Lesson 7

Comparing Numbers and Distance from Zero

Goals

- Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values.
- Generate values that meet given conditions for their relative positions and absolute values, and justify (using words and symbols) the comparisons.
- Recognize that the value of $-a$ can be positive or negative, depending on the value of a.

Learning Targets

- I can explain what absolute value means in situations involving elevation.
- I can use absolute values to describe elevations.
- I can use inequalities to compare rational numbers and the absolute values of rational numbers.

Lesson Narrative

In this lesson, students distinguish order when working with rational numbers and their absolute values. Students begin by using a vertical number line to visualize the possible elevations of four characters in relation to a submarine. They use precise language when describing a character's elevation versus their distance from sea level.

Next, students use the *Information Gap* structure to locate points on a number line. Students will need to request information about the absolute values, opposites, and relative locations of the points.

An optional activity gives students additional practice comparing rational numbers by having partners take turns using numbers and symbols to write comparison statements and then using words to describe the same comparison.

Student Learning Goal

Let's use absolute value and negative numbers to think about elevation.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)

Instructional Routines

- MLR4: Information Gap Cards
- Poll the Class
- Take Turns

Required Materials

Materials To Copy

- Points on the Number Line Cards (1 copy for every 4 students): Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	20 min	15 min	10 min	5 min
				Cool-down

Instructional Routines

Poll the Class

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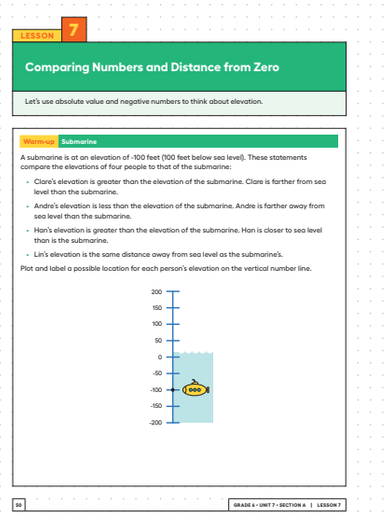
Student Workbook


LESSON 7

Comparing Numbers and Distance from Zero

Let's use absolute value and negative numbers to think about elevation.

Warm-up Submarine

A submarine is at an elevation of -100 feet (100 feet below sea level). These statements compare the elevations of four people to that of the submarine:

- Clare's elevation is greater than the elevation of the submarine. Clare is farther from sea level than the submarine.
- Andre's elevation is less than the elevation of the submarine. Andre is farther away from sea level than the submarine.
- Han's elevation is greater than the elevation of the submarine. Han is closer to sea level than is the submarine.
- Lin's elevation is the same distance away from sea level as the submarine's.

Plot and label a possible location for each person's elevation on the vertical number line.

200 150 100 50 0 -50 -100 -150 -200

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Warm-up

Submarine

10 min

Activity Narrative

In this *Warm-up*, students analyze statements about the elevations of four people in relation to a submarine. Considering possible elevations for each person and placing them on the number line serves as a transition to thinking about solutions to inequalities.

Monitor for students who choose positive and negative elevations for Han and Lin.

Launch

Arrange students in groups of 2–3. Display the image from the *Task Statement*, and read aloud the stem and first bullet about Clare, or have a student read them aloud. Poll the class for possible elevations for Clare. Display the results of the poll for all to see.

Then give students 3 minutes to complete the rest of the task. Give groups 1–2 minutes to compare where they plotted each person on the number line before the whole-class discussion.

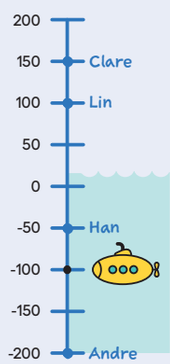
Student Task Statement

A submarine is at an elevation of -100 feet (100 feet below sea level). These statements compare the elevations of four people to that of the submarine:

- Clare's elevation is greater than the elevation of the submarine. Clare is farther from sea level than the submarine.
- Andre's elevation is less than the elevation of the submarine. Andre is farther away from sea level than the submarine.
- Han's elevation is greater than the elevation of the submarine. Han is closer to sea level than is the submarine.
- Lin's elevation is the same distance away from sea level as the submarine's.

Plot and label a possible location for each person's elevation on the vertical number line.

Sample response:



Activity Synthesis

The goal of this discussion is to emphasize the difference between elevation and distance from sea level. Begin by displaying the vertical number line from the *Task Statement* for all to see. Invite students to share possible elevations for each person and to share their reasoning about how the elevation satisfies each part of the given description. Record student responses on the display for all to see. If possible, record responses for Clare, Andre, Han, and Lin in different colors.

Then display this table for all to see.

	possible elevation	compare to submarine	distance from sea level
Clare	150 feet	150 > -100	|150| or 150 feet
Andre			
Han			
Lin			

Explain how the first row for Clare has been filled in with one possible elevation of 150 feet. Fill in the rest of the table as a class, emphasizing the language of "greater than" and "less than" when completing the middle column and "absolute value" when completing the right column.

If time allows, ask students if it would be possible for a person to have an elevation less than the submarine's and also be closer to sea level. (No, it would not be possible because any elevation less than that of the submarine would be below the submarine and must also be farther away from sea level.)

Activity 1

Info Gap: Points on the Number Line

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to determine the location of points on a number line.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Instructional Routines

MLR4: Information Gap Cards

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Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity, or provide students with a physical copy.

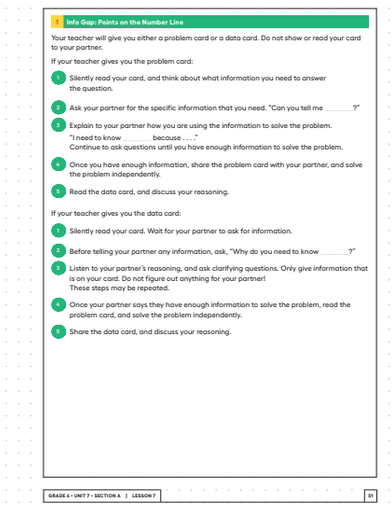
Supports accessibility for: Memory, Organization

Building on Student Thinking

If some students struggle to make sense of the abstract information they are given, rather than specifically instructing them to draw a number line, consider asking:

"What information do you know about the points?"
"How could you keep track of the information you've learned about the points so far?"

Student Workbook



1 Info Gap: Points on the Number Line

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . ." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

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Launch

Tell students they will be locating points on a number line. Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: Point A is at $-\frac{1}{2}$

Problem Card 2: Point Z is at 3

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"What strategies were helpful in determining the location of the point?"*

  Drawing a number line.

- *"Which clues were most helpful? Least helpful?"*

Highlight for students the following glossary and vocabulary terms that were used in this activity and how they are related but different: "absolute value," "sign," "opposite," "greater than," "less than."

Instructional Routines

Take Turns

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Activity 2: Optional

Inequality Mix and Match

15 min

Activity Narrative

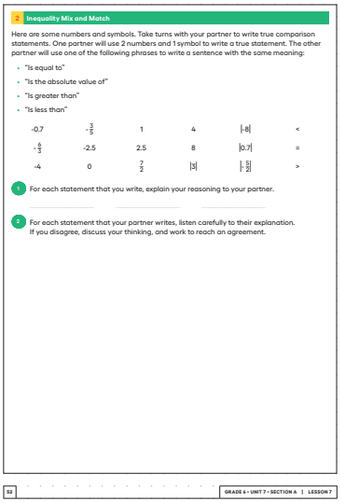
The purpose of this activity is to give students additional practice comparing rational numbers. In this partner activity, students take turns writing a true statement using numbers and symbols and translating that statement into words. Students must attend to precision as they communicate precisely with their partner.

Launch

Arrange students in groups of 2. Display the task for all to see. Tell students that they will work with their partner to write true comparison statements. If time allows, choose a student as a partner, and demonstrate how to do the activity. Otherwise, share these steps:

- One partner selects two numbers and one comparison symbol and uses them to write a true statement.
- The other partner listens and makes sure they agree with the statement.
- If they don't agree, the partners discuss until they come to an agreement.
- The other partner then uses one of the following phrases to write a sentence with the same meaning:
- "Is equal to"
- "Is the absolute value of"
- "Is greater than"
- "Is less than"
- For the next statement, the students swap roles.

Student Workbook



2 Inequality Mix and Match

Here are some numbers and symbols. Take turns with your partner to write true comparison statements. One partner will use 2 numbers and 1 symbol to write a true statement. The other partner will use one of the following phrases to write a sentence with the same meaning:

- "Is equal to"
- "Is the absolute value of"
- "Is greater than"
- "Is less than"

-0.7	$-\frac{3}{5}$	1	4	|-8|	<
$-\frac{6}{3}$	-2.5	2.5	8	|0.7|	=
-4	0	$\frac{7}{2}$	|3|	$\left\|-\frac{5}{2}\right\|$	>

1. For each statement that you write, explain your reasoning to your partner.
2. For each statement that your partner writes, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

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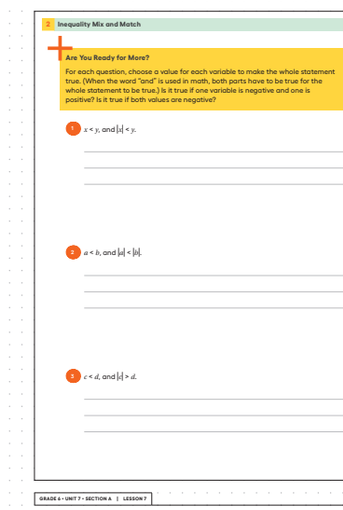
Student Workbook



2 Inequality Mix and Match

Are You Ready for More?

For each question, choose a value for each variable to make the whole statement true. (When the word "and" is used in math, both parts have to be true for the whole statement to be true.) Is it true if one variable is negative and one is positive? Is it true if both values are negative?

1. $x < y$, and $|x| < y$.
2. $a < b$, and $|a| < |b|$.
3. $c < d$, and $|c| > d$.

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Student Task Statement

Here are some numbers and symbols. Take turns with your partner to write true comparison statements. One partner will use 2 numbers and 1 symbol to write a true statement. The other partner will use one of the following phrases to write a sentence with the same meaning:

- "Is equal to"
- "Is the absolute value of"
- "Is greater than"
- "Is less than"

-0.7	$-\frac{3}{5}$	1	4	|-8|	<
$-\frac{6}{3}$	-2.5	2.5	8	|0.7|	=
-4	0	$\frac{7}{2}$	|3|	$\left\|-\frac{5}{2}\right\|$	>

1. For each statement that you write, explain your reasoning to your partner.
2. For each statement that your partner writes, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

Sample responses:

- $-0.7 > -\frac{6}{3}$
- $\frac{7}{2} < 4$
- $|-8| = 8$
- -0.7 is greater than $-\frac{6}{3}$
- $\frac{7}{2}$ is less than 4
- The absolute value of -8 is 8

Are You Ready for More?

For each question, choose a value for each variable to make the whole statement true. (When the word "and" is used in math, both parts have to be true for the whole statement to be true.) Is it true if one variable is negative and one is positive? Is it true if both values are negative?

1. $x < y$, and $|x| < y$.

   Sample response: $-1 < 5$, and $|-1| < 5$.
2. $a < b$, and $|a| < |b|$.

   Sample response: $-2 < 3$, and $|-2| < |3|$.
3. $c < d$, and $|c| > d$.

   Sample response: $-12 < -8$, and $|-12| > -8$.
4. $t < u$, and $|t| > |u|$.

   Sample response: $-10 < -1$, and $|-10| > |-11|$.

Activity Synthesis

Much discussion takes place between partners. Invite students to share how they wrote their true statements comparing two numbers or how they translated the statement into words. Consider discussing the following questions:

- *"What were some ways you handled disagreements?"*

  "Describe any difficulties you experienced and how you resolved them."

Lesson Synthesis

The goal of this discussion is for students to use precise language to articulate their understanding of absolute value. Display |-8| for all to see, and discuss the following questions:

- *"How do you say this?"*

  The absolute value of -8.

- *"What does it mean in an elevation situation?"*

  It represents the distance from 8 feet below sea level to sea level.

- *"What does it mean on a number line?"*

  It's the distance from -8 to 0 on the number line.

- *"What is its value?"*

  8

Next, display the statement |-8| > 5 for all to see, and discuss the following questions:

- *"How do you say this?"*

  The absolute value of -8 is greater than 5.

- *"What does it mean on a number line?"*

  -8 is more than 5 units away from 0.

- *"Is it true?"*

  Yes

Then, display the statement |-4| < 3 for all to see, and discuss the following questions:

- *"How do you say this?"*

  The absolute value of -4 is less than 3.

- *"What does it mean on a number line?"*

  -4 is less than 3 units away from 0.

- *"Is it true?"*

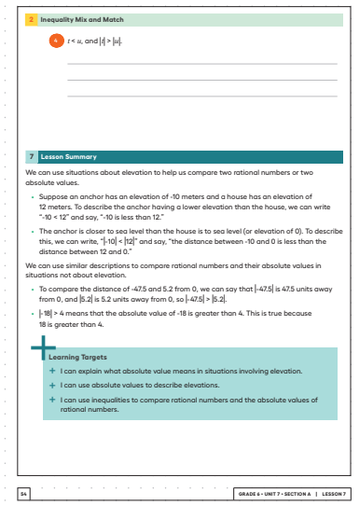
  No, -4 is more than 3 units away from 0.

Responding To Student Thinking

Points to Emphasize
If most students struggle with interpreting absolute value as different from the value, or with understanding the meaning of inequality symbols, revisit this idea in the practice problem referred to here.

Grade 6, Unit 7, Lesson 8, Practice Problem 4

Student Workbook


7 Inequality Mix and Match

A. $r < u$, and $|s| > |u|$.

7 Lesson Summary

We can use situations about elevation to help us compare two rational numbers or two absolute values.

- Suppose an anchor has an elevation of -10 meters and a house has an elevation of 12 meters. To describe the anchor having a lower elevation than the house, we can write "-10 < 12" and say, "-10 is less than 12."
- The anchor is closer to sea level than the house is to sea level (or elevation of 0). To describe this, we can write, "|-10| < |12|" and say, "the distance between -10 and 0 is less than the distance between 12 and 0."

We can use similar descriptions to compare rational numbers and their absolute values in situations not about elevation.

- To compare the distance of -47.5 and 5.2 from 0, we can say that |-47.5| is 47.5 units away from 0, and |5.2| is 5.2 units away from 0, so |-47.5| > |5.2|.
- |-18| > 4 means that the absolute value of -18 is greater than 4. This is true because 18 is greater than 4.

Learning Targets
- I can explain what absolute value means in situations involving elevation.
- I can use absolute values to describe elevations.
- I can use inequalities to compare rational numbers and the absolute values of rational numbers.

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Lesson Summary

We can use situations about elevation to help us compare two rational numbers or two absolute values.

- Suppose an anchor has an elevation of -10 meters and a house has an elevation of 12 meters. To describe the anchor having a lower elevation than the house, we can write "-10 < 12" and say, "-10 is less than 12."
- The anchor is closer to sea level than the house is to sea level (or elevation of 0). To describe this, we can write, "|-10| < |12|" and say, "the distance between -10 and 0 is less than the distance between 12 and 0."

We can use similar descriptions to compare rational numbers and their absolute values in situations not about elevation.

- To compare the distance of -47.5 and 5.2 from 0, we can say that |-47.5| is 47.5 units away from 0, and |5.2| is 5.2 units away from 0, so |-47.5| > |5.2|.
- |-18| > 4 means that the absolute value of -18 is greater than 4. This is true because 18 is greater than 4.

Cool-down

True or False?

5 min

Student Task Statement

Mark each statement as true or false, and explain your reasoning.

1. -5 < 3

True

Sample reasoning: -5 is farther to the left on the number line than 3.

2. -5 > 3

False

Sample reasoning: -5 is farther to the left on the number line than 3.

3. |-5| < 3

False

Sample reasoning: |-5| = 5, and 5 > 3.

4. |-5| > 3

True

Sample reasoning: |-5| = 5, and 5 > 3.

Lesson 7

Practice Problems

6 Problems

Problem 1

In the context of elevation, what would |-7| feet mean?

the vertical distance between the point at -7 feet and sea level (0 feet)

Problem 2

Match each written statement with a mathematical statement.

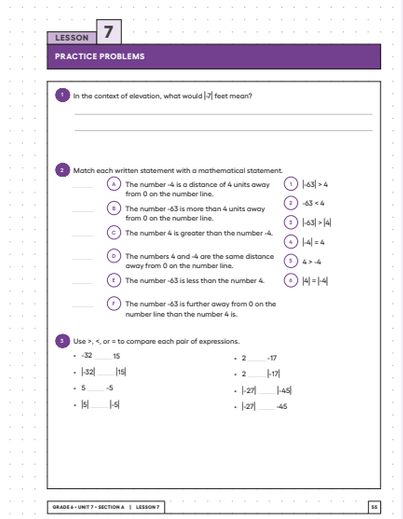
4	**A.** The number -4 is a distance of 4 units away from 0 on the number line.	**1.** |-63| > 4
1	**B.** The number -63 is more than 4 units away from 0 on the number line.	**2.** -63 < 4
5	**C.** The number 4 is greater than the number -4.	**3.** |-63| > |4|
6	**D.** The numbers 4 and -4 are the same distance away from 0 on the number line.	**4.** |-4| = 4
2	**E.** The number -63 is less than the number 4.	**5.** 4 > -4
3	**F.** The number -63 is further away from 0 on the number line than the number 4 is.	**6.** |4| = |-4|

Problem 3

Use >, <, or = to compare each pair of expressions.

- -32 < 15
- 2 > -17
- |-32| > |15|
- 2 < |-17|
- 5 > -5
- |-27| < |-45|
- |5| = |-5|
- |-27| > -45

Student Workbook


LESSON 7

PRACTICE PROBLEMS

1. In the context of elevation, what would |-7| feet mean?

2. Match each written statement with a mathematical statement.

A. The number -4 is a distance of 4 units away from 0 on the number line.
B. The number -63 is more than 4 units away from 0 on the number line.
C. The number 4 is greater than the number -4.
D. The numbers 4 and -4 are the same distance away from 0 on the number line.
E. The number -63 is less than the number 4.
F. The number -63 is further away from 0 on the number line than the number 4 is.

1. |-63| > 4
2. -63 < 4
3. |-63| > |4|
4. |-4| = 4
5. 4 > -4
6. |4| = |-4|

3. Use >, <, or = to compare each pair of expressions.

- -32 ___ 15
- |-32| ___ |15|
- 5 ___ -5
- |5| ___ |-5|
- 2 ___ -17
- 2 ___ |-17|
- |-27| ___ |-45|
- |-27| ___ -45

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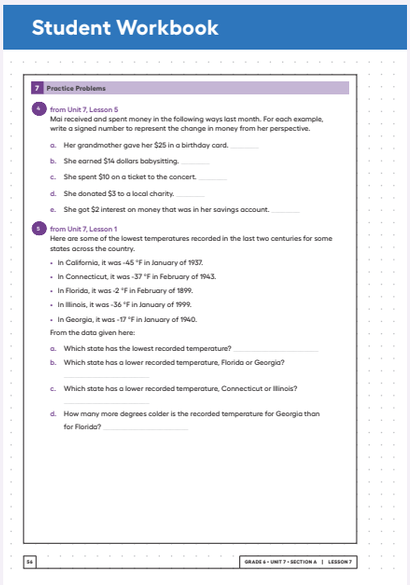
Student Workbook



7 Practice Problems

4 from Unit 7, Lesson 5
Mai received and spent money in the following ways last month. For each example, write a signed number to represent the change in money from her perspective.

a. Her grandmother gave her $25 in a birthday card. _____
b. She earned $14 dollars babysitting. _____
c. She spent $10 on a ticket to the concert. _____
d. She donated $3 to a local charity. _____
e. She got $2 interest on money that was in her savings account. _____

5 from Unit 7, Lesson 1
Here are some of the lowest temperatures recorded in the last two centuries for some states across the country.

- In California, it was -45 °F in January of 1937.
- In Connecticut, it was -37 °F in February of 1943.
- In Florida, it was -2 °F in February of 1899.
- In Illinois, it was -36 °F in January of 1999.
- In Georgia, it was -17 °F in January of 1940.

From the data given here:

a. Which state has the lowest recorded temperature? _____
b. Which state has a lower recorded temperature, Florida or Georgia? _____
c. Which state has a lower recorded temperature, Connecticut or Illinois? _____
d. How many more degrees colder is the recorded temperature for Georgia than for Florida? _____

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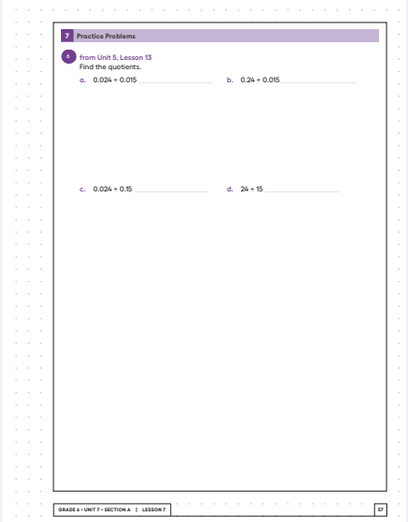
Student Workbook



7 Practice Problems

6 from Unit 5, Lesson 13
Find the quotients.

a. 0.024 ÷ 0.015 _____
b. 0.24 ÷ 0.015 _____
c. 0.024 ÷ 0.15 _____
d. 24 ÷ 15 _____

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Problem 4

from Unit 7, Lesson 5

Mai received and spent money in the following ways last month. For each example, write a signed number to represent the change in money from her perspective.

a. Her grandmother gave her $25 in a birthday card. +25 or 25

b. She earned $14 dollars babysitting. +14 or 14

c. She spent $10 on a ticket to the concert. -10

d. She donated $3 to a local charity. -3

e. She got $2 interest on money that was in her savings account. +2 or 2

Problem 5

from Unit 7, Lesson 1

Here are some of the lowest temperatures recorded in the last two centuries for some states across the country.

- In California, it was -45 °F in January of 1937.
- In Connecticut, it was -37 °F in February of 1943.
- In Florida, it was -2 °F in February of 1899.
- In Illinois, it was -36 °F in January of 1999.
- In Georgia, it was -17 °F in January of 1940.

From the data given here:

a. Which state has the lowest recorded temperature?

California

b. Which state has a lower recorded temperature, Florida or Georgia?

Georgia

c. Which state has a lower recorded temperature, Connecticut or Illinois?

Connecticut

d. How many more degrees colder is the recorded temperature for Georgia than for Florida?

15 °F

Problem 6

from Unit 5, Lesson 13

Find the quotients.

a. 0.024 ÷ 0.015 1.4

b. 0.24 ÷ 0.015 14

c. 0.024 ÷ 0.15 0.14

d. 24 ÷ 15 1.4