Lesson 6

Methods for Multiplying Decimals

Goals

- Interpret different methods for computing the product of decimals, and evaluate (orally) their usefulness.
- Justify (orally and in writing) where to place the decimal point in the product of two decimals with multiple non-zero digits.

Learning Targets

- I can use area diagrams to represent and reason about multiplication of decimals.
- I know and can explain more than one way to multiply decimals using fractions and place value.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Lesson Narrative

In this lesson, students continue to develop methods for computing products of decimals by reasoning about properties of base-ten numbers, as well as by reasoning about the area of a rectangle.

Previously, students multiplied decimals by first expressing them as fractions with a power of 10 for the denominator. Here, they see that they can also multiply decimals by powers of 10 to get whole numbers, multiply the whole numbers, and then divide the result by the same powers of 10. For example, to multiply $(0.25) \cdot (1.6)$, students may compute $(0.25) \cdot 100$ and $(1.6) \cdot 10$ to have whole numbers 25 and 16, multiply these to get 400, and then divide 400 by 1,000 to invert the initial multiplication by 1,000.

In an optional activity, students use the area of a rectangle that is decomposed into unit squares to represent and find products of decimals. They see that a rectangle that represents $4 \cdot 2$, for instance, can also represent $(0.4) \cdot (0.2)$, $(0.004) \cdot (0.002)$, or $40 \cdot 20$ because they all share a common structure. Because each pair of factors is related to $4 \cdot 2$ by a power of 10 or a power of $\frac{1}{10}$, we can multiply 8 by these powers to find the other products.

Student Learning Goal

Let’s look at some ways we can represent multiplication of decimals.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	15 min	10 min	5 min

Warm-up
Which Three Go Together: Multiplication Expressions

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare multiplication expressions. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terminologies that students know and how they talk about multiplication and equivalent expressions.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see.

Give students 1 minute of quiet think time. and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $(0.1) \cdot 2 \cdot 3$

B. $3 \cdot (0.2)$

C. $(0.1) \cdot 3$

D. $6 \cdot \frac{1}{10}$

A, B, C go together because they all have 3 and a decimal as factors.

A, B, D go together because they all have a value of 6 tenths or 0.6.

A, C, D go together because they all have 1 tenth as a factor.

B, C, D go together because they all involve only one operation.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure that the reasons given are correct.

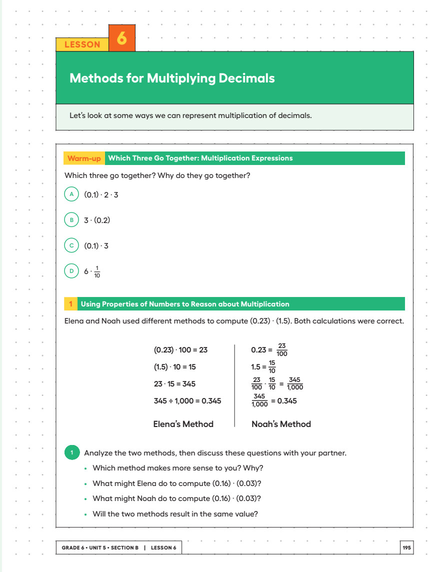
During the discussion, prompt students to explain the meaning of any term that they use, such as "factor," "product," "equivalent," or "commutative," and to clarify their reasoning as needed. Consider asking:

"How do you know ... ?"

"What do you mean by ... ?"

"Can you say that in another way?"

Student Workbook



LESSON 6

Methods for Multiplying Decimals

Let's look at some ways we can represent multiplication of decimals.

Warm-up Which Three Go Together: Multiplication Expressions

Which three go together? Why do they go together?

A $(0.1) \cdot 2 \cdot 3$

B $3 \cdot (0.2)$

C $(0.1) \cdot 3$

D $6 \cdot \frac{1}{10}$

1 Using Properties of Numbers to Reason about Multiplication

Elena and Noah used different methods to compute $(0.23) \cdot (1.5)$. Both calculations were correct.

Elena's Method	Noah's Method
$(0.23) \cdot 100 = 23$	$0.23 = \frac{23}{100}$
$(1.5) \cdot 10 = 15$	$1.5 = \frac{15}{10}$
$23 \cdot 15 = 345$	$\frac{23}{100} \cdot \frac{15}{10} = \frac{345}{1,000}$
$345 \div 1,000 = 0.345$	$\frac{345}{1,000} = 0.345$

1 Analyze the two methods, then discuss these questions with your partner.

- Which method makes more sense to you? Why?
- What might Elena do to compute $(0.16) \cdot (0.03)$?
- What might Noah do to compute $(0.16) \cdot (0.03)$?
- Will the two methods result in the same value?

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Activity 1

Using Properties of Numbers to Reason about Multiplication

20 min

Activity Narrative

This activity builds on the two methods for computing products of decimals introduced in a previous lesson, namely by:

- Thinking in terms of division. Multiplying a number by 0.1 or $\frac{1}{10}$ is the same as dividing the number by 10, multiplying by 0.01 or $\frac{1}{100}$ is the same as dividing by 100, and so on.
- Thinking in terms of fractions. We can write decimal factors as fractions, compute the product, and then convert the product to a decimal.

Students make sense of both methods and use a preferred method to find the products of other decimals. As they reason about the relationship between whole numbers, decimals, and fractions and apply it to multiply decimals, students practice looking for and making use of the structure of the base-ten system.

To reason correctly about the products of decimals, students also need to keep careful track of the digits and their place value. In doing so, they practice attending to precision.

Launch

Arrange students in groups of 2. Tell students that they will analyze two methods for multiplying decimals. Ask each student to study one method and then explain it to their partner. After making sense of both methods together, each partner applies one method to find a new product.

Give partners 5–6 minutes to complete the first set of questions. Ask students to pause for a brief discussion before moving on to the second half of the activity.

Invite a student who used each method to share their reasoning. If not already mentioned in students' explanations, ask:

"Why might Elena have multiplied by 0.23 by 100 and 1.5 by 10?"

To get the decimals into whole numbers, which are easier to multiply.

"What might be Elena's reason for dividing 345 by 1,000?"

She multiplied the original factors by $(100 \cdot 10)$ or 1,000, so the product 345 is 1,000 times the original product and must be divided by 1,000.

"How is Noah's method different from Elena's?"

Noah converted each decimal into fractions and multiplied the fractions.

Give students another 5 minutes of quiet time to work on the second set of questions.

Student Task Statement

Elena and Noah used different methods to compute (0.23) · (1.5). Both calculations were correct.

Elena's Method	Noah's Method
$(0.23) \cdot 100 = 23$	$0.23 = \frac{23}{100}$
$(1.5) \cdot 10 = 15$	$1.5 = \frac{15}{10}$
$23 \cdot 15 = 345$	$\frac{23}{100} \cdot \frac{15}{10} = \frac{345}{1{,}000}$
$345 \div 1{,}000 = 0.345$	$\frac{345}{1{,}000} = 0.345$

1. Analyze the two methods, then discuss these questions with your partner.
   - Which method makes more sense to you? Why?
   - What might Elena do to compute (0.16) · (0.03)?

     Elena might multiply 0.16 by 100 to get 16 and 0.03 by 100 to get 3, multiply 16 and 3 to get 48, and then divide 48 by 10,000 (because $100 \cdot 100 = 10{,}000$).
   - What might Noah do to compute (0.16) · (0.03)?

     Noah might write 0.16 as $\frac{16}{100}$ and 0.03 as $\frac{3}{100}$, and then multiply $\frac{16}{100} \cdot \frac{3}{100}$ to get $\frac{48}{10{,}000}$.
   - Will the two methods result in the same value?

     Both methods would result in 0.0048.
2. Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain or show your reasoning.

   a. (2.1) · (4.7)

   9.87. Multiply each of the original factors by 10 (making a product of 100), and then divide 987 by 100. $987 \div 100 = 9.87$.

   b. 21 · (0.047)

   0.987. Multiply the second original factor by 1,000, then divide 987 by 1,000. $987 \div 1{,}000 = 0.987$.

   c. (0.021) · (4.7)

   0.0987. Multiply the original factors by 1,000 and 10 (making a product of 10,000), then divide 987 by 10,000. $987 \div 10{,}000 = 0.0987$.

Activity Synthesis

Focus the discussion on how the two methods are mathematically equivalent. Highlight that:

- In Noah's method, the product of the numerators of his fractions (23 and 15) is the same as the product of Elena's whole numbers.
- In both methods, the product of 23 and 15 is divided by 1,000. For Noah, 1,000 is the denominator of his fraction. For Elena, the division by 1,000 reverts the initial multiplication, which was by 1,000.

Access for Multilingual Learners (Activity 1, Synthesis)

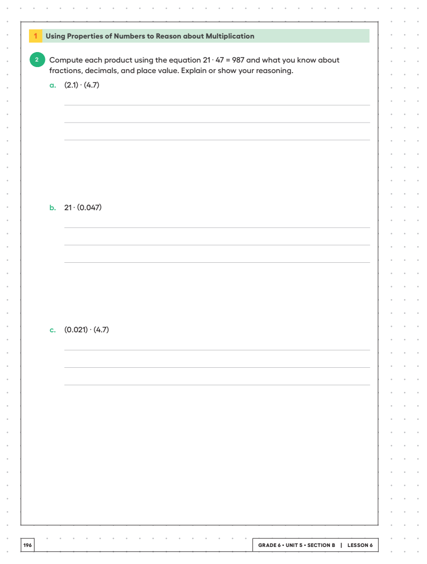
MLR8: Discussion Supports. Invite students to begin partner interactions by repeating the question,

"What did Elena (or Noah) do first to find (0.23) · (1.5)? What did Elena (or Noah) do next?"

This gives both students an opportunity to produce language.

Advances: Conversing

Student Workbook


1 Using Properties of Numbers to Reason about Multiplication

2 Compute each product using the equation 21 · 47 = 987 and what you know about fractions, decimals, and place value. Explain or show your reasoning.

a. (2.1) · (4.7)

b. 21 · (0.047)

c. (0.021) · (4.7)

Activity 2: Optional
Using Area Diagrams to Reason about Multiplication

15 min

Activity Narrative

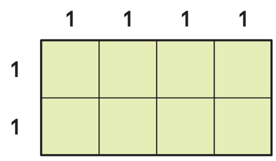
This activity prepares students to use area diagrams to find products of decimals in upcoming lessons. The reasoning here revisits students' work in grade 5, during which they used area diagrams to find products of two numbers, including whole numbers, fractions, and decimals through tenths.

In addition to using the structure of base-ten numbers in their reasoning, students also use the structure of the area diagram to help them find products of decimals.

As students work, monitor for the different ways students reason about the area of each unit square and the area of the large rectangle given particular side lengths. Identify a few students with correct reasoning so they can share later.

Launch

Display a diagram of a rectangle composed of 8 squares. Tell students that the side length of each square is 1 unit, as shown:



Ask students:

- *"What is the area of each square?"*

  1 square unit

- *"How do you know?"*

  $1 \cdot 1 = 1$

- *"What is the area of the rectangle?"*

  8 square units

- *"How do you know?"*

  It is made up of 8 squares and $8 \cdot 1 = 8$. The length is 4 units, the width is 2 units, and $4 \cdot 2 = 8$.

Give students 5 minutes of quiet work time and 2–3 minutes to discuss their responses with a partner. Follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the side lengths of each square to correspond to the factors of each calculation, and the area of each square to correspond to the product of each calculation.

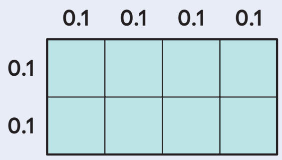
Supports accessibility for: Visual-Spatial Processing

Building on Student Thinking

Some students might think that $(0.1) \cdot (0.1) = 0.1$ (just like $1 \cdot 1 = 1$). If this happens, have them write 0.1 in fraction form so they see that $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$, or 0.01.

Student Task Statement

1. In the diagram, the side length of each square is 0.1 unit.



a. Explain why the area of each square is *not* 0.1 square unit.

The area is not 0.1 square unit because $(0.1) \cdot (0.1)$ is 1 tenth times 1 tenth, which is 1 hundredth, or 0.01, square unit.

b. Explain how you can use the area of each square to find the area of the rectangle.

There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot (0.01) = 0.08$ square unit.

c. What does the equation $(0.4) \cdot (0.2) = 0.08$ represent in this situation?

The 0.4 and 0.2 represent the side lengths of the rectangle, and the 0.08 represents its area. Multiplying the side lengths of a rectangle gives its area.

2. Label the squares with their side lengths so the area of this rectangle represents $40 \cdot 20$.

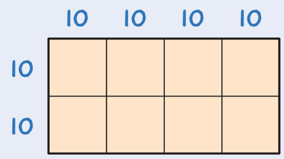
The side length of each square is 10 units.

a. What is the area of each square?

The area of each square is $(10 \cdot 10)$, or 100, square units.

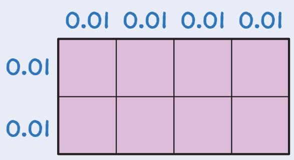
b. Use the squares to help you find $40 \cdot 20$. Be prepared to explain your reasoning.

$40 \cdot 20 = 800$
Sample reasoning: There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot 100$, or 800, square units. Multiplying the side lengths of the rectangle, which are 40 units and 20 units, gives an area of 800 square units.



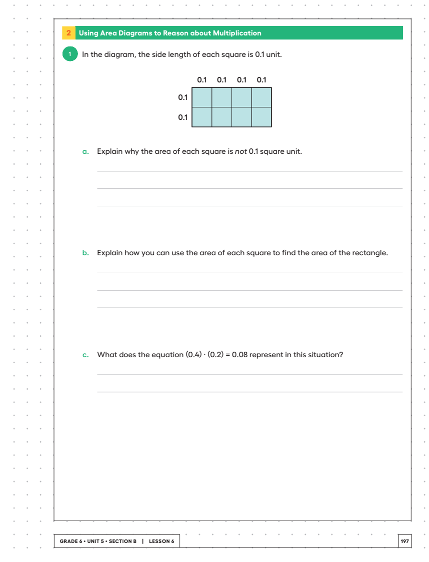
3. Label the squares with their side lengths so the area of this rectangle represents $(0.04) \cdot (0.02)$.

Next, use the diagram to help you find $(0.04) \cdot (0.02)$. Be prepared to explain your reasoning.



The side length of each square is 0.01 unit. $(0.04) \cdot (0.02) = 0.0008$
Sample reasoning: The area of each square is $(0.01) \cdot (0.01)$, or 0.0001, square unit. There are 8 squares in the rectangles, so the area of the rectangle is $8 \cdot (0.0001)$ square unit, which is 0.0008 square unit.

Student Workbook



2 Using Area Diagrams to Reason about Multiplication

1 In the diagram, the side length of each square is 0.1 unit.

a. Explain why the area of each square is *not* 0.1 square unit.

b. Explain how you can use the area of each square to find the area of the rectangle.

c. What does the equation $(0.4) \cdot (0.2) = 0.08$ represent in this situation?

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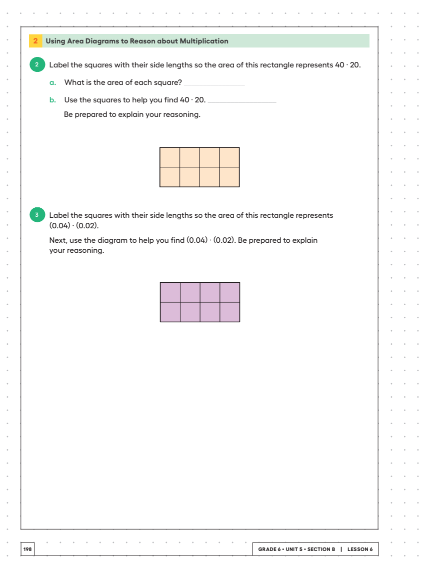
Student Workbook



2 Using Area Diagrams to Reason about Multiplication

2 Label the squares with their side lengths so the area of this rectangle represents $40 \cdot 20$.

a. What is the area of each square?

b. Use the squares to help you find $40 \cdot 20$. Be prepared to explain your reasoning.

3 Label the squares with their side lengths so the area of this rectangle represents $(0.04) \cdot (0.02)$.

Next, use the diagram to help you find $(0.04) \cdot (0.02)$. Be prepared to explain your reasoning.

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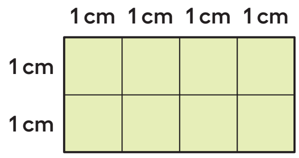
Activity Synthesis

The goal of this discussion is to highlight that the sides of a rectangle can represent lengths in different values, so its area can represent many different products. (This idea is not unlike how a collection of base-ten blocks can represent 103, 10.3, 1.03, and so on.)

If time permits, consider discussing the idea of the same diagram representing different values in the context of measurement units. Display a gridded 2-by-4 rectangle with the side length of squares marked "1 cm." Ask students:



- *"What is the area of each square in cm^2?"*

  1

- *"What is the area of the rectangle in cm^2?"*

  8

- *"What equation can represent the relationship between the side lengths and area?"*

  $4 \cdot 2 = 8$

- *"What is the area of each square in mm^2?"*

  100

- *"What is the area of the rectangle in mm^2?"*

  800

- *"What equation can represent the relationship between the side lengths and area?"*

  $40 \cdot 20 = 800$

- *"What is the area of each square in m^2?"*

  $\frac{1}{10,000}$, or 0.0001

- *"What is the area of the rectangle in m^2?"*

  $\frac{8}{10,000}$, or 0.0008

- *"What equation can represent the relationship between the side lengths and area?"*

  $(0.04) \cdot (0.02) = 0.0008$

Reiterate that the diagram hasn't changed, but the side lengths and area of the same diagram can represent different values depending on the unit of measurement we use.

Tell students that they will continue to use area diagrams to multiply decimals in an upcoming lesson.

Lesson Synthesis

The goal of the discussion is to highlight some properties of base-ten numbers that can facilitate multiplication of decimals. Ask questions such as:

- *"We can multiply two decimals by first writing them as fractions. What is it about the fractions that make multiplying easier?"*

  The numerators and denominators are whole numbers, so they're easier to multiply. The denominator is often a power of 10, which makes division easier.

- *"Another way to multiply decimals is to first multiply the factors by powers of 10. Why might it be helpful to multiply the decimals that way?"*

  It allows us to have whole-number factors.

- *"Why is it okay to multiply the factors by powers of 10?"*

  Afterward, we'd divide the product by the same powers of 10 to undo the multiplication, so we're still multiplying the two original factors.

Lesson Summary

Here are three other ways to calculate a product of two decimals, such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

$$(0.04) \cdot 100 = 4$$
$$(0.07) \cdot 100 = 7$$

  Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

$$4 \cdot 7 = 28$$
$$28 \div 10{,}000 = 0.0028$$

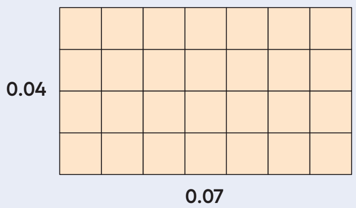
- Second, we can write each decimal as a fraction and multiply them.

$$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10{,}000} = 0.0028$$

- Third, we can use an area diagram. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.

  In this diagram, each small square is 0.01 unit by 0.01 unit. The area of each square, in square units, is therefore

$$\left(\frac{1}{100} \cdot \frac{1}{100}\right), \text{ which is } \frac{1}{10{,}000}.$$

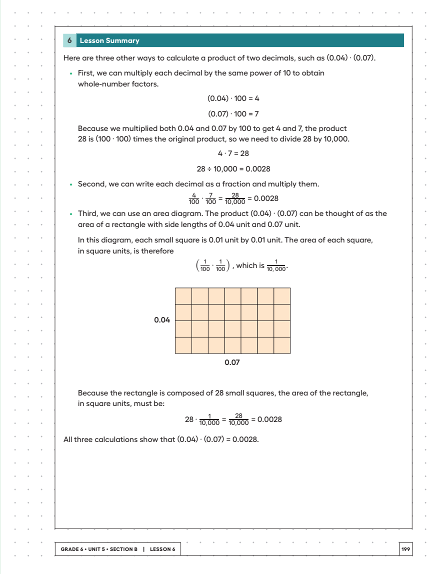


  Because the rectangle is composed of 28 small squares, the area of the rectangle, in square units, must be:

$$28 \cdot \frac{1}{10{,}000} = \frac{28}{10{,}000} = 0.0028$$

All three calculations show that $(0.04) \cdot (0.07) = 0.0028$

Student Workbook



Responding To Student Thinking

Points to Emphasize
If students do not see the relationship between the decimals (1.35 and 4.2) and their related whole numbers (135 and 42), focus on making such relationships explicit as opportunities arise in upcoming lessons. For example, before students calculate the value of the multiplication expressions in the practice problem referred to here, ask students to name the whole number that is related to each decimal factor and explain the relationship.

Grade 6, Unit 5, Lesson 6, Practice Problem 1

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Cool-down

A Product of Two Decimals

5 min

Student Task Statement

Explain or show how you would find the value of $(1.35) \cdot (4.2)$ if you know that $135 \cdot 42 = 5,670$.

Sample responses:

- $135 = (1.35) \cdot 100$ and $42 = (4.2) \cdot 10$, so $135 \cdot 42$ is $100 \cdot 10$, or 1,000, times $(1.35) \cdot (4.2)$. This means $(1.35) \cdot (4.2)$ is $5,670 \div 1,000$, which is 5.67.
- $(1.35) \cdot (4.2) = \frac{135}{100} \cdot \frac{42}{10} = \frac{135 \cdot 42}{10 \cdot 100} = \frac{5,670}{1,000}$, which is 5.67.

Lesson 6

Practice Problems

7 Problems

Problem 1

Find each product. Show your reasoning.

a. (1.2) · (0.11) 0.132

Sample reasoning: 1.2 is one tenth of 12 and 0.11 is one hundredth of 11, so the product of 1.2 and 0.11 is one thousandth of 12 · 11, or $\frac{1}{1{,}000} \cdot 132$, which is 0.132.

b. (0.34) · (0.02) 0.0068

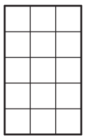
Sample reasoning: $\frac{34}{100} \cdot \frac{2}{100} = \frac{68}{1{,}000}$, or 0.0068.

c. 120 · (0.002) 0.24

Sample reasoning: 0.002 is 2 thousandths, or $2 \cdot \frac{1}{1{,}000}$, so the product of 120 and 0.002 is $120 \cdot 2 \cdot \frac{1}{1{,}000}$, which equals $\frac{240}{1{,}000}$, or 0.24.

Problem 2

You can use a rectangle to represent (0.3) · (0.5).



a. What must the side length of each square represent for the rectangle to correctly represent (0.3) · (0.5)?

0.1

b. What area is represented by each square?

0.01 square units

c. What is (0.3) · (0.5)? Show your reasoning.

0.15

Sample reasoning: There are 15 squares, and 15 · (0.01) = 0.15.

Problem 3

Suppose gasoline costs \$2.29 per gallon in Buffalo, New York and \$0.91 per liter in Toronto, Canada. There are 3.8 liters in one gallon.

a. How much does one gallon of gas cost in Toronto? Round your answer to the nearest cent.

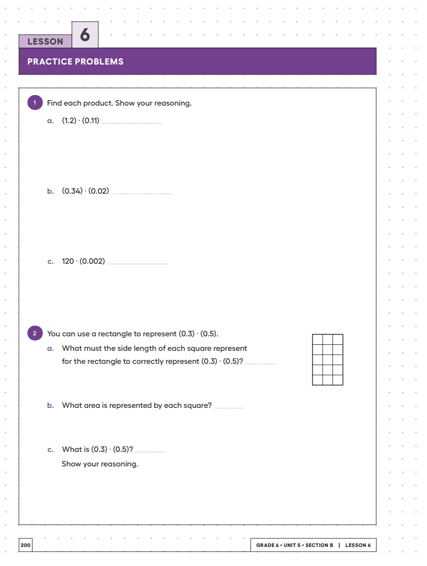
\$3.46

Sample reasoning: (3.8) · (0.91) = 3.458, and this is closer to 3.46 than to 3.45.

b. Is the cost of gas greater in Buffalo or in Toronto? How much greater?

The cost of one gallon of gas is \$1.17 more in Toronto.

Student Workbook



LESSON 6
PRACTICE PROBLEMS

1 Find each product. Show your reasoning.
a. (1.2) · (0.11)
b. (0.34) · (0.02)
c. 120 · (0.002)

2 You can use a rectangle to represent (0.3) · (0.5).
a. What must the side length of each square represent for the rectangle to correctly represent (0.3) · (0.5)?
b. What area is represented by each square?
c. What is (0.3) · (0.5)? Show your reasoning.

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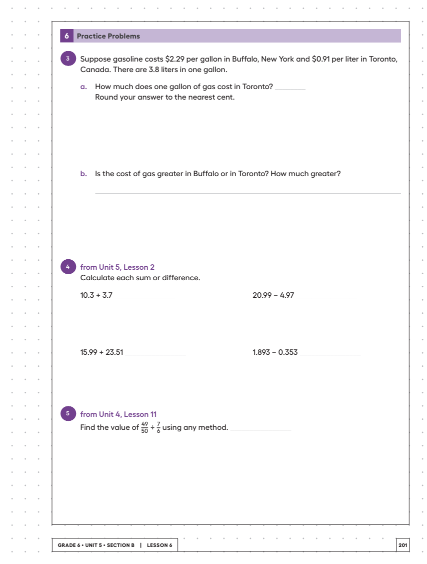
Student Workbook



6 Practice Problems

3 Suppose gasoline costs \$2.29 per gallon in Buffalo, New York and \$0.91 per liter in Toronto, Canada. There are 3.8 liters in one gallon.
a. How much does one gallon of gas cost in Toronto? Round your answer to the nearest cent.
b. Is the cost of gas greater in Buffalo or in Toronto? How much greater?

4 from Unit 5, Lesson 2
Calculate each sum or difference.
10.3 + 3.7
20.99 − 4.97
15.99 + 23.51
1.893 − 0.353

5 from Unit 4, Lesson 11
Find the value of $\frac{49}{50} \div \frac{7}{5}$ using any method.

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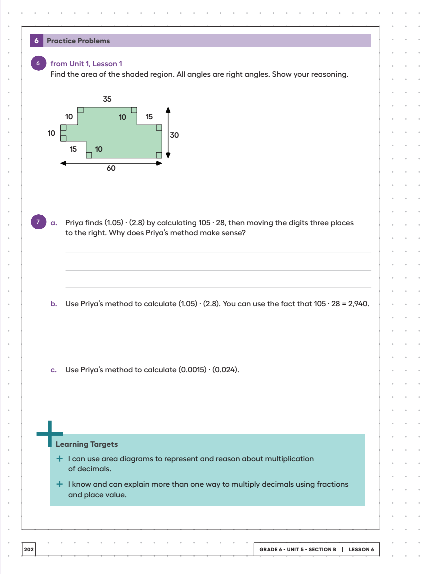
Student Workbook



Problem 4

from Unit 5, Lesson 2

Calculate each sum or difference.

10.3 + 3.7 14

20.99 − 4.97 16.02

15.99 + 23.51 39.5

1.893 − 0.353 1.54

Problem 5

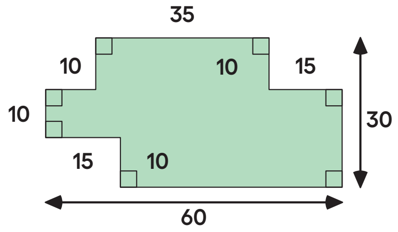
from Unit 4, Lesson 11

Find the value of $\frac{49}{50} \div \frac{7}{6}$ using any method. $\frac{21}{25}$ (or equivalent)

Problem 6

from Unit 1, Lesson 1

Find the area of the shaded region. All angles are right angles. Show your reasoning.



1,400 square units

Sample reasoning: The region can be enclosed with a 60-by-30 rectangle, which has an area of 1,800 square units. Three of the corners of that rectangle have a rectangular region removed. The removed areas are 100 square units (upper left), 150 square units (lower left), and 150 square units (upper right). The area of the shaded region, in square units, is 1,800 − (100 + 150 + 150) or 1,800 − 400, which is 1,400.

Problem 7

a. Priya finds (1.05) · (2.8) by calculating 105 · 28, then moving the digits three places to the right. Why does Priya's method make sense?

$1.05 = \frac{1}{100} \cdot 105$ and $2.8 = \frac{1}{10} \cdot 28$, so $(1.05) \cdot (2.8) = \frac{1}{1{,}000} \cdot (105 \cdot 28)$. This is the same as finding 105 · 28 and then dividing by 1,000, which moves the digits three places to the right.

b. Use Priya's method to calculate (1.05) · (2.8). You can use the fact that 105 · 28 = 2,940.

2.94

c. Use Priya's method to calculate (0.0015) · (0.024).

0.0000360 or 0.000036