## **Interpreting Rates**

#### Goals

- Calculate and interpret the two unit rates associated with a ratio, i.e.,  $\frac{a}{b}$  and  $\frac{b}{a}$ for the ratio a:b.
- Choose which unit rate to use to solve a given problem, and explain the choice (orally and in writing).
- Comprehend that the term "unit rate" (in spoken and written language) refers to a rate per 1.

## **Learning Targets**

- I can choose which unit rate to use based on how I plan to solve the problem.
- When I have a ratio, I can calculate its two unit rates and explain what each of them means in the situation.

This lesson introduces the term **unit rate** and the two unit rates,  $\frac{a}{b}$  and  $\frac{b}{a}$ . associated with a ratio a:b. Students learn that each unit rate tells us how much of one quantity in the ratio there is per unit of the other quantity. They also see that, although both unit rates describe the same situation, one or the other might be preferable for answering a given question about the situation.

The work in this lesson also reinforces the idea that dividing one number in a ratio by another is an efficient way to find a unit rate. (It is not necessary to start with a table or use another representation as an intermediate step.)

In explaining how two unit rates can describe the same situation, students practice justifying their reasoning and critiquing the reasoning of others. As they calculate unit rates, interpret them in context, and decide on a unit rate to use to solve a problem, students practice reasoning abstractly and quantitatively.

## Student Learning Goal

Let's explore unit rates.

## **Lesson Timeline**

Warm-up

15

**Activity 1** 

15

**Activity 2** 

10

**Lesson Synthesis** 

#### **Access for Students with Diverse Abilities**

- Engagement (Activity 1)
- Representation (Activity 2)

#### **Access for Multilingual Learners**

- MLR6: Three Reads (Activity 2)
- MLR8: Discussion Supports (Warm-up)

#### **Instructional Routines**

- Math Talk
- · MLR3: Critique, Correct, Clarify
- MLR6: Three Reads

5

Cool-down

### **Inspire Math**

#### **Climbing Mount Everest video**



#### Go Online

Before the lesson, show this video to introduce the real-world connection.

#### ilclass.com/l/614136

Please log in to the site before using the QR code or URL.



#### **Instructional Routines**

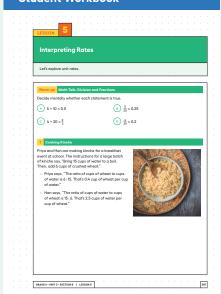
#### **Math Talk**

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## Student Workbook



#### Warm-up

#### **Math Talk: Division and Fractions**



#### **Activity Narrative**

This *Math Talk* focuses on the connections between division, fractions, and decimals. It encourages students to think about the relationship between a fraction and division (an idea from grade 5) and to rely on what they know about equivalent fractions and decimals to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students interpret or find unit rates in a situation given a ratio of two quantities.

To mentally determine if the statements involving division, fractions, and decimals are true, students need to look for and make use of structure.

### Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

#### **Student Task Statement**

Decide mentally whether each statement is true.

**A.** 
$$5 \div 10 = 0.5$$

True

Sample reasoning:

- 5 ÷ 10 =  $\frac{5}{10}$  =  $\frac{1}{2}$ , which can be written as 0.5 in decimal form.
- $\circ$  50 ÷ 10 = 5, so 5 ÷ 10 is a tenth of 5, which is 0.5.

**B.** 
$$\frac{5}{20}$$
 = 0.25

True

Sample reasoning:

- $\circ \frac{5}{20}$  is equivalent to  $\frac{1}{4}$ , which is 0.25 in decimal form.
- $\frac{5}{20}$  is half of  $\frac{5}{10}$ , so it is half of 0.5, which is 0.25.

**C.** 
$$4 \div 20 = \frac{2}{5}$$

**False** 

Sample reasoning:

- 4 ÷ 20 is  $\frac{4}{20}$  or  $\frac{1}{5}$ , not  $\frac{2}{5}$ .
- $4 \div 20$  is less than  $\frac{1}{4}$  (or 0.25) while  $\frac{2}{5}$  is close to  $\frac{1}{2}$  (or 0.5).

**D.**  $\frac{4}{20}$  = 0.2

True

Sample reasoning:

- $4 \div 20$  is  $\frac{4}{20}$  or  $\frac{1}{5}$ , which is 0.2 in decimal form.
- $40 \div 20 = 2$ , so  $4 \div 20$  is one tenth of 2, which is 0.2.

## **Activity Synthesis**

To involve more students in the conversation, consider asking:

"Who can restate \_\_\_\_\_\_'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to \_\_\_ \_\_\_\_\_'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Highlight that a fraction can be seen as division of the numerator by the denominator:  $\frac{a}{b}$  =  $a \div b$ . By the same token, the result of dividing a number by another number,  $a \div b$ , can be expressed as a fraction  $\frac{a}{b}$ .

## **Activity 1**

#### **Cooking Kinche**

15

## **Activity Narrative**

In this activity, students explore two unit rates associated with the ratio, think about their meanings, and use both to solve problems. The goals are to:

- Help students see that for every situation that can be represented with a ratio a:b and an associated unit rate  $\frac{b}{a}$ , there is another unit rate  $\frac{a}{b}$  that also has meaning and purpose within the situation.
- Encourage students to choose a unit rate flexibly depending on the question at hand.

Students begin by reasoning whether the two rates per 1 (cups of wheat per 1 cup of water, or cups of water per 1 cup of wheat) accurately convey a given recipe. After students conclude that both rates are valid, they use the rates to determine unknown amounts of wheat or water.

As students work and discuss, monitor for those who use different representations (a table or a double number line diagram) and those who construct logical arguments in different ways.

#### **Access for Multilingual Learners** (Warm-up, Synthesis)

#### MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, \_\_ because ..." or \_ so I ..." Some "I noticed \_\_\_ students may benefit from the opportunity to rehearse what they will say with a partner before they

Advances: Speaking, Representing

share with the whole class.

#### **Instructional Routines**

MLR3: Critique, **Correct, Clarify** 

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## Access for Students with Diverse Abilities (Activity 1, Launch)

## Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

## Launch 22

Tell students to close their books or devices (or to keep them closed). Display the instructions for making *kinche* (an Ethiopian breakfast dish made with cracked wheat and pronounced *KEEN*-jay) for all to see.

#### KINCHE

To make a large batch, bring 15 cups of water to a boil. Then, add 6 cups of crushed wheat.

Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing that they notice and one that thing they wonder about.

Students are likely to notice that this is a recipe involving water and crushed wheat in a ratio of 15 to 6. If the amount or ratio of water and crushed wheat does not come up during the conversation, ask students to discuss this idea.

Students may wonder about what kinche is. Ask students if they have had oatmeal, porridge, or another kind of dish that involves mixing boiling water or milk with a kind of grain or a ground ingredient. Invite a few students to briefly share some information about the dish. (What grain does it use? Does it come in large bags or small packets? Is it served with seasonings or toppings?)

Tell students that kinche is a traditional Ethiopian breakfast dish. It is popular because it is easy to make, healthy, and can be flavored in different ways. Like oatmeal or porridge, kinche can be cooked to different consistencies using different ratios of wheat or water. Sometimes milk is also added into the mixture.

Arrange students in groups of 2. Give students 2 minutes of quiet think time for the first question. Ask them to pause and share their response with their partner afterward. Encourage partners to reach a consensus and to be prepared to justify their thinking.

After partners have conferred, select several students to explain their reasoning, and display their work for all to see. When the class is convinced that both Priya and Han are correct, ask students to complete the rest of the activity.

#### **Student Task Statement**

Priya and Han are making kinche for a breakfast event at school. The instructions for a large batch of kinche say, "Bring 15 cups of water to a boil. Then, add 6 cups of crushed wheat."

- Priya says, "The ratio of cups of wheat to cups of water is 6:15. That's 0.4 cup of wheat per cup of water."
- Han says, "The ratio of cups of water to cups of wheat is 15:6. That's 2.5 cups of water per cup of wheat."
- **1.** Who is correct? Explain your reasoning. If you get stuck, consider using the table.

The tables include fractions and their decimal equivalents but students are not required to write both.

Priya and Han are both correct.

Sample reasoning:

- 15:6 is equivalent to 5:2, which is equivalent to 2.5:1 and to 1: $\frac{2}{5}$  (or 1:0.4).
- Using a table:

water (cups)	wheat (cups)
15	6
1	$\frac{6}{15}$ or $\frac{2}{5}$ or 0.4
15/6 or 2.5	1

- 2. Priya and Han are each making a pot of kinche.
  - **a.** Priya has 5 cups of wheat. How many cups of water should she boil? Show your reasoning.

12.5 cups of water

Sample reasoning: Since I cup of wheat needs 2.5 cups of water, 5 cups of wheat need  $5 \cdot (2.5)$ , or I2.5, cups of water.

**b.** Han boils 10 cups of water. How many cups of wheat should he add into the water? Show your reasoning.

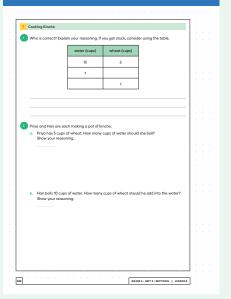
4 cups of wheat

Sample reasoning: Since I cup of water is mixed with  $\frac{2}{5}$  or 0.4 cup of wheat, IO cups of water is to be mixed with IO  $\cdot$  (0.4), or 4, cups of wheat.

#### **Building on Student Thinking**

Some students may think that Priya and Han cannot both be right because they came up with different numbers. Ask them to explain what each number means, so that they have a chance to notice that the numbers mean different things. Point out that the positioning of the number 1 appears in different columns within the table.

#### **Student Workbook**



## **Activity Synthesis**

Focus the discussion on how students found the amount of water for 5 cups of wheat and the amount of wheat for 10 cups of water. Ask questions such as:

"To find the amount of water for 5 cups of wheat, which 'per cup' rate did you use: Priya's (0.4 cup of wheat per cup of water) or Han's (2.5 cups of water per cup of wheat)? Why?"

"To find the amount of wheat for 10 cups of water, which 'per cup' rate did you use? Why?"

If not uncovered in students' explanations, highlight that when the amount of wheat is known but the amount of water is not, it helps to use the "per 1 cup of wheat" rate. In that situation, a simple multiplication will tell us the missing quantity. Conversely, if the amount of water is known, it helps to use the "per 1 cup of water" rate. Consider using additional rows in the given table to illustrate this line of reasoning.

water (cups)	wheat (cups)
15	6
1	6/15 or 0.4
15 or 2.5	1
12.5	5
10	4

Leave the table for this activity displayed as a reference in the next activity.

# Activity 2 Laundry Detergent and Raffle Tickets

15 min

## **Activity Narrative**

In this activity, students are introduced to the term unit rate as a general name for describing the amount of one quantity in a ratio per 1 of the other quantity. Given a ratio a:b, they calculate  $\frac{a}{b}$  and  $\frac{b}{a}$  and interpret both unit rates in context. Students also practice selecting the better unit rate to use  $(\frac{a}{b} \text{ or } \frac{b}{a})$  based on the question posed.

Though the activity prompts students to think in terms of unit rate, some students may still reason in other ways that are more intuitive to them. For example, to find out how many weeks it would take to use 3 gallons of detergent, students may:

- Triple the 10:6 ratio of gallons to weeks to arrive at 30:18, and then divide each 30 and 18 by 10 to get 3 gallons and 1.8 weeks.
- Divide both 10 gallons and 6 weeks by 5 to get 2 gallons in 1.2 weeks, and then add half of each quantity (1 gallon and 0.6 week) to get 3 gallons in 1.8 weeks.

Such lines of reasoning show depth of understanding and should be celebrated. Students should also see, however, that some problems (such as in the context of detergent use and ticket price) may be more efficiently solved using unit rates.

As they decide which unit rates to use, students have an opportunity to interpret numbers in context and practice reasoning quantitatively and abstractly. For example, they may comment that  $\frac{1}{4}$  of a ticket costing a dollar

doesn't make sense since it is not possible to purchase a fraction of a ticket. Encourage such observations and use them as entry points for discussing when or how each unit rate could make sense.

As students work, identify 1–2 students per question to share their choice of unit rate and how it was used to answer the question.

#### Launch



Recap that, in the previous activity, the ratio of 15 cups of water for every 6 cups of wheat can be expressed as two rates "per 1":

- $\frac{6}{15}$  or 0.4 cup of wheat per cup of water
- $\frac{5}{2}$  or 2.5 cups of water per cup of wheat.

Emphasize that, in a table, each of these rates reflects a value paired with a "1" in a row, and that both can be useful depending on the problem at hand. Tell students that we call the amounts 0.4 and 2.5 "unit rates." Explain that a **unit rate** means "the amount of one quantity for 1 of another quantity."

Arrange students in groups of 2. Tell students that they will now solve some problems using unit rates. For each question, ask them to answer the first two parts about unit rates, discuss their responses, and come to an agreement before answering the last part.

### **Student Task Statement**

For each situation, find the **unit rates**. Then, choose a unit rate to solve the problem in Part c. Show your reasoning.

- 1. A laundry service uses 10 gallons of detergent every 6 weeks.
  - a. How many gallons of detergent does the laundry service use per week?

The laundry service uses  $\frac{10}{6}$  or  $\frac{5}{3}$  or  $1\frac{2}{3}$  gallons per week.

**b.** How many weeks does it take the laundry service to use 1 gallon of detergent?

It takes  $\frac{6}{10}$  or  $\frac{3}{5}$  or 0.6 week to use I gallon.

**c.** How many weeks will it take the laundry service to finish 3 gallons of detergent?

It will take I.8 weeks. Sample reasoning: 3 · (0.6) = I.8

- 2. Tyler paid \$16 for 4 raffle tickets.
  - a. What is the price per ticket?

Tyler paid \$4 per ticket.

b. How many tickets is that per dollar?

That is  $\frac{1}{4}$  or 0.25 ticket per dollar.

c. How much would 1.000 raffle tickets cost?

A thousand tickets would cost \$4,000. Sample reasoning: Multiplying the cost of one ticket, \$4, by 1,000 gives \$4,000.

## Access for Multilingual Learners (Activity 2, Student Task)

#### MLR6: Three Reads.

Keep books or devices closed. For the first situation in the activity, display only the problem stem ("A laundry service uses 10 gallons of detergent every 6 weeks"), without revealing the questions. "We are going to read this statement 3 times."

- → After the 1st read: "Tell your partner what this situation is about."
- → After the 2nd read: "List the quantities. What can be counted or measured?"
- → Reveal and read the questions. Ask "What are some ways we might get started on this?"

If necessary, repeat the routine for the second situation.

Advances: Reading, Representing

## Access for Students with Diverse Abilities (Activity 2, Student Task)

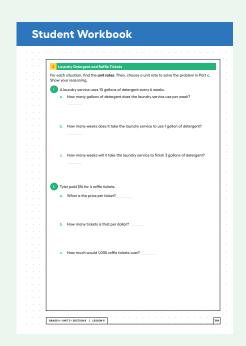
## Representation: Internalize Comprehension.

Provide students with a blank or partially labeled double number line diagram to record the ratio of quantities in each situation. Prompt students to identify the amount of each quantity when the other quantity is 1 and to interpret each pair of values in context.

Supports accessibility for: Visual-Spatial Processing, Organization

#### **Building on Student Thinking**

If students are not sure how to use the unit rates they found for each situation to answer the second half of the task, remind them of how the oatmeal problem was solved. Suggest that this problem is similar because they can scale up from a unit rate to answer the questions.



#### **Are You Ready for More?**

Write a "deal" on tickets for Tyler's raffle that sounds good, but is actually a little worse than just buying tickets at the normal price.

Sample response: One bad deal is getting 6 tickets for \$25, when 6 tickets normally cost \$24.

### **Activity Synthesis**

Invite previously identified students to share their work on the last question about each situation. Highlight observations about if or when one unit rate might be more helpful than the other for answering each question, and why it might be more helpful.

If no students made observations about the ticket-per-dollar unit rate, ask: "How can  $\frac{1}{4}$  of a ticket costing a dollar make sense?" Students may argue that the quantity, on its own, does not make sense. Challenge them to figure out how the rate could be used in the context of the problem. For example, consider asking:

 $\bigcirc$  "If I had \$80, how many tickets could I buy? What if I had \$75?" "Can the  $\frac{1}{4}$  of a ticket per dollar' unit rate help answer these questions?"

### **Lesson Synthesis**

The important takeaways from this lesson are:

- Any ratio has two associated unit rates.
- Unit rates can often be calculated efficiently with a single operation (division or multiplication).
- Depending on the problem being solved, one unit rate might be more useful than the other.

Display the table from earlier in the lesson:

water (cups)	oats (cups)
15	6
1	
	1

Ask students the following questions, and complete the table along the way:

"What is a quick way to compute the number of cups of water for 1 cup of wheat?"

$$15 \div 6 = \frac{15}{6}$$

"What is a quick way to compute the number of cups of wheat for 1 cup of water?"

$$6 \div 15 = \frac{6}{15}$$

 $\bigcirc$  "For what types of problems would it be easier to use  $\frac{15}{6}$ ?"

Finding how many cups of water when we know the number of cups of wheat.

 $\bigcirc$  "For what types of problems would it be easier to use  $\frac{6}{15}$ ?"

Finding how many cups of wheat when we know the number of cups of water.

#### **Lesson Summary**

Suppose a farm lets us pick 2 pounds of blueberries for 5 dollars. We can say:

- We get  $\frac{2}{5}$  pound of blueberries per dollar.
- The blueberries cost  $\$\frac{5}{2}$  or  $\$2\frac{1}{2}$  dollars per pound.

weight of blueberries (pounds)	price (dollars)
2	5
1	<u>5</u> 2
<u>2</u> 5	1

The "price per pound of blueberries" and the "weight of blueberries per dollar" are the two unit rates describing this situation.

A **unit rate** tells us how much of one quantity for 1 of the other quantity. Each of these numbers is useful in the right situation.

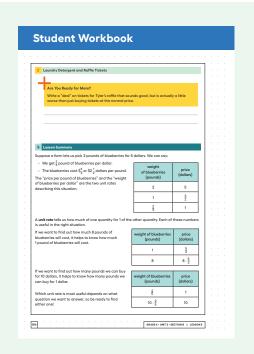
If we want to find out how much 8 pounds of blueberries will cost, it helps to know how much 1 pound of blueberries will cost.

weight of blueberries (pounds)	price (dollars)
1	<u>5</u> 2
8	$8 \cdot \frac{5}{2}$

If we want to find out how many pounds we can buy for 10 dollars, it helps to know how many pounds we can buy for 1 dollar.

weight of blueberries (pounds)	price (dollars)
<u>2</u> 5	1
$10 \cdot \frac{2}{5}$	10

Which unit rate is most useful depends on what question we want to answer, so be ready to find either one!



## **Responding To Student Thinking**

## **More Chances**

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

## Cool-down

## Gasoline by the Gallon

## 5 min

#### **Student Task Statement**

Two gallons of gasoline cost \$6.

1. Complete the table with the missing volume of gasoline or missing price.

gasoline (gallons)	price (dollars)
2	6
1/3	1
1	3

2. Explain the meaning of each of the numbers you found.

The price of  $\frac{1}{3}$  gallon of gasoline is \$1. One gallon of gasoline costs \$3.

**Practice Problems** 

7 Problems

## Problem 1

A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint. The table shows different quantities of red and white paint for the same shade of pink. Complete the table.

Equivalent values are also acceptable.

white paint (cups)	red paint (cups)
4	3
4/3	1
1	3 4
<u>16</u> 3	4
5	1 <u>5</u> 4

#### **Problem 2**

A farm lets you pick 3 pints of raspberries for \$12.00.

a. What is the cost per pint?

$$\frac{12}{3}$$
 or \$4

**b.** How many pints do you get per dollar?

 $\frac{3}{12}$  or  $\frac{1}{4}$  or 0.25 pints per dollar

c. At this rate, how many pints can you buy for \$20.00?

5 pints, because  $20 \div 4 = 5$  and  $(0.25) \cdot 20 = 5$ .

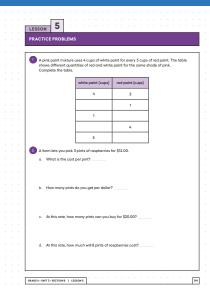
d. At this rate, how much will 8 pints of raspberries cost?

\$32.00, because  $8 \cdot 4 = 32$ .

Students may use a table to reason about all questions:

pints of raspberries	cost in dollars
3	12
I	4
1/4	I
5	20
8	32

## Student Workbook





#### **Problem 3**

Han and Tyler are following a polenta recipe that uses 5 cups of water for every 2 cups of cornmeal.

- Han says, "I am using 3 cups of water. I will need  $1\frac{1}{5}$  cups of cornmeal."
- Tyler says, "I am using 3 cups of cornmeal. I will need  $7\frac{1}{2}$  cups of water."

Do you agree with either of them? Explain your reasoning.

Agree with both

## Sample reasoning:

- For every cup of water,  $\frac{2}{5}$  cup of cornmeal is used. For every cup of cornmeal,  $2\frac{1}{2}$  cups of water are used.
- Using a table:

water (cups)	cornmeal (cups)
5	2
I	<u>2</u> 5
2 <u>1</u>	I
3	l <mark>-</mark> 5
7 <u>1</u>	3

#### **Problem 4**

One gallon of paint can cover 350 square feet. Mai is working on a large art project and needs enough paint to cover 1,750 square feet. She thinks she should use the rate  $\frac{1}{350}$  of a gallon per square foot to find how much paint is needed.

Do you agree with Mai? Explain or show your reasoning.

#### Sample responses:

- I agree with Mai. She needs enough paint for 1,750 square feet. Since each square foot requires  $\frac{1}{350}$  gallons of paint, Mai needs 5 gallons of paint because  $(1,750) \cdot \frac{1}{350} = 5$ .
- I disagree with Mai. It is easier to use the rate 350 square feet per gallon. This table shows that she needs 5 gallons of paint:

gallons of paint	area in square feet
I	350
5	1,750

LESSON 5 • PRACTICE PROBLEMS

#### Problem 5

from Unit 3, Lesson 4

Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Who types faster? Explain your reasoning.

Noah types faster. Sample reasoning: Andre types at a rate of 52 words per minute, because  $208 \div 4 = 52$ . Noah types at a rate of 57 words per minute, because  $342 \div 6 = 57$ . Noah can type 5 more words per minute than Andre.

#### Problem 6

from Unit 3, Lesson 4

A corn vendor at a farmer's market was selling a bag of 20 ears of corn for \$8. Another vendor was selling a bag of 25 for \$9. Which bag is the better deal? Explain or show your reasoning.

The bag of 25 is better. Sample reasoning:  $8 \div 20 = 0.4$ , so each ear of corn is 40 cents. In the bag of 25, each ear of corn is 36 cents because  $9 \div 25 = 0.36$ .

#### **Problem 7**

from Unit 3, Lesson 2

A soccer field is 100 meters long. What could be its length in yards?

- **A.** 33.3
- **B.** 91
- **C.** 100
- **D.** 109

