# **Calculating Products of Decimals**

#### Goals

- Draw and label a diagram to check the answer to a decimal multiplication problem.
- Generalize (orally and in writing) that the number of decimal places in a product is the sum of the number of decimal places in the factors.
- Use an algorithm to calculate the product of two decimals, and explain (orally) the solution method.

### **Learning Targets**

- I can describe and apply a method for multiplying decimals.
- I know how to use a product of whole numbers to find a product of decimals.

# Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

#### Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Warm-up)

#### **Instructional Routines**

- Math Talk
- MLR1: Stronger and Clearer Each Time

#### **Required Materials**

#### **Materials to Gather**

• Graph paper: Activity 2

#### **Lesson Narrative**

In this final lesson on multiplication, students consolidate their understanding of the ideas from the previous lessons and generalize toward an algorithm that can be used to find the product of any pair of decimals.

Previously, students saw that we can multiply decimals by:

- Thinking of the decimals as products of whole numbers and fractions and multiplying those numbers.
- Multiplying the decimal factors by powers of 10 to make them whole numbers, finding their product, and dividing it by the same powers of 10.
- Representing decimal factors as side lengths of a rectangle, decomposing them by place value, finding partial products, and adding them.

Here, students make a generalization that we can also: multiply whole numbers with the same digits as the decimal factors, count the total number of decimal places in the factors, and place the decimal point in the product so that it has that number of decimal places. They see that the steps of this algorithm can be explained by prior reasoning about properties of base-ten numbers and the calculation result can be checked using other methods.

# Lesson Timeline

5 min

25 min

**Activity 1** 

15 min

Activity 2

10 min

**Lesson Synthesis** 

**Assessment** 

5 min

Cool-down

# **Calculating Products of Decimals**

# Lesson Narrative (continued)

The lesson includes an optional activity that allows students to apply the multiplication algorithm both in and out of context.

# **Student Learning Goal**

Let's multiply decimals.

#### **Instructional Routines**

#### **Math Talk**

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# Access for Students with Diverse Abilities (Warm-up, Launch)

# Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

# 

#### Warm-up

#### Math Talk: Twenty Times a Number



#### **Activity Narrative**

This *Math Talk* focuses on multiplication of a whole number and a decimal. It encourages students to think about properties of operations and to rely on what they know about place value to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students decompose and multiply decimals beyond tenths later in the lesson.

To multiply decimals to the hundredths, students need to look for and make use of the structure of base-ten numbers.

#### Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

#### **Student Task Statement**

Find the value of each expression mentally.

A.20 · 5 100

Sample reasoning:

- $2 \cdot 5 = 10$ , so  $20 \cdot 5$  is 10 times 10, which is 100.
- $\circ$  20 · 10 = 200, so 20 · 5 is half of 200, which is 100.

B.20 · (0.8) 16

Sample reasoning:

•  $20 \cdot 8 = 160$  and 0.8 is one-tenth of 8, so  $20 \cdot (0.8)$  is one-tenth of 160, which is 16.

C.20 · (0.04) 0.8

Sample reasoning:

- 20 · 4 = 80 and 0.04 is one-hundredth of 4, so 20 · (0.04) is one-hundredth of 80, which is 0.8.
- The decimal 0.4 is half of 0.8, so 20 · (0.4) is half of 16, which is 8.

  The decimal 0.04 is a tenth of 0.4, so 20 · (0.04) is a tenth of 8, or 0.8.

D.20 · (5.84) II6.8

Sample reasoning: 5.84 is 5 + 0.8 + 0.04, so  $20 \cdot (5.84)$  is the sum of,  $20 \cdot 5$ ,  $20 \cdot (0.8)$ , and  $20 \cdot (0.04)$ , or 100 + 16 + 0.8, which is 116.8.

### **Activity Synthesis**

To involve more students in the conversation, consider asking:

"Who can restate \_\_\_\_\_\_'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to \_\_\_\_\_\_'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

#### **Activity 1**

#### **Calculating Products of Decimals**

25 min

#### **Activity Narrative**

In this activity, students see that they can find the product of any pair of decimals by using the standard algorithm to multiply a related pair of whole numbers and then placing the decimal point. For example, to calculate (2.5)  $\cdot$  (1.2), they can first find 25  $\cdot$  12 and then place the decimal point such that the result has two decimal places.

Students begin by analyzing an example and formulating a draft explanation for the placement of the decimal point in a product of decimals. Then, they take turns sharing their initial ideas with different partners. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and to critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language that they use to describe their thinking.

Later, students apply the same method to multiply other decimals in and out of context. They also have an opportunity to check their reasoning using an area diagram.

### Launch



Arrange students in groups of 3–4. Display the calculations in the first question. Read aloud the problem stem.

Give students 2–3 minutes of quiet time to make sense of the calculations and to write an explanation for the placement of the decimal point.

Use Stronger and Clearer Each Time to give students an opportunity to revise and refine their response for why the decimal point of the product of  $(2.5) \cdot (1.2)$  is where it is. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help clarify and strengthen their partner's ideas and writing.

If time allows, display these prompts for feedback:

- "\_\_\_\_\_ makes sense, but what do you mean when you say ... ?"
- "Can you describe that another way?"

# Access for Multilingual Learners (Warm-up, Synthesis)

#### MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_\_\_\_\_ because ..." or "I notice \_\_\_\_\_\_ so I ..." Some students may benefit from the opportunity to rehearse with a partner what they will say before they share with the whole class.

Advances: Speaking, Representing

#### **Instructional Routines**

# MLR1: Stronger and Clearer Each Time

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# Access for Multilingual Learners (Activity 1)

#### MLR1: Stronger and Clearer Each Time

This activity uses the Stronger and Clearer Each Time math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

# Access for Students with Diverse Abilities (Activity 1, Launch)

# Representation: Develop Language and Symbols.

Activate or supply background knowledge. Review the standard multiplication algorithm. To help students recall the terms "factor" and "product," ask,

"What is the name for each part of a multiplication calculation?"

Supports accessibility for: Memory, Language

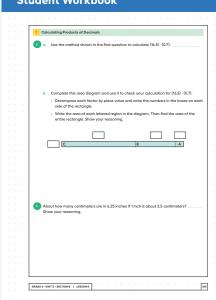
Warm-up

#### **Building on Student Thinking**

When completing the diagram to represent  $(16.5) \cdot (0.7)$ , students might recognize that 16.5 needs to be decomposed into three parts but be unsure how to do so. Encourage them to think about the place value of each digit in 16.5. Ask:

"What values do the 1, 6, and 5 represent? How do those values correspond to the three parts on the long side of the diagram?"

#### **Student Workbook**



Close the partner conversations, and give students 2–3 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

After Stronger and Clearer Each Time, give students 8–10 minutes to complete the activity individually or with their group.

Some students might find it helpful to use a grid to align the digits in vertical calculations. Provide access to graph paper.

#### **Student Task Statement**

**1.** A common way to multiply decimals is to multiply whole numbers, then place the decimal point in the product.

Here is an example for  $(2.5) \cdot (1.2)$ .

$$25 \cdot 12 = 300$$
  
(2.5) · (1.2) = 3.00

Use what you know about place value to explain why the decimal point of the product is placed where it is.

#### Sample responses:

- 2.5 is 0.1 times 25, and 1.2 is 0.1 times I2, so the product of 25 and I2, which is 300, needs to be multiplied by (0.1)  $\cdot$  (0.1), or 0.01. Multiplying 300 by 0.01, or  $\frac{1}{100}$ , is the same as dividing it by 100, which moves the digits of 300 two places to the right.
- 2.5 is  $\frac{25}{10}$  and I.2 is  $\frac{12}{10}$ . Multiplying these fractions gives  $\frac{300}{100}$ , which is 3, so the decimal point follows 3.
- **2. a.** Use the method shown in the first question to calculate (16.5)  $\cdot$  (0.7).

11.55

- **b.** Complete this area diagram and use it to check your calculation for  $(16.5) \cdot (0.7)$ .
  - Decompose each factor by place value and write the numbers in the boxes on each side of the rectangle.
  - Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.



- A:  $(0.5) \cdot (0.7) = 0.35$ , B:  $6 \cdot (0.7) = 4.2$ , C:  $10 \cdot (0.7) = 7$
- $\circ$  0.35 + 4.2 + 7 = II.55, so (16.5)  $\cdot$  (0.7) = II.55
- **3.** About how many centimeters are in 6.25 inches if 1 inch is about 2.5 centimeters? Show your reasoning.

About 15.625 centimeters

Sample reasoning:  $(6.25) \cdot (2.5) = 15.625$ .

#### **Activity Synthesis**

Focus the discussion on how students calculated the products of decimals in the last two questions. Ask questions such as:

 $\bigcirc$  "How did you know where to place the decimal point in the product of (16.5)  $\cdot$  (0.7)?"

Because I65 is IO times I6.5 and 7 is IO times O.7, the result of I65  $\cdot$  7, or I,155, needs to be divided by IO  $\cdot$  IO, or IOO. This means moving the digits of I,155 two places to the right, giving II.55.

 $625 \cdot 25$  is 1,000 times the product of  $(6.25) \cdot (2.5)$ , so we can divide the former by 1,000—or move the digits 3 places to the right—to get the latter.

If time permits, invite students to reflect on their reasoning, asking questions such as:

"Which method—drawing an area diagram or using vertical calculations do you prefer when finding products such as (16.5) · (0.7)? Why?"

Drawing an area diagram, because the visual representation helps to break up the calculation into smaller, more manageable pieces:  $10 \cdot (0.7)$ ,  $6 \cdot (0.7)$ , and  $(0.5) \cdot (0.7)$ . Vertical calculation, because it is quicker to just multiply whole numbers and place the decimal point.



#### **Activity 2: Optional**

#### **Practicing Multiplication of Decimals**



#### **Activity Narrative**

This activity allows students to practice using the methods in this lesson to calculate products of decimals, including a problem in a real-world context. While students can choose to use area diagrams to help organize their work and support their reasoning, they should also practice using the multiplication algorithm on decimals.

# Launch



Keep students in groups of 3-4.

Give them 5–7 minutes of quiet work time and then time to discuss their responses with their group. If time is limited, consider asking students to convert only one length in the last question from meters to feet.

Provide continued access to graph paper for students who wish to use a grid to align the digits in vertical calculations.

#### **Student Task Statement**

- **1.** Calculate each product. Show your reasoning. If you get stuck, consider drawing a diagram.
  - $\mathbf{a.}(5.6) \cdot (1.8)$

10.08

Sample reasoning:

- 56 · 18 = 1,008. 56 is 10 times 5.6, and 18 is 10 times 1.8, so 1,008 needs to be divided by 100. 1,008 ÷ 100 = 10.08
- 56 · 18 = 1,008. Each factor has I decimal place, so the product has 2 decimal places. The decimal point will be between the two zeros in 1008.
- **b.** (0.008) · (7.2)

0.0576

Sample reasoning:  $\frac{8}{1,000} \cdot \frac{72}{10} = \frac{576}{10,000}$ 

- **2.** A rectangular playground is 18.2 meters by 12.75 meters.
  - **a.** Find its area in square meters. Show your reasoning.

232.05 square meters

Sample reasoning:  $(18.2) \cdot (12.75) = 232.05$ 

**b.** If 1 meter is approximately 3.28 feet, what are the approximate side lengths of the playground in feet? Show your reasoning.

59.696 feet and 41.82 feet

Sample reasoning:  $(18.2) \cdot (3.28) = 59.696$  and  $(12.75) \cdot (3.28) = 41.82$ 

#### **Are You Ready for More?**

1. Find the value of each expression.

a.1 - 0.1

0.9

**b.**1 - 0.1 + 10 - 0.01

10.89

c.1 - 0.1 + 10 - 0.01 + 100 - 0.001

110.889

**2.** Suppose we keep writing expressions that follow the same pattern of adding a whole number and subtracting a decimal. What would the values of those expressions look like? Make a prediction.

Sample response: The decimal would consist of 0.8 sandwiched between I's on the left and 8's on the right, ending with a 9 in the smallest decimal place.

3. What would the value of the expressions be if all of the addition and subtraction symbols became multiplication symbols? Explain your reasoning.

The decimal would be equal to just the last number in each expression. Sample reasoning: Each pair of numbers after the first number, I, has a product of I. For example, in  $I \cdot (0.I) \cdot I0 \cdot (0.0I) \cdot I00 \cdot (0.00I)$ , the expression  $(0.I) \cdot I0$  has a value of I and so does  $(0.0I) \cdot I00$ . That leaves I times the last number, 0.00I, which is 0.00I.

#### **Activity Synthesis**

Invite students to share their responses and reasoning. Display their calculations and diagrams (if used) for all to see. For each product, discuss how students determined the number of decimal places that the product should have.

When discussing the area of the rectangular playground, students may record the product of the side lengths as 232.050 or 232.05. If so, discuss why both values are correct.

#### **Lesson Synthesis**

Focus the discussion on the steps that can be taken to multiply any two decimals and why they work. Discuss questions such as:

 $\bigcirc$  "We can calculate (1.9)  $\cdot$  (0.08) by first calculating 19  $\cdot$  8. How are the factors in the two expressions related?"

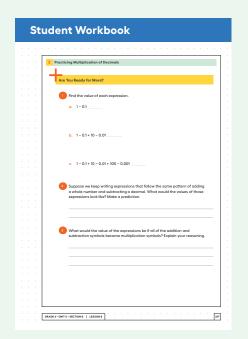
19 is 10 times 1.9, and 8 is 100 times 0.08.

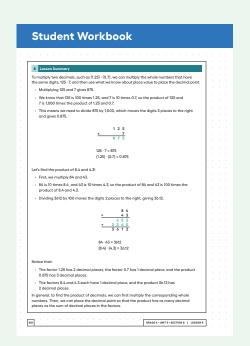
Once we have the product of 19 and 8, which is 152, how do we know where to put the decimal point?"

0.9 has one decimal place and 0.08 has two, so the product will have three decimal places.

"Why does it make sense for this product to have three decimal places?"

We multiplied the factors by 10 and 100 to get 19 and 8, so the product, 152, needs to be divided by 1,000. Doing that moves the digits three places to the right, giving it three decimal places.





"How can we check if the product we have is correct?"

We can use another strategy. We can estimate to see if it makes sense. For example, I.9 is a little less than 2 and 0.08 is a little less than 0.1. The answer must be less than 0.2.

#### **Lesson Summary**

To multiply two decimals, such as (1.25)  $\cdot$  (0.7), we can multiply the whole numbers that have the same digits, 125  $\cdot$  7, and then use what we know about place value to place the decimal point.

- Multiplying 125 and 7 gives 875.
- We know that 125 is 100 times 1.25, and 7 is 10 times 0.7, so the product of 125 and 7 is 1,000 times the product of 1.25 and 0.7.
- This means we need to divide 875 by 1,000, which moves the digits 3 places to the right and gives 0.875.

$$125 \cdot 7 = 875$$
$$(1.25) \cdot (0.7) = 0.875$$

Let's find the product of 8.4 and 4.3!

- First, we multiply 84 and 43.
- 84 is 10 times 8.4, and 43 is 10 times 4.3, so the product of 84 and 43 is 100 times the product of 8.4 and 4.3.
- Dividing 3612 by 100 moves the digits 2 places to the right, giving 36.12.

#### Notice that:

- The factor 1.25 has 2 decimal places, the factor 0.7 has 1 decimal place, and the product 0.875 has 3 decimal places.
- The factors 8.4 and 4.3 each have 1 decimal place, and the product 36.12 has 2 decimal places.

In general, to find the product of decimals, we can first multiply the corresponding whole numbers. Then, we can place the decimal point so that the product has as many decimal places as the sum of decimal places in the factors.

5 min

#### Cool-down

### **Calculate This!**

#### **Student Task Statement**

Calculate  $(1.6) \cdot (0.215)$ . Show your reasoning.

#### 0.344

#### Sample reasoning:

#### **Responding To Student Thinking**

#### Points to Emphasize

If most students struggle to use what they know about base-ten numbers, place value, and fractions or to use diagrams or partial products to multiply decimals, make time to revisit these ideas before exploring division of decimals. For example, complete the optional activity and practice problem referred to here. For each multiplication problem, elicit as many ways to reason about the product as time permits and discuss the connections across strategies.

Grade 6, Unit 5, Lesson 7, Activity 1 Connecting Area Diagrams to Calculations with Whole Numbers

Grade 6, Unit 5, Lesson 5, Practice Problem 2

### **Practice Problems**

8

7 Problems

# **Problem 1**

Here are an unfinished calculation of  $(0.54) \cdot (3.8)$  and a 0.54-by-3.8 rectangle.

a. Which part of the rectangle has an area of 0.432? Which part of the rectangle has an area of 1.62? Show your reasoning.

0.432 is the area of the 0.8 by 0.54 rectangle because  $(0.8) \cdot (0.54) = 0.432 \cdot 1.62$ is the area of the 3 by 0.54 rectangle because  $3 \cdot (0.54) = 1.62$ .

	3	8.0
0.54		0.432

**b.** What is (0.54) · (3.8)? **2.16** 

Sample response: 0.54 + 1.62 = 2.052

### Problem 2

Explain how the product of 3 and 65 could be used to find  $(0.03) \cdot (0.65)$ .

Sample response: We can use vertical calculation to find 3 times 65, which equals 195. Because 0.03 is 3 hundredths and 0.65 is 65 hundredths, 195 will need to be multiplied by (0.01) · (0.01), or 0.0001. Multiplying 195 by 0.0001 gives 0.0195.

#### **Problem 3**

Use vertical calculation to find each product.

**a.** (5.4) · (2.4) **12.96** 

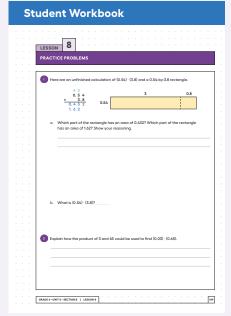
**b.** (1.67) · (3.5) **5.845** 

### **Problem 4**

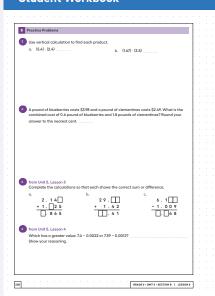
A pound of blueberries costs \$3.98 and a pound of clementines costs \$2.49. What is the combined cost of 0.6 pound of blueberries and 1.8 pounds of clementines? Round your answer to the nearest cent.

### \$6.87

Sample reasoning:  $(3.98) \cdot (0.6) = 2.388$ , or about \$2.39.  $(2.49) \cdot (1.8) = 4.482$ , or about \$4.48. The combined cost is 2.39 + 4.48, or 6.87.



#### Student Workbook



Problem 5

from Unit 5, Lesson 3

Complete the calculations so that each shows the correct sum or difference.

a.

b.

c.

#### Problem 6

from Unit 5, Lesson 4

Which has a greater value: 7.4 - 0.0022 or 7.39 - 0.0012? Show your reasoning.

7.4 - 0.0022 has a greater value. 7.4 - 0.0022 = 7.3978 and 7.39 - 0.0012 = 7.3888.

### Problem 7

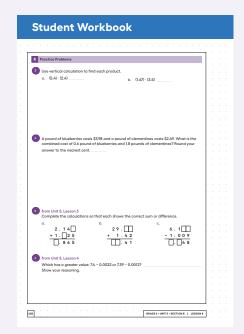
from Unit 2, Lesson 12

Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings? If you get stuck, consider using the table.

140 minutes (or equivalent)

Sample response:

number of saplings	time in minutes	
3	30	
1	10	
14	140	



#### Student Workbook

