Lesson 12

Meaning of Exponents

Goals

- Describe (orally and in writing) a pattern that could be expressed using repeated multiplication.
- Generate and evaluate numerical expressions involving whole-number exponents.
- Interpret expressions with exponents greater than 3, and comprehend the phrase "to the power" or "to the" (in spoken language).

Learning Targets

- I can find the value of expressions with exponents and write expressions with exponents that are equal to a given number.
- I understand the meaning of an expression with an exponent like 3^5.

Access for Students with Diverse Abilities

- Representation (Warm-up, Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Narrative

In a previous unit, students were introduced to the term exponent in the context of area and volume. At that time, they worked with expressions in which a number was squared or cubed. This lesson extends that work by defining **exponent** more generally and considering exponent notation for any positive whole-number exponent.

In the *Warm-up*, students first consider a dot pattern where the number of dots is repeatedly multiplied by 3 in each successive level. Students next consider a situation involving grains of rice that repeatedly doubles, and connect the repeated calculations to expressions involving exponents. Students then make use of the new shorthand notation to write expressions with exponents that have particular values.

Student Learning Goal

Let's see how exponents show repeated multiplication.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Warm-up
Notice and Wonder: Dots and Lines

5 min

Activity Narrative

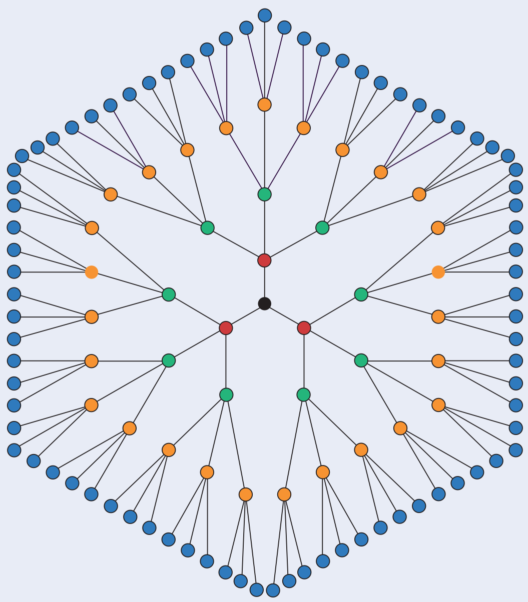
The purpose of this *Warm-up* is to give students an opportunity to look for multiplication patterns in an image, which will be useful when students interpret exponential relationships and notation in later activities. While students may notice and wonder many things about the image, including patterns in the dots, lines, and color, how repeated multiplication is shown in the image is the most important discussion point. Students see and make use of structure to describe the fact that each dot branches out to three more dots of a different color. These connections mean we have repeatedly growing groups of 3, so we can multiply by 3 to find the number of dots and lines at various stages.

Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- It looks like a hexagon.
- There are different colored dots.
- The black dot is the center.
- Each dot has three lines off of it.
- There is a vertical line of symmetry (and two other lines of symmetry).

Instructional Routines

Notice and Wonder

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Access for Students with Diverse Abilities (Warm-up, Launch)

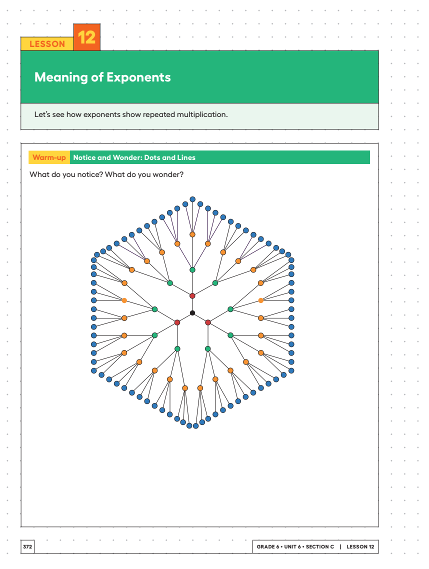
Representation: Internalize Comprehension.
Guide information processing and visualization. To support working memory, show the image for a longer period of time. Students may also benefit from being explicitly told not to count the dots, but instead to look for helpful structure within the image.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Some students may try to count the dots in the two outer layers. To encourage students to use the patterns in the image, ask them if there is an easier way they could use their count from the layer before to determine the next one.

Student Workbook


LESSON 12
Meaning of Exponents
Let's see how exponents show repeated multiplication.
Warm-up Notice and Wonder: Dots and Lines
What do you notice? What do you wonder?

- Each dot branches outward to three more dots of a different color.
- We can multiply the number of dots in one layer by 3 to find the number of dots in the next.

Students may wonder:

- How many dots are in the outer layer?
- How many dots would there be in the next layer if we drew it?
- What would happen if there were only two dots connected to each one?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the idea of each dot branching out into 3 more dots to form the new layer does not come up during the conversation, ask students to discuss this idea. Consider asking:

"How many red dots branch out from the black dot in the center?"

"How many green dots branch out from each red dot?"

"How could you use these facts to calculate how many green dots there are?"

"Does this pattern continue? How do you know?"

Activity 1

The Rice and Chessboard Problem

20 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to introduce a context where exponent notation is naturally useful. The task lends itself to connecting repeated calculations with an expression involving exponents. The situation described in the task is based on a legend that has a rich mathematical history. If time permits, consider inviting students to research the different versions of the legend and their connections to mathematical communities in Asia over time.

In the digital version of the activity, students use an applet to see how the number of grains of rice increases each day through day 16. The applet allows students to stop at any time and see the total count. Use the digital version to help students visualize how quickly the amount of rice increases or to simply provide an additional tool for all students to better understand the nature of exponential growth.

If students don't have individual access, displaying the applet for all to see would be helpful after students answer the third question.

Launch

Ask students to close their books or devices. Then ask students to share what they know about chess. Invite them to describe a chessboard. Explain that many historians believe chess comes from a strategy board game called chaturanga (cheh-tor-AHN-gah) that people in India played more than 1,000 years ago. Over time, the game became very popular and versions of it spread across the world.

Ask students,

"What do you notice? What do you wonder?"

It is natural for students to agree with the king and be skeptical of the inventor's proposal. Poll the class on what reward they would take and record the results. If possible, display the first few screens from the applet at *ggbm.at/hvJbDbjg* to help students see how the rice doubles each day, keeping the "Count" hidden. Use the Play and Pause buttons in the lower left corner of the screen. If not possible, ask students to describe what the first four days of the inventor's prize would look like. Draw their descriptions for all to see.

Distribute scientific calculators to students or be ready to display a scientific calculator from a device. Follow with 5 minutes of quiet work time for students to complete the first two questions and then pause for discussion. Encourage students to describe any patterns they see in the table.

Then draw students' attention to the third question. Ask,

"How would you use the calculator to figure this out?"

After a minute of quiet think time, solicit responses. Tell students how to calculate with exponents on the calculator and make the point that exponent notation is much more convenient for calculation and communication than writing out all the repeated factors.

Give students 5 minutes to complete the last two questions, followed by whole-class discussion.

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the story, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 1, Student Task)

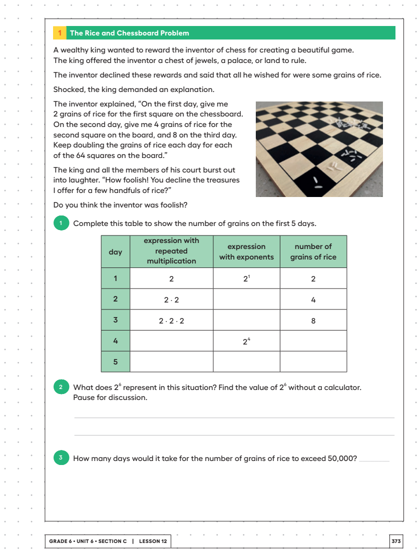
Representation: Internalize Comprehension.
Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content. Display an image or video of a chessboard to activate prior knowledge of the context of the doubling problem.

Supports accessibility for: Language, Conceptual Processing

Building on Student Thinking

Students might evaluate 2^6 as $2 \cdot 6$. If this happens, instruct students to review the table showing the number of grains accumulated each day. It will be apparent that many more than 12 grains will be accumulated after 6 days.

Student Workbook


1 The Rice and Chessboard Problem

A wealthy king wanted to reward the inventor of chess for creating a beautiful game. The king offered the inventor a chest of jewels, a palace, or land to rule.

The inventor declined these rewards and said that all he wished for were some grains of rice.

Shocked, the king demanded an explanation.

The inventor explained, “On the first day, give me 2 grains of rice for the first square on the chessboard. On the second day, give me 4 grains of rice for the second square on the board, and 8 on the third day. Keep doubling the grains of rice each day for each of the 64 squares on the board.”

The king and all the members of his court burst out into laughter. “How foolish! You decline the treasures I offer for a few handfuls of rice?”

Do you think the inventor was foolish?

1. Complete this table to show the number of grains on the first 5 days.

day	expression with repeated multiplication	expression with exponents	number of grains of rice
1	2	2^1	2
2	$2 \cdot 2$		4
3	$2 \cdot 2 \cdot 2$		8
4		2^4	
5			

2. What does 2^6 represent in this situation? Find the value of 2^6 without a calculator. Pause for discussion.

3. How many days would it take for the number of grains of rice to exceed 50,000? _____

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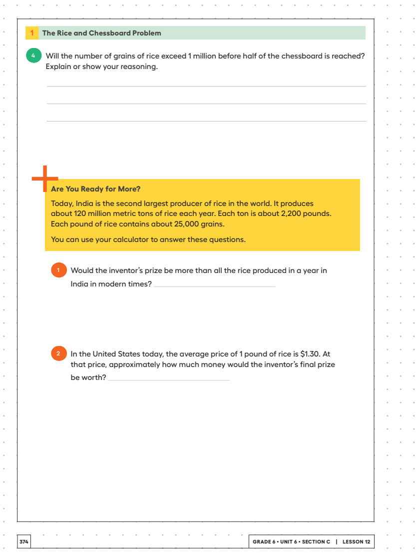
Student Workbook


1 The Rice and Chessboard Problem

4. Will the number of grains of rice exceed 1 million before half of the chessboard is reached? Explain or show your reasoning.

Are You Ready for More?

Today, India is the second largest producer of rice in the world. It produces about 120 million metric tons of rice each year. Each ton is about 2,200 pounds. Each pound of rice contains about 25,000 grains.

You can use your calculator to answer these questions.

1. Would the inventor’s prize be more than all the rice produced in a year in India in modern times? _____

2. In the United States today, the average price of 1 pound of rice is \$1.30. At that price, approximately how much money would the inventor’s final prize be worth? _____

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Student Task Statement

A wealthy king wanted to reward the inventor of chess for creating a beautiful game. The king offered the inventor a chest of jewels, a palace, or land to rule.

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2	$2 \cdot 2$	2^2	4
3	$2 \cdot 2 \cdot 2$	2^3	8
4	$2 \cdot 2 \cdot 2 \cdot 2$	2^4	16
5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^5	32

2. What does 2^6 represent in this situation? Find the value of 2^6 without a calculator. Pause for discussion.

   2^6 represents the number of grains on the 6th day. $2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$

3. How many days would it take for the number of grains of rice to exceed 50,000?

   16 days

4. Will the number of grains of rice exceed 1 million before half of the chessboard is reached? Explain or show your reasoning.

   Yes

   Sample reasoning:

   On the 16th day, there are 65,536 grains of rice. On the 17th day, there will be more than 130,000. There will be more than 260,000 on the 18th day, and more than half a million on the 19th day. On the 20th day, it will be more than 1 million. It will only take 4 more days to get to a million. $2^{20} = 1,048,576$. Since the chessboard has 64 squares, the number of grains of rice will exceed 1 million long before half the chessboard is reached.

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You can use your calculator to answer these questions.

1. Would the inventor's prize be more than all the rice produced in a year in India in modern times?

   Yes

   India produces about 6,600,000,000,000,000 grains of rice each year, which is less than the 18,446,744,073,709,551,616 grains of rice the inventor would get.

2. In the United States today, the average price of 1 pound of rice is $1.30. At that price, approximately how much money would the inventor's final prize be worth?

   about $960,000,000,000,000 or $960 trillion

Activity Synthesis

The goal of the discussion is for students to connect the idea of multiplying n factors of 2 to get the expression 2^n and to define **exponent** more generally. Display the expression 2^3, and explain that this notation is convenient for communicating and computing repeated multiplication. Remind students that the 3 in 2^3 is called an "exponent." Explain that it is used to indicate how many factors of 2 to multiply, so 2^3 means: $3 \cdot 3 \cdot 3$.

Invite students to share their responses. Ask questions such as:

"How did you know what 2^6 represents? How did you find its value?"

"How many times as much as 2^7 is 2^8? Could you answer this without finding the value of both expressions?"

"How did you organize your work to answer questions 3 and 4?"

"How many grains of rice would the inventor receive at the end of 32 days (half the chessboard)? After 64 days? Does this surprise you?"

Activity 2
Make 81

10 min

Activity Narrative

In this activity, students practice finding the value of exponential expressions and writing equivalent expressions. To determine equivalence, students need to apply the meaning of exponents and to look for structure.

Instructional Routines

MLR8: Discussion Supports

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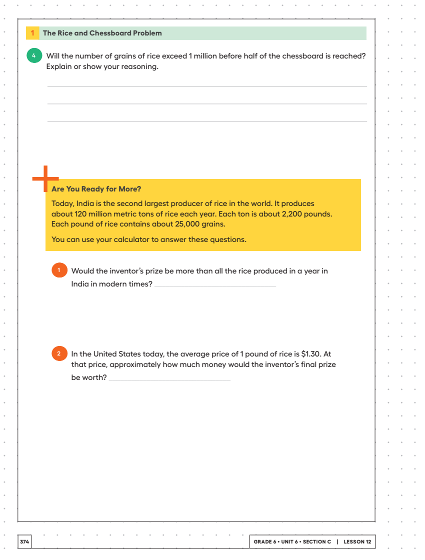
Student Workbook


1 The Rice and Chessboard Problem

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Are You Ready for More?

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1 Would the inventor's prize be more than all the rice produced in a year in India in modern times? ____

2 In the United States today, the average price of 1 pound of rice is $1.30. At that price, approximately how much money would the inventor's final prize be worth? ____

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports

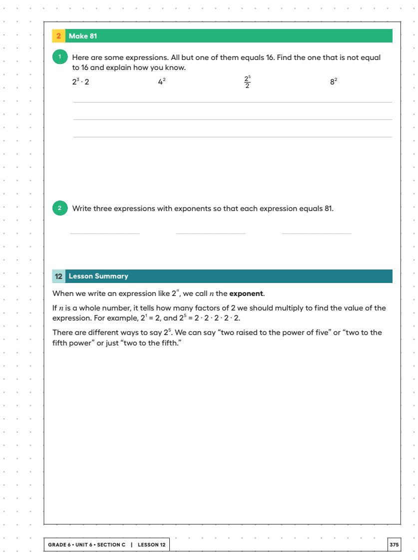
To support the transfer of new vocabulary and mathematical language to long-term memory, invite students to chorally repeat ways to read expressions with exponents. Display the expression $2^3 \cdot 2$ and the phrase "two to the third power times two." Read the phrase once, and invite students to repeat as you point to each part of the expression. Ask students to chorally repeat in unison 1–2 times.

Advances: Reading, Speaking

Building on Student Thinking

When evaluating exponential expressions, students might apply the order of operations incorrectly. For example, they might interpret $2^3 \cdot 2$ as 4^3 instead of 2 multiplied by itself 4 times. Students might also multiply the base and exponent, for instance, writing $9^2 = 18$ instead of $9^2 = 81$. If so, ask students to rewrite the expressions without exponents, for example, $2^3 \cdot 2 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

Student Workbook



2 Make 81

1 Here are some expressions. All but one of them equals 16. Find the one that is not equal to 16 and explain how you know.

$2^3 \cdot 2$ 4^2 $\frac{2^5}{2}$ 8^2

2 Write three expressions with exponents so that each expression equals 81.

12 Lesson Summary

When we write an expression like 2^n, we call n the **exponent**.

If n is a whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. We can say "two raised to the power of five" or "two to the fifth power" or just "two to the fifth."

Launch

Remind students that an exponent indicates how many times a number is being multiplied. Display 2^4. Ask,

"What does this expression mean?"

$2 \cdot 2 \cdot 2 \cdot 2$

Explain that there are different ways to read expressions with exponents. Point to 2^4 and explain that we can read this expression as "two raised to the power of four" or "two to the fourth power" or just "two to the fourth."

Give students 5 minutes of quiet work time, followed by whole-class discussion.

Student Task Statement

1. Here are some expressions. All but one of them equals 16. Find the one that is *not* equal to 16 and explain how you know.

   $2^3 \cdot 2$

   4^2

   $\frac{2^5}{2}$

   8^2

   Sample reasoning: $2^3 \cdot 2$ equals 16 because 2 is a factor 4 times. The expression 4^2 is 16 because it is $4 \cdot 4$. The expression $\frac{2^5}{2}$ is 16 because it is $32 \div 2$. The expression 8^2 is not equal to 16 because it is $8 \cdot 8$, not $8 \cdot 2$.

2. Write three expressions with exponents so that each expression equals 81.

   Sample responses:

   - 9^2
   - $\frac{9^3}{9}$
   - 3^4
   - $\frac{3^5}{3}$
   - $3^3 \div \frac{1}{3}$
   - $3 \cdot 3^3$
   - $3^2 \cdot 3^2$
   - 81^1
   - $\frac{81^2}{81}$

Activity Synthesis

The purpose of the discussion is to highlight that we can use exponents to express numbers in multiple ways.

Review students' responses to the first question and address any misconceptions. Then invite students to share some of their expressions for the second question. Look for expressions that go beyond the more obvious choices and that involve other operations. If students only share expressions that include 9^2 or 3^4, challenge them to come up with one more expression that involves an exponent and another operation. For any expression presented, ask other students confirm that it has the value of 81.

Lesson Synthesis

Display the image from the *Warm-up* and ask students to think about some of the things they may have wondered:

- *"The first layer has 3 red dots. How can we write an expression with an exponent to represent the number of dots in each layer?"*

  3 with an exponent that is the number of the layer

- *"Based on those expressions, how many dots are in the outer layer?"*

  3^4

- *"How many would be in the next layer?"*

  3^5

- *"In the one after that?"*

  3^6

- *"What part of the expression stays the same as we move from layer to layer?"*

  The 3 stays the same.

- *"What part changes? In what way does it change?"*

  The exponent changes. It increases by one for each higher layer.

- *"What would happen if there were 2 dots or 4 dots connected to each one? What part of the expression would change?"*

  The number that is being repeatedly multiplied would change to 2 or 4.

Lesson Summary

When we write an expression like 2^n, we call n the **exponent**.

If n is a whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. We can say "two raised to the power of five" or "two to the fifth power" or just "two to the fifth."

Cool-down

More 3's

5 min

Student Task Statement

1. What is the value of the expression 3^5?

   243

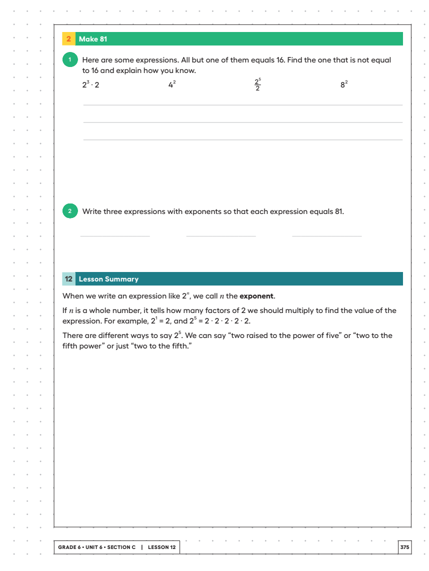
2. Explain how to use that value to quickly find the value of 3^6.

   Sample response: $3^6 = 3^5 \cdot 3 = 729$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



2 Make 81

1. Here are some expressions. All but one of them equals 16. Find the one that is not equal to 16 and explain how you know.

   $2^3 \cdot 2$ $\quad 4^2 \quad$ $\frac{2^5}{2}$ $\quad 8^2$

2. Write three expressions with exponents so that each expression equals 81.

12 Lesson Summary

When we write an expression like 2^n, we call n the **exponent**.

If n is a whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

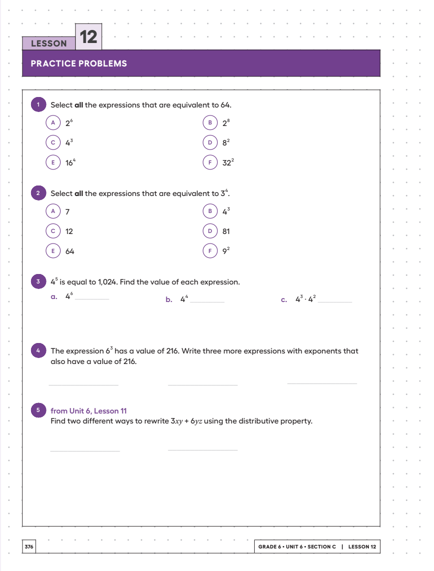
There are different ways to say 2^5. We can say "two raised to the power of five" or "two to the fifth power" or just "two to the fifth."

Lesson 12

Practice Problems

8 Problems

Student Workbook


LESSON 12
PRACTICE PROBLEMS

1 Select **all** the expressions that are equivalent to 64.
A 2^6 B 2^8
C 4^3 D 8^2
E 16^4 F 32^2

2 Select **all** the expressions that are equivalent to 3^4.
A 7 B 4^3
C 12 D 81
E 64 F 9^2

3 4^5 is equal to 1,024. Find the value of each expression.
a. 4^6 b. 4^4 c. $4^3 \cdot 4^2$

4 The expression 6^3 has a value of 216. Write three more expressions with exponents that also have a value of 216.

5 from Unit 6, Lesson 11
Find two different ways to rewrite $3xy + 6yz$ using the distributive property.

Problem 1

Select **all** the expressions that are equivalent to 64.

A. 2^6

B. 2^8

C. 4^3

D. 8^2

E. 16^4

F. 32^2

Problem 2

Select **all** the expressions that are equivalent to 3^4.

A. 7

B. 4^3

C. 12

D. 81

E. 64

F. 9^2

Problem 3

4^5 is equal to 1,024. Find the value of each expression.

a. 4^6

4,096

Sample reasoning: $4^6 = 4^5 \cdot 4 = 1{,}024 \cdot 4 = 4{,}096$

b. 4^4

256

Sample reasoning: $4^4 = 4^5 \div 4 = 1{,}024 \div 4 = 256$

c. $4^3 \cdot 4^2$

1,024

Sample reasoning: $4^3 \cdot 4^2 = 4^5 = 1{,}024$

Problem 4

The expression 6^3 has a value of 216. Write three more expressions with exponents that also have a value of 216.

Sample responses: $6^2 \cdot 6$, $\frac{6^4}{6}$, $2^3 \cdot 3^3$

Problem 5

from Unit 6, Lesson 11

Find two different ways to rewrite $3xy + 6yz$ using the distributive property.

Sample responses: $3(xy + 2yz)$, $3y(x + 2z)$, $y(3x + 6z)$

Problem 6

from Unit 6, Lesson 4

Solve each equation.

$a - 2.01 = 5.5$

$a = 7.51$

$b + 2.01 = 5.5$

$b = 3.49$

$10c = 13.71$

$c = 1.371$

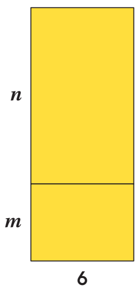
$100d = 13.71$

$d = 0.1371$

Problem 7

from Unit 6, Lesson 10

Which expressions represent the total area of the large rectangle? Select **all** that apply.



A. $6(m + n)$

B. $6n + m$

C. $6n + 6m$

D. $6mn$

E. $(n + m)6$

Problem 8

from Unit 3, Lesson 16

Is each statement true or false? Explain your reasoning.

a. $\frac{45}{100} \cdot 72 = \frac{45}{72} \cdot 100$

False

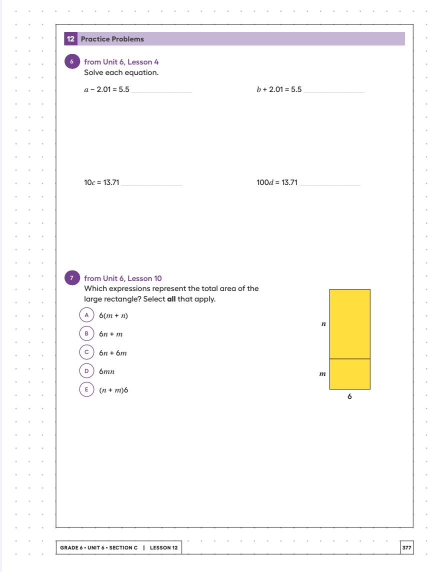
Sample reasoning: The left side equals $45 \cdot \frac{72}{100}$ and the right side equals $45 \cdot \frac{100}{72}$. The left side is less than 45 and the right side is greater than 45.

b. 16% of 250 is equal to 250% of 16.

True

Sample reasoning: 16% of 250 equals $\frac{16}{100} \cdot 250$, and 250% of 16 is $\frac{250}{100} \cdot 16$. Each of these is equal to $\frac{16 \cdot 250}{100}$.

Student Workbook



12 Practice Problems

6 from Unit 6, Lesson 4
Solve each equation.

$a - 2.01 = 5.5$ $b + 2.01 = 5.5$

$10c = 13.71$ $100d = 13.71$

7 from Unit 6, Lesson 10
Which expressions represent the total area of the large rectangle? Select **all** that apply.

A. $6(m + n)$
B. $6n + m$
C. $6n + 6m$
D. $6mn$
E. $(n + m)6$

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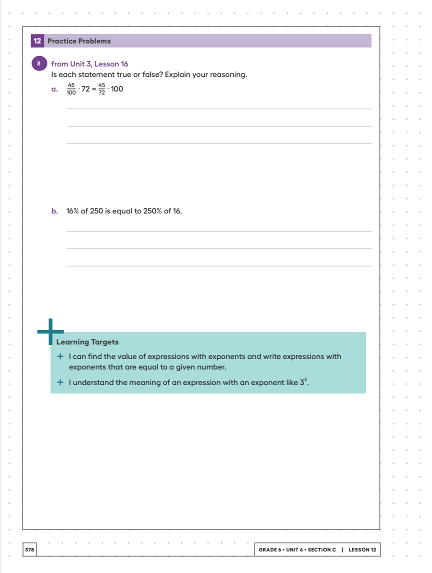
Student Workbook



12 Practice Problems

8 from Unit 3, Lesson 16
Is each statement true or false? Explain your reasoning.

a. $\frac{45}{100} \cdot 72 = \frac{45}{72} \cdot 100$

b. 16% of 250 is equal to 250% of 16.

Learning Targets
+ I can find the value of expressions with exponents and write expressions with exponents that are equal to a given number.
+ I understand the meaning of an expression with an exponent like 3^5.

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