

More than Two Choices

Goals

- Choose and justify (orally) which voting system seems the fairest for dealing with more than two choices.
- Compare and contrast (orally and in writing) different voting systems for dealing with more than two choices, i.e., plurality, runoff, and instant runoff.
- Interpret (orally and in writing) voting situations involving two choices through the use of ratios and percentages.

Lesson Narrative

This optional lesson introduces the complications that arise in elections with more than two choices. Students explore different rules for determining the winner—plurality, runoff, and ranked—and discover that the rules can give different results from the same set of voter preferences. They construct arguments about which voting rule more fairly represents the opinions of the voters. The mathematics in these activities emphasizes quantitative reasoning in a real-world situation as students compare the results of different voting systems.

Student Learning Goal

Let's explore different ways to determine a winner.

Lesson Timeline

5
min

Warm-up

20
min

Activity 1

20
min

Activity 2

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Warm-up
Field Day

5 min

Activity Narrative

This is the first of 6 activities about elections in which there are more than two choices. This introductory activity gets students thinking about the fairness of a voting rule. If the choice with the most votes wins, it’s possible that the winning choice was preferred by only a small percentage of the voters.

Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Students in a sixth-grade class were asked, “What activity would you most like to do for field day?” The results are shown in the table.

activity	number of votes
softball game	16
scavenger hunt	10
dancing talent show	8
marshmallow throw	4
no preference	2

1. What percentage of the class voted for softball? **40%**
Sample response: $\frac{16}{40} = \frac{2}{5} = 40\%$

2. What percentage did not vote for softball as their first choice? **60%**

Activity Synthesis

The goal of this discussion is to show that in elections with more than two options sometimes the winning choice is not the majority’s choice. Here are some questions for discussion:

- Which activity has the most votes?
softball
- Does softball have a majority of the votes?
No, a majority would be more than 50%, but softball only has 40%.
- If the class can do only one activity for field day, explain why they should or should not do softball.
They should do softball because more people voted for it than any other activity. They should not do softball because most people did not pick softball.

Student Workbook

LESSON 6

More than Two Choices

Let's explore different ways to determine a winner.

Warm-up Field Day

Students in a sixth-grade class were asked, "What activity would you most like to do for field day?" The results are shown in the table.

activity	number of votes
softball game	16
scavenger hunt	10
dancing talent show	8
marshmallow throw	4
no preference	2

1. What percentage of the class voted for softball?

2. What percentage did not vote for softball as their first choice?

GRADE 6 • UNIT 9 • SECTION B | LESSON 6

Access for Students with Diverse Abilities (Activity 1, Launch)
Engagement: Provide Access by Recruiting Interest.

Invite students to share experiences when they have participated in a vote or seen a family member vote in an election.

Supports accessibility for: Conceptual Processing, Memory

Tell students that this system of voting, where everyone gets one vote and the option with the most votes wins, is called plurality voting. Explain that, while 60% of the people did not vote for softball, that does not mean those people were against softball, just that they liked a different option more. In the upcoming activities, students will see other ways of voting, including some that can take this situation into account.

Activity 1
Field Trip (Part 1)
20
min

Activity Narrative

This activity compares plurality voting with a different method for deciding the winner of an election that has more than two choices: runoff voting. If no choice has a majority of votes, then the choice or choices with the fewest votes are eliminated, and another vote is held between the remaining choices. This process is repeated until one choice gets a majority of the votes.

Students learn the technique of analyzing the results by holding their own vote. A fictitious story (choosing a field trip) is provided for students to vote on a situation with four choices, each of which may have some positive and negative aspects. They follow two different systems of voting rules to see how results can differ depending on the rule system used. Students use quantitative reasoning to analyze and compare two different voting rules.

Note: This activity includes a lot of teacher-directed voting activity. Students periodically stop to record information and determine the winner, or the need to do another round of voting, or reflect on the results.

Launch


Tell students that they will be exploring a few ways of voting, using a fictitious situation: voting for a sixth grade field trip. The first voting system they will use is plurality.

Voting System #1: Plurality

Remind students that in this system everyone votes once, and the option with the most votes wins, even if that option does not have a majority. Display the four field trip options and the voting table:

Option 1: Baseball Game

- During school hours
- Buy lunch at the game
- Free souvenir

Option 2: Amusement Park

- During and after school
- Lunch included
- Unlimited rides

Option 3: Museum

- During school hours
- Lunch included
- Free souvenir

Option 4: Dance at School

- After school hours
- Snacks included
- DJ games and prizes

options	vote 1			
baseball game				
amusement park				
museum				
dance				

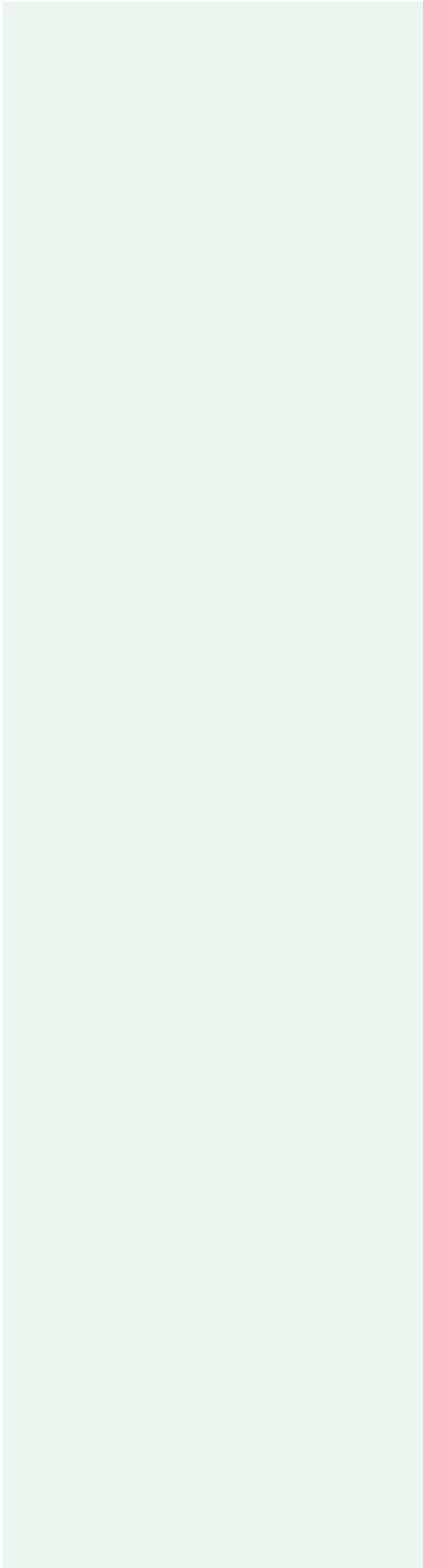
Before students vote, ask the class how many people are voting and how many votes it would take to win a majority. Then give students 30 seconds to decide which option they will vote for. Poll the class and record their votes in the table. If there are an even number of students the teacher should vote as well, to prevent a tie.

After the voting is complete, ask the class how many votes the top option received and if it was a majority of the votes. Have students calculate what percentage of the class is satisfied (by calculating the percentage that voted for the winning option) and what percentage of the class is not satisfied (did not vote for the winning option), and record this information near the “Vote 1” column. Ask the class if there is anyone who did not vote for the winning option but would be fine with that choice.

Voting System #2: Runoff

One way of voting that allows people to give more input is runoff voting. In this system, if no option wins the majority of the votes, the option with the least votes is eliminated and a new vote is held. Go back to the results of the first vote and eliminate the least popular option. Hold a second vote with only the 3 remaining options and record the results in the table.

Arrange students in groups of 2–4 to complete the first question. Continue rounds of voting until an option has a majority of the votes, then give students 2–3 minutes to answer the remaining questions.



Student Workbook

Field Trip (Part 1)

Suppose students at our school are voting for the sixth-grade field trip. The following is a list of options for your class to vote on.

Option 1: Baseball Game

- During school hours
- Buy lunch at the game
- Free souvenir

Option 2: Amusement Park

- During and after school
- Lunch included
- Unlimited rides

Option 3: Museum

- During school hours
- Lunch included
- Free souvenir

Option 4: Dance at School

- After school hours
- Snacks included
- DJ games and prizes

options	vote 1				
baseball game					
amusement park					
museum					
dance					

1

After the second round of voting, did any choice get a majority? If so, is it the same choice that got a plurality in Voting System #1?

2

Which choice won?

3

What percentage of voters are satisfied with these results (voted for the winning option)? What percentage were not satisfied?

4

Compare the satisfaction results for the plurality voting rule and the runoff rule. Did one produce satisfactory results for more people than the other?

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Student Task Statement

Suppose students at our school are voting for the sixth-grade field trip. The following is a list of options for your class to vote on.

- Option 1: Baseball Game**
 - During school hours
 - Buy lunch at the game
 - Free souvenir
- Option 2: Amusement Park**
 - During and after school
 - Lunch included
 - Unlimited rides
- Option 3: Museum**
 - During school hours
 - Lunch included
 - Free souvenir
- Option 4: Dance at School**
 - After school hours
 - Snacks included
 - DJ games and prizes

options	vote 1				
baseball game					
amusement park					
museum					
dance					

1. After the second round of voting, did any choice get a majority? If so, is it the same choice that got a plurality in Voting System #1?
Answers vary.
2. Which choice won?
Answers vary.
3. What percentage of voters are satisfied with these results (voted for the winning option)? What percentage were not satisfied?
Answers vary.
4. Compare the satisfaction results for the plurality voting rule and the runoff rule. Did one produce satisfactory results for more people than the other?
Answers vary.

Activity Synthesis

The goal of this discussion is to compare the two voting methods. Here are some questions for discussion:

- 💬

“Which voting system seems more fair, plurality or runoff?”

The plurality system accounts for voters’ first choice only, but the runoff system allows people to pick their next favorite choice if their first choice is eliminated.
- 💬

“Which voting system would be easier to implement on a large scale, like for a presidential election?”

The runoff system can take multiple voting sessions, so it takes more time than does the plurality system, in which only one voting session takes place.

Activity 2

Field Trip (Part 2)

20 min

Activity Narrative

In this activity students revisit the situation from the previous activity, but they analyze the votes of a different class. In this case, the members of different student clubs all voted for the same field trip option. Students analyze a ranked-choice voting system using the provided data and compare it to a plurality voting system using the same data. They use quantitative reasoning to analyze and compare the two different voting systems.

Students must think through the voting process and determine which choice is eliminated at each round, and what vote totals the club presidents will turn in at every round of voting.

Launch

Arrange students in groups of 2–4. Tell students that they will analyze the results of the vote from a different class for the same field trip situation from the previous activity. In this situation there are four clubs in the class, and everyone in each club agrees to vote for the same choice each time they vote. The table shows the symbols used to record their choices.

Each club ranked the options on their ballot.

1st choice

2nd choice

3rd choice

4th choice

Show the ranking symbols and this sample ballot, submitted by the members of the Adventure Club:

option	vote
baseball game	
amusement park	
museum	
dance	

Student Workbook

Field Trip (Part 2)

Let's analyze a different election.

In another class, there are four clubs. They plan to use the ranked-voting system. Everyone in each club agrees to make the same choice for the field trip for each round of voting. That means that everyone's choices are set by their first vote. The table shows their choices. They used these symbols to rank their choices for the field trip.

★

😊

☹️

✖️

1st choice

2nd choice

3rd choice

4th choice

	sports club (21 members)	adventure club (13 members)	art club (7 members)	music club (9 members)
baseball game	★	✖️	✖️	✖️
amusement park	😊	★	☹️	☹️
museum	☹️	☹️	😊	★
dance	✖️	😊	★	😊

Figure out which option won the election by answering these questions.

1

2

3

4

5

1

2

3

4

5

Ask students about the Adventure Club’s choices, to make sure that they understand the meaning of the symbols.

Give students 5–7 minutes to work through the questions with their group, and follow up with a whole-class discussion.

Student Task Statement

Let’s analyze a different election.

In another class, there are four clubs. They plan to use the ranked-voting system. Everyone in each club agrees to make the same choice for the field trip for each round of voting. That means that everyone’s choices are set by their first vote. The table shows their choices.

They used these symbols to rank their choices for the field trip.

★

😊

☹️

✖️

1st choice

2nd choice

3rd choice

4th choice

	sports club (21 members)	adventure club (13 members)	art club (7 members)	music club (9 members)
baseball game	★	✖️	✖️	✖️
amusement park	😊	★	☹️	☹️
museum	☹️	☹️	😊	★
dance	✖️	😊	★	😊

- Figure out which option won the election by answering these questions.
- 1

2

3

4

5
- 1

2

3

4

5
- Baseball would get 21 votes, Amusement Park would get 13 votes, Museum would get 9 votes, and Dance would get 7 votes. No option has a majority of the votes.
- Dance
- Baseball would get 21 votes, Amusement Park would get 13 votes, and Museum would get 16 votes. No option has a majority of the votes.
- Amusement Park
- Baseball would get 21 votes and Museum would get 29 votes. Museum won.

Activity Synthesis

The goal of this discussion is to compare multiple voting systems and how using different methods of voting affects voter satisfaction with the result. Notice that after the first round, baseball seemed to be winning. After the second round, baseball was still winning, but museum's votes almost doubled. Only after 3 rounds of voting did museum overtake baseball to win the vote.

Ask students:

💬 “How did the results of this class compare to our own class?”

“In this situation, everyone ranked their choices before the voting began. How does this impact the voting process?”

In the runoff system, people could change their votes between each round, but in the ranked-voting system, everyone's choices are locked in before any rounds of voting begin.

💬 “What are some advantages and disadvantages of plurality, runoff, and ranked voting?”

Plurality takes less effort but could be less fair. Runoff can be influenced between rounds of voting but allows people to vote beyond their first choice. Ranked voting locks in choices at the beginning but allows people to vote beyond their first choice.

💬 “If this class used plurality voting, what percentage of the class would be satisfied with the result? If this class used ranked voting, what percentage of the class would be satisfied with the result?”

42% are satisfied with plurality voting, 58% are satisfied with ranked voting

💬 “Which system seems more fair—plurality, runoff, or ranked?”