Representing Data Graphically

Goals

- Comprehend that the word "frequency" refers to the number of times that a particular value occurs in a data set.
- Create and interpret a dot plot to answer statistical questions about a numerical data set.
- Justify (in writing) whether a dot plot is an appropriate way to display a given data set, paying attention to whether the data set is numerical or categorical.

Learning Targets

- I can describe the information presented in tables, dot plots, and bar graphs.
- I can use tables, dot plots, and bar graphs to represent distributions of data.

Access for Students with Diverse Abilities

- Engagement (Activity 3)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Warm-up)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

• MLR5: Co-Craft Questions

Required Materials

Materials to Gather

- · Dot stickers: Activity 1
- · Sticky notes: Activity 1
- Straightedges: Activity 2, Activity 3

Required Preparation

Lesson:

1 sticky note and 1 dot sticker for each student. Straightedges should be made available to create dot plots.

Lesson Narrative

In this lesson, students represent distributions of numerical data with dot plots after organizing them into frequency tables. Using graphical representations of distributions, they continue to develop a spatial understanding of distributions in preparation for understanding the concepts of center and spread in future lessons. Students make use of the structure of dot plots to describe distributions and to draw conclusions about the data.

An optional activity is included to remind students how bar graphs can be used to visualize categorical data.

Student Learning Goal

Let's represent data with dot plots and bar graphs.

Assessment

5 min

Lesson Timeline

15 min

Activity 1

15 min

Activity 2

15 min

Activity 3

min

5_{min}

Cool-down

Lesson Synthesis

10

Warm-up

Curious about Caps



Activity Narrative

The purpose of this *Warm-up* is to give students a chance to generate questions about an image. This leads to a discussion about statistical and non-statistical questions. The context here is used again in another activity, so this also gets students familiar with this situation.

Launch



Arrange students in groups of 2. Introduce the context by telling students that Clare collects bottle caps and keeps them in containers. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement



Sample questions:

- · In general, how many caps fit in a container?
- · What is the most common diameter of the bottle caps in the collection?
- What is the most common bottle-cap color in the collection?
- How long did it typically take to fill up a container?

Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "statistical questions," "non-statistical questions," "numerical data," and "categorical data."

Instructional Routines

MLR5: Co-Craft Questions

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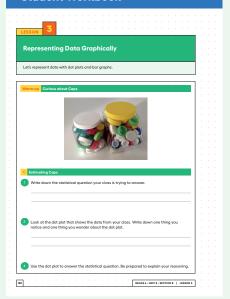


Access for Multilingual Learners (Warm-up)

MLR5: Co-Craft Questions.

This activity uses the Co-Craft Questions math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook



Activity 1

Estimating Caps



Activity Narrative

In this activity, students are motivated to create a dot plot by identifying a statistical question and collecting data from the class to answer the question. When the data is initially presented, it is messy and difficult to analyze in an unorganized form. Students are then asked to choose an appropriate representation to use for the data.

Launch

If any groups in the *Warm-up* asked a statistical question that would require counting the items in the container to answer, use that as a way to transition into this activity.

Tell students to keep their materials closed for this discussion. Give each student a sticky note and a dot sticker. Display the image and questions for all to see.

- Question 1: How many caps are in the jar?
- Question 2: In this class, what is a common estimate for the number of caps in the jar?
- Question 3: What is the teacher's estimate for the number of caps in the jar?



Give students a minute of quiet think time to determine if each question is a statistical question and their reasoning. Invite students to share their responses. When the class agrees that Question 2 is the only statistical question and has a good reason, ask students to write down on a sticky note an estimate for the number of caps in the jar and to remember their answer. Next, ask them to affix their sticky notes on a wall or a board for all to see, and then use the display to answer the statistical question.

Students should recognize that the data cannot be easily interpreted in this format. Discuss other ways in which the estimates can be displayed so that they can be easily seen and understood. If no students suggest placing the sticky notes along a number line like a dot plot, suggest this idea to them.

Next, look for the smallest and largest estimates (either by asking students or skimming through the sticky notes). Draw or display a number line that spans those two numbers, large enough so that students could affix their dot stickers along it. Have each student put their dot sticker on the right place along the number line. If more than one person made the same estimate, the second person should put theirs higher up on the board in the same horizontal position. When the dot plot is complete, tell students to open their materials and answer the questions.

Student Task Statement

- **1.** Write down the statistical question your class is trying to answer.
 - In this class, what is a common estimate for the number of caps in the jar?
- **2.** Look at the dot plot that shows the data from your class. Write down one thing you notice and one thing you wonder about the dot plot.

Sample responses:

I notice that:

- · The dots are mostly grouped around a certain number.
- There is a wide range of guesses.
- · Nobody guessed fewer than 20.

Iwonder:

- Why are the guesses so spread out?
- · Where is the middle of the dot plot?
- Where does the real answer fit on the dot plot?
- **3.** Use the dot plot to answer the statistical question. Be prepared to explain your reasoning.

Sample response: There are a lot of guesses near 60.

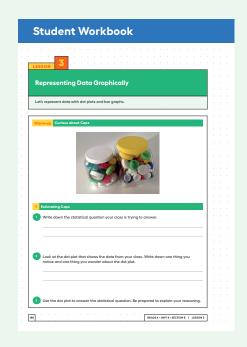
Activity Synthesis

The purpose of this discussion is to see why a dot plot can be useful to visualize and quickly understand a large amount of data.

Ask students what they noticed about the display and for any questions they had about the display. Ask students what advantages this display might have over other ways of arranging the sticky notes.

Tell students that the actual number of bottle caps in the jars is 82. Ask students how this number relates to the dot plot created by the class.

In general, the average from a group of individual guesses can be more accurate than any individual guess. This phenomenon is called "wisdom of the crowd" and is relied on for things such as reviews for products sold online.



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

At the appropriate time, give groups 2–3 minutes to plan what they will say when they present to the class.

"Practice what you will say when you share your advantages of a dot plot with the class. Talk about what is important to say, and decide who will share."

Advances: Speaking, Conversing, Representing

Activity 2

Been There, Done That!

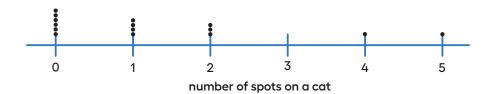


Activity Narrative

In this activity, students create a dot plot on their own. The work of drawing dot plots is not new, but students are asked to describe their analysis of the data broadly. They also learn to use the terms **frequency** and **distribution**.

Launch

Keep students in groups of 3–4. Display the dot plot for all to see.



Ask students how many cats have 1 spot. (4 cats) Explain that the term frequency associated with a particular item represents the number of times an item occurs in the data set. The value 1 in this data set has a frequency of 4. Ask,

"What is the frequency of cats with 2 spots?"

3

"What is the frequency of cats with 3 spots?"

0

Then ask students to describe how the values are arranged in the dot plot. (There are a lot of cats with only a couple of spots, and then 2 with 4 or more spots.) Explain that the distribution refers to a description of how the data are arranged (or distributed) in the dot plot.

Give groups 5–6 minutes to work together to organize the given data and draw their dot plots. Then, give them 2–3 minutes of quiet time to analyze the dot plots and answer the last two questions, and time for a brief small-group discussion about their responses. Reconvene as a whole class afterward.

As students work and discuss, identify those who are able to describe the distribution of the data clearly and succinctly, as well as students who can articulate why a dot plot is an appropriate representation. Ask them to share later.

Student Task Statement

Priya wants to know if basketball players on 2 teams have had prior experience in international competitions. She gathers data on the number of times the players were on a team before 2016.

Team 1

Te

3	0	0	0	0	1	0	0	0	0	0	0
eam 2											

0

1

0

3

1

1. Did Priya collect categorical or numerical data? numerical data

0

2. Organize the information on the two basketball teams into these tables.

2

Team 1

2

3

3

1

number of prior competitions	frequency (number)
0	10
1	1
2	0
3	T.
4	0

Team 2

number of prior competitions	frequency (number)
0	3
1	4
2	2
3	3
4	0

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Develop Language and Symbols.

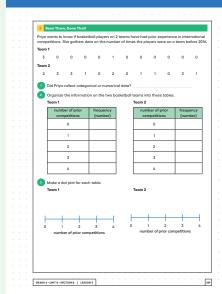
Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of distributions. Terms may include "spread," "distribution." and "frequency."

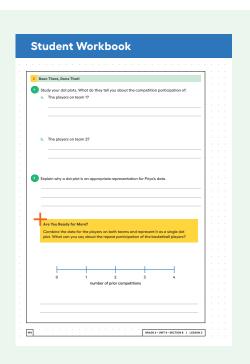
Supports accessibility for: Conceptual Processing, Language

Building on Student Thinking

When drawing dot plots, some students might use dots of different sizes or neglect to stack the dots in a straight column. Remind students to use uniform dots and to stack them vertically, using a straightedge as a guide, if needed. For some students, the use of graph paper may be helpful.

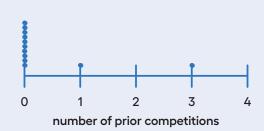
Student Workbook



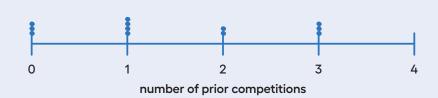


3. Make a dot plot for each table.

Team 1



Team 2



- **4.** Study your dot plots. What do they tell you about the competition participation of:
 - a. The players on team 1?

Sample response: The vast majority of players on team I are athletes for which this is their first competition. Only 2 of the I2 are returning athletes.

b. The players on team 2?

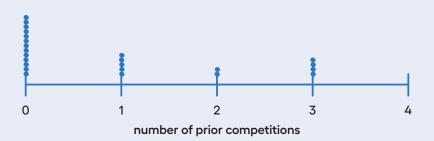
Sample response: Three-quarters of the players on team 2 are returning athletes. Only 3 players are at their first competition.

5. Explain why a dot plot is an appropriate representation for Priya's data.

Sample response: Priya's data are numerical, so they can be organized along a number line with a dot plot.

Are You Ready for More?

Combine the data for the players on both teams and represent it as a single dot plot. What can you say about the repeat participation of the basketball players?



Sample response: About half of the basketball players did not play in a previous competition. About 25% of the players had been at one prior competition, and another 25% had 2 or 3 prior competitions.

Activity Synthesis

Select previously identified students to share their comments on the competition participation of the two teams and why a dot plot is an appropriate representation. Then, invite them to compare the features and merits of the representations that they used in this lesson. Discuss:

- "In what ways might dot plots be more useful than lists and tables?"
 Dot plots give us a visual snapshot of the data so that we can see patterns or anything unusual.
- Could a dot plot be used to represent the first letter of the last names of the players represented on these teams? Why or why not?"

No, because a dot plot uses a number line. Each point on the number line represents a number, so where a data point is placed matters. A bar graph could be used where the categories could be shown in any order, and the width of the bars is flexible.

Explain to students that a frequency table, bar graph, and dot plot all tell us about the distribution of a data set: each of them lists or shows all the possible values or categories in a data set and how often each one occurs.

Activity 3: Optional

Favorite Summer Sports

15 min

Activity Narrative

This activity is marked optional because it goes beyond the expectations of the Alignments. The activity helps students think about how bar graphs are useful and how they compare to dot plots. In later lessons, students examine histograms, and the emphasis on categorical and numerical data here can help distinguish histograms from bar graphs.

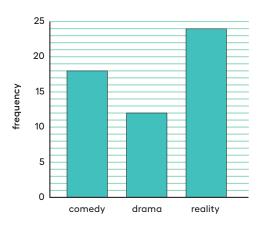
In this activity, students organize categorical data into frequency tables and represent them as a bar graph. Neither task is new to the grade level, but students approach data analysis with a new awareness of data types, and they use data to answer questions that are more open-ended. Later, they will contrast the representations for categorical data with those for numerical data. Students also learn to use the term "frequency" with categorical data.

As students draw and analyze their bar graphs, listen for the questions they might ask one another about how to draw the bar graph. For example, they might wonder about the order in which the categories are displayed, the width of the bars, or whether certain sports might belong in the same category (for example, swimming and diving are related sports that could be grouped). Also notice the arguments that students make about whether a dot plot would be a suitable representation for the data.

Launch

Keep students in groups of 3–4. Provide access to straightedges.

Display this dot plot for all to see. Tell students that this represents the types of shows that Noah watched over the past few months.



Ask students to share what they know or remember about bar graphs. Then, ask:

"What does the bar labeled 'drama' tell us?"

that Noah watched 12 drama shows over the past few months

"What does the frequency label mean?"

It means how many times each category is represented in the data set.

"What type of show did Noah watch the most over the past few months? Explain how you know."

Reality. That bar is the tallest.

"Why do you think the bars are arranged in this way? Could they be arranged in another way?"

The categories are listed alphabetically left to right. They could be rearranged to go from the shortest to the tallest bar or the other way around.

Give groups 5–6 minutes to work together to organize the given data and draw their bar graph. Then, give them 2–3 minutes of quiet time to analyze the bar graph and answer the last two questions, and time for a brief small-group discussion about their responses. Reconvene as a whole class afterward.

Student Task Statement

Kiran wants to know which three summer sports are most popular in his class. He surveyed his classmates on their favorite summer sport. Here are their responses.

swimming	gymnastics	track and field	volleyball
swimming	swimming	diving	track and field
gymnastics	basketball	basketball	volleyball
track and field	track and field	volleyball	gymnastics
diving	gymnastics	volleyball	rowing
track and field	track and field	soccer	swimming
gymnastics	track and field	swimming	rowing
diving	soccer		

- **1.** Did Kiran collect categorical or numerical data?
 - categorical data
- **2.** Organize the responses in a table to help him find which summer sports are most popular in his class.

Answers vary based on the data. Sample responses:

sport	frequency
basketball	2
diving	3
gymnastics	5
rowing	2
soccer	2
swimming	5
track and field	7
volleyball	4

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

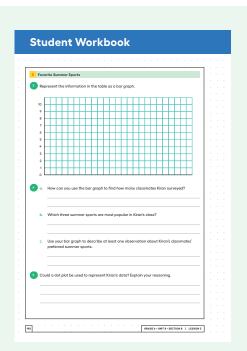
Chunk this task into more manageable parts. After the first 2–3 minutes of work time, check in with students to provide feedback and encouragement after each chunk. Check to make sure that students have developed a way to keep track of their counting.

Supports accessibility for: Attention, Social-Emotional Functioning

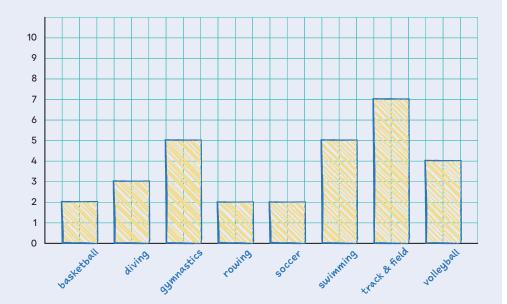
Building on Student Thinking

When determining the frequencies of different sports, students might lose track of their counting. If this happens, urge students to check off each sport as they account for it and then to double-check their counts afterward.





3. Represent the information in the table as a bar graph.



- **4. a.** How can you use the bar graph to find how many classmates Kiran surveyed?
 - Add the heights of each bar to find the total number of classmates surveyed. Kiran surveyed 30 classmates.
 - **b.** Which three summer sports are most popular in Kiran's class?
 - Track and field is the most popular summer sport. Swimming and gymnastics tied for the second most popular summer sport.
 - **c.** Use your bar graph to describe at least one observation about Kiran's classmates' preferred summer sports.
 - Sample response: More than half the class chose one of these top three sports as their favorite.
- **5.** Could a dot plot be used to represent Kiran's data? Explain your reasoning.
 - No, a dot plot could not be used to represent categorical data. A dot plot is built on a number line and is to be used with numerical data.

Activity Synthesis

The purpose of the discussion is for students to recognize when it is appropriate to use a bar graph and how bar graphs compare to dot plots.

Focus the whole-class discussion on the decisions that students made when drawing their bar graph and on the last two questions. Some discussion questions:

- "How did you know which sport to represent first, second, and so on? Does the order in which the sports are listed matter?"
 - It doesn't matter. We arranged the sports alphabetically.
- O "Does the width of the bars matter?"

The width does not matter as long as all of the bars are the same. If some are wider than others, it will have a relatively larger area and can be misleading.

\bigcirc	If some students combined two or more sports into a single category:
	"How might the bar graph change if we combine two sports, say
	swimming and diving, into a single bar?"

There would be fewer bars, and the height of the combined category would have the height of the sum of the 2 individual categories included.

Select a couple of students to share their observations about favorite summer sports in Kiran's class. Ask the class to see if they agree with those observations. If they don't find those conclusions to be reasonable, ask for their reasoning and alternative conclusions.

Ask students.

"How can you decide whether or not to use a dot plot?"

Dot plots should be used to represent only numerical data along a number line and not categorical data.

Lesson Synthesis

The purpose of the discussion is to talk about distributions. Ask students:

"What is the distribution of data?"

Data and how often they occur. This can be seen in a dot plot or bar graph.

"How do we tell the frequency of a data value from a data set? What about from a frequency table?"

If all the data are listed, we can count how often a value occurs to find the frequency. In a frequency table, the frequency is listed next to the value.

"How do we tell the frequencies of different data values from a dot plot?"

The number of dots stacked over the value on a number line is the frequency.

If the optional activity was completed:

"What kind of graphical representation can we use to show the distribution of categorical data? What about for numerical data?"

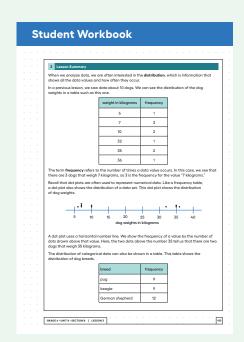
A bar graph can be used for categorical data. Numerical data can be shown with a dot plot.

"How do we tell the frequencies of different data values from a bar graph?"

The height of the bar graph represents the frequency of values in a category.

"Both the dot plot and the bar graph are built on a horizontal line. How are the horizontal lines different?"

In a dot plot, the horizontal line is a number line with values arranged in order. In a bar graph, the categories are listed along the horizontal line, but can be rearranged to any position.



Lesson Summary

When we analyze data, we are often interested in the **distribution**, which is information that shows all the data values and how often they occur.

In a previous lesson, we saw data about 10 dogs. We can see the distribution of the dog weights in a table such as this one.

weight in kilograms	frequency
6	1
7	3
10	2
32	1
35	2
36	1

The term **frequency** refers to the number of times a data value occurs. In this case, we see that there are 3 dogs that weigh 7 kilograms, so 3 is the frequency for the value "7 kilograms."

Recall that dot plots are often used to represent numerical data. Like a frequency table, a dot plot also shows the distribution of a data set. This dot plot shows the distribution of dog weights.

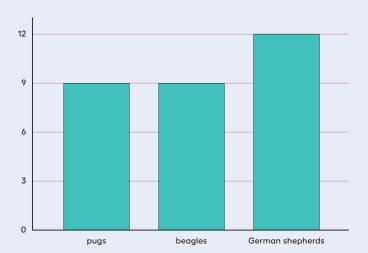


A dot plot uses a horizontal number line. We show the frequency of a value by the number of dots drawn above that value. Here, the two dots above the number 35 tell us that there are two dogs that weigh 35 kilograms.

The distribution of categorical data can also be shown in a table. This table shows the distribution of dog breeds.

breed	frequency
pug	9
beagle	9
German shepherd	12

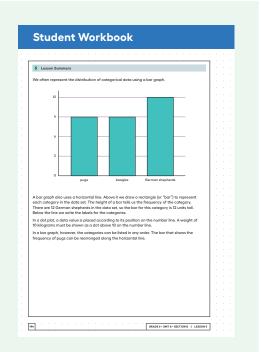
We often represent the distribution of categorical data using a bar graph.



A bar graph also uses a horizontal line. Above it we draw a rectangle (or "bar") to represent each category in the data set. The height of a bar tells us the frequency of the category. There are 12 German shepherds in the data set, so the bar for this category is 12 units tall. Below the line we write the labels for the categories.

In a dot plot, a data value is placed according to its position on the number line. A weight of 10 kilograms must be shown as a dot above 10 on the number line.

In a bar graph, however, the categories can be listed in any order. The bar that shows the frequency of pugs can be rearranged along the horizontal line.



Responding To Student Thinking

Press Pause

If students struggle with determining if data sets are numerical or categorical and thus what data display to use, make time to revisit this Cool-down before moving forward. If students struggle with interpreting the dot plot, there are many chances in upcoming lessons to use questions to support students' understanding of dot plots.

Cool-down

Swimmers and Swimming Class

5 min

Student Task Statement

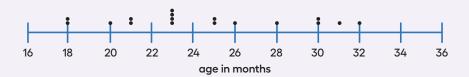
1. Noah gathered information on the home states of the swimmers on a national team. He organized the data in a table. Would a dot plot be appropriate to display his data? Explain your reasoning.

No

He could use a bar graph because the home states of swimmers are categorical data.

2. This dot plot shows the ages of students in a swimming class. How many students are in the class?

16 students are in the class.



- **3.** Based on the dot plot, do you agree with each of the statements? Explain your reasoning.
 - a. The frequency of each age represented is never greater than 3. Agree Sample reasoning: The number of dots stacked over a number is never greater than 3 for this dot plot.
 - b. Half of the students are between 2 and 3 years old. Agree
 On the number line, eight of the 16 data points, or half of the class, are
 placed to the right of 24 months and to the left of 36 months.

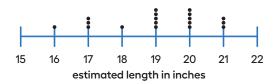
Practice Problems

6 Problems

2

Problem 1

A teacher draws a line segment that is 20 inches long on the blackboard. She asked each of her students to estimate the length of the segment and uses their estimates to draw this dot plot.



- a. How many students are represented in the dot plot? I8
- **b.** Are students generally accurate in their estimates of the length of the line? Explain your reasoning.

Sample response: Students tended to underestimate the length of the line, with more than half giving an estimate that was smaller than the actual length of 20. No one overestimated by more than I inch, but some students underestimated by as much as 3 or 4 inches.

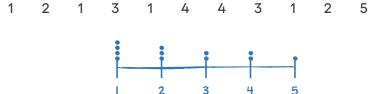
Problem 2

Here are descriptions of data sets. Select all descriptions of data sets that could be graphed as dot plots.

- **A.** Class size for the classes at an elementary school
- B. Colors of cars in a parking lot
- C. Favorite sport of each student in a sixth-grade class
- **D.** Birth weights for the babies born during October at a hospital
- **E.** Number of goals scored in each of 20 games played by a school soccer team

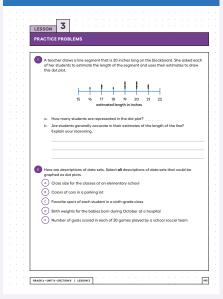
Problem 3

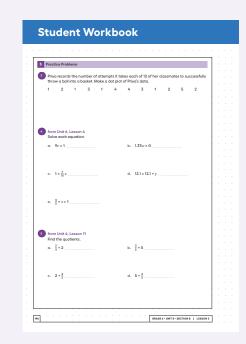
Priya records the number of attempts it takes each of 12 of her classmates to successfully throw a ball into a basket. Make a dot plot of Priya's data.

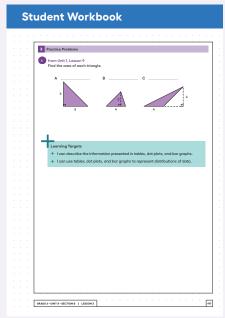


number of attempts

Student Workbook







Problem 4

Solve each equation.

a.
$$9v = 1$$
 $v = \frac{1}{9}$

b.
$$1.37w = 0$$

$$w = 0$$

c.
$$1 = \frac{7}{10} x$$

 $x = \frac{10}{7}$

d.
$$12.1 = 12.1 + y$$

e.
$$\frac{3}{5} + z = 1$$

Problem 5

Find the quotients.

a.
$$\frac{2}{5} \div 2$$

b.
$$\frac{2}{5} \div 5$$

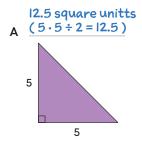
c.
$$2 \div \frac{2}{5}$$

5

d.
$$5 \div \frac{2}{5}$$

Problem 6

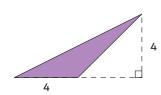
Find the area of each triangle.







8 square units C $(4 \cdot 4 \div 2 = 8)$



from Unit 6, Lesson 4

from Unit 4, Lesson 11

from Unit 1, Lesson 9