Lesson 8

Picking Representatives

Goals

- Create a method for distributing representatives, and justify (orally) why it is fair.
- Critique (orally and in writing) whether a method for distributing representatives is fair.

Lesson Narrative

In this optional lesson, students investigate how elections use representatives to decide results. This lesson explores mathematical difficulties that arise in a representative democracy, where people do not vote individually, but vote for representatives who vote for all their constituents. The activities explore ways to "share" the representatives fairly between groups of people. However, sometimes the groups to be represented are predetermined, such as classrooms or states. It's not always possible to have the same numbers of constituents per representative. The lesson explores simple and more complex sharing situations, first through giving computers to families and through representing students to the school board.

Most of the activities use students' skills from earlier units to reason about ratios and proportional relationships in the context of real-world problems. While some of the activities do not involve much computation, they all require thorough consideration and decision making as students create and justify their plans. Students model with mathematics as they make assumptions, decide what is important, and determine reasonable quantities.

Student Learning Goal

Let's think about fair representation.

Lesson Timeline

10 min	15 min	20 min
Activity 1	Activity 2	Activity 3

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Four-function calculators: Activity 2, Activity 3

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 1)

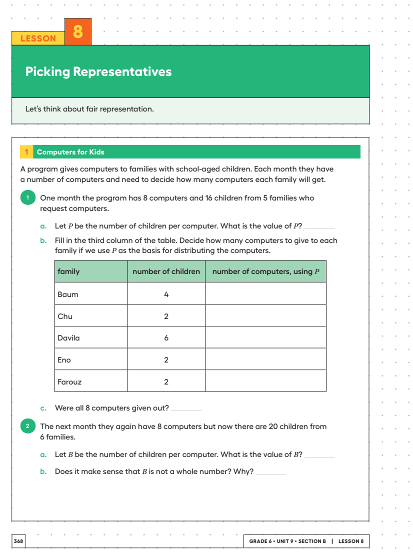
MLR2: Collect and Display. This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Provide Access by Recruiting Interest. Invite students to share experiences when they have needed to share something. Highlight the difference between times when the item being shared can be divided among all of the people (like a cake) and when the item cannot be divided (like a stuffed toy).

Supports accessibility for: Conceptual Processing, Memory

Student Workbook


LESSON 8

Picking Representatives

Let's think about fair representation.

1 Computers for Kids

A program gives computers to families with school-aged children. Each month they have a number of computers and need to decide how many computers each family will get.

1 One month the program has 8 computers and 16 children from 5 families who request computers.

a. Let P be the number of children per computer. What is the value of P?

b. Fill in the third column of the table. Decide how many computers to give to each family if we use P as the basis for distributing the computers.

family	number of children	number of computers, using P
Baum	4	
Chu	2	
Davila	6	
Eno	2	
Farouz	2	

c. Were all 8 computers given out?

2 The next month they again have 8 computers but now there are 20 children from 6 families.

a. Let R be the number of children per computer. What is the value of R?

b. Does it make sense that R is not a whole number? Why?

Activity 1

Computers for Kids

10 min

Activity Narrative

This activity introduces the question for the remaining activities on voting: How can we fairly share a small number of representatives between several groups of people?

This activity invites students to consider a simpler situation about how to distribute computers fairly to families with children. In the first question, computers can be shared so that the same number of children share a computer in each family. Later in the activity, fair sharing is not possible, so students need to construct arguments to explain which alternative is the fairest, or the least unfair.

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet think time, and then ask them to compare their work with a partner.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to create and discuss different methods of distributing computers. Display words and phrases such as "divide," "split," "fair," "unfair," "________ per ________," and "whole number."

Student Task Statement

A program gives computers to families with school-aged children. Each month they have a number of computers and need to decide how many computers each family will get.

1. One month the program has 8 computers and 16 children from 5 families who request computers.

   a. Let P be the number of children per computer. What is the value of P?

   $P = 2$ children per computer

   b. Fill in the third column of the table. Decide how many computers to give to each family if we use P as the basis for distributing the computers.

family	number of children	number of computers, using P
Baum	4	2
Chu	2	1
Davila	6	3
Eno	2	1
Farouz	2	1

   c. Were all 8 computers given out? Yes

   Sample response: $2 + 1 + 3 + 1 + 1 = 8$, so 8 computers have been given out.

2. The next month they again have 8 computers but now there are 20 children from 6 families.

a. Let B be the number of children per computer. What is the value of B?

2.5 children or $\frac{5}{2}$ children

b. Does it make sense that B is not a whole number? Why?

Sample responses:

- $B = 2.5$ makes sense. It's an average, not an actual amount for any children or families.
- B not being a whole number does not make sense because it is based on the number of children, which must be a whole number.

c. Fill in the third column of the table. Decide how many computers to give to each family if we use B as the basis for giving the computers.

family	number of children	number of computers, using B	number of computers, your way	children per computer, your way
Gray	3	1.2 or $\frac{6}{5}$	1	3
Hernandez	1	0.4 or $\frac{2}{5}$	1	1
Ito	2	0.8 or $\frac{4}{5}$	1	2
Jones	5	2	2	2.5
Krantz	1	0.4 or $\frac{2}{5}$	1	1
Lo	8	3.2 or $\frac{16}{5}$	2	4

Divide the number of children by 2.5 or $\frac{5}{2}$ children per computer for each family to get the number of computers.

d. Were all 8 computers given out?

Yes

e. Does it make sense that the number of computers for one family is not a whole number? Explain your reasoning.

It doesn't make sense for a family to get a fractional or decimal amount of a computer. Computers work only when they are whole. A fraction of a computer is a broken computer.

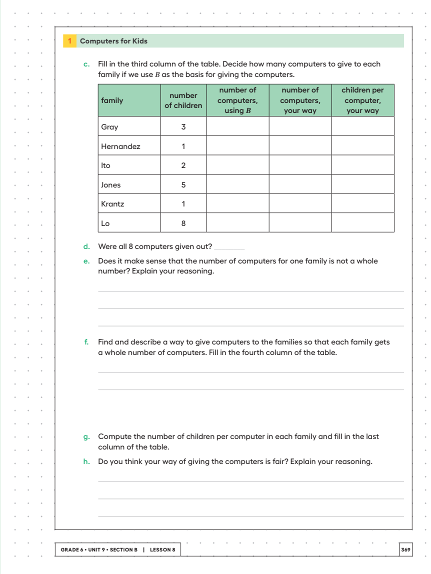
f. Find and describe a way to give computers to the families so that each family gets a whole number of computers. Fill in the fourth column of the table.

Sample response: This distribution first gives one computer to each family, which uses 6 computers. Then the two last computers are given to the two largest families.

g. Compute the number of children per computer in each family and fill in the last column of the table.

See table.

Student Workbook


1 Computers for Kids

c. Fill in the third column of the table. Decide how many computers to give to each family if we use B as the basis for giving the computers.

family	number of children	number of computers, using B	number of computers, your way	children per computer, your way
Gray	3			
Hernandez	1			
Ito	2			
Jones	5			
Krantz	1			
Lo	8			

d. Were all 8 computers given out?

e. Does it make sense that the number of computers for one family is not a whole number? Explain your reasoning.

f. Find and describe a way to give computers to the families so that each family gets a whole number of computers. Fill in the fourth column of the table.

g. Compute the number of children per computer in each family and fill in the last column of the table.

h. Do you think your way of giving the computers is fair? Explain your reasoning.

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h. Do you think your way of giving the computers is fair? Explain your reasoning.

Sample response: It's not completely fair because the Hernandez and Krantz children get the computer all to themselves, while the Lo children need to share with 3 others.

Activity Synthesis

The goal of this discussion is to see how distributions are not always equal. First, direct students' attention to the reference created using *Collect and Display*. Ask students to share their response to "Do you think your way of giving the computers is fair?" Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

Invite students to share their answers. Here are some questions for discussion:

- ***"In the first situation, how many children share each computer?"***

  2

- ***"In the second situation, how many children share each computer?"***

  "Are there multiple ways to give out the computers? Which is the most fair?"

Activity 2

School Mascot (Part 1)

15 min

Activity Narrative

This activity uses a voting situation with one vote per class. Similarly to the families with computers, three classrooms need to share three votes. The vote for the class is whichever choice wins a majority in the class election. Students discover that this system is unfair because a class voting heavily for one choice counts for the same as a class barely voting for the choice ("yessiness"). They use division to try to devise a more fair system. Monitor for different ways to assign votes. As students evaluate the fairness of the principal's plan and propose alternatives, they are creating and working with models of the situation.

Note: A banana slug is a bright yellow snail without a shell that lives in redwood forests. In 1986, students at the University of California Santa Cruz voted for the banana slug as their mascot. The administration thought sea lions were more dignified.

Launch

Arrange students in groups of 2. Students start with 5 minutes of quiet think time, followed by comparing work with a partner. Provide access to four-function calculators.

Student Task Statement

A school is deciding on a school mascot. They have narrowed the choices down to a Banana Slug or a Sea Lion.

The principal decided that each class gets one vote. Each class held an election, and the winning choice was the one vote for the whole class. The table shows how three classes voted.

	banana slugs	sea lions	class vote
class A	9	3	banana slug
class B	14	10	
class C	6	30	

1. Which mascot won, according to the principal's plan? What percentage of the votes did the winner get under this plan?

   The banana slugs win with $\frac{2}{3}$, or 67% of the representatives' vote (rounded), since Classes A and B voted for banana slugs.

2. Which mascot received the most student votes in all? What percentage of the votes did this mascot receive?

   Sea lions had the most student votes at 43, which is about 60% of the student vote.

3. The students thought this plan was not very fair. They suggested that bigger classes should have more votes to send to the principal. Create a proposal for the principal where there are as few votes as possible, but the votes proportionally represent the number of students in each class.

   Sample responses: The smallest number of representatives to give proportional representation has Class A with 1 vote, Class B with 2 votes, and Class C with 3 votes.

   Using fractions, Class A has 12 students, or $\frac{12}{72} = \frac{1}{6}$ of all the students, Class B has 24 students for $\frac{24}{72} = \frac{2}{6}$ of the students, and Class C has 36 students for $\frac{36}{72} = \frac{3}{6}$ of the students. So 6 representatives can be shared fairly among the three classes.

   Another method: The greatest common divisor of 12, 24, and 36 is 12. A proportional system of votes would give one vote to every 12 students, so Class A would get 1 vote, Class B 2 votes, and Class C 3 votes.

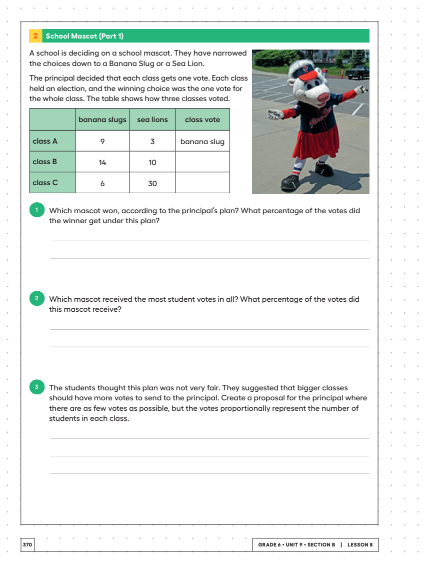
   Other choices are possible, but they will need more than 6 representatives. For representation to be exactly proportional, there must be a multiple of 6 representatives.

Building on Student Thinking

If students are confused about the two-step process (each class votes, then the representatives vote again), consider asking:

> *"Who is involved in the final vote?"*
> *"How do they know how to represent their class?"*

Student Workbook



2 School Mascot (Part 1)

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1. Which mascot won, according to the principal's plan? What percentage of the votes did the winner get under this plan?
2. Which mascot received the most student votes in all? What percentage of the votes did this mascot receive?
3. The students thought this plan was not very fair. They suggested that bigger classes should have more votes to send to the principal. Create a proposal for the principal where there are as few votes as possible, but the votes proportionally represent the number of students in each class.

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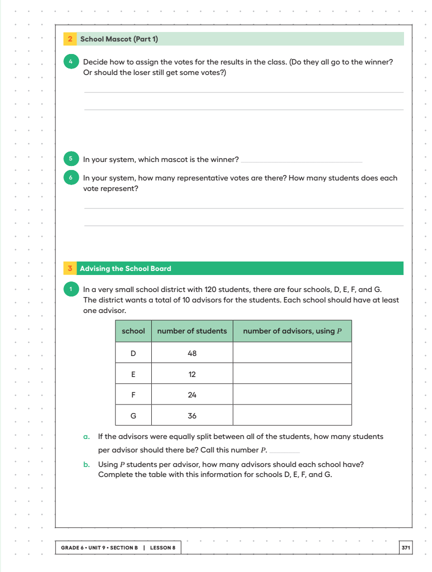
Student Workbook


2 School Mascot (Part 1)

4 Decide how to assign the votes for the results in the class. (Do they all go to the winner? Or should the loser still get some votes?)

5 In your system, which mascot is the winner?

6 In your system, how many representative votes are there? How many students does each vote represent?

3 Advising the School Board

1 In a very small school district with 120 students, there are four schools, D, E, F, and G. The district wants a total of 10 advisors for the students. Each school should have at least one advisor.

school	number of students	number of advisors, using P
D	48	
E	12	
F	24	
G	36	

a. If the advisors were equally split between all of the students, how many students per advisor should there be? Call this number P.

b. Using P students per advisor, how many advisors should each school have? Complete the table with this information for schools D, E, F, and G.

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4. Decide how to assign the votes for the results in the class. (Do they all go to the winner? Or should the loser still get some votes?)

Sample responses: If the choice winning a majority in a class determines all the votes for the class, then A gives its one vote to banana slugs, B gives 2 votes for banana slugs, and all 3 of C's votes go to sea lions. It's a tie!

If you try to assign votes proportionally within classes, then Class B should probably give one vote for banana slugs and one for sea lions, since the numbers are fairly close. This would give 2 total class votes for banana slugs and 4 for sea lions. Now sea lions win.

5. In your system, which mascot is the winner? Sea lions

Sample responses: Sea lions win if B's votes are split between banana slugs and sea lions.

It's a tie if the winner of the class election gets all the votes for that class.

6. In your system, how many representative votes are there? How many students does each vote represent?

Sample response: There are 6 representatives who vote. Each vote represents 12 students.

Activity Synthesis

Invite students to share their ways to assign votes and the reasons for their system. Select students who have different methods.

Students should recognize, after discussion, that the system the principal proposed is unfair. A majority in a small class voting for banana slugs can overwhelm a larger number voting for sea lions in a larger class.

A more fair system should take the sizes of the classes into account.

Activity 3

Advising the School Board

20 min

Activity Narrative

This activity includes two problems of assigning representatives proportionally, with schools sending students to advise the school board. In the first problem, school sizes have been carefully planned so that each school has the same number of students per representative as the district as a whole. In the second, this is not possible, in part because of a very large and a very small school.

The mathematics is the same as in the previous activity distributing computers to families. It is also the same as in the problem of assigning congressional representatives to states. Some states have very large populations, and others have very small populations. As students grapple with finding a fair representation they reason abstractly and quantitatively.

Launch

Arrange students in groups of 2–4. Provide access to four-function calculators, if desired.

Explain the situation:

“The school board (the elected people who make major decisions about all the schools) wants students from the schools to help them make a decision, and to give the board advice about what the students at each school think.”

“The school board would like 10 students to be chosen to come to school board meetings. These students will be called advisors.”

“Big schools should send more advisors than small schools, but even the tiniest school should send at least one advisor.”

“If possible, the number of advisors should be proportional to the number of students at the school.”

As students work, select students who assign advisors in different ways. Ask them to share later.

Student Task Statement

1. In a very small school district with 120 students, there are four schools, D, E, F, and G. The district wants a total of 10 advisors for the students. Each school should have at least one advisor.

school	number of students	number of advisors, using P
D	48	4
E	12	1
F	24	2
G	36	3

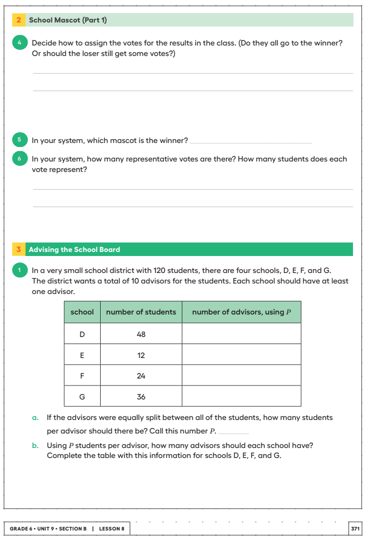
a. If the advisors were equally split between all of the students, how many students per advisor should there be? Call this number P.

12

b. Using P students per advisor, how many advisors should each school have? Complete the table with this information for schools D, E, F, and G.

See table.

Student Workbook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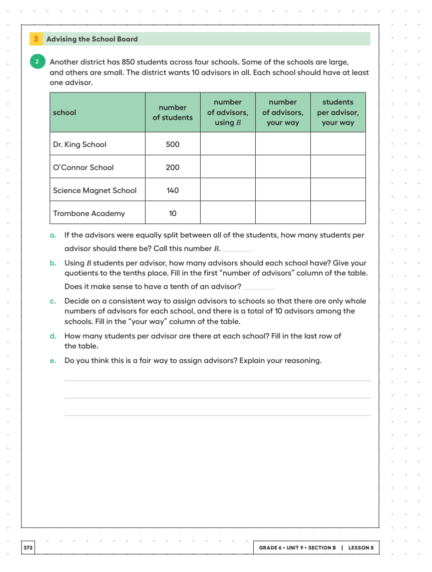
Student Workbook



2. Another district has 850 students across four schools. Some of the schools are large, and others are small. The district wants 10 advisors in all. Each school should have at least one advisor.

school	number of students	number of advisors, using B	number of advisors, your way	students per advisor, your way
Dr. King School	500	5.9	6	83.3
O'Connor School	200	2.4	2	100
Science Magnet School	140	1.6	1	140
Trombone Academy	10	0.1	1	10

a. If the advisors were equally split between all of the students, how many students per advisor should there be? Call this number B.

85

b. Using B students per advisor, how many advisors should each school have? Give your quotients to the tenths place. Fill in the first "number of advisors" column of the table.

See table.

Does it make sense to have a tenth of an advisor?

It doesn't make sense to have a tenth of an advisor because you can't have a fraction of a person.

c. Decide on a consistent way to assign advisors to schools so that there are only whole numbers of advisors for each school, and there is a total of 10 advisors among the schools. Fill in the "your way" column of the table.

Sample response: Round to the nearest whole number, and then adjust. The first attempt gives the Trombone Academy no advisors. The second attempt gives them one of the two from the Science Magnet.

d. How many students per advisor are there at each school? Fill in the last row of the table.

See table.

e. Do you think this is a fair way to assign advisors? Explain your reasoning.

Sample response: This method is unfair because the Trombone Academy is very small but still gets one advisor, and there are 10 students per advisor, compared to the Science Magnet, with 140 students per advisor. Dr. King School gets almost the ideal number of students per advisor since 83.3 is close to 85.

Activity Synthesis

Invite previously selected students to present their plans for assigning advisors.

If not brought up by students, highlight that the ideal number of students per representative is 85, an average. The Dr. King School is very close to this ideal. The other schools have much higher or lower numbers.

Make sure that students understand that the big idea here is that it’s impossible to be completely fair. The students from Trombone Academy will have more influence with their representatives than will the students at the bigger schools because their advisor is representing only 10 students. On the other hand, if the Trombone Academy gets no advisors, then their views aren’t represented at all, which also isn’t fair.