

Equivalent Ratios Have the Same Unit Rates

Goals

- Apply reasoning about unit rates to complete a table of equivalent ratios, and explain (orally and in writing) the solution method.
- Explain (orally) that if two ratios are equivalent, they have the same rate per 1.
- Generalize that the unit rate is the factor that relates the value in one column to the value in the other column in a table of equivalent ratios.

Learning Targets

- I can give an example of two equivalent ratios and show that they have the same unit rates.
- I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios.

Lesson Narrative

The purpose of this lesson is to make it explicit to students that equivalent ratios have the same unit rates. To arrive at this insight, students reason about equivalent ratios and unit rates in a direction opposite of that in prior work.

Previously, students calculated unit rates from given ratios and then identified the ratios with matching unit rates as equivalent. For instance, they saw that 4,200 meters in 30 minutes and 6.5 kilometers in 45 minutes both have 140 meters per minute as a unit rate. This means both distance-to-time ratios are equivalent: they describe two things happening at the same rate (traveling at the same speed). Here, students reason the other way. They see that if two or more ratios are equivalent, then they have the same unit rates. For instance, $10:4$, $15:6$, and $20:8$, which are equivalent, all have $\frac{2}{5}$ and $\frac{5}{2}$ as their unit rates.

This understanding offers a new insight for reasoning with tables of equivalent ratios. In addition to reasoning across rows (understanding that a factor relates the values in any two rows), we can also reason across columns (understanding that another factor—a unit rate—relates the values in one column to the values in the other column.)

Access for Students with Diverse Abilities

- Engagement (Activity 2)
- Representation (Warm-up)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Which Three Go Together?

Lesson Timeline

5
min

Warm-up

15
min

Activity 1

20
min

Activity 2

10
min

Activity 3

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Equivalent Ratios Have the Same Unit Rates**Lesson Narrative (continued)**

<i>a</i>		<i>b</i>
10	$\div 5$	4
15	$\div 5$	6
20	$\div 5$	8

Students use these insights to find unknown quantities and to compare rates. Later in the lesson, students work to generalize their observations:

All ratios that are equivalent to $a:b$ have $\frac{a}{b}$ and $\frac{b}{a}$ as their unit rates.

Student Learning Goal

Let's revisit equivalent ratios.

Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL



Student Workbook

LESSON 6

Equivalent Ratios Have the Same Unit Rates

Let's revisit equivalent ratios.

Warm-up

Which Three Go Together: Distance and Time

Which three go together? Why do they go together?

Ⓐ 5 miles in 15 minutes

Ⓑ 3 minutes per mile

Ⓒ 20 miles per hour

Ⓓ 3 kilometers per minute

Binders and Notebooks

Two binders cost \$14 and 5 binders cost \$35.

a. Complete the table to show the cost for 4 binders and 10 binders at that rate. Next, find the cost for a single binder in each case, and record those values in the third column.

number of binders	cost (dollars)	unit price (dollars per binder)
2	14	
4		
5	35	
10		

b. What do you notice about the values in this table?

GRADE 6 • UNIT 3 • SECTION B | LESSON 6

Warm-up

Which Three Go Together: Distance and Time

10 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four rates. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terms that students know and how students talk about rates. During the discussion, listen for the terms “unit rate,” “speed,” and “pace” (which at this point students may use informally as a synonym for speed), and ways in which students reason about whether two rates indicate the same speed.

Launch



Arrange students in groups of 2–4. Display the four statements for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three statements that go together and can explain why. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. 5 miles in 15 minutes

B. 3 minutes per mile

C. 20 miles per hour

D. 3 kilometers per minute

Sample responses:

A, B, and C go together because:

- They show the same speed of 1 mile in 3 minutes or $\frac{1}{3}$ mile per minute.
- They have equivalent ratios of distance in miles to time in minutes.
- They all use miles as the unit for distance.

A, B, and D go together because:

- They all use minutes as the unit for time.

A, C, and D go together because:

- They all show a speed (distance in some amount of time) instead of a pace (amount of time per some unit of distance).

B, C, and D go together because:

- They are all unit rates.
- They all use the word “per” followed by 1 unit of something.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students’ explanations and ensure that the reasons given are correct.

During the discussion, ask students to explain the meaning of any terms that they use, such as “unit rate,” “speed,” or “pace,” and to clarify their reasoning as needed. Consider asking:

- “How do you know ...?”
- “What do you mean by ...?”
- “Can you say that in another way?”

Activity 1

Binders and Notebooks

15 min

Activity Narrative

In this activity, students see that when two ratios are equivalent, they have the same unit rate (if $a:b$ is equivalent to $c:d$, then $\frac{a}{b} = \frac{c}{d}$).

Students explore this idea by analyzing the values in a table that represent two quantities in a ratio. They notice that in addition to the values in the rows being equivalent ratios, the values in the columns have a multiplicative relationship. Students learn that a unit rate is a factor that relates the values in one column to those in another column. They also recognize that this unit rate can be used to reason about one quantity of the ratio when the other is known.

In investigating the relationships in the table, students practice looking for and making use of structure. As they use variables to generalize the pattern seen in numerical examples, they also practice expressing regularity in repeated reasoning.

As students work and discuss, identify those who observed and can describe the structure in the table. Select them to share during discussion later.

Launch

Arrange students in groups of 2. Give students a few minutes of quiet think time to complete the first set of questions, and then 1–2 minutes to discuss with a partner their observations about the values in the table. Ask them to complete the rest of the activity afterward.

Access for Students with Diverse Abilities (Warm-up, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the number of items and the cost in dollars in each table. Color code them in the same way when calculating the unit rate of dollars per item.
Supports accessibility for: Visual-Spatial Processing

Building on Student Thinking


Students may not realize that the third column asks for dollars per 1 binder and instead may write 14 dollars per 2 binders or 28 dollars per 8 binders. If this happens, remind students that “per binder” means “per 1 binder.”

Student Workbook

1 Binders and Notebooks

2 This table shows the cost of notebooks. Complete the table. Be prepared to explain your reasoning.

number of notebooks	cost (dollars)	unit price (dollars per notebook)
20		3
50		3
	21	3
	34	3
n		3



GRADE 6 • UNIT 3 • SECTION B | LESSON 6

95

Student Task Statement

1. Two binders cost \$14 and 5 binders cost \$35.
- a. Complete the table to show the cost for 4 binders and 10 binders at that rate. Next, find the cost for a single binder in each case, and record those values in the third column.

number of binders	cost (dollars)	unit price (dollars per binder)
2	14	7
4	28	7
5	35	7
10	70	7

- b. What do you notice about the values in this table?

Sample responses:

- The ratios of number of binders to cost in dollars are equivalent.
- The unit price is always \$7.
- Multiplying the numbers in the first column by 7 gives the numbers in the second column.

2. This table shows the cost of notebooks. Complete the table. Be prepared to explain your reasoning.

number of notebooks	cost (dollars)	unit price (dollars per notebook)
20	60	3
50	150	3
7	21	3
28	84	3
n	$3 \cdot n$	3

Activity Synthesis

Focus the discussion on the relationships that students noticed in the tables.

Display a completed version of the first table for all to see. Select previously identified students to share their observations. As they explain, illustrate their comments on the table. Students may bring up that:

- The ratios shown in the first two columns are equivalent across all rows.
- The ratios in all rows have the same unit price.
- When the number of binders doubles from 2 to 4 (or from 5 to 10), the cost also doubles, but the unit price is still 7.
- Multiplying the number of binders by the unit price gives the cost.
- Dividing the cost in dollars by the number of binders gives the unit price.

Highlight the first three observations, or bring them up if students do not mention them. (The last two observations can be emphasized when discussing the second table about notebooks.)

Next, invite selected students to share how they completed the second table. To highlight that the unit price relates the values in the columns by multiplication, ask questions such as:

☞ “How are the values in the first two columns related to each other?”

The values in the second column are 3 times those in the first column.

The value in the first column is $\frac{1}{3}$ of that in the second column.

☞ “How is the 3 in the last column related to the values in the first two columns?”

It’s the number that relates the two columns. Multiplying the 3 by the number of notebooks gives the cost in dollars. Dividing the 3 by the cost in dollars gives the number of notebooks.

☞ “What is the cost for buying n notebooks?”

$3 \cdot n$ dollars

☞ “Why does it make sense to express that cost as $3 \cdot n$?”

No matter how many notebooks are bought, the cost is 3 times that number.

Activity 2

Making Bracelets

15 min

Activity Narrative

This activity further develops students’ understanding that equivalent ratios have the same unit rate. Students practice finding an unknown value in a ratio given the other value and a unit rate, reinforcing their understanding of the relationships among the values. Then, they work to generalize this relationship using variables, by reasoning about whether the ratios such $5:d$ and $10:2d$, or $b:c$ and $3b:3c$, have the same unit rate.

As students work, monitor for different approaches taken. Some students may be inclined to create a different table—such as the one shown here—as an intermediate step for completing the given table.

time in hours	number of bracelets	speed (bracelets per hour)
1	6	6
2	12	6

Though appropriate, this intermediate strategy is less efficient than directly dividing or multiplying the value of one column by the unit rate to get the value of the other column.

Also monitor for students who can explain how they know the per-item cost is the same given two ratios with one or more variables.

Launch



Display the first sentence of the activity (“Clare makes bracelets at a constant speed”) and the first row of the table.

time spent (hours)	number of bracelets	speed (bracelets per hour)
2		6

Read the sentence aloud (or invite a student to do so). Ask students to interpret the sentence and the partial table. In particular, ask them to think about what the 2, the 6, and the missing value tell us about the situation.

Give students 1 minute of quiet think time. Then, invite a few students to share. If not apparent from students’ responses, clarify that the 2 and 6 in the first row tells us that Clare spends 2 hours making bracelets, at the speed of 6 bracelets per hour. The missing value is how many bracelets she makes at that rate and in 2 hours’ time.

Arrange students in groups of 2. Give students 5–6 minutes of quiet work time for the first two questions and then time to discuss their responses with a partner. Ask students to be mindful of how they go about completing the table and to be prepared to explain their thinking. Tell students to leave the last question unanswered for now.

Student Task Statement

1. Clare makes bracelets at a constant speed. Complete the table. Be prepared to explain your reasoning.

time spent (hours)	number of bracelets	speed (bracelets per hour)
2	12	6
5	30	6
7	42	6
11	66	6
15	90	6
h	$6 \cdot h$	6

2. Noah and Lin each bought some bracelets that Clare made.

a. Noah bought 5 bracelets for d dollars. How much did he pay per bracelet? Record it in the table.

	number of bracelets	cost (dollars)	unit price (dollars per bracelet)
Noah	5	d	$\frac{d}{5}$
Lin			

b. Lin bought twice as many bracelets as Noah bought and paid twice the amount Noah paid. Complete the table to show her purchase.

	number of bracelets	cost (dollars)	unit price (dollars per bracelet)
Noah	5	d	$\frac{d}{5}$
Lin	$2 \cdot 5$ or 10	$2 \cdot d$	$\frac{2 \cdot d}{10} = \frac{d}{5}$

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Students may benefit from feedback that emphasizes effort, and time on task. For example, provide positive feedback about student effort on the first parts of this task to encourage students to continue with the later, more abstract sections.

Supports accessibility for: Social-Emotional Functioning, Organization

Student Workbook

2 Making Bracelets

1 Clare makes bracelets at a constant speed. Complete the table. Be prepared to explain your reasoning.

time spent (hours)	number of bracelets	speed (bracelets per hour)
2		6
5		6
	42	6
	66	6
	90	6
h		6

2 Noah and Lin each bought some bracelets that Clare made.

a. Noah bought 5 bracelets for d dollars. How much did he pay per bracelet? Record it in the table.

	number of bracelets	cost (dollars)	unit price (dollars per bracelet)
Noah	5	d	
Lin			

b. Lin bought twice as many bracelets as Noah bought and paid twice the amount Noah paid. Complete the table to show her purchase.

c. Did Lin pay twice the unit price that Noah paid? Pause here for a class discussion before moving on.

3 Mai bought h bracelets for c dollars. Diego bought 3 times as many bracelets and paid 3 times as much as Mai did. Complete the table.

	number of bracelets	cost (dollars)	unit price (dollars per bracelet)
Mai	h	c	
Diego			

Building on Student Thinking

If students recognize the expressions $\frac{d}{5}$ and $\frac{2 \cdot d}{10}$ as fractions but do not see them as equivalent or are unsure how to make sense of them, consider using numerical examples to elicit what they know about the size of fractions. Ask them to explain what they know about numbers such as $\frac{1}{3}$ and $\frac{2}{6}$. They might say that these fractions are the same size, are both “a third,” or are equivalent fractions. Ask them to name one or more fractions that are equivalent to these and explain how they know the fractions are equivalent. If needed, clarify that the numerators and denominators of two equivalent fractions are related by the same factor.

Instructional Routines

MLR8: Discussion Supports

ilclassroom.com/r/10695617

Please log in to the site before using the QR code or URL



Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the question: “Did Lin pay twice the unit price that Noah paid? Explain why or why not.” Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback that they receive.

Advances: Writing, Speaking, Listening

c. Did Lin pay twice the unit price that Noah paid?

No, Lin did not pay twice the unit price.

Sample reasoning:

- The ratios $5:d$ and $10:2 \cdot d$ are equivalent (since 10 is 5 doubled and $2 \cdot d$ is d doubled), so they have the same unit price.
- The fraction $\frac{2 \cdot d}{2 \cdot 5}$ or $\frac{2 \cdot d}{10}$ is equivalent to $\frac{d}{5}$. (Multiplying the numerator and denominator of $\frac{d}{5}$ by 2 gives $\frac{2 \cdot d}{2 \cdot 5}$.)
- Multiplying 5 by the unit price $\frac{d}{5}$ gives d and multiplying 5 by the unit price $\frac{2 \cdot d}{10}$ also gives d . This means that the two unit prices are equivalent.

Pause here for a class discussion before moving on.

3. Mai bought b bracelets for c dollars. Diego bought 3 times as many bracelets and paid 3 times as much as Mai did. Complete the table.

	number of bracelets	cost (dollars)	unit price (dollars per bracelet)
Mai	b	c	$\frac{c}{b}$
Diego	$3 \cdot b$	$3 \cdot c$	$\frac{3 \cdot c}{3 \cdot b} = \frac{c}{b}$

Activity Synthesis

Select a few students to share with the class their strategies for completing the first table. If any students created an additional row to show 1 hour and another row to show 1 bracelet, invite them to share first. Progress toward strategies that use the given unit rate to navigate from column to column in efficient ways, such as multiplying the time in hours by 6 to find the number of bracelets, and multiplying the number of bracelets by $\frac{1}{6}$ (or dividing it by 6) to find time in hours.

Then, focus the discussion on how students know whether Lin and Noah paid the same unit price. Display the table for all to see and annotate it with students’ responses and reasoning. Highlight explanations that make clear that $\frac{2 \cdot d}{10}$ and $\frac{d}{5}$ are equivalent fractions or have the same value.

number of bracelets	cost in dollars	unit price
5	d	$\frac{d}{5}$
10	$2 \cdot d$	$\frac{2 \cdot d}{10}$

To help students further generalize this relationship, direct students’ attention to the last problem. Tell students: “Mai bought b bracelets and paid c dollars. Diego bought 3 times as many bracelets as Mai did and paid 3 times what Mai paid.” Give students 1–2 minutes to complete the table. Then, discuss:

☞ “What is the unit price Mai paid? How do you know?”

$\frac{c}{b}$ dollars

Dividing the amount paid by the number of items gives the unit price.

- “How many bracelets did Diego buy and for how much?”
- $3 \cdot b$ bracelets for $3 \cdot c$
- “Did Mai and Diego pay the same unit price? Why or why not?”
- Yes
- Diego paid $\frac{3 \cdot c}{3 \cdot b}$ dollars, which is equivalent to $\frac{c}{b}$.

Activity 3: Optional

How Much Applesauce?

10 min

Activity Narrative

This activity is a chance to apply newly-learned techniques to answer questions in another situation involving equivalent ratios. None of the given numbers are multiples of each other from row to row (for example, 7 isn’t a multiple of 4), which encourages students to reason about unit rates. The activity also prompts students to generalize how to use unit rates to find an unknown value in equivalent ratios and to describe their understanding in words.

Launch

If any students are familiar with making applesauce, ask them to explain how it is made. If not, explain: To make applesauce, peel, core, and chop apples. Then, heat the apples gently in a saucepan for a while until they break down into a sauce. Finally, add flavors like lemon juice and cinnamon.

Explain that if we know how many pounds of apples we start with, we can predict how many cups of applesauce we would have.

Give students 4–5 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

It takes 4 pounds of apples to make 6 cups of applesauce.

1. At this rate, how much applesauce can you make with:

a. 7 pounds of apples?

10.5 cups

b. 10 pounds of apples?

15 cups

2. How many pounds of apples would you need to make:

a. 9 cups of applesauce?

6 pounds

b. 20 cups of applesauce?

13⅓ pounds

Student Workbook

3 How Much Applesauce?

It takes 4 pounds of apples to make 6 cups of applesauce.

1 At this rate, how much applesauce can you make with:

a. 7 pounds of apples?

b. 10 pounds of apples?

2 How many pounds of apples would you need to make:

a. 9 cups of applesauce?

b. 20 cups of applesauce?

3 In general, how would you:

a. Find the number of cups of applesauce that can be made from a given number of pounds of apples.

b. Find the number of pounds of apples needed to make a given number of cups of applesauce.

Are You Ready for More?

Andre and his neighbor are using their garden hoses to fill a 750-gallon pool. Andre's hose can fill a 5-gallon bucket in 2 minutes. His neighbor's hose can fill a 10-gallon bucket in 8 minutes.

The hoses start filling the pool at the same time and work at the same rate as when filling a bucket. How long will they take to fill the pool?

GRADE 6 • UNIT 3 • SECTION B | LESSON 6

Access for Multilingual Learners
(Activity 3, Synthesis)

MLR8: Discussion Supports.
For each response that is shared, ask, “Who can restate what _____ shared using mathematical language?” or “Who can restate what _____ shared using the term ‘unit rate’?”
Advances: Listening, Speaking

3. In general, how would you:
- a. Find the number of cups of applesauce that can be made from a given number of pounds of apples?
Multiply the number of pounds of apples by 1.5 or $\frac{3}{2}$ because that's the unit rate (or the number of cups of applesauce that can be made from 1 pound of apples).
 - b. Find the number of pounds of apples needed to make a given number of cups of applesauce?
Divide the number of cups of applesauce by 1.5 or multiply it by $\frac{2}{3}$ to find the pounds of apples needed because 1 cup of applesauce requires $\frac{2}{3}$ pound of apples.

pounds of apples	cups of applesauce
4	6
7	10.5
10	15
6	9
$13\frac{1}{3}$	20

Are You Ready for More?

Andre and his neighbor are using their garden hoses to fill a 750-gallon pool. Andre’s hose can fill a 5-gallon bucket in 2 minutes. His neighbor’s hose can fill a 10-gallon bucket in 8 minutes.

The hoses start filling the pool at the same time and work at the same rate as when filling a bucket. How long will they take to fill the pool?

200 minutes or 3 hours and 20 minutes. Sample reasoning: The rate for Andre’s hose is 2.5 gallons per minute, and the rate for his neighbor’s hose is 1.25 gallons per minute. If they use the hoses at the same time, the pool will fill at a rate of 3.75 gallons per minute. $750 \div 3.75 = 200$, so it will take 200 minutes for the hoses to emit 750 gallons of water.

Activity Synthesis

Focus the discussion on the questions that prompted students to generalize their reasoning. Invite students to share how they could compute the number of cups of applesauce given the number of pounds of apples, and vice versa. Highlight explanations that incorporate unit rates and multiplicative reasoning across columns.

Lesson Synthesis

- The two important takeaways from the lesson are:
- Equivalent ratios have the same unit rate.
 - In a table of equivalent ratios, unit rates are the factors that relate the values in one column to the values in the other.

To summarize these ideas, consider displaying a table as shown and completing it as students answer the following questions. Leave the last column blank for now.

time spent (hours)	number of bracelets	number of bracelets per hour	
2	12		
4			
10			

“Clare made bracelets at a constant speed. She made 12 bracelets in 2 hours. How many would she make in 4 hours?”

24 bracelets

“In 10 hours?”

60 bracelets

“What do we know about the ratios 2 to 12, 4 to 24, and 10 to 60?”

They are equivalent.

“One unit rate tells us the number of bracelets made per hour. What is that number?”

6

“How can we show that it is always 6?”

Dividing 12 by 6, 24 by 4, and 60 by 10 all give 6.

“What does the other unit rate tell us?”

The hours spent per bracelet

Label the last column “hours spent per bracelet.” Ask students:

“What is the second unit rate?”

$\frac{1}{6}$

“How can you show that it is always $\frac{1}{6}$?”

Dividing 2 by 12, 4 by 24, and 10 by 60 all give $\frac{1}{6}$.

Student Workbook

Lesson Summary

The table shows different amounts of apples selling at the same rate. This means that all of the ratios of weight (in pounds) to price (in dollars) are equivalent.

We can find the unit price in dollars per pound by dividing the price (in dollars) by the weight of apples (in pounds).

In each case, the unit price is always the same. Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, the apples cost 2.50 dollars per pound.

weight of apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

We can also find the number of pounds of apples we can buy per dollar by dividing the weight of apples (in pounds) by the price (in dollars).

The number of pounds we can buy for a dollar is the same as well! Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, we are getting 0.4 pound per dollar.

weight of apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

This is true in all situations: When two ratios are equivalent, their unit rates will be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
5 · a	5 · b	$\frac{5 \cdot a}{5 \cdot b} = \frac{a}{b}$	$\frac{5 \cdot b}{5 \cdot a} = \frac{b}{a}$
x · a	x · b	$\frac{x \cdot a}{x \cdot b} = \frac{a}{b}$	$\frac{x \cdot b}{x \cdot a} = \frac{b}{a}$

GRADE 6 • UNIT 3 • SECTION B | LESSON 6

Point out that multiplying 6 by the value in the first column gives the value in the second column, and multiplying $\frac{1}{6}$ by the value in the second column gives the value in the first. Consider annotating the table to show how unit rates relate the values across columns:

time spent (hours)		number of bracelets
2	$\cdot 6$	12
4	$\cdot 6$	24
10	$\cdot 6$	60

time spent (hours)		number of bracelets
2	$\cdot \frac{1}{6}$	12
4	$\cdot \frac{1}{6}$	24
10	$\cdot \frac{1}{6}$	60

Lesson Summary

The table shows different amounts of apples selling at the same rate. This means that all of the ratios of weight (in pounds) to price (in dollars) are equivalent.

We can find the *unit price* in dollars per pound by dividing the price (in dollars) by the weight of apples (in pounds).

In each case, the unit price is always the same. Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, the apples cost 2.50 dollars per pound.

weight of apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

We can also find the number of pounds of apples we can buy per dollar by dividing the weight of apples (in pounds) by the price (in dollars).

weight of apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a dollar is the same as well! Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, we are getting 0.4 pound per dollar.

This is true in all situations: When two ratios are equivalent, their unit rates will be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
$5 \cdot a$	$5 \cdot b$	$\frac{5 \cdot a}{5 \cdot b} = \frac{a}{b}$	$\frac{5 \cdot b}{5 \cdot a} = \frac{b}{a}$
$s \cdot a$	$s \cdot b$	$\frac{s \cdot a}{s \cdot b} = \frac{a}{b}$	$\frac{s \cdot b}{s \cdot a} = \frac{b}{a}$

Responding To Student Thinking

Points to Emphasize
If students struggle to calculate a unit rate because they misinterpret the quantities being divided, or if they mix up the dividend and the divisor, focus on interpreting both the ratios in a situation and the result of any division. For example, in this lesson, prompt students to clarify what the ratios and rates tell us in each practice problem:

Unit 3, Lesson 6 Equivalent Ratios Have the Same Unit Rates

Cool-down

Cheetah Speed

5 min

Student Task Statement

Complete the table to represent a cheetah running at a constant speed. Explain or show your reasoning.

time (seconds)	distance (meters)	speed (meters per second)
4	120	30
25	750	30
9	270	30

Sample reasoning:

The cheetah ran 120 meters in 4 seconds, so its speed was 30 meters per second ($120 \div 4 = 30$). Since the table represents a cheetah running at a constant speed, the speed in each row is 30 meters per second.

- To find the distance run in 25 seconds, multiply 25 by 30.
- To find the time it takes to run 270 meters, divide 270 by 30.

Practice Problems

7 Problems

Student Workbook

LESSON 6
PRACTICE PROBLEMS

1 A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$		
$1\frac{1}{2}$		
	110	

2 The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result. What is the meaning of the number that Elena has calculated?

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

3 A restaurant is offering two specials on *horchata*, a sweet cinnamon-infused rice milk: \$3 for a 12-oz cup, or \$5 for a 20-oz cup. Which *horchata* special is a better deal? Explain your reasoning.

GRADE 4 • UNIT 3 • SECTION 3 • LESSON 6

Problem 1

A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$	27.5	55
$1\frac{1}{2}$	82.5	55
2	110	55

Problem 2

The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result.

What is the meaning of the number that Elena has calculated?

The number, 8, represents how many ounces of tomatoes to use per ounce of onions.

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

Problem 3

A restaurant is offering two specials on *horchata*, a sweet cinnamon-infused rice milk: \$3 for a 12-oz cup, or \$5 for a 20-oz cup. Which *horchata* special is a better deal? Explain your reasoning.

Neither is a better deal. Sample reasoning: $3 \div 12 = 0.25$ and $5 \div 20 = 0.25$.

The two specials have the same unit rate of \$0.25 per oz of *horchata*.

Problem 4

Complete the table so that the cost per banana remains the same.

number of bananas	cost in dollars	unit price (dollars per banana)
4	2.00	0.50
6	3.00	0.50
7	3.50	0.50
10	5.00	0.50
20	10.00	0.50
33	16.50	0.50

Problem 5

from Unit 3, Lesson 4

Two planes travel at a constant speed. Plane A travels 2,800 miles in 5 hours. Plane B travels 3,885 miles in 7 hours. Which plane is faster? Explain your reasoning.

Plane A is faster. Sample reasoning: Plane A travels $2800 \div 5 = 560$ or 560 miles per hour. Plane B travels $3,885 \div 7 = 555$, or 555 miles per hour. Plane A travels a farther distance in one hour.

Problem 6

from Unit 3, Lesson 5

A car can travel 35 miles per gallon of gas. It uses $\frac{1}{35}$ of a gallon of gas to go 1 mile.

- a. How far can the car travel with 15 gallons? Show your reasoning.

525 miles

Sample reasoning:

gallons of gas	miles car can travel
1	35
5	175
15	525

Student Workbook

4 Practice Problems

1 Complete the table so that the cost per banana remains the same.

number of bananas	cost in dollars	unit price (dollars per banana)
4		0.50
6		0.50
7		0.50
10		0.50
	10.00	0.50
	16.50	0.50

2 From Unit 3, Lesson 4:
Two planes travel at a constant speed. Plane A travels 2,800 miles in 5 hours. Plane B travels 3,885 miles in 7 hours. Which plane is faster? Explain your reasoning.

3 From Unit 3, Lesson 5:
A car can travel 35 miles per gallon of gas. It uses $\frac{1}{35}$ of a gallon of gas to go 1 mile.

a. How far can the car travel with 15 gallons? Show your reasoning.

b. How much gas does the car use to go 100 miles? Show your reasoning.

106 GRADE 4 • UNIT 3 • SECTION B LESSON 6

$\frac{100}{35}$ (or $\frac{20}{7}$ or $2\frac{6}{7}$) gallons

gallons of gas	miles car can travel
$\frac{1}{35}$	1
$\frac{10}{35}$	10
$\frac{100}{35}$	100

from Unit 3, Lesson 3

a. How much does a 3-kilogram object weigh in pounds?

Sample reasoning: If 10 kilograms is 22 pounds, then 1 kilogram is 2.2 pounds and 3 kilograms is $3 \cdot (2.2)$ or 6.6 pounds.

About 1.32 pounds

- Since 3 kilograms is 6.6 pounds, 6 kilograms is 13.2 pounds, and 600 grams is one-tenth of that, which is 1.32 pounds.
- Using a table, with 10 kilograms written as 10,000 grams:

grams	pounds
10,000	22
1,000	2.2
100	0.22
600	1.32