Lesson 6

Absolute Value of Numbers

Goals

- Compare rational numbers and their absolute values, and explain (orally and in writing) the reasoning.
- Comprehend the phrase "absolute value" and the symbol "| |" to refer to a number's distance from zero on the number line.
- Interpret rational numbers and their absolute values in the context of elevation or temperature.

Learning Targets

- I can explain what the absolute value of a number is.
- I can find the absolute values of rational numbers.
- I can recognize and use the notation for absolute value.

Lesson Narrative

In this lesson, students formalize the concept of **absolute value** as a number's distance from zero. Students begin by determining which of two numbers is closer to zero. Next, they consider a bug that is jumping left or right on a number line. They reason about how far the bug jumps versus how far the bug is from zero. Lastly, students use the concept and notation of absolute value to reason abstractly about the familiar contexts of temperature and elevation.

Student Learning Goal

Let's explore distances from zero more closely.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR7: Compare and Connect

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	15 min	10 min	10 min	5 min
				Cool-down

Instructional Routines

Math Talk

ilclass.com/r/10694967

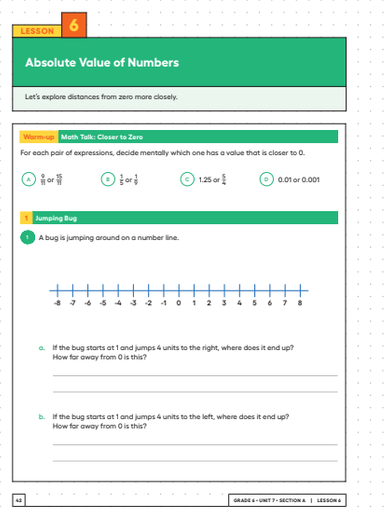
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



LESSON 6

Absolute Value of Numbers

Let's explore distances from zero more closely.

Warm-up Math Talk: Closer to Zero

For each pair of expressions, decide mentally which one has a value that is closer to 0.

A. $\frac{9}{11}$ or $\frac{15}{11}$ B. $\frac{1}{5}$ or $\frac{1}{9}$ C. 1.25 or $\frac{5}{4}$ D. 0.01 or 0.001

1 Jumping Bug

A bug is jumping around on a number line.

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

a. If the bug starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

b. If the bug starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

Warm-up

Math Talk: Closer to Zero

5 min

Activity Narrative

This *Math Talk* focuses on comparing two positive values to determine which is closer to 0. It encourages students to think about distance on a number line and to rely on the structure of factions and decimals to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students find the distance of both positive and negative rational numbers from zero.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

For each pair of expressions, decide mentally which one has a value that is closer to 0.

A. $\frac{9}{11}$ or $\frac{15}{11}$

Sample reasoning: $\frac{9}{11}$ is positive and less than 1, while $\frac{15}{11}$ is greater than 1, so $\frac{9}{11}$ is closer to 0.

B. $\frac{1}{5}$ or $\frac{1}{9}$

Sample reasoning: Ninths are smaller than fifths, so $\frac{1}{9}$ is closer to 0.

C. 1.25 or $\frac{5}{4}$

Neither

Sample reasoning: They are equal, so they are equally close to 0.

D. 0.01 or 0.001

Sample reasoning: 1 thousandth is 10 times smaller than 1 hundredth, so 0.001 is closer to 0.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Activity 1

Jumping Bug

15 min

Activity Narrative

There is a digital version of this activity.

In this task, students are introduced to the absolute value of a number as its distance from zero on the number line. The context is not realistic, but helps students visualize relationships on the number line in a more concrete way.

In the digital version of the activity, students use an applet to visualize a bug jumping on a number line. The applet allows students to pick a starting point for the bug, choose the direction it jumps, and then check where it lands.

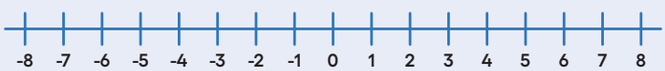
If students don't have individual access, displaying the applet for all to see would be helpful during the *Launch* and *Activity Synthesis*.

Launch

Give students 7–8 minutes of quiet work time, and followed with a whole-class discussion.

Student Task Statement

1. A bug is jumping around on a number line.



a. If the bug starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

5; this is 5 units from 0.

b. If the bug starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

-3; this is 3 units from 0.

c. If the bug starts at 0 and jumps 3 units away, where might it land?

3 or -3

d. If the bug jumps 7 units and lands at 0, where could it have started?

7 or -7

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports
Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

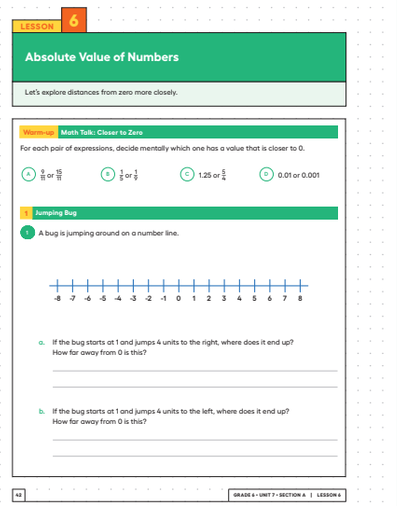
Building on Student Thinking

If students confuse absolute value with opposites and think that finding the absolute value of a number changes the sign of the number, consider sharing these examples and explaining how they contrast opposites and absolute value:

A car can travel 5 miles east or 5 miles west. Either way, the car travels a distance of 5 miles. However, the car can move in opposite directions.

In a board game, a playing piece can move 3 spaces forward or 3 spaces backward in the opposite direction. Both moves involve a distance of 3 spaces. However, they go in opposite directions.

Student Workbook


LESSON 6

Absolute Value of Numbers

Let's explore distances from zero more closely.

Warm-up Math Talk: Closer to Zero

For each pair of expressions, decide mentally which one has a value that is closer to 0.

A $\frac{9}{10}$ or $\frac{15}{11}$ B $\frac{1}{2}$ or $\frac{1}{9}$ C 1.25 or $\frac{5}{4}$ D 0.01 or 0.001

1 Jumping Bug

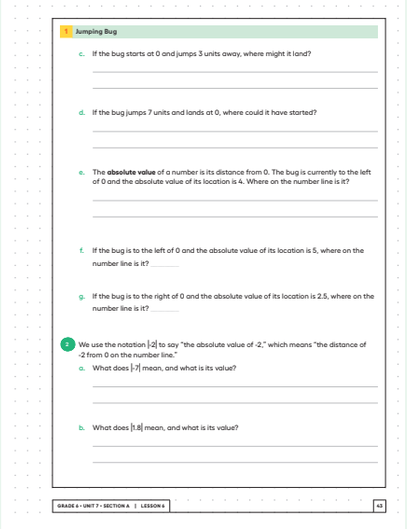
1 A bug is jumping around on a number line.

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

a. If the bug starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

b. If the bug starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

Student Workbook



1 Jumping Bug

c. If the bug starts at 0 and jumps 3 units away, where might it land?

d. If the bug jumps 7 units and lands at 0, where could it have started?

e. The **absolute value** of a number is its distance from 0. The bug is currently to the left of 0 and the absolute value of its location is 4. Where on the number line is it?

f. If the bug is to the left of 0 and the absolute value of its location is 5, where on the number line is it? _____

g. If the bug is to the right of 0 and the absolute value of its location is 2.5, where on the number line is it? _____

2 We use the notation |-2| to say "the absolute value of -2," which means "the distance of -2 from 0 on the number line."

a. What does |-7| mean, and what is its value?

b. What does |1.8| mean, and what is its value?

e. The **absolute value** of a number is its distance from 0. The bug is currently to the left of 0 and the absolute value of its location is 4. Where on the number line is it?

-4

f. If the bug is to the left of 0 and the absolute value of its location is 5, where on the number line is it?

-5

g. If the bug is to the right of 0 and the absolute value of its location is 2.5, where on the number line is it?

2.5

2. We use the notation |-2| to say "the absolute value of -2," which means "the distance of -2 from 0 on the number line."

a. What does |-7| mean, and what is its value?

|-7| means the distance of -7 from 0 on the number line. Its value is 7.

b. What does |1.8| mean, and what is its value?

|1.8| means the distance of 1.8 from 0 on the number line. Its value is 1.8.

Activity Synthesis

The purpose of this discussion is to formally introduce the term absolute value. The **absolute value** of a number is its distance from zero. Begin by asking students to contrast |-8| and |8|. (They are both finding the distance from a number to zero. They both have the same value, but represent the distance between two different points and zero.) If needed, draw a number line, and demonstrate how -8 and 8 are both 8 units away from 0.

Here are some additional questions for discussion:

- *"Does finding a number's absolute value always mean changing the sign?"*

  No, absolute value represents a number's distance from zero. Distances are always positive, so absolute value will always be positive.

- *"What is the absolute value of 12? Why?"*

  The absolute value of 12 is 12 because 12 is 12 units away from 0 on the number line.

- *"Are there any other numbers that have an absolute value of 12?"*

  Yes, the absolute value of -12 is also 12 because it is also 12 units away from 0, just on the opposite side of the number line.

- *"Can the absolute value of a number ever be negative?"*

  No, it can't be negative because absolute value represents a distance, and we can't have a negative distance.

Conclude the discussion by asking students to think of situations that use the concept of absolute value.

A car's odometer tracks the miles driven regardless of the direction. A swimmer swimming laps in a pool will end up in the same location, but absolute value can be used to measure the total distance swam. People can get on or off a city bus. Absolute value measures the total number of riders.

Activity 2

Absolute Elevation and Temperature

10 min

Activity Narrative

In this activity, students continue to develop their understanding that $|x|$ represents the distance from zero to x and that $|-x|$ and $|x|$ are equal.

Launch

Tell students that a part of the city of New Orleans is 6 feet below sea level. Explain that we can use "-6 feet" to describe its elevation, and "|-6| feet" to describe its vertical distance from sea level.

Remind students that we write "-5 °C" to describe a temperature that is 5 degrees Celsius below freezing and "5 °C" for a temperature that is 5 degrees above freezing.

Give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. Match each number with a description of a situation.

A. 23 °C	**1.** The indoor temperature is above freezing.
B. -7 °C	**2.** The indoor temperature is away from the freezing point.
C. |23| °C	**3.** The outdoor temperature is below freezing.
D. |-7| °C	**4.** The outdoor temperature is away from the freezing point.

2. a. Which temperature is colder: -6 °C or 3 °C?

-6 °C

b. Which temperature is closer to the freezing point: -6 °C or 3 °C?

3 °C

c. Which temperature has a smaller absolute value? Explain how you know.

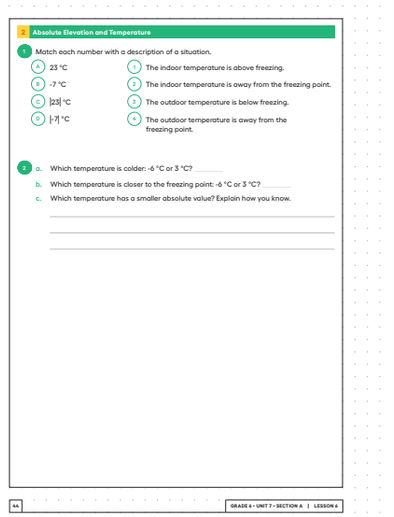
3 °C

Sample reasoning: 3 is closer to 0 than -6 is.

3. Match each number with a description of an elevation situation.

A. 12 feet	**1.** The elevation of a fish swimming below sea level
B. |-800| feet	**2.** The elevation of a bird flying above sea level
C. -51 feet	**3.** The distance between the surface of the water and a submerged submarine
D. |30| feet	**4.** The distance between the top of a tree and the ground

Student Workbook


2 Absolute Elevation and Temperature

1 Match each number with a description of a situation.

A 23 °C — 1 The indoor temperature is above freezing.
B -7 °C — 2 The indoor temperature is away from the freezing point.
C |23| °C — 3 The outdoor temperature is below freezing.
D |-7| °C — 4 The outdoor temperature is away from the freezing point.

2 a. Which temperature is colder: -6 °C or 3 °C? _____
b. Which temperature is closer to the freezing point: -6 °C or 3 °C? _____
c. Which temperature has a smaller absolute value? Explain how you know.

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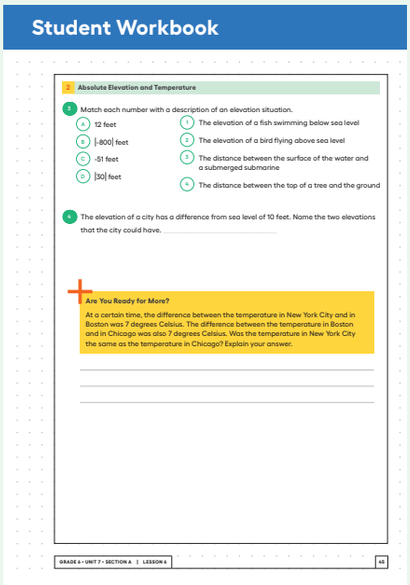
Student Workbook


2 Absolute Elevation and Temperature

3 Match each number with a description of an elevation situation.

A 12 feet
B |-800| feet
C -51 feet
D |30| feet

1 The elevation of a fish swimming below sea level
2 The elevation of a bird flying above sea level
3 The distance between the surface of the water and a submerged submarine
4 The distance between the top of a tree and the ground

4 The elevation of a city has a difference from sea level of 10 feet. Name the two elevations that the city could have.

Are You Ready for More?

At a certain time, the difference between the temperature in New York City and in Boston was 7 degrees Celsius. The difference between the temperature in Boston and in Chicago was also 7 degrees Celsius. Was the temperature in New York City the same as the temperature in Chicago? Explain your answer.

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4. The elevation of a city has a difference from sea level of 10 feet. Name the two elevations that the city could have.

-10 feet or 10 feet

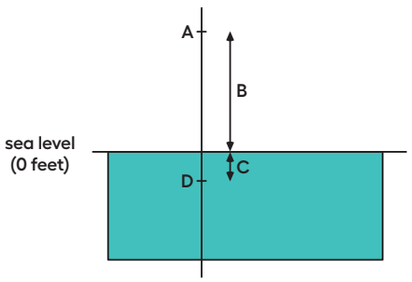
Are You Ready for More?

At a certain time, the difference between the temperature in New York City and in Boston was 7 degrees Celsius. The difference between the temperature in Boston and in Chicago was also 7 degrees Celsius. Was the temperature in New York City the same as the temperature in Chicago? Explain your answer.

Sample response: It is not possible to say for sure. There are two temperatures that differ from the temperature in Boston by 7 degrees. The temperature in New York City could be either 7 degrees above or below the temperature in Boston. The temperature in Chicago could also be either 7 degrees above or below the temperature in Boston. So the temperatures in New York City and Chicago could be equal, or they could differ by 14 degrees.

Activity Synthesis

The goal of this discussion is for students to explain how absolute value relates to distance and elevation in a situation. Arrange students in groups of 2, and display the image and expressions for all to see.



- -4 feet
- |15| feet
- |-4| feet
- 15 feet

Give students 1 minute to study the image and match each letter with one of the expressions (A: 15 feet, B: |15| feet, C: |-4| feet, D: -4 feet).

Have students discuss their matches with a partner before inviting students to share their responses and reasoning. Record and display student responses on the image for all to see.

Lesson Synthesis

The goal of this discussion is to give students an opportunity to revise and refine their explanation of the difference between the absolute value of a number and the number's opposite. Arrange students in groups of 2, and display this prompt for all to see:

> *"How is the absolute value of a number the same and different from a number's opposite?"*

Give students 2–3 minutes to write their first draft and 2–3 minutes to share their draft with a partner. Close the partner conversations, and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Here is an example of a second draft:
The opposite of a number is the number that is the same distance away from 0 but on the opposite side of the number line. The opposite of a number could be positive or negative depending on what the original number was. The absolute value of a number tells how far that number is from 0, but it doesn't matter which side of 0 the original number is on since all we are looking for is a distance.

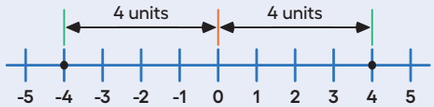
Lesson Summary

We can compare two numbers by looking at their positions on the number line: The number farther to the right is greater. The number farther to the left is less.

Sometimes we want to compare which one is closer to or farther from 0. For example, we may want to know how far away the temperature is from the freezing point of 0 °C, regardless of whether it is above or below freezing.

The **absolute value** of a number tells us its distance from 0.

For example, the absolute value of -4 is 4, because -4 is 4 units to the left of 0. The absolute value of 4 is also 4, because 4 is 4 units to the right of 0. Opposites always have the same absolute value because they are both the same distance from 0.



The distance from 0 to itself is 0, so the absolute value of 0 is 0. Zero is the *only* number whose distance to 0 is 0. For all other absolute values, there are always two numbers—one positive and one negative—that have that distance from 0.

To say, "the absolute value of 4," we write "|4|."

To say, "the absolute value of -8 is 8," we write "|-8| = 8."

Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect
Lead a discussion comparing, contrasting, and connecting the diagram in the *Activity Synthesis* with the numerical values shown in the bullets and phrases that connect to the situation (for example, "4 feet below sea level," "15 feet from sea level," "4 feet from sea level," and "15 feet above sea level"). Emphasize the language needed to say what the numerical values mean and how those values connect to the diagram and the phrases. Ask "How are these representations the same? How are they different?"

Advances: Representing, Conversing

Access for Students with Diverse Abilities (Activity 2, Synthesis)

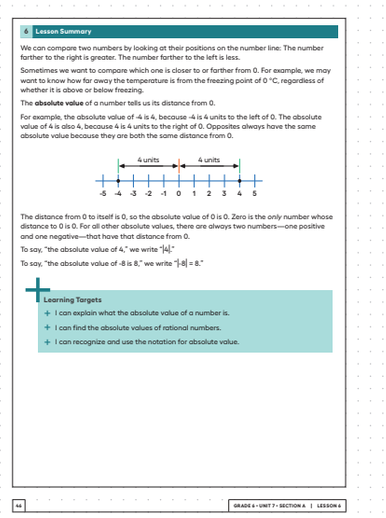
Representation: Develop Language and Symbols.
Maintain a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of the absolute value.

Supports accessibility for: Language, Memory

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



6 Lesson Summary

We can compare two numbers by looking at their positions on the number line: The number farther to the right is greater. The number farther to the left is less.

Sometimes we want to compare which one is closer to or farther from 0. For example, we may want to know how far away the temperature is from the freezing point of 0 °C, regardless of whether it is above or below freezing.

The **absolute value** of a number tells us its distance from 0.

For example, the absolute value of -4 is 4, because -4 is 4 units to the left of 0. The absolute value of 4 is also 4, because 4 is 4 units to the right of 0. Opposites always have the same absolute value because they are both the same distance from 0.

The distance from 0 to itself is 0, so the absolute value of 0 is 0. Zero is the *only* number whose distance to 0 is 0. For all other absolute values, there are always two numbers—one positive and one negative—that have that distance from 0.

To say, "the absolute value of 4," we write "|4|."

To say, "the absolute value of -8 is 8," we write "|-8| = 8."

Learning Targets
- I can explain what the absolute value of a number is.
- I can find the absolute values of rational numbers.
- I can recognize and use the notation for absolute value.

Cool-down

Greater, Less, the Same

5 min

Student Task Statement

1. Write a number that has the same value as each expression:
   a. |5|

      5 or |-5|

   b. |-12.9|

      12.9 or |12.9|
2. Write a number that has a value less than |4.7|

   Sample responses:
   - 4.5,
   - |-4.5|
   - -10
3. Write a number that has a value greater than |-2.6|

   Sample responses: 2.7 or |-2.7|

Lesson 6

Practice Problems

6 Problems

Problem 1

On the number line, plot and label all numbers with an absolute value of $\frac{3}{2}$.

Problem 2

The temperature at dawn is 6 °C away from 0. Select **all** the temperatures that are possible.

A. -12 °C

B. -6 °C

C. 0 °C

D. 6 °C

E. 12 °C

Problem 3

Order these numbers from least to greatest.

|-2.7| 0 1.3 |-1| 2

0 |-1| 1.3 2 |-2.7|

Problem 4

from Unit 5, Lesson 11

Lin's family needs to travel 325 miles to reach her grandmother's house.

a. At 26 miles, what percentage of the trip's distance have they completed?

8% of the trip, because 26 ÷ 325 = 0.08

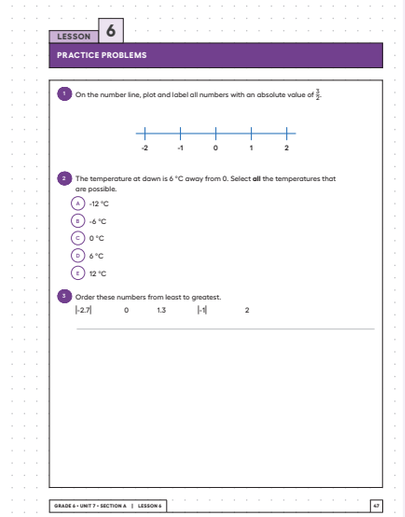
b. How far have they traveled when they have completed 72% of the trip's distance?

234 miles, because 0.72 · 325 = 234

c. At 377 miles, what percentage of the trip's distance have they completed?

116% of the trip, because 377 ÷ 325 = 1.16

Student Workbook


LESSON 6

PRACTICE PROBLEMS

1 On the number line, plot and label all numbers with an absolute value of $\frac{3}{2}$.

-2 -1 0 1 2

2 The temperature at dawn is 6 °C away from 0. Select **all** the temperatures that are possible.

A -12 °C
B -6 °C
C 0 °C
D 6 °C
E 12 °C

3 Order these numbers from least to greatest.

|-2.7| 0 1.3 |-1| 2

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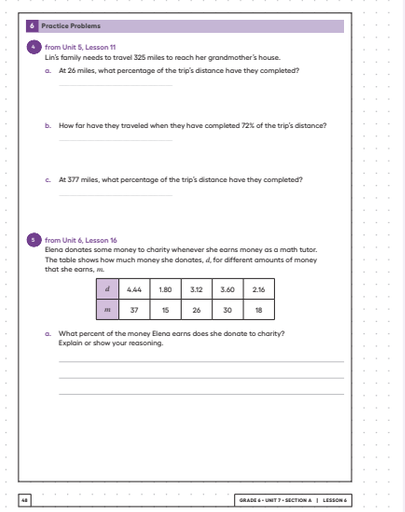
Student Workbook


6 Practice Problems

4 from Unit 5, Lesson 11

Lin's family needs to travel 325 miles to reach her grandmother's house.

a. At 26 miles, what percentage of the trip's distance have they completed?

b. How far have they traveled when they have completed 72% of the trip's distance?

c. At 377 miles, what percentage of the trip's distance have they completed?

5 from Unit 6, Lesson 16

Elena donates some money to charity whenever she earns money as a math tutor. The table shows how much money she donates, d, for different amounts of money that she earns, m.

d	4.44	1.80	3.12	3.60	2.16
m	37	15	26	30	18

a. What percent of the money Elena earns does she donate to charity? Explain or show your reasoning.

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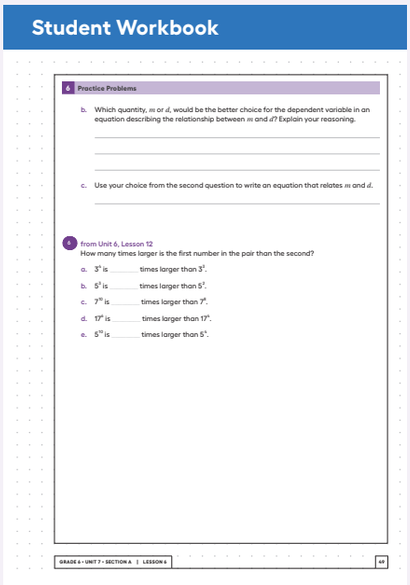
Student Workbook



6 Practice Problems

b. Which quantity, m or d, would be the better choice for the dependent variable in an equation describing the relationship between m and d? Explain your reasoning.

c. Use your choice from the second question to write an equation that relates m and d.

A from Unit 6, Lesson 12
How many times larger is the first number in the pair than the second?

a. 3^4 is _____ times larger than 3^3.
b. 5^3 is _____ times larger than 5^2.
c. 7^{10} is _____ times larger than 7^8.
d. 17^6 is _____ times larger than 17^4.
e. 5^{10} is _____ times larger than 5^4.

GRADE 6 • UNIT 7 • SECTION A | LESSON 6

Problem 5

from Unit 6, Lesson 16

Elena donates some money to charity whenever she earns money as a math tutor. The table shows how much money she donates, d, for different amounts of money that she earns, m.

d	4.44	1.80	3.12	3.60	2.16
m	37	15	26	30	18

a. What percent of the money Elena earns does she donate to charity? Explain or show your reasoning.

Elena donates 12% of the money she earns to charity.

Sample reasoning: We want to know what percent of 30 is 3.6, so we can write $30p = 3.6$.
To solve this, divide 3.6 by 30, which is 0.12. So 12% of 30 is 3.6.

b. Which quantity, m or d, would be the better choice for the dependent variable in an equation describing the relationship between m and d? Explain your reasoning.

Sample response: Since the amount of the donation depends on how much money she earns, d would be better as the dependent variable. If she wants to donate a certain amount and needs to figure out how much she needs to earn to achieve that donation, then m would be better as the dependent variable.

c. Use your choice from the second question to write an equation that relates m and d.

$d = .12m$ (or equivalent), $m = \frac{100}{12}d$, or $m = d \div .12$ (or equivalent)

Problem 6

from Unit 6, Lesson 12

How many times larger is the first number in the pair than the second?

a. 3^4 is 3 times larger than 3^3.

b. 5^3 is 5 times larger than 5^2.

c. 7^{10} is 7^2 or 49 times larger than 7^8.

d. 17^6 is 17^2 or 289 times larger than 17^4.

e. 5^{10} is 5^6 or 15,625 times larger than 5^4.