Lesson 17

Common Multiples

Goals

- Comprehend (orally and in writing) the terms "multiple," "common multiple," and "least common multiple."
- Explain (orally and in writing) how to calculate the least common multiple of two whole numbers.
- List the multiples of a number, and identify common multiples for two numbers in a real-world situation.

Learning Targets

- I can explain what a common multiple is.
- I can explain what the least common multiple is.
- I can find the least common multiple of two whole numbers.

Lesson Narrative

In this lesson, students are introduced to **common multiples** of two numbers as products that result from multiplying each of the two numbers by some whole number. They are also introduced to the **least common multiples** of two whole numbers as the smallest multiple shared by both numbers. Students reason abstractly when they use common multiples to solve problems in contextual situations.

Student Learning Goal

Let's use multiples to solve problems.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 3)

Instructional Routines

- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	10 min	10 min	10 min	10 min	5 min
					Cool-down

Warm-up

Notice and Wonder: Multiples

5 min

Activity Narrative

The purpose of this *Warm-up* is to review multiples, which will be useful when students find the least common multiple in a later activity. While students may notice and wonder many things about the circled numbers, the idea that some numbers are circled twice is the important discussion point.

Launch

Arrange students in groups of 2. Display the *Task Statement* for all to see. Ask students to circle all the multiples of 4 and 6 and then to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

Circle all the multiples of 4 in this list.

1	2	3	(4)	5	6	7	(8)	9	10	11	(12)	13
14	15	(16)	17	18	19	(20)	21	22	23	(24)	25	26

Circle all the multiples of 6 in this list.

1	2	3	4	5	(6)	7	8	9	10	11	(12)	13
14	15	16	17	(18)	19	20	21	22	23	(24)	25	26

What do you notice? What do you wonder?

Students may notice:

- 4 and 6 have common multiples of 12 and 24.
- 4 has multiples not in common with 6, like 8, 16, and 20.
- 6 has multiples not in common with 4, like 6 and 18.
- All of the common multiples of 4 and 6 are multiples of 12.

Students may wonder:

- What other multiples would 4 and 6 have in common if we kept counting?
- Why are all of the common multiples of 4 and 6 also multiples of 12?
- Could we multiply 4 by any numbers to match all of the multiples of 6?

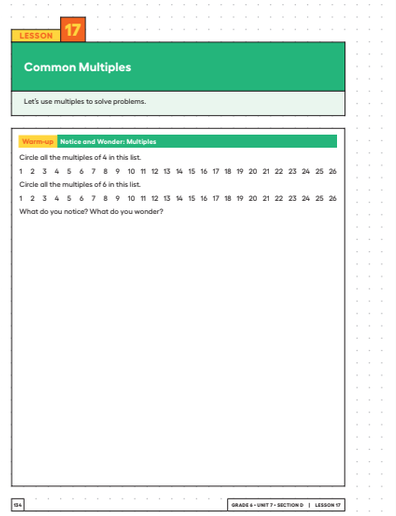
Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 17

Common Multiples

Let's use multiples to solve problems.

Warm-up Notice and Wonder: Multiples

Circle all the multiples of 4 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

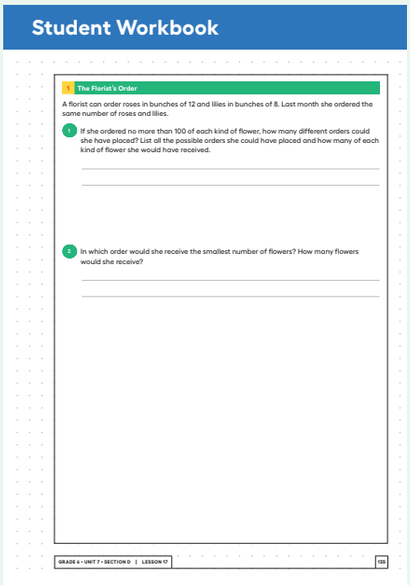
Circle all the multiples of 6 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

What do you notice? What do you wonder?

194 GRADE 6 • UNIT 7 • SECTION D | LESSON 17

Student Workbook



1 The Florist's Order

A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

1. If she ordered no more than 100 of each kind of flower, how many different orders could she have placed? List all the possible orders she could have placed and how many of each kind of flower she would have received.

2. In which order would she receive the smallest number of flowers? How many flowers would she receive?

GRADE 6 • UNIT 7 • SECTION D | LESSON 17 135

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the lists of numbers. Next, ask students,

"*Is there anything on this list that you are wondering about now?*"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Activity 1

The Florist's Order

10 min

Activity Narrative

In this activity, students begin to think about common multiples and the least common multiple when finding ways to order two types of flowers that come in bunches of different sizes. Students find all multiples up to 100 for two different numbers. They compare these multiples to determine which ones are the same, representing an order with the same numbers of both flowers, and then they determine the least common multiple.

Monitor for different strategies and representations students use to describe the situation. Some students may draw pictures of groups of flowers, other students may use tables or lists, and other students may do a combination of these.

Launch

Arrange students into groups of 2.

Give students 5–7 minutes of quiet work time, then 2 minutes of partner discussion.

Follow with a whole-class discussion.

Student Task Statement

A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

1. If she ordered no more than 100 of each kind of flower, how many different orders could she have placed? List all the possible orders she could have placed and how many of each kind of flower she would have received.

   She could have placed 4 possible orders:

   - 2 bunches of roses and 3 bunches of lilies (24 of each kind of flower)
   - 4 bunches of roses and 6 bunches of lilies (48 of each kind of flower)
   - 6 bunches of roses and 9 bunches of lilies (72 of each kind of flower)
   - 8 bunches of roses and 12 bunches of lilies (96 of each kind of flower)

2. In which order would she receive the smallest number of flowers? How many flowers would she receive?

   She would receive the smallest number of flowers (24 of each kind) if she ordered 2 bunches of roses and 3 bunches of lilies.

Activity Synthesis

The purpose of this activity is for students to share how they organized information when finding different combinations of flowers. Invite students to share their strategies for the first questions. Display or record their responses for all to see. Here are some strategies students may have used:

- Draw a picture to represent bunches of roses and bunches of lilies.
- Make an organized list or table.

If not brought up in students' explanations, confirm that there are 4 different order combinations, and each time, there are 24 more of each flower added. Discuss why the smallest number of flowers of each type is 24, and explain that it takes 2 bunches of roses to equal 24, and 3 bunches of lilies to equal 24. If possible, use student responses to create a visual display of the concept of least common multiple, and display it for all to see throughout the unit.

Activity 2

Least Common Multiple

10 min

Activity Narrative

In this activity, students are introduced to the terms common multiple and least common multiple.

Launch

Arrange students in groups of 2. Ask students to list the first 6 multiples of 2 and 3 (2: 2, 4, 6, 8, 10, 12; 3: 3, 6, 9, 12, 15, 18). Then ask students to discuss with a partner what they think a **common multiple** of two numbers is, and select 1–2 groups to share their thinking. If not brought up in students' explanations, tell students that a common multiple is a product that is the result of multiplying each of the two numbers by some whole number. For example, 6 and 12 are common multiples of 2 and 3.

Then give students 5 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

The **least common multiple** of 6 and 8 is 24.

1. What do you think the term "least common multiple" means?

   Sample response: Least common multiple is the smallest multiple that numbers share.

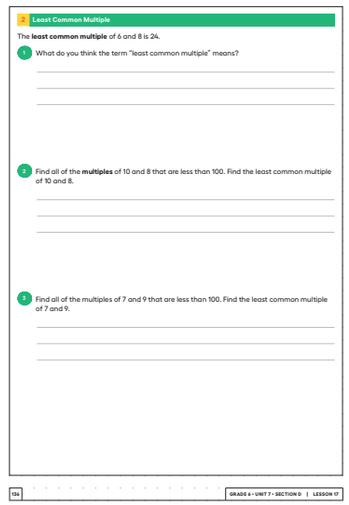
2. Find all of the **multiples** of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.
   - Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90.
   - Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.
   - Common multiples: 40, 80.
   - The least common multiple of 10 and 8 is 40.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators for finding common multiples to ensure inclusive participation in the activity.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


2 Least Common Multiple

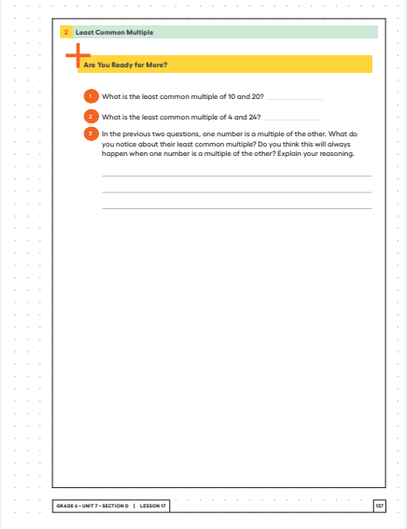
The **least common multiple** of 6 and 8 is 24.

1 What do you think the term "least common multiple" means?

2 Find all of the **multiples** of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.

3 Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.

Student Workbook


2 Least Common Multiple

Are You Ready for More?

1. What is the least common multiple of 10 and 20?
2. What is the least common multiple of 4 and 24?
3. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

GRADE 6 • UNIT 7 • SECTION D | LESSON 17 137

3. Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.
   - Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.
   - Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.
   - Common multiples: only 63.
   - The least common multiple of 7 and 9 is 63.

Are You Ready for More?

1. What is the least common multiple of 10 and 20?

   The multiples of 10 are 10, 20, 30, 40, ... The multiples of 20 are 20, 40, 60, 80, ... 20 is the least common multiple.

2. What is the least common multiple of 4 and 24?

   The multiples of 4 are 4, 8, 12, 16, 20, 24, ... The multiples of 12 are 12, 24, 36, 48, 60, 72, ... 12 is the least common multiple.

3. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

   If one number is a multiple of the other, then it is a common multiple.

   For example, in the previous question with 4 and 12, we know that 12 is a multiple of 4 ($12 = 4 \cdot 3$) and 12 is a multiple of 12 ($12 = 12 \cdot 1$). If one number is a multiple of another, then it must also be the least common multiple because no multiples of a number are less than the number itself. In the previous question, there are no multiples of 12 that are less than 12.

Activity Synthesis

The purpose of discussion is to formally introduce the least common multiple of two numbers, and to clarify the process of finding common multiples and identifying the **least common multiple**. Begin by inviting students to share their responses to the first question. If not brought up in students' explanations, tell students that the least common multiple of two numbers is the smallest product that results from multiplying each of the two numbers by some whole number.

Next, ask students to discuss a way to find the least common multiple of any two numbers with a partner. Invite groups to share their responses, and record them for all to see. If time allows, display pairs of numbers, and ask students to find the least common multiple.

Activity 3

Prizes on Grand Opening Day

10 min

Activity Narrative

In this activity, students continue to explore common multiples in context. Prizes are being given away to every 5th, 9th, and 15th customer. Students list the multiples of each number when determining which customers get prizes and when customers get more than one prize. Customers who get more than one prize represent pairwise least common multiples. It is also true that the first customer who gets all three prizes represents the least common multiple of all three numbers, but this idea goes beyond the standards being addressed, and there aren't enough customers for this to happen. Students reason abstractly about common multiples and the least common multiple to solve problems in context.

Monitor for students using these strategies:

- List numbers from 1 to 50, and skip count to identify common multiples.
- Analyze common multiples of pairs of numbers rather than all three numbers at once.
- Denote multiples of different numbers with different shapes, colors, or other notations. Identify common multiples as numbers that have multiple designations.

Launch

Arrange students in groups of 2. Encourage students to discuss their reasoning with their partner as they work.

Give students 10 minutes work time, and follow with a whole-class discussion.

Student Task Statement

Lin's uncle is opening a bakery. On the bakery's grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every 5th customer will get a free bagel. Every 9th customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

1. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Do you agree?? If so, how far back should he go to get at least one prize? Explain your reasoning.

   I agree with Diego.

   Sample reasoning: Diego is correct that he will not get a prize because 23 is not a multiple of 5, 9, or 12, and he could get a prize if he went backward in line. If he goes back 1 spot to be the 24th customer, he will get a slice of carrot cake, because 24 is a multiple of 12. If he goes back 2 spots, he will be the 25th customer and get a bagel, because 25 is a multiple of 5. If he goes back 4 spots, he will be the 27th customer and get a blueberry muffin, because 27 is a multiple of 9.

Access for Multilingual Learners (Activity 3, Launch)

MLR6: Three Reads

Keep books or devices closed. Display only the problem stem, without revealing the questions.

"We are going to read this problem stem 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

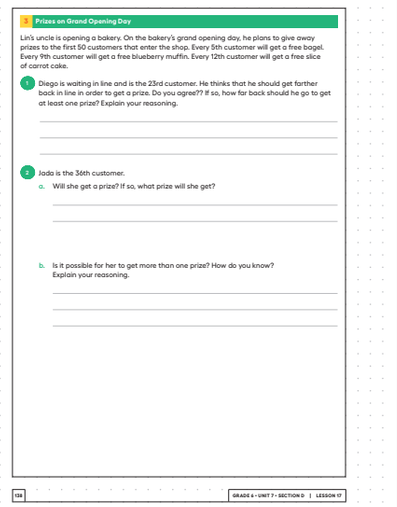
"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Student Workbook



3 Prizes on Grand Opening Day

Lin's uncle is opening a bakery. On the bakery's grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every 5th customer will get a free bagel. Every 9th customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

1. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Do you agree?? If so, how far back should he go to get at least one prize? Explain your reasoning.

2. Jada is the 36th customer.
   a. Will she get a prize? If so, what prize will she get?

   b. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

2. Jada is the 36th customer.

a. Will she get a prize? If so, what prize will she get?

Jada will get a muffin and a slice of carrot cake, because 36 is a multiple of 9 and of 12.

b. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

Yes, it's possible.

Her number in line is a common multiple of both 9 and 12, so she will get both prizes.

Activity Synthesis

The purpose of this discussion is for students to make connections between multiples and a real-world context. Invite students to share their strategies for determining if Jada would get any prizes as the 36th customer. Then ask students how many prizes in total Lin's uncle will give away, and discuss the following questions:

- *"How can multiples help us solve this problem?"*

  We can list out the multiples of 5, 9, and 12 up to 50 and add up how many prizes will be given away.

- *"What are the multiples of 5, 9, and 12 up to 50?"*

  The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50. The multiples of 9 are 9, 18, 27, 36, and 45. The multiples of 12 are 12, 24, 36, and 48.

- *"How many total prizes will be given away?"*

  19 prizes

If time allows, continue discussing the following questions:

- *"How many customers will win 2 prizes?"*

  Two. Customer 36 will win a muffin and a slice of carrot cake. Customer 45 will win a bagel and a muffin.

- *"How many customers will win all 3 prizes?"*

  Zero. None of the multiples are common to all three numbers.

Lesson Synthesis

The purpose of this discussion is for students to describe the differences between multiples and factors. Here are some questions for discussion:

- *"What is the same and different about multiples and factors?"*

  Both have to do with multiplication or division. Factors are numbers that multiply to get a certain product. A number also divides evenly into its factors. Multiples are the product that results from multiplying by a whole number.

- *"What are some situations when finding the least common multiple is helpful?"*

  It is helpful when forming the smallest amount of equal groups or determining when both events first happen at the same time.

Lesson Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \cdot 4$.

A **common multiple** for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The **least common multiple** (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each number in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44,...
- Multiples of 10: 10, **20**, 30, **40**, 50, ...

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, ...), but 20 is the smallest number that is on *both* lists, so 20 is the least common multiple.

Cool-down

In Your Own Words Again

5 min

Student Task Statement

1. What is the least common multiple of 6 and 9? Explain or show your reasoning.

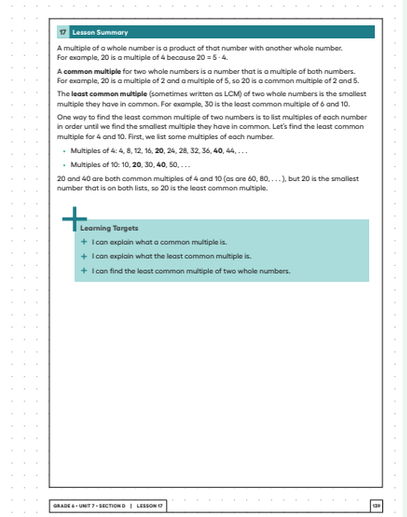
   The least common multiple of 6 and 9 is 18.

   Sample reasoning: The first few multiples of 6 are 6, 12, 18, 24, 30, and 36. The first few multiples of 9 are 9, 18, 27, and 36. The number 18 is the first to appear on both lists.

2. In your own words, what is the least common multiple of two whole numbers? How can you find it?

   Sample response: The least common multiple of two numbers is the smallest multiple that the numbers share. You can find the least common multiple by listing the multiples of each number until you find one that is common to both lists. The first multiple that is common to both lists is the least common multiple.

Student Workbook



17 Lesson Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \cdot 4$.

A **common multiple** for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The **least common multiple** (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each number in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44, . . .
- Multiples of 10: 10, **20**, 30, **40**, 50, . . .

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, . . .), but 20 is the smallest number that is on both lists, so 20 is the least common multiple.

Learning Targets

- I can explain what a common multiple is.
- I can explain what the least common multiple is.
- I can find the least common multiple of two whole numbers.

GRADE 6 • UNIT 7 • SECTION D | LESSON 17 139

Responding To Student Thinking

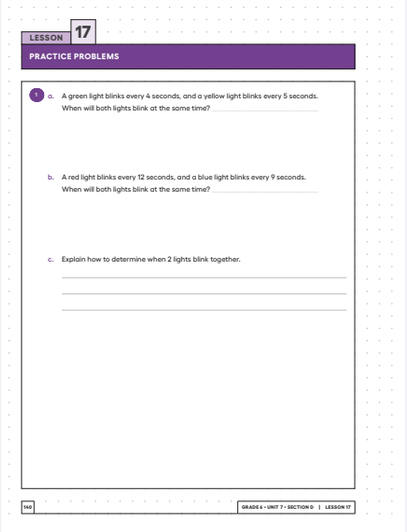
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 17

Practice Problems

6 Problems

Student Workbook


LESSON 17

PRACTICE PROBLEMS

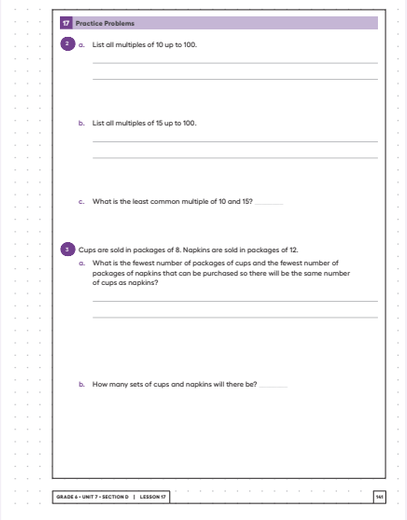
1 a. A green light blinks every 4 seconds, and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

b. A red light blinks every 12 seconds, and a blue light blinks every 9 seconds. When will both lights blink at the same time?

c. Explain how to determine when 2 lights blink together.

140 GRADE 6 • UNIT 7 • SECTION D | LESSON 17

Student Workbook


17 Practice Problems

2 a. List all multiples of 10 up to 100.

b. List all multiples of 15 up to 100.

c. What is the least common multiple of 10 and 15?

3 Cups are sold in packages of 8. Napkins are sold in packages of 12.

a. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

b. How many sets of cups and napkins will there be?

GRADE 6 • UNIT 7 • SECTION D | LESSON 17 141

Problem 1

a. A green light blinks every 4 seconds, and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

20, 40, 60, 80, 100, ... seconds, because these are common multiples of 4 and 5

b. A red light blinks every 12 seconds, and a blue light blinks every 9 seconds. When will both lights blink at the same time?

36, 72, 108, ... seconds, because these are common multiples of 15 and 9

c. Explain how to determine when 2 lights blink together.

Sample response: They blink together every common multiple.

Problem 2

a. List all multiples of 10 up to 100.

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

b. List all multiples of 15 up to 100.

15, 30, 45, 60, 75, 90

c. What is the least common multiple of 10 and 15?

30

Problem 3

Cups are sold in packages of 8. Napkins are sold in packages of 12.

a. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

3 packages of cups ($3 \cdot 8 = 24$) and 2 packages of napkins ($2 \cdot 12 = 24$)

b. How many sets of cups and napkins will there be?

24 sets

Problem 4

from Unit 7, Lesson 15

a. Plot and connect these points to form a polygon.

(-5, 3),(3, 3),(1, -2),(-3, -2)

A trapezoid is plotted.

b. Find the lengths of the two horizontal sides of the polygon.

The longer horizontal side is 8 units long, and the shorter horizontal side is 4 units long.

Problem 5

from Unit 7, Lesson 14

Rectangle $ABCD$ is drawn on a coordinate plane. $A(-6, 9)$ and $B(5, 9)$. What could be the locations of points C and D?

Sample responses:

- C could be $(-6, -9)$ and D could be $(5, -9)$
- C could be $(-6, 18)$, and D could be $(5, 18)$.

Problem 6

from Unit 3, Lesson 14

A school wants to raise $2,500 to support its music program.

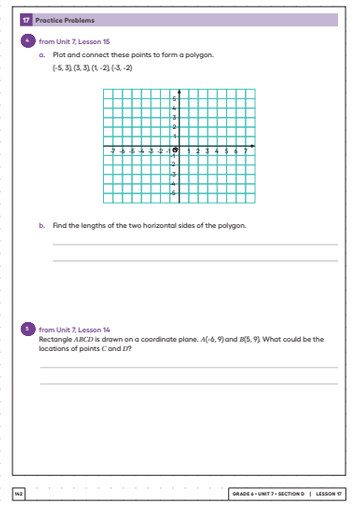
a. If it has met 20% of its goal so far, how much money has it raised?

$500, because $2,500 \cdot 0.2 = 500$

b. If it raises 175% of its goal, how much money will the music program receive? Show your reasoning.

$4,375 because $2,500 \cdot 1.75 = 4,375$

Student Workbook


17 Practice Problems

4 from Unit 7, Lesson 15

a. Plot and connect these points to form a polygon.

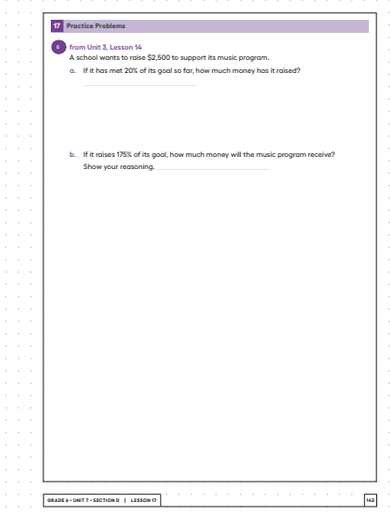
(-5, 3), (3, 3), (1, -2), (-3, -2)

b. Find the lengths of the two horizontal sides of the polygon.

5 from Unit 7, Lesson 14

Rectangle $ABCD$ is drawn on a coordinate plane. $A(-6, 9)$ and $B(5, 9)$. What could be the locations of points C and D?

Student Workbook


17 Practice Problems

6 from Unit 3, Lesson 14

A school wants to raise $2,500 to support its music program.

a. If it has met 20% of its goal so far, how much money has it raised?

b. If it raises 175% of its goal, how much money will the music program receive? Show your reasoning.