Lesson 3

# Staying in Balance

## Goals

- Interpret hanger diagrams (orally and in writing) and write equations that represent relationships between the weights on a balanced hanger diagram.
- Use balanced hangers to explain (orally and in writing) how to find solutions to equations of the form $x + p = q$ or $px = q$.

## Learning Targets

- I can compare the process of removing or grouping weights to keep a hanger diagram balanced and the process of subtracting or dividing numbers to solve an equation.
- I can explain what a balanced hanger diagram and a true equation have in common.
- I can write equations that could represent the weights on a balanced hanger diagram.

## Lesson Narrative

The goal of this lesson is for students to understand why we can generally approach solving $p + x = q$ by subtracting the same value from each side and why we can generally approach solving $px = q$ by dividing each side by the same value. This understanding is accomplished by considering what can be done to a hanger to keep it balanced.

Previously, students solved a variable equation by thinking about values for the variable that made the equation true. Here, they use the image of a balanced hanger to reason about things they could "do" to the equation to make the solution easier to see while still keeping both sides of the equation equal (balanced). In the next lesson, students will apply this strategy to solve equations algebraically.

## Student Learning Goal

Let's use balanced hangers to help us solve equations.

### Access for Students with Diverse Abilities

- Representation (Activity 2)

### Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 2)

### Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

## Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Assessment: Cool-down |
|---|---|---|---|---|
| 10 min | 10 min | 15 min | 10 min | 5 min |

## Warm-up
## Hanging Around

10 min

### Activity Narrative

Students encounter and reason about a concrete situation, two clothes hangers with equal and unequal weights on each side. Students then see diagrams representing balanced and unbalanced hangers and think about what must be true and false about the situations. In subsequent activities, students will use the hanger diagrams to develop general strategies for solving equations.

If possible and if time allows, demonstrate the balancing concept with a real clothes hanger, clothespins, socks, and different weights, as shown in this *Warm-up* image. Or provide those materials for groups of students to experiment as they work through the activities in this lesson.

### Launch

Tell students to close their books or devices (or to keep them closed). Display the photo of the socks for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses without editing or commentary. If possible, record the relevant reasoning on or near the photo.

Things students may notice:

- There are two pink socks and two blue socks.
- The socks are clipped to either ends of two clothes hangers. The hangers are hanging from a rod.
- The hanger holding the pink socks is level, but the hanger holding the blue socks is not level.

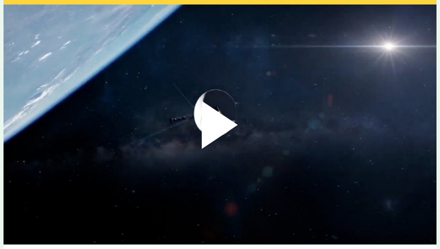
Things students may wonder:

- Why is the hanger holding the blue socks not level?
- Is something inside one of the blue socks to make it heavier than the other sock?
- What does this picture have to do with math?

Use the word "balanced" to describe the hanger on the left and "unbalanced" to describe the hanger on the right. Tell students that the hanger on the left is balanced because the two pink socks have an equal weight, and the hanger on the right is unbalanced because one blue sock has something in it that makes it heavier than the other blue sock.

**Inspire Math**

**Voyager video**



**Go Online**

Before the lesson, show this video to introduce the real-world connection.

**ilclass.com/l/614162**

Please log in to the site before using the QR code or URL.

**Instructional Routines**

**Notice and Wonder**

**ilclass.com/r/10694948**

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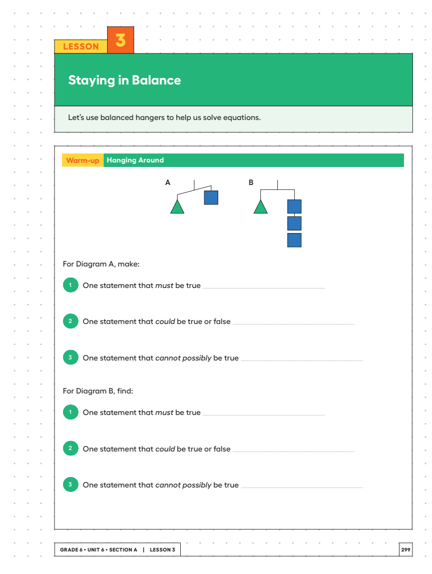
**Student Workbook**


LESSON 3

Staying in Balance

Let's use balanced hangers to help us solve equations.

Warm-up Hanging Around

For Diagram A, make:

1 One statement that *must* be true

2 One statement that *could* be true or false

3 One statement that *cannot possibly* be true

For Diagram B, find:

1 One statement that *must* be true

2 One statement that *could* be true or false

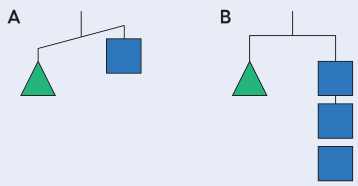
3 One statement that *cannot possibly* be true

GRADE 6 • UNIT 6 • SECTION A | LESSON 3

Tell students to open their books or devices and look at the two diagrams with shapes. Point out that the diagrams are like the clothes hangers in the photo except that they have shapes instead of socks. Explain that students will now reason about the weights of the shapes just like they reasoned about the weights of the socks.

Give students 3 minutes of quiet work time followed by whole-class discussion.

### Student Task Statement




For Diagram A, make:

1. One statement that *must* be true

   Sample response: The triangle is heavier than the square.

2. One statement that *could* be true or false

   Sample response: The triangle could weigh 10 ounces and the square could weigh 6 ounces.

3. One statement that *cannot possibly* be true

   Sample response: The square and the triangle weigh the same.

For Diagram B, find:

1. One statement that *must* be true

   Sample response: One triangle weighs the same as three squares.

2. One statement that *could* be true or false

   Sample response: The triangle weighs three pounds and each square weighs one pound.

3. One statement that *cannot possibly* be true

   Sample response: One square is heavier than one triangle.

### Activity Synthesis

Ask students to share some things that must be true, could be true, and cannot possibly be true about each diagram. Ask students to explain their reasoning.

The purpose of this discussion is to help students understand how the hanger diagrams work.

- When the diagram is balanced, or level, there is equal weight on each side. For example, since Diagram B is balanced, we know that one triangle weighs the same as three squares.
- When the diagram is unbalanced, one side is heavier than the other, making that side lower. For example, since Diagram A is unbalanced, and the side with a triangle is lower, so we know that one triangle is heavier than one square.

## Activity 1
## Match Equations and Hangers

**10 min**

### Activity Narrative

Students are presented with three hanger diagrams and are asked to match an equation to each diagram. They analyze relationships and find correspondences between the two representations (diagrams and equations), and use them to find the unknown value in each diagram. This value is a solution of the equation.

### Launch

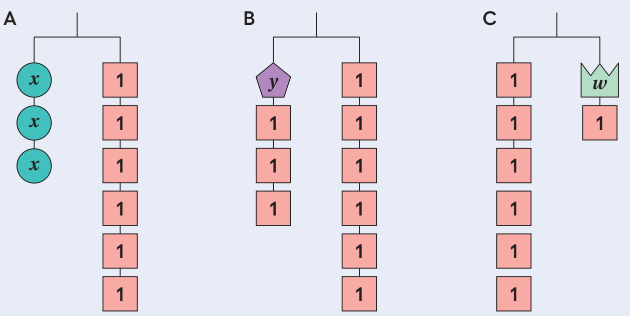
Display the diagrams for all to see. Explain that each square labeled with a "1" weighs 1 unit, and each shape labeled with a letter has an unknown weight in the same unit.

Arrange students in groups of 2.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language that students use to talk about hanger diagrams and equations. Display words and phrases, such as "level," "equal," "the same as," "balanced," "tilted," "more than," "less than," "unbalanced," and "grouping."

### Student Task Statement




1. Match each hanger diagram to an equation. Complete the equation by writing $x$, $y$, or $w$ in the empty box.

   B $\boxed{y} + 3 = 6$ A $3 \cdot \boxed{x} = 6$ C $6 = \boxed{w} + 1$

2. Find a solution to each equation. Use the diagrams to explain what each solution means.

   A: $x$ is 2

   Sample reasoning: Each circle weighs the same as 2 squares.

   B: $y$ is 3

   Sample reasoning: The pentagon weighs as much as 3 squares.

   C: $w$ is 5

   Sample reasoning: The $w$ shape weighs as much as 5 squares.

**Access for Multilingual Learners (Activity 1)**

**MLR2: Collect and Display**
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

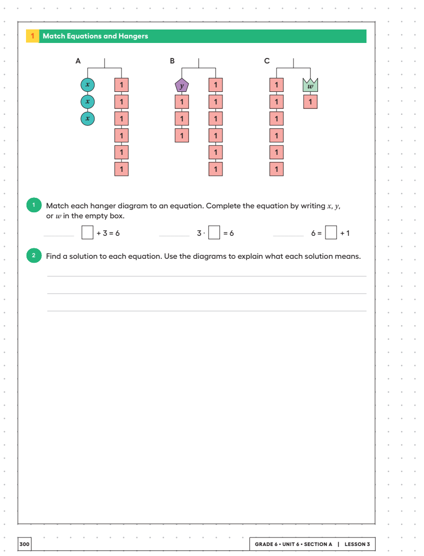
**Instructional Routines**

**MLR2: Collect and Display**

**ilclass.com/r/10690754**

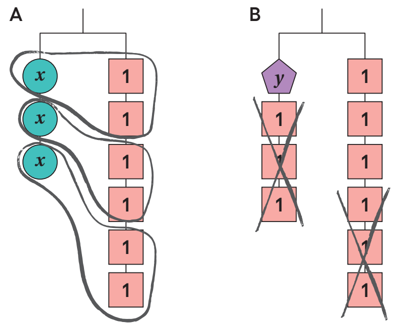
Please log in to the site before using the QR code or URL.

**Student Workbook**



## Activity Synthesis

Draw on the diagrams to help students reason about the "weights" of the shapes. In Diagram A, circle groups of shapes on the two sides that show equal weights. In Diagram B, remove shapes on either side that show equal weights. When you are done drawing, your diagrams might look like this:




Emphasize that, in each case, the result shows the weight of one lettered shape, which represents the value of the variable in the matching equation. In Diagram A, we see that each $x$ weighs the same as 2 squares. So, the solution to the matching equation $3 \cdot x = 6$ is 2. Verify that 2 is the solution by substituting 2 for $x$ in the diagram and equation.

Repeat for Diagram B showing $y$ is equal to 3. Consider asking the following questions:

*"Why does the equation $3 \cdot x = 6$ represent Diagram A?"*

*"How does the grouping we drew on Diagram A show the solution to that equation?"*

*"For Diagram B, how is crossing off 3 squares on each side of the diagram like subtracting 3 from each side of its matching equation?"*

*"What are some moves that ensure that a balanced hanger diagram stays balanced?"*

## Activity 2

## Connecting Diagrams to Equations and Solutions

15 min

## Activity Narrative

This activity continues the work of using hanger diagrams to develop strategies for solving equations. Students write equations that represent given diagrams explain how to use the diagrams and equations to reason about a solution. As students complete this activity, they notice correspondences in the structure of equations and diagrams and in the solution strategies.

## Launch

Draw students' attention to the diagrams in the *Task Statement*. Ensure they notice that the hangers are balanced and that each object is labeled with its weight in the same units. Some weights are labeled with numbers, but some are unknown, so they are labeled with variables.

Keep students in the same groups.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Some good free resources for using or creating hanger diagrams are available on the Internet. Consider searching "algebra mobile puzzles" for good examples and, if time permits, encouraging students to use them to practice reasoning about unknown values and solving equations.

### Student Task Statement

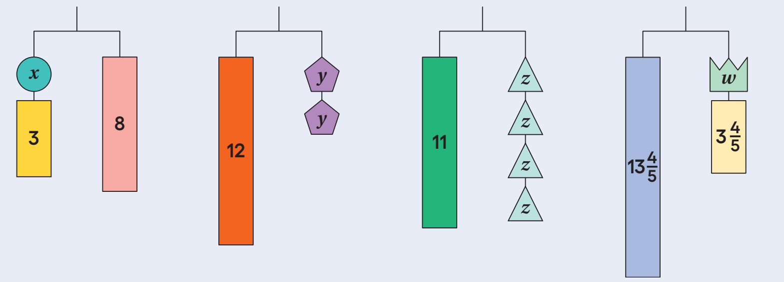
Here are some balanced hanger diagrams. Each piece is labeled with its weight in the same units.

A $x + 3 = 8$

B $12 = 2y$

C $10 = 4z$

D $13\frac{4}{5} = w + 3\frac{4}{5}$




For each diagram:

- Write an equation to represent the relationship between the weights.
- Explain how to reason with the diagram or the equation to find the value of the variable.

Sample response:

Diagram A:

Together $x$ and 3 weigh 8, so $x$ weighs 5.

Diagram B:

12 is twice the weight of $y$, so $y$ weighs half of 12.

Diagram C:

Dividing each side of the equation by 4 leaves $\frac{10}{4} = z$.

Diagram D:

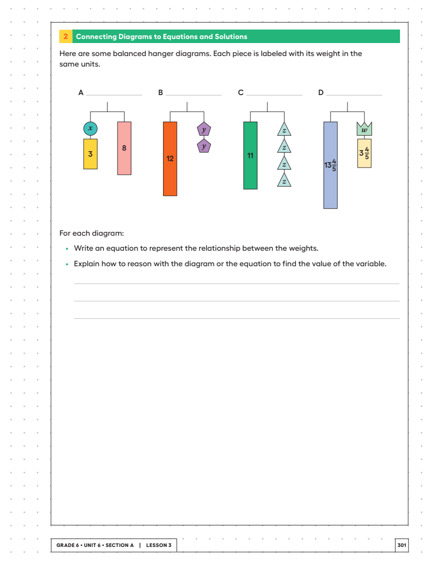
Subtracting $3\frac{4}{5}$ from each side of the equation leaves $10 = w$.

**Access for Students With Diverse Abilities (Activity 2, Student Task)**

**Representation: Internalize Comprehension.**
Encourage students to use color coding and annotations to highlight connections between representations, as done in a previous *Activity Synthesis*.

*Supports accessibility for: Visual-Spatial Processing*

**Student Workbook**



**Access for Multilingual Learners (Activity 2, Synthesis)**

**MLR7: Compare and Connect.** Lead a discussion comparing, contrasting, and connecting the different representations students have used so far in this unit. Ask,

*"How are hanger diagrams and equations similar? How are they different?"*
*"How does an unknown value show up in each representation?"*
*"Which do you like better for representing relationships: tape diagrams, hanger diagrams, or equations?"*
*"Are there any benefits or drawbacks to one representation compared to another?"*
*"How do these different representations show the same information?"*

*Advances: Representing, Conversing*

## Activity Synthesis

Draw Diagram A next to the equation $x + 3 = 8$. If needed, show dividing the "8" piece into 8 equal parts and then cross out 3 of them. Under the equation, write $x + 3 - 3 = 8 - 3$, followed by $x = 5$. Ask:

*"Why does the diagram stay balanced when we remove 3 from both sides?"*

The weight of both sides are still equal.

*"Why does the equation stay balanced when we subtract 3 from both sides?"*

The value of both sides are still equal.

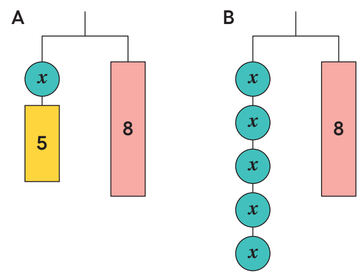
*"Is a balanced equation true? How do you know?"*

Yes. An equation is balanced when both sides have the same value. This is also what it means for an equation to be true.

Invite students to demonstrate, side by side, how they reasoned with both the diagram and the equation for Diagrams B, C, and D. For the diagrams represented by a multiplication equation, prompt students to show dividing each side into the same number of equal-size groups.

## Lesson Synthesis

Display these two balanced hanger diagrams side by side. Ask students to identify how the diagrams are alike and how they are different.




Write the matching equation below each diagram: $5x = 8$ and $5 + x = 8$. Ask students how the equations are alike and how they are different. Then write the solution below each equation: $x = 3$ and $x = \frac{8}{5}$. Ask:

*"When is the equation $5 + x = 8$ true?"*

when $x$ is 3

*"Since Diagram A is balanced, how much must this piece with an $x$ weigh?"*

3 units

*"When is the equation $5x = 8$ true?"*

when $x$ is $\frac{8}{5}$

*"Since Diagram B is balanced, how much must this piece with an $x$ weigh?"*

$\frac{8}{5}$ units

*"What do a balanced hanger diagram and a true equation have in common?"*

They have two sides with the same value.

## Lesson Summary

A hanger stays balanced when the weights on both sides are equal. We can change the weights and the hanger will stay balanced as long as both sides are changed in the same way. For example, adding 2 pounds to each side of a balanced hanger will keep it balanced. Removing half of the weight from each side will also keep it balanced.

An equation can be compared to a balanced hanger. We can change the equation, but for a true equation to remain true, the same thing must be done to both sides of the equal sign. If we add or subtract the same number on each side, or multiply or divide each side by the same number, the new equation will still be true.

This way of thinking can help us find solutions to equations. Instead of checking different values for the variable, we can think about subtracting the same amount from each side or dividing each side by the same number.

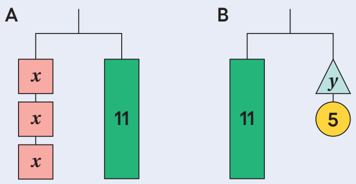



Diagram A can be represented by the equation $3x = 11$.

If we break the 11 into 3 equal parts, each part will have the same weight as 1 block with an $x$.

Splitting each side of the diagram into 3 equal parts is the same as dividing each side of the equation by 3.

- $3x$ divided by 3 is $x$.
- 11 divided by 3 is $\frac{11}{3}$.
- If $3x = 11$ is true, then $x = \frac{11}{3}$ is true.
- The solution to $3x = 11$ is $\frac{11}{3}$.

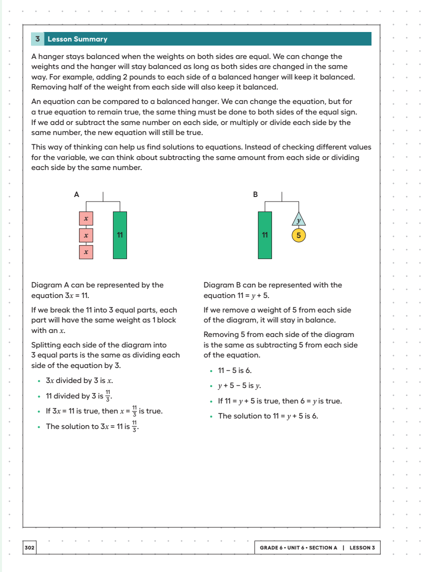
Diagram B can be represented with the equation $11 = y + 5$.

If we remove a weight of 5 from each side of the diagram, it will stay in balance.

Removing 5 from each side of the diagram is the same as subtracting 5 from each side of the equation.

- $11 - 5$ is 6.
- $y + 5 - 5$ is $y$.
- If $11 = y + 5$ is true, then $6 = y$ is true.
- The solution to $11 = y + 5$ is 6.

**Student Workbook**



**Responding To Student Thinking**

**More Chances**
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

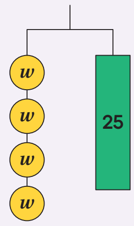
**Cool-down**

## Weight of the Circle

5 min

### Student Task Statement

Here is a balanced hanger diagram.




1. Write an equation that represents the diagram.

   $4w = 25$

2. Find the weight of one circle. Explain or show your reasoning.

   $\frac{25}{4}$ or $6\frac{1}{4}$ units

   Sample reasoning: The left side of the diagram has 4 circles, so I divided the right side into 4 equal pieces. Each of those pieces weighs $6\frac{1}{4}$ units. This shows that each circle piece weighs $6\frac{1}{4}$ units.

3. What is the solution to your equation?

   $w = \frac{25}{4}$ or $6\frac{1}{4}$

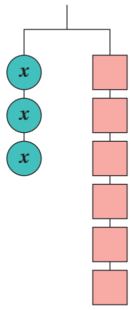
# Lesson 3

## Practice Problems

7 Problems

### Problem 1

Select **all** the equations that represent the hanger diagram.




**A.** $x + x + x = 1 + 1 + 1 + 1 + 1 + 1$

**B.** $x \cdot x \cdot x = 6$

**C.** $3x = 6$

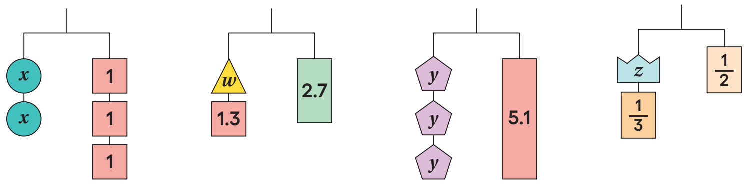
**D.** $x + 3 = 6$

**E.** $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

### Problem 2

Write an equation to represent each hanger diagram.

A ____________ B ____________ C ____________ D ____________



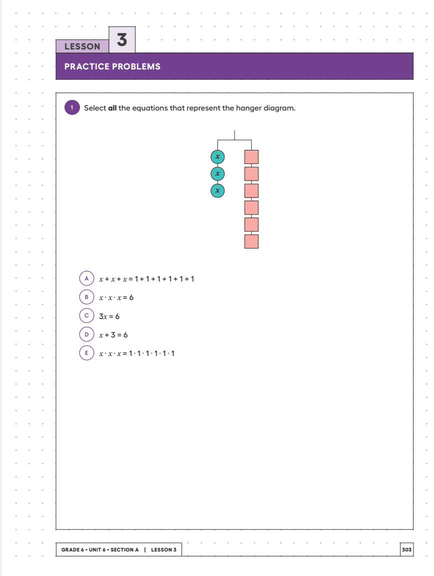

A. $2x = 3$ (or equivalent)

B. $w + 1.3 = 2.7$ (or equivalent)

C. $3y = 5.1$ (or equivalent)

D. $z + \frac{1}{3} = \frac{1}{2}$ (or equivalent)

**Student Workbook**



LESSON 3

PRACTICE PRODUCTS

1 Select **all** the equations that represent the hanger diagram.

A $x+x+x=1+1+1+1+1+1$

B $x \cdot x \cdot x = 6$

C $3x = 6$

D $x + 3 = 6$

E $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

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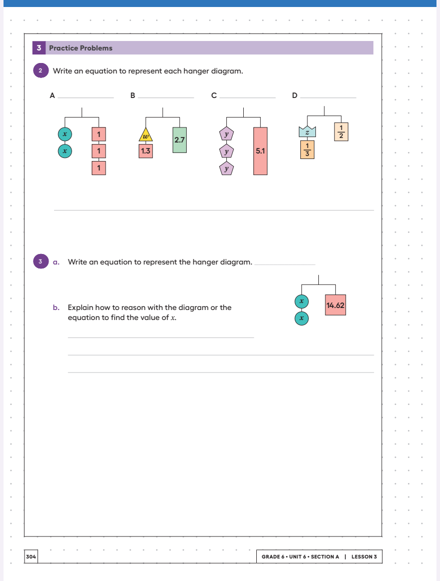
**Student Workbook**



3 Practice Problems

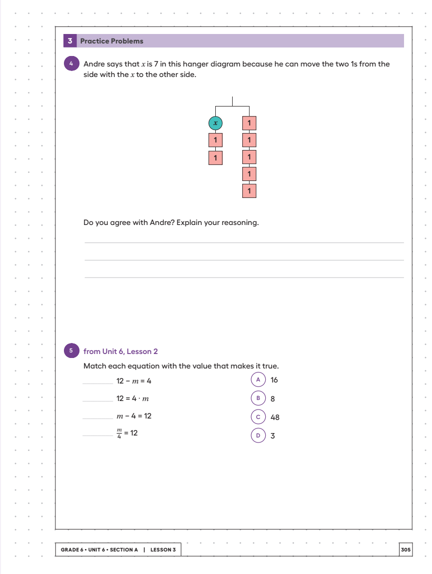
2 Write an equation to represent each hanger diagram.

3 a. Write an equation to represent the hanger diagram.

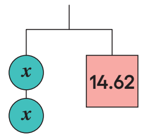
b. Explain how to reason with the diagram or the equation to find the value of $x$.

GRADE 6 • UNIT 6 • SECTION A | LESSON 3

**Student Workbook**




## Problem 3




**a.** Write an equation to represent the hanger diagram.

$2x = 14.62$ (or equivalent)

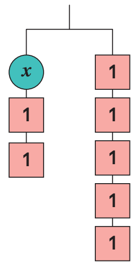
**b.** Explain how to reason with the diagram or the equation to find the value of $x$.

Sample responses:

- Each $x$ can be grouped with half of the other side, so that means $x$ is half of 14.62, or 7.31.
- 14.62 is twice $x$, so $x$ must be 7.31, since $2 \cdot (7.31) = 14.62$.

## Problem 4

Andre says that $x$ is 7 in this hanger diagram because he can move the two 1s from the side with the $x$ to the other side.




Do you agree with Andre? Explain your reasoning.

Andre is not correct.

Sample reasoning: Each 1 on the left balances with a 1 on the right. So taking away the two 1s from the left side only makes the diagram unbalanced. To balance it, we also need to take away two 1s from the right side. This leaves $x$ alone on the left side and three 1s on the right side, so $x = 3$.

## Problem 5

from Unit 6, Lesson 2

Match each equation with the value that makes it true.

| | |
|---|---|
| B $12 - m = 4$ | **A.** 16 |
| D $12 = 4 \cdot m$ | **B.** 8 |
| A $m - 4 = 12$ | **C.** 48 |
| C $\frac{m}{4} = 12$ | **D.** 3 |

## Problem 6

from Unit 4, Lesson 13

The area of a rectangle is 14 square units. It has side lengths $x$ and $y$. Given each value for $x$, find the value for $y$.

**a.** $x = 2\frac{1}{3}$

$y = 6$ $(14 \div 2\frac{1}{3} = 14 \div \frac{7}{3}$, and $14 \cdot \frac{3}{7} = 6)$

**b.** $x = 4\frac{1}{5}$

$y = 3\frac{1}{3}$ $(14 \div 4\frac{1}{5} = 14 \div \frac{21}{5}$, and $14 \cdot \frac{5}{21} = 3\frac{1}{3})$

**c.** $x = \frac{7}{6}$

$y = 12$ $(14 \div \frac{7}{6} = 14 \cdot \frac{6}{7} = 12)$

## Problem 7

from Unit 3, Lesson 11

Lin needs to save $20 for a new game. How much money does she have if she has saved each percentage of her goal. Explain or show your reasoning.

**a.** 25%

$5

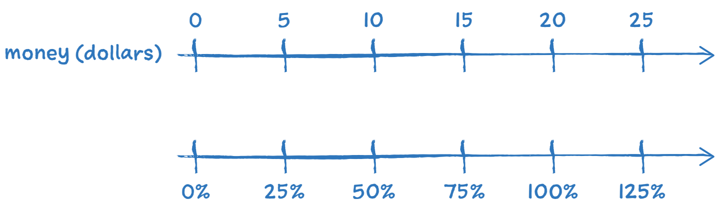
**b.** 75%

$15

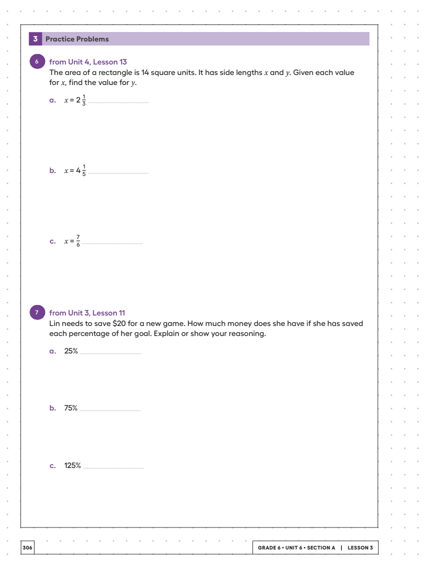
**c.** 125%

$25.

Sample reasoning:




**Student Workbook**


3 Practice Problems

from Unit 4, Lesson 13

The area of a rectangle is 14 square units. It has side lengths $x$ and $y$. Given each value for $x$, find the value for $y$.

a. $x = 2\frac{1}{3}$

b. $x = 4\frac{1}{5}$

c. $x = \frac{7}{6}$

from Unit 3, Lesson 11

Lin needs to save $20 for a new game. How much money does she have if she has saved each percentage of her goal. Explain or show your reasoning.

a. 25%

b. 75%

c. 125%