Lesson 2

Points on the Number Line

Goals

- Comprehend that two numbers are called "opposites" when they are the same distance from zero, but on different sides of the number line.
- Interpret a point on the number line that represents a positive or negative rational number.
- Plot a point on a number line to represent a positive or negative rational number.

Learning Targets

- I can determine or approximate the value of any point on a number line.
- I can represent negative numbers on a number line.
- I understand what it means for numbers to be opposites.

Lesson Narrative

In this lesson, students examine and make use of the structure of the number line. Students begin by considering which values could be represented by a point on a horizontal number line by reasoning about the position in relation to other known values on the number line. Then they revisit the context of temperature, represented on a vertical number line, extending previous work interpreting equally spaced divisions to the negative part of the number line.

Next, students create folded number lines out of tracing paper to reinforce the symmetry of the number line and to reason about **opposites**—numbers that are the same distance from zero but on opposite sides of the number line.

Lastly, students are introduced to the term **rational number** as a number that can be expressed as a positive or negative fraction. Students will have more practice placing rational numbers on the number line in future lessons. The focus in this lesson is the concept of opposites rather than plotting different kinds of rational numbers.

Student Learning Goal

Let's plot positive and negative numbers on the number line.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	20 min	10 min	5 min

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Rulers marked with centimeters: Activity 2
- Tracing paper: Activity 2

Warm-up

A Point on the Number Line

5 min

Activity Narrative

The purpose of this activity is to prime students to locate negative fractions on a number line by analyzing a number's position relative to landmarks on the number line. In later activities, students will use the same process of discerning which two numbers a value is between to describe negative rational numbers.

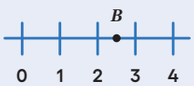
Launch

Arrange students in groups of 2.

Give students 2 minutes of quiet think time, and follow with a whole-class discussion.

Student Task Statement

Which of the following numbers could be represented by point B?



A. 2.45

B. $\frac{2}{5}$

C. $\frac{5}{2}$

D. $\frac{35}{10}$

E. 2.11

F. -2.5

Sample reasoning: Both values are halfway or about halfway between 2 and 3. The fraction $\frac{2}{5}$ is less than 1 and is too small to be point B. The fraction $\frac{35}{10}$ is equivalent to 3.5, which would be between 3 and 4. The number 2.11 is also located between 2 and 3 on the number line, but it would be closer to 2.

Activity Synthesis

The goal of this discussion is for students to understand how they can use landmarks on the number line (in this case, 2 and 3) and their knowledge of fractions to correctly place a number on a number line. Ask students:

- *"Were there any responses you could tell right away were not point B? How?"*

  $\frac{2}{5}$ is less than 1, but B is between 2 and 3.

- *"Were there any responses you had to think harder about?"*

  2.11 is also between 2 and 3.

- *"What part of the number line was useful when trying to decide which numbers could be point B?"*

  The numbers 2 and 3 on the number line told me that the number had to have a value somewhere between 2 and 3.

Inspire Math

Search and Rescue video

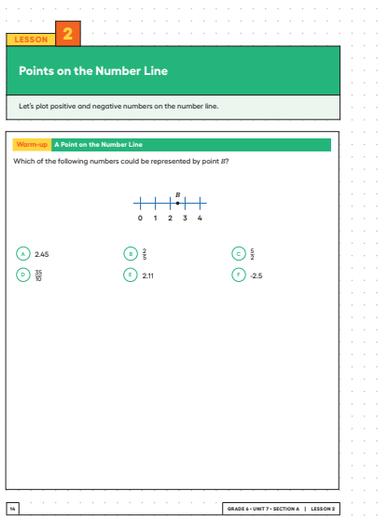
Go Online

Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614170

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 2

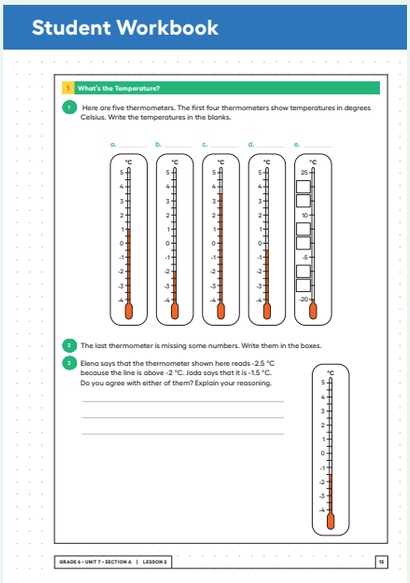
Points on the Number Line

Let's plot positive and negative numbers on the number line.

Warm-up A Point on the Number Line

Which of the following numbers could be represented by point B?

Student Workbook


1 What's the Temperature?

1. Here are five thermometers. The first four thermometers show temperatures in degrees Celsius. Write the temperatures in the blanks.

a. b. c. d. e.

2. The last thermometer is missing some numbers. Write them in the boxes.

3. Elena says that the thermometer shown here reads -2.5 °C because the line is above -2 °C. Jada says that it is -1.5 °C. Do you agree with either of them? Explain your reasoning.

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Activity 1

What's the Temperature?

10 min

Activity Narrative

The purpose of this task is to build understanding of the negative side of the number line, both by reading values and assigning values to equally spaced divisions. Non-integer negative numbers are also used. Students reason abstractly and quantitatively as they interpret positive and negative numbers in context.

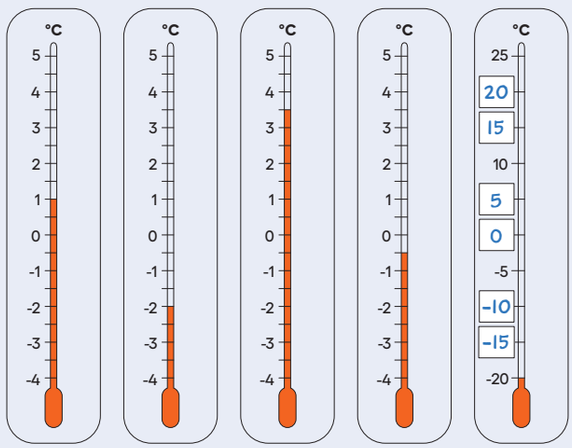
Launch

Give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. Here are five thermometers. The first four thermometers show temperatures in degrees Celsius. Write the temperatures in the blanks.

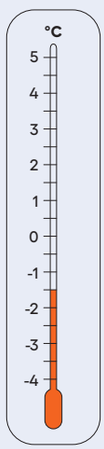
a. 1°C b. -2°C c. 3.5°C d. -0.5°C e. -20°C



2. The last thermometer is missing some numbers. Write them in the boxes.

See image.

3. Elena says that the thermometer shown here reads -2.5 °C because the line is above -2 °C. Jada says that it is -1.5 °C. Do you agree with either of them? Explain your reasoning.



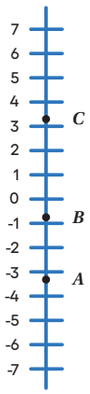
I agree with Jada.

Sample reasoning: The temperature is -1.5 °C because the line is between the numbers -1 and -2, and -2.5 would be between -2 and -3.

Activity Synthesis

The purpose of the discussion is for students to share their strategies for making sense of negative values on the number line. Begin by inviting students to share their reasoning as to whether they agreed with Jada or Elena in the last question. If not mentioned by students, connect this question to the *Warm-up* by pointing out that the temperature is halfway between -1 and -2 on the number line, so it must be -1.5 degrees.

Then display this number line for all to see.



Discuss the following questions:

- *"What could be the value of point C? Explain your reasoning."*

  A value between 3 and 3.5 is reasonable. The point is between 3 and 4 but looks closer to 3.

- *"What could be the value of point A? Explain your reasoning."*

  A value between -3 and -3.5 is reasonable. The point is between -3 and -4 but looks closer to -3.

- *"Do you think the value of point B is closer to -0.75 or -1.25? Why?"*

  Point B is closer to -0.75 because it is located between 0 and -1.

- *"What do you notice about the location of negative values on a vertical number line?"*

  Negative numbers are at the bottom. The negative numbers are like a mirror of the positive numbers.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports
Display sentence frames to support all students to participate in the discussion. Examples: "I agree/disagree because ..." "I noticed _____, so I ..."

Advances: Speaking, Conversing

Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Provide students with a partially completed number line, such as a blank number line with just the tick marks.

Supports accessibility for: Visual-Spatial Processing, Organization

Activity 2

Folded Number Lines

20 min

Activity Narrative

The purpose of this activity is to build an understanding of the structure of the number line by observing symmetry across zero on the number line. This is also the first time students work with negative numbers on a horizontal number line. If students have difficulty, consider displaying a vertical number from a previous activity to connect with.

While this activity also introduces the notion of distance on a number line, students do not need to know the term "absolute value," as it will be introduced in a following lesson.

Monitor for students who use these different strategies when finding the values of points P, X, and Y on the number line:

- Use tracing paper to recreate the number line and fold it at 0
- Use a ruler, tracing paper, or other tool to measure distances

Launch

Provide access to tracing paper and rulers marked with centimeters. If the tracing paper is less than 20 centimeters wide, instruct students to make their number lines from -7 to 7 instead of -10 to 10, or instruct them to make their number line on the diagonal of the tracing paper.

Give students 10 minutes to construct their folded number line and answer the first question. Check student work, then give students 5 more minutes to complete the last question. Follow with a whole-class discussion.

Select work for the last question from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Your teacher will give you a sheet of tracing paper. Follow the steps to make your own number line.

- Use a straightedge or a ruler to draw a horizontal line. Mark the middle point of the line, and label it 0.
- To the right of 0, draw tick marks that are 1 centimeter apart. Label the tick marks 1, 2, 3, ..., 10. This represents the positive side of your number line.
- Fold your paper so that a vertical crease goes through 0 and the two sides of the number line match up perfectly.
- Use the fold to help you trace the tick marks that you already drew onto the opposite side of the number line. Unfold and label the tick marks -1, -2, -3... -10. This represents the negative side of your number line.

1. Use your number line to answer these questions:

a. Which number is the same distance away from 0 as is the number 4?

-4

b. Which number is the same distance away from 0 as is the number -7?

7

c. Two numbers that are the same distance from 0 on the number line are called **opposites.** Find another pair of opposites on the number line.

Sample response: 6 and -6.

d. Determine how far away the number 5 is from 0. Then, choose a positive number and a negative number that is each farther away from 0 than is the number 5.

Sample response: 7 and -8

e. Determine how far away the number -2 is from 0. Then, choose a positive number and a negative number that is each farther away from 0 than is the number -2.

Sample response: 3 and -5

Pause here so your teacher can review your work.

2. Here is a number line with some points labeled with letters. Determine the location of points P, X, and Y. Be prepared to show or explain your reasoning.

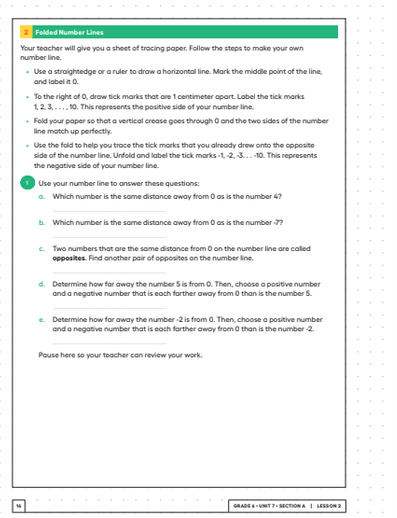
P is located at -2; X is at -4.5; and Y is at -7.5.

Are You Ready for More?

At noon, the temperatures in Portland, Maine, and Phoenix, Arizona, had opposite values. The temperature in Portland was 18 °C lower than in Phoenix. What was the temperature in each city? Explain your reasoning.

The temperature in Portland was -9 °C. The temperature in Phoenix was 9 °C.

Student Workbook


2 Folded Number Lines

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- Fold your paper so that a vertical crease goes through 0 and the two sides of the number line match up perfectly.
- Use the fold to help you trace the tick marks that you already drew onto the opposite side of the number line. Unfold and label the tick marks -1, -2, -3. . . -10. This represents the negative side of your number line.

1 Use your number line to answer these questions:

a. Which number is the same distance away from 0 as is the number 4?

b. Which number is the same distance away from 0 as is the number -7?

c. Two numbers that are the same distance from 0 on the number line are called **opposites**. Find another pair of opposites on the number line.

d. Determine how far away the number 5 is from 0. Then, choose a positive number and a negative number that is each farther away from 0 than is the number 5.

e. Determine how far away the number -2 is from 0. Then, choose a positive number and a negative number that is each farther away from 0 than is the number -2.

Pause here so your teacher can review your work.

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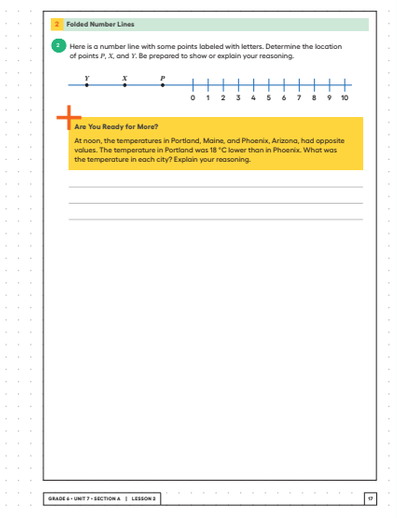
Student Workbook


2 Folded Number Lines

2 Here is a number line with some points labeled with letters. Determine the location of points P, X, and Y. Be prepared to show or explain your reasoning.

Y X P 0 1 2 3 4 5 6 7 8 9 10

Are You Ready for More?

At noon, the temperatures in Portland, Maine, and Phoenix, Arizona, had opposite values. The temperature in Portland was 18 °C lower than in Phoenix. What was the temperature in each city? Explain your reasoning.

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Activity Synthesis

The goal of this discussion is to compare different ways students determined the locations of points P, X, and Y on the number line and to reinforce the meaning of the term "opposites."

Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

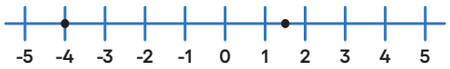
- *"What do the approaches have in common? How are they different?"*

  They are all about measuring distance, just in different ways.

- *"Did anyone solve the problem the same way, but would explain it differently?"*

  "Are there any benefits or drawbacks to one approach compared to another?"

The main idea for students to understand is that opposites are the same distance from 0 and on different sides of the number line. Display the horizontal number line and these sentence frames for all to see:



- *"The opposite of _____ is _____."*

  "The opposite of the opposite of _____ is _____."

Ask students to use the first sentence frame to describe the opposites of the labeled points and add them to the number line.

The opposite of -4 is 4. The opposite of 1.5 is -1.5.

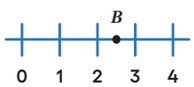
Repeat this with the second sentence frame.

The opposite of the opposite of -4 is -4. The opposite of the opposite of 1.5 is 1.5.

Point out that the opposite of the opposite of a number is always the number itself.

Lesson Synthesis

The goal of this discussion is to introduce rational numbers. Begin by displaying this image from the *Warm-up* for all to see.



Remind students that point B looks about halfway between 2 and 3 and some possible values are 2.45 or $\frac{5}{2}$. Tell students that both of these values are **rational numbers**, which are numbers that can be written as positive or negative fractions. Ask students why 2.45 is considered a positive or negative fraction. (It can be rewritten as the fraction $\frac{245}{100}$ or $2\frac{9}{20}$.)

Display some additional examples of rational numbers, like 4, -3.8, $-\frac{4}{3}$, and $\frac{1}{2}$, for all to see. Ask students how 4 and -3.8 could be rewritten as fractions $\left(\frac{4}{1}\right.$ and $\left.-\frac{38}{10}\right)$. Explain that all rational numbers can be plotted as points on the number line and can be positive, zero, or negative.

Tell students that they have spent most of their mathematical careers studying positive numbers called fractions. Now that we can find their opposites, we are studying rational numbers, which are fractions and their opposites. The "ratio" in "rational number" comes from the fact that ratios and fractions are closely related.

Lesson Summary

Two numbers that are the same distance from 0 and on different sides of the number line are **opposites.** For example, points A and B are opposites because they are both 2.5 units away from 0 and on opposite sides of 0.

We can also say that the opposite of 8.3 is -8.3, and the opposite of $-\frac{3}{2}$ is $\frac{3}{2}$. The opposite of 0 is itself.

Here is another labeled number line with some **rational numbers**. A rational number is a number that can be written as a positive or negative fraction.

The number 4 is positive, and its location is 4 units to the right of 0 on the number line. The number 4 can be written as $\frac{4}{1}$ or $\frac{16}{4}$ or any other equivalent fraction.

The number $-\frac{2}{3}$ is negative, and its location is $\frac{2}{3}$ units to the left of 0 on the number line. To locate $-\frac{2}{3}$ on the number line, we can divide the distance between 0 and -1 into thirds and then count 2 thirds to the left of 0.

All fractions and their opposites are rational numbers.

Cool-down

Positive, Negative, and Opposite

5 min

Student Task Statement

1. Plot a point on the number line to represent the value -3.2.

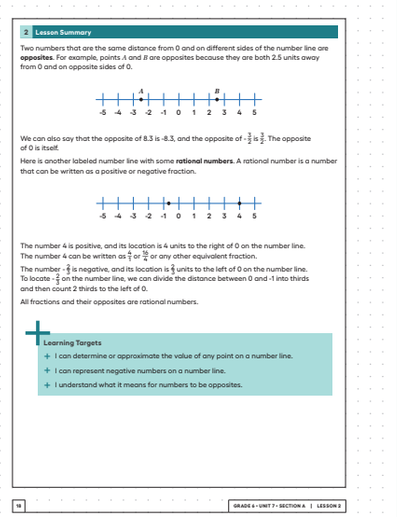
2. What is the opposite of -3.2?

   3.2

3. What is the opposite of the opposite of -3.2?

   -3.2

Student Workbook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All fractions and their opposites are rational numbers.

Learning Targets
- I can determine or approximate the value of any point on a number line.
- I can represent negative numbers on a number line.
- I understand what it means for numbers to be opposites.

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Responding To Student Thinking

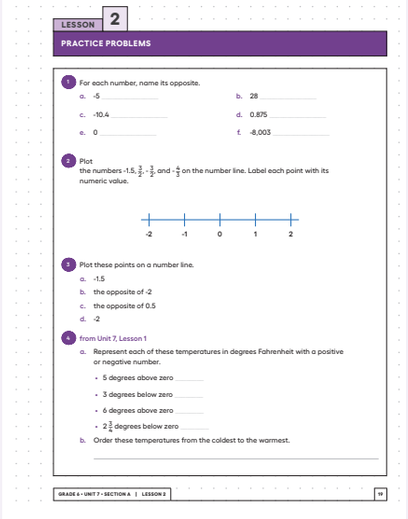
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 2

Practice Problems

7 Problems

Student Workbook


LESSON 2

PRACTICE PROBLEMS

1. For each number, name its opposite.
   a. -5 _____ b. 28 _____
   c. -10.4 _____ d. 0.875 _____
   e. 0 _____ f. -8,003 _____
2. Plot the numbers -1.5, $\frac{3}{2}$, $-\frac{3}{2}$, and $-\frac{4}{3}$ on the number line. Label each point with its numeric value.

   -2 -1 0 1 2
3. Plot these points on a number line.
   a. -1.5
   b. the opposite of -2
   c. the opposite of 0.5
   d. -2
4. from Unit 7, Lesson 1
   a. Represent each of these temperatures in degrees Fahrenheit with a positive or negative number.
      - 5 degrees above zero _____
      - 3 degrees below zero _____
      - 6 degrees above zero _____
      - $2\frac{3}{4}$ degrees below zero _____
   b. Order these temperatures from the coldest to the warmest.

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Problem 1

For each number, name its opposite.

a. -5

5

b. 28

-28

c. -10.4

10.4

d. 0.875

-0.875

e. 0

0

f. -8,003

8,003

Problem 2

Plot the numbers -1.5, $\frac{3}{2}$, $-\frac{3}{2}$, and $-\frac{4}{3}$ on the number line. Label each point with its numeric value.

Problem 3

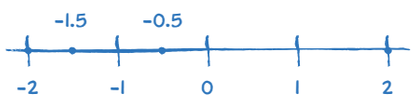
Plot these points on a number line.

a. -1.5

b. the opposite of -2

c. the opposite of 0.5

d. -2



Problem 4

from Unit 7, Lesson 1

a. Represent each of these temperatures in degrees Fahrenheit with a positive or negative number.

- 5 degrees above zero 5
- 3 degrees below zero -3
- 6 degrees above zero 6
- $2\frac{3}{4}$ degrees below zero $-2\frac{3}{4}$

b. Order these temperatures from the coldest to the warmest.

$-3, -2\frac{3}{4}, 5, 6$

Problem 5

from Unit 6, Lesson 5

Solve each equation.

a. $8x = \frac{2}{3}$

$x = \frac{2}{24}$ (or equivalent)

b. $1\frac{1}{2} = 2x$

$x = \frac{3}{4}$ (or equivalent)

c. $5x = \frac{2}{7}$

$x = \frac{2}{35}$ (or equivalent)

d. $\frac{1}{4}x = 5$

$x = 20$

e. $\frac{1}{5} = \frac{2}{3}x$

$x = \frac{3}{10}$ (or equivalent)

Problem 6

from Unit 6, Lesson 5

Write the solution to each equation as a fraction and as a decimal.

a. $2x = 3$

$x = \frac{3}{2}$ or $x = 1.5$

b. $5y = 3$

$y = \frac{3}{5}$ or $y = 0.6$

c. $0.3z = 0.009$

- $z = \frac{0.009}{0.3}$
- $z = 0.03$
- $z = \frac{3}{100}$

Problem 7

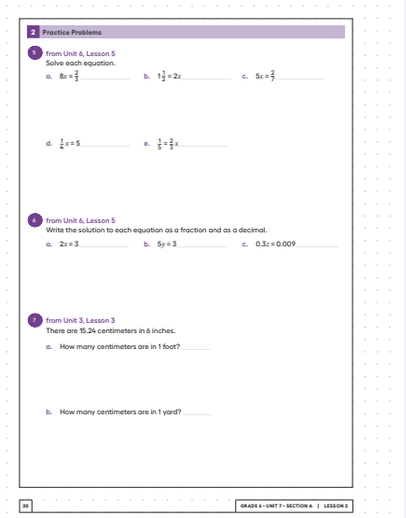
from Unit 3, Lesson 3

There are 15.24 centimeters in 6 inches.

a. How many centimeters are in 1 foot? 30.48 centimeters

b. How many centimeters are in 1 yard? 91.44 centimeters

Student Workbook


2 Practice Problems

5 from Unit 6, Lesson 5
Solve each equation.
a. $8x = \frac{2}{3}$ b. $1\frac{1}{2} = 2x$ c. $5x = \frac{2}{7}$
d. $\frac{1}{4}x = 5$ e. $\frac{1}{5} = \frac{2}{3}x$

6 from Unit 6, Lesson 5
Write the solution to each equation as a fraction and as a decimal.
a. $2x = 3$ b. $5y = 3$ c. $0.3z = 0.009$

7 from Unit 3, Lesson 3
There are 15.24 centimeters in 6 inches.
a. How many centimeters are in 1 foot?
b. How many centimeters are in 1 yard?

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