Lesson 13

Polyhedra

Goals

- Compare and contrast (orally and in writing) features of prisms and pyramids.
- Comprehend and use the words “face”, “edge”, “vertex”, and “base” to describe polyhedra (in spoken and written language).
- Understand that the word “net” refers to a two-dimensional figure that can be assembled into a polyhedron, and create a net for a given polyhedron.

Learning Targets

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson Narrative

In this lesson, students learn about polyhedra and their **nets**. They also study **prisms** and **pyramids** as types of **polyhedra** with certain defining features.

Students begin by identifying the defining characteristics of polyhedra. They learn or review terminology such as faces, edges, and vertices as they develop a working definition of polyhedra.

Next, students explore the defining characteristics of prisms and pyramids. They consider the polygons that constitute the faces of a given prism or pyramid and how to arrange them into nets that can be assembled into the given polyhedron.

As students analyze polyhedra, prisms, and pyramids for defining characteristics and use their observations to distinguish these figures, they practice looking for and making use of structure. In communicating the geometric attributes that they see, students practice attending to precision.

An optional activity is included to give students an opportunity to assemble a net into a polyhedron and identify the number of vertices, edges, and faces.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Chart paper: Warm-up
- Math Community Chart: Warm-up
- Pre-assembled or commercially produced polyhedra: Warm-up
- Nets of polyhedra: Activity 2
- Scissors: Activity 1, Activity 2
- Tape: Activity 1, Activity 2
- Geometry toolkits: Activity 2
- Glue or glue sticks: Activity 2

Materials to Copy

- Assembling Polyhedra Cutouts (1 copy for every 12 students): Warm-up
- Assembling Polyhedra Cutouts (1 copy for every 6 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	25 min	20 min	10 min	5 min

Lesson 13

Polyhedra

Lesson Narrative (continued)

A note about polygons and polyhedra:

Here are some important aspects of polygons:

- They are made out of line segments called edges.
- Edges meet at a vertex.
- The edges meet only at vertices.
- Polygons always enclose a two-dimensional region.

Here is an analogous way to characterize polyhedra:

- They are made out of filled-in polygons called faces.
- Faces meet at an edge.
- The faces meet only at edges.
- Polyhedra always enclose a three-dimensional region.

Students do not need to memorize a formal definition of a polyhedron, but recognizing its defining characteristics can help students make sense of nets and surface area.

Math Community

In today's activities, students are introduced to the idea of math norms as expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Students then consider what norms would connect and support the math actions that the class recorded so far in the Math Community Chart.

Student Learning Goal

Let's investigate polyhedra.

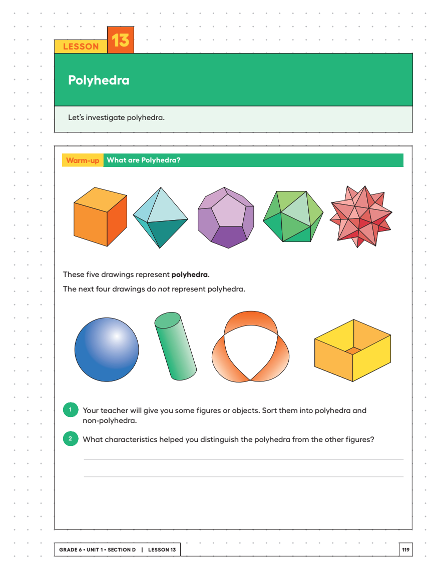
Required Preparation

Warm-up:
Assemble collections of geometric figures that each contains at least 2 familiar polyhedra, 2 unfamiliar polyhedra, and 2 non-polyhedra. Prepare one collection for each group of 3–4 students. If pre-made polyhedra are unavailable, assemble some from the nets in the blackline master.

Activity 1:
Every student workbook contains a set of nets and polygons for the students to cut out. Each group of 3–4 students will need 1 set, along with tape to join the polygons into a net.

Activity 2:
Print the nets from the same blackline master as the one used for the *Warm-up*. For each student, prepare 2 copies of one net and tape or glue to assemble the net.

Student Workbook


LESSON 13

Polyhedra

Let's investigate polyhedra.

Warm-up What are Polyhedra?

These five drawings represent **polyhedra**.

The next four drawings do *not* represent polyhedra.

1 Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.

2 What characteristics helped you distinguish the polyhedra from the other figures?

GRADE 6 • UNIT 1 • SECTION D | LESSON 13

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Warm-up

What are Polyhedra?

10 min

Activity Narrative

In this *Warm-up*, students analyze examples and counterexamples of polyhedra, observe their defining characteristics, and use their insights to sort objects into polyhedra and non-polyhedra. They then start developing a working definition of “polyhedron.”

Prepare physical examples of polyhedra and non-polyhedra for students to sort. These examples should be geometric figures rather than real-world objects such as shoe boxes or canisters. If such figures are not available, make some ahead of time using the nets in the blackline master.

As students work and discuss, notice those who can articulate defining features of a polyhedron and invite them to share later.

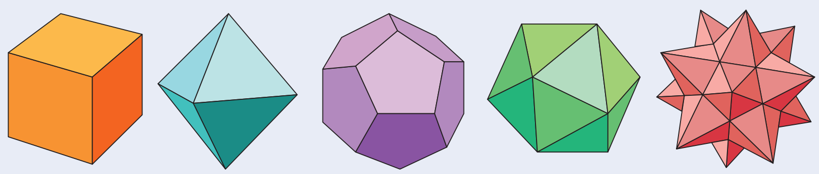
Launch

Arrange students in groups of 3–4. Give students 1 minute of quiet time to study the examples and non-examples in the task statement. Ask them to be ready to share at least one thing that they notice and one thing that they wonder. Give the class a minute to share some of their observations and questions.

Next, give each group a physical set of three-dimensional figures. The set should include some familiar polyhedra, some unfamiliar ones, and some non-polyhedra.

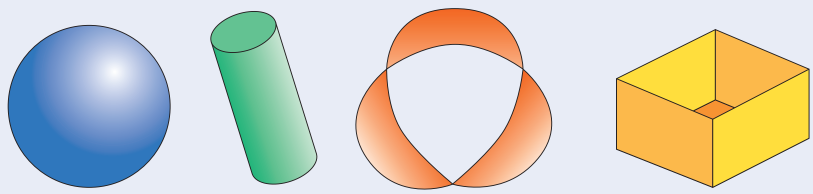
Ask groups to sort the figures into polyhedra and non-polyhedra (the first question). If members disagree about whether a figure is a polyhedron, prompt them to discuss the disagreements with their group. When the group has come to an agreement, give them 2–3 minutes of quiet time to complete the second question.

Student Task Statement



These five drawings represent **polyhedra**.

The next four drawings do *not* represent polyhedra.



1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.

   No response required.

2. What characteristics helped you distinguish the polyhedra from the other figures?

   Sample responses:

   - Polyhedra are made from polygons.
   - Polyhedra don't have any unattached edges.
   - Non-polyhedra sometimes have curved or round surfaces.
   - Some non-polyhedra have a face that is not a polygon.

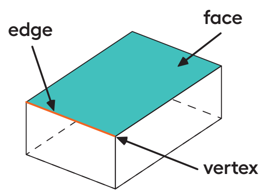
Activity Synthesis

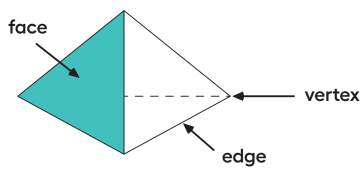
Invite students to share what they see as characteristics of polyhedra. Record their responses for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate on to make the description clearer or more precise. If they disagree, ask for an explanation or a counterexample.

Students will have a chance to refine their definition of polyhedra later in the lesson—after exploring prisms and pyramids and learning about nets, so it is not important to compile a complete or precise set of descriptions or features.

Use a sample polyhedron or a diagram as shown here to introduce or reinforce the terminology surrounding polyhedra.

- The polygons that make up a polyhedron are called "faces."
- The places where the sides of the faces meet are called edges.
- The "corners" are called vertices. (Clarify that the singular form is "vertex" and the plural form is "vertices.")





Math Community

At the end of the *Warm-up*, display the Math Community Chart.
Tell students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Using the Math Community Chart, offer an example of how the "Doing Math" actions can be used to create norms. For example, "In the last exercise, many of you said that our math community sounds like 'sharing ideas.' A norm that supports that is 'We listen as others share their ideas.' For a teacher norm, 'questioning vs telling' is very important to me, so a norm to support that is 'Ask questions first to make sure I understand how someone is thinking.'"

Invite students to reflect on both individual and group actions. Ask, "As we work together in our mathematical community, what norms, or expectations, should we keep in mind?"

Give 1–2 minutes of quiet think time and then invite as many students as time allows to share either their own norm suggestion or to "+1" another student's suggestion. Record student thinking in the student and teacher "Norms" sections on the Math Community Chart.

Conclude the discussion by telling students that what they made today is only a first draft of math community norms and that they can suggest other additions during the *Cool-down*. Throughout the year, students will revise, add, or remove norms based on those that are and are not supporting the community.

Activity 1

Prisms and Pyramids

25 min

Activity Narrative

This activity serves two goals: to uncover the defining features of **prisms** and **pyramids** as well as to introduce **nets** as two-dimensional representations of polyhedra.

Students first analyze prisms and pyramids and try to define their characteristics. Next, they learn about nets and think about the polygons needed to compose the nets of given prisms and pyramids. They then use their experience with the nets of prisms and pyramids to sharpen and refine their definitions of these polyhedra.

As students discuss the features of prisms and pyramids, encourage them to use the terms face, edge, and vertex (vertices) in their descriptions.

Launch

Arrange students in groups of 3–4. Tell students that there are 3 questions in this activity and that the class will pause for a discussion after responding to each question and before moving on to the next one.

Display images of the prisms and pyramids in the activity, or display and pass around physical representations of those polyhedra, if available. Tell students that Polyhedra A–F are all prisms and Polyhedra P–S are all pyramids.

For the first question:

- Give students 2–3 minutes of quiet think time and 1–2 minutes to discuss their observations in their groups.
- Solicit students' ideas about characteristics that distinguish prisms and pyramids. Record students' responses in two columns—one for prisms and the other for pyramids. It is not important that the lists are complete at this point.

Next, tell students that we are going to use nets to better understand prisms and pyramids. Explain that a **net** is a two-dimensional representation of a polyhedron.

Display a cube assembled for the *Warm-up*, as well as a cutout of an unfolded net (consider removing the flaps). Demonstrate how the net with squares could be folded and assembled into a cube. Point out how the number and the shape of the faces on the cube correspond to the number and the shape of the polygons in the net.

Instructional Routines

MLR2: Collect and Display

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Please log in to the site before using the QR code or URL

Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display. Circulate, and listen for and collect the language that students use as they talk about characteristics of prisms and pyramids. On a visible display, record words and phrases such as: "faces," "edges," "vertex," "parallel," "rectangular (or triangular) faces," and names of polygons. Invite students to borrow language from the display as needed and update it throughout the lesson.

Advances: Speaking, Conversing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide prisms and pyramids for students to view or manipulate. These hands-on models help students identify characteristics of polyhedra and support net building.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

For the second question:

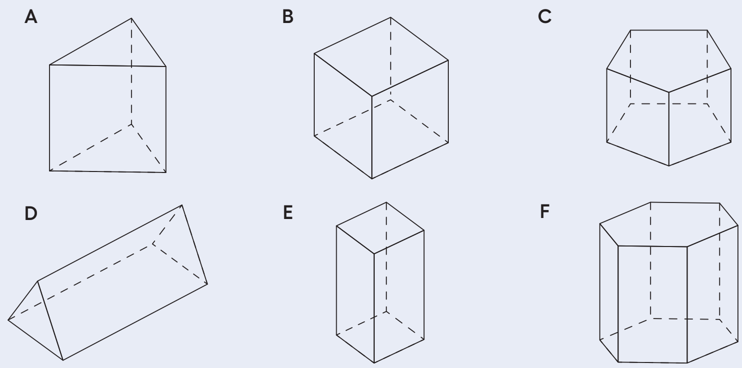
- Give students 1 minute of quiet think time and 1 minute to discuss their response in their groups.
- Tell students that they will now verify their answer. Assign each group one of the three nets from the second page of the Prisms and Pyramids Cutouts in the student workbook. Ask them to try to assemble a triangular pyramid from their net.
- Invite groups to share with the class whether it can be done. Discuss why Net 3 cannot be assembled into Pyramid P (two of the triangles would overlap).

For the last question, tell students that they will create a net of another prism or pyramid:

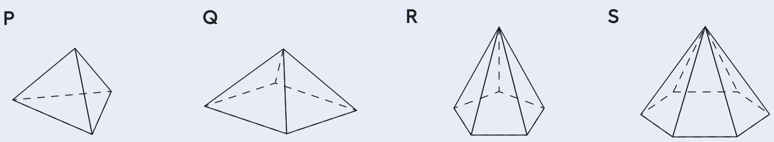
- Assign each group a prism or a pyramid from the task statement (except for Prism B and Pyramid P).
- Ask each group to cut out the polygons from the last two pages of the Prisms and Pyramids Cutouts.
- Tell students to choose the right kind and number of polygons that make up their polyhedron. Then, arrange the polygons so that, when taped and folded, the arrangement is a net and could be assembled into their prism or pyramid. Encourage them to think of more than one net, if possible.

Student Task Statement

1. Here are some polyhedra called **prisms**.



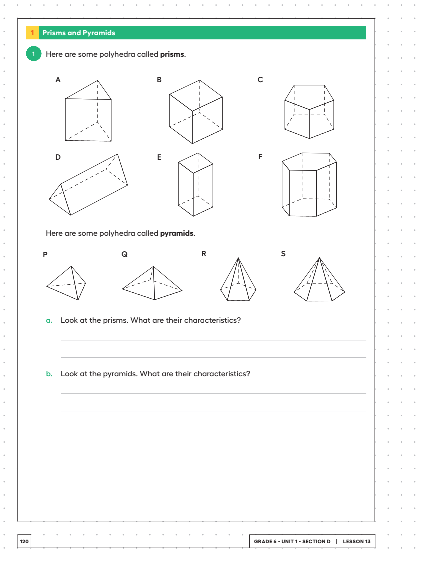
Here are some polyhedra called **pyramids**.



a. Look at the prisms. What are their characteristics?

Sample responses: A prism has rectangular faces. Some of the faces are parallel to one another. A prism may have two faces that are not rectangles.

Student Workbook

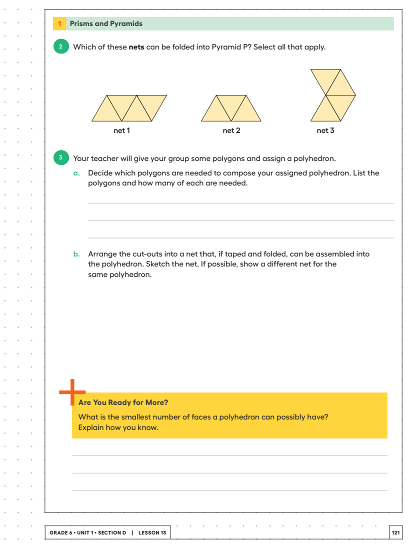

1 Prisms and Pyramids
1 Here are some polyhedra called prisms.
Here are some polyhedra called pyramids.
a. Look at the prisms. What are their characteristics?
b. Look at the pyramids. What are their characteristics?

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Provide Access for Physical Action. Provide access to pre-cut nets to reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Supports accessibility for: Fine Motor Skills, Organization, Visual-Spatial Processing

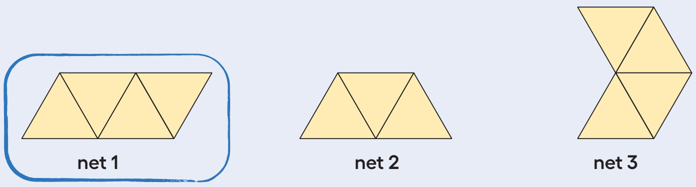
Student Workbook



b. Look at the pyramids. What are their characteristics?

Sample responses: A pyramid has a triangle for all or all but one of its faces. That one face might be a different polygon, and all triangles share an edge with it. All triangles also meet at a single vertex.

2. Which of these **nets** can be folded into Pyramid P? Select all that apply.



3. Your teacher will ask your group to cut out some polygons and assign you a polyhedron.

a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.

Sample responses:

- Prism A: 2 triangles, 3 squares
- Prism C: 2 pentagons, 5 rectangles
- Prism D: 2 triangles, 3 rectangles
- Prism E: 2 squares, 4 rectangles
- Prism F: 2 hexagons, 6 rectangles
- Pyramid Q: 1 square, 4 triangles
- Pyramid R: 1 pentagon, 5 triangles
- Pyramid S: 1 hexagon, 6 triangles

b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, show a different net for the same polyhedron.

Drawings vary.

Are You Ready for More?

What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

Four faces (a triangular pyramid, also known as a tetrahedron)
Sample reasoning: The triangle is the polygon with the fewest number of sides. If there is one triangular face, there must be another polygon attached at each edge of it, so it is impossible to use fewer than four faces. It is possible to create a polyhedron with four triangular faces. (Pyramid P in this activity is an example.)

Activity Synthesis

Select groups to share their arrangements of polygons. If time permits and if possible, have students tape their polygons and fold the net to verify that it could be assembled into the intended polyhedron. Discuss:

"What do the nets of prisms have in common?"

They all have rectangles. They have a pair of polygons that may not be rectangles.

"What do the nets of pyramids have in common?"

They all have triangles. They have one polygon that may not be a triangle.

"Is there only one possible net for a prism or a pyramid?"

No, the polygons can be arranged in different ways and still be assembled into the same prism or pyramid.

Explain the following points about prisms and pyramids:

- A prism has two parallel, identical faces called **bases** and a set of rectangles connecting the bases.
- Prisms are named for the shape of the bases. For example, if the base of a prism is a pentagon, then the prism is called a "pentagonal prism."
- A pyramid has one face called the **base** that can be any polygon and a set of faces that are all triangles. Each edge of the base is shared with an edge of a triangle. All of these triangles meet at a single vertex.
- Pyramids are named for the shape of their base. For example, if the base of a pyramid is a square, then the pyramid is called a "square pyramid."

Activity 2: Optional
Assembling Polyhedra

20 min

Activity Narrative

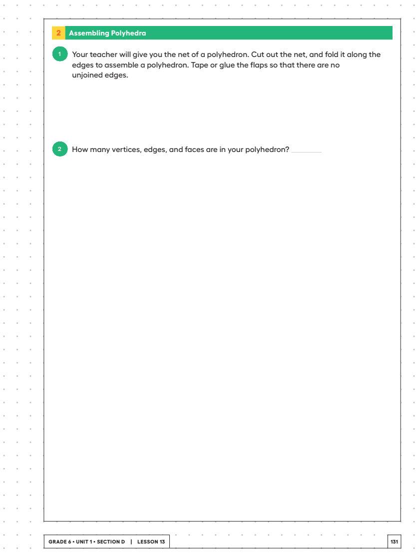
This optional activity gives students the experience of assembling polyhedra from nets. Counting the vertices, edges, and faces of a polyhedra helps to reinforce their understanding of the vocabulary. You will need the same blackline master as the one provided for the *Warm-up*.

Students are likely to need assistance in assembling their polyhedra. Circulate, and support students as needed.

Building on Student Thinking

Students may have trouble getting an accurate count of faces, edges, and vertices. Suggest that they set the figure on the table and then separately count the amount on the top, bottom, and lateral sides of the figure. Or recommend that they label each face with a number or a name and keep track of the parts associated with each face, taking care not to double count edges and vertices.

Student Workbook


2 Assembling Polyhedra

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.

2. How many vertices, edges, and faces are in your polyhedron? ______

Launch

Tell the class that they are going to assemble polyhedra from nets. Point out that the net has shaded and unshaded polygons. Display an example and explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are "flaps" to make it easier to glue or tape the polygons together; they will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold up the net and attach the various polygons together. A straightedge can be very helpful for making the creases.

Give each student two copies of a net so they can compare the assembled version with the unfolded net. Provide access to geometry toolkits and glue or tape. Ask students to build their figures and complete the question, and then to discuss their responses with another student who has the same polyhedron.

Student Task Statement

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.

   No answer required.

2. How many vertices, edges, and faces are in your polyhedron?

   Sample responses:

   - A: 6 vertices, 9 edges, 5 faces
   - B: 8 vertices, 12 edges, 6 faces
   - C: 8 vertices, 12 edges, 6 faces
   - D: 8 vertices, 12 edges, 6 faces
   - E: 4 vertices, 6 edges, 4 faces
   - F: 5 vertices, 8 edges, 5 faces
   - G: 6 vertices, 10 edges, 6 faces
   - H: 8 vertices, 14 edges, 8 faces
   - J: 10 vertices, 20 edges, 12 faces
   - K: 9 vertices, 15 edges, 8 faces

Activity Synthesis

After students have conferred with another student and agreed on the number of vertices, edges, and faces of their polyhedron, tell the class they will now share their completed polyhedra and the unfolded version of the net with the class. Consider either asking students to pass their two items around, or to leave their the polyhedra and nets displayed while students circulate around the room to view others' work.

Lesson Synthesis

Review the features of prisms and pyramids by selecting 1 or 2 polyhedra used in the *Warm-up*. Ask students to explain (using the terminology they learned, if possible) why each one is or is not a prism or a pyramid. If it is a prism or pyramid, ask students to name it.

Revisit the working definition of polyhedra generated earlier in the lesson and ask students to see if or how it might be refined. Ask if there is anything they should add, remove, or adjust given their work with prisms, pyramids, and nets.

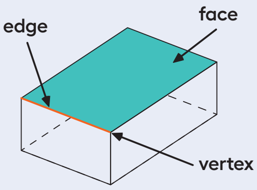
Highlight the following points about polyhedra. Ask students to illustrate each point using a figure or a net.

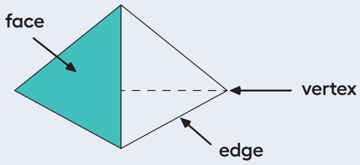
- A "polyhedron" is a three-dimensional figure built from filled-in polygons. We call the polygons "faces." (The plural of polyhedron is polyhedra.)
- Every "edge" of a polygon meets another polygon along a complete edge.
- Each polygon meets one and only one polygon on each of the edges.
- The polygons enclose a three-dimensional region.

Consider displaying in a visible place the key ideas from the students' list and from this discussion so that they key ideas can serve as a reference later.

Lesson Summary

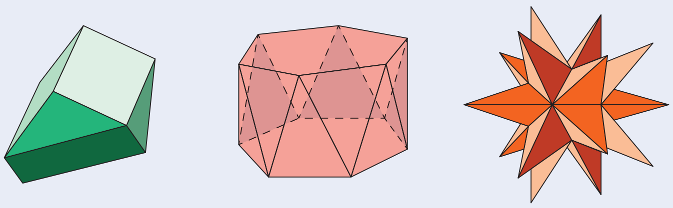
A **polyhedron** is a three-dimensional figure composed of faces. Each face is a polygon and meets only one other face along a complete edge. The ends of the edges meet at points that are called vertices.





A polyhedron always encloses a three-dimensional region.

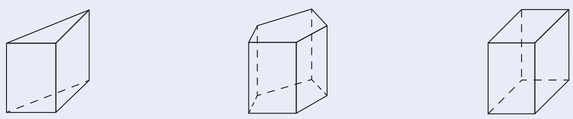
The plural of polyhedron is polyhedra. Here are some drawings of polyhedra:



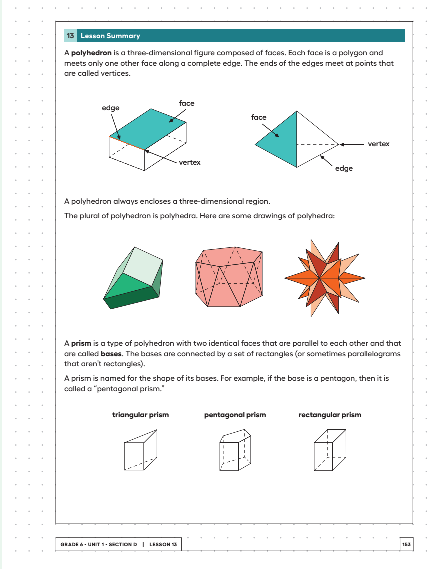
A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called **bases**. The bases are connected by a set of rectangles (or sometimes parallelograms that aren't rectangles).

A prism is named for the shape of its bases. For example, if the base

triangular prism | **pentagonal prism** | **rectangular prism**



Student Workbook



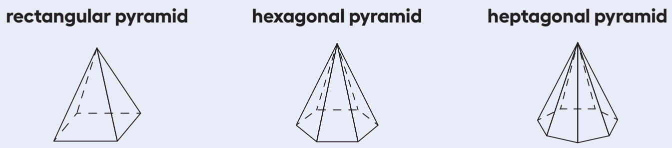
Responding To Student Thinking

Points to Emphasize
If students struggle with identifying polyhedra, revisit this idea when opportunities arise over the next several lessons. For example, in this activity, emphasize the properties of polyhedra when students are asked to match nets with their respective polyhedra:

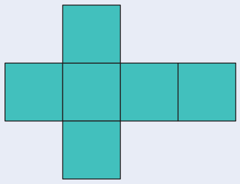
Unit 1, Lesson 14, *Warm-up* Matching Nets

A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single vertex.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a "pentagonal pyramid."



A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, each of these open edges will join another edge.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the norms question "Which norm has not already been listed that you'd like to add to our chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms question. Use that information to add to the initial draft of the "Norms" sections of the Math Community Chart.

Cool-down

Three-Dimensional Shapes

5 min

Student Task Statement

1. Write your best definition or description of a polyhedron. If possible, use the terms you learned in this lesson.

   Answers might include one or more of these elements:

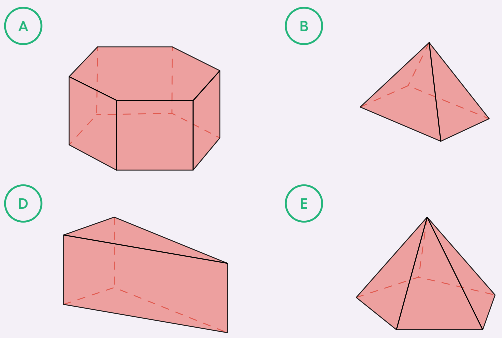
   - A polyhedron is a three-dimensional figure made from faces that are filled-in polygons
   - Each face meets one and only one other face along a complete edge
   - The points where edges meet are called vertices.

2. Which of these five polyhedra are prisms?

   A, C, and D are prisms

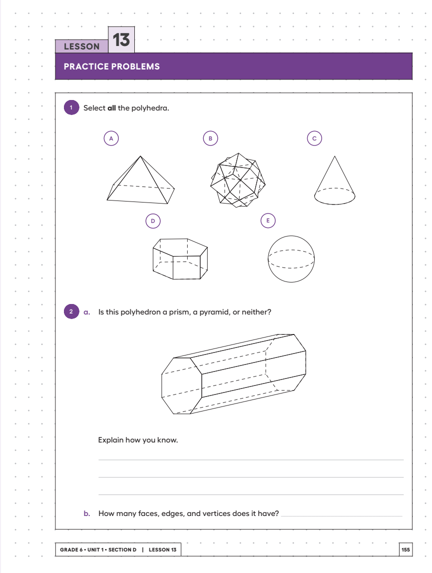
   Which are pyramids?

   B and E are pyramids



Practice Problems

6 Problems


Student Workbook

LESSON 13

PRACTICE PROBLEMS

1 Select **all** the polyhedra.

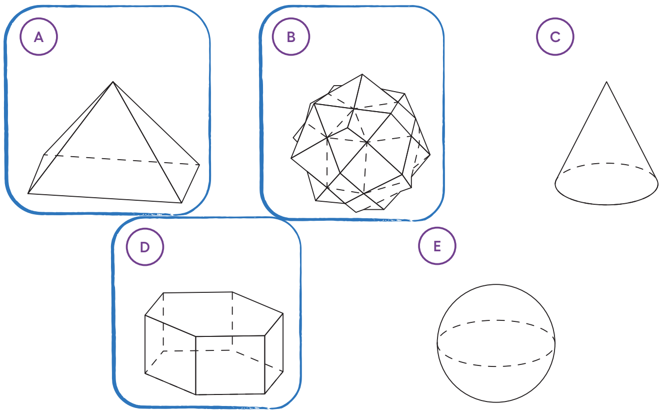
2 a. Is this polyhedron a prism, a pyramid, or neither?

Explain how you know.

b. How many faces, edges, and vertices does it have?

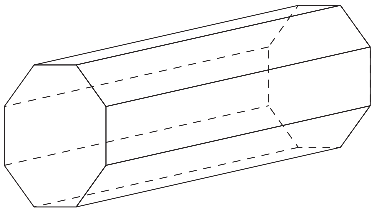
Problem 1

Select **all** the polyhedra.



Problem 2

a. Is this polyhedron a prism, a pyramid, or neither? Explain how you know.



Prism

Sample reasoning: It has two parallel octagonal bases that match up exactly.

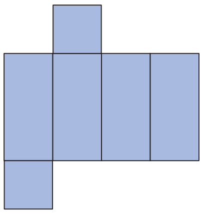
b. How many faces, edges, and vertices does it have?

10 faces, 24 edges, 16 vertices

Problem 3

Tyler said this net cannot be a net for a square prism because not all the faces are squares.

Do you agree with Tyler? Explain your reasoning.



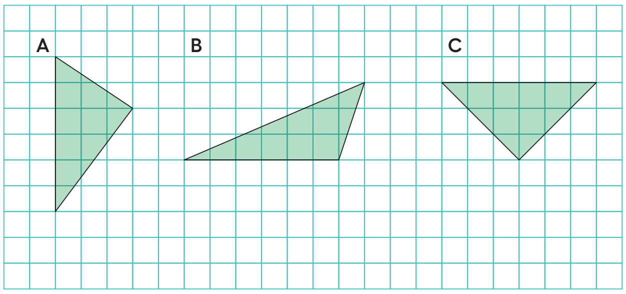
Disagree

Sample reasoning: A square prism must have two bases that are squares, but the other faces can be non-square rectangles. There are two squares in the net, and the net can be folded into a square prism.

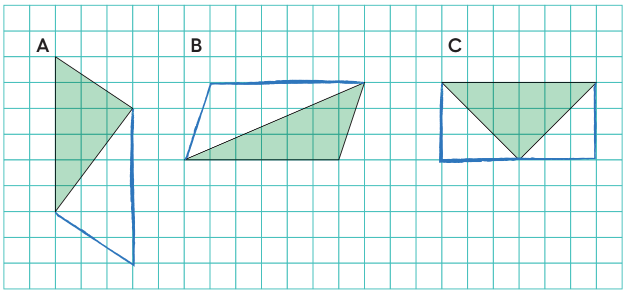
Problem 4

from Unit 1, Lesson 8

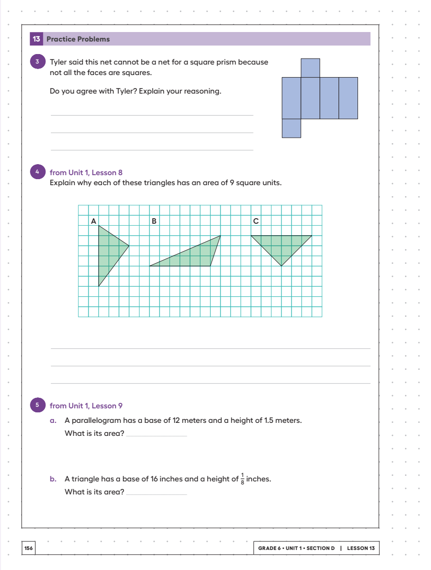
Explain why each of these triangles has an area of 9 square units.



Sample reasoning: Each triangle is half of a parallelogram with an area of 18 square units (with a base of 6 units and a height of 3 units), as shown in these diagrams.



Student Workbook



13 Practice Problems

3 Tyler said this net cannot be a net for a square prism because not all the faces are squares.

Do you agree with Tyler? Explain your reasoning.

4 from Unit 1, Lesson 8

Explain why each of these triangles has an area of 9 square units.

5 from Unit 1, Lesson 9

a. A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area? ____

b. A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area? ____

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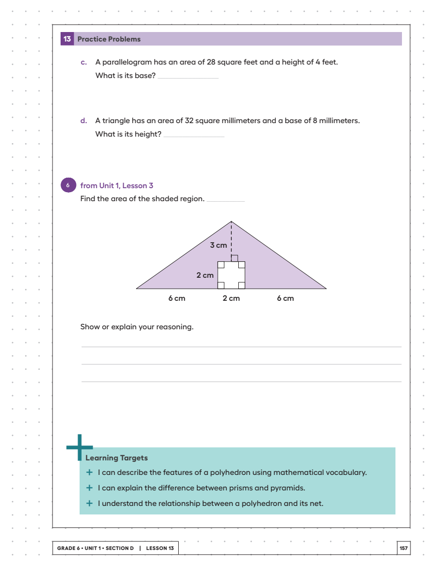
Student Workbook


13 Practice Problems

c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?

d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

6 from Unit 1, Lesson 3

Find the area of the shaded region.

Show or explain your reasoning.

Learning Targets

+ I can describe the features of a polyhedron using mathematical vocabulary.
+ I can explain the difference between prisms and pyramids.
+ I understand the relationship between a polyhedron and its net.

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Problem 5

from Unit 1, Lesson 9

a. A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area?

18 square meters

b. A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area?

1 square inch

c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?

7 feet

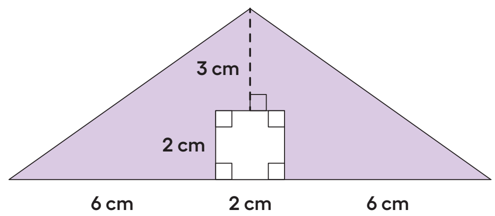
d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

8 millimeters

Problem 6

from Unit 1, Lesson 3

Find the area of the shaded region. Show or explain your reasoning.



31 sq cm

Sample reasoning: The two right triangles can be put together to make a 7 cm-by-5 cm rectangle whose area is 35 sq cm. However, a 2 cm-by-2 cm square is removed. The shaded area is 31 sq cm.