

Introducing Double Number Line Diagrams

Goals

- Compare and contrast (orally and in writing) discrete diagrams and double number line diagrams representing the same situation.
- Explain (orally) how to use a double number line diagram to find equivalent ratios.
- Label and interpret a double number line diagram that represents a familiar context.

Learning Targets

- I can label a double number line diagram to represent batches of a recipe or color mixture.
- When I have a double number line that represents a situation, I can explain what it means.

Lesson Narrative

This lesson introduces the **double number line diagram**, a useful and efficient tool for reasoning about equivalent ratios.

The number lines that make up this new representation are similar to those seen in earlier grades in that:

Each number corresponds to a point located that many units away from 0 on the number line.

Because numbers correspond to distances, the distance between, for example, 0 and 12 is three times the distance between 0 and 4.

We can choose the scale to use—whether each interval represents 1 unit, 2 units, 5 units, and so on.

The lines can be extended as needed.

In a double number line diagram, we use two parallel number lines—one line for each quantity in the ratio—and line up the 0s vertically. We choose a scale on each line so that equivalent ratios also line up vertically.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- 5 Practices
- Math Talk

Required Materials

Materials to Gather

- Rulers: Lesson

Lesson Timeline

10
mins

Warm-up

15
mins

Activity 1

15
mins

Activity 2

10
mins

Activity 3

10
min

Lesson Synthesis

Assessment

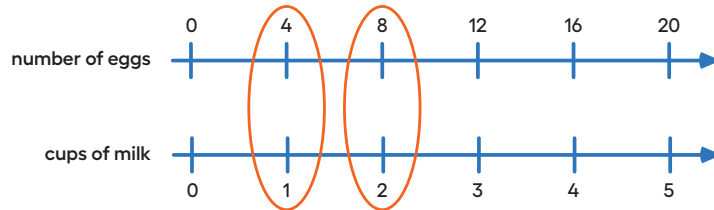
5
mins

Cool-down

Introducing Double Number Line Diagrams

Lesson Narrative (continued)

For example, if the ratio of the number of eggs to the cups of milk in a recipe is 4 to 1, we can draw one number line to represent the number of eggs and another for the cups of milk. On the two number lines, the positions of 4 eggs and 1 cup of milk would line up vertically, as would 8 eggs and 2 cups of milk, and so on.



Double number lines are more abstract and general than are discrete diagrams because quantities are represented by locations on a number line rather than with counts of objects. Relating the concrete to the abstract allows students to practice reasoning quantitatively and abstractly.

Student Learning Goal

Let's use number lines to represent equivalent ratios.

Warm-up

Math Talk: Adjusting Another Factor

10
min

Activity Narrative

This *Math Talk* focuses on multiplication of a whole number and a decimal. It encourages students to observe the impact of adjusting a factor and to rely on structure, patterns, and the properties of operations to find products.

Each expression is designed to elicit slightly different reasoning. In explaining their strategies, students need to be precise in their word choice and use of language. Although many ways of reasoning may emerge, it may not be feasible to discuss every strategy. Consider gathering only 2–3 different strategies per expression. As students explain their strategies, ask them how the factors affected their approach.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.

- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each product mentally.

A. $2 \cdot 45$

90

Sample reasoning:

$$(2 \cdot 40) + (2 \cdot 5) = 80 + 10 = 90$$

- Two times 50 is 100, so 2 times 45 is 2 times 5, or 10, less than 100, which is 90.

B. $2 \cdot (4.5)$

9

Sample reasoning:

- The first factor, 4.5, is one-tenth of 45, so $2 \cdot (4.5)$ is one-tenth of $2 \cdot 45$ or one-tenth of 90, which is 9.

$$(2.4) + (2 \cdot (0.5)) = 8 + 1 = 9$$

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities
(Warm-up, Student Task)Action and Expression: Internalize
Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook

LESSON 6

Introducing Double Number Line Diagrams

Let's use number lines to represent equivalent ratios.

Warm-up: Math Talk: Adjusting Another Factor

Find the value of each product mentally.

- Ⓐ $2 \cdot 45$
- Ⓑ $2 \cdot (4.5)$
- Ⓒ $6 \cdot (4.5)$
- Ⓓ $(0.2) \cdot 45$

Drink Mix on a Double Number Line

A drink recipe says to mix 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:

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Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, “First, I _____ because ...” or “I noticed _____ so I ...” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Student Workbook

LESSON 6

Introducing Double Number Line Diagrams

Let's use number lines to represent equivalent ratios.

Warm-up Math Talk: Adjusting Another Factor

Find the value of each product mentally.

- 2 · 45
- 2 · (4.5)
- 6 · (4.5)
- (0.2) · 45

Drink Mix on a Double Number Line

A drink recipe says to mix 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:

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C. $6 \cdot (4.5)$

27

Sample reasoning:

- Six is 3 times 2, so $6 \cdot (4.5)$ is 3 times the result of $2 \cdot (4.5)$, or 3 times 9, which is 27.
- $6 \cdot 4 = 24$ and $6 \cdot (0.5) = 3$, so $6 \cdot (4.5)$ is the sum of 24 and 3, which is 27.

D. $(0.2) \cdot 45$

9

Sample reasoning: The first factor, 0.2, is one-tenth of 2, so $(0.2) \cdot 45$ is one-tenth of $2 \cdot 45$, or one-tenth of 90, which is 9. Two-tenths is one-fifth, and one-fifth of 45 is 9.

Activity Synthesis

To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ___’s strategy?”
- “Do you agree or disagree? Why?”
- “What connections to previous problems do you see?”

Activity 1

Drink Mix on a Double Number Line

15
min

Activity Narrative

In this activity, a double number line (a new representation) is presented and interpreted alongside the more familiar discrete diagrams and in the familiar context of recipes.

Students learn that, just like discrete diagrams, double number lines represent equivalent ratios. They see that alignment between the numbers of the two lines matters, and that it is through the alignment that the association of two quantities is shown. Students notice that pairs of numbers that “line up” vertically are equivalent ratios.

Because double number lines are quicker to draw and can be extended easily to show many more equivalent ratios, they are more efficient than discrete diagrams are, especially for dealing with larger quantities.

As students work, monitor for those who contrast the two representations in terms of using graphic symbols versus numbers, and those who think about equivalent ratios in terms of the alignment of numbers in the double number line diagram.

Launch

Ask students to recall the mixture of powdered drink mix and water from a previous lesson. Ask:

“How much drink mix and water was in one batch?”

4 teaspoons of drink mix and 1 cup of water.

“What would you need to mix a double batch?”

8 teaspoons of drink mix and 2 cups of water.

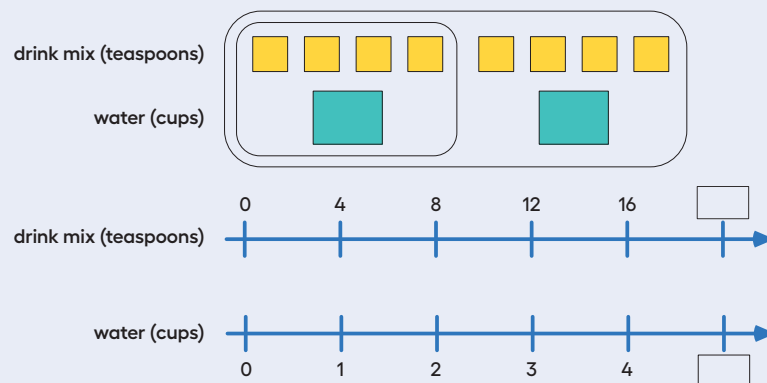
Explain that they are going to show batches of a mixture using a **double number line diagram**.

Give students 5 minutes of quiet think time to make sense of the new representation and to answer the questions, and then give them time to share their response with a partner.

Follow up with a whole-class discussion.

Student Task Statement

A drink recipe says to mix 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:



1. How can we tell that 4:1 and 12:3 are equivalent ratios?

12 and 3 are 3 times 4 and 1, respectively. On the number line diagram, we can see that 4 and 1 line up vertically, as do 12 and 3.

2. How are these representations the same? How are these representations different?

Sample response:

- Same: Each representation shows the amount of drink mix and water for one batch and two batches. They each show teaspoons of drink mix along the top and cups of water along the bottom.
- Different: The first diagram uses squares to represent each teaspoon of drink mix and cup of water, but the number line diagram has these amounts written with numbers. The first diagram shows only two batches and the number line diagram shows 0, 1, 2, 3, and 4 batches (with space for 5 batches).

3. How many teaspoons of drink mix should be used with 3 cups of water?

12 teaspoons

Access for Students with Diverse Abilities
(Activity 1, Launch)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “The representations are the same because ...” and “The representations are different because ...”

Supports accessibility for: Language, Social-Emotional Functioning

Access for Multilingual Learners
(Activity 1, Launch)

MLR8: Discussion Supports.

If necessary, remind students of the meaning of these terms: “recipe,” “batch,” “mixture,” and “diagram.” This will support student understanding of the context so that they can make sense of the double number line.

Advances: Reading, Writing

Building on Student Thinking

While the double number line diagram is given here, some students may not feel comfortable with seeing the same numbers (the 4’s) in different positions. Remind students that each number line represents a different quantity, and that the two 4’s have different meanings.

Student Workbook

LESSON 6

Introducing Double Number Line Diagrams

Let's use number lines to represent equivalent ratios.

Math Task: Adjusting Another Factor
Find the value of each product mentally.
2 · 45
2 · (4 · 5)
5 · (4 · 5)
(5 · 2) · 45

Drink Mix on a Double Number Line
A drink recipe says to mix 4 teaspoons of powdered drink mix for every cup of water.
Here are two ways to represent multiple batches of this recipe.
drink mix (teaspoons) [diagram with 4 yellow squares] [diagram with 12 yellow squares]
water (cups) [diagram with 1 blue square] [diagram with 3 blue squares]
drink mix (teaspoons) 0 4 8 12 16
water (cups) 0 1 2 3 4

Student Workbook

Drink Mix on a Double Number Line
1. How can we tell that 4 : 1 and 12 : 3 are equivalent ratios?
2. How are these representations the same? How are these representations different?
3. How many teaspoons of drink mix should be used with 3 cups of water?
4. How many cups of water should be used with 16 teaspoons of drink mix?
5. What numbers should go in the empty boxes on the double number line diagram? What do these numbers mean?

Are You Ready for More?
Recall that a perfect square is a number of objects that can be arranged into a square. For example, 9 is a perfect square because 9 objects can be arranged into 3 rows of 3. 16 is also a perfect square, because 16 objects can be arranged into 4 rows of 4. In contrast, 12 is not a perfect square because you can't arrange 12 objects into a square.
1. How many whole numbers starting with 1 and ending with 100 are perfect squares?
2. What about whole numbers starting with 1 and ending with 1,000?

4. How many cups of water should be used with 16 teaspoons of drink mix?
4 cups
5. What numbers should go in the empty boxes on the double number line diagram? What do these numbers mean?

The numbers 20 and 5 should go in the missing places. These numbers mean that the result of mixing 20 teaspoons of drink mix with 5 cups of water would taste the same as the other mixtures, or that these amounts would make 5 batches of the recipe.

Are You Ready for More?

Recall that a perfect square is a number of objects that can be arranged into a square. For example, 9 is a perfect square because 9 objects can be arranged into 3 rows of 3. 16 is also a perfect square, because 16 objects can be arranged into 4 rows of 4. In contrast, 12 is not a perfect square because you can't arrange 12 objects into a square.

1. How many whole numbers starting with 1 and ending with 100 are perfect squares?
There are 10 perfect squares between 1 and 100, because $1^2 = 1$ and $10^2 = 100$.
2. What about whole numbers starting with 1 and ending with 1,000?
There are 31 perfect squares between 1 and 1,000, because $31^2 = 961$, but $32^2 = 1,024$.

Activity Synthesis

Select students to share their observations about how the two representations are alike and how they differ. As students discuss solutions to the questions, circle pairs of associated quantities on the double number line. Help students connect information as it is represented in the different diagrams.

On the last question, ask students how they knew that 20 was the next number on the line representing teaspoons of drink mix? (Skip counting by 4, or multiply the next number of cups of water by 4.)

Ask students to think more generally for a minute about the representations at hand:

- “What is a double number line diagram? What do they do? What do the numbers on the tick marks represent and how should they be scaled?”
- “What might be some benefits of using double number lines instead of diagrams?”
- We can use them to show many more batches; they are quicker to draw.

Activity 2

Blue Paint on a Double Number Line

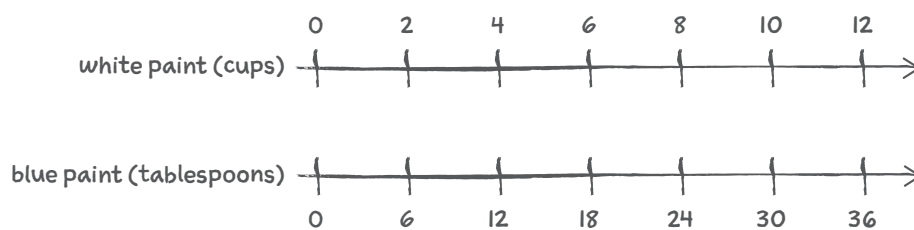
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min

Activity Narrative

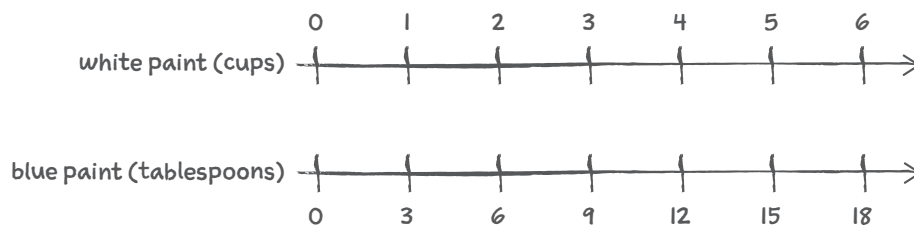
In this activity, students apply their understanding of different-sized batches of a recipe to label the tick marks on a double number line diagram with equivalent ratios. The context is familiar so students can focus on further making sense of the new representation.

Monitor for the different ways in which students interpret the discrete diagram and represent the ratio of white paint to blue paint on a double number line diagram. Here are some approaches that students may take, from more common to less common:

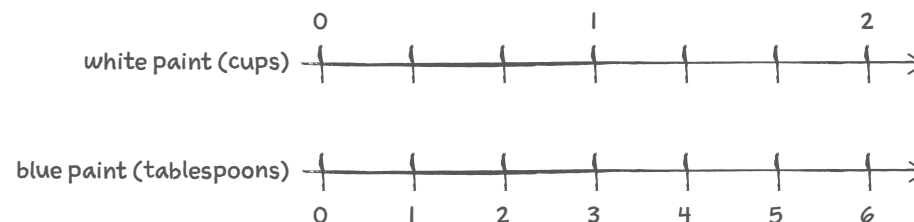
- Label the tick marks on the line representing white paint with multiples of 2 and those on the other line with multiples of 6.



- Label the tick marks on the line representing white paint with multiples of 1 and those on the other line with multiples of 3.



- Use the given diagram to represent a single batch (by labeling the third and sixth tick marks on the line representing white paint with 1 and 2, respectively, and labeling the tick marks for blue paint in increments of 1).



During partner discussions, students may need guidance in recognizing that each of the listed approaches is valid. Encourage students to explain the rationale behind their labels.

The last two representations may be less common and may affect students' responses to subsequent questions about the number of batches for 12 tablespoons of blue paint or for 6 cups of white paint.

Instructional Routines

5 Practices

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Student Workbook

Blue Paint on a Double Number Line

Here is a diagram showing Elena's recipe for one batch of light blue paint.

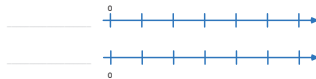
white paint (cups)



blue paint (tablespoons)



1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint.



2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram.

3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make?

4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?

5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint.

6. How do you know that these mixtures would make the same shade of light blue paint?

155

GRADE 6 • UNIT 2 • SECTION C | LESSON 6

Launch

Give students 4 minutes of quiet work time and time to share their response with a partner.

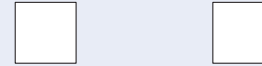
Follow up with a whole-class discussion.

Select students who created different representations to share.

Student Task Statement

Here is a diagram showing Elena's recipe for one batch of light blue paint.

white paint (cups)



blue paint (tablespoons)



1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint.



2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram.

No response required.

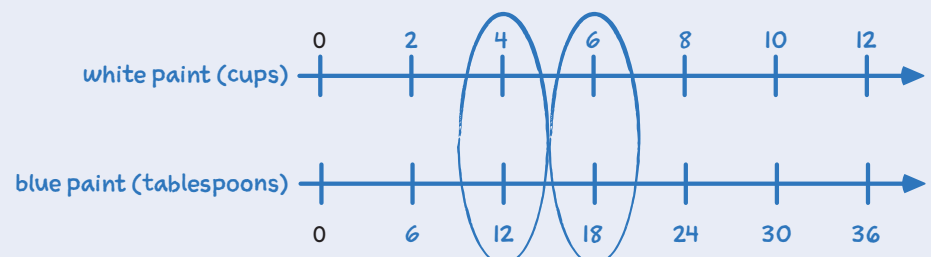
3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make?

4 cups of white paint, because that is the number on the top line that lines up with the 12 on the bottom line. This would make 2 batches of paint.

4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?

18 tablespoons of blue paint. This would make 3 batches of paint.

Sample reasoning:



5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint.

Sample responses:

- 8 cups of white paint and 24 tablespoons of blue paint
- 10 cups of white paint and 30 tablespoons of blue paint
- 3 cups of white paint and 9 tablespoons of blue paint
- 1 cup of white paint and 3 tablespoons of blue paint

6. How do you know that these mixtures would make the same shade of light blue paint?

Sample response: I know these mixtures would make the same shade of light blue paint because the ratios of the amounts of each paint color are equivalent to the ratio in the original recipe.

Activity Synthesis

Select students to present their solutions. Sequence the discussion of students’ completed diagrams in the order listed in the *Activity Narrative*. Connect students’ representations to the ratio of 2 pints of white paint to 6 cups of blue paint and to the discrete diagrams. Highlight the different ways in which this ratio (or its equivalence) is visible in the diagrams, extending or annotating the number lines as needed. Ask questions such as:

- “Where do you see one batch in each diagram?”
- “What information do we gain when we label the number line for tablespoons of blue paint by 3s or by 1s instead of by 6s?”
- “How do we show larger batches using drawings? Using a double number line?”

Emphasize the importance of labeling everything clearly so our representations are easier to interpret.

Student Workbook

Blue Point on a Double Number Line

Here is a diagram showing Elena's recipe for one batch of light blue paint.

white paint (cups)

blue paint (tablespoons)

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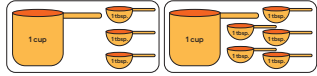
Building on Student Thinking

Students may suggest adding 2 more cups of milk since Andre accidentally added 2 more tablespoons of cocoa. Ask students what the ratio of milk to cocoa would be in that case (3 cups of milk to 5 tablespoons of cocoa) and whether the mixture would taste the same as the original recipe. Encourage them to create a diagram (or amend the given diagram) to represent their proposed fix and reason about its taste compared to the hot cocoa that follows the recipe.


Student Workbook

Fix It!

Andre likes a hot cocoa recipe with 1 cup of milk and 3 tablespoons of cocoa. He poured 1 cup of milk but accidentally added 5 tablespoons of cocoa.



- How can you fix Andre's mistake and make his hot cocoa taste like the recipe?
- Explain how you know your adjustment will make Andre's hot cocoa taste the same as the one in the recipe.



GRADE 6 • UNIT 2 • SECTION C | LESSON 6

Activity 3: Optional

Fix It!

10
min

Activity Narrative

In this optional activity, students apply their understanding of equivalent ratios to solve a problem they might encounter naturally outside a mathematics classroom. The reasoning is more involved, as the question is open-ended and the response needs to be validated by mathematical reasoning.

Launch



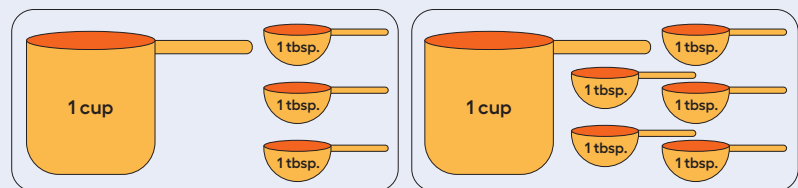
Arrange students in groups of 2. Display the image for all to see. Alternatively, introduce the context using actual ingredients in the problem. Prepare a clear glass, milk, and cocoa powder. Tell students that Andre likes his hot cocoa with 1 cup of milk and 3 tablespoons of cocoa. Then, demonstrate the mistake Andre made: Pour 1 cup of milk into the glass and add 5 tablespoons of cocoa powder.

Ask students how they would fix Andre's mistake and make the hot cocoa taste like the recipe. Tell students to give a signal when they have an answer and a strategy.

Give students 2 minutes of quiet think time.

Student Task Statement

Andre likes a hot cocoa recipe with 1 cup of milk and 3 tablespoons of cocoa. He poured 1 cup of milk but accidentally added 5 tablespoons of cocoa.



- How can you fix Andre's mistake and make his hot cocoa taste like the recipe?

Sample responses:

- Add 1 more tablespoon of cocoa and 1 cup of milk.
- Add $\frac{2}{3}$ cup of milk.

- Explain how you know your adjustment will make Andre's hot cocoa taste the same as the one in the recipe.

Sample response:

- Adding 1 tablespoon of cocoa and 1 cup of milk would double the cups of milk and tablespoons of cocoa and make two batches of hot cocoa, which would taste the same as one batch. The ratio 1:3 is equivalent to 2:6.
- In his recipe, Andre uses $\frac{1}{3}$ cup of milk for every tablespoon of cocoa. Since he added 2 extra tablespoons of hot cocoa, he'll need to add an extra $\frac{2}{3}$ cup of milk to make his mixture taste the way he likes it.

Activity Synthesis

Invite students to share how they would make the hot cocoa taste like the recipe. Record the strategies for all to see. After each explanation, ask the class if they agree or disagree and how they know the resulting hot cocoa will taste the same. Highlight appropriate use of ratio language (such as “3 tablespoons of cocoa for every cup of milk” or “ $\frac{1}{3}$ cup of milk per tablespoon of cocoa”) and explanations in terms of equivalent ratios (such as “the ratio 1 cup to 3 tablespoons and 2 cups to 6 tablespoons are equivalent”).

Lesson Synthesis

To highlight the features of a double number line diagram and reasons for using the representation, consider asking students:

“How does a double number line diagram show equivalent ratios? How is it different from the way other diagrams show equivalent ratios?”

In a double number line, the numbers in each equivalent ratio line up vertically. In earlier diagrams, we need to count the objects or symbols to see the numbers in the ratios and then see if the ratios are equivalent.

“When might it be helpful to use a double number line diagram?”

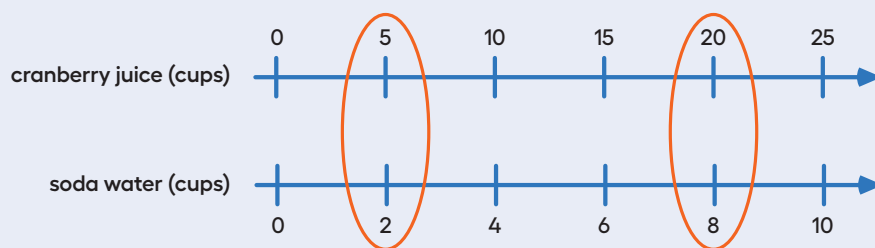
A double number line can be used when drawing a discrete diagram is cumbersome or impossible, such as when the numbers involved are very large.

“What are some things to pay attention to when using a double number line diagram?”

Make sure that the label and numbers representing each quantity are shown correctly and that equivalent ratios align vertically. Make sure that the 0s on the number lines are aligned.

Lesson Summary

You can use a **double number line diagram** to find many equivalent ratios. For example, a recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” The ratio of cranberry juice to soda water is 5:2. Multiplying both ingredients by the same number creates equivalent ratios.

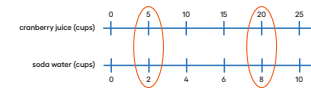


This double number line shows that the ratio 20:8 is equivalent to 5:2. If you mix 20 cups of cranberry juice with 8 cups of soda water, it makes 4 times as much fizzy juice that tastes the same as the original recipe.

Student Workbook

Lesson Summary

You can use a **double number line diagram** to find many equivalent ratios. For example, a recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” The ratio of cranberry juice to soda water is 5:2. Multiplying both ingredients by the same number creates equivalent ratios.



This double number line shows that the ratio 20:8 is equivalent to 5:2. If you mix 20 cups of cranberry juice with 8 cups of soda water, it makes 4 times as much fizzy juice that tastes the same as the original recipe.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

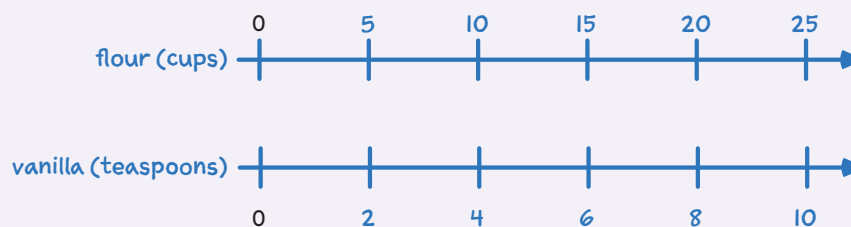
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Batches of Cookies on a Double Number Line

Student Task Statement

A recipe for one batch of cookies uses 5 cups of flour and 2 teaspoons of vanilla.

1. Complete the double number line diagram to show the amount of flour and vanilla needed for 1, 2, 3, 4, and 5 batches of cookies.



2. If you use 20 cups of flour, how many teaspoons of vanilla should you use?

8 teaspoons of vanilla

3. If you use 6 teaspoons of vanilla, how many cups of flour should you use?

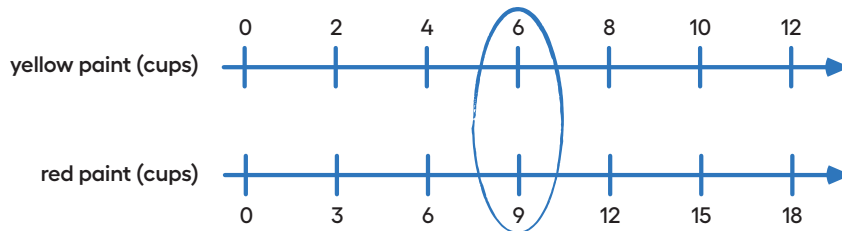
15 cups of flour

Practice Problems

5 Problems

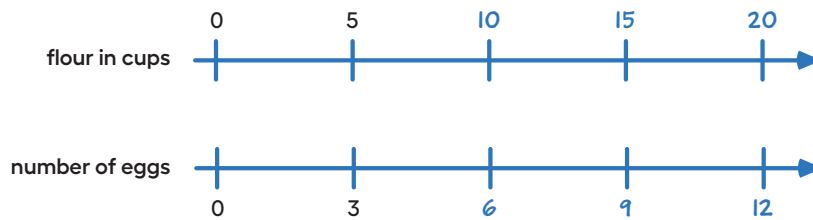
Problem 1

A particular shade of orange paint has 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of orange paint.



Problem 2

This double number line diagram shows the amount of flour and eggs needed for 1 batch of cookies.

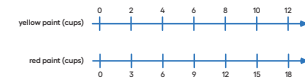


- Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of cookies.
- What is the ratio of cups of flour to eggs?
5:3 or equivalent
- How much flour and how many eggs are used in 4 batches of cookies?
20 cups of flour and 12 eggs
- How much flour is used with 6 eggs?
10 cups
- How many eggs are used with 15 cups of flour?
9 eggs

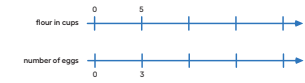
Student Workbook

LESSON 6
PRACTICE PROBLEMS

- 1 A particular shade of orange paint has 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of orange paint.



- 2 This double number line diagram shows the amount of flour and eggs needed for 1 batch of cookies.

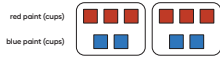


- Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of cookies.
- What is the ratio of cups of flour to eggs?
- How much flour and how many eggs are used in 4 batches of cookies?
- How much flour is used with 6 eggs?
- How many eggs are used with 15 cups of flour?

Student Workbook

Practice Problems

Here is a representation showing the amount of red and blue paint that makes 2 batches of purple paint.



On the double number line, label the tick marks to represent amounts of red and blue paint used to make batches of this shade of purple paint.



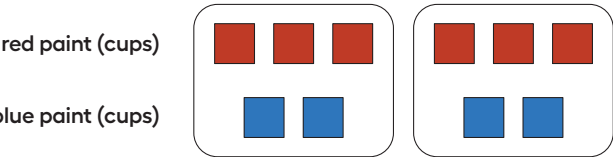
How many batches are made with 12 cups of red paint?

How many batches are made with 6 cups of blue paint?

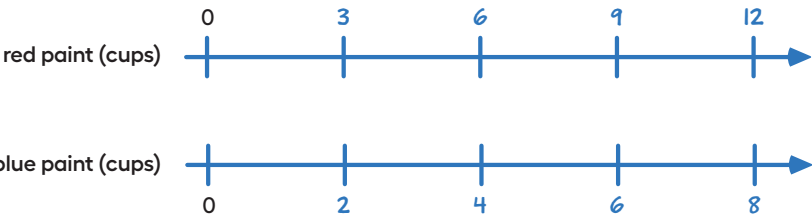
- from Unit 2, Lesson 1
Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select **all** the statements that express this ratio.
- ☒ The ratio of kids to pizzas is 7:3.
 - ☒ The ratio of pizzas to kids is 3 to 7.
 - ☒ The ratio of kids to pizzas is 3:7.
 - ☒ The ratio of pizzas to kids is 7 to 3.
 - ☒ For every 7 kids there need to be 3 pizzas.

Problem 3

Here is a representation showing the amount of red and blue paint that makes 2 batches of purple paint.



a. On the double number line, label the tick marks to represent amounts of red and blue paint used to make batches of this shade of purple paint.



b. How many batches are made with 12 cups of red paint?

4 batches

c. How many batches are made with 6 cups of blue paint?

3 batches

Problem 4

from Unit 2, Lesson 1

Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select **all** the statements that express this ratio.

A. The ratio of kids to pizzas is 7:3.

B. The ratio of pizzas to kids is 3 to 7.

C. The ratio of kids to pizzas is 3:7.

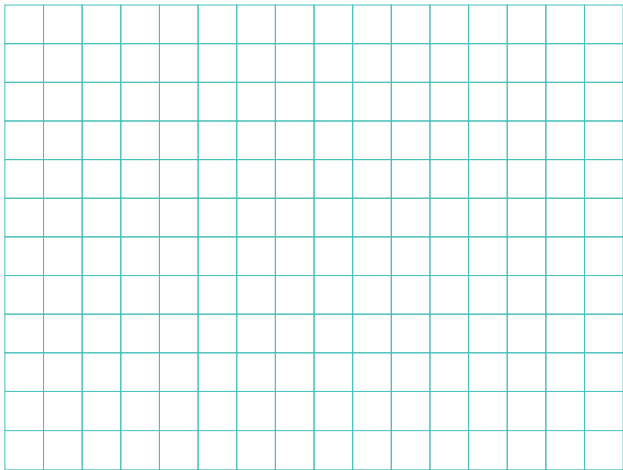
D. The ratio of pizzas to kids is 7 to 3.

E. For every 7 kids there need to be 3 pizzas.

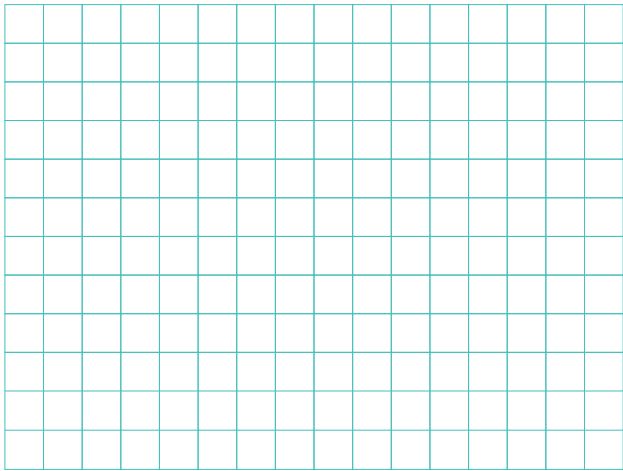
Problem 5

from Unit 1, Lesson 6

- a. Draw a parallelogram that is not a rectangle that has an area of 24 square units. Explain or show how you know that the area is 24 square units.



- b. Draw a triangle that has an area of 24 square units. Explain or show how you know that the area is 24 square units.



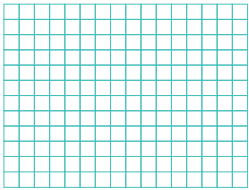
Answers vary. There are many possible pairs of base and height lengths to make an area of 24 square units.

Student Workbook

Practice Problems

From Unit 1, Lesson 6

a. Draw a parallelogram that is not a rectangle that has an area of 24 square units.



Explain or show how you know that the area is 24 square units.

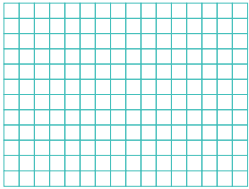
GRADE 4 • UNIT 1 • SECTION C | LESSON 6

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Student Workbook

Practice Problems

b. Draw a triangle that has an area of 24 square units.



Explain or show how you know that the area is 24 square units.

Learning Targets

+ I can label a double number line diagram to represent batches of a recipe or color mixture.

+ When I have a double number line that represents a situation, I can explain what it means.

GRADE 4 • UNIT 1 • SECTION C | LESSON 6

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