

Distinguishing Between Surface Area and Volume (Optional)

Goals

- Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- Describe (orally and in writing) shapes built out of cubes, including observations about their surface area and volume.
- Determine the surface area and volume of shapes made out of cubes.

Learning Targets

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson Narrative

This optional lesson reminds students of the distinctions between measures of one-, two-, and three-dimensional attributes. It reinforces the idea that length is a one-dimensional attribute of geometric figures, surface area is a two-dimensional attribute, and **volume** is a three-dimensional attribute.

Students take a closer look at the distinction between surface area and volume. They build or draw representations of polyhedra and then calculate both the surface area and volume. In doing so, students see that different three-dimensional figures can have the same volume but different surface areas, and vice versa. This is analogous to the fact that two-dimensional figures can have the same area but different perimeters, and vice versa. Throughout the lesson, students practice attending to precision as they consider the geometric attributes being studied and the corresponding units of measurement.

Student Learning Goal

Let's contrast surface area and volume.

Lesson Timeline

10
min

Warm-up

25
min

Activity 1

20
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR8: Discussion Supports (Activity 1, Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Snap cubes: Activity 1, Activity 2
- Sticky notes: Activity 2
- Geometry toolkits: Activity 2

Required Preparation

Warm-up:

Prepare solutions to the first question of the *1-2-3 Dimensional Attributes* activity on a large visual display.

Activity 1:

Prepare sets of 16 snap cubes and two sticky notes for each student.

For the digital version of the activity, acquire devices that can run the applet.

Warm-up

Attributes and Their Measures

10 min

Activity Narrative

This activity strengthens students’ awareness of one-, two-, and three-dimensional attributes and the units commonly used to measure them. Students decide on the units based on the attributes being measured and on the size of the units and how appropriate they would be for describing given quantities.

As students work, select a few students to share their responses to the last two questions of the activity (on the quantities that could be measured in miles and in cubic meters).

Launch

Consider a quick review of metric and standard units of measurement before students begin work. Include some concrete examples that could help illustrate the size of each unit.

Then, pick an object in the classroom for which surface area and volume could be measured (for example, a desk). Ask students,

☞ “What units might we use to measure the surface area of the desktop? What units might we use to measure the volume of a drawer?”

Clarify the relative sizes of the different units that come up in the conversation. For instance, discuss how a meter is a little over three feet, a yard is three feet, a kilometer is about two-thirds of a mile, a millimeter is one tenth of a centimeter, and so on.

Give students 4–5 minutes of quiet think time and then a couple of minutes to share their responses with a partner. Prepare to display the answers to the first six questions for all to see.

Student Task Statement

For each quantity, choose one or more appropriate units of measurement. For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

Units

1. Perimeter of a parking lot:
Meters, feet

2. Volume of a semi truck:
Cubic yards

3. Surface area of a refrigerator:
Square inches, square feet

4. Length of an eyelash:
Millimeters

5. Area of a state:
Square miles

• millimeters (mm)

• feet (ft)

• meters (m)

• square inches (sq in)

• square feet (sq ft)

• square miles (sq mi)

• cubic kilometers (cu km)

• cubic yards (cu yd)

Building on Student Thinking

Depending on the students’ familiarity with metric and standard units, there may be some confusion about the size of each unit. Consider displaying measuring tools or a reference sheet that shows concrete examples of items measured in different-sized units.

Student Workbook

LESSON 16

Distinguishing Between Surface Area and Volume

Let's contrast surface area and volume.

Warm-up

Attributes and Their Measures

For each quantity, choose one or more appropriate units of measurement. For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

Units

1. Perimeter of a parking lot:

2. Volume of a semi truck:

3. Surface area of a refrigerator:

4. Length of an eyelash:

5. Area of a state:

6. Volume of an ocean:

7. _____ : miles

8. _____ : cubic meters

• millimeters (mm)

• feet (ft)

• meters (m)

• square inches (sq in)

• square feet (sq ft)

• square miles (sq mi)

• cubic kilometers (cu km)

• cubic yards (cu yd)

Building with 8 Cubes

Your teacher will give you 16 cubes. Build two different figures using 8 cubes for each. For each shape, complete these three steps and record your responses on a sticky note:

1. Give it a name or a label (such as Ma's First Shape or Diego's Steps).

2. Find the volume.

3. Find the surface area.

GRADE 6 • UNIT 1 • SECTION D | LESSON 16

Instructional Routines

MLR8: Discussion Supports

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6. Volume of an ocean:

Cubic kilometers, cubic yards

7. _____: miles

Sample responses: distance between home and school, length of a river

8. _____: cubic meters

Sample responses: volume of a room, volume of a swimming pool

Activity Synthesis

Display the solutions to the first six questions for all to see and to use for checking. Then, select a few students to share their responses to the last two questions.

Ask students what they notice about the units for area and the units for volume. If not already mentioned by students, highlight that area is always measured in square units and volume in cubic units.

Activity 1: Optional

Building with 8 Cubes

25
min

Activity Narrative

There is a digital version of this activity.

This activity clarifies the distinction between volume and surface area and illustrates that two polyhedra can have the same volume but different surface areas.

Students build figures using two sets of 8 cubes and determine their volumes and surface areas. Because all of the designs are made of the same number of cubes, they all have the same volume. Students then examine all of the designs and discuss what distinguishes figures with smaller surface areas from those with greater ones.

As students work, monitor the range of surface areas for the figures that students built.

This activity works best when each student has access to snap cubes. If physical cubes are not available, consider using the digital version of the activity. In the digital version, students use an applet that has 16 cubes with which to build the two figures.

Launch



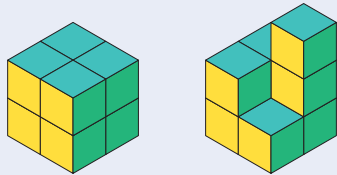
Give each student (or group of 2 students) 16 snap cubes and two sticky notes. Explain that their job is to design and build two figures—using 8 cubes for each—and find the volume and surface area of each figure. Ask them to give each figure a name or a label and then record the name, surface area, and volume of each figure on a sticky note.

Give students 8–10 minutes of work time. Select several students whose designs collectively represent a range of surface areas to share their work later.

Student Task Statement

Your teacher will give you 16 cubes. Build two different figures using 8 cubes for each.

Sample response:



For each shape, complete these three steps and record your responses on a sticky note:

1. Give it a name or a label (such as Mai’s First Shape or Diego’s Steps).

Answers vary.

2. Find the volume.

They both have a volume of 8 cubic units.

3. Find the surface area.

The first has a surface area of 24 square units. The second has a surface area of 28 square units. (The smallest possible surface area for an 8-cube construction is 24 square units, and the largest is 34 square units.)

Activity Synthesis

Ask all students to display their designs and their sticky notes and give students a couple of minutes to circulate and view one another’s work. Then, ask previously identified students to arrange their designs in the order of their surface area, from least to greatest, and display their designs for all to see. Record the information about the designs in a table, in the same sequence. Display the table for all to see. Here is an example.

shapes	volume	surface area
Andre’s cube	8	24
Lin’s steps	8	28
Jada’s first shape	8	28
...
Noah’s tower	8	34

Give students a minute to examine the designs and the information in the table. Then, discuss the following questions:

“What do all of the shapes have in common?”

They have the same volume.

“Why are all the volumes the same?”

Volume measures the number of unit cubes that can be packed into a figure. All the designs are built using the same number of cubes.

Building on Student Thinking

Even though students are dealing with only 8 cubes at a time, they may make counting errors by inadvertently omitting or double-counting squares or faces. This is especially likely for designs that are non-prisms. Encourage students to think of a systematic way to track the number of square units they are counting.

Some students may associate volume only with prisms and claim that the volume of non-prism designs cannot be determined. Remind them of the definition of volume.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

To help students describe and explain their comparisons, provide language that students can use. Demonstrate contrasting words such as “spread out” and “compact,” “exposed” and “hidden,” or “visible” and “covered” using one or more cube designs. Display sentence frames to support students with their explanations, for example: “The surface area of Shape _____ is larger (or smaller) than the surface area of Shape _____ because ...”

Advances: Speaking, Representing

Access for Multilingual Learners (Activity 2, Synthesis)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Instructional Routines

MLR3: Critique, Correct, Clarify

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MLR8: Discussion Supports

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Access for Multilingual Learners (Activity 2, Student Task)

Conversing: MLR8 Discussion Supports

Use this routine to help students describe what they noticed about the volume and surface areas of their prisms. Students should take turns stating an observation and their reasoning with their partner. Invite Partner A to begin with this sentence frame: “I noticed ____, so ...” Invite the listener, Partner B, to press for additional details referring to specific features of the prisms. Students should switch roles until they have listed all observations.

Design Principle(s): Support sense-making; Cultivate conversation

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Develop Expression and Communication.

Invite students to show thinking using snap cubes or blocks.

Supports accessibility for: Language, Visual-Spatial Processing

“Why do some shapes have larger surface areas than others? What do shapes with larger surface areas look like?”

The cubes are more spread out and have more of their faces exposed.

“What about those with smaller surface areas?”

They are more compact and have more of their faces hidden or shared with another cube.

“Is it possible to build a shape with a different volume? How?”

Yes, but it would involve using fewer or more cubes.

If students have trouble visualizing how surface area changes when the design changes, demonstrate the following:

- Make a cube made of 8 smaller cubes. Point to one cube and ask how many of its faces are exposed.
3
- Pop that cube off and move it to another place.
- Point out that, in the “hole” left by the cube that was moved, 3 previously interior faces now contribute to the surface area. At the same time, the relocated cube now has 5 faces exposed.

Activity 2: Optional

Comparing Prisms

20
min

Activity Narrative

Previously, students studied shapes with the same volume but different surface areas. Here they see that it is also possible for shapes to have the same surface area but different volumes. Students think about how the appearance of these shapes might compare visually.

Students are given the side lengths of three rectangular prisms and asked to find the surface area and the volume of each. Some students can visualize these, but others may need to draw nets, sketch the figures on isometric grid paper, or build physical prisms. Prepare cubes for students to use. Each of the three prisms can be built with 15 or fewer cubes, but 40 cubes are needed to build all three simultaneously. (If the cubes are not centimeter cubes, ask students to treat them as if the edge length of each cube was 1 cm.)

As students work, look out for errors in students’ calculations in the first question, which will affect the observations they make in the second question. Select a few students who notice that the volumes of the prisms are all different but the surface areas are the same.

Launch



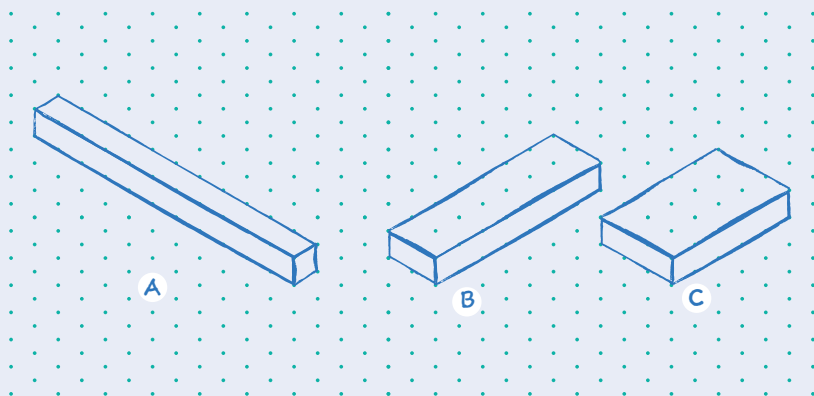
Arrange students in groups of 2. Provide access to snap cubes and geometry toolkits. Give students 6–7 minutes of quiet think time and then 2–3 minutes to discuss their responses with their partner. Ask partners to agree upon one key observation to share with the whole class.

Student Task Statement

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.



Surface areas:

- Prism A: $4(11 \cdot 1) + 2(1 \cdot 1) = 46$ square centimeters
- Prism B: $2(7 \cdot 2) + 2(7 \cdot 1) + 2(2 \cdot 1) = 46$ square centimeters
- Prism C: $2(5 \cdot 3) + 2(5 \cdot 1) + 2(3 \cdot 1) = 46$ square centimeters

Volumes:

- Prism A: 11 cubic centimeters ($11 \cdot 1 \cdot 1 = 11$)
- Prism B: 14 cubic centimeters ($7 \cdot 2 \cdot 1 = 14$)
- Prism C: 15 cubic centimeters ($5 \cdot 3 \cdot 1 = 15$)

2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

Sample responses:

- The surface areas of the prisms are all the same, but the volumes are all different.
- The polygons that make up the faces of each prism are different-sized rectangles, but their areas all add up to the same total of square centimeters.
- Prism C can fit the most centimeter cubes, but because the cubes would fit together in a compact way, some of the cubes would only have two square centimeters of exposed faces.
- Prism A can fit the fewest centimeter cubes, but because the cubes would be more spread out, more of their faces would be exposed.

Are You Ready for More?

Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Building on Student Thinking

Students may miss or double-count one or more faces of the prisms and miscalculate surface areas. Encourage students to be systematic in their calculations and to use organizational strategies that they learned when they used nets to find surface areas.

Students may need reminders to use square units for area and cubic units for volume.

Student Workbook

2 Comparing Prisms

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1 Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.

2 Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

Are You Ready for More?

Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Sample responses:

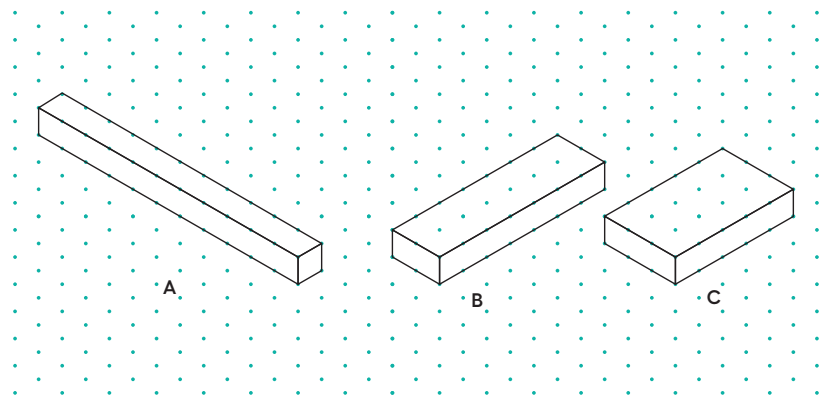
- A prism that is 4 units by 5 units by 1 unit and one that is 2 units by 9 units by 1 unit have the same surface area but different volumes.
- Generate examples by finding different pairs of factors of the same number and subtracting 1 from each factor. However, there are other ways. For example, $60 = 6 \cdot 10$ and $60 = 5 \cdot 12$. The 5-by-9-by-1 and 4-by-11-by-1 prisms have the same surface areas but different volumes.

Activity Synthesis

The goal of this discussion is to highlight volume and surface area as distinct measurements of a polyhedron.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written statement about volume and surface area by correcting errors, clarifying meaning, and adding details.

- Display an image of Prisms A, B, and C for all to see.



- Display this first draft:

Prism C has a greater volume than do Prisms A and B, so Prism C also has a greater surface area than the other two prisms.

- Ask,

“What parts of this statement are unclear, incorrect, or incomplete?”

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, and then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If needed, consider referring to the filing cabinet activity in an earlier lesson to help students conceptualize the idea of figures with different volumes having the same surface area. The number of square sticky notes needed to cover all of the faces of the filing cabinet was its surface area. If we use all of those square notes (no more, no less) to completely cover (without overlapping sticky notes) a cabinet that has a different volume, we can say that the two pieces of furniture have the same surface area and different volumes.

Lesson Synthesis

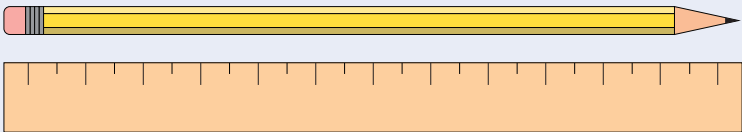
In this lesson, students refreshed their memory of measures of one-, two-, and three-dimensional attributes. Reiterate that length is a one-dimensional attribute of geometric figures, area is a two-dimensional attribute, and volume is a three-dimensional attribute. Revisit a few examples of units for length, area, and volume.

To reiterate the distinctions between surface areas and volumes of polyhedra, consider asking students:

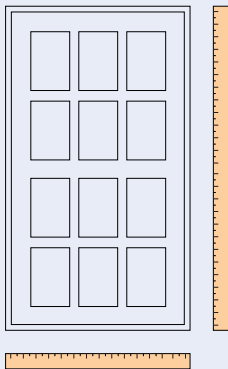
- “How could two figures that both have a volume of 4 cubic units have different surface areas?”
Surface area and volume measure different attributes of a three-dimensional shape.
- “How is surface area different from volume?”
Surface area is a two-dimensional attribute. It measures how many unit squares cover all the faces of a figure. Volume is a three-dimensional attribute. It measures how many unit cubes fill the figure.
- “Are the two measures related? Does a greater volume necessarily mean a greater surface area, and vice versa?”
No, one measure does not affect the other. A figure that has a greater volume than another may not necessarily have a greater surface area.

Lesson Summary

Length is a one-dimensional attribute of a geometric figure. We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.

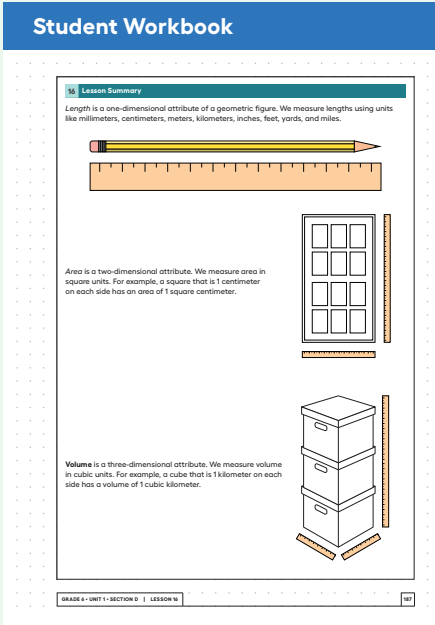


Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



A note about materials for an upcoming unit:

For the first lesson in the unit on ratios, students will need to bring in a collection of 10–20 small, inexpensive objects. Examples include rocks, seashells, erasers, stickers, bottle caps, bread clips, or hair clips. Inform students about this and remind them in the days leading to that lesson. If any students are concerned about having enough items to bring, brainstorm possibilities with them or suggest making a collection using paper or other materials available at school.



Student Workbook

16 Lesson Summary

Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

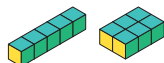
Two figures can have the same volume but different surface areas. For example:

- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.



Similarly, two figures can have the same surface area but different volumes.

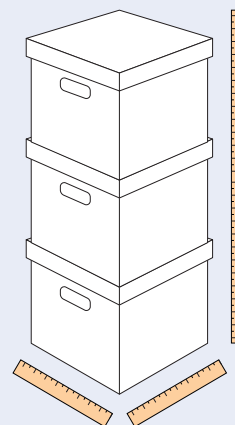
- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.



161

GRADE 6 • UNIT 1 • SECTION D | LESSON 16

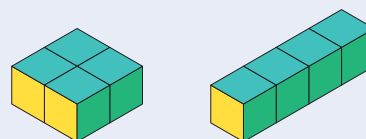
Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.



Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

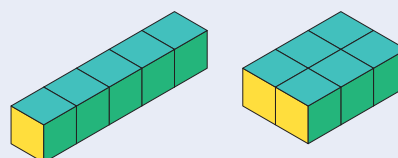
Two figures can have the same volume but different surface areas. For example:

- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.



Similarly, two figures can have the same surface area but different volumes.

- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.



Cool-down

Same Surface Area, Different Volumes

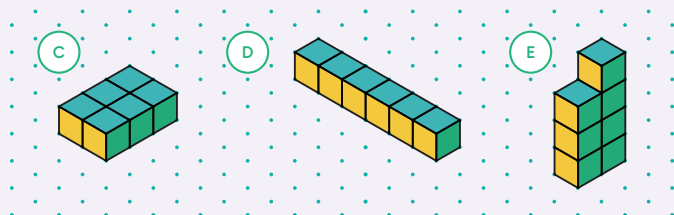
5
min

Launch

Encourage students to refer to the class list of observations from the previous activity.

Student Task Statement

Choose two figures that have the same surface area but different volumes. Show your reasoning.



Figures D and E both have a surface area of 26 square units, but D has a volume of 6 cubic units, and E has a volume of 7 cubic units.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

5 Problems

Student Workbook

LESSON 16
PRACTICE PROBLEMS

1. Match each quantity with an appropriate unit of measurement.

- | | |
|---------------------------------------|----------------------|
| Ⓐ The surface area of a tissue box | Ⓛ Square meters |
| Ⓑ The amount of soil in a planter box | Ⓜ Yards |
| Ⓒ The area of a parking lot | Ⓨ Cubic inches |
| Ⓓ The length of a soccer field | Ⓟ Cubic feet |
| Ⓔ The volume of a fish tank | Ⓢ Square centimeters |

2. Here is a figure built from cubes.



a. Find the volume of the figure in cubic units.

b. Find the surface area of the figure in square units.

c. True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.

GRADE 4 • UNIT 1 • SECTION D | LESSON 16

161

Problem 1

Match each quantity with an appropriate unit of measurement.

- | | |
|---|------------------------------|
| A. The surface area of a tissue box | 1. Square meters |
| B. The amount of soil in a planter box | 2. Yards |
| C. The area of a parking lot | 3. Cubic inches |
| D. The length of a soccer field | 4. Cubic feet |
| E. The volume of a fish tank | 5. Square centimeters |

- A matches 5
- B matches 3
- C matches 1
- D matches 2
- E matches 4

Problem 2

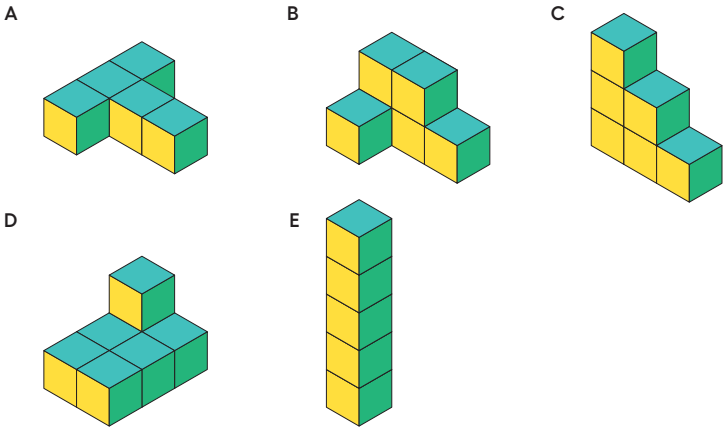
Here is a figure built from snap cubes.



- a.** Find the volume of the figure in cubic units.
4 cubic units. $(1 \cdot 1 \cdot 4) = 4$
- b.** Find the surface area of the figure in square units.
18 square units. $(4 \cdot 4) + (2 \cdot 1) = 18$
- c.** True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.
False
Sample reasoning: The volume will double to 8 cubic units, but the surface area will not. Only the side faces will double in area, to $(4 \cdot 8)$ or 32 square units, but the top and bottom faces will not double, so the surface area will be 34, not 36, square units.

Problem 3

Lin said, “Two figures with the same volume also have the same surface area.”



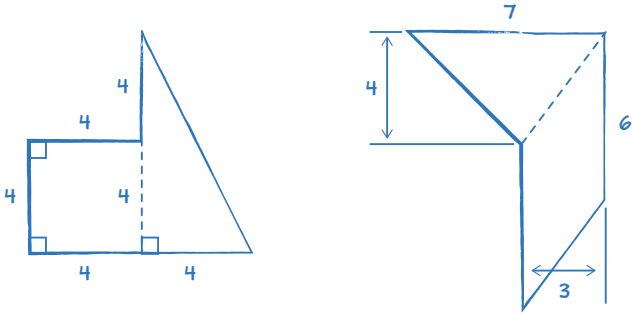
- a. Which two figures suggest that her statement is true?
B and C
- b. Which two figures could show that her statement is not true?
A and B, or A and C

Problem 4

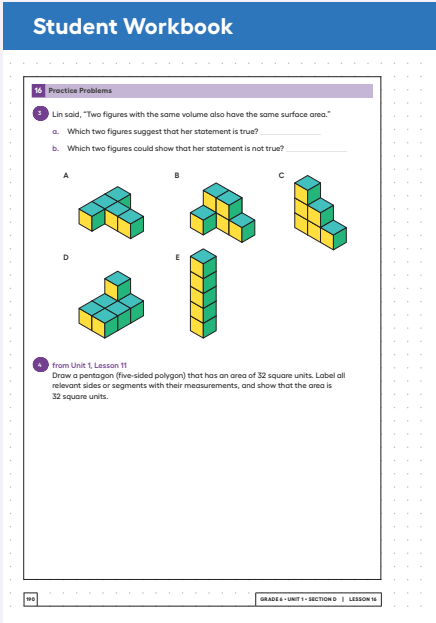
from Unit 1, Lesson 11

Draw a pentagon (five-sided polygon) that has an area of 32 square units. Label all relevant sides or segments with their measurements, and show that the area is 32 square units.

Sample responses:



- The first pentagon is composed of a square and a right triangle. The square has an area of 16 square units. The triangle has a base of 4 and a height of 4, so its area is 8 square units. The combined area is 16 + 8 or 24 square units.
- The second pentagon is composed of a parallelogram with a base of 6 and a height of 3, and a triangle with a base of 7 and a height of 4. The area of the parallelogram is 18 square units. The area of the triangle is $\frac{1}{2} \cdot 7 \cdot 4$ or 14 square units. The combined area is 18 + 14 or 32 square units.

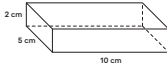


Student Workbook

Practice Problems

from Unit 1, Lesson 15

a. Draw a net for this rectangular prism.



b. Find the surface area of the rectangular prism.

Learning Targets

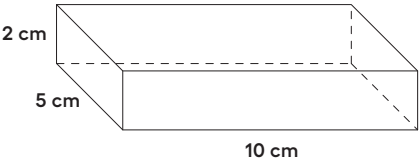
- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

GRADE 4 • UNIT 1 • SECTION D | LESSON 16

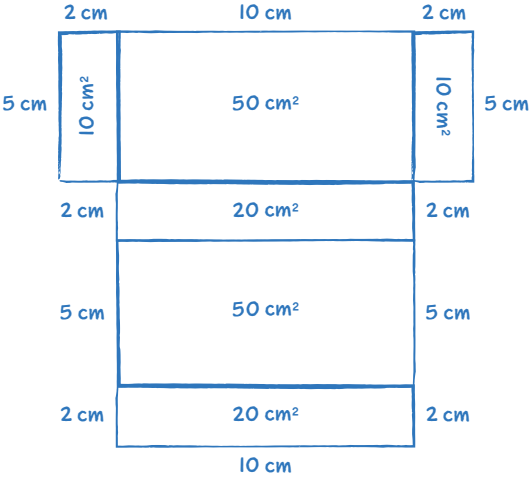
Problem 5

from Unit 1, Lesson 15

a. Draw a net for this rectangular prism.



Sample response:



b. Find the surface area of the rectangular prism.

160 square units

Sample reasoning: There are two faces with an area of 50 square cm, two faces with an area of 20 square cm, and two faces with an area of 10 square cm.