Lesson 13

Expressions with Exponents

Goals

- Critique (orally and in writing) arguments that claim two different numerical expressions are equal.
- Justify (orally and in writing) whether numerical expressions involving whole-number exponents are equal.

Learning Target

I can decide if expressions with exponents are equal by finding the values of the expressions or by understanding what exponents mean.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Take Turns
- Which Three Go Together?

Required Materials

Materials to Gather

- Math Community Chart: Activity 2

Lesson Narrative

In this lesson, students deepen their understanding of the meaning of exponents by reasoning about the values of exponential expressions. They write, interpret, compare, and evaluate exponential expressions in which the base may or may not be a whole number.

Students also practice looking for and making use of structure to determine if two exponential expressions are equivalent, as well as to write expressions that are equivalent to a given one. As they discuss their way of evaluating and comparing expressions, students have opportunities to construct logical arguments and critique the reasoning of others.

Student Learning Goal

Let's use the meaning of exponents to decide if equations are true.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up
Which Three Go Together: Twos

5 min

Activity Narrative

This *Warm-up* prompts students to compare four expressions. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $2 \cdot 2 \cdot 2 \cdot 2$

B. 16

C. 2^4

D. $4 \cdot 2$

Sample responses:

A, B, and C go together because:

- They are equivalent expressions.
- They are equal to 16.

A, B, and D go together because:

- They don't use exponents.

A, C, and D go together because:

- They represent multiplication.
- They show 2 and 4. (C and D show the numbers, and A shows 4 twos.)

B, C, and D go together because:

- They each use only 2 numbers or digits.

Instructional Routines

Which Three Go Together?

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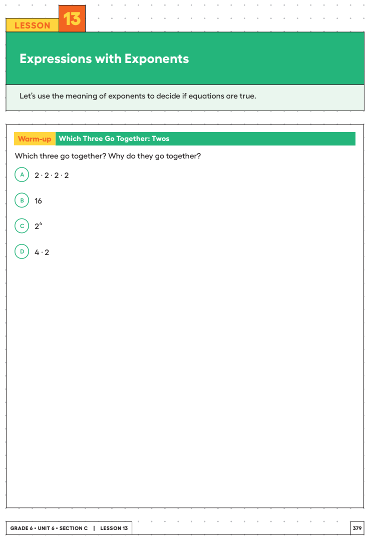
Student Workbook


LESSON 13

Expressions with Exponents

Let's use the meaning of exponents to decide if equations are true.

Warm-up Which Three Go Together: Twos

Which three go together? Why do they go together?

A $2 \cdot 2 \cdot 2 \cdot 2$

B 16

C 2^4

D $4 \cdot 2$

GRADE 6 • UNIT 6 • SECTION C | LESSON 13

Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Instructional Routines

MLR7: Compare and Connect

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Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Begin with the demonstration as described in the *Launch* to support connections between new situations that involve evaluating exponential expressions and prior understandings. Use color or annotations to highlight what changes and what stays the same at each step.

Supports accessibility for: Conceptual processing, Visual-Spatial Processing

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "numbers," "digits," or "exponents," and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Activity 1

Is the Equation True?

15 min

Activity Narrative

The purpose of this activity is to give students opportunities to look for and make use of structure to compare exponential expressions and determine if statements about them are true. These problems can also be reasoned by evaluating the expressions, which is fine at this point, students should also look for other ways to reason about each statement.

Monitor for students who use these different strategies:

- Find the value of each expression (for instance, calculate both 5^3 and $5 \cdot 5 \cdot 5$, and see that they both have a value of 125).
- Rewrite expression in an equivalent form to compare structure on both sides of an equation (for instance, rewrite $5^3 = 5 \cdot 5 \cdot 5$ as $5^3 = 5^3$.
- Reason about the meaning of the operations on both sides of an equation (for instance, reason that 5^3 means $5 \cdot 5 \cdot 5$.

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time, followed by time to discuss their thinking with a partner before completing the remainder of the activity.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later. If most students are evaluating every given expression, consider asking:

"Are there ways to compare the expressions on the two sides of the equal sign without calculating their values?"

Student Task Statement

Decide whether each equation is true or false. Explain or show how you know.

1. $2^4 = 2 \cdot 4$

   False

   Sample reasoning: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ and $2 \cdot 4 = 2 \cdot 2 \cdot 2$.

2. $3 + 3 + 3 + 3 + 3 = 3^5$

   False

   Sample reasoning: $3 + 3 + 3 + 3 + 3 = 15$ and $3^5 = 243$.

3. $5^3 = 5 \cdot 5 \cdot 5$

   True

   Sample reasoning: $5 \cdot 5 \cdot 5$ is what 5^3 means.

4. $2^3 = 3^2$

   False

   Sample reasoning: $2^3 = 8$ and $3^2 = 9$.

5. $16^1 = 8^2$

   False

   Sample reasoning: $16^1 = 16$ and $8^2 = 64$.

6. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$

   False

   Sample reasoning: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$ and $4 \cdot \frac{1}{2} = 2$.

7. $\left(\frac{1}{2}\right)^4 = \frac{1}{8}$

   False

   Sample reasoning: $\frac{1}{8}$ is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$.

8. $8^2 = 4^3$

   True

   Sample reasoning: Both sides of the equation equal 64.

Activity Synthesis

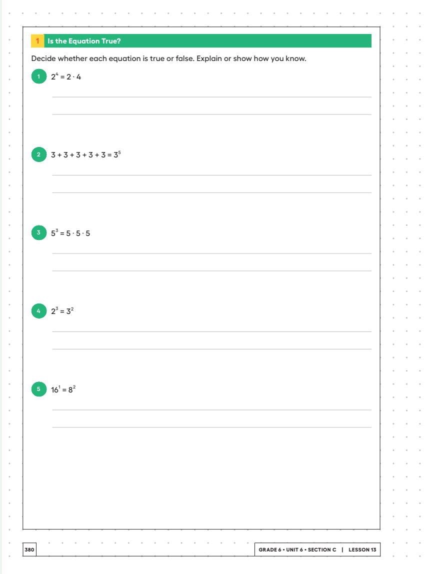
The goal of this discussion is to draw students' attention to the structure of expressions and the meaning of exponent notation. Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do these approaches have in common? How are they different?"

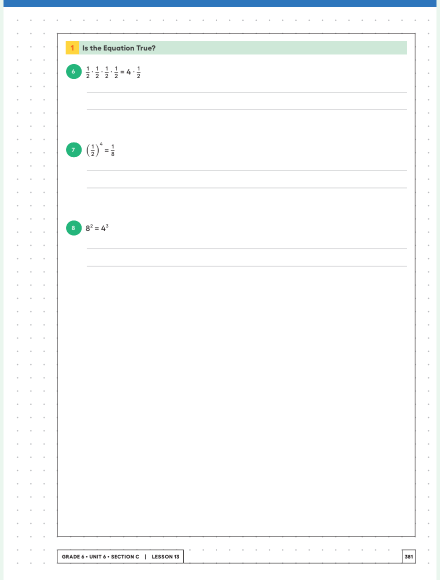
"How does repeated multiplication show up in each method?"

"Are there any benefits or drawbacks to one approach compared to another?"

Student Workbook



Student Workbook



Instructional Routines

Take Turns

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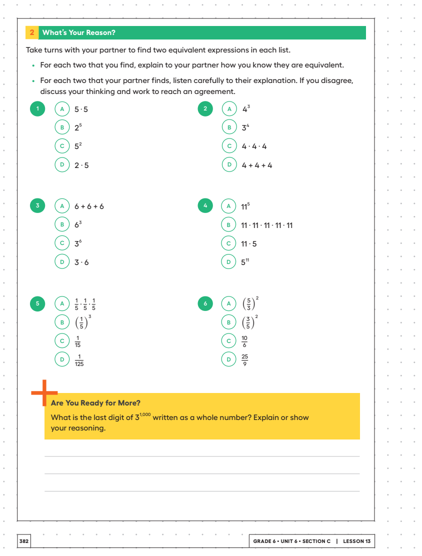
Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames, such as "_____ and _____ are equivalent because …" and "I agree/disagree because …"

Supports accessibility for: Language, Organization

Student Workbook



Activity 2

What's Your Reason?

15 min

Activity Narrative

In this partner activity, students take turns explaining why they think two expressions are equivalent. The reasoning here allows students to practice looking for and making use of structure in analyzing exponential expressions. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms, or invite a student to read them out loud. Tell students that during this activity, they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norms supported the mathematical community during the activity.

Arrange students in groups of 2. Display the task for all to see. Tell students that for each group of 4 expressions, there are two equivalent expressions, and maybe more. If time allows, choose a student to be your partner and demonstrate how to set up and do the activity, otherwise share these steps:

- One partner identifies two equivalent expressions from a group of four expressions and explains why they think those expressions are equivalent.
- The other partner listens to the speaker's reasoning and makes sure it makes sense.
- If there is a disagreement, the partners discuss until coming to an agreement.
- Both partners look for any other expressions in the group that are also equivalent and explain their reasoning.
- For the next group of expressions, the students swap roles.

Remind students that we can often tell if two expressions are equivalent by looking for structure in the expressions and applying the meaning of exponents. It is not always necessary to calculate the values of the expressions.

Student Task Statement

Take turns with your partner to find two equivalent expressions in each list.

- For each two that you find, explain to your partner how you know they are equivalent.
- For each two that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

1.

A. $5 \cdot 5$

B. 2^5

C. 5^2

D. $2 \cdot 5$

2.
A. 4^3
B. 3^4
C. $4 \cdot 4 \cdot 4$
D. $4 + 4 + 4$

3.
A. $6 + 6 + 6$
B. 6^3
C. 3^6
D. $3 \cdot 6$

4.
A. 11^5
B. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
C. $11 \cdot 5$
D. 5^{11}

5.
A. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$
B. $\left(\frac{1}{5}\right)^3$
C. $\frac{1}{15}$
D. $\frac{1}{125}$

6.
A. $\left(\frac{5}{3}\right)^2$
B. $\left(\frac{3}{5}\right)^2$
C. $\frac{10}{6}$
D. $\frac{25}{9}$

Are You Ready for More?

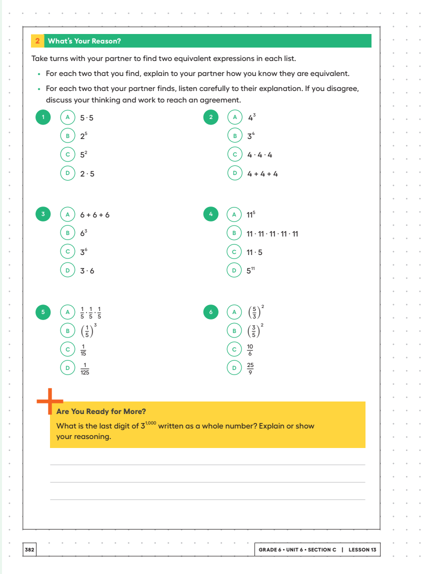
What is the last digit of $3^{1,000}$ written as a whole number? Explain or show your reasoning.

The last digit is 1.

Sample reasoning:

Some experimentation reveals a pattern—the first few powers of 3 are 3, 9, 27, 81, 243, 729, 2,187, 6,561, 19,683, etc. Specifically, the pattern of last digits goes 3, 9, 7, 1, 3, 9, 7, 1, 3, etc., repeating every four terms. So every exponent that is a multiple of 4, as in $3^{1,000}$, has the value of a number whose last digit is a 1.

Student Workbook


2 What's Your Reason?
Take turns with your partner to find two equivalent expressions in each list.
- For each two that you find, explain to your partner how you know they are equivalent.
- For each two that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

1. A $5 \cdot 5$ B 2^5 C 5^2 D $2 \cdot 5$
2. A 4^3 B 3^4 C $4 \cdot 4 \cdot 4$ D $4 + 4 + 4$
3. A $6 + 6 + 6$ B 6^3 C 3^6 D $3 \cdot 6$
4. A 11^5 B $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ C $11 \cdot 5$ D 5^{11}
5. A $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ B $\left(\frac{1}{5}\right)^3$ C $\frac{1}{15}$ D $\frac{1}{125}$
6. A $\left(\frac{5}{3}\right)^2$ B $\left(\frac{3}{5}\right)^2$ C $\frac{10}{6}$ D $\frac{25}{9}$

Are You Ready for More?
What is the last digit of $3^{1,000}$ written as a whole number? Explain or show your reasoning.

Activity Synthesis

Invite students to share their responses and strategies for finding equivalent expressions. Highlight strategies that rely on the structure of the expressions. As students respond, record the equivalent expressions using an equal sign.

Consider asking students:

"Were there any expressions that you and your partner disagreed about and needed to discuss to come to an agreement? If so, which ones?"

"Is it possible to figure out the expressions that are equivalent in each set without calculating the value of any of the expressions?"

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

"I noticed our norm '__________' in action today, and it really helped me/my group because __________."

Lesson Synthesis

The purpose of this discussion is to analyze some of the typical errors and misconceptions that often come up when working with exponent notation. Based on student responses and discussions in the activities, consider asking questions such as:

"Are a^b and b^a equivalent? How do you know?"

No, we can try different values of a and b or use the meaning of exponents to see that a multiplied b times is not always the same as b multiplied a times.

"What change can we make to the equation $3 + 3 + 3 + 3 + 3 = 3^5$ to make it true?"

Change addition to multiplication on the left, or change the exponent to multiply by 5 on the right.

"Your friend claims the equation $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$ is true. What do you think they are misunderstanding? How can you convince them it is false?"

They likely are thinking that multiplying four of a number is the same as multiplying that number by 4. I would explain that $4 \cdot \frac{1}{2}$ means $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, rather than $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$. One expression has a value of 2, while the other is $\frac{1}{16}$. I could show similar examples with other numbers, but the best way to convince them is to talk about what exponents and multiplication mean.

"Can we show that $8^2 = 4^3$ is true without calculating the value of 8^2 or 4^3? How?"

Yes

8^2 means $8 \cdot 8$ or $4 \cdot 2 \cdot 4 \cdot 2$, which also equals $4 \cdot 4 \cdot 2 \cdot 2$ or $4 \cdot 4 \cdot 4$. Another way to write $4 \cdot 4 \cdot 4$ is 4^3. We know we can sometimes write a number as its factors, rearrange the factors, and multiply them in any order.

Lesson Summary

When adding or multiplying numbers, the order of the numbers doesn't affect the result of addition or multiplication. When working with exponents, each number means something specific, so its placement does matter.

$$3 + 4 \text{ equals } 4 + 3.$$

$$a + b \text{ always equals } b + a.$$

$$3 \cdot 4 \text{ equals } 4 \cdot 3.$$

$$a \cdot b \text{ always equals } b \cdot a.$$

$$3^4 \text{ does not equal } 4^3.$$

$$a^b \text{ does not always equal } b^a.$$

$$3^4 \text{ means } 3 \cdot 3 \cdot 3 \cdot 3, \text{ while } 4^3 \text{ means } 4 \cdot 4 \cdot 4.$$

It is also important to remember that we use multiplication as a quicker way to express repeated addition and we use exponent as a quicker way to express repeated multiplication.

$$3 + 3 + 3 + 3 = 4 \cdot 3$$

$$3 \cdot 3 \cdot 3 \cdot 3 = 3^4$$

When working with exponents, the numbers being multiplied don't have to always be whole numbers. They can also be other kinds of numbers, like fractions, decimals, and even variables. For example, we can use exponents in each of the following ways:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$(1.7)^3 = (1.7) \cdot (1.7) \cdot (1.7)$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Cool-down

Grain Calculation

5 min

Student Task Statement

Recall the story about the inventor of chess who asked for grains of rice to be doubled every day, starting with 2 grains on the first day. Andre and Elena knew that after 28 days, the inventor would have 2^{28} grains of rice, but they wanted to find out how many grains that actually is.

- Andre wrote:
  $2^{28} = 2 \cdot 28 = 56$.
- Elena said, "No, exponents mean repeated multiplication. It should be $28 \cdot 28$, which works out to be 784."

Do you agree with either of them? Explain your reasoning.

I disagree with both Andre and Elena.

Sample reasoning:

Andre thinks exponents are just a different way of writing multiplication of two numbers. Elena calculates 28^2 rather than 2^{28}. To find the value of 2^{28}, we have to multiply 2 by itself 28 times.

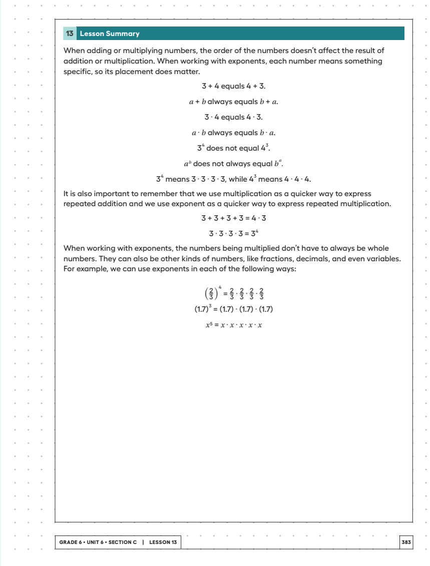
Responding To Student Thinking

Points to Emphasize

If students struggle with understanding the connection between repeated multiplication and exponents, focus on this idea when opportunities arise over the next several lessons. For example, consider inviting students to write expressions in expanded form to make visible the repeated multiplication of the base in exponential expressions in:

Grade 6, Unit 6, Lesson 14 Evaluating Expressions with Exponents

Student Workbook

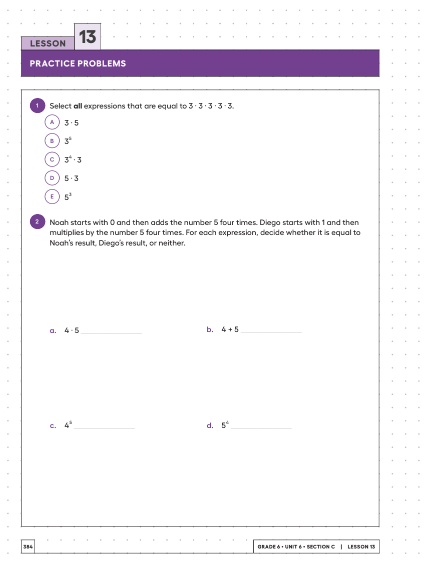


Lesson 13

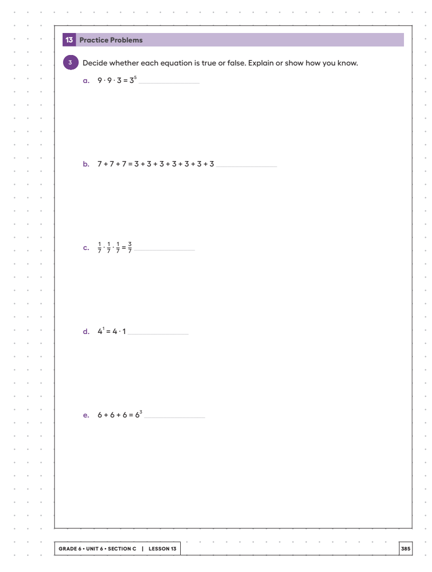
Practice Problems

7 Problems

Student Workbook


LESSON 13
PRACTICE PROBLEMS
1 Select **all** expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.
A $3 \cdot 5$
B 3^5
C $3^4 \cdot 3$
D $5 \cdot 3$
E 5^3
2 Noah starts with 0 and then adds the number 5 four times. Diego starts with 1 and then multiplies by the number 5 four times. For each expression, decide whether it is equal to Noah's result, Diego's result, or neither.
a. $4 \cdot 5$ b. $4 + 5$
c. 4^5 d. 5^4
384 GRADE 6 • UNIT 6 • SECTION C | LESSON 13

Student Workbook


13 Practice Problems
3 Decide whether each equation is true or false. Explain or show how you know.
a. $9 \cdot 9 \cdot 3 = 3^5$
b. $7+7+7=3+3+3+3+3+3+3$
c. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{3}{7}$
d. $4^1 = 4 \cdot 1$
e. $6+6+6=6^3$
GRADE 6 • UNIT 6 • SECTION C | LESSON 13 385

Problem 1

Select **all** expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

A. $3 \cdot 5$

B. 3^5

C. $3^4 \cdot 3$

D. $5 \cdot 3$

E. 5^3

Problem 2

Noah starts with 0 and then adds the number 5 four times. Diego starts with 1 and then multiplies by the number 5 four times. For each expression, decide whether it is equal to Noah's result, Diego's result, or neither.

a. $4 \cdot 5$

Noah's

b. $4 + 5$

Neither

c. 4^5

Neither

d. 5^4

Diego's

Problem 3

Decide whether each equation is true or false. Explain or show how you know.

a. $9 \cdot 9 \cdot 3 = 3^5$ True

Sample reasoning: The expression on the left is equivalent to $(3 \cdot 3) \cdot (3 \cdot 3) \cdot 3 = 3^5$.

b. $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$ True

Sample reasoning: Both sides of the equation are ways of writing $3 \cdot 7$.

c. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{3}{7}$ False

Sample reasoning: $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{1}{7^3}$ or $\frac{1}{343}$ which does not equal $\frac{3}{7}$

d. $4^1 = 4 \cdot 1$ True

Sample reasoning: Both sides equal 4.

e. $6 + 6 + 6 = 6^3$ False

Sample reasoning: $6^3 = 216$, but $6 + 6 + 6 = 18$

Problem 4

a. What is the area of a square with side lengths of $\frac{3}{5}$ units?

$\frac{9}{25}$ square units

$\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

b. What is the side length of a square with area $\frac{1}{16}$ square units?

$\frac{1}{4}$ units

$\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

c. What is the volume of a cube with edge lengths of $\frac{2}{3}$ units?

$\frac{8}{27}$ cubic units

$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$

d. What is the edge length of a cube with volume $\frac{27}{64}$ cubic units?

$\frac{3}{4}$ units

$\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$

Problem 5

from Unit 6, Lesson 10

Select **all** the expressions that represent the area of the shaded rectangle.

A. $3(10 - c)$

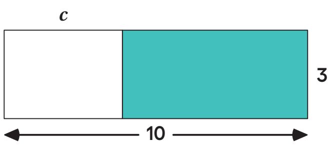
B. $3(c - 10)$

C. $10(c - 3)$

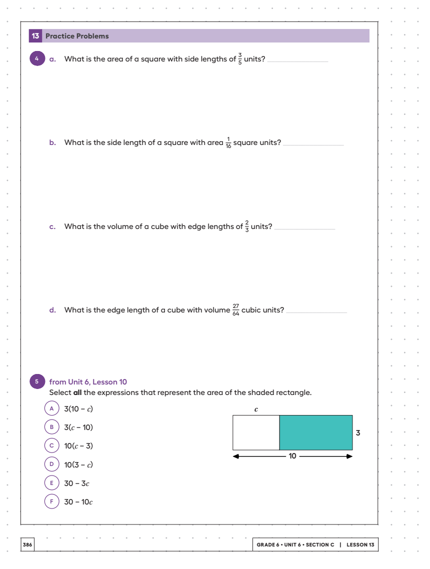
D. $10(3 - c)$

E. $30 - 3c$

F. $30 - 10c$



Student Workbook


13 Practice Problems

4 a. What is the area of a square with side lengths of $\frac{3}{5}$ units?

b. What is the side length of a square with area $\frac{1}{16}$ square units?

c. What is the volume of a cube with edge lengths of $\frac{2}{3}$ units?

d. What is the edge length of a cube with volume $\frac{27}{64}$ cubic units?

5 from Unit 6, Lesson 10

Select **all** the expressions that represent the area of the shaded rectangle.

A $3(10 - c)$

B $3(c - 10)$

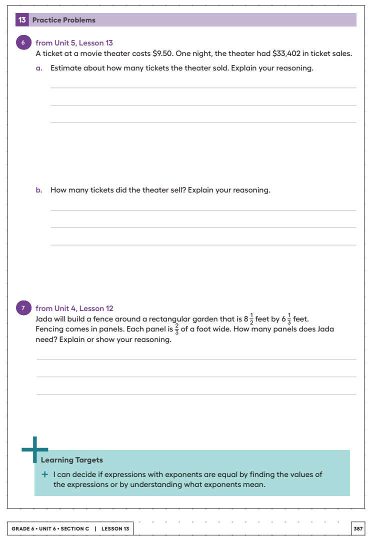
C $10(c - 3)$

D $10(3 - c)$

E $30 - 3c$

F $30 - 10c$

Student Workbook


13 Practice Problems

6 from Unit 5, Lesson 13

A ticket at a movie theater costs $9.50. One night, the theater had $33,402 in ticket sales.

a. Estimate about how many tickets the theater sold. Explain your reasoning.

b. How many tickets did the theater sell? Explain your reasoning.

7 from Unit 4, Lesson 12

Jada will build a fence around a rectangular garden that is $8\frac{1}{2}$ feet by $6\frac{1}{3}$ feet. Fencing comes in panels. Each panel is $\frac{2}{3}$ of a foot wide. How many panels does Jada need? Explain or show your reasoning.

Learning Targets

+ I can decide if expressions with exponents are equal by finding the values of the expressions or by understanding what exponents mean.

GRADE 6 • UNIT 6 • SECTION C | LESSON 13 387

Problem 6

from Unit 5, Lesson 13

A ticket at a movie theater costs $9.50. One night, the theater had $33,402 in ticket sales.

a. Estimate about how many tickets the theater sold. Explain your reasoning.

More than 3,000 tickets were sold.

Sample reasoning:

If there were $30,000 in sales and the tickets were $10 each, then it would be 3,000. The actual tickets are less than $10 and $30,000 is less than the total sales, so the actual answer should be more than 3,000.

b. How many tickets did the theater sell? Explain your reasoning.

3,516 tickets were sold.

Sample reasoning:

$33{,}402 \div 9.5 = 3{,}516$

Problem 7

from Unit 4, Lesson 12

Jada will build a fence around a rectangular garden that is $8\frac{1}{2}$ feet by $6\frac{1}{3}$ feet. Fencing comes in panels. Each panel is $\frac{2}{3}$ of a foot wide. How many panels does Jada need? Explain or show your reasoning.

Sample responses:

- Not reusing panel pieces: 46 panels. For the sides of length $8\frac{1}{2}$ feet, Jada needs $8\frac{1}{2} \div \frac{2}{3}$ panels. This is $\frac{51}{4}$, or $12\frac{3}{4}$, so each of those two sides will use 13 panels of fencing. The other two sides each use $6\frac{1}{3} \div \frac{2}{3}$ panels of fencing, which is $9\frac{1}{2}$ panels. This is 10 panels each.
- Reusing panel pieces: 45 panels. The sides of length $8\frac{1}{2}$ feet each use $12\frac{3}{4}$ panels of fencing, for a total of $25\frac{1}{2}$. The other two sides each use $9\frac{1}{2}$ panels for a total of 19 panels. Jada needs $44\frac{1}{2}$ panels, which means she needs 45 whole panels.