Lesson 14

Solving Problems Involving Decimals

Goals

- Interpret a description or a question (orally and in written language) of a real-world situation involving operations of decimals, and write expressions to represent it.
- Use operations with decimals to solve problems in context, and explain (orally, in writing, and using other representations) the solution method.

Learning Target

I can use addition, subtraction, multiplication, and division on decimals to solve problems.

Lesson Narrative

In this lesson, students apply their knowledge of operations on decimals, equal-size groups, and rates to solve a variety of contextual problems.

The lesson begins with a *Math Talk* that prompts students to estimate quotients of decimals. In the main activity, an *Information Gap*, students practice identifying and asking for information that they need to solve problems about decimal lengths, distances, and constant speed. The two optional activities that follow allow students to interpret decimal quantities in sporting contexts (hurdle race and tennis), decide which operations are relevant, and perform calculations to find unknown measurements or information.

Students also draw or use a diagram to help them make sense of measurements, and practice communicating information with precision. The numbers used in the problems reflect measurements that can be accurately measured on site, so the decimals can all be calculated by hand.

Student Learning Goal

Let's solve some problems using decimals.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 2)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)
- MLR5: Co-Craft Questions (Activity 3)
- MLR6: Three Reads (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Materials to Copy

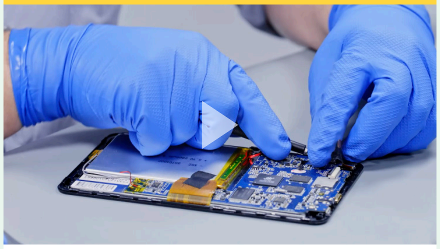
- Two Ropes and a Traveling Tortoise Cards (1 copy for every 4 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
10 min	25 min	25 min	25 min	10 min	5 min

Inspire Math

Gold Mine video



Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614239

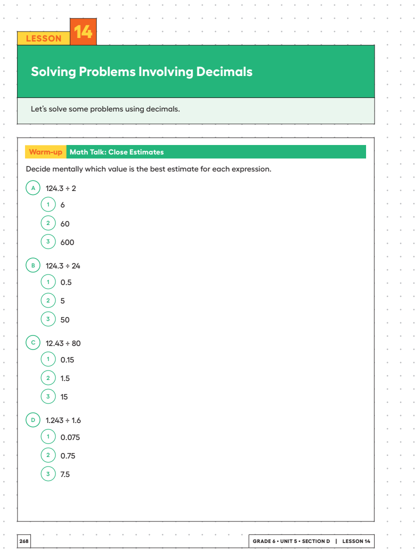
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 14
Solving Problems Involving Decimals
Let's solve some problems using decimals.
Warm-up Math Talk: Close Estimates
Decide mentally which value is the best estimate for each expression.
A 124.3 ÷ 2
1 6
2 60
3 600
B 124.3 ÷ 24
1 0.5
2 5
3 50
C 12.43 ÷ 80
1 0.15
2 1.5
3 15
D 1.243 ÷ 1.6
1 0.075
2 0.75
3 7.5

Warm-up

Math Talk: Close Estimates

10 min

Activity Narrative

This *Math Talk* focuses on division of a decimal. It encourages students to think about the reasonableness of a quotient by looking closely at the values of the dividend and divisor and to rely on what they know about base-ten numbers to mentally solve problems. The reasoning elicited here will be helpful later in the lesson when students solve contextual problems involving division.

To find the value of the last two expressions, students need to look for and make use of structure. In explaining their reasoning, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally which value is the best estimate for each expression.

A. $124.3 \div 2$

1. 6

2. 60

3. 600

Sample reasoning:

- 124.3 is close to 120, and $120 \div 2 = 60$.
- $2 \cdot 6$ is only 12, which is much less than 124.3, and $2 \cdot 600$ is $1{,}200$, which is much greater than 124.3. $2 \cdot 60$ is 120, which is very close to 124.3.

B. 124.3 ÷ 24

1. 0.5
2. 5
3. 50

Sample reasoning:

- 124.3 is close to 125 and 24 is close to 25. 125 ÷ 25 is 5.
- There are more than 1 group of 24 and fewer than 10 groups of 24 in 124, so the quotient is between 1 and 10.
- The divisor 24 is 12 times the divisor in the first problem, so the quotient is one-twelfth of the first quotient. One-twelfth of 60 is 5.

C. 12.43 ÷ 80

1. 0.15
2. 1.5
3. 15

Sample reasoning:

- The quotient is less than 1 because there is less than 1 group of 80 in 12.43.
- 12 is about 1.5 times 8, so 1.2 is about 0.15 times 8.

D. 1.243 ÷ 1.6

1. 0.075
2. 0.75
3. 7.5

Sample reasoning:

- The divisor is slightly greater than the dividend, so the quotient is close to 1.
- 1.243 is close to 1.2, and 1.2 ÷ 1.6 is equivalent to 12 ÷ 16, which is $\frac{12}{16}$, or $\frac{3}{4}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _________'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on t _________'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

If time permits, ask students if the actual value of each expression would be greater than or less than their estimate, and ask them to explain how they know.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because …" or "I noticed _____ so I …" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR4: Information Gap Cards

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Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Activity 1, Narrative)

MLR4: Information Gap
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity 1

Information Gap: Two Ropes and a Traveling Tortoise

25 min

Activity Narrative

In this activity, students calculate quotients of decimals in the context of length and distance but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and to ask increasingly more precise questions until they get the information they need.

The problems allow students to practice dividing decimals by making use of the structure they observe in the numbers, or by multiplying both the divisor and the dividend by an appropriate power of 10. The questions in Problem Card 1 reiterate the idea of division as a way to answer questions about the size of a group and the number of groups. Those in Problem Card 2 revisit concepts about rate and constant speed. Students may choose to reason with a ratio table, but it would be less efficient than dividing directly, and students would still need to divide decimals somewhere along the way.

Launch

Tell students they will continue to solve problems involving decimals. Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice. This norm should be one that they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _________ because ..."
   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _________?"
3. Listen to your partner's reasoning, and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
   These steps may be repeated.
4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1:

1. 0.2875 meter

   Sample reasoning: $5.75 \div 2 = 2.875$, so $5.75 \div 20$ is one-tenth of that, which is 0.2875.

2. 115 pieces of rope

   Sample reasoning: $5.75 \div 0.05$ is equivalent to $575 \div 5$, which is 115.

Problem Card 2:

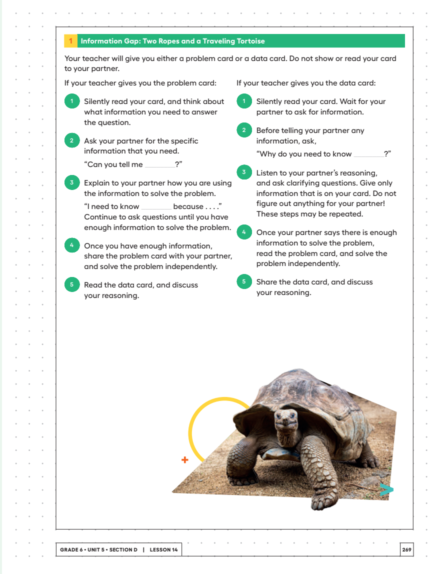
1. 0.27 mile per hour

   Sample reasoning: $0.945 \div 3.5$ is equivalent to $945 \div 3{,}500$, which is 0.27.

2. 18 hours

   Sample reasoning: $4.86 \div 0.27$ is equivalent to $486 \div 27$, which is 18.

Student Workbook



1 Information Gap: Two Ropes and a Traveling Tortoise

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ________ because . . . ." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ________?"
3. Listen to your partner's reasoning, and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"To solve the problems about the blue rope and red rope, did you multiply the dividend and divisor by powers of 10 before dividing, or did you reason another way?"

"How did you know what information to ask for? Once you have the information, how did you go about finding the tortoise's speed in miles per hour?"

"How did you find the time it would take to travel 4.86 miles?"

Highlight for students that it may not always be necessary to multiply the dividend and divisor by a power of 10 when a division involves one or more decimals. In some cases, we can reason about the relationship between the divisor and dividend directly. For example:

- When finding 5.75 ÷ 20, we can first think of 5.75 ÷ 2 and then divide the result by 10, instead of calculating 575 ÷ 2,000.
- When dividing 5.75 ÷ 0.05, we can use partial quotients: First find 5 ÷ 0.05, which is 100, and then find 0.75 ÷ 0.05, or how many 5 hundredths are in 75 hundredths, which is 15.

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

- "I picked the norm '_________.' It really helped me/my group because _________."
- "I picked the norm '_________.' During the activity, I realized the norm '_________' would be a better focus because _________."

Activity 2: Optional
Distance between Hurdles

25 min

Activity Narrative

In this activity, students use arithmetic with decimals to study the 110-meter hurdle race. The first question prompts students to draw a diagram to capture and make sense of information about measurements in a hurdle race. The last two questions prompt students to calculate the distance between hurdles and the size of a runner's stride, both of which involve subtracting and dividing decimals.

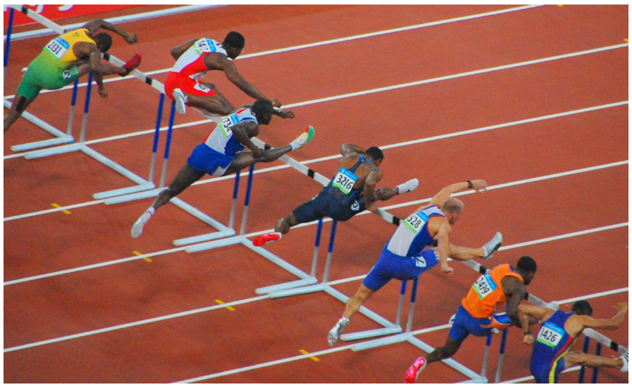
Throughout the activity, students need to make sense of problems and persevere in solving them. As they represent given measurements diagrammatically, use them in calculations, and interpret the results in terms of the situation, students engage in abstract and quantitative reasoning.

As students work, monitor for a variety of diagrams and reasoning strategies. Select students who organize their understanding of the situation and reasoning in different ways, and ask them to share later.

Launch

Ask students if they have had the chance to watch track-and-field competitions and whether they are familiar with hurdle races. Invite students to describe what a hurdle race is. Display images such as shown here or show a short video of a hurdle race.

Tell students that they will now use what they have learned about decimals to solve a couple of problems involving hurdles.



Arrange students in groups of 2. Give students about 5–6 minutes to draw a diagram for the first question and discuss their drawing with their partner. Then, pause for a class discussion.

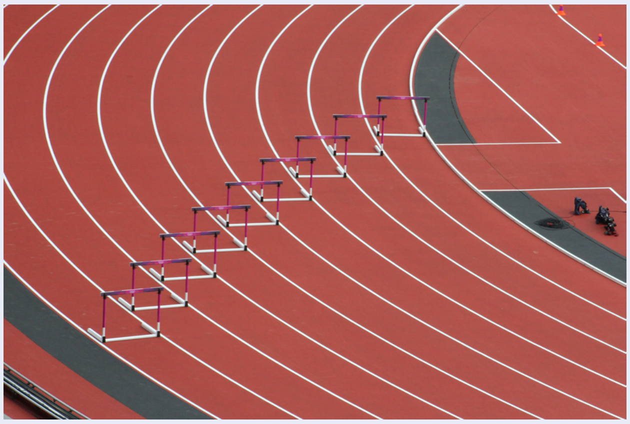
Select one or more previously identified students to share their diagrams. Ask other students if theirs are comparable to these, and if not, where differences exist. Emphasize the importance of proper labeling of the parts of the diagram to support problem solving.

Give students another 8–10 minutes to complete the other two questions. Follow with a whole-class discussion.

Student Task Statement

Here is some information about a race track:

- There are 10 equally-spaced hurdles on the track.
- The first hurdle is 13.72 meters from the start line.
- The final hurdle is 14.02 meters from the finish line.
- The race track is 110 meters long.



Access for Multilingual Learners (Activity 2, Launch)

MLR6: Three Reads.
Keep books or devices closed. Display only the first sentence and the information about the race track, without revealing the questions. Say,

"We are going to read this information 3 times."

After the 1st read, say,

"Tell your partner what this situation is about."

After the 2nd read, say,

"List the quantities. What can be counted or measured?"

For the 3rd read, reveal and read the question(s). Ask,

"What are some ways we might get started on this?"

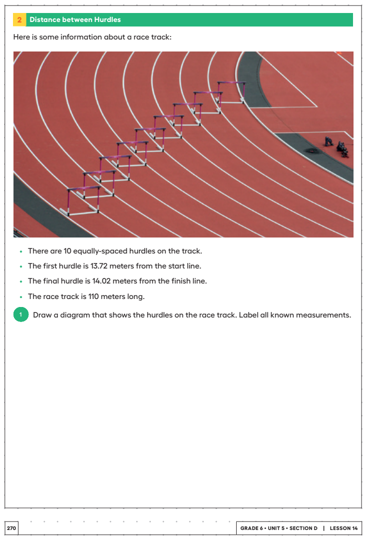
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Students may benefit from feedback that emphasizes effort, and time on task. For example, provide feedback on student diagrams and reasoning that highlights positive qualities of student work and encourages continued effort.

Supports accessibility for: Social-Emotional Functioning, Organization

Student Workbook



2 Distance between Hurdles

Here is some information about a race track:

- There are 10 equally-spaced hurdles on the track.
- The first hurdle is 13.72 meters from the start line.
- The final hurdle is 14.02 meters from the finish line.
- The race track is 110 meters long.

1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.

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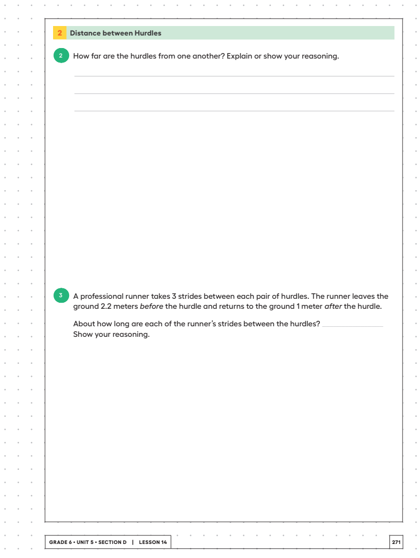
Student Workbook



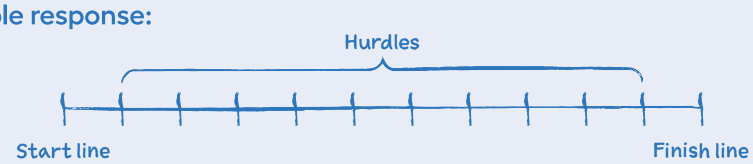
Building on Student Thinking

Students may not realize that there are only 9 spaces—not 10 spaces—between 10 hurdles, leading them to miscalculate the distance between hurdles. Urge students to study the number of spaces in their diagram, or ask them to think about how many spaces are between 2 hurdles, 3 hurdles, 4 hurdles, and so on. and extend the pattern to 10 hurdles.

A calculation error in dividing may lead to a quotient with a non-terminating decimal. Look out for arithmetic errors when students calculate the distance between the first and last hurdles (82.26 meters) and when students perform division. If students end up with a non-terminating decimal for their answer, ask them to revisit each step and see where an error might have occurred.

1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.

   Sample response:



2. How far are the hurdles from one another? Explain or show your reasoning.

   9.14 meters

   Sample reasoning: The distance between the first and last hurdles, in meters, is 110 – 13.72 – 14.02 = 82.26. Since they are equally spaced, the ten equally spaced hurdles divide the track into 9 equal parts between the first and last hurdles. The distance between them is 82.26 ÷ 9 = 9.14 meters.

3. A professional runner takes 3 strides between each pair of hurdles. The runner leaves the ground 2.2 meters *before* the hurdle and returns to the ground 1 meter *after* the hurdle.

   About how long are each of the runner's strides between the hurdles? Show your reasoning.

   1.98 meters (or about 2 meters)

   Sample reasoning: There are 9.14 meters between the hurdles. The runner comes down 1 meter beyond the previous hurdle and takes off 2.2 meters before the following hurdle so that means the 3 strides cover a distance of 9.14 – 1 – 2.2 = 5.94 meters. Each of the three strides covers 5.94 ÷ 3 = 1.98 meters, just under 2 meters.

Activity Synthesis

Select one or more previously identified students to share their diagrams for the first question. Ask other students if theirs are comparable to these, and if not, where differences exist. If not mentioned by students, be sure to highlight proper labeling of the parts of the diagram. Then, discuss the second and third questions. For the second question, discuss:

- How many 'gaps' are there between the hurdles?

  9

- What is the distance from the first hurdle to the last hurdle?

  82.26 meters

- What arithmetic operation is applied to the two numbers, 9 and 82.26? Why?

  Division, because the 10 equally spaced hurdles divide 82.26 meters into 9 equal groups.

For the third question, discuss:

- How far does the runner go in three strides? How do you know?
- Is 1.98 meters (the exact answer) an appropriate answer for the question? Why or why not?

  Most likely not, because runners are not going to control their strides and jumps to the nearest centimeter. About 2 meters would be a more appropriate answer.

Consider asking a general question about hurdle races: Is it important for the runner that the hurdles be placed as closely as possible to the correct location? The answer is yes, because runners train to take a precise number of strides and to hone their jumps to be as regular as possible. Moving a hurdle a few centimeters is unlikely to create a problem, but moving a hurdle by a meter would ruin the runners' regular rhythm in the race.

Activity 3: Optional
Examining a Tennis Court

25 min

Activity Narrative

In this activity, students apply their understanding of decimals to solve problems in another sporting context. They study the measurements of a tennis court and answer questions about lengths and area.

Visually, it appears as if each half of the tennis court (divided by the net) is a square. Similarly, it appears as if the service line is about halfway between the net and the baseline. Calculations show that in both cases, however, neither half of the tennis court is a square, and that the service line is not half way between the baseline and the net.

Just as with the distances between hurdles in an earlier activity, the measurements of a tennis court are very precisely determined. It is also very important for professional tennis players that the courts on which they play have consistent measurements, as the smallest differences could affect whether the ball is in or out.

There are some subtleties related to measurement in the real world and the idealized version in the task. On a tennis court, the lines have width. For the first two questions of the task, the strips can be taken as dimensionless lines. In the final questions, students deal explicitly with these strips, an opportunity to attend to precision.

Launch

Ask students if they know of or have played sports that involve moving a ball or another object around within a prescribed space. Solicit examples from students. If tennis doesn't come up, ask students if they play tennis or have ever watched a tennis match.

Display a picture of the tennis court, and discuss the purpose of the boundary lines. Consider asking students to locate various sections of the court by pointing to rectangles, parallel segments, right angles, and the service box. (More detailed measurements for the parts of a tennis court can be found online.) Tell students that a tennis court used for two players ("doubles") per side is wider than a court used for one player ("singles") per side.

Keep students in groups of 2.

Give students 5–7 minutes of quiet time to complete the first two questions and then time to discuss their responses with their partner. Then, give students another 8–10 minutes to complete the last set of questions. Follow with a whole-class discussion.

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 3, Launch)

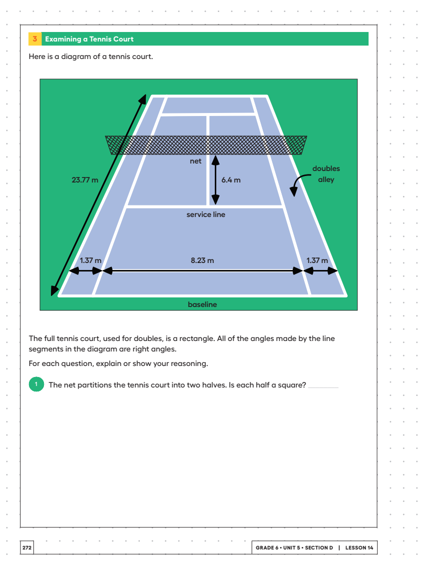
MLR5: Co-Craft Questions. Display only the image of the tennis court, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Student Workbook


3 Examining a Tennis Court

Here is a diagram of a tennis court.

The full tennis court, used for doubles, is a rectangle. All of the angles made by the line segments in the diagram are right angles.

For each question, explain or show your reasoning.

1 The net partitions the tennis court into two halves. Is each half a square?

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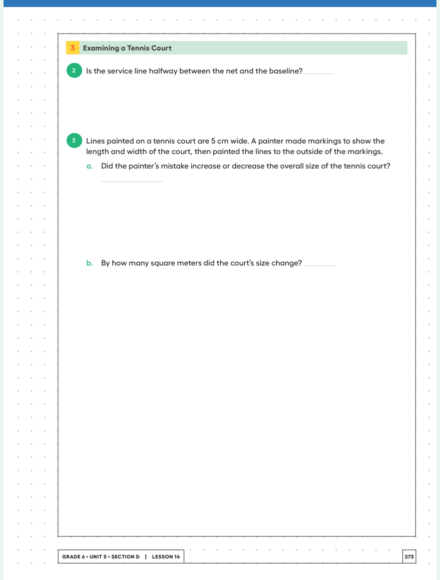
Student Workbook


3 Examining a Tennis Court

2 Is the service line halfway between the net and the baseline?

3 Lines painted on a tennis court are 5 cm wide. A painter made markings to show the length and width of the court, then painted the lines to the outside of the markings.

a. Did the painter's mistake increase or decrease the overall size of the tennis court?

b. By how many square meters did the court's size change?

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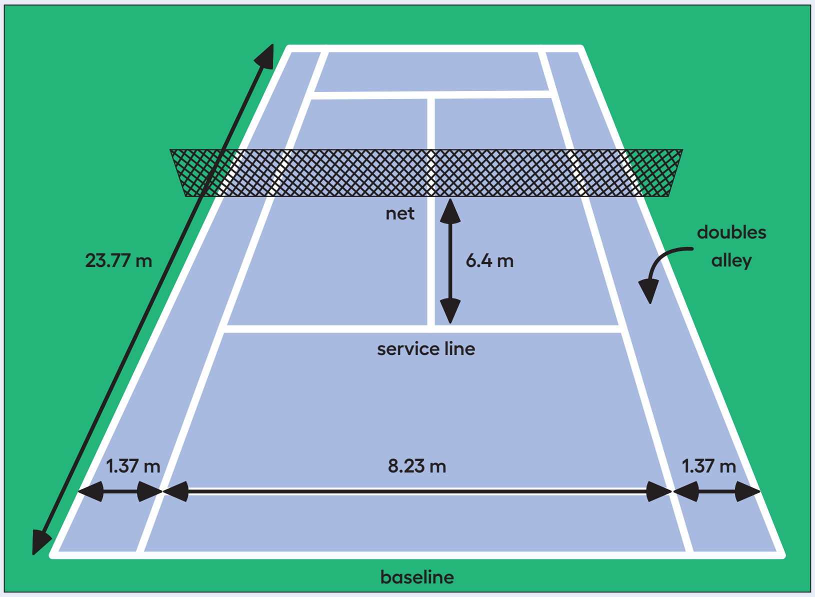
Student Task Statement

Here is a diagram of a tennis court.



The full tennis court, used for doubles, is a rectangle. All of the angles made by the line segments in the diagram are right angles.

For each question, explain or show your reasoning.

1. The net partitions the tennis court into two halves. Is each half a square?

   No, the tennis court is 23.77 meters long, and the net is in the middle. This means that each half of the court is $\frac{23.77}{2}$ meters long. This is 11.885 meters. Since the tennis court is only 10.97 meters wide, it is not a square.

2. Is the service line halfway between the net and the baseline?

   No, the service line is 6.4 meters from the net. Since the length of the (half) court is 11.885 meters, this means that the service line is 11.885 − 6.4 meters from the baseline. This is 5.485 meters. So the service line is almost a meter closer to the baseline than it is to the net.

3. Lines painted on a tennis court are 5 cm wide. A painter made markings to show the length and width of the court, then painted the lines to the outside of the markings.

   a. Did the painter's mistake increase or decrease the overall size of the tennis court?

      The painter's mistake made the court larger. The painter's outline of the court *begins* where the outline of the court is supposed to *end*.

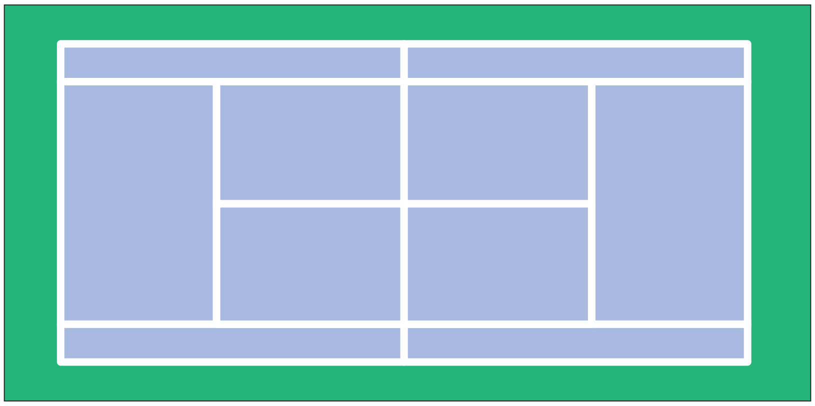
   b. By how many square meters did the court's size change?

      The painter added two extra 23.77 m by 0.05 m strips and two 10.97 m by 0.05 m strips along the sides, and four 0.05 m by 0.05 m squares in the corners. All measurements are in meters. The two long strips make 2.377 square meters, while the two shorter strips add 1.097 square meters. The four small squares add another 0.01 square meter. This adds up to an extra 3.484 square meters.

Activity Synthesis

Focus the discussion on how students use calculations involving decimals to answer each question.

Before discussing each question, poll the class on whether they concluded that each half of the court is a square, the service line is halfway between the baseline and the net, and the court is larger or smaller than it should be given a painter's mistake. Record and display the responses for all to see.



Consider displaying a picture of a tennis court from directly above, as shown, to facilitate discussion. Though it is possible to judge from the picture that the service line is closer to the baseline than it is to the net, it is not as easy to tell, just by looking, whether each half of the court is a square.

Ask students to share their responses and reasoning. Record or display their calculations for all to see. Invite other students to agree or disagree, or to offer alternative ways to reason.

If time permits, consider asking questions such as:

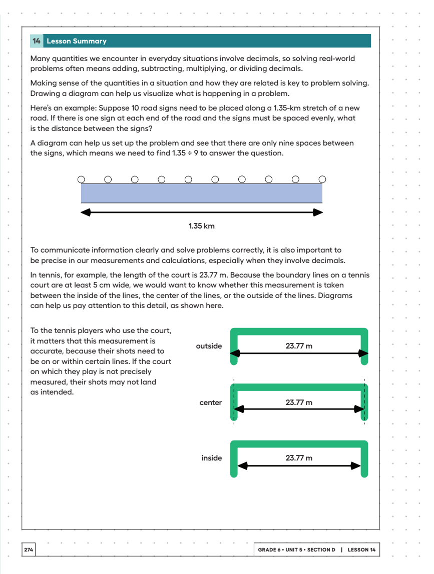
- *"Would the painter's mistake change your answers to the first two questions?"*

  No, the sides of the quadrilaterals would all increase by 10 cm and they still would not be squares.

- *"Would you notice the painter's mistake if you were playing on the mis-painted court?"*

  A professional player may notice, but an unseasoned player may not because 5 cm is very little compared to 11 meters and 24 meters.

Student Workbook


14 Lesson Summary

Many quantities we encounter in everyday situations involve decimals, so solving real-world problems often means adding, subtracting, multiplying, or dividing decimals.

Making sense of the quantities in a situation and how they are related is key to problem solving. Drawing a diagram can help us visualize what is happening in a problem.

Here's an example: Suppose 10 road signs need to be placed along a 1.35-km stretch of a new road. If there is one sign at each end of the road and the signs must be spaced evenly, what is the distance between the signs?

A diagram can help us set up the problem and see that there are only nine spaces between the signs, which means we need to find 1.35 ÷ 9 to answer the question.

1.35 km

To communicate information clearly and solve problems correctly, it is also important to be precise in our measurements and calculations, especially when they involve decimals.

In tennis, for example, the length of the court is 23.77 m. Because the boundary lines on a tennis court are at least 5 cm wide, we would want to know whether this measurement is taken between the inside of the lines, the center of the lines, or the outside of the lines. Diagrams can help us pay attention to this detail, as shown here.

To the tennis players who use the court, it matters that this measurement is accurate, because their shots need to be on or within certain lines. If the court on which they play is not precisely measured, their shots may not land as intended.

outside 23.77 m

center 23.77 m

inside 23.77 m

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Lesson Synthesis

The goal of this discussion is to highlight key aspects of problem solving, such as the importance of understanding what is happening in a situation, creating a visual representation to make sense of relationships, estimating or rounding, and attending to units of measurement and precision.

Select and display 2–3 problems that students solved in the lesson. Ask questions such as:

"In each problem, how did you know what calculations to make or what to do?"

"For which problems did you use a diagram? How did the diagram help you solve the problems?"

"Were there times when you rounded a decimal or made an estimate as a part of your process? If so, how did it help you?"

"For which problems was it necessary to calculate precisely? Why might that be?"

Lesson Summary

Many quantities we encounter in everyday situations involve decimals, so solving real-world problems often means adding, subtracting, multiplying, or dividing decimals.

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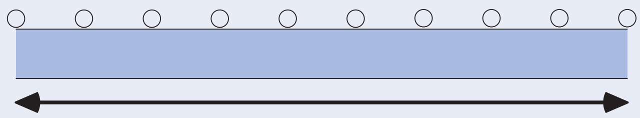

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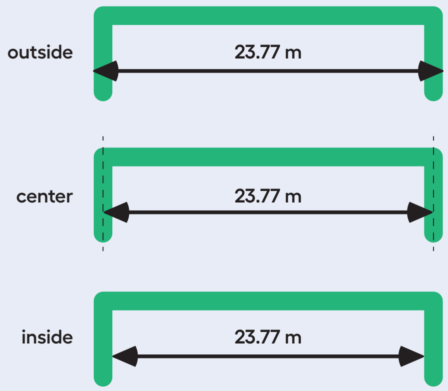


To the tennis players who use the court, it matters that this measurement is accurate, because their shots need to be on or within certain lines. If the court on which they play is not precisely measured, their shots may not land as intended.

Cool-down

Ribbon for Sharing

5 min

Student Task Statement

Jada and Han are sharing a piece of ribbon that is 1.905 meters long for a craft project. Jada cuts 0.82 meter from one end of the ribbon and Han cuts 0.175 meter from the other end.

Afterward, they split the ribbon that is left into equal-size pieces that are 0.13-meter long each. How many pieces will they have?
Show your reasoning.

7 pieces

Sample reasoning: Jada and Han cut a total of 0.82 + 0.175, or 0.995 meter, from the two ends. This leaves 1.905 − 0.995, or 0.91 meter. Dividing 0.91 by 0.13 gives 7.

$$\begin{array}{r} 0.175 \\ +\,0.82 \\ \hline 0.995 \end{array} \qquad \begin{array}{r} 1.905 \\ -\,0.995 \\ \hline 0.910 \end{array} \qquad \begin{array}{r} 7 \\ 13\overline{)91} \\ -\,91 \\ \hline 0 \end{array}$$

Responding To Student Thinking

Points to Emphasize

If students struggle to make sense of the operations that correspond to the situation, support students in interpreting information in contextual problems. For example, when working on the practice problems referred to here, discuss how to represent the given measurements and what is happening in each situation, using a diagram or another representation.

Grade 6, Unit 5, Lesson 14, Practice Problem 1

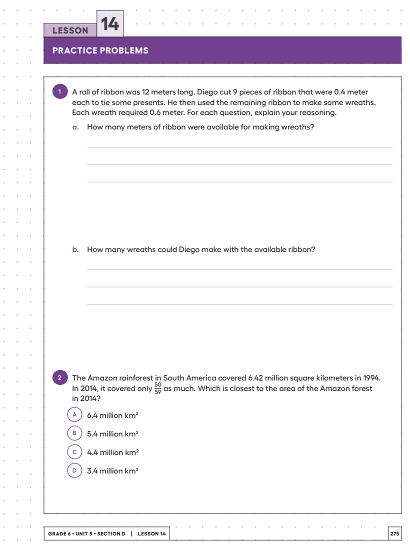
Grade 6, Unit 5, Lesson 14, Practice Problem 3

Lesson 14

Practice Problems

7 Problems

Student Workbook



LESSON 14

PRACTICE PROBLEMS

1. A roll of ribbon was 12 meters long. Diego cut 9 pieces of ribbon that were 0.4 meter each to tie some presents. He then used the remaining ribbon to make some wreaths. Each wreath required 0.6 meter. For each question, explain your reasoning.
   a. How many meters of ribbon were available for making wreaths?
   b. How many wreaths could Diego make with the available ribbon?

2. The Amazon rainforest in South America covered 6.42 million square kilometers in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which is closest to the area of the Amazon forest in 2014?
   A. 6.4 million km^2
   B. 5.4 million km^2
   C. 4.4 million km^2
   D. 3.4 million km^2

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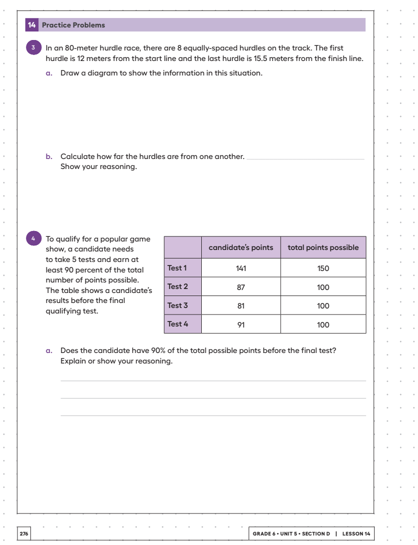
Student Workbook



14 Practice Problems

3. In an 80-meter hurdle race, there are 8 equally-spaced hurdles on the track. The first hurdle is 12 meters from the start line and the last hurdle is 15.5 meters from the finish line.
   a. Draw a diagram to show the information in this situation.
   b. Calculate how far the hurdles are from one another. Show your reasoning.

4. To qualify for a popular game show, a candidate needs to take 5 tests and earn at least 90 percent of the total number of points possible. The table shows a candidate's results before the final qualifying test.

	candidate's points	total points possible
Test 1	141	150
Test 2	87	100
Test 3	81	100
Test 4	91	100

   a. Does the candidate have 90% of the total possible points before the final test? Explain or show your reasoning.

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Problem 1

A roll of ribbon was 12 meters long. Diego cut 9 pieces of ribbon that were 0.4 meter each to tie some presents. He then used the remaining ribbon to make some wreaths. Each wreath required 0.6 meter. For each question, explain your reasoning.

a. How many meters of ribbon were available for making wreaths?

8.4 meters

Sample reasoning: Diego used $9 \cdot (0.4)$ or 3.6 meters for the presents, which leaves 8.4 meters in the roll, because $12 - 3.6 = 8.4$.

b. How many wreaths could Diego make with the available ribbon?

14 wreaths

Sample reasoning: $8.4 \div 0.6 = 14$

Problem 2

The Amazon rainforest in South America covered 6.42 million square kilometers in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which is closest to the area of the Amazon forest in 2014?

A. 6.4 million km^2

B. 5.4 million km^2

C. 4.4 million km^2

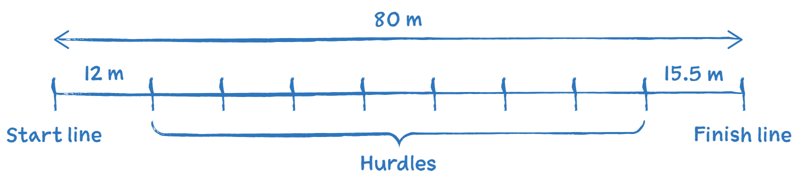
D. 3.4 million km^2

Problem 3

In an 80-meter hurdle race, there are 8 equally-spaced hurdles on the track. The first hurdle is 12 meters from the start line and the last hurdle is 15.5 meters from the finish line.

a. Draw a diagram to show the information in this situation.

Sample response:



b. Calculate how far the hurdles are from one another. Show your reasoning.

7.5 meters

Sample reasoning: The distance from the first to the last hurdle is $80 - 12 - 15.5$, which is 52.5 meters. There are 7 equal gaps between hurdles, so each of these gaps is $52.5 \div 7$, which is 7.5.

Problem 4

To qualify for a popular game show, a candidate needs to take 5 tests and earn at least 90 percent of the total number of points possible. The table shows a candidate's results before the final qualifying test.

	candidate's points	total points possible
Test 1	141	150
Test 2	87	100
Test 3	81	100
Test 4	91	100

a. Does the candidate have 90% of the total possible points before the final test? Explain or show your reasoning.

No

Sample reasoning: Before the final test, the candidate has 400 points out of 450. 90% of 450 is $(0.9) \cdot 450$, or 405, so the candidate is 5 points short.

b. The candidate thinks that if they get at least 92 out of 100 on the final test, they will qualify. Do you agree? Explain your reasoning.

No

Sample reasoning: After the final test, the candidate needs 495 points out of 550 because 90% of 550 is $(0.9) \cdot 550$, or 495. They only have 492 points.

Problem 5

from Unit 5, Lesson 4

Find the following differences. Show your reasoning.

a. 0.151 – 0.028 0.123

Sample reasoning: 0.151 is 151 thousandths and 0.028 is 28 thousandths. 151 – 28 = 123, so the difference is 123 thousandths.

b. 0.106 – 0.0315 0.0745

Sample reasoning: 0.106 can be written as 0.1060 before subtracting 0.0315 from it using vertical calculation.

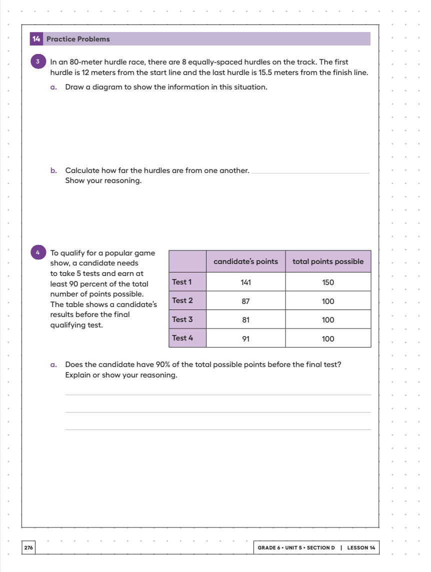
$$\begin{array}{r} {\scriptstyle 0\;10\;5\;10} \\ 0\,.\,\cancel{1}\,\cancel{0}\,\cancel{6}\,0 \\ -\;0\,.\,0\,3\,1\,5 \\ \hline 0\,.\,0\,7\,4\,5 \end{array}$$

c. 3.572 – 2.6014 0.9706

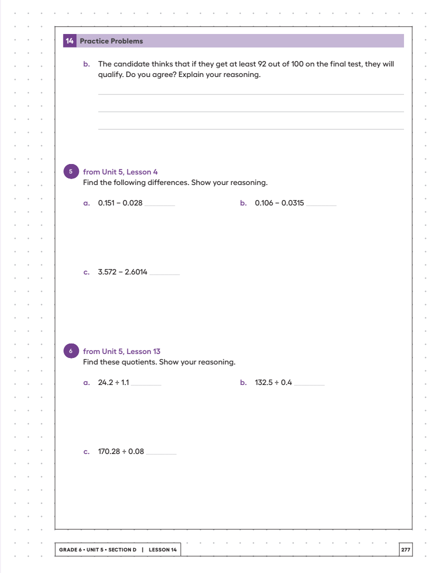
Sample reasoning: 3.572 – 2.6014 can be thought of as 2.6014 + ? = 3.572, and the missing number can be found using vertical calculation.

$$\begin{array}{r} {\scriptstyle 1\quad\;\; 1\;\;} \\ 2\,.\,6\,0\,1\,4 \\ +\;\boxed{0}\,.\,\boxed{9}\,\boxed{7}\,\boxed{0}\,\boxed{6} \\ \hline 3\,.\,5\,7\,2\,0 \end{array}$$

Student Workbook



Student Workbook



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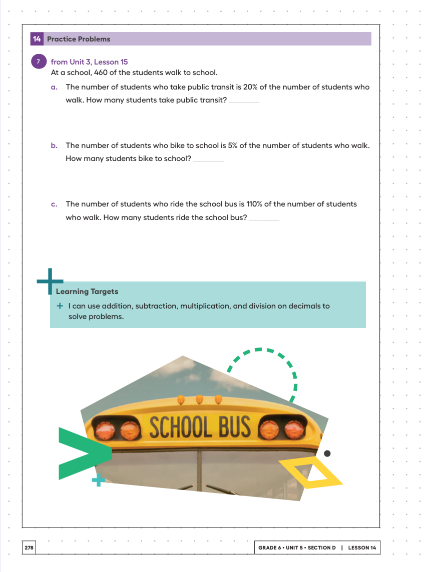

14 Practice Problems

7 from Unit 3, Lesson 15

At a school, 460 of the students walk to school.

a. The number of students who take public transit is 20% of the number of students who walk. How many students take public transit? ______

b. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school? ______

c. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus? ______

Learning Targets

+ I can use addition, subtraction, multiplication, and division on decimals to solve problems.

Problem 6

from Unit 5, Lesson 13

Find these quotients. Show your reasoning.

a. $24.2 \div 1.1$ 22

Sample reasoning:

$$\begin{array}{r} 220 \div 11 = 20 \\ 22 \div 11 = 2 \\ \hline 242 \div 11 = 22 \end{array}$$

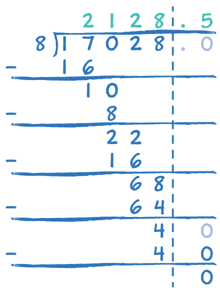
b. $132.5 \div 0.4$ 331.25

Sample reasoning:

$$\begin{array}{r} 120 \div 0.4 = 300 \\ 12 \div 0.4 = 30 \\ 0.4 \div 0.4 = 1 \\ 0.1 \div 0.4 = 0.25 \\ \hline 132.5 \div 0.4 = 331.25 \end{array}$$

c. $170.28 \div 0.08$ 2,128.5

Sample reasoning (using long division):



Problem 7

from Unit 3, Lesson 15

At a school, 460 of the students walk to school.

a. The number of students who take public transit is 20% of the number of students who walk. How many students take public transit?

92 students

Sample reasoning: $460 \cdot 0.2 = 92$

b. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?

23 students

Sample reasoning: $460 \cdot 0.05 = 23$

c. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

506 students

Sample reasoning: $460 \cdot 1.10 = 506$