to make coins of many different values. For each coin, state its worth as a percentage of \$1.



Problem 4

Complete the double number line diagram to show percentages of \$50.



Problem 5

from Unit 3, Lesson 7

Elena bought 8 tokens for \$4.40. At this rate:

a. How many tokens could she buy with \$6.05?

II tokens

b. How much do 19 tokens cost?

\$10.45

Problem 6

from Unit 3, Lesson 9

A snail travels 10 cm in 4 minutes. At this rate:

a. How long will it take the snail to travel 24 cm?

9.6 minutes (or equivalent)

b. How far does the snail travel in 6 minutes?

15 cm

Problem 7

from Unit 3, Lesson 6

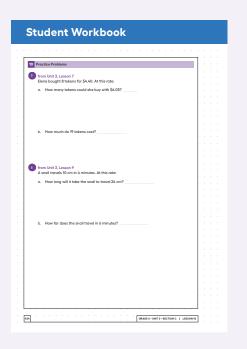
a. Three tickets for a train ride cost \$18. Complete the table to show the cost of 4, 5, and 6 tickets at the same rate.

number of tickets	cost in dollars	unit price in dollars per ticket
3	18	6
4	24	6
5	30	6
6	36	6

b. t tickets cost c dollars. What is a unit rate that describes ticket costs? What does it tell us about the situation?

Sample responses:

- $\frac{c}{t}$ tells us the unit price or the cost in dollars per ticket.
- $\circ \frac{t}{c}$ tells us the number of tickets that can be bought per dollar.



Student Workbook

