## **More Rate Comparisons**

#### Goal

## **Learning Target**

Apply reasoning about ratios and unit rates to compare prices and justify (orally) whether a given price is a good deal.

I can choose how to use unit rates to solve problems.

In this lesson, students gain fluency in working with unit rates by solving problems without scaffolding. They choose which unit rate to use to solve a problem, divide to find the desired unit rate, and multiply or divide by the unit rate to answer questions. Students may choose to create diagrams to represent the situations but are not prompted to do so.

In the main activity about shopping deals, students are given two rates—an original rate and a new rate—and are to decide whether to accept the latter. In the optional activity, students compare distances given in different units of length to determine which animal ran the farthest. To make comparisons, students need to use multiple conversion rates in a sequence to convert all the measurements to the same unit. This activity offers an additional opportunity to practice using unit rates in converting measurement units.

## Student Learning Goal

Let's use unit rates to solve problems.

#### **Access for Students with Diverse Abilities**

• Engagement (Activity 1, Activity 2)

#### **Access for Multilingual Learners**

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 2)

#### **Instructional Routines**

- · Card Sort
- · MLR2: Collect and Display
- MLR7: Compare and Connect
- Poll the Class
- · Take Turns

#### **Required Materials**

#### **Materials to Gather**

• Four-function calculators: Activity 2

## **Materials to Copy**

• Is It a Deal Cards (1 copy for every 2 students): Activity 1

### **Required Preparation**

#### Warm-up:

If desired, purchase a 4-pack beverage to display when launching the Warm-up.

#### Lesson:

Consider providing access to calculators. All of the calculations in this lesson can be done using strategies learned in grade 5. If arithmetic practice is desired, don't offer calculators. If calculations are likely to present a significant barrier to gradelevel work, make calculators available.

### **Lesson Timeline**

Warm-up **Activity 1** 

20

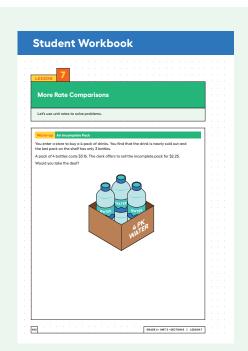
**Activity 2** 

10

**Lesson Synthesis** 

### **Assessment**

Cool-down



#### Warm-up

## **An Incomplete Pack**



## **Activity Narrative**

In this Warm-up, students are given two rates—one for a complete pack of a drink and one for an incomplete pack—and asked to decide whether to take the incomplete pack. The work prompts students to consider what makes a good deal and prepares them to make rate comparisons in a shopping context later in the lesson.

## Launch

Tell students that you will show them a 10 x 10 grid for 3 seconds and that the entire grid represents 1. Their job is to find how much is shaded in the image and explain how they saw it.

Display the image for 3 seconds and then hide it. Do this twice. Give students 15 seconds of quiet think time between each flash of the image. Encourage students who have one way of seeing the grid to consider another way to determine the size of the shaded portion while they wait.

#### **Student Task Statement**

You enter a store to buy a 4-pack of drinks. You find that the drink is nearly sold out and the last pack on the shelf has only 3 bottles.

A pack of 4 bottles costs \$3.16. The clerk offers to sell the incomplete pack for \$2.25.

Would you take the deal?



Answers vary.

## **Activity Synthesis**

Poll the class for their response and display how many students would and would not take the deal.

Then, ask students:

☐ "How could you figure out if the deal is good or not?"

Give students a minute of quiet think time to come up with strategies for solving such a problem. Invite a few students to share.

Tell students that in the next activity, they'll decide whether to take or reject some deals.

## **Activity 1**

### Card Sort: Is it a Deal?



## **Activity Narrative**

In this activity, students are given cards, each of which contains an original price and a new price, as shown.

B. Juice Boxes

Original: 10 for \$3.50

New Deal: 6 for \$2.40

Their job is to sort the cards into two piles: one pile for deals they would take, and another for those they would reject. A sorting task gives students opportunities to analyze statements and structures closely and make connections.

There are many paths that students could use to reason about whether or not to accept a deal. For example, in the case of juice boxes, they could:

 Find and compare the unit rates for both the original pack and the new pack. If the unit rate is the same, the deal is fair. If the unit rate is lower, the clerk is offering a discount. If the unit rate is higher, the clerk is not being fair.

number of juice boxes	cost in dollars	dollars per box
10	3.50	0.35
6	2.40	0.40

 Find the unit rate in the original pack, apply it to the number of items in the new pack, and compare the costs for the same number of items in the original and new pricing schemes. This can be done in two ways, one focused more on column reasoning and the other on row reasoning, as shown.

number of juice boxes	cost in dollars
10 -0	3.50
6 -0	.35 -> 2.10

#### **Instructional Routines**

#### **Card Sort**

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#### **Instructional Routines**

#### **Poll the Class**

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#### **Instructional Routines**

#### **Take Turns**

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## Access for Multilingual Learners (Activity 1, Student Task)

### MLR2: Collect and Display.

Circulate, listen for, and collect the language that students use to reason about the deals and to compare unit rates. On a visible display, record words and phrases such as "the same," "equal," "unit rate," "cost for the same number of items," and so on. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

## Access for Students with Diverse Abilities (Activity 1, Student Task)

## Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards after students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

number of juice boxes	cost in dollars
10	3.50
1 4	0.35
6	2.10

Use an abbreviated table and bypass calculating the unit rate. Find the
multiplier to get from the original to the new number of items, and use
the same multiplier to find what the price would be if the deal has not
changed. Compare the actual new price to this projected price.

number of juice boxes	cost in dollars
10 6	3.50
6	?

Monitor for different ways that students reason about the deals and decide whether to take or decline them.



Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2 and distribute to each group a set of five pre-cut cards, Cards A–E. If you want students to complete the "Are You Ready for More?" activity, distribute all six pre-cut cards, Cards A–F, and instruct the students to set aside Card F for now. Allow students to familiarize themselves with the representations on the cards:

- Give students 1 minute to place the first five cards face up and to start thinking about possible ways to sort the cards into categories.
- Pause the class, and select 1–3 students to share the categories that they identified.
- Discuss as many different categories as time allows.

After a brief discussion, tell students that their job is to sort the five cards into a "Deal" pile and a "No Deal" pile. Instruct partners to collaborate in finding the answer for Card A and to divide up the remaining cards between them. Ask students to first work on their cards individually, then to share their reasoning with their partner, and lastly, to sort all of the cards into two piles.

#### **Student Task Statement**

Your teacher will give you a set of cards showing different offers.

**1.** Find Card A, and work with your partner to decide whether the offer on Card A is a good deal. Explain or show your reasoning.

Card A: Deal

Sample reasoning: The original price per bottle is \$0.79 so a 3 pack should be \$2.37. The new deal, 3 for \$2.25, is cheaper.

- 2. Next, split Cards B–E so that you and your partner each have two.
  - **a.** Decide individually if your two cards are good deals. Be prepared to explain your reasoning.
  - **b.** For each of your cards, tell your partner whether you think it is a good deal or not, and explain why. Listen to your partner's explanations for the other cards. If you disagree, explain your thinking.
  - **c.** Revise any decisions about your cards based on the feedback from your partner.

Card B: No Deal

Sample reasoning: The original price per juice box is \$0.35. The new price per juice box is \$0.40, which is more expensive.

Card C: Deal

Sample reasoning: The original price per bar is \$0.88 so 4 bars should be \$3.52. The new deal, 4 for \$3.12, costs less.

Card D: No Deal

Sample reasoning: The original price per hummus container is \$0.75. In the new deal, the unit price is \$0.90, which is higher.

Card E: No Deal

Sample reasoning: The price per yogurt container is \$0.85 so 6 containers should cost \$5.10. The new deal, 6 for \$5.22, is more expensive.

**3.** When you and your partner are in agreement about Cards B–E, place all the cards that you think are a good deal in one stack and all the cards that you think are a bad deal in another stack. Be prepared to explain your reasoning. No response required.

#### **Are You Ready for More?**

It's time to make your own deal!

Read the information on Card F, and then decide what you would charge if you were the clerk. When your teacher signals, trade cards with another group and decide whether or not you would take the other group's offer.

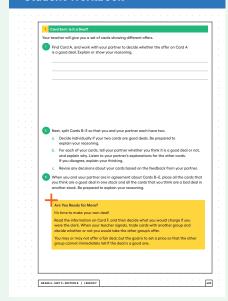
You may or may not offer a fair deal, but the goal is to set a price so that the other group cannot immediately tell if the deal is a good one.

Card F: Answers vary. Any price of \$6.84 or less for a pack of 9 (or \$0.76 per unit) would be a fair deal.

#### **Building on Student Thinking**

Some students may default to dividing the larger number by the smaller number as they investigate the deals. Encourage them to consider which quantity represents a price for multiple items and therefore needs to be divided. (For example, the \$3.50 price is for all 10 juice boxes so the 3.50 is the amount to be divided.)

#### **Student Workbook**



## **Activity Synthesis**

Select 2–3 students who used different but effective strategies to share their thinking with the class. Record the different strategies in one place, and display them for all to see. Highlight any similarities and differences, such as whether a unit rate was used, or whether students compared the original unit rate to the new quantity or the other way around.

The goal of the discussion is for students to see that in situations involving prices, we can make comparisons by reasoning about equivalent ratios or by using unit rates. Depending on what information is given and what is sought, some strategies may be more practical than others.

## **Activity 2: Optional**

The Fastest of All



## **Activity Narrative**

In this activity, students perform conversions to compare lengths given in customary and metric units. They apply what they know about ratios formed by different units of measurements, equivalent ratios, and strategies for converting units. To make some measurements comparable to others, students may need to perform multiple conversions and activate arithmetic skills from previous grades. Support students with computations as needed, and provide access to calculators as appropriate.

Students may choose to convert the given measurements into any of the given units. Although their calculations are likely to vary based on the choice made and the strategy used, students should arrive at the same conclusion.

Monitor for students who opt to use the same unit in making comparisons (for example, to convert all units to feet) so that they can be grouped together for discussion.

## Launch 2



Tell students to close their books or devices. Display the problem stem and the table with sprint distances for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing that they notice and one thing that they wonder about. Invite students to share their observations and questions. Record and display their responses for all to see, without editing or commentary.

If no students noticed that units of measurement are all different or wondered which animal ran the farthest or the fastest, discuss these ideas with them.

Then, display the question and prompt for all to see, and read them aloud: "Which animal ran the farthest in 1 minute? Order the sprint distances from greatest to least." Ask questions such as:

☐ "How might you compare the distances?"

"What information would you need?"

"What calculations would you need to perform?"

Arrange students in groups of 2–4. Give students 1–2 minutes to discuss with their group which unit (or units) to use to make comparisons. Encourage them to consider what their choice might mean in terms of the calculations to be made. (For example, students might notice that converting all distances into inches would mean performing more than one multiplication for most distances.)

Give students 8–10 minutes to work on the problem. Consider splitting up the conversion work and assigning each group member at least two measurements: one that needs to be converted to a larger unit and another to a smaller unit, in the same measurement system and across different ones. Provide access to calculators. Then, ask groups to discuss their work with another group who made comparisons using the same unit of measurement.

#### **Student Task Statement**

Six wild animals sprinted for 1 minute. The table shows how far they ran. Which animal ran the farthest in 1 minute?

#### The antelope

Order the sprint distances from greatest to least. Show your reasoning.

animal	sprint distance
cougar	1,408 yards
antelope	1 mile
hare	49,632 inches
kangaroo	1,073 meters
ostrich	1.15 kilometers
coyote	3,773 feet

# Distances in order from greatest to least: antelope, cougar, hare, coyote, ostrich, kangaroo

animal	original distance	distance in feet	distance in yards	distance in meters
antelope	l mile	5,280	1,760	1,609.34
cougar	1,408 yards	4,224	1,408	1,287.48
hare	49,632 inches	4,136	1,378 <sup>2</sup> / <sub>3</sub>	1,260.65
coyote	3,773 feet	3,773	1,257 <sup>2</sup> / <sub>3</sub>	1,150.01
ostrich	I.I5 kilometers	3,772.97	1,257 <sup>2</sup> / <sub>3</sub>	1,150
kangaroo	I,073 meters	3,520.34	I,173 <sup>1</sup> / <sub>3</sub>	1,073

## Access for Students with Diverse Abilities (Activity 2, Student Task)

## Engagement: Provide Access by Recruiting Interest.

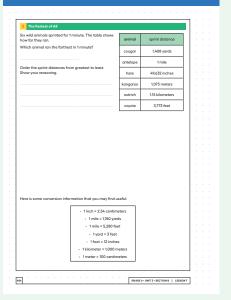
Leverage choice around perceived challenge. Invite students to select 4 of the 6 animals to order from greatest to least.

Supports accessibility for: Organization, Social-Emotional Functioning

### **Building on Student Thinking**

Students may not recognize that the conversion between two units may require one or more intermediate steps, and they might get stuck when not seeing the desired conversion information. Ask them if there is another unit that they could use as a stepping stone between the starting unit and the final unit.

## Student Workbook



Sample reasoning when converting all measurements to feet:

- Antelope sprinted 5,280 feet. I · I,760 · 3 = 5,280
- Cougar sprinted 4,224 feet. (1,408) · 3 = 4,224
- Hare sprinted 4,136 feet. 49,632 ÷ 12 = 4,136
- Coyote sprinted 3,773 feet.
- Ostrich sprinted almost 3,773 feet. (I.15)  $\cdot$  (I,000)  $\cdot$  100  $\div$  2.54  $\div$  12 = 3, 772.97
- Kangaroo sprinted 3,520 feet. (1,073) · 100 ÷ 2.54 ÷ 12 = 3,520.34

Here is some conversion information that you may find useful:

1 inch = 2.54 centimeters

1 mile = 1,760 yards

1 mile = 5,280 feet

1 yard = 3 feet

1 foot = 12 inches

1 kilometer = 1.000 meters

1 meter = 100 centimeters

### **Activity Synthesis**

Invite at least two students to share the ordered distances—one student who converted all measurements to feet or yards, and another who converted them to meters. Consider compiling the converted distances in a table such as shown in sample student responses and adding additional columns for other units used to make comparisons.

Ask questions such as:

- "How did you use the information given to convert a distance from a smaller unit to a larger unit, such as from inches to feet or to yards?"
  - Divide the distance in inches by the number of inches in I foot, and then divide the distance in feet by the number of feet in I yard.
- "How did you convert a distance from a larger unit to a smaller one?"
  Multiply the distance by the number of smaller units in I larger unit.
- "For each animal, how do the measurements in feet, yards, and meters compare?

The number is the smallest in meters and the largest in feet.

© "Why might that be?"

A foot is smaller than a yard or a meter, so it takes more feet to measure the same distance.

"Does the order change when the distances are measured in different units? Why or why not?"

No, because each animal ran one distance, which doesn't change. It's just measured in units of different sizes.

## **Lesson Synthesis**

A key point to emphasize is that comparing unit rates is a straightforward way to compare rates. Choose an example from one of the cards, for instance: 10 juice boxes for \$3.50 or 6 juice boxes for \$2.40. Discuss questions such as:

"What is an efficient way to compare these rates?"

Find their unit rates and compare them

"How can we compute those unit rates?"

Divide 3.5 by 10 and divide 2.4 by 6

"What do these numbers mean in this situation?"

They are each the price per bottle: \$0.35 in the first offer and \$0.40 in the second offer.

If time permits, consider asking students:

Could we divide 10 by 3.50 and divide 6 by 2.40 and use those unit rates instead?"

Yes

○ "What would those numbers tell us?"

They would tell us numbers of boxes for I dollar.

"Would you compare these unit rates instead of the unit prices found earlier? Why or why not?"

No, these unit rates are not as easy to calculate. We usually think about unit price or how much a single item costs.

## Access for Multilingual Learners (Activity 2, Synthesis)

#### MLR7: Compare and Connect.

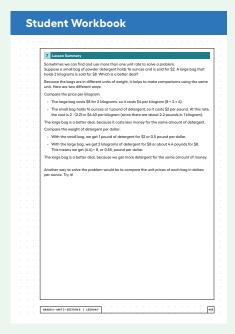
After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask:

"How are the strategies for converting to another unit the same?

How are they different?"
"Why did the conversions lead
to the same order even though
different units were used for
comparison?"

"Are there any benefits or drawbacks to using one unit of measurement or another?"

Advances: Representing, Conversing



## **Responding To Student Thinking**

#### Points to Emphasize

If students struggle with interpreting and comparing rates in context, focus on identifying the rates in each situation, what they mean, and possible ways for comparing them. For example, in each activity in Lesson 9, in which there are at least two rates (relating distance and time), discuss what each rate tells us and strategies for finding out which of the two moving objects is faster.

Unit 3, Lesson 9 More about Constant Speed

### **Lesson Summary**

Sometimes we can find and use more than one unit rate to solve a problem. Suppose a small bag of powder detergent holds 16 ounces and is sold for \$2. A large bag that holds 2 kilograms is sold for \$8. Which is a better deal?

Because the bags are in different units of weight, it helps to make comparisons using the same unit. Here are two different ways:

Compare the price per kilogram:

- The large bag costs \$8 for 2 kilograms, so it costs \$4 per kilogram  $(8 \div 2 = 4)$ .
- The small bag holds 16 ounces or 1 pound of detergent, so it costs \$2 per pound. At this rate, the cost is  $2 \cdot (2.2)$  or \$4.40 per kilogram (since there are about 2.2 pounds in 1 kilogram).

The large bag is a better deal, because it costs less money for the same amount of detergent.

Compare the weight of detergent per dollar:

- With the small bag, we get 1 pound of detergent for \$2 or 0.5 pound per dollar.
- With the large bag, we get 2 kilograms of detergent for \$8 or about 4.4 pounds for \$8. This means we get (4.4) ÷ 8, or 0.55, pound per dollar.

The large bag is a better deal, because we get more detergent for the same amount of money.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

#### Cool-down

## **Rulers by the Pack**

#### **Student Task Statement**

A store sells wooden rulers in packs of 10 and packs of 6. A pack of 10 rulers costs \$8.49 and a pack of 6 rulers costs \$5.40.

Which is the better deal? Explain how you know.

The pack of 10 rulers is a better deal.

#### Sample reasoning:

- In a pack of 10, each ruler is about \$0.85 because  $8.49 \div 10 = 0.849$ . In a pack of 6, each ruler is \$0.90 because  $5.40 \div 6 = 0.90$ .
- If we need 30 rulers, we can buy 3 packs of 10 or 5 packs of 6 rulers. Buying 3 packs of 10 would cost less than \$26 because  $3 \cdot (8.49) = 25.47$ . Buying 5 packs of 6 would cost \$27 because  $5 \cdot (5.40) = 27$ .

### **Practice Problems**

## 6 Problems

## **Problem 1**

The 11 slices of cheddar cheese in this package weigh 308 grams.

At the same weight per slice, how much would 18 slices of cheddar cheese weigh? Explain or show your reasoning.



## 504 grams

Sample reasoning: 308  $\div$  II = 28 so each slice weighs 28 grams.  $18 \cdot 28 = 504$ 

## **Problem 2**

A copy machine can print 480 copies every 4 minutes. For each question, explain or show your reasoning.

a. How many copies can it print in 10 minutes?

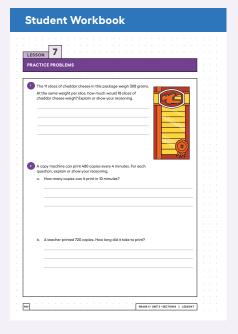
1,200 copies

Sample reasoning: The rate is 120 copies per minute, and  $120 \cdot 10 = 1,200$ .

**b.** A teacher printed 720 copies. How long did it take to print?

6 minutes

Sample reasoning:  $720 \div 120 = 6$ 





### Problem 3

Order these objects from heaviest to lightest.

(Note: 1 pound is 16 ounces. 1 ton is 2,000 pounds. 1 kilogram is approximately 2.2 pounds.)

item	weight	weight in pounds
school bus	9 tons	18,000
horse	1,100 pounds	1,100
elephant	5,500 kilograms	12.100
grand piano	15,840 ounces	990

school bus, elephant, horse, grand piano

Problem 4

Clare sometimes mows lawns on the weekend to make extra money.

• Two weeks ago, she mowed a neighbor's lawn for  $\frac{1}{2}$  hour and earned \$10.

from Unit 3, Lesson 4

- Last week, she mowed her uncle's lawn for  $\frac{3}{2}$  hours and earned \$30.
- This week, she mowed the lawn of a community center for 2 hours and earned \$30.

Which jobs paid better than others? Explain your reasoning.

The first two jobs paid better. Sample reasoning: Her neighbor and her uncle both paid \$20 per hour. For her neighbor, an hour of lawn mowing pays  $10 \cdot 2$  or \$20. Her uncle paid \$30 per  $\frac{3}{2}$  hours, which means \$10 every  $\frac{1}{2}$  hour and \$20 every hour. The third job at the community center paid \$15 per hour, since  $30 \div 2 = 15$ .

LESSON 7 • PRACTICE PROBLEMS 119

### **Problem 5**

from Unit 3, Lesson 8

Calculate and express your answer in decimal form.

- **a.**  $\frac{1}{2} \cdot 17$ 
  - 8.5
- **b.**  $\frac{3}{4} \cdot 200$ 
  - 150
- **c.** (0.2) · 40
  - 8
- **d.** (0.25) · 60

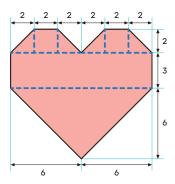
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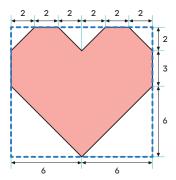
## **Problem 6**

from Unit 1, Lesson 11

Here is a polygon. All measurements are in centimeters.

**a.** Decompose this polygon so that its area can be calculated.





## Sample responses:

- Decompose the polygon into triangles and rectangles and add up their areas.
- Enclose the polygon with a rectangle, find its area, and subtract the unshaded right triangles from it.
- **b.** Calculate its area. Organize your work so that it can be followed by others.

### 88 square centimeters

Sample reasoning (using enclose-and-subtract approach):

• Area of the rectangle enclosing the polygon: I2 · II = I32 Area of 2 large triangles in the lower half:  $2 \cdot \left(\frac{1}{2} \cdot 6 \cdot 6\right) = 36$ Area of 3 triangles in the top half:  $4 \cdot \left(\frac{1}{2} \cdot 2 \cdot 2\right) = 8$ 

Subtracting 36 and 8 square centimeters from I32 gives 88 square centimeters (I32 - 36 - 8 = 88).

