Lesson 4

Comparing Speeds and Prices

Goals

- Explain (orally and in writing) that if two ratios have the same rate per 1, they are equivalent ratios.
- Justify (orally and in writing) comparisons of speeds or prices.
- Recognize that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates (in spoken and written language) using the word "per" and specifying the unit.

Learning Targets

- I understand that if two ratios have the same rate per 1, they are equivalent ratios.
- When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing "how much for 1" of the same unit.

Lesson Narrative

In this lesson, students continue to work with ratios in context and reason about rates per 1. They consider situations and questions about speed and prices, such as:

- A runner ran 4,200 meters in 30 minutes. Another runner ran 6,300 meters in 45 minutes. Did they run at the same speed?
- Which shopping deal, 2 for $3 or 8 for $6, is better?

Students see that expressing ratios as rates per 1 is an effective way to compare them. For instance:

- Calculating the distance run in 1 hour (or 1 minute) shows that both runners ran at a rate of 8,400 meters per hour (or 140 meters per minute).
- Calculating the price for 1 item shows that it costs $1.50 per item in the first deal and $0.75 per item in the second deal.

Students work toward the general understanding that when two ratios have the same rate per 1, they are equivalent ratios. They also notice that dividing one quantity in a ratio by the other is an efficient way to find a rate per 1.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR5: Co-Craft Questions (Warm-up)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- MLR5: Co-Craft Questions

Required Preparation

Activity 2:
If possible, gather some examples of grocery store advertisements from newspapers or weekly fliers for deals like "3 for $5."

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Lesson 4

Comparing Speeds and Prices

Lesson Narrative (continued)

As students reason about ratios in situations, find rates per 1, and interpret the meanings of numbers in context, they practice reasoning abstractly and quantitatively.

Student Learning Goal

Let's compare some speeds and some prices.

Warm-up

Treadmill Workouts

5 min

Activity Narrative

This *Warm-up* activates ideas about rates and constant speed before students solve problems about them later in the lesson. Students are presented with a situation with some numerical values and prompted to develop mathematical questions that can be asked about the situation. The routine allows students to make sense of a context before feeling pressure to produce answers, and it develops students' awareness of the language used in mathematics problems.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Remind students of the context of running on a treadmill, which they had encountered in an earlier unit. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Some students did treadmill workouts.

- Tyler ran 4,200 meters in 30 minutes.
- Kiran ran 6,300 meters in $\frac{1}{2}$ hour.
- Mai ran 6.3 kilometers in 45 minutes.

Sample responses:

- Were the students running at the same rate?
- Were the students running at a constant speed?
- What was each student's speed?
- Who ran the fastest?
- How far would Tyler and Kiran run in 45 minutes?
- How far did Mai run in the first 30 minutes?

Activity Synthesis

Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "constant speed," "at the same rate," and "___ meters per hour (or minute)."

Tell students that they will now look more closely at Tyler's, Kiran's, and Mai's workouts and compare them.

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544

Please log in to the site before using the QR code or URL

Access for Multilingual Learners (Warm-up)

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Access for Multilingual Learners (Warm-up, Launch)

Speaking, Reading Representing: MLR8: Discussion Supports. Display a table that shows different representations and language used for $\frac{1}{2}$, 1, and $1\frac{1}{2}$. Highlight differences between similar-looking or similar-sounding language like "one", "one half", "a half", "one and a half", etc.

Design Principle(s): Support sense-making; Maximize meta-awareness

Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Activity 1

More Treadmills

15 min

Activity Narrative

In this activity, students analyze the workouts of several people on a treadmill given time-distance ratios, and they work toward more efficient ways to compare speeds, namely, by computing rates per 1. Students see that when such ratios can be expressed with the same number of meters per minute (or per hour), the ratios are equivalent and the moving objects (such as people or cars) have the same speed.

Speed is typically expressed as a distance per 1 unit of time, so the context lends itself for comparing rates per 1. The numbers have been chosen such that any two workouts being compared have the same time, same distance, or same speed.

Encourage students to use "per 1" and "for each" language throughout, as this language supports the development of the concept of unit rate.

As students discuss the problems, listen closely for those who use these terms as well as descriptions of speed (such as "same speed," "faster," or "slower"). Also monitor for students who first make the connections between the rates per 1 that they calculated in the first half of the task and then use them to answer questions in the second half. Invite some of these students or groups to share later.

Launch

Arrange students in groups of 3. Give students 2–3 minutes of quiet think time to complete the first set of questions (comparisons of pairs of workouts) and then time to discuss their responses. Specify that each group member should take the lead on analyzing one sub-problem and sharing how the workouts of the two given runners are similar or different.

Afterward, ask students to complete the remainder of the task with their group.

Student Task Statement

Some students did treadmill workouts, each one running at a constant speed. Answer the questions about their workouts. Explain or show your reasoning.

- Tyler ran 4,200 meters in 30 minutes.
- Kiran ran 6,300 meters in $\frac{1}{2}$ hour.
- Mai ran 6.3 kilometers in 45 minutes.

1. What is the same about the workouts done by:

   a. Tyler and Kiran?

   Tyler and Kiran both ran for the same amount of time: 30 minutes. Kiran ran a greater distance in 30 minutes, so Kiran was running faster than Tyler.

   b. Kiran and Mai?

   Kiran and Mai ran the same distance, 6,300 meters, but Mai took more time than Kiran to run 6,300 meters, so Mai was running slower than Kiran.

Access for Students with Diverse Abilities (Activity 1, Launch)

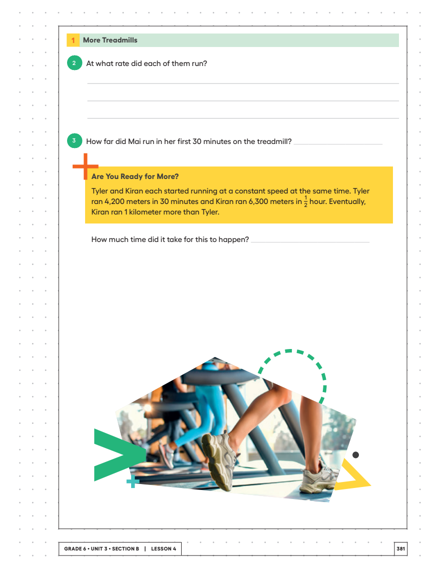
Action and Expression: Develop Expression and Communication. Identify connections between strategies that result in the same outcomes but use differing approaches. Monitor for students who use double number line diagrams, tables, or other organizational strategies. During the discussion, invite students to share how the same information can be found in each.

Supports accessibility for: Conceptual Processing, Language

Building on Student Thinking

If students are not sure how to begin, suggest that they try using a table or a double number line diagram that associates meters and minutes.

Student Workbook



c. Mai and Tyler?

Mai and Tyler both ran 140 meters per minute, so Mai and Tyler were running at the same speed. However, they ran different distances and took different amounts of time to do so.

2. At what rate did each of them run?

Tyler ran 140 meters per minute (or 8,400 meters per hour).

distance (meters)	time (minutes)
4,200	30
1,400	10
140	1

Kiran ran 210 meters per minute (or 12,600 meters per hour).

distance (meters)	time (minutes)
6,300	30
2,100	10
1,050	5
210	1

Mai ran 140 meters per minute (or 8,400 meters per hour).

distance (meters)	time (minutes)
6,300	45
140	1

Students may also reason with double number line diagrams or simply calculate $\frac{b}{a}$ for the given ratio $a:b$.

3. How far did Mai run in her first 30 minutes on the treadmill?

Mai ran 4,200 meters in 30 minutes, because she is going the same speed as Tyler and that is how far Tyler ran in 30 minutes.

Are You Ready for More?

Tyler and Kiran each started running at a constant speed at the same time. Tyler ran 4,200 meters in 30 minutes and Kiran ran 6,300 meters in $\frac{1}{2}$ hour. Eventually, Kiran ran 1 kilometer more than Tyler. How much time did it take for this to happen?

Just over 14 minutes. Sample reasoning: Kiran runs 2100 meters more than Tyler in 30 minutes. Each minute he runs 70 meters more, so it will take $\frac{1,000}{70} = 14\frac{2}{7}$ minutes for him to run 1 kilometer more.

different (meters)	time (minutes)
2,100	30
70	1
1,000	$\frac{1,000}{70}$

Activity Synthesis

Focus the conversation on the speed of each runner and the idea of "same speed," including clues that two objects are moving equally fast or slow. Ask questions such as:

"How can you tell when runners are going at the same speed?"

They keep up with one another while moving. They run the same distance in the same amount of time.

"How can you tell when two cars are moving at the same speed?"

They move at the same miles per hour or kilometers per hour.

Invite a few students to share their analyses of how the runners compare, starting with how Tyler's workout compares to Kiran's, and how Kiran's compares to Mai's. Descriptions such as "slower," "faster," or "higher or lower speed" should begin to emerge.

After students share their analyses of Mai's and Tyler's workouts, emphasize that even though they ran different distances in different amounts of time, they each ran 140 meters per minute (or 8,400 meters per hour) so we can say "they ran at the same speed." This also means that Mai and Tyler's original ratios—4,200 : 30 and 6,300 : 45—are equivalent ratios.

In the last problem, students need to understand that since Mai and Tyler ran at the same speed they traveled the same distance for the first 30 minutes on the treadmill. This may be difficult for students to articulate with precision, so allowing multiple students to share their thinking may be beneficial.

Activity 2

The Best Deal on Beans

15 min

Activity Narrative

In this activity, students compare rates per 1 in a shopping context as they look for "the best deal." The work reminds students how unit price works and encourages them to look for efficient ways to compare unit prices.

The phrase "the best deal" may have different meanings to students and should be discussed. For instance, students may account for various factors besides price, such as distance to store, transportation costs, store preference, loyalty points, and range of item selections and sizes in a store. Discussing these real-life considerations, and choosing which to prioritize and which to disregard, is an important part of modeling with mathematics. For the purposes of this activity, however, it is also appropriate to clarify that we are looking for "the best deal" in the sense of the lowest cost per can.

As students work, monitor for students who use representations like double number line diagram or tables of equivalent ratios. These are useful for making sense of a strategy that divides the price by the number of cans to find the price per 1. Also monitor for students using more efficient strategies, such as dividing price by the number of cans or the number of ounces.

Launch

Tell students that this activity is about finding the best deal when shopping for a grocery item. Ask students to share what "a good deal" and "the best deal" mean to them. Many students are likely to interpret these in terms of low prices (per item or otherwise) or "getting more for less money," but some may have other practical or personal considerations. (For example, it is not a good deal to pay less for a large amount of food but not be able to consume it before it spoils, or to have to travel a long distance to the store or pay more for transportation.) Acknowledge students' perspectives and how "messy" such seemingly simple terms can be. Clarify that in this task, we are looking for "the best deal" in the sense of the lowest cost per can.

Give students 6–8 minutes of quiet work time, and follow that with a whole-class discussion. Clarify that "oz" is an abbreviation for "ounce."

Student Task Statement

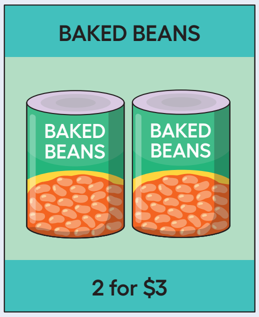
Four different stores posted ads about special sales on 15-oz cans of baked beans.

1. Which store is offering the best deal? Explain your reasoning.

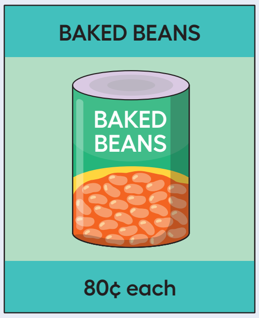
Store A

Store B

Store C



Store D



8 for $6 is the best deal at $0.75 per can. 2 for $3 is the worst deal at $1.50 per can.

Sample reasoning:

- Store A: 8 for $6

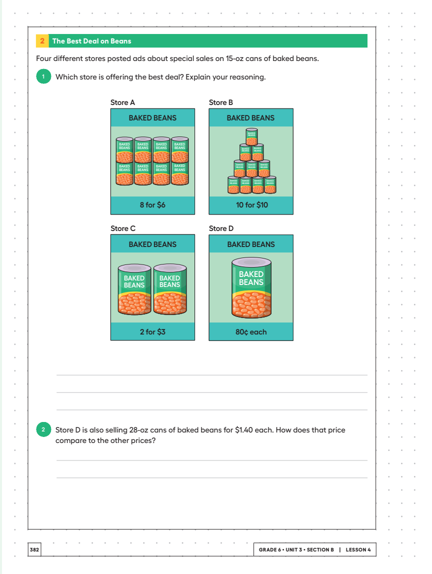
price (dollars)	cans
6	8
3	4
0.75	1

- Store B: $10 for 10 cans means $1 per can.
- Store C: 2 for $3:

price (dollars)	cans
3	2
1.50	1

- Store D: 80 cents per can is the same as $0.80 per can.

Student Workbook


2 The Best Deal on Beans

Four different stores posted ads about special sales on 15-oz cans of baked beans.

1 Which store is offering the best deal? Explain your reasoning.

2 Store D is also selling 28-oz cans of baked beans for $1.40 each. How does that price compare to the other prices?

Building on Student Thinking

Students may look only at the number of cans in each offer or only at the price. For example, they may say that the deal offered by Store D is the best one because it is the only price that is under $1. Ask students how many cans 80 cents buys at that store and whether the other prices shown are also for that many cans. Ask them how they might compare the prices for the same number of cans rather than for different numbers of cans.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the question on which store is offering the best deal and their explanation. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

2. Store D is also selling 28-oz cans of baked beans for $1.40 each. How does that price compare to the other prices?

Sample responses:

- The price of $1.40 for 28 ounces means $0.05 per ounce (1.40 ÷ 28 = 0.05). This is the same price per ounce as in Store A, which charges $0.75 for 15 ounces (0.75 ÷ 15 = 0.05).
- The larger 28-ounce can for $1.40 is a better deal than the smaller 15-ounce can for $0.80 at Store D. For the larger can, the price is 5 cents per ounce. For the smaller can, it is more than 5 cents per ounce. (If it were also 5 cents per ounce for the 15-ounce can, then the price for 1 can would've been $0.75.)

Activity Synthesis

The purpose of this discussion is to help students see that computing and comparing the price per 1 is an efficient way to compare rates in a price context.

Select students who supported their reasoning with a double number line diagram or table to share their responses first. Keep these representations visible and make connections with subsequent explanations from students who used more efficient strategies. Highlight the use of division to compute the price per can and the use of "per 1" language.

Lesson Synthesis

To summarize the different ways that ratios in situations can be expressed as rates per 1, display the given ratios in each activity and the associated rates per 1, as shown here.

From the activities about running on a treadmill:

given ratio	rate per 1
4,200 meters in 30 minutes	140 meters per minute or 8,400 meters per hour
6,300 meters in 30 minutes	210 meters per minute or 12,600 meters per hour
6.3 kilometers in 45 minutes	140 meters per minute or 8,400 meters per hour

From the activity about finding the best deal:

given ratio	rate per 1
8 cans for \$6	\$0.75 per can
10 cans for \$10	\$1.00 per can
2 cans for \$3	\$1.50 per can
80 cents per can	\$0.80 per can

Give students some quiet time to read through the list. Then, ask questions such as:

"Which way of making comparisons do you find more helpful: using the given ratios or using rates per 1? Why?"

- Using rates per 1. Since the value for one quantity is 1, we can just compare the numbers in the rates.
- It depends on the ratios. Sometimes we can quickly multiply the values in one ratio by a number and compare the result to another ratio. For example, doubling 2 cans for \$3 gives 4 cans for \$6, which tells us that it is a worse deal than 8 cans for \$6.

"Are any of the given ratios equivalent? How do you know?"

Only two are equivalent: 4,200 meters in 30 minutes and 6.3 kilometers in 45 minutes. They have the same rate per 1.

"What is a quick way to find a rate per 1 when given a ratio?"

Divide one value in the ratio by the other

Lesson Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

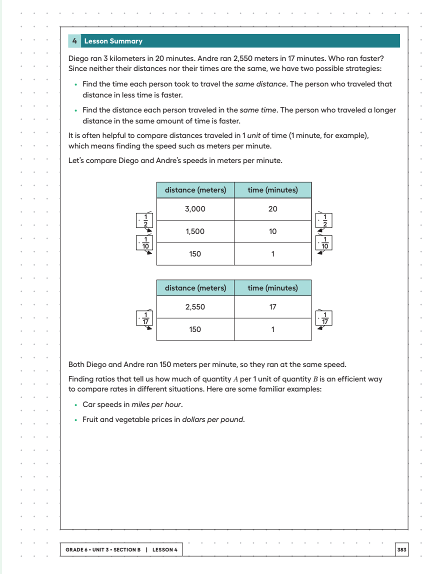
- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

It is often helpful to compare distances traveled in *1 unit* of time (1 minute, for example), which means finding the speed such as meters per minute.

Let's compare Diego and Andre's speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

Student Workbook


Lesson Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

It is often helpful to compare distances traveled in 1 unit of time (1 minute, for example), which means finding the speed such as meters per minute.

Let's compare Diego and Andre's speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

distance (meters)	time (minutes)
2,550	17
150	1

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity *A* per 1 unit of quantity *B* is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in miles *per hour*.
- Fruit and vegetable prices in dollars *per pound*.

GRADE 6 • UNIT 3 • SECTION B | LESSON 4

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

distance (meters)	time (minutes)
2,550	17
150	1

$\cdot \frac{1}{17}$ $\cdot \frac{1}{17}$

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity A per 1 unit of quantity B is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *dollars per pound*.

Cool-down

Sparkling Water

5 min

Student Task Statement

Bottles of sparkling water usually cost \$1.69 each. This week, 4 bottles cost \$5.

Are bottles of sparkling water cheaper or more expensive this week? How much cheaper or more expensive? Show your reasoning.

They are \$0.44 cheaper per bottle

Sample reasoning: Since 4 bottles cost \$5, each bottle costs 5 ÷ 4, or \$1.25, this week. The difference is \$0.44, because 1.69 − 1.25 = 0.44.

Lesson **4**

Practice Problems

6 Problems

Problem 1

Mai and Priya were on scooters. Mai traveled 15 meters in 6 seconds. Priya travels 22 meters in 10 seconds. Who was moving faster? Explain your reasoning.

Mai's scooter is faster

Sample reasoning: $22 \div 10 = 2.2$, so Priya's scooter travels at a rate of 2.2 meters per second. $15 \div 6 = 2.5$, so Mai's scooter travels at a rate of 2.5 meters per second.

Problem 2

Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

Store X: 4 cans for $2.48

Store Y: 5 cans for $3.00

Store Z: 59 cents per can

Store Z has the best deal

Sample reasoning: $2.48 \div 4 = 0.62$ or 62 cents per can. $3 \div 5 = 0.6$ or 60 cents per can. Of the 3 options, 59 cents is the least expensive.

Problem 3

Costs of homes can be very different in different parts of the United States.

a. A 450-square-foot apartment in New York City costs $540,000. What is the price per square foot? Explain or show your reasoning.

$1,200

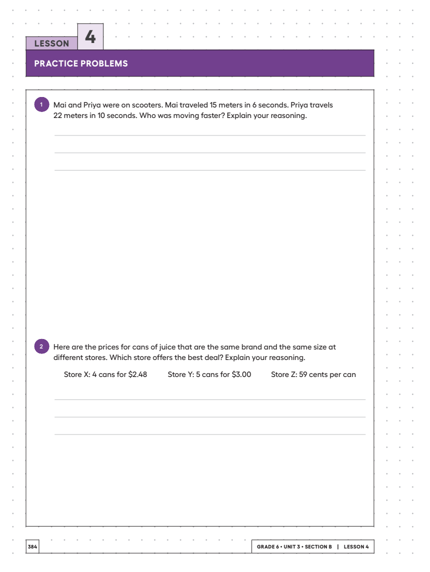
Sample reasoning: $540,000 \div 450 = 1,200$

b. A 2,100-square-foot home in Cheyenne, Wyoming, costs $110 per square foot. How much does this home cost? Explain or show your reasoning.

$231,000

Sample reasoning: $2,100 \cdot 110 = 231,000$

Student Workbook



LESSON 4

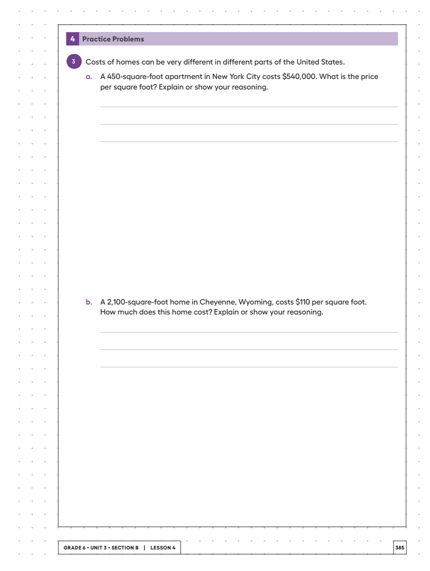
PRACTICE PROBLEMS

1 Mai and Priya were on scooters. Mai traveled 15 meters in 6 seconds. Priya travels 22 meters in 10 seconds. Who was moving faster? Explain your reasoning.

2 Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

Store X: 4 cans for $2.48 Store Y: 5 cans for $3.00 Store Z: 59 cents per can

Student Workbook



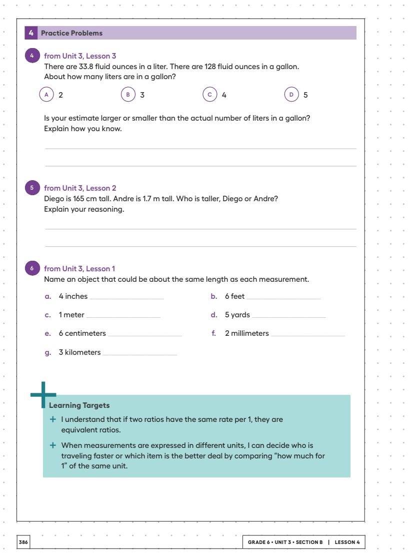
4 Practice Problems

3 Costs of homes can be very different in different parts of the United States.

a. A 450-square-foot apartment in New York City costs $540,000. What is the price per square foot? Explain or show your reasoning.

b. A 2,100-square-foot home in Cheyenne, Wyoming, costs $110 per square foot. How much does this home cost? Explain or show your reasoning.

Student Workbook


4 Practice Problems

4 from Unit 3, Lesson 3
There are 33.8 fluid ounces in a liter. There are 128 fluid ounces in a gallon. About how many liters are in a gallon?

A 2 B 3 C 4 D 5

Is your estimate larger or smaller than the actual number of liters in a gallon? Explain how you know.

5 from Unit 3, Lesson 2
Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

6 from Unit 3, Lesson 1
Name an object that could be about the same length as each measurement.

a. 4 inches b. 6 feet
c. 1 meter d. 5 yards
e. 6 centimeters f. 2 millimeters
g. 3 kilometers

Learning Targets
+ I understand that if two ratios have the same rate per 1, they are equivalent ratios.
+ When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing "how much for 1" of the same unit.

Problem 4 from Unit 3, Lesson 3

There are 33.8 fluid ounces in a liter. There are 128 fluid ounces in a gallon. About how many liters are in a gallon?

A. 2

B. 3

C. 4

D. 5

Is your estimate larger or smaller than the actual number of liters in a gallon? Explain how you know.

Sample reasoning: This estimate is too big: $4 \cdot 32 = 128$, so $4 \cdot (33.8)$ is larger than 128.

Problem 5 from Unit 3, Lesson 2

Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

Andre is taller. Sample reasoning: 1.7 m is 170 cm, and 170 is greater than 165.

Problem 6 from Unit 3, Lesson 1

Name an object that could be about the same length as each measurement.

a. 4 inches

Sample response: pencil

b. 6 feet

Sample response: ladder

c. 1 meter

Sample response: person's leg

d. 5 yards

Sample response: tablecloth

e. 6 centimeters

Sample response: insect

f. 2 millimeters

Sample response: grain of rice

g. 3 kilometers

Sample response: foot race