Lesson 14

Evaluating Expressions with Exponents

Goals

- Evaluate numerical expressions that have an exponent and one other operation, and explain (orally) the process.
- Explain (orally and in writing) that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols.

Learning Targets

- I know how to find the value of expressions that have both an exponent and addition or subtraction.
- I know how to find the value of expressions that have both an exponent and multiplication or division.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Lesson Narrative

In this lesson, students encounter expressions that involve exponents as well as another operation. They are prompted to consider whether to first compute the value of the part of the expression with an exponent, or carry out the other operation before raising the result to a certain power.

Students begin by recalling ideas about the surface area and volume of a cube, which provides a concrete context for writing expressions with exponents 2 and 3, as well as to think about whether to square or cube a number before or after multiplying or adding. Then, they consider two ways of finding the value of the expression $6 \cdot 10^2$ and learn about the conventional order of operations. Finally, students practice applying the convention to evaluate expressions involving exponents, grouping symbols, and another operation.

Student Learning Goal

Let's find the values of expressions with exponents.

Lesson Timeline

10 min	10 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down (5 min)

Warm-up
Revisiting the Cube

10 min

Activity Narrative

The purpose of this *Warm-up* is for students to recall previous understandings of area, volume, and surface area of cubes, and how to record these measurements as expressions with exponents. Given the side length of a square and the edge length of a cube, students are prompted to describe other measurements that could be determined. Students might respond with either verbal or numerical descriptions, saying, for example, "We can find the area of the square," or "The area of the square is 9 square units."

Launch

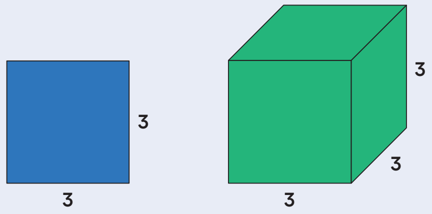
Arrange students in groups of 2. Give students 2 minutes to read the problem and discuss it with their partner. After students share their responses, display the following table for all to see and give students time to discuss the information with a partner.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3			
as an expression using an exponent	3^1			

Give students 1 minute of quiet work time to complete as much of the table as they can independently. Then ask them to discuss their responses with their partner and complete the rest of the table.

Student Task Statement

Based on the given information, what other measurements of the square and cube could we find?



Sample responses:

- We can find the area of the square.
- The area of the square is 9 square units.
- The perimeter is 12 units.
- We can find the volume of the cube.
- The volume of the cube is 27 cubic units.
- The surface area is 54 square units.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3	9	27	54
as an expression using an exponent	3^1	3^2	3^3	$6(3^2)$

Activity Synthesis

Ask students to share their responses for the first row in the table and their reasoning. Record and display the responses for all to see. Clarify their answers with questions such as:

- *"What calculation did you do to arrive at that answer?"*

  "Where are those measurements in the image?"

Then invite students to share their responses for the second row in the table. Ask questions such as:

- *"How did you decide on the exponent for your answer?"*

  "Where are those measurements in the image?"

In the next activity, students will analyze calculations of the surface area of a cube. If the reason that $6(3^2)$ expresses the surface area of the cube is not yet discussed, ask students:

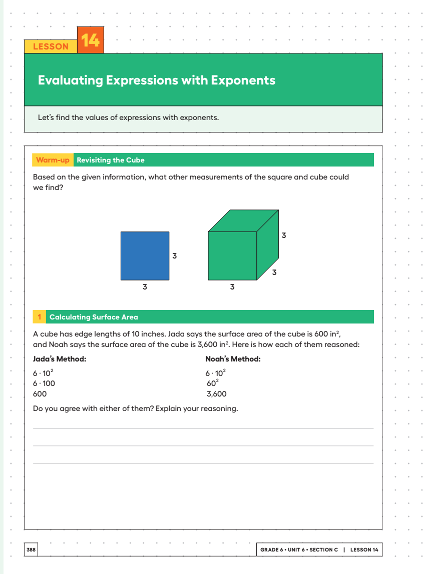
- *"Where did the 3^2 come from?"*

  It's the area of one face of the cube.

- *"Why are we multiplying by 6?"*

  The cube has 6 faces, and the surface area of the cube is the total of those 6 areas. Multiplying 3^2 by 6 is the same as adding six 3^2s.

Student Workbook



LESSON 14

Evaluating Expressions with Exponents

Let's find the values of expressions with exponents.

Warm-up Revisiting the Cube

Based on the given information, what other measurements of the square and cube could we find?

1 Calculating Surface Area

A cube has edge lengths of 10 inches. Jada says the surface area of the cube is 600 in², and Noah says the surface area of the cube is 3,600 in². Here is how each of them reasoned:

Jada's Method:	Noah's Method:
$6 \cdot 10^2$	$6 \cdot 10^2$
$6 \cdot 100$	60^2
600	3,600

Do you agree with either of them? Explain your reasoning.

Access for Multilingual Learners (Activity 1, Student Task)

MLR6: Three Reads.
Keep books or devices closed. Display only the problem stem and Jada and Noah's work, without revealing the question. Say,

"We are going to read this problem 3 times."

- After the 1st read:
  "Tell your partner what this situation is about."
  Jada and Noah have different solutions for the surface area of the same cube.
- After the 2nd read:
  "List the quantities. What can be counted or measured?"
  the side length of the cube, the number of faces of a cube, the area of each face of the cube
- For the 3rd read: Reveal and read the question. Ask,
  "What are some ways we might get started on this?"

Advances: Reading, Representing

Activity 1
Calculating Surface Area

10 min

Activity Narrative

In this activity, students extend their understanding of the order of operations to include expressions with exponents. They do so in the context of surface area, which provides a reason to find the value of an exponential expression before performing the multiplication.

Launch

Keep students in groups of 2. Give students 2–3 minutes of quiet work time and 1–2 minutes to share their thinking with their partner, followed by a class discussion.

Student Task Statement

A cube has edge lengths of 10 inches. Jada says the surface area of the cube is 600 in^2, and Noah says the surface area of the cube is 3,600 in^2. Here is how each of them reasoned:

Jada's Method:

$6 \cdot 10^2$ $\quad\quad$ $6 \cdot 100$ $\quad\quad$ 600

Noah's Method:

$6 \cdot 10^2$ $\quad\quad$ 60^2 $\quad\quad$ $3{,}600$

Do you agree with either of them? Explain your reasoning.

I agree with Jada.

Sample reasoning:

The cube has 6 faces and each has an area of 10^2 or 100. The area calculation comes before multiplying by 6.

Activity Synthesis

The goal of this discussion is to make explicit the order of operations when finding the value of an expression involving exponents.

Invite students to share their responses and reasoning. Point out that in finding the surface area, we need to find the area of one face of the cube, which is 10^2, before multiplying that number by 6.

Tell students that sometimes it is not so clear in which order to perform the operations in an expression. However, there is an order that we all generally agree on, and when we want something done in a different order, we use parentheses, brackets, or other grouping symbols to communicate what to do first. In general:

- When an exponent occurs in the same expression as another operation, we evaluate the exponent first. For example, to find the value of $3 \cdot 4^2$, we calculate 4^2 first, which is 16, and then multiply it by 3, which gives 48.
- When an expression involves grouping symbols, we perform the operation inside them first. For example, for $(3 \cdot 4)^2$, we first multiply 3 and 4, which gives 12, and then square the 12 to get 144.

If students bring up PEMDAS or another mnemonic for remembering the order of operations, point out that PEMDAS can be misleading in indicating multiplication before division, and addition before subtraction. Discuss the convention:

- First, find the value of any expression in brackets or parentheses.
- Next, find the value of any expression with an exponent.
- Then, perform multiplication or division, from left to right.
- Lastly, perform addition or subtraction, from left to right.

Activity 2

Row Game: Expression Explosion

15 min

Activity Narrative

In this activity, students use the order of operations to find the value of expressions with exponents. As they exchange explanations for their responses and work to reach an agreement, students practice constructing logical arguments, listening, and critiquing the reasoning of others.

Launch

Keep students in groups of 2. Tell partners to each choose a column and find the values of all the expressions in that column. Explain that they would work individually on their expression in each row, discuss their answers (which should be the same for both partners), and come to an agreement before moving on to the next row.

Give partners 8–10 minutes to complete the activity. Follow with a whole-class discussion.

Student Task Statement

Find the value of the expressions in one of the columns. Your partner will work on the other column.

Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

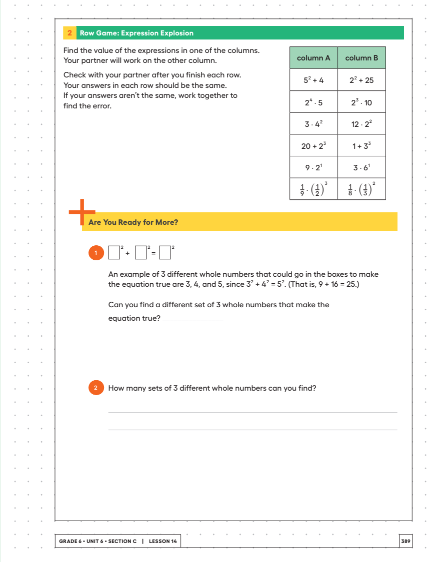
	column A	column B
1. 29	$5^2 + 4$	$2^2 + 25$
2. 80	$2^4 \cdot 5$	$2^3 \cdot 10$
3. 48	$3 \cdot 4^2$	$12 \cdot 2^2$
4. 28	$20 + 2^3$	$1 + 3^3$
5. 18	$9 \cdot 2^1$	$3 \cdot 6^1$
6. $\frac{1}{72}$	$\frac{1}{9} \cdot \left(\frac{1}{2}\right)^2$	$\frac{1}{8} \cdot \left(\frac{1}{3}\right)^2$

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they work together to find errors. Examples:

"How did you ...?"
"First, I _____ because..."
"I agree/disagree because ..."
Supports accessibility for: Language, Social-Emotional Skills

Student Workbook



2 Row Game: Expression Explosion

Find the value of the expressions in one of the columns. Your partner will work on the other column.

Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$5^2 + 4$	$2^2 + 25$
$2^4 \cdot 5$	$2^3 \cdot 10$
$3 \cdot 4^2$	$12 \cdot 2^2$
$20 + 2^3$	$1 + 3^3$
$9 \cdot 2^1$	$3 \cdot 6^1$
$\frac{1}{9} \cdot \left(\frac{1}{2}\right)^3$	$\frac{1}{8} \cdot \left(\frac{1}{3}\right)^2$

Are You Ready for More?

1. $\square^2 + \square^2 = \square^2$

An example of 3 different whole numbers that could go in the boxes to make the equation true are 3, 4, and 5, since $3^2 + 4^2 = 5^2$. (That is, $9 + 16 = 25$.)

Can you find a different set of 3 whole numbers that make the equation true? ______

2. How many sets of 3 different whole numbers can you find?

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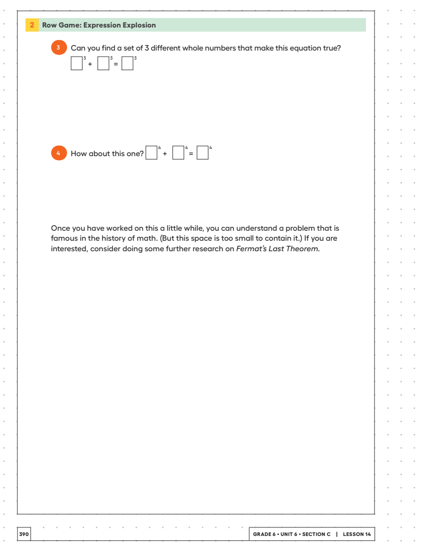
Student Workbook


2 Row Game: Expression Explosion

3 Can you find a set of 3 different whole numbers that make this equation true?
$\square^3 + \square^3 = \square^3$

4 How about this one? $\square^4 + \square^4 = \square^4$

Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (But this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

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Are You Ready for More?

1. $\square^2 + \square^2 = \square^2$

   An example of 3 different whole numbers that could go in the boxes to make the equation true are 3, 4, and 5, since $3^2 + 4^2 = 5^2$.
   (That is, $9 + 16 = 25$.)

   Can you find a different set of 3 whole numbers that make the equation true?

   Sample responses:

   - 6, 8, 10
   - 5, 12, 13

2. How many sets of 3 different whole numbers can you find?

   Answers vary.

3. Can you find a set of 3 different whole numbers that make this equation true? $\square^3 + \square^3 = \square^3$

   No

   (No such triple exists.)

4. How about this one? $\square^4 + \square^4 = \square^4$

   No

   (No such triple exists.)

Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (But this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

Activity Synthesis

The purpose of the discussion is to ensure that students understand and can apply the conventional rules for order of operations when working with expressions with exponents. Consider asking questions such as:

"Were there any expressions whose values were difficult to find? Why were they difficult?"

"Did you disagree with your partner about the value in any of the rows? How did you settle the disagreement?"

"What is something new that you learned about working with expressions with exponents?"

Lesson Synthesis

The purpose of this discussion is to highlight the convention of evaluating the part of an expression with an exponent first, before other operations (unless grouping symbols indicate otherwise).

Display $20 + 2^2$ and $(20 + 2)^2$ (or use another expression from the lesson and an expression with grouping symbols for comparison). Consider asking:

- *"How are these expressions alike?"*

  They have the same numbers. They both include an addition of 20.

- *"How are they different?"*

  Different numbers are being squared. In the first expression, 2 is being squared. In the second, 22 is being squared. One expression uses grouping symbols.

- *"Which expression has a greater value? How do you know without evaluating the expressions?"*

  $(20 + 2)^2$, because it shows a greater number being raised to the second power.

- *"Why is it important to pay attention to or to use grouping symbols?"*

  If there are no parentheses, we evaluate the part of the expression with an exponent first. It can make a big difference if that's not what is intended.

Lesson Summary

Exponents give us a new way to describe operations with numbers, so we need to understand how exponents work with other operations.

When we write an expression such as $6 \cdot 4^2$, we want to make sure everyone agrees about how to find its value. Otherwise, some people might multiply first and others compute the exponent first, and different people would get different values for the same expression!

Earlier we saw situations in which $6 \cdot 4^2$ represented the surface area of a cube with edge lengths of 4 units. When computing the surface area, we compute 4^2 first (or find the area of one face of the cube first) and then multiply the result by 6 (because the cube has 6 faces).

In many other expressions that use exponents, the part with an exponent is intended to be computed first.

To make everyone agree about the value of expressions like $6 \cdot 4^2$, we follow the convention to *find the value of the part of the expression with the exponent first*. Here are a couple of examples:

$6 \cdot 4^2$	$45 + 5^2$
$6 \cdot 16$	$45 + 25$
96	70

If we want to communicate that 6 and 4 should be multiplied first and then squared, then we can use parentheses to group parts of the expression together:

$(6 \cdot 4)^2$	$(45 + 5)^2$
24^2	502
576	2,500

Student Workbook

Responding To Student Thinking

Points to Emphasize
If most students struggle with making the connection between the context of volume and the corresponding expressions, focus on this idea when opportunities arise over the next several lessons. For example, ask students how the second and fourth equations could connect to the volume of a cube in:

Grade 6, Unit 6, Lesson 15, Activity 2
Exponent Experimentation

In general, to find the value of expressions, we use this order of operations:

- Do any operations in parentheses.
- Apply any exponents.
- Multiply or divide from left to right in the expression.
- Add or subtract from left to right in the expression.

Cool-down

Calculating Volumes

5 min

Student Task Statement

Jada and Noah want to find the combined volume of two gift boxes. One is shaped like a cube and the other is shaped like a rectangular prism that is not a cube. The prism has a volume of 20 cubic inches. The cube has edge lengths of 10 inches.

Jada says the total volume is 27,000 cubic inches. Noah says it is 1,020 cubic inches. Here is how each of them reasoned:

Jada's method:

$20 + 10^3$

30^3

27,000

Noah's method:

$20 + 10^3$

$20 + 1{,}000$

1,020

Do you agree with either of them? Explain your reasoning.

I agree with Noah.

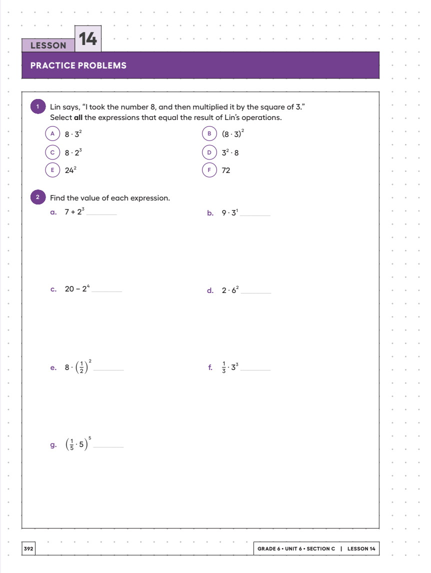
Sample reasoning:

The cube has a volume of 1,000 cubic inches, and the additional 20 cubic inches from the prism makes the total volume 1,020 cubic inches. The exponent calculation comes before addition.

Practice Problems

8 Problems

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 Lin says, "I took the number 8, and then multiplied it by the square of 3." Select **all** the expressions that equal the result of Lin's operations.

A $8 \cdot 3^2$ B $(8 \cdot 3)^2$
C $8 \cdot 2^3$ D $3^2 \cdot 8$
E 24^2 F 72

2 Find the value of each expression.

a. $7 + 2^3$ b. $9 \cdot 3^1$
c. $20 - 2^4$ d. $2 \cdot 6^2$
e. $8 \cdot \left(\frac{1}{2}\right)^2$ f. $\frac{1}{3} \cdot 3^3$
g. $\left(\frac{1}{5} \cdot 5\right)^5$

Problem 1

Lin says, "I took the number 8, and then multiplied it by the square of 3." Select **all** the expressions that equal the result of Lin's operations.

A. $8 \cdot 3^2$

B. $(8 \cdot 3)^2$

C. $8 \cdot 2^3$

D. $3^2 \cdot 8$

E. 24^2

F. 72

Problem 2

Find the value of each expression.

a. $7 + 2^3$

15

b. $9 \cdot 3^1$

27

c. $20 - 2^4$

4

d. $2 \cdot 6^2$

72

e. $8 \cdot \left(\frac{1}{2}\right)^2$

2

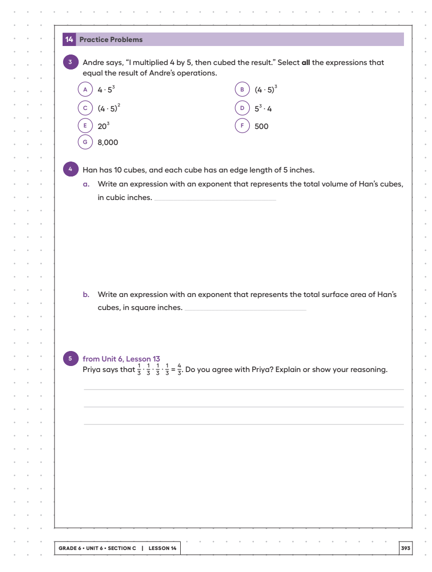
f. $\frac{1}{3} \cdot 3^3$

9

g. $\left(\frac{1}{5} \cdot 5\right)^5$

1

Student Workbook


14 Practice Problems

3 Andre says, "I multiplied 4 by 5, then cubed the result." Select **all** the expressions that equal the result of Andre's operations.

A. $4 \cdot 5^3$
B. $(4 \cdot 5)^3$
C. $(4 \cdot 5)^2$
D. $5^3 \cdot 4$
E. 20^3
F. 500
G. 8,000

4 Han has 10 cubes, and each cube has an edge length of 5 inches.

a. Write an expression with an exponent that represents the total volume of Han's cubes, in cubic inches. ______

b. Write an expression with an exponent that represents the total surface area of Han's cubes, in square inches. ______

5 from Unit 6, Lesson 13
Priya says that $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{4}{3}$. Do you agree with Priya? Explain or show your reasoning.

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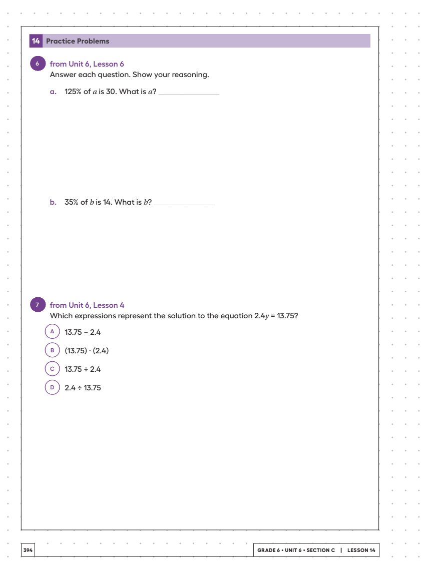
Student Workbook


14 Practice Problems

6 from Unit 6, Lesson 6
Answer each question. Show your reasoning.

a. 125% of a is 30. What is a? ______

b. 35% of b is 14. What is b? ______

7 from Unit 6, Lesson 4
Which expressions represent the solution to the equation $2.4y = 13.75$?

A. $13.75 - 2.4$
B. $(13.75) \cdot (2.4)$
C. $13.75 \div 2.4$
D. $2.4 \div 13.75$

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Problem 3

Andre says, "I multiplied 4 by 5, then cubed the result." Select **all** the expressions that equal the result of Andre's operations.

A. $4 \cdot 5^3$

B. $(4 \cdot 5)^3$

C. $(4 \cdot 5)^2$

D. $5^3 \cdot 4$

E. 20^3

F. 500

G. 8,000

Problem 4

Han has 10 cubes, and each cube has an edge length of 5 inches.

a. Write an expression with an exponent that represents the total volume of Han's cubes, in cubic inches.

$10 \cdot 5^3$ (or equivalent)

b. Write an expression with an exponent that represents the total surface area of Han's cubes, in square inches.

$10 \cdot 6 \cdot 5^2$ or $60 \cdot 5^2$ (or equivalent)

Problem 5

from Unit 6, Lesson 13

Priya says that $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{4}{3}$. Do you agree with Priya? Explain or show your reasoning.

I disagree with Priya.

Sample reasoning:

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ is really $\left(\frac{1}{3}\right)^4$, or $\frac{1}{81}$.

Problem 6

from Unit 6, Lesson 6

Answer each question. Show your reasoning.

a. 125% of a is 30. What is a?

24

Sample reasoning:

$\frac{125}{100} \cdot a = 30$, so $a = 30 \div \frac{125}{100}$, and $a = 30 \cdot \frac{100}{125}$

b. 35% of b is 14. What is b?

40

Sample reasoning:

$(0.35) \cdot b = 14$, so $b = 14 \div 0.35$

Problem 7

from Unit 6, Lesson 4

Which expressions represent the solution to the equation $2.4y = 13.75$?

A. 13.75 – 2.4

B. (13.75) · (2.4)

C. 13.75 ÷ 2.4

D. 2.4 ÷ 13.75

Problem 8

from Unit 5, Lesson 7

Jada explains how she finds 15 · 23:

"I know that ten 23s is 230, so five 23s will be half of 230, which is 115. 15 is 10 plus 5, so 15 · 23 is 230 plus 115, which is 345."

a. Do you agree with Jada? Explain your reasoning.

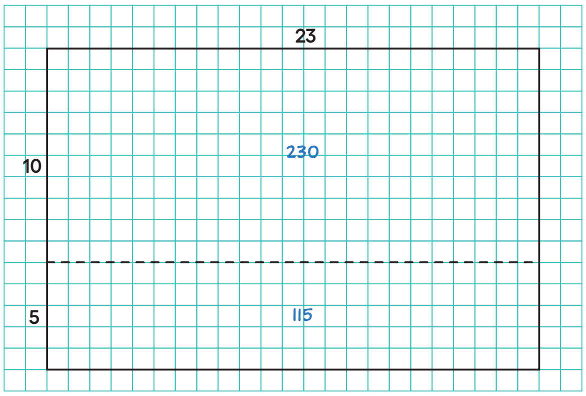
Yes

Sample reasoning:

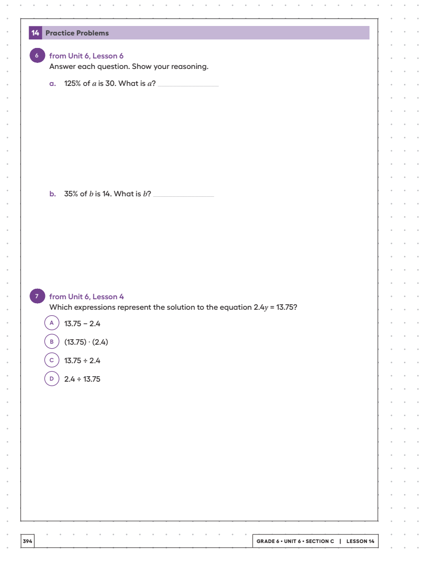
Jada is calculating 15 · 23 by writing it as (10 + 5) · 23 (using the distributive property). To find 5 · 23, she thinks of 5 as $\frac{10}{2}$. So Jada needs to multiply 23 by 10 (which gives her 230) and add half of this product (which is 115) to find the value of (10 + 5) · 23.

b. Draw a 15-by-23 rectangle. Partition the rectangle into two rectangles and label them to show Jada's reasoning.

Sample response:



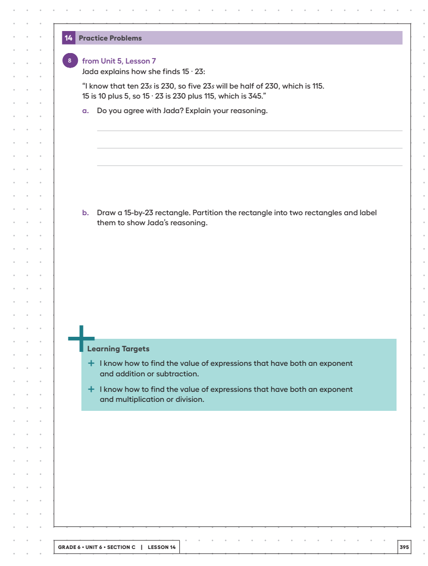
Student Workbook


14 Practice Problems

6 from Unit 6, Lesson 6
Answer each question. Show your reasoning.
a. 125% of a is 30. What is a?
b. 35% of b is 14. What is b?

7 from Unit 6, Lesson 4
Which expressions represent the solution to the equation $2.4y = 13.75$?
A 13.75 – 2.4
B (13.75) · (2.4)
C 13.75 ÷ 2.4
D 2.4 ÷ 13.75

Student Workbook


14 Practice Problems

8 from Unit 5, Lesson 7
Jada explains how she finds 15 · 23:
"I know that ten 23s is 230, so five 23s will be half of 230, which is 115. 15 is 10 plus 5, so 15 · 23 is 230 plus 115, which is 345."
a. Do you agree with Jada? Explain your reasoning.
b. Draw a 15-by-23 rectangle. Partition the rectangle into two rectangles and label them to show Jada's reasoning.

Learning Targets
+ I know how to find the value of expressions that have both an exponent and addition or subtraction.
+ I know how to find the value of expressions that have both an exponent and multiplication or division.