Lesson 15

Making and Measuring Boxes (Optional)

Goals

- Compare the measurements of different paper boxes multiplicatively, and determine, by estimation or calculation, how many times as great one set of measurements are as another.
- Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting measurements or results of calculation.
- Measure the decimal edge lengths of paper boxes and calculate their surface area.

Learning Target

I can use the four operations on decimals to find surface areas and reason about real-world problems.

Lesson Narrative

This culminating lesson is optional. Students apply their understanding of decimals to record, calculate, and analyze measurements of two- and three-dimensional objects: open-top paper boxes constructed by folding square paper of different sizes.

Before folding, students measure the side lengths of square pieces of paper and notice their relative sizes. Then, they make conjectures about how the length and area measurements would compare after the paper is folded into boxes.

Next, students measure the edge lengths of the paper boxes and calculate the surface area of each box, applying their knowledge of operations on decimals.

Finally, in an optional activity, students test their conjectures by making multiplicative comparisons—dividing the measurements of one box by those of another—and reflect on the accuracy of their predictions.

Lesson Timeline

25 min	20 min	20 min	10 min
Activity 1	Activity 2	Activity 3	Lesson Synthesis

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Warm-up, Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Origami paper: Activity 1
- Rulers: Activity 1, Activity 2
- Graph paper: Activity 2, Activity 3

Required Preparation

Activity 1:

Choose at least three different sizes of origami paper for students to use. Common length and width sizes of square origami paper include 6 inch, 7 inch, 8 inch, 9 inch, and 9.75 inch. Though less common, larger sizes such as 10 inches and 12 inches are also available. To see the mathematical structure more clearly, using 6-inch paper for the smallest size and 12-inch paper for the largest is recommended. If origami paper is not accessible, cut squares of paper from available paper (thinner is better). Prepare at least 1 sheet for each student.

Pre-make sample boxes of different sizes to show students. To prepare to demonstrate the folding, consider practicing the steps and the verbal instructions.

To help students fold their own origami boxes, an embedded video is provided and there are printed instructions in the student workbook.

Lesson 15

Making and Measuring Boxes (Optional)

Lesson Narrative (continued)

While arithmetic operations are central to the work in this lesson, students also build on concepts of geometry and ratios from earlier units. In each step along the way, as they deal with decimal measurements and calculations, students also need to consider the appropriate level of precision in the numbers and units they use and report.

Depending on the instructional choices made and students' experience with paper folding, this lesson could take one or more class meetings.

Student Learning Goal

Let's use what we know about decimals to make and measure boxes.

Activity 1

Folding Paper Boxes

25 min

Activity Narrative

In this activity, students are given square sheets of paper to measure and then fold into open-top boxes. Students work with decimals as they measure the side lengths of paper of different sizes, and estimate how the heights and surface areas of the resulting boxes would compare.

As students take measurements and make estimates to describe the relationship between two measurements, they have opportunities to attend to precision. For example:

- If one sheet of square paper is very close to twice the length of another, it is reasonable to predict that the height of the box folded from the former to be 2 times the height of the box folded from the latter given the possible error in measurement.
- If the relationship is very close to a fraction, such as $\frac{3}{2}$ for the 8-inch and 12-inch squares, it makes sense to report the number as a fraction.
- If a decimal is used to describe an estimated quotient of two lengths or surface areas, then using tenths may be appropriate.

Launch

Ask students if they have ever done origami or folded sheets of paper into three-dimensional objects. Ask a few students to share their experience.

Tell students that in this activity they will measure pieces of paper and make some predictions about the measurements of the boxes to be created from the paper. Later, they will fold the paper into boxes and measure the boxes to check their predictions.

Arrange students in groups of 3–4. Provide each group with at least three different sizes of paper and metric rulers that can measure in millimeters. Make some extra squares available for each group, in case they are needed.

Read the prompts as a class and answer any clarifying questions.

Then give students 5–7 minutes to measure their paper and complete the activity. Leave at least 10 minutes for discussion and paper folding.

Access for Multilingual Learners (Warm-up, Student Task)

MLR2: Collect and Display.
Circulate to listen for and collect the language that students use as they communicate about attributes of the paper or the folded box, as well as about the measurements and their relationships. On a visible display, record words and phrases such as "length," "height," "surface area," "right angle," "twice," "approximate," "estimate," "rounding," "precise," "as long (or tall, or large, or much) as," "units," "centimeters," and "millimeters." Invite students to borrow language from the display as needed, and update it throughout the lesson.

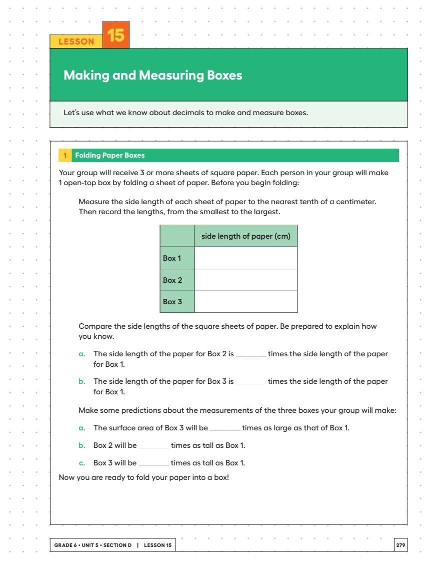
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Provide Access for Physical Action.
Give students who need support with fine-motor skills the option of representing the experience in the activity kinesthetically on a larger scale. For example, cut squares from large, easily folded paper such as chart paper or newsprint for students to use in place of origami paper.

Supports accessibility for: Fine Motor Skills, Visual-Spatial Processing

Student Workbook


LESSON 15

Making and Measuring Boxes

Let's use what we know about decimals to make and measure boxes.

1 Folding Paper Boxes

Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 open-top box by folding a sheet of paper. Before you begin folding:

Measure the side length of each sheet of paper to the nearest tenth of a centimeter. Then record the lengths, from the smallest to the largest.

	side length of paper (cm)
Box 1	
Box 2	
Box 3	

Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.

a. The side length of the paper for Box 2 is ______ times the side length of the paper for Box 1.

b. The side length of the paper for Box 3 is ______ times the side length of the paper for Box 1.

Make some predictions about the measurements of the three boxes your group will make:

a. The surface area of Box 3 will be ______ times as large as that of Box 1.

b. Box 2 will be ______ times as tall as Box 1.

c. Box 3 will be ______ times as tall as Box 1.

Now you are ready to fold your paper into a box!

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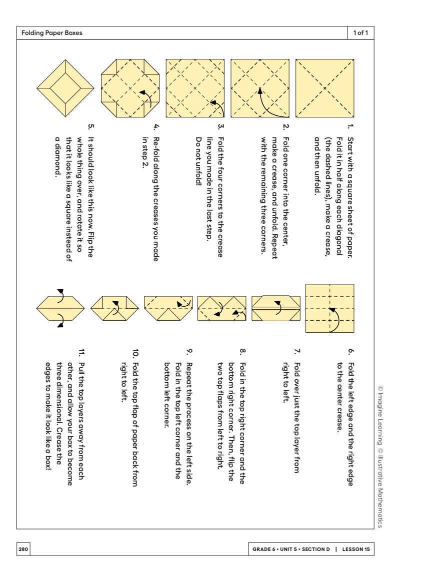
Student Workbook



Building on Student Thinking

When measuring the side length of their paper, students might not remember to align the ruler to the edge of the paper or to start at the 0 mark of the ruler. Remind them to do so or demonstrate as needed.

If students round their measurements to the nearest centimeter, ask them how the measurements would change if the lengths are measured to the nearest tenth of a centimeter (or the nearest millimeter). Urge them to repeat the measurement at the specified level of precision.

Student Task Statement

Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 open-top box by folding a sheet of paper. Before you begin folding:

1. Measure the side length of each sheet of paper to the nearest tenth of a centimeter. Then record the lengths, from the smallest to the largest.

   Sample responses based on 6-inch, 8-inch, and 12-inch square sheets of paper:

	side length of paper (cm)
Box 1	15.2
Box 2	20.8
Box 3	30.5

2. Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.

   a. The side length of the paper for Box 2 is ______ times the side length of the paper for Box 1.

   The paper for Box 2 is about 1.3 times as long as the paper for Box 1, because 20.3 ÷ 15.2 is a little less than 1.5. In inches, 8 ÷ 6 is $\frac{4}{3}$ or $1\frac{1}{3}$.

   b. The side length of the paper for Box 3 is ______ times the side length of the paper for Box 1.

   The paper for Box 3 is 2 times as long as the paper for Box 1 because 12 inches is twice 6 inches, or 30.5 ÷ 15.2 is about 2.

3. Make some predictions about the measurements of the three boxes your group will make:

   a. The surface area of Box 3 will be about 4 times as large as that of Box 1.

   b. Box 2 will be about 1.3 times as tall as Box 1.

   c. Box 3 will be about 2 times as tall as Box 1.

Now you are ready to fold your paper into a box!

Activity Synthesis

The goal of this discussion is for students to think critically about the accuracy of their measurements and predictions. Consider asking the following questions (assuming use of paper squares with side lengths 6 inches, 8 inches, and 12 inches):

- *"What did you find for the side length of the smallest square?"*

  15.2 cm or 15.3 cm (Expect a range of values).

- *"How confident are you about the accuracy of your measurements?"*

  For the first square, very confident about the 15 in 15.2 cm, but not confident about the 0.2.

- *"When you make predictions about how some measurements of the boxes would compare, what numbers did you use?"*

  Whole numbers such as 2 or 3, fractions such as $\frac{3}{2}$ or $1\frac{1}{2}$, and decimals such as 1.5 or 1.3.

- *"You predicted some measurements of one box to be 2 times those of another box. In those cases, how confident are you that the size of one box will be exactly twice the size of the other when measured? Why is that?"*

  Confident, because the length of one paper is exactly twice the length of the other, and if the folding is very precise, then the result will also be exact. Not confident, because the measurement and the folding may not be very precise.

- *"Did you use numbers such as 1.95 or 2.08 in predicting how the boxes would compare? Why or why not?"*

  No, because the measuring wasn't that precise, and we don't have the measurements of the boxes yet to calculate the relationship accurately.

After discussion, demonstrate how to fold a paper square into a box, explaining each step so students can follow along. Alternatively, demonstrate the folding once, then ask each group to follow the printed instructions in their student workbooks or provide them with access to the demonstration video. Encourage students to make strong creases when folding their paper. Suggest that they use the side of a thumbnail or a ruler to flatten the crease after making each initial fold.

Note that these particular instructions make a box with a square base. The following activity, which prompts students to record the length and width of the box's base, is based on this premise. If a different origami construction is used, the instructions and possibly the *Task Statement* will need to be adjusted.

Video 'Origami Box' available here: ilclass.com/r/16624779

Access for Multilingual Learners (Activity 1, Launch)

MLR2: Collect and Display.
Direct attention to words collected and displayed from the previous activity. Invite students to borrow language from the display as needed, and update it throughout the lesson.

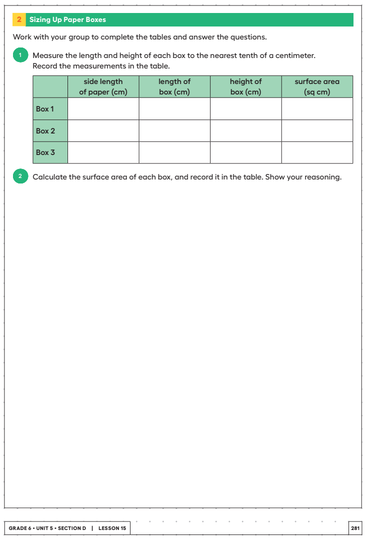
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Engagement: Internalize Self Regulation.
Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to compare either Box 2 or Box 3 to Box 1.

Supports accessibility for: Organization, Attention

Student Workbook


2 Sizing Up Paper Boxes

Work with your group to complete the tables and answer the questions.

1 Measure the length and height of each box to the nearest tenth of a centimeter. Record the measurements in the table.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1				
Box 2				
Box 3				

2 Calculate the surface area of each box, and record it in the table. Show your reasoning.

Activity 2

Sizing Up Paper Boxes

20 min

Activity Narrative

In this activity, students measure the length, height, and surface area of the boxes they built earlier. This work requires students to apply their knowledge of operations with decimal numbers and to exercise care in measurement. In measuring the lengths of a three-dimensional object, some imprecision can be expected, for instance:

- The lengths will not be an exact number of millimeters, so students round to the nearest millimeter. In some cases, this may be a guess between two different values.
- The folding process is not exact, so the box is not exactly a rectangular prism with a square base. Measurements of the length, width, and height vary depending on which part of the box is measured.
- When finding the surface area of their box, students will add and multiply their measurements. In performing operations, any errors in the measurements propagate.

Launch

Keep students in the same groups of 3–4.

Give students 8–10 minutes to collaborate with their group to complete the first table, with each group member to complete one row. (If a group has more than three paper sizes, adjust the tables in the activity accordingly.) Remind students to account for the fact that each box has an open top when calculating surface area.

Provide access to rulers and graph paper, in case students wish to use them to align the digits when multiplying numbers using partial products or the standard algorithm.

Student Task Statement

Work with your group to complete the tables and answer the questions.

1. Measure the length and height of each box to the nearest tenth of a centimeter. Record the measurements in the table.
2. Calculate the surface area of each box, and record it in the table. Show your reasoning.

Sample response based on use of 6-inch, 8-inch, and 12-inch square pieces of paper:

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1	15.2	5.4	2.7	87.5
Box 2	20.8	7.2	3.6	156
Box 3	30.5	10.8	5.4	350

Activity Synthesis

Focus the discussion on the variation in the values students recorded, possible factors that contributed to the variation, and appropriate degree of precision in this context.

Display a blank table and invite groups to share their measurements of the length, height, and surface area of the boxes. Record variations in each measurement. Ask questions such as:

"Why do you think the measurements were not all the same?"

Some likely reasons:

- Measuring challenges: It was tricky to line up the edges of a paper box with the ruler and to measure to the nearest millimeter.
- Folding challenges: The sides of the box were not completely flat, the height of the box was different at different places, or the edges weren't fully creased.
- Rounding variations: Different students might have rounded differently or to different numbers of decimal places.
- Calculation variations: Some students might round before multiplying, and others might round afterward. Using slightly different factors would lead to different products.

"For the surface area of each box, how precise was the value you recorded? Did you record it to the nearest ten square centimeters, square centimeter, or one-tenth of a square centimeter?"

"Did anyone record their answer to hundredths of a square centimeter? Why or why not?"

Yes, because the lengths were in tenths of a centimeter, so the products were in hundredths of a square centimeter. No, it was rounded to the nearest tenth of a square centimeter.

Activity 3: Optional
Comparing Paper Boxes

20 min

Activity Narrative

In this activity, students check their predictions about how the measurements of the boxes they built compare. Doing so involves reasoning about quotients of decimals. Students may opt to estimate the quotients, calculate them more accurately, or do both depending on the numbers involved. Each approach is acceptable provided that students can offer an explanation to support their decision. The work here offers students an opportunity to consider the level of precision in communicating their answers.

Launch

Keep students in the same groups of 3–4. Tell them they will now check their predictions about how the measurements of the boxes compare.

Give students 8–10 minutes of work time. Provide access to graph paper, in case students wish to use it to align the digits when dividing numbers using partial quotients or long division.

Building on Student Thinking

Students may neglect to attend to units of measurement when calculating area. Remind them to include the units being used in their responses.

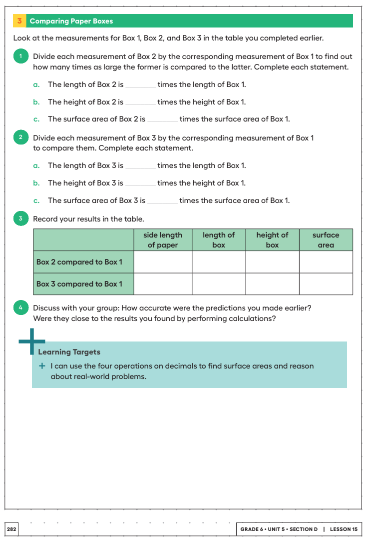
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support small-group discussion about students' predictions. Examples:

1. "My measurements matched (or did not match) my predictions because ..."
2. "Why did you make that prediction?"
3. "I made that prediction because ..."

Advances: Speaking, Conversing

Student Workbook



3 Comparing Paper Boxes

Look at the measurements for Box 1, Box 2, and Box 3 in the table you completed earlier.

1. Divide each measurement of Box 2 by the corresponding measurement of Box 1 to find out how many times as large the former is compared to the latter. Complete each statement.
   a. The length of Box 2 is ______ times the length of Box 1.
   b. The height of Box 2 is ______ times the height of Box 1.
   c. The surface area of Box 2 is ______ times the surface area of Box 1.
2. Divide each measurement of Box 3 by the corresponding measurement of Box 1 to compare them. Complete each statement.
   a. The length of Box 3 is ______ times the length of Box 1.
   b. The height of Box 3 is ______ times the height of Box 1.
   c. The surface area of Box 3 is ______ times the surface area of Box 1.
3. Record your results in the table.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1				
Box 3 compared to Box 1				

4. Discuss with your group: How accurate were the predictions you made earlier? Were they close to the results you found by performing calculations?

Learning Targets
+ I can use the four operations on decimals to find surface areas and reason about real-world problems.

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Student Task Statement

Look at the measurements for Box 1, Box 2, and Box 3 in the table you completed earlier.

1. Divide each measurement of Box 2 by the corresponding measurement of Box 1 to find out how many times as large the former is compared to the latter. Complete each statement.
   a. The length of Box 2 is _________ times the length of Box 1.
   b. The height of Box 2 is _________ times the height of Box 1.
   c. The surface area of Box 2 is _________ times the surface area of Box 1.

   See completed table.
2. Divide each measurement of Box 3 by the corresponding measurement of Box 1 to compare them. Complete each statement.
   a. The length of Box 3 is _________ times the length of Box 1.
   b. The height of Box 3 is _________ times the height of Box 1.
   c. The surface area of Box 3 is _________ times the surface area of Box 1.

   See completed table.
3. Record your results in the table.

   Sample response based on use of 6-inch, 8-inch, and 12-inch square pieces of paper:

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1	1.33	1.33	1.33	1.78
Box 3 compared to Box 1	2	2	2	4

4. Discuss with your group: How accurate were the predictions you made earlier? Were they close to the results you found by performing calculations?

   No response required.

Activity Synthesis

Invite students to share their findings on how the measurements of Box 2 and Box 3 compare to those of Box 1, as well as how accurate their earlier predictions were, especially the prediction for how the surface area of Box 3 compares to that of Box 1. Ask questions such as:

"Were your predictions for how the edge lengths of the boxes would compare correct? Were they close?"

"Were your predictions for how the surface areas of the boxes would compare correct? Were they close?"

"What do you notice about the numbers in each row of the second table?"

When the side length of the paper is 1.3 times or 2 times as long as that of another sheet of paper, the edge lengths of the box are also 1.3 times or 2 times as long, respectively, but the surface area is 1.78 times or 4 times greater, respectively.

Lesson Synthesis

A key focus of this lesson is on considering appropriate levels of accuracy and precision when working with measurements and using mathematics in a real-world context. Invite students to reflect on this idea.

Consider displaying the table that shows students' measurements and calculations for all three boxes, as well as reflect variations across groups. Ask questions such as:

"How are the numbers you used in the problems in this lesson like the numbers in other lessons?"

They are mostly decimals in tenths or hundredths. There are few whole numbers.

"How are the numbers different from those in earlier lessons?"

They are not given to us. We had to measure objects to obtain the numbers. The numbers are slightly different for different groups. They are rounded to different decimal places. Some are estimates, so they are not exact.

"When might it be appropriate to estimate or round a number when measuring or calculating? For example, when finding the side length of a piece of paper or the edge length of a paper box, what's more appropriate: rounding to the nearest centimeter, a tenth of a centimeter, or a hundredth of a centimeter?"

It depends on the situation, the tools being used, and the purpose of measuring or calculating. When measuring paper using a ruler, it's not possible to measure accurately to a hundredth of a centimeter. For the purposes of the activity, it is also not necessary.