Lesson 10

Interpreting Inequalities

Goals

- Critique (orally and in writing) possible values given for a situation with more than one constraint, including whether fractional or negative values are reasonable.
- Interpret (orally and in writing) unbalanced hanger diagrams, and write inequality statements to represent relationships between the weights on an unbalanced hanger diagram.
- Write and interpret inequality statements that include more than one variable.

Learning Targets

- I can explain what the solution to an inequality means in a situation.
- I can write inequalities that involve more than one variable.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Lesson Narrative

In this lesson, students explore situations where some of the solutions to inequalities do not make sense in the context of the situation. They reason carefully about a situation's constraints when determining reasonable solutions.

Students begin by addressing a common mistake made when the variable is written on the right side of an inequality, such as $15 < n$, thinking that n must be less than 15.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Lesson 10

Interpreting Inequalities

Lesson Narrative (continued)

Next, students consider the context of points scored in a basketball game. While many numbers can be solutions to an inequality describing the number of points scored by a player, only some of those solutions make sense in context. Lastly, students look at inequalities by using the familiar context of hanger diagrams, contrasting the balanced hangers of equations with the unbalanced hangers of inequalities.

Student Learning Goal

Let's examine what inequalities can tell us.

Warm-up

Andre's Number Line

5 min

Activity Narrative

In this activity, students critique a statement or response that is intentionally incorrect. The example given represents a common mistake made by students and prompts students to consider the meaning and placement of inequality symbols in relation to a number line.

Launch

Give students 2–3 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Andre drew this number line to represent $15 < n$.

Do you agree with Andre's number line? Explain your reasoning.

I do not agree with Andre.

Sample reasoning: The number 20 makes the inequality true since $15 < 20$, but 20 is not in the shaded region of the number line.

Activity Synthesis

The purpose of this discussion is to address a common mistake made by students when graphing or interpreting an inequality where the variable is on the right side of the inequality. Begin by inviting 2–3 students to share their responses and reasoning. If not mentioned by students, discuss the following questions:

- *"How do we say the inequality $15 < n$?"*

  15 is less than n.

- *"What are some values that n could be?"*

  16, 20, 34.9, 983

- *"What are some values that n could not be?"*

  15, 14.9, 10, 1, -3

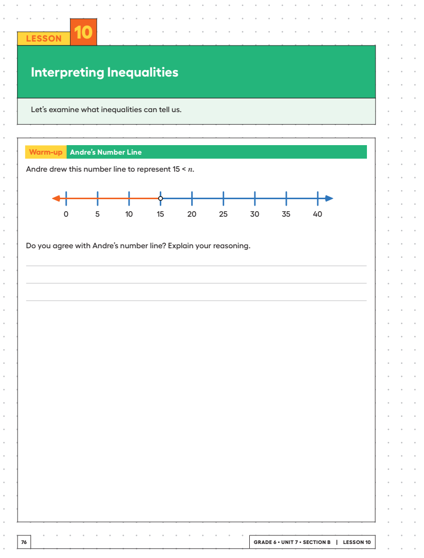
- *"Since n could not be equal to 15, did Andre draw the correct type of circle at 15?"*

  Yes, it should be an open, non-shaded circle.

- *"How could Andre's number line be changed to correctly represent $15 < n$?"*

  The numbers to the right of 15 should be shaded instead of to the left.

Student Workbook

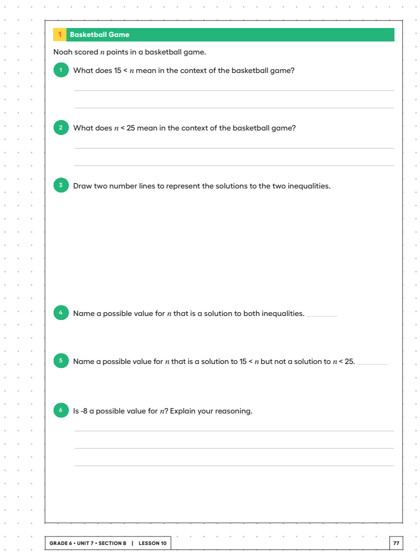


Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Develop Expression and Communication. Give students access to blank number lines.

Supports accessibility for: Conceptual Processing, Organization

Student Workbook


1 Basketball Game

Noah scored n points in a basketball game.

1 What does $15 < n$ mean in the context of the basketball game?

2 What does $n < 25$ mean in the context of the basketball game?

3 Draw two number lines to represent the solutions to the two inequalities.

4 Name a possible value for n that is a solution to both inequalities.

5 Name a possible value for n that is a solution to $15 < n$ but not a solution to $n < 25$.

6 Is -8 a possible value for n? Explain your reasoning.

GRADE 6 • UNIT 7 • SECTION 8 | LESSON 10 77

Activity 1

Basketball Game

15 min

Activity Narrative

In this activity, students interpret inequalities that represent constraints or conditions in a real-world situation. They find solutions to an inequality and reason about the context's limitations on solutions.

Launch

Give students 10 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Noah scored n points in a basketball game.

1. What does $15 < n$ mean in the context of the basketball game?

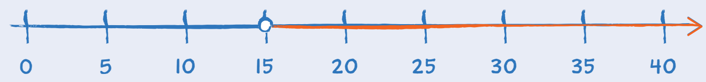
   Noah scored more than 15 points.

2. What does $n < 25$ mean in the context of the basketball game?

   Noah scored less than 25 points.

3. Draw two number lines to represent the solutions to the two inequalities.

   $n > 15$:



   $n < 25$:

4. Name a possible value for n that is a solution to both inequalities.

   Sample response: $n = 17$.

5. Name a possible value for n that is a solution to $15 < n$ but not a solution to $n < 25$.

   Sample response: $n = 30$.

6. Is -8 a possible value for n? Explain your reasoning.

   No, -8 cannot be a solution in this context.

   Sample reasoning: The score of a basketball game cannot be below 0.

Activity Synthesis

The goal of this discussion is to extend the context of scored points in a basketball game by considering additional situations. Invite students to share their responses to each question. If necessary, have a student share what they know about how scoring works in basketball (free throws are worth 1 point, regular baskets are worth 2 points, baskets shot from outside a certain line are worth 3 points). Then discuss the following questions:

- *"Could Noah have scored 0 points? 1 point?"*

  Yes, if Noah did not make any baskets. Yes, if Noah made a free throw.

- *"Could Noah have scored 2.5 points? -3 points?"*

  No, baskets are only worth whole numbers of points. No, a player cannot score negative points [though a student may argue that scoring for the opposing team may be the same as scoring negative points].

- *"Is it reasonable for a player to score 200 points?"*

  No, because there has never been an NBA game with a score that high.

Activity 2

Unbalanced Hangers

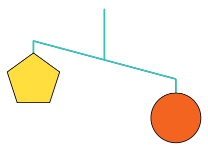
15 min

Activity Narrative

In this activity, students describe unbalanced hanger diagrams with inequalities. Students construct viable arguments and critique the reasoning of others during partner and whole-class discussions about how unknown values relate to each other.

Launch

Display this diagram of an unbalanced hanger:



Discuss the following questions:

- *"If p is the weight of one pentagon and c is the weight of one circle, how can we represent this diagram with an inequality?"*

  $p < c$ or $c > p$

- *"How could this inequality be written if we knew the circle weighs 12 ounces?"*

  $p < 12$ or $12 > p$

- *"What does this new inequality mean for this situation?"*

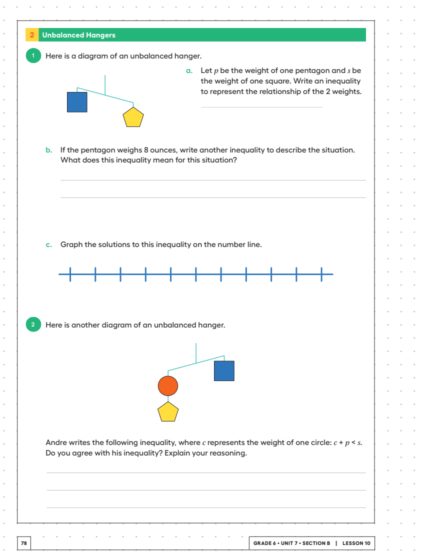
  The pentagon weighs less than 12 ounces.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports
Encourage students to begin partner discussions by reading aloud their written responses. If time allows, invite students to revise or add to their responses based on the conversation that follows.

Advances: Conversing, Speaking

Student Workbook


2 Unbalanced Hangers

1 Here is a diagram of an unbalanced hanger.

a. Let p be the weight of one pentagon and s be the weight of one square. Write an inequality to represent the relationship of the 2 weights.

b. If the pentagon weighs 8 ounces, write another inequality to describe the situation. What does this inequality mean for this situation?

c. Graph the solutions to this inequality on the number line.

2 Here is another diagram of an unbalanced hanger.

Andre writes the following inequality, where c represents the weight of one circle: $c + p < s$. Do you agree with his inequality? Explain your reasoning.

78 GRADE 6 • UNIT 7 • SECTION 8 | LESSON 10

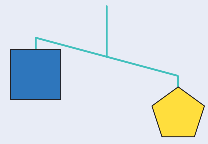
Arrange students in groups of 2.

Give students 5 minutes of quiet work time, followed by 2 minutes for a partner discussion.

Tell students to discuss their thinking with their partners and that if there are disagreements, work to reach an agreement. Follow with a whole-class discussion.

Student Task Statement

1. Here is a diagram of an unbalanced hanger.



   a. Let p be the weight of one pentagon and s be the weight of one square. Write an inequality to represent the relationship of the 2 weights.

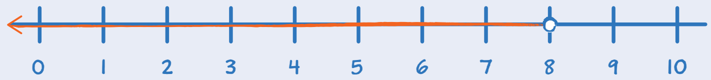
   $s < p$ or $p > s$

   b. If the pentagon weighs 8 ounces, write another inequality to describe the situation. What does this inequality mean for this situation?

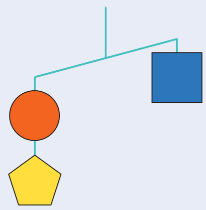
   $s < 8$ or $8 > s$

   This means that the square weighs less than 8 ounces.

   c. Graph the solutions to this inequality on the number line.



2. Here is another diagram of an unbalanced hanger.



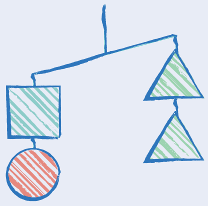
   Andre writes the following inequality, where c represents the weight of one circle: $c + p < s$. Do you agree with his inequality? Explain your reasoning.

   No, I do not agree

   Sample response: The combined weight of the circle and the pentagon is greater than the weight of the square, so the inequality should be $c + p > s$.

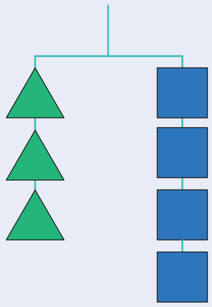
3. Jada looks at another diagram of an unbalanced hangar and writes the following inequality: $s + c > 2t$, where t represents the weight of one triangle. Draw a sketch of the diagram.

Sample response:



Are You Ready for More?

Here is a picture of a balanced hanger. It shows that the total weight of the three triangles is the same as the total weight of the four squares.



1. What does this tell you about the weight of one square when compared to one triangle? Explain how you know.

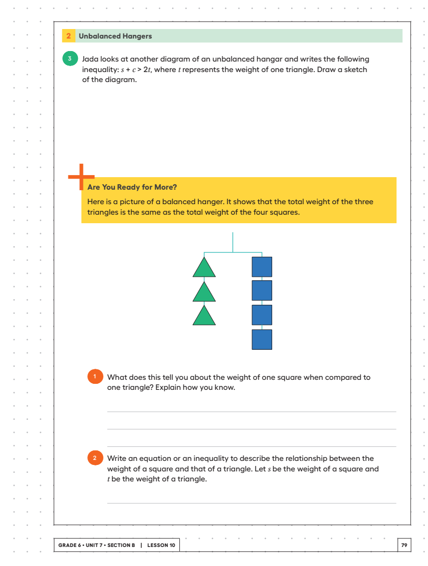
   The weight of a square is less than the weight of a triangle, or the weight of a triangle is more than the weight of a square.

   Sample reasoning: It takes 4 squares to weigh the same as 3 triangles, so each square must be lighter than each triangle.

2. Write an equation or an inequality to describe the relationship between the weight of a square and that of a triangle. Let s be the weight of a square and t be the weight of a triangle.

   - $s < t$
   - $t > s$
   - $3t = 4s$

Student Workbook



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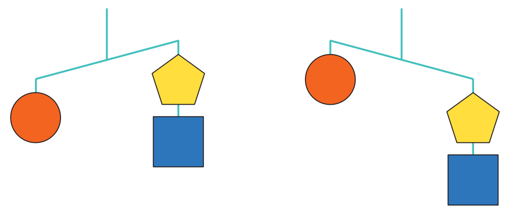
GRADE 6 • UNIT 7 • SECTION 8 | LESSON 10

Activity Synthesis

The purpose of the discussion is for students to explain how they used inequalities to compare the weights of different shapes on the hanger diagrams. Invite groups share their responses to the questions. If time allows, ask students to describe any disagreements or difficulties they had and how they resolved them.

Remind students that a circle weighs 12 ounces and a pentagon weighs 8 ounces and display these two diagrams for all to see:



Ask students:

- *"How much would the square have to weigh for the side with the circle to be heavier? How can we represent that with an inequality?"*

  The square would have to weigh less than 4 ounces: $s < 4$ or $4 > s$

- *"How much would the square have to weigh for the side with the pentagon and square to be heavier? How can we represent that with an inequality?"*

  The square would have to weigh more than 4 ounces: $s > 4$ or $4 < s$.

Lesson Synthesis

The purpose of this discussion is for students to practice writing inequalities to represent real world situations involving public health or safety. Begin by asking students to think of situations where limits or ranges of values can be important to public health or safety. Some examples include weight limitations on an elevator, safe dosages for medication, tire pressure, speed limit, and temperatures for growing carrots.

Then ask students to define variables and write inequalities to represent these situations. Select 2 or 3 students to share their responses. Record and display those responses for all to see, using the appropriate symbols. Here are some questions to consider during discussion:

- *"Do solutions that are not whole numbers make sense in this situation?"*

  "Do solutions that are negative numbers make sense in this situation?"

  "Do the numbers on the boundary count as solutions? For example, if an elevator has a maximum capacity of 2,500 pounds, can it handle exactly 2,500 pounds?"

Lesson Summary

An inequality that describes a real-world situation may have number solutions that make the inequality true, but those solutions may not always make sense in real life.

For example:

- A basketball player scored more than 11 points in a game. This can be represented by the inequality $s > 11$, where s is the number of points scored. Numbers such as 12, $14\frac{1}{2}$, and 130.25 are all solutions to the inequality because they each make the inequality true.

  $$12 > 11 \qquad 14\frac{1}{2} > 11 \qquad 130.25 > 11$$

  In a basketball game, however, it is only possible to score a whole number of points, so fractional and decimal scores are not possible. It is also highly unlikely that one person would score more than 130 points in a single game.

  This particular situation limits the solutions.

  Here is another example:

- It rained for less than 30 minutes yesterday (but it did rain). This can be represented by the inequality $r < 30$, where r represents the number of minutes of rain yesterday. Even though numbers such as $27\frac{3}{4}$, 18.2, and -7 are all less than 30, our solutions are limited to positive numbers since 0 or a negative number of minutes would not make sense in this context.

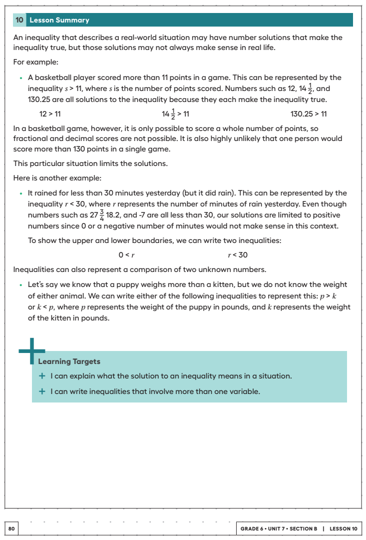
  To show the upper and lower boundaries, we can write two inequalities:

  $$0 < r \qquad r < 30$$

  Inequalities can also represent a comparison of two unknown numbers.

- Let's say we know that a puppy weighs more than a kitten, but we do not know the weight of either animal. We can write either of the following inequalities to represent this: $p > k$ or $k < p$, where p represents the weight of the puppy in pounds, and k represents the weight of the kitten in pounds.

Student Workbook


10 Lesson Summary

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Learning Targets

+ I can explain what the solution to an inequality means in a situation.
+ I can write inequalities that involve more than one variable.

80 | GRADE 6 • UNIT 7 • SECTION B | LESSON 10

Responding To Student Thinking

Press Pause
If most students struggle with interpreting inequalities in context and identifying solutions to a given inequality, make time to revisit the ideas of this section before the end of the unit. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 6, Unit 7, Section B Inequalities

Cool-down

Lin and Andre's Heights

5 min

Student Task Statement

1. Lin says that the inequalities $h > 150$ and $h < 160$ describe her height in centimeters. What do the inequalities tell us about her height?

   These inequalities tell us that Lin is between 150 and 160 cm tall.

2. Andre notices that he is a little taller than Lin but is shorter than their math teacher, who is 164 centimeters tall. Write two inequalities to describe Andre's height. Let a be Andre's height in centimeters.

   - $a < 164$
   - $a > h$ or $h < a$

3. Select **all** heights in centimeters that could be Andre's height. If you get stuck, consider drawing a number line to help you.

   A. 150
   B. 154.5
   C. 160
   D. 162.5
   E. 164

Lesson 10

Practice Problems

6 Problems

Problem 1

There is a closed carton of eggs in Mai's refrigerator. The carton contains e eggs, and it can hold 12 eggs.

a. What does the inequality $e < 12$ mean in this context?

Sample response: There are fewer than 12 eggs in the carton; the carton is not full.

b. What does the inequality $e > 0$ mean in this context?

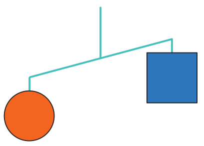
Sample response: There are more than 0 eggs in the carton; the carton is not empty.

c. What are some possible values of e that will make both $e < 12$ and $e > 0$ true?

Sample response: There could be as few as 1 egg or as many as 11 eggs in the carton: any whole number of eggs from 1 up to 11.

Problem 2

Here is a diagram of an unbalanced hanger.



a. Write an inequality to represent the relationship of the weights. Use s to represent the weight of the square in grams and c to represent the weight of the circle in grams.

$s < c$ (or equivalent)

b. One red circle weighs 12 grams. Write an inequality to represent the weight of one blue square.

$s < 12$ (or equivalent)

c. Could 0 be a value of s? Explain your reasoning.

No

Sample reasoning: 0 could not be a value of s because the square represents an object. It must have some weight, even if it is very small.

Problem 3

from Unit 7, Lesson 8

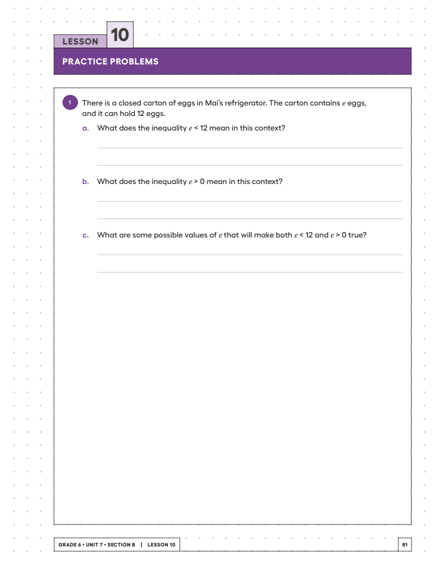
a. Jada is taller than Diego. Diego is 54 inches tall (4 feet, 6 inches). Write an inequality that compares Jada's height in inches, j, to Diego's height.

$j > 54$

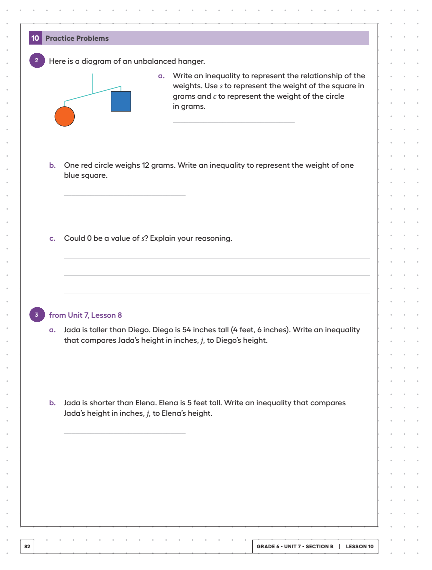
b. Jada is shorter than Elena. Elena is 5 feet tall. Write an inequality that compares Jada's height in inches, j, to Elena's height.

$j < 60$

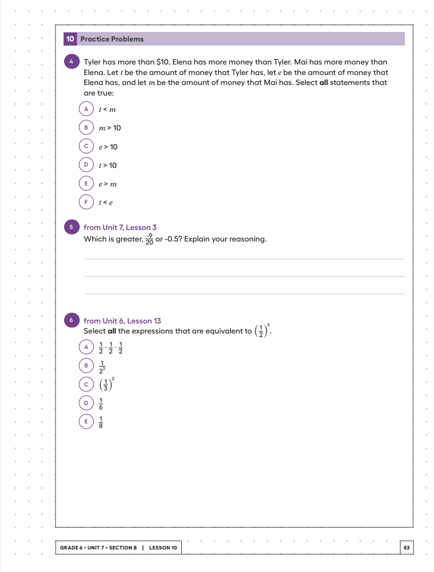
Student Workbook



Student Workbook



Student Workbook


10 Practice Problems

4 Tyler has more than $10. Elena has more money than Tyler. Mai has more money than Elena. Let t be the amount of money that Tyler has, let e be the amount of money that Elena has, and let m be the amount of money that Mai has. Select **all** statements that are true:

A $t < m$
B $m > 10$
C $e > 10$
D $t > 10$
E $e > m$
F $t < e$

5 from Unit 7, Lesson 3
Which is greater, $\frac{-9}{20}$ or -0.5? Explain your reasoning.

6 from Unit 6, Lesson 13
Select **all** the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.

A $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
B $\frac{1}{2^3}$
C $\left(\frac{1}{3}\right)^2$
D $\frac{1}{6}$
E $\frac{1}{8}$

GRADE 6 • UNIT 7 • SECTION B | LESSON 10

Problem 4

Tyler has more than $10. Elena has more money than Tyler. Mai has more money than Elena. Let t be the amount of money that Tyler has, let e be the amount of money that Elena has, and let m be the amount of money that Mai has. Select **all** statements that are true:

A. $t < m$

B. $m > 10$

C. $e > 10$

D. $t > 10$

E. $e > m$

F. $t < e$

Problem 5

from Unit 7, Lesson 3

Which is greater, $\frac{-9}{20}$ or -0.5? Explain your reasoning.

$\frac{-9}{20}$ is larger

Sample reasoning: $\frac{-9}{20} = -0.45$, and this is to the right of -0.5 on the number line. So $\frac{-9}{20}$ is larger.

Problem 6

from Unit 6, Lesson 13

Select **all** the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.

A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

B. $\frac{1}{2^3}$

C. $\left(\frac{1}{3}\right)^2$

D. $\frac{1}{6}$

E. $\frac{1}{8}$