Lesson 8

Writing and Graphing Inequalities

Goals

- Coordinate verbal, algebraic, and number line representations of inequalities.
- Critique (orally and in writing) possible values given for a situation with a constraint, including determining whether the boundary value is included and making sense of situations with discrete quantities.
- Interpret phrases that describe a quantity constrained by a maximum or minimum acceptable value, e.g. “at least,” “at most,” “up to,” “more than,” “less than,” etc., and write an inequality statement to represent the constraint.

Learning Targets

- I can graph inequalities on a number line.
- I can write an inequality to represent a situation.

Lesson Narrative

In this lesson, students extend their work with inequality statements by considering comparisons with an unknown quantity. Students begin by examining an image of a person standing next to a basketball hoop and determine possible minimum and maximum heights of the hoop. They express possible heights as a variable in an inequality statement.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Warm-up)

Instructional Routines

- Card Sort
- MLR5: Co-Craft Questions

Required Materials

Materials to Gather

- Blank paper: Lesson

Materials to Copy

- Stories about 9 Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Lesson 8

Writing and Graphing Inequalities

Lesson Narrative (continued)

Next, students are presented with two types of scenarios and must reason about the quantities involved. In one type, the variable represents a measurement, and the possible values can usually be any number within the range satisfied by the constraint. In the second type, where the variable represents a count of people or objects, the possible values are restricted to whole numbers within the range. Students also consider whether the constraint itself is included or excluded in the set of possible values, and they learn how to indicate this result on the number line representation with open or closed circles. Note that inequality symbols in grade 6 are limited to < and >. For situations where the constraint itself is included, students write 2 statements, such as $x > 5$ and $x = 5$.

Student Learning Goal

Let's write inequalities.

Instructional Routines

MLR5: Co-Craft Questions

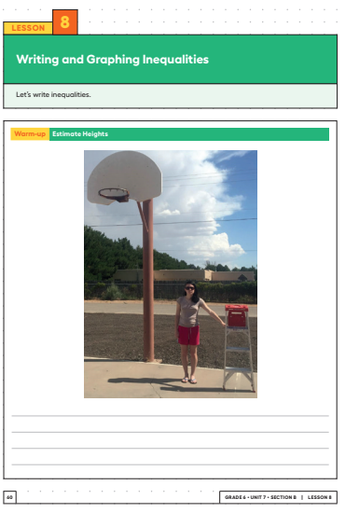
ilclass.com/r/10695544

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Warm-up)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook


LESSON 8

Writing and Graphing Inequalities

Let's write inequalities.

Warm-up Estimate Heights

Warm-up

Estimate Heights

5 min

Activity Narrative

In this *Warm-up,* students consider an image of a person standing next to a ladder and a basketball hoop. Students will see this image again in a following activity.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Introduce the image. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Sample responses:

- How tall is the person?
- What is the elevation of the basketball hoop?
- If the person stands on the ladder, will they reach the top of the basketball hoop?

Activity Synthesis

Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "more than," "less than," "at least," and "at most."

vActivity 1

How High and How Low Can It Be?

15 min

Activity Narrative

In this activity, students extend the use of inequalities to describe maximum and minimum possible values. Though students are thinking about whether or not a particular value makes an inequality true, the term "solution" will not be formally introduced until a future lesson.

Students estimate the maximum and minimum height of a basketball hoop in a given picture, represent these estimates on a number line, and use inequality symbols to write statements. Note that the symbols ≤ and ≥ will be introduced in a later course and are not used in this unit. To describe a range of values that includes the boundary value, such as "x is 6 or greater," two statements are used: $x = 6$ and $x > 6$.

Launch

Arrange students in groups of 2. Give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Here is a picture of a person next to a basketball hoop and a ladder. Use the picture to make reasonable estimates for the minimum and maximum heights of the basketball hoop.

1. Complete the first blank in each sentence with an estimate, and complete the second blank with "taller" or "shorter."

   Sample responses:

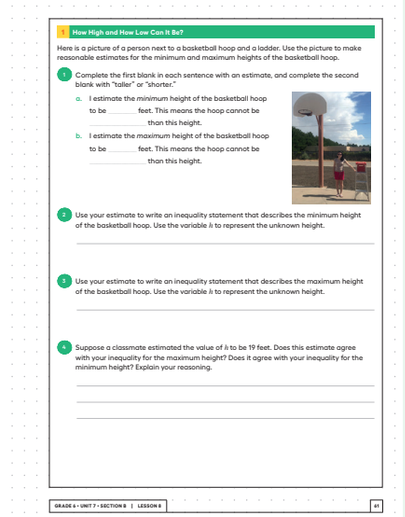
   a. I estimate the *minimum* height of the basketball hoop to be 6 feet. This means the hoop cannot be shorter than this height.

   b. I estimate the *maximum* height of the basketball hoop to be 15 feet. This means the hoop cannot be taller than this height.

2. Use your estimate to write an inequality statement that describes the minimum height of the basketball hoop. Use the variable h to represent the unknown height.

   Sample response: $h > 6$

Student Workbook



1 How High and How Low Can It Be?

Here is a picture of a person next to a basketball hoop and a ladder. Use the picture to make reasonable estimates for the minimum and maximum heights of the basketball hoop.

1 Complete the first blank in each sentence with an estimate, and complete the second blank with "taller" or "shorter."

a. I estimate the *minimum* height of the basketball hoop to be _____ feet. This means the hoop cannot be _____ than this height.

b. I estimate the *maximum* height of the basketball hoop to be _____ feet. This means the hoop cannot be _____ than this height.

2 Use your estimate to write an inequality statement that describes the minimum height of the basketball hoop. Use the variable h to represent the unknown height.

3 Use your estimate to write an inequality statement that describes the maximum height of the basketball hoop. Use the variable h to represent the unknown height.

4 Suppose a classmate estimated the value of h to be 19 feet. Does this estimate agree with your inequality for the maximum height? Does it agree with your inequality for the minimum height? Explain your reasoning.

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3. Use your estimate to write an inequality statement that describes the maximum height of the basketball hoop. Use the variable h to represent the unknown height.

   Sample response: $h < 15$

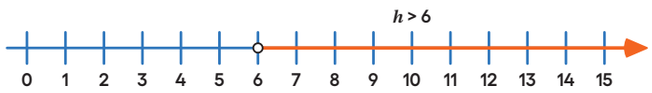
4. Suppose a classmate estimated the value of h to be 19 feet. Does this estimate agree with your inequality for the maximum height? Does it agree with your inequality for the minimum height? Explain your reasoning.

   Answers vary depending on previous question. Sample response: Since 19 feet is greater than 15 feet, this estimate does not agree with my inequality for the maximum height of the hoop. Since 19 feet is greater than 6 feet, this answer does agree with my inequality for the minimum height of the hoop.

Activity Synthesis

The purpose of this activity is to introduce graphing inequalities on a number line and to discuss boundary values. Begin by inviting students to share the inequality statements they wrote to describe the minimum height of the basketball hoop, recording their responses for all to see. Then display a blank number line for all to see.

Choose a student's inequality statement to use as an example, such as $h > 6$, and demonstrate how to represent this on the number line by labeling a tick mark as 6, drawing an open circle at 6, and shading the area to the right of the circle. Label the shaded part of the number line with the statement $h > 6$.



Then ask students what the word "minimum" means to them. (the smallest value that something can be) Ask students:

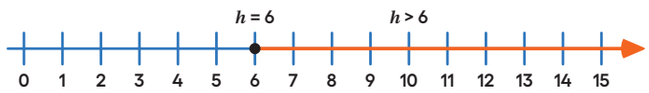
- *"If the minimum height of the basketball hoop is 6 feet, could the height be equal to 6 feet?"*

  Yes, the minimum refers to the smallest value the height could be equal to.

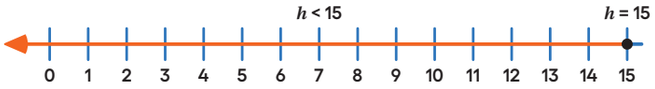
- *"How do we use numbers and symbols to write that the height could be equal to 6 feet?"*

  $h = 6$

Remind students that to represent $h = 6$ on the number line, we would plot a point at 6. Adding this to the number line would make it look like this:



Repeat the explanation, and draw another number line to represent the maximum height of the basketball hoop. Make sure students understand that $h < 15$ is graphed to the left of 15 and does not include the value 15. But the term "maximum" implies inclusion of 15, so the statement $h = 15$ can also be graphed on the same number line as a point at 15.



Activity 2

Stories about 9

15 min

Activity Narrative

Students sort different number lines, descriptions of real-world situations, and descriptions of possible numerical values during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Students also consider when to include or exclude the endpoints. They learn how to represent this on the number line with a closed (include) or open (exclude) circle at the boundary of the constraint.

Launch

Arrange students in groups of 2, and give each group a set of 16 pre-cut slips. Give groups 6–8 minutes to sort the cards into groups of 4. Each group of cards should include a story and question, a description of values that matches the story, math statements that represent the situation, and a number line.

Give students 1–2 minutes to compare their matching decisions with another group and come to an agreement before recording their sorted representations in the second problem of the *Task Statement*.

Student Task Statement

1. Your teacher will give you a set of cards containing stories and questions, descriptions of values, math statements, and number lines. Match each story and question with 3 other representations. Be prepared to explain your reasoning.
2. Compare your matching decisions with another group. If you disagree, work to reach an agreement. Then, record your final matching decisions here.

   Cards are shown in the correct groupings in the blackline master.

   a. A fishing boat can hold fewer than 9 people. How many people can it hold?

   - Description of values:
   - Number line:

   - Equation and/or inequality:

Instructional Routines

Card Sort

ilclass.com/r/10783726

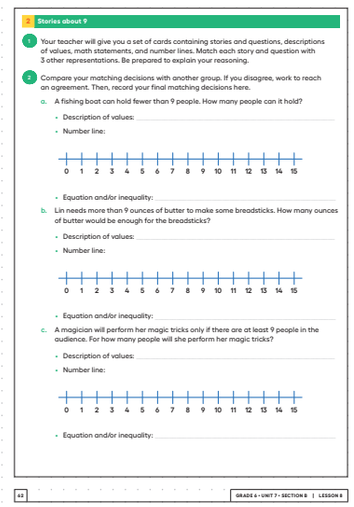
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 2, Student Task)

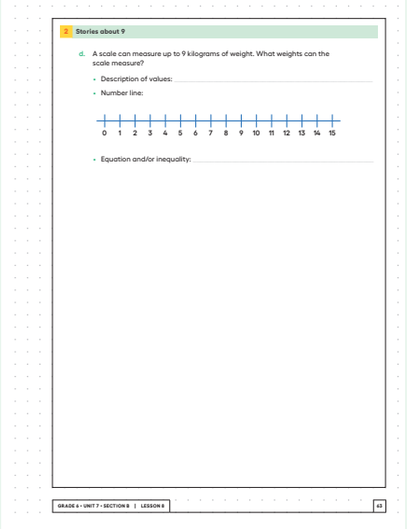
Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Student Workbook



Student Workbook


2 Stories about 9

d. A scale can measure up to 9 kilograms of weight. What weights can the scale measure?

- Description of values:
- Number line:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

- Equation and/or inequality:

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b. Lin needs more than 9 ounces of butter to make some breadsticks. How many ounces of butter would be enough for the breadsticks?

- Description of values:
- Number line:

- Equation and/or inequality:

c. A magician will perform her magic tricks only if there are at least 9 people in the audience. For how many people will she perform her magic tricks?

- Description of values:
- Number line:

- Equation and/or inequality:

d. A scale can measure up to 9 kilograms of weight. What weights can the scale measure?

- Description of values:
- Number line:

- Equation and/or inequality:

Activity Synthesis

The goal of this discussion is for students to consider how the context of a situation can affect different aspects of the solution, such as whether the solution is continuous or discrete, whether negative values make sense, and how the boundaries behave. Discuss the following questions:

- *"Why did some stories have number lines with just dots?"*

  Those stories were referring to numbers of people. Since there can't be a fraction of a person, it makes sense to represent those situations with only whole numbers.

- *"Why did the graph representing the food scale stop at 0 instead of having an arrow pointing to the left?"*

  It doesn't make sense to measure something with a negative weight, so negative numbers don't make sense in this situation.

- *"Why did the graph representing the butter Lin needed for cookies have an unshaded, or open, circle at 9?"*

  Lin needed more than 9 ounces of butter. Since exactly 9 ounces of butter would not have been enough, we leave the circle at 9 open, or unshaded.

- *"Why does the graph representing the butter Lin needs have an arrow*

pointing to the right? Is this reasonable?"

There is an arrow because Lin needs more than 9 ounces, but we don't know the maximum amount, so any amount greater than 9 would work. But realistically, values greater than 9 that are very large are not reasonable.

If time allows, display this statement for all to see:
"Jada built a robot that can push heavy boxes from one place to another. The robot is meant to be used only for pushing boxes heavier than 100 pounds. For what box weights (w) should the robot push the box?"

Ask students to represent all possible answers in three ways:

1. List or describe all numbers that answer the question.

   any weight that is greater than 100 pounds

2. Plot or graph the values that work on a number line.

   open circle at 100 and shaded to the right

3. Write an inequality.

   $w > 100$

Lesson Synthesis

The purpose of this discussion is for students to come up with situations where a quantity might have a maximum or minimum value and to analyze the context of the situation. Provide each student with a sheet of blank paper, and begin by asking students to think of an example where a quantity has a minimum and maximum value. Some examples include the number of students in a class, the pounds of fruit purchased by the school cafeteria, and the budget for a trip.

For each example shared, record the example for all to see, and ask the following questions:

- *"What are some reasonable minimum and maximum values?"*
- *"Are the minimum or maximum values included in the range of possible values? For example, does the number of students in a class have to be less than 30, or can it also be equal to 30?"*
- *"What are some equations or inequalities that could represent this situation?"*

Then ask students to draw a number line on the blank paper and graph the possible values. If time allows, invite 1–2 students to share their situation, math statement(s), and graph.

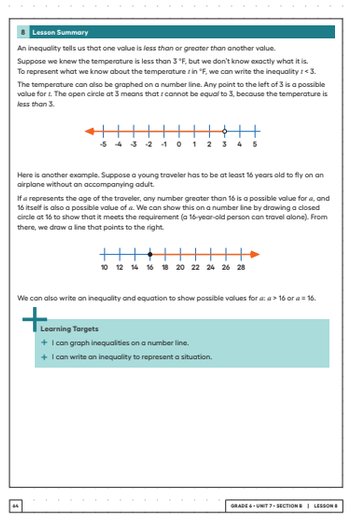
Lesson Summary

An inequality tells us that one value is *less than* or *greater than* another value.

Suppose we knew the temperature is *less than* 3 °F, but we don't know exactly what it is. To represent what we know about the temperature t in °F, we can write the inequality $t < 3$.

The temperature can also be graphed on a number line. Any point to the left of 3 is a possible value for t. The open circle at 3 means that t cannot be *equal* to 3, because the temperature is *less than* 3.

Student Workbook


8 Lesson Summary

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The temperature can also be graphed on a number line. Any point to the left of 3 is a possible value for t. The open circle at 3 means that t cannot be equal to 3, because the temperature is less than 3.

-5 -4 -3 -2 -1 0 1 2 3 4 5

Here is another example. Suppose a young traveler has to be at least 16 years old to fly on an airplane without an accompanying adult.

If a represents the age of the traveler, any number greater than 16 is a possible value for a, and 16 itself is also a possible value of a. We can show this on a number line by drawing a closed circle at 16 to show that it meets the requirement (a 16-year-old person can travel alone). From there, we draw a line that points to the right.

10 12 14 16 18 20 22 24 26 28

We can also write an inequality and equation to show possible values for a: $a > 16$ or $a = 16$.

Learning Targets
- I can graph inequalities on a number line.
- I can write an inequality to represent a situation.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Here is another example. Suppose a young traveler has to be at least 16 years old to fly on an airplane without an accompanying adult.

If a represents the age of the traveler, any number greater than 16 is a possible value for a, and 16 itself is also a possible value of a. We can show this on a number line by drawing a closed circle at 16 to show that it meets the requirement (a 16-year-old person can travel alone). From there, we draw a line that points to the right.

We can also write an inequality and equation to show possible values for a: $a > 16$ or $a = 16$.

Cool-down

A Box of Paper Clips

5 min

Student Task Statement

Andre looks at a box of paper clips. He says: "I think the number of paper clips in the box is less than 1,000."

Lin also looks at the box. She says: "I think the number of paper clips in the box is more than 500."

1. Write an inequality to show Andre's statement, using p for the number of paper clips.

   $p < 1,000$

2. Write another inequality to show Lin's statement, also using p for the number of paper clips.

   $p > 500$

3. Do you think both Lin and Andre would agree that there could be 487 paper clips in the box? Explain your reasoning.

   No

   Sample reasoning: Andre would agree because the inequality, $487 < 1,000$ is a true statement. However, Lin would not agree because the inequality $487 > 500$ is a false statement.

4. Do you think both Lin and Andre would agree that there could be 742 paper clips in the box? Explain your reasoning.

   Yes

   Sample reasoning: Both inequalities are true for 742 paper clips: $742 < 1,000$, and $742 > 500$. This means that according to Lin and Andre, there could be 742 paper clips in the box.

Lesson 8

Practice Problems

5 Problems

Problem 1

At the book sale, all books cost less than $5.

a. What is the most expensive a book could be?

$4.99

b. Write an inequality to represent the costs of a book at the sale.

Sample response: If p is the price of a book, then $p < 5$

c. Draw a number line to represent the inequality.

The number line has an open circle and an arrow starting at 5 and drawn to the left.

Problem 2

Kiran started his homework *before* 7:00 p.m. and finished his homework *after* 8:00 p.m. Let h represent the number of hours Kiran worked on his homework.

Decide if each statement is definitely true, definitely not true, or possibly true. Explain your reasoning.

a. $h > 1$

Definitely true

Sample reasoning: Kiran worked from 7:00 until 8:00 and some additional time.

b. $h > 2$

Possibly true

Sample reasoning: It would be true if Kiran started his homework at 6:15 and finished at 8:30. It would be false if Kiran started at 6:45 and finished at 8:15.

c. $h < 1$

Definitely false

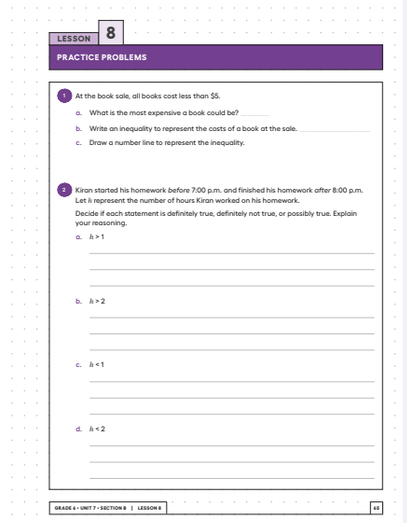
Sample reasoning: $h > 1$ is true.

d. $h < 2$

Possibly true

Sample reasoning: Kiran could work for 1.5 hours, which is greater than 1 hour but less than 2 hours.

Student Workbook


LESSON 8

PRACTICE PROBLEMS

1. At the book sale, all books cost less than $5.
   a. What is the most expensive a book could be?
   b. Write an inequality to represent the costs of a book at the sale.
   c. Draw a number line to represent the inequality.

2. Kiran started his homework *before* 7:00 p.m. and finished his homework *after* 8:00 p.m. Let h represent the number of hours Kiran worked on his homework.
   Decide if each statement is definitely true, definitely not true, or possibly true. Explain your reasoning.
   a. $h > 1$
   b. $h > 2$
   c. $h < 1$
   d. $h < 2$

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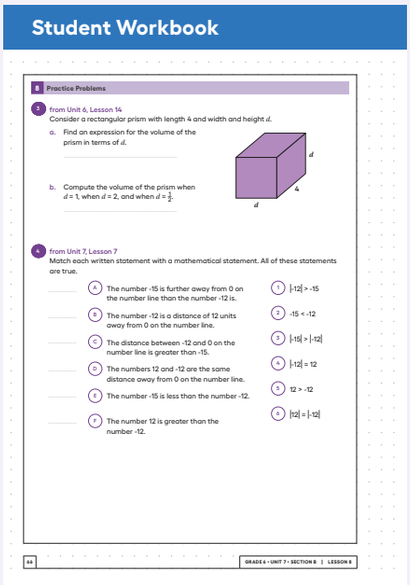
Student Workbook



Practice Problems

3 from Unit 6, Lesson 14

Consider a rectangular prism with length 4 and width and height d.

a. Find an expression for the volume of the prism in terms of d.

b. Compute the volume of the prism when $d = 1$, when $d = 2$, and when $d = \frac{1}{2}$.

4 from Unit 7, Lesson 7

Match each written statement with a mathematical statement. All of these statements are true.

A. The number -15 is further away from 0 on the number line than the number -12 is.
B. The number -12 is a distance of 12 units away from 0 on the number line.
C. The distance between -12 and 0 on the number line is greater than -15.
D. The numbers 12 and -12 are the same distance away from 0 on the number line.
E. The number -15 is less than the number -12.
F. The number 12 is greater than the number -12.

1. |-12| > -15
2. -15 < -12
3. |-15| > |-12|
4. |-12| = 12
5. 12 > -12
6. |12| = |-12|

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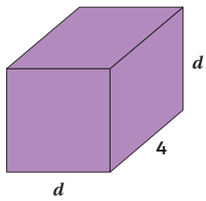
Problem 3

from Unit 6, Lesson 14

Consider a rectangular prism with length 4 and width and height d.



a. Find an expression for the volume of the prism in terms of d.

4 d^2 (or equivalent)

b. Compute the volume of the prism when $d = 1$, when $d = 2$, and when $d = \frac{1}{2}$.

When $d = 1$, the volume is 4.

When $d = 2$, the volume is 16.

When $d = \frac{1}{2}$, the volume is 1.

Problem 4

from Unit 7, Lesson 7

Match each written statement with a mathematical statement. All of these statements are true.

3 **A.** The number -15 is further away from 0 on the number line than the number -12 is.

4 **B.** The number -12 is a distance of 12 units away from 0 on the number line.

1 **C.** The distance between -12 and 0 on the number line is greater than -15.

6 **D.** The numbers 12 and -12 are the same distance away from 0 on the number line.

2 **E.** The number -15 is less than the number -12.

5 **F.** The number 12 is greater than the number -12.

1. |-12| > -15

2. -15 < -12

3. |-15| > |-12|

4. |-12| = 12

5. 12 > -12

6. |12| = |-12|

Problem 5

from Unit 6, Lesson 11

Here are five sums. Use the distributive property to write each sum as a product with two factors.

Sample responses:

a. $2a + 7a$

(2 + 7)a or 9a

b. $5z - 10$

5(z − 2)

c. $c - 2cd$

c(1 − 2d)

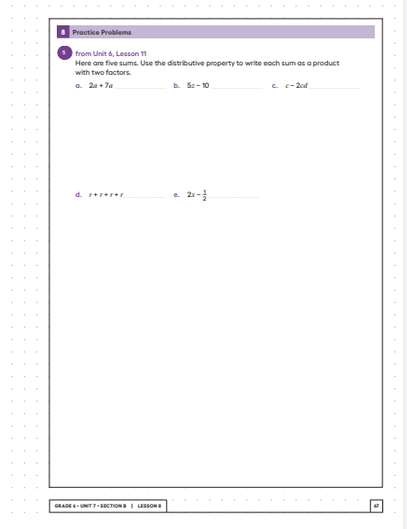
d. $r + r + r + r$

- (1 + 1 + 1 + 1)r
- 4r

e. $2x - \frac{1}{2}$

- $2\left(x - \frac{1}{4}\right)$
- $\frac{1}{2}(4x - 1)$

Student Workbook


8 Practice Problems

5 from Unit 6, Lesson 11
Here are five sums. Use the distributive property to write each sum as a product with two factors.

a. 2a + 7a b. 5z − 10 c. c − 2cd

d. r + r + r + r e. $2x - \frac{1}{2}$

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