Lesson 10

Finding and Interpreting the Mean as the Balance Point

Goals

- Calculate and interpret (orally and in writing) distances between data points and the mean of the data set.
- Interpret diagrams that represent the mean as a "balance point" for both symmetrical and non-symmetrical distributions.
- Represent the mean of a data set on a dot plot and interpret it in the context of the situation.

Learning Targets

- I can describe what the mean tells us in the context of the data.
- I can explain how the mean represents a balance point for the data on a dot plot.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)

Instructional Routines

- MLR5: Co-Craft Questions
- Which Three Go Together?

Lesson Narrative

In this lesson, students use the structure of the data to interpret the mean as the balance point of a numerical distribution. They calculate how far away each data point is from the mean and recognize that the distances on either side of the mean have the same sum.

Students connect this interpretation to why we call the mean a **measure of the center** of a distribution and, through this interpretation, begin to see how the mean is useful in characterizing a typical value for the group. Students continue to practice calculating the mean of a data set and interpreting it in context.

Student Learning Goal

Let's look at another way to understand the mean of a data set.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Instructional Routines

Which Three Go Together?

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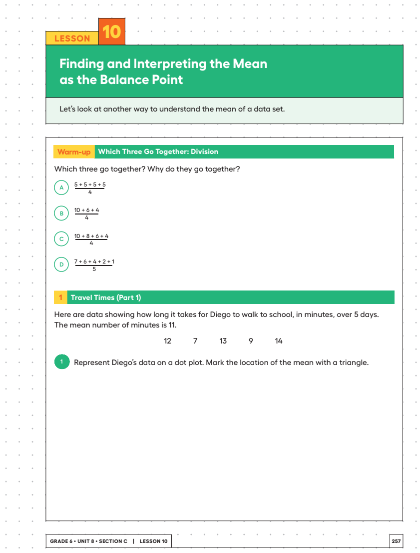
Student Workbook


LESSON 10

Finding and Interpreting the Mean as the Balance Point

Let's look at another way to understand the mean of a data set.

Warm-up Which Three Go Together: Division

Which three go together? Why do they go together?

A $\frac{5+5+5+5}{4}$

B $\frac{10+6+4}{4}$

C $\frac{10+8+6+4}{4}$

D $\frac{7+6+4+2+1}{5}$

1 Travel Times (Part 1)

Here are data showing how long it takes for Diego to walk to school, in minutes, over 5 days. The mean number of minutes is 11.

12 7 13 9 14

1 Represent Diego's data on a dot plot. Mark the location of the mean with a triangle.

GRADE 6 • UNIT 8 • SECTION C | LESSON 10 257

Warm-up

Which Three Go Together: Division

5 min

Activity Narrative

This *Warm-up* prompts students to compare four expressions. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group, and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $\frac{5+5+5+5}{4}$

B. $\frac{10+6+4}{4}$

C. $\frac{10+8+6+4}{4}$

D. $\frac{7+6+4+2+1}{5}$

Sample responses:

- A, B, and C go together because they have 4 in the denominator.
- A, B, and D go together because the sum in the numerator is 20.
- A, C, and D go together because they could be used to calculate a mean.
- B, C, and D go together because they have different numbers in the numerator.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology that they use, such as "numerator," "denominator," "mean," or "sum," and to clarify their reasoning as needed. Consider asking:

"How do you know ... ?"

"What do you mean by ... ?"

"Can you say that in another way?"

If it is not mentioned, bring up that options A, C, and D can go together because they could be used to calculate a mean.

Activity 1

Travel Times (Part 1)

15 min

Activity Narrative

In this activity, students explore the idea of the mean as a **measure of center** of all the values in the data, using a dot plot to help them visualize this idea. Students determine the distance between each data point and the mean, and notice that the sum of distances to the left is equal to the sum of distances to the right. In this sense, the mean can been seen as "balancing" the sets of points with smaller values than it and those with larger values. They make use of the structure to calculate the distance between each data point and another point that is not the mean to see that the sums on the two sides are *not* equal. The idea of the mean as a measure of center of a distribution is introduced in this context.

As students work and discuss, identify those who could articulate why the mean can be considered a balancing point of a data set.

Launch

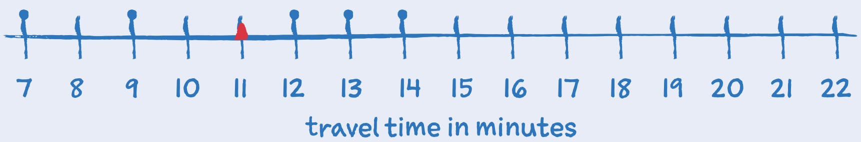
Arrange students in groups of 2. Give students 5 minutes to complete the first two questions with a partner, and then 5 minutes of quiet work time to complete the last two questions. Follow with a whole-class discussion.

Student Task Statement

Here are data showing how long it takes for Diego to walk to school, in minutes, over 5 days. The mean number of minutes is 11.

12 7 13 9 14

1. Represent Diego's data on a dot plot. Mark the location of the mean with a triangle.



2. The mean can also be seen as a **measure of center** that balances the points in a data set. If we find the distance between every point and the mean, add the distances on each side of the mean, and compare the two sums, we can see this balancing.

Access for Students with Diverse Abilities (Activity 1, Student Task)

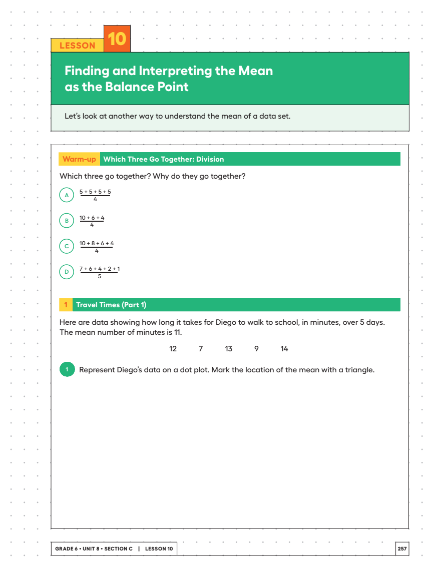
Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Within the first 3–5 minutes, check in with students to provide feedback and encouragement. Check to see how they calculate the distance between each point and the mean. If necessary, remind students that distances are positive.

Supports accessibility for: Attention, Social-Emotional Functioning

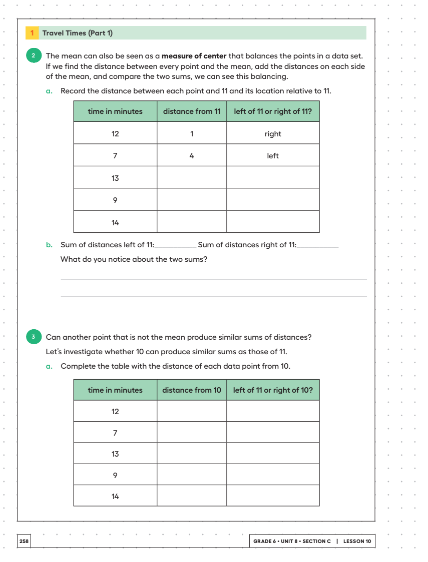
Building on Student Thinking

Some students might write negative values for distances between the mean and points to the left of the mean. They might recall looking at distances between 0 and numbers to the left of it in a previous unit and mistakenly think that numbers to the left of the mean would have a negative distance from the mean. Remind students that distances are always positive—the answer to "How far away?" or "How many units away?" cannot be a negative number.

Student Workbook


LESSON 10
Finding and Interpreting the Mean as the Balance Point
Let's look at another way to understand the mean of a data set.
Warm-up Which Three Go Together: Division
Which three go together? Why do they go together?
1 Travel Times (Part 1)
Here are data showing how long it takes for Diego to walk to school, in minutes, over 5 days. The mean number of minutes is 11.
12 7 13 9 14
1 Represent Diego's data on a dot plot. Mark the location of the mean with a triangle.
GRADE 6 • UNIT 8 • SECTION C | LESSON 10 257

Student Workbook


1 Travel Times (Part 1)

2 The mean can also be seen as a **measure of center** that balances the points in a data set. If we find the distance between every point and the mean, add the distances on each side of the mean, and compare the two sums, we can see this balancing.

a. Record the distance between each point and 11 and its location relative to 11.

time in minutes	distance from 11	left of 11 or right of 11?
12	1	right
7	4	left
13		
9		
14		

b. Sum of distances left of 11: ______ Sum of distances right of 11: ______

What do you notice about the two sums?

3 Can another point that is not the mean produce similar sums of distances?

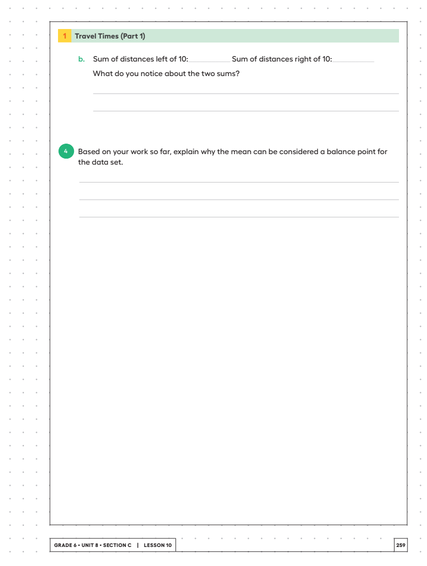
Let's investigate whether 10 can produce similar sums as those of 11.

a. Complete the table with the distance of each data point from 10.

time in minutes	distance from 10	left of 11 or right of 10?
12		
7		
13		
9		
14		

258 GRADE 6 • UNIT 8 • SECTION C | LESSON 10

Student Workbook


1 Travel Times (Part 1)

b. Sum of distances left of 10: ______ Sum of distances right of 10: ______

What do you notice about the two sums?

4 Based on your work so far, explain why the mean can be considered a balance point for the data set.

GRADE 6 • UNIT 8 • SECTION C | LESSON 10 259

a. Record the distance between each point and 11 and its location relative to 11.

time in minutes	distance from 11	left of 11 or right of 11?
12	1	right
7	4	left
13	2	right
9	2	left
14	3	right

b. Sum of distances left of 11: 6
Sum of distances right of 11: 6

What do you notice about the two sums?

The sum of the distances to the left of 11 is 4 + 2 = 6. The sum of the distances to the right of 11 is 1 + 2 + 3 = 6. The two sums are equal.

3. Can another point that is not the mean produce similar sums of distances?

Let's investigate whether 10 can produce similar sums as those of 11.

a. Complete the table with the distance of each data point from 10.

time in minutes	distance from 10	left of 10 or right of 10?
12	2	right
7	3	left
13	3	right
9	1	left
14	4	right

b. Sum of distances left of 10: 4
Sum of distances right of 10: 9

What do you notice about the two sums?

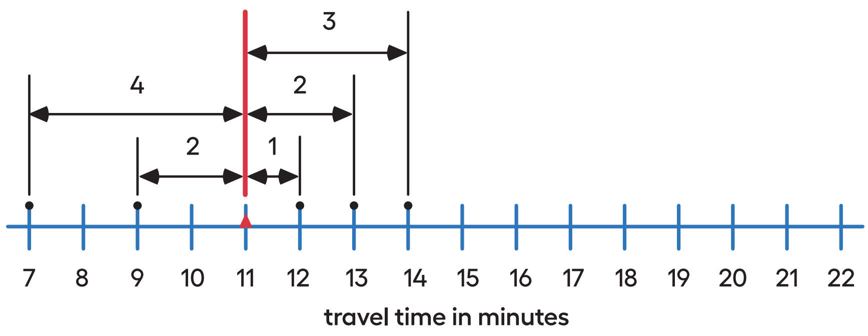
Sample reasoning: The sum of the distances to the left of 10 is 3 + 1 = 4. The sum of the distances to the right of 10 is 2 + 3 + 4 = 9. The two sums are not equal.

4. Based on your work so far, explain why the mean can be considered a balance point for the data set.

The sum of distances to the left of the mean is equal to the sum of distances to the right of the mean, so the mean balances the data values that are larger and those that are smaller. If a number is not the mean of the data set, then the sum of distances to the left and the sum of the distances to the right of it are not equal.

Activity Synthesis

Select a couple of students to share their observations on the distances between Diego's mean travel time and other points. To facilitate discussion, display this dot plot (with the distances labeled) for all to see. Discuss how the sums of distances change when different points are chosen as a reference from which deviations are measured.



Ask:

"What did you notice about the sums of distances to the mean?"

The sums of the distances to the mean are the same for each side.

"If you choose another point or location on the number line, would it produce equal sums of distances to the left and to the right?"

No, for any other point, one of the sums will be greater.

Highlight the idea that only the mean could produce an equal sum of distances. Remind students that they have previously described centers of data sets. Explain that the mean is used as a *measure of center* of a distribution because it balances the values in a data set. Because data points that are greater than the mean balance with those that are less than the mean, the mean is used to describe what is typical for a data set.

Activity 2
Travel Times (Part 2)

15 min

Activity Narrative

This activity reinforces the idea of the mean as a balance point and a measure of center of a distribution. It also introduces the idea that distances of data points from the mean can help us describe variability in data, which prepares students to think about mean absolute deviation in the next lesson. In addition, students practice both calculating the mean of a distribution and interpreting it in context.

As students work, notice those who may need additional prompts to perform these tasks. Also listen for students' explanations on what a larger mean tells us in this context. Identify those who can clearly distinguish how the mean differs from deviations from the mean.

Instructional Routines

MLR5: Co-Craft Questions

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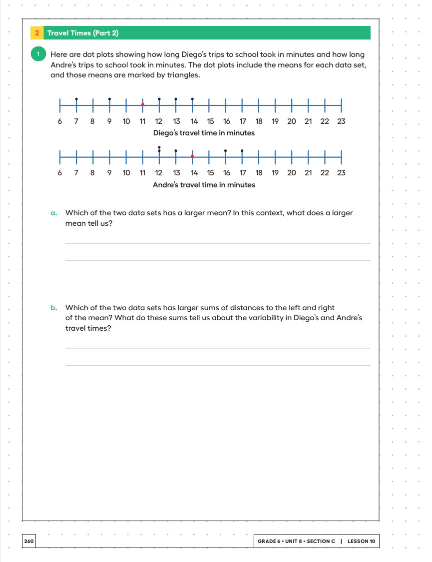
Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions. This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook



Launch

Arrange students in groups of 2. Remind students of the context of travel time to school. Use *Co-Craft Questions* to orient students to the context, and elicit possible mathematical questions.

Display only the problem stem and image of Diego's and Andre's dot plots, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "mean," "distribution," and "spread."

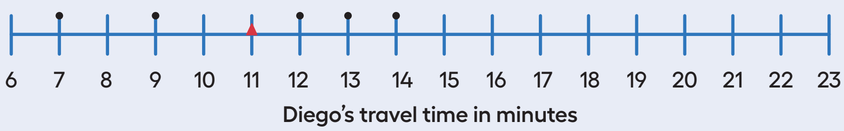
Reveal the questions about the mean and sums of distances to the left and right of the mean, and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

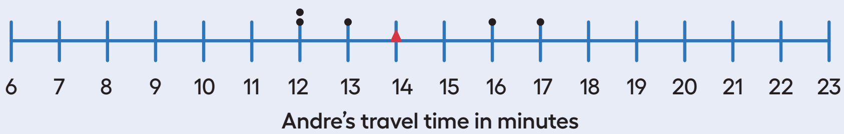
"Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it."

"How do your questions relate to spread?"

Student Task Statement

1. Here are dot plots showing how long Diego's trips to school took in minutes and how long Andre's trips to school took in minutes. The dot plots include the means for each data set, and those means are marked by triangles.





a. Which of the two data sets has a larger mean? In this context, what does a larger mean tell us?

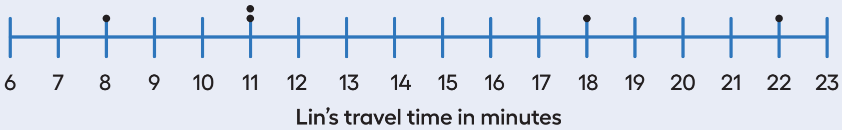
Andre's data set has a larger mean, because Andre's mean is 14 minutes, and Diego's mean is 11 minutes. In the context of this problem, this implies that Andre's average travel time is longer than Diego's average travel time.

b. Which of the two data sets has larger sums of distances to the left and right of the mean? What do these sums tell us about the variability in Diego's and Andre's travel times?

The sum of distances to the left and right of the mean for Diego's data set is $(11-7)+(11-9)+(12-11)+(13-11)+(14-11)=4+2+1+2+3=12$

The sum of distances for Andre's data set is
$2(14-12)+(14-13)+(16-14)+(17-14)=2 \cdot 2+1+2+3=10$
Diego's sum is greater than Andre's, which implies that the variability of Diego's travel times is greater.

2. Here is a dot plot showing lengths of Lin's trips to school.



a. Calculate the mean of Lin's travel times.

The mean is $\frac{8+11+11+18+22}{5} = \frac{70}{5} = 14$.

b. Complete the table with the distance between each point and the mean as well whether the point is to the left or right of the mean.

time in minutes	distance from the mean	left or right of the mean?
22	8	right
18	4	right
11	3	left
8	6	left
11	3	left

c. Find the sum of distances to the left of the mean and the sum of distances to the right of the mean.

Sum of distances to the left: $3 + 3 + 6 = 12$

Sum of distances to the right: $8 + 4 = 12$

d. Use your work to compare Lin's travel times to Andre's. What can you say about their average travel times? What about the variability in their travel times?

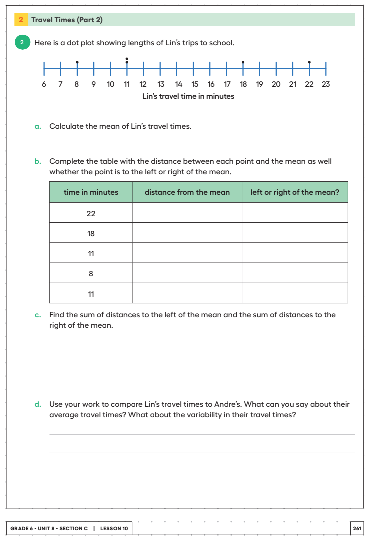
Sample response: Lin's average travel time is the same as Andre's (both have a mean of 14 minutes), and these are both greater than Diego's. The sum of the distances of data points from Lin's mean is much larger than Andre's and Diego's sums. I think this implies that Lin's travel times are much more varied than Andre's and Diego's travel times.

Activity Synthesis

The two big ideas to emphasize during discussion are: what the means tell us in this context, and what the sums of distances to either side of each mean tell us about the travel times.

Select a couple of students to share their analyses of Diego and Andre's travel times. After each student explains, briefly poll the class for agreement or disagreement. If one or more students disagree with an analysis, ask for their reasoning and alternative explanations.

Student Workbook


2 Travel Times (Part 2)

2 Here is a dot plot showing lengths of Lin's trips to school.

a. Calculate the mean of Lin's travel times. ____

b. Complete the table with the distance between each point and the mean as well whether the point is to the left or right of the mean.

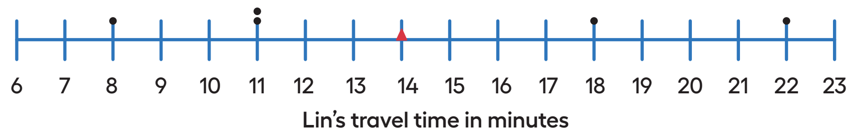
time in minutes	distance from the mean	left or right of the mean?
22		
18		
11		
8		
11		

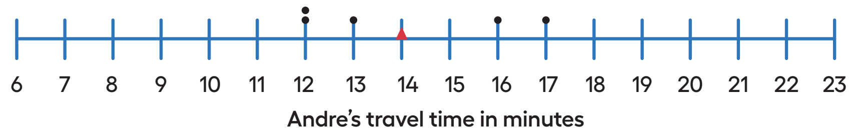
c. Find the sum of distances to the left of the mean and the sum of distances to the right of the mean.

d. Use your work to compare Lin's travel times to Andre's. What can you say about their average travel times? What about the variability in their travel times?

GRADE 6 • UNIT 8 • SECTION C | LESSON 10 261

Then, focus the conversation on how Lin and Andre's travel times compare. Display the dot plots of their travel times for all to see.





Discuss:

- *"How do the data points in Lin's dot plot compare to those in Andre's?"*

  Lin's dot plot shows the dots much more spread out than the dots in Andre's dot plot.

- *"How do their means compare? How do their sums of distances from the mean compare?"*

  They have the same mean, but the sum on each side for Andre is 5 while the sum on each side for Lin is 12.

- *"What do the sums of distances tell us about the travel times?"*

  This tells us that Andre's travel times are more consistent. He often travels around 14 minutes. Lin's travel times are much more variable.

- *"If more than half of Lin's data points are far from the mean of 14 minutes, is the mean still a good description of her typical travel time? Why or why not?"*

  Probably not. The mean still gives a central value, but because the times are so variable, it does not give a good idea of what her usual travel is like.

Lesson Synthesis

In this lesson, we learn that the mean can be interpreted as the balance point of a distribution.

- *"How does the mean balance the distribution of a data set?"*

  Thinking of each point on the dot plot as having the same weight, the mean would be where the dot plot is balanced so that the weights are evenly distributed on either side of the mean.

- *"How can a dot plot help us make sense of this interpretation?"*

  Because balance and weight are more natural to think about, it can make it easy to estimate where the balance point is.

- *"Could another value—besides the mean—balance a data distribution? How can we tell?"*

  No. If another point is used, the total distances on the left and right will not be equal.

We also learn that the mean is used as a **measure of center** of a distribution, or a number that summarizes the center of a distribution.

"Why might it make sense for the mean to be a number that describes the center of a distribution?"

In order to balance, the mean needs to be somewhere near the center of the distribution.

"In earlier lessons, we had used an estimate of the center of a distribution to describe what is typical or characteristic of a group. Why might it make sense to use the mean to describe a typical feature of a group?"

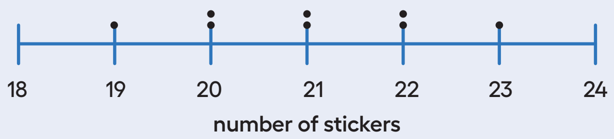
The mean is a measure of center, so it could be used as a typical value to describe a distribution.

Lesson Summary

The mean is often used as a **measure of center** of a distribution. One way to see this is that the mean of a distribution can be seen as the "balance point" for the distribution. Why is this a good way to think about the mean? Let's look at a very simple set of data on the number of stickers that are on 8 pages:

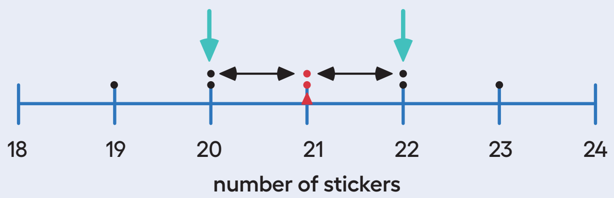
19 20 20 21 21 22 22 23

Here is a dot plot showing the data set.

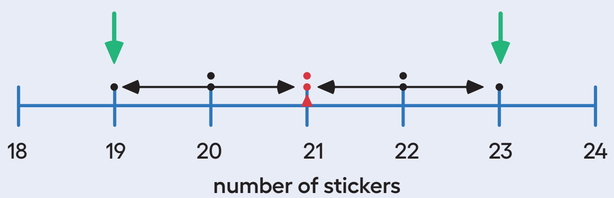


The distribution shown is completely symmetrical. The mean number of stickers is 21, because $(19 + 20 + 20 + 21 + 21 + 22 + 22 + 23) \div 8 = 21$. If we mark the location of the mean on the dot plot, we can see that the data points could balance at 21.

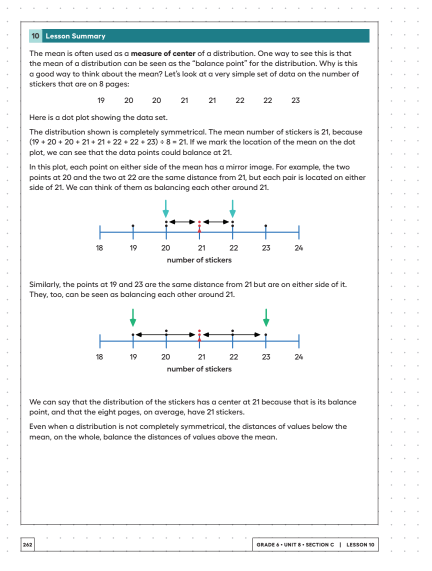
In this plot, each point on either side of the mean has a mirror image. For example, the two points at 20 and the two at 22 are the same distance from 21, but each pair is located on either side of 21. We can think of them as balancing each other around 21.



Similarly, the points at 19 and 23 are the same distance from 21 but are on either side of it. They, too, can be seen as balancing each other around 21.



Student Workbook



Responding To Student Thinking

Points to Emphasize
If students struggle with understanding variability, then revisit and clarify the idea of variability in this lesson: Unit 8, Lesson 11 Variability and MAD

We can say that the distribution of the stickers has a center at 21 because that is its balance point, and that the eight pages, on average, have 21 stickers.

Even when a distribution is not completely symmetrical, the distances of values below the mean, on the whole, balance the distances of values above the mean.

Cool-down

Text Messages

5 min

Student Task Statement

The three data sets show the number of text messages sent to their parents by Jada, Diego, and Lin over 6 days. One of the data sets has a mean of 4, one has a mean of 5, and one has a mean of 6.

Jada

4	4	4	6	6	6

Diego

4	5	5	6	8	8

Lin

1	1	2	2	9	9

1. Which data set has which mean?

   Jada's mean is 5, since $\frac{4+4+4+6+6+6}{6} = \frac{30}{6} = 5$.

   Diego's mean is 6, since $\frac{4+5+5+6+8+8}{6} = \frac{36}{6} = 6$.

   Lin's mean is 4, since $\frac{1+1+2+2+9+9}{6} = \frac{24}{6} = 4$.

   What does this tell you about the text messages sent by the three students?

   On average, Diego sent the most text messages to his parents per day, and Lin sent the fewest text messages per day to her parents.

2. Which data set has the greatest variability? Explain your reasoning.

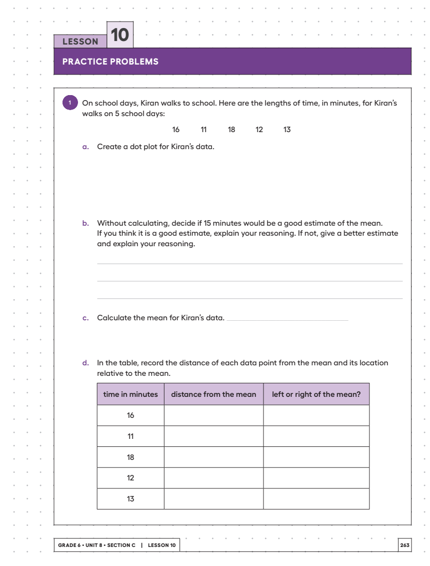
   Sample response: Lin's data has the highest variability. The sum of the distances to each side of the mean is the greatest.

Lesson 10

Practice Problems

5 Problems

Student Workbook


LESSON 10

PRACTICE PROBLEMS

1 On school days, Kiran walks to school. Here are the lengths of time, in minutes, for Kiran's walks on 5 school days:

16 11 18 12 13

a. Create a dot plot for Kiran's data.

b. Without calculating, decide if 15 minutes would be a good estimate of the mean. If you think it is a good estimate, explain your reasoning. If not, give a better estimate and explain your reasoning.

c. Calculate the mean for Kiran's data.

d. In the table, record the distance of each data point from the mean and its location relative to the mean.

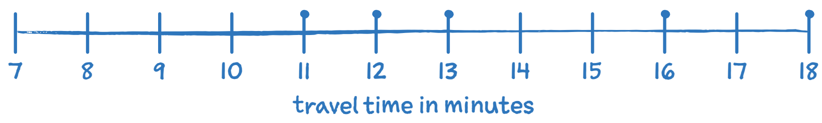
time in minutes	distance from the mean	left or right of the mean?
16		
11		
18		
12		
13		

Problem 1

On school days, Kiran walks to school. Here are the lengths of time, in minutes, for Kiran's walks on 5 school days:

16 11 18 12 13

a. Create a dot plot for Kiran's data.



b. Without calculating, decide if 15 minutes would be a good estimate of the mean. If you think it is a good estimate, explain your reasoning. If not, give a better estimate and explain your reasoning.

Sample response: 15 minutes seems to be a little too high an estimate for the mean, because (looking at the dot plot) the sum of the distances between the mean and the points to its left seems to be greater than the sum of the distances between it and the points to its right. A better estimate would be 14 minutes, because the sums of distances to its left and the right would be more balanced.

c. Calculate the mean for Kiran's data.

$\frac{11 + 12 + 13 + 16 + 18}{5} = 14.$

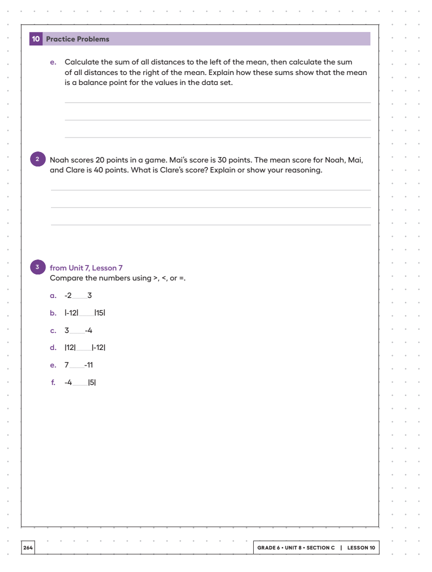
d. In the table, record the distance of each data point from the mean and its location relative to the mean.

time in minutes	distance from the mean	left or right of the mean?
16	2	right
11	3	left
18	4	right
12	2	left
13	1	left

e. Calculate the sum of all distances to the left of the mean, then calculate the sum of all distances to the right of the mean. Explain how these sums show that the mean is a balance point for the values in the data set.

The sum of the distances to the left: $3 + 2 + 1 = 6$. The sum of the distances to the right: $4 + 2 = 6$. The sum of distances on the left of the mean is equal to the sum of distances to the right of the mean, which tells us that the data values are balanced on the mean.

Student Workbook


10 Practice Problems

e. Calculate the sum of all distances to the left of the mean, then calculate the sum of all distances to the right of the mean. Explain how these sums show that the mean is a balance point for the values in the data set.

2 Noah scores 20 points in a game. Mai's score is 30 points. The mean score for Noah, Mai, and Clare is 40 points. What is Clare's score? Explain or show your reasoning.

3 from Unit 7, Lesson 7

Compare the numbers using >, <, or =.

a. -2 ___ 3

b. |-12| ___ |15|

c. 3 ___ -4

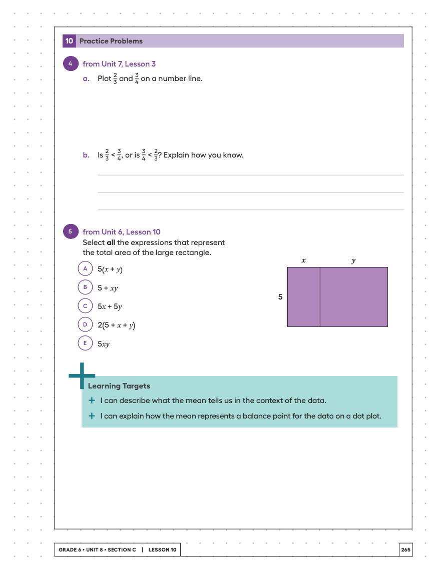
d. |12| ___ |-12|

e. 7 ___ -11

f. -4 ___ |5|

264 GRADE 6 • UNIT 8 • SECTION C | LESSON 10

Student Workbook


10 Practice Problems

4 from Unit 7, Lesson 3

a. Plot $\frac{2}{3}$ and $\frac{3}{4}$ on a number line.

b. Is $\frac{2}{3} < \frac{3}{4}$, or is $\frac{3}{4} < \frac{2}{3}$? Explain how you know.

5 from Unit 6, Lesson 10

Select **all** the expressions that represent the total area of the large rectangle.

A. $5(x + y)$

B. $5 + xy$

C. $5x + 5y$

D. $2(5 + x + y)$

E. $5xy$

Learning Targets

+ I can describe what the mean tells us in the context of the data.

+ I can explain how the mean represents a balance point for the data on a dot plot.

GRADE 6 • UNIT 8 • SECTION C | LESSON 10 265

Problem 2

Noah scores 20 points in a game. Mai's score is 30 points. The mean score for Noah, Mai, and Clare is 40 points. What is Clare's score? Explain or show your reasoning.

70 points

Sample reasoning:

- Clare needs to have a score that would be 30 points to the right of the mean score of 40. This score balances the 30 points at the left of the mean.
- If the mean score is 40 points, Noah's score is 20 points short and Mai's is 10 points short. Clare's score must be 30 points above the mean so that when the points are distributed each person's share is 40 points.

Problem 3

from Unit 7, Lesson 7

Compare the numbers using >, <, or =.

a. -2 <u><</u> 3

b. |-12| <u><</u> |15|

c. 3 <u>></u> -4

d. |12| <u>=</u> |-12|

e. 7 <u>></u> -11

f. -4 <u><</u> |5|

Problem 4

from Unit 7, Lesson 3

a. Plot $\frac{2}{3}$ and $\frac{3}{4}$ on a number line.

The number line should show $\frac{2}{3}$ to the left of $\frac{3}{4}$, and both closer to 1 than to 0.

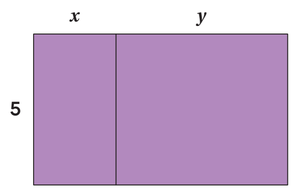
b. Is $\frac{2}{3} < \frac{3}{4}$, or is $\frac{3}{4} < \frac{2}{3}$? Explain how you know.

$\frac{2}{3} < \frac{3}{4}$ because $\frac{2}{3}$ is to the left of $\frac{3}{4}$ on the number line.

Problem 5

from Unit 6, Lesson 10

Select **all** the expressions that represent the total area of the large rectangle.



A. $5(x + y)$

B. $5 + xy$

C. $5x + 5y$

D. $2(5 + x + y)$

E. $5xy$