Lesson 11

Variability and MAD

Goals

- Calculate the mean absolute deviation (MAD) for a data set and interpret what it tells us about the situation.
- Compare and contrast (in writing) distributions that have the same mean, but different amounts of variability.
- Comprehend that "mean absolute deviation (MAD)" is a measure of variability, that is, a single number summarizing how spread out the data set is.

Learning Targets

- I can find the MAD for a set of data.
- I know what the mean absolute deviation (MAD) measures and what information it provides.

Lesson Narrative

In this lesson, students make sense of the **mean absolute deviation (MAD).** Students learn that the MAD is the average distance of data points from the mean. They use their knowledge of how to calculate and interpret the mean to calculate and interpret the MAD.

Students also learn that we think of the MAD as a measure of variability or a measure of spread of a distribution. They compare distributions with the same mean but different MADs, and recognize that the centers are the same but the distribution with the larger MAD has greater variability or spread.

Student Learning Goal

Let's study distances between data points and the mean and see what they tell us.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2, Activity 3)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Required Materials

Materials to Gather

- Decks of playing cards: Activity 3

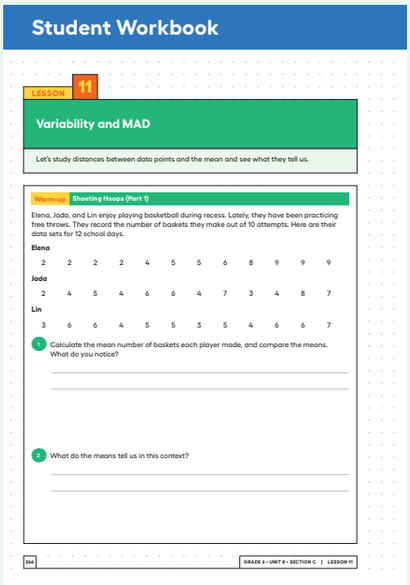
Required Preparation

Lesson:

If students are playing the optional Game of 22, prepare one standard deck of 52 playing cards for every 2–3 students.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	15 min	15 min	15 min	10 min	5 min
					Cool-down


Student Workbook

LESSON 11

Variability and MAD

Let's study distances between data points and the mean and see what they tell us.

Warm-up Shooting Hoops (Part 1)

Elena, Jada, and Lin enjoy playing basketball during recess. Lately, they have been practicing free throws. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena
2 2 2 2 4 5 5 6 8 9 9 9

Jada
2 4 5 4 6 6 4 7 3 4 8 7

Lin
3 6 6 4 5 5 3 5 4 6 6 7

1. Calculate the mean number of baskets each player made, and compare the means. What do you notice?

2. What do the means tell us in this context?

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Warm-up

Shooting Hoops (Part 1)

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to first reason about the mean of a data set without calculating and then to practice calculating the mean. The context will be used in an upcoming activity in this lesson, so this Warm-up familiarizes students with the context for talking about deviation from the mean.

In their predictions, students may think that Elena will have the highest mean, because she has a few very high scores (7, 8, and 9 points). They may also think that Lin and Jada will have very close means because they each have 5 higher scores than one another, and their other scores are the same. Even though each player has the same mean, all of these ideas are reasonable things for students to consider when looking at the data. Record and display their predictions without further questions until they have calculated and compared the mean of their individual data sets.

Launch

Arrange students in groups of 3.

Tell each group member to calculate the mean of the data set for one player in the task, share their work in the small group, and complete the remaining questions.

Student Task Statement

Elena, Jada, and Lin enjoy playing basketball during recess. Lately, they have been practicing free throws. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena

2 2 2 2 4 5 5 6 8 9 9 9

Jada

2 4 5 4 6 6 4 7 3 4 8 7

Lin

3 6 6 4 5 5 3 5 4 6 6 7

1. Calculate the mean number of baskets each player made, and compare the means. What do you notice?

Elena's mean score is $\frac{2+2+2+2+4+5+5+6+8+9+9+9}{12} = 5.25$.

Jada's mean score is $\frac{2+4+5+4+6+6+4+7+3+4+8+7}{12} = 5$.

Lin's mean score is $\frac{3+6+6+4+5+5+3+5+4+6+6+7}{12} = 5$.

I noticed that all three players have close to the same mean score, but Elena's is a little greater.

2. What do the means tell us in this context?

Sample explanation: The means show that all 3 students make, on average, about half of the 10 attempts to get the basketball in the hoop.

Activity Synthesis

Ask students to share the mean for each player's data set. Record and display their responses for all to see. After each student shares, ask the class if they agree or disagree and what the mean tells us in this context. If the idea that the means show that all three students make, on average, half of the 10 attempts to get the basketball in the hoop does not arise, make that idea explicit.

If there is time, consider revisiting the predictions and asking how the mean of Elena's data set can be the same as the others when she has more high scores?

Activity 1
Shooting Hoops (Part 2)

15 min

Activity Narrative

In this activity, students turn their focus to variability while continuing to develop their understanding of what could be considered typical for a group. Students compare distributions with the same mean but different spreads and interpret them in the context of a situation. The context given here (basketball scores) prompts them to connect the mean to the notion of how well a player plays in general, and deviations from the mean to how "consistently" that player plays.

They encounter the idea of calculating the average absolute deviation from the mean as a way to describe variability in data.

Launch

Arrange students in groups of 3–4. Give groups 6–7 minutes to answer the questions, and follow with a whole-class discussion.

Student Task Statement

Here are the dot plots showing the number of baskets that Elena, Jada, and Lin each made over 12 school days.

1. On each dot plot, mark the location of the mean with a triangle. Then, contrast the dot plot distributions. Write 2–3 sentences to describe the shape and spread of each distribution.

number of baskets Elena made

number of baskets Jada made

number of baskets Lin made

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Invite students to verbalize their strategy for calculating the mean from a dot plot before they begin. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Access for Multilingual Learners (Activity 1, Student Task)

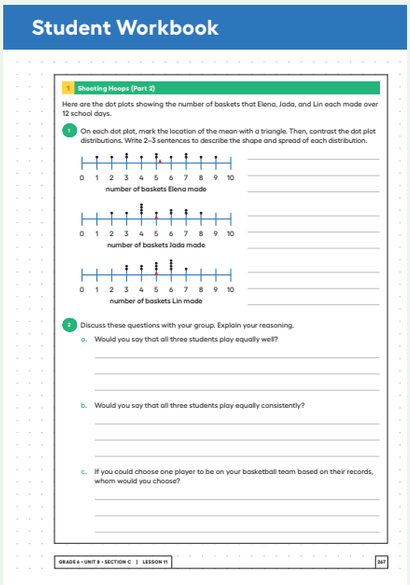
MLR2: Collect and Display.
Collect the language that students use to describe how well the players play. Display words and phrases such as "consistent," "typical," and "most." During the synthesis, invite students to suggest ways to update the display:

"What are some other words or phrases we should include?"

Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student Workbook



1 Shooting Hoops (Part 2)

Here are the dot plots showing the number of baskets that Elena, Jada, and Lin each made over 12 school days.

1. On each dot plot, mark the location of the mean with a triangle. Then, contrast the dot plot distributions. Write 2–3 sentences to describe the shape and spread of each distribution.

0 1 2 3 4 5 6 7 8 9 10
number of baskets Elena made

0 1 2 3 4 5 6 7 8 9 10
number of baskets Jada made

0 1 2 3 4 5 6 7 8 9 10
number of baskets Lin made

2. Discuss these questions with your group. Explain your reasoning.
   a. Would you say that all three students play equally well?
   b. Would you say that all three students play equally consistently?
   c. If you could choose one player to be on your basketball team based on their records, whom would you choose?

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Elena's mean is about 5.25. Jada and Lin's mean are 5.

Sample response: The distributions for all players are centered close to 5 with Elena's mean slightly greater. Elena's distribution is approximately symmetric and has a large spread. Jada's distribution is not symmetric, but has a lower spread than Elena's. Lin's distribution has the narrowest spread.

2. Discuss these questions with your group. Explain your reasoning.

a. Would you say that all three students play equally well?

Sample response: Elena typically makes slightly more baskets, but has a lot of really good games and a lot of not so good games. The other 2 players typically make the same number of baskets, so they play equally well.

b. Would you say that all three students play equally consistently?

Sample response: Lin is the most consistent because she always makes between 3 and 7 baskets and most often makes around 5 or 6. Elena is the least consistent of this group.

c. If you could choose one player to be on your basketball team based on their records, whom would you choose?

Sample response: I would choose Jada. She is more consistent than Elena and sometimes has a really good game with 7 or 8 baskets made.

Activity Synthesis

The purpose of the discussion is to highlight that the center of the distribution is not always the only consideration when discussing data. The variability or spread can also influence how we understand the data.

There are many ways to answer the second set of questions. Invite students or groups who have different interpretations of playing well and playing consistently to share their thinking. Allow as many interpretations to be shared as time permits. Discuss:

- *"How might we use the given data to quantify playing well and playing consistently?"*

  The mean can represent how many baskets a player typically makes. The greater the mean, the better a player typically plays. The spread or variability indicates how consistent a player is. The greater the spread, the less consistent a player is.

- *"There are many considerations when selecting a teammate. What is not captured by the data here? How do data help with the decision?"*

  Position on the team, how they get along with the other players and coach, and other skills like assists or rebounds are important for basketball. If the other aspects are the same or similar, then these data can help make a choice that is not based only on something like who I like.

Activity 2
Shooting Hoops (Part 3)

15 min

Activity Narrative

In this activity, students learn the term **mean absolute deviation (MAD)** as a way to quantify variability and calculate it by finding distances between the mean and each data value. Students compare data sets with the same mean but different MADs and interpret the variability in context.

While this process of calculating MAD involves taking the absolute value of the difference between each data point and the mean, this formal language is downplayed here. Instead, the idea of "finding the distance," which is always positive, is used. This is done for a couple of reasons. One reason is to focus students' attention on the statistical work rather than on terminology or symbolic work. Another reason is that finding these differences may involve operations with signed numbers, which are not expected in this course.

Launch

Remind students that earlier they found the distance between each data point and the mean, and found that the sum of those distances on the left and the sum on the right are equal, which allows us to think of the mean as the balancing point, or the center, of the data. Explain that the distance between each point and the mean can be used to tell us something else about a distribution.

Arrange students in groups of 2. Give students 4–5 minutes to complete the first set of questions with their partner, and then 4–5 minutes of quiet time to complete the remaining questions. Follow with a whole-class discussion.

Student Task Statement

The tables show the number of baskets made by Jada and Lin in several games. Recall that the mean of Jada and Lin's data is 5.

1. Record the distance between the number of baskets Jada made in each game and the mean.

Jada	2	4	5	4	6	6	4	7	3	4	8	7
distance from 5	3	1	0	1	1	1	1	2	2	1	3	2

Now find the average of the distances in the table. Show your reasoning, and round your answer to the nearest tenth.
This value is the mean absolute deviation (MAD) of Jada's data.

Jada's MAD: $\frac{15.3+1+0+1+1+1+1+2+2+1+3+2}{12} = 1.5$

2. Find the mean absolute deviation of Lin's data. Round it to the nearest tenth.

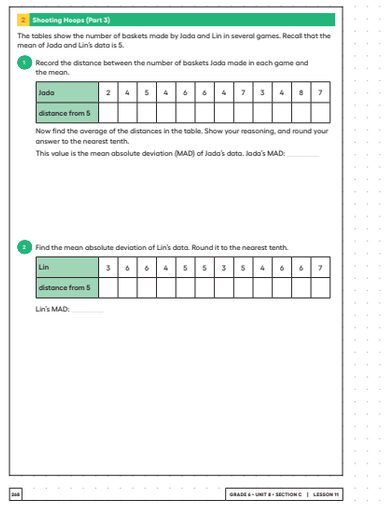
Lin	3	6	6	4	5	5	3	5	4	6	6	7
distance from 5	2	1	1	1	0	0	2	0	1	1	1	2

Lin's MAD: $\frac{1.2+1+1+1+0+0+2+0+1+1+1+2}{12} = 1$

Building on Student Thinking

Students may recall an earlier lesson about thinking of the mean as a balance point and think that the MAD should always be zero because the left and right distances should be equal. Remind them that distances are always positive, so the average of these distances to the mean can be zero only if all the data points are exactly at the mean.

Student Workbook


2 Shooting Hoops (Part 3)

The tables show the number of baskets made by Jada and Lin in several games. Recall that the mean of Jada and Lin's data is 5.

1 Record the distance between the number of baskets Jada made in each game and the mean.

Jada	2	4	5	4	6	6	4	7	3	4	8	7
distance from 5												

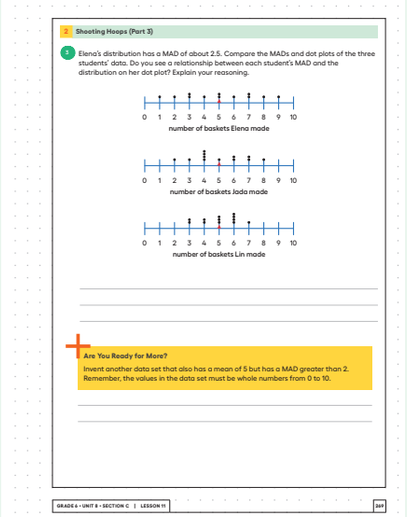
Now find the average of the distances in the table. Show your reasoning, and round your answer to the nearest tenth.
This value is the mean absolute deviation (MAD) of Jada's data. Jada's MAD: _____

2 Find the mean absolute deviation of Lin's data. Round it to the nearest tenth.

Lin	3	6	6	4	5	5	3	5	4	6	6	7
distance from 5												

Lin's MAD: _____

Student Workbook


2 Shooting Hoops (Part 3)

3 Elena's distribution has a MAD of about 2.5. Compare the MADs and dot plots of the three students' data. Do you see a relationship between each student's MAD and the distribution on her dot plot? Explain your reasoning.

0 1 2 3 4 5 6 7 8 9 10
number of baskets Elena made

0 1 2 3 4 5 6 7 8 9 10
number of baskets Jada made

0 1 2 3 4 5 6 7 8 9 10
number of baskets Lin made

Are You Ready for More?

Invent another data set that also has a mean of 5 but has a MAD greater than 2. Remember, the values in the data set must be whole numbers from 0 to 10.

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Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Develop Language and Symbols.
Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of variability. Terms may include "mean absolute deviation" (MAD).

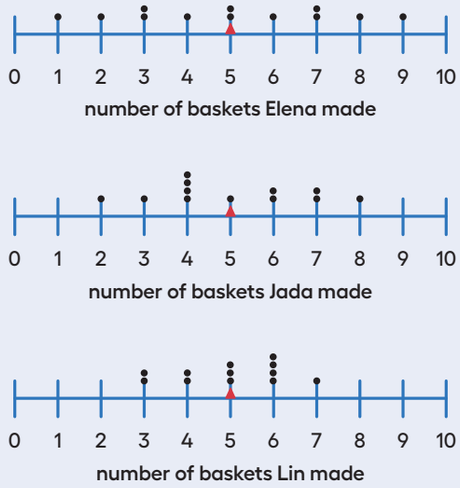
Supports accessibility for: Conceptual Processing, Language

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display a sentence frame to support a whole-class discussion: "The largest (or smallest) mean absolute deviation value matches with the dot plot with the _____ spread because ..."

Advances: Conversing, Speaking

3. Elena's distribution has a MAD of about 2.5. Compare the MADs and dot plots of the three students' data. Do you see a relationship between each student's MAD and the distribution on her dot plot? Explain your reasoning.



Sample response: Yes, I see a relationship between the MAD and the distribution of data. The largest MAD value corresponds to the dot plot with the widest spread. The smallest MAD value corresponds to the dot plot with the narrowest spread.

Are You Ready for More?

Invent another data set that also has a mean of 5 but has a MAD greater than 2. Remember, the values in the data set must be whole numbers from 0 to 10.

Sample response: 0, 0, 0, 0, 0, 0, 10, 10, 10, 10, 10, 10

Activity Synthesis

During discussion, highlight that finding how far away, on average, the data points are from the mean is a way to describe the variability of a distribution. Discuss:

- *"What can we say about a data set whose data points have very small distances from the mean?"*

  That means most of the points are near the mean, so there is not much spread.

- *"What about a data set with points that show large distances from the mean?"*

  That means most of the points are not close to the mean, so there is a large spread.

- *"Does a data set with smaller distances (and therefore smaller average distances) show less or more variability?"*

  There is less variability when the points have smaller distances from the mean.

Activity 3: Optional

Game of 22

15 min

Activity Narrative

This optional activity uses a game to help students develop the idea of variability. Use this activity if students could benefit from more concrete experiences with the idea of distance from the mean. Students will draw from a standard deck of playing cards and find the sum of the values of the drawn cards. They will determine how far their sum is from "22." After playing 5 rounds, the player with the least mean distance from 22 wins the game.

Launch

Have students work in groups of 2–3. Provide a deck of standard playing cards to each group. Play 5 rounds.

Student Task Statement

Your teacher will give your group a deck of cards. Shuffle the cards, and put the deck face down on the playing surface.

- To play: Draw 3 cards and add up the values. An ace is a 1. A jack, queen, and king are each worth 10. Cards 2–10 are each worth their face value. If your sum is anything other than 22 (either above or below 22), say: "My sum deviated from 22 by _____," or "My sum was off from 22 by _____."
- To keep score: Record each sum and each distance from 22 in the table. After five rounds, calculate the average of the distances. The player with the lowest average distance from 22 wins the game.

Playing results vary.

player A	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

player B	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

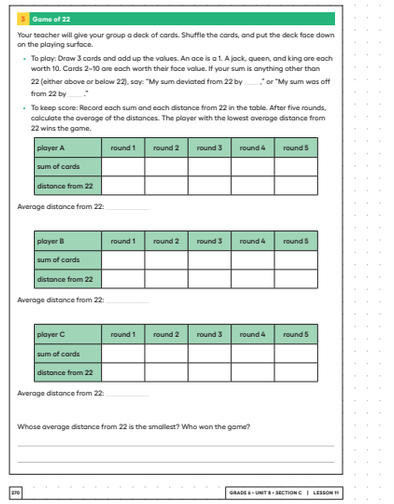
Average distance from 22: _____

Access for Students with Diverse Abilities (Activity 3, Student Task)

Representation: Access for Perception.
Begin by demonstrating the game for the whole class and doing the steps in the activity to support understanding of the context.

Supports accessibility for: Conceptual Processing, Language

Student Workbook



3 Game of 22

Your teacher will give your group a deck of cards. Shuffle the cards, and put the deck face down on the playing surface.

- To play: Draw 3 cards and add up the values. An ace is a 1. A jack, queen, and king are each worth 10. Cards 2–10 are each worth their face value. If your sum is anything other than 22 (either above or below 22), say: "My sum deviated from 22 by ____," or "My sum was off from 22 by ____."
- To keep score: Record each sum and each distance from 22 in the table. After five rounds, calculate the average of the distances. The player with the lowest average distance from 22 wins the game.

player A	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

player B	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

player C	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

Whose average distance from 22 is the smallest? Who won the game?

_____

_____

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player C	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

Whose average distance from 22 is the smallest? Who won the game?

Activity Synthesis

Ask students to think about how average distance from a number can be used to summarize variability, and invite a couple of students to share their thinking.

In the game, we can think of the player with the least average distance from 22 as having cards that are, on the whole, closest to 22 or the "least different" from 22. By the same token, a player with the greatest average distance from 22 can be seen as having cards that are, on the whole, farthest away from 22 or the "most different" from 22. Connect this to the idea that a data set with a large MAD means it has many values that vary from what we could consider a typical member of the group.

Lesson Synthesis

The purpose of the discussion is to understand how a measure of spread can be used to quantify variability for a distribution. Ask students:

- *"How did we measure spread?"*

  By finding the average of how far each value is from the balance point.

- *"We learned about a measure called the 'mean absolute deviation' (or MAD). What is the meaning of this term?"*

  The typical distance from each data value to the mean.

- *"What does the MAD tell us?"*

  The average distance between data points and the mean. It tells us how spread out the data values are.

- *"How do we find the MAD?"*

  Find the distance from each point to the mean. Then, find the mean of those distances.

- *"How is MAD related to the variability of a data set?"*

  The greater the MAD, the more variability in the data.

Access for Multilingual Learners (Activity 3, Synthesis)

Writing, Conversing: MLR1: Stronger and Clearer Each Time.
Use this routine with 2–3 successive pair shares to give students a structured opportunity to revise and refine their response to "How can the average distance from a number to the mean be used to summarize variability?" Provide students with prompts for feedback (for example,

"Could you give a specific example from a round of 'Game of 22'?"

or,

"Could you use the term 'mean absolute deviation' to explain your example further?").

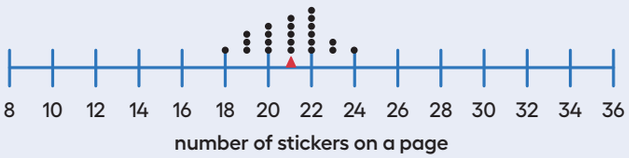
Students can borrow ideas and language from each other to strengthen their final product. This will help students strengthen their ideas and clarify their language.

Design Principle(s): Optimize output (for explanation); Cultivate conversation

Lesson Summary

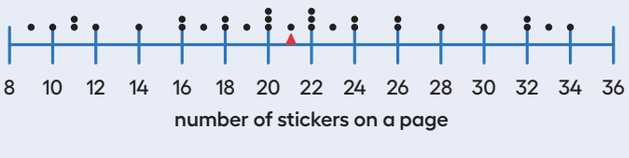
We use the mean of a data set as a 'measure of center' of its distribution, but two data sets with the same mean could have very different distributions.

This dot plot shows the number of stickers on 30 pages.



The mean number of stickers is 21. All the pages have within 3 stickers of the mean, and most of them are even closer. These pages are all fairly close in the number of stickers on them.

This dot plot shows the number of stickers on a different group of 30 pages.

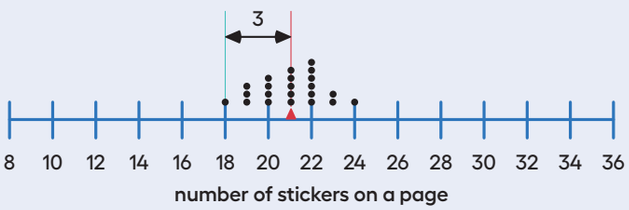


The mean number of stickers for this set of pages is also 21, but some pages have half that number of stickers and others have one-and-a-half times as many. There is a lot more variability in the number of stickers.

There is a number that we can use to describe how far away, or how spread out, data points generally are from the mean. This measure of spread is called the **mean absolute deviation (MAD).**

In this distribution, the MAD is 3. That tells us that the number of stickers on a typical page is 3 numbers away from the mean, which is 21. To find the MAD, we find the distance between each data value and the mean, and then calculate the mean of those distances.

For instance, the point that represents 18 stickers is 3 units away from the mean of 21 stickers. We can find the distance between each point and the mean of 21 stickers and then organize the distances into a table, as shown.

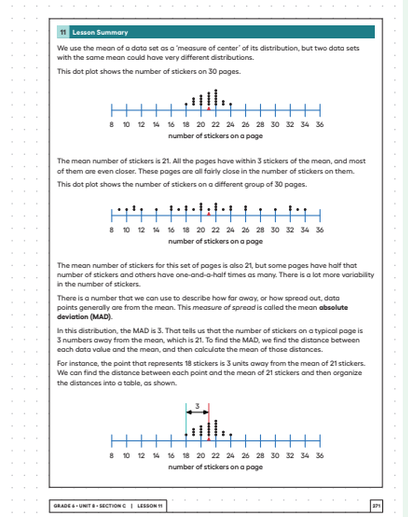


number of stickers	18	19	19	19	20	20	20	20	21	21	21	21	21	22	22	22	22	22	22	23	23	24
distance from mean	3	2	2	2	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	2	2	3

The values in the first row of the table are the number of stickers on a page for the first set of pages. Their mean, 21, is the mean number of stickers on a page.

The values in the second row of the table are the distances, or absolute deviation, between the values in the first row and 21. The mean of these distances is the MAD of the number of stickers on a page, about 1.2 stickers.

Student Workbook


11 Lesson Summary

We use the mean of a data set as a 'measure of center' of its distribution, but two data sets with the same mean could have very different distributions.

This dot plot shows the number of stickers on 30 pages.

8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
number of stickers on a page

The mean number of stickers is 21. All the pages have within 3 stickers of the mean, and most of them are even closer. These pages are all fairly close in the number of stickers on them.

This dot plot shows the number of stickers on a different group of 30 pages.

8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
number of stickers on a page

The mean number of stickers for this set of pages is also 21, but some pages have half that number of stickers and others have one-and-a-half times as many. There is a lot more variability in the number of stickers.

There is a number that we can use to describe how far away, or how spread out, data points generally are from the mean. This *measure of spread* is called the mean **absolute deviation (MAD)**.

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For instance, the point that represents 18 stickers is 3 units away from the mean of 21 stickers. We can find the distance between each point and the mean of 21 stickers and then organize the distances into a table, as shown.

3
8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
number of stickers on a page

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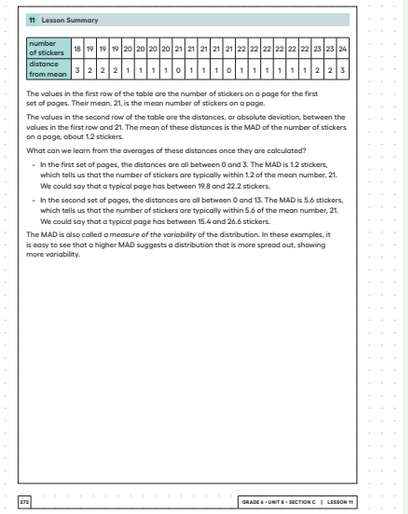
Student Workbook


11 Lesson Summary

number of stickers	18	19	19	19	20	20	20	20	21	21	21	21	21	22	22	22	22	22	22	23	23	24
distance from mean	3	2	2	2	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	2	2	3

The values in the first row of the table are the number of stickers on a page for the first set of pages. Their mean, 21, is the mean number of stickers on a page.

The values in the second row of the table are the distances, or absolute deviation, between the values in the first row and 21. The mean of these distances is the MAD of the number of stickers on a page, about 1.2 stickers.

What can we learn from the averages of these distances once they are calculated?

- In the first set of pages, the distances are all between 0 and 3. The MAD is 1.2 stickers, which tells us that the number of stickers are typically within 1.2 of the mean number, 21. We could say that a typical page has between 19.8 and 22.2 stickers.
- In the second set of pages, the distances are all between 0 and 13. The MAD is 5.6 stickers, which tells us that the number of stickers are typically within 5.6 of the mean number, 21. We could say that a typical page has between 15.4 and 26.6 stickers.

The MAD is also called a *measure of the variability* of the distribution. In these examples, it is easy to see that a higher MAD suggests a distribution that is more spread out, showing more variability.

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

What can we learn from the averages of these distances once they are calculated?

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The MAD is also called a *measure of the variability* of the distribution. In these examples, it is easy to see that a higher MAD suggests a distribution that is more spread out, showing more variability.

Cool-down

Text Messages, Again

5 min

Student Task Statement

These three data sets show the number of text messages sent to their parents by Jada, Diego, and Lin over 6 days as well as the mean number of text messages sent to their parents by each student per day.

Jada	4	4	4	6	6	6
mean: 5						
Diego	4	5	5	6	8	8
mean: 6						
Lin	1	1	2	2	9	9
mean: 4						

1. Predict which data set has the largest MAD and which has the smallest MAD.

   Lin's data set has the largest MAD, because the data have the most variability. Jada's data set has the smallest MAD, because the data have the least variability.

2. Compute the MAD for each data set to check your prediction.

   Jada's MAD is $\frac{1+1+1+1+1+1}{6} = 1$.

   Diego's MAD is $\frac{2+1+1+0+2+2}{6} = 1.33$.

   Lin's MAD is $\frac{3+3+2+2+5+5}{6} = 3.33$.

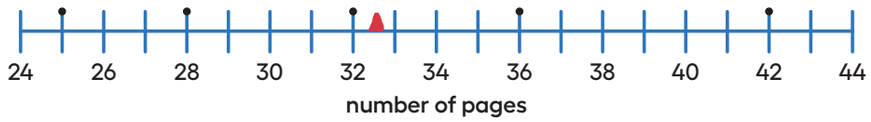
Lesson 11

Practice Problems

3 Problems

Problem 1

Han records the number of pages that he read each day for five days. The dot plot shows his data.



a. Is 30 pages a good estimate of the mean number of pages that Han read each day? Explain your reasoning.

The value of 30 is not a good estimate of the mean because it would not balance the other values.

b. Find the mean number of pages that Han read during the five days. Draw a triangle to mark the mean on the dot plot.

The mean is 32.6 pages.

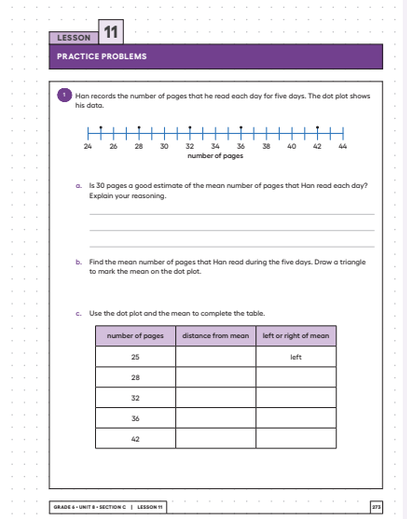
c. Use the dot plot and the mean to complete the table.

number of pages	distance from mean	left or right of mean
25	7.6	left
28	4.6	left
32	0.6	left
36	3.4	right
42	9.4	right

d. Calculate the mean absolute deviation (MAD) of the data. Explain or show your reasoning.

The MAD is 5.12 pages: $\frac{7.6 + 4.6 + 0.6 + 3.4 + 9.4}{5} = 5.12$

Student Workbook


LESSON 11

PRACTICE PROBLEMS

1. Han records the number of pages that he read each day for five days. The dot plot shows his data.

24 26 28 30 32 34 36 38 40 42 44

number of pages

a. Is 30 pages a good estimate of the mean number of pages that Han read each day? Explain your reasoning.

b. Find the mean number of pages that Han read during the five days. Draw a triangle to mark the mean on the dot plot.

c. Use the dot plot and the mean to complete the table.

number of pages	distance from mean	left or right of mean
25		left
28		
32		
36		
42		

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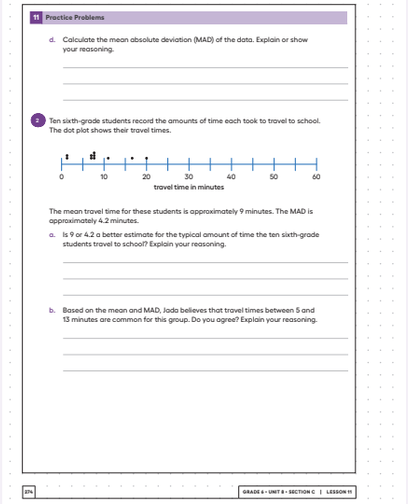
Student Workbook


11 Practice Problems

d. Calculate the mean absolute deviation (MAD) of the data. Explain or show your reasoning.

2 Ten sixth-grade students record the amounts of time each took to travel to school. The dot plot shows their travel times.

0 10 20 30 40 50 60
travel time in minutes

The mean travel time for these students is approximately 9 minutes. The MAD is approximately 4.2 minutes.

a. Is 9 or 4.2 a better estimate for the typical amount of time the ten sixth-grade students travel to school? Explain your reasoning.

b. Based on the mean and MAD, Jada believes that travel times between 5 and 13 minutes are common for this group. Do you agree? Explain your reasoning.

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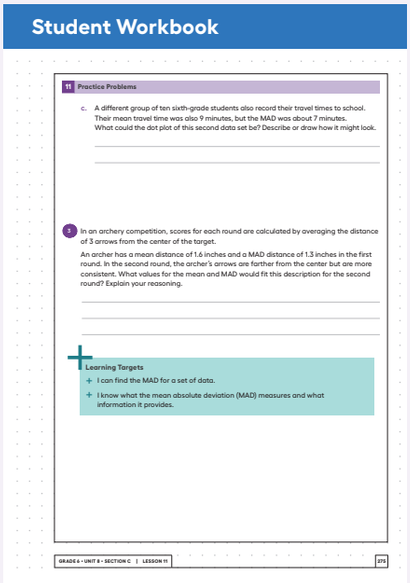
Student Workbook


11 Practice Problems

c. A different group of ten sixth-grade students also record their travel times to school. Their mean travel time was also 9 minutes, but the MAD was about 7 minutes. What could the dot plot of this second data set be? Describe or draw how it might look.

3 In an archery competition, scores for each round are calculated by averaging the distance of 3 arrows from the center of the target.

An archer has a mean distance of 1.6 inches and a MAD distance of 1.3 inches in the first round. In the second round, the archer's arrows are farther from the center but are more consistent. What values for the mean and MAD would fit this description for the second round? Explain your reasoning.

Learning Targets
+ I can find the MAD for a set of data.
+ I know what the mean absolute deviation (MAD) measures and what information it provides.

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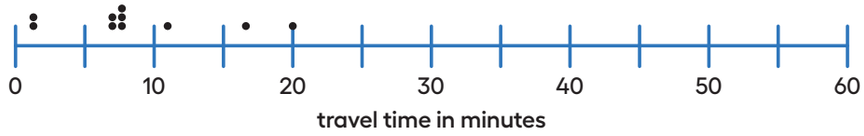
Problem 2

Ten sixth-grade students record the amounts of time each took to travel to school. The dot plot shows their travel times.



The mean travel time for these students is approximately 9 minutes. The MAD is approximately 4.2 minutes.

a. Is 9 or 4.2 a better estimate for the typical amount of time the ten sixth-grade students travel to school? Explain your reasoning.

9 minutes

Sample reasoning: On the dot plot, the center or balance point of the data set is located at or near 9, so that value is a good description of a typical travel time.

b. Based on the mean and MAD, Jada believes that travel times between 5 and 13 minutes are common for this group. Do you agree? Explain your reasoning.

Agree

Sample reasoning: The MAD tells us that, on average, the travel times of the students in this group are 4.2 minutes below the mean (about 5 minutes) or 4.2 minutes above the mean (about 13 minutes), so travel times between 5 and 13 minutes are common.

c. A different group of ten sixth-grade students also record their travel times to school. Their mean travel time was also 9 minutes, but the MAD was about 7 minutes. What could the dot plot of this second data set be? Describe or draw how it might look.

Sample response: The data points on the second dot plot would be more spread out, with more points farther from 9, because the MAD is larger. Travel times between 2 minutes and 16 minutes would be typical for this group.

Problem 3

In an archery competition, scores for each round are calculated by averaging the distance of 3 arrows from the center of the target.

An archer has a mean distance of 1.6 inches and a MAD distance of 1.3 inches in the first round. In the second round, the archer's arrows are farther from the center but are more consistent. What values for the mean and MAD would fit this description for the second round? Explain your reasoning.

Correct responses should have a mean greater than 1.6 inches and a MAD less than 1.3 inches.