Lesson 1

Fermi Problems

Goals

- Estimate quantities in a real-world situation, and explain (orally and in writing) the estimation strategy.
- Justify (orally) why it is unreasonable to have an exact answer for a situation that involves estimation, and critique (orally) different estimates.
- Make simplifying assumptions, and determine what information is needed to solve a Fermi problem about distance, volume, or surface area.

Student Learning Goal

Let's make some estimates.

Lesson Narrative

The activities in this optional lesson are sometimes called "Fermi problems" after the famous physicist Enrico Fermi. A Fermi problem requires students to make a rough estimate for quantities that are difficult or impossible to measure directly. Often, they use rates and require several calculations with fractions and decimals, making them well-aligned to grade 6 work. Fermi problems are examples of mathematical modeling, because one must make simplifying assumptions, make estimates, conduct research, and make decisions about which quantities are important and what mathematics to use. They also encourage students to attend to precision, because one must think carefully about how to appropriately report estimates and choose words carefully to describe the quantities.

Lesson Timeline

30 min	20 min	20 min
Activity 1	Activity 2	Activity 3

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR7: Compare and Connect (Activity 3)

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Poll the Class

Required Materials

Materials to Gather

- Four-function calculators: Activity 1, Activity 2, Activity 3
- Internet-enabled device: Activity 1, Activity 2, Activity 3
- Tools for creating a visual display: Activity 1, Activity 2, Activity 3

Required Preparation

Activity 1:
Internet-enabled devices are necessary only if students will conduct research to find quantities that they need to know. As an alternative, you can supply the information when they ask for it.

Tools for creating a visual display are needed only if you would like students to present their work in an organized way and have the option of conducting a *Gallery Walk*.

Activity 2:
Internet-enabled devices are necessary only if students will conduct research to find quantities that they need to know. As an alternative, you can supply the information when they ask for it.

Tools for creating a visual display are only needed if you would like students to present their work in an organized way and have the option of conducting a *Gallery Walk*.

Lesson 1

Fermi Problems

Lesson Narrative (continued)

The three problems in this lesson involve measurement conversion, calculation of volumes and surface areas of three-dimensional figures, or the relationship of distance, rate, and time. Each of these activities can stand on its own and teachers can select how many of the activities to complete. As students work on any of these activities, monitor for different approaches to solving the problem, and select students to share during discussion. In particular look for:

- A range of estimates.
- Different levels of precision in reporting the final estimate.
- Different representations (for example, double number line diagrams and tables).

Activity 3:
Internet-enabled devices are only necessary if students will conduct research to find quantities that they need to know. As an alternative, you can supply the information when they ask for it.

Tools for creating a visual display are only needed if you would like students to present their work in an organized way and have the option of conducting a *Gallery Walk*.

Activity 1
Ant Trek

30 min

Activity Narrative

This activity is an opportunity for students to apply their understanding of rates to solve a Fermi problem estimating the time it would take an ant to run from Los Angeles to New York City. To do so, students need to consider various factors that could affect the time of travel, such as the distance between the two cities, the speed of travel, whether breaks are involved, and so on. As they consider relevant factors and make a plan to approach the problem, students make sense of problems and persevere in solving them. In making estimates and performing calculations involving large numbers, students practice attending to precision.

Monitor for different approaches to solving the problem, and select students to share during the discussion. In particular, look for:

- The distance students use as the basis for their calculations.
- The assumptions students make about the ant's speed, how it travels, and other external factors.
- The units of measurement students choose for speed and distance.
- Whether or how measurements or estimates are rounded and converted in the process of problem solving.

Launch

Arrange students in groups of 2. Introduce the context of an ant running from Los Angeles to New York City. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

- Display only the statement "an ant is running from Los Angeles to New York City" without revealing the question. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation, and then ask them to compare their questions with a partner.
- Invite several partners to share one question with the class, and record the responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as finding reasonable estimates.

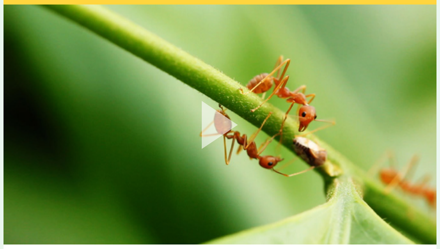
- Reveal the question "How long will the journey take?" and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

"Which of your questions is most similar to or different from the one given? Why?"

Tell students that they will now answer the question about how much time it would take an ant to travel between the two cities. Remind students that a problem like this is called a Fermi problem—a problem that cannot be solved by measuring directly but can be answered by estimating and reasoning. Ask students to brainstorm the information they need to answer the question. Give the information provided when students ask for it, or provide access to internet-enabled devices so that students can find the information they need.

Inspire Math

Ants video



Go Online
Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614191

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544

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Access for Multilingual Learners (Activity 1)

MLR5: Co-Craft Questions.
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.
Activate or supply background knowledge. Provide a Fermi problem graphic organizer for students to use as a reference.

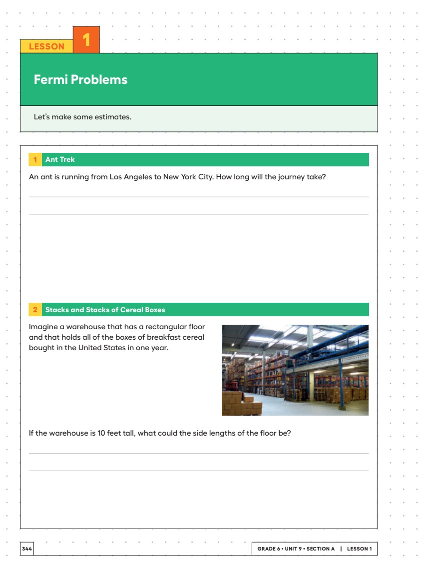
Supports accessibility for: Memory, Organization

Building on Student Thinking

If students have trouble starting with no given information, consider asking:

> *"What are some things you need to know to solve this problem?"*
> *"What subquestions do you need to answer to solve this problem?"*

Student Workbook


LESSON 1

Fermi Problems

Let's make some estimates.

1 Ant Trek

An ant is running from Los Angeles to New York City. How long will the journey take?

2 Stacks and Stacks of Cereal Boxes

Imagine a warehouse that has a rectangular floor and that holds all of the boxes of breakfast cereal bought in the United States in one year.

If the warehouse is 10 feet tall, what could the side lengths of the floor be?

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Provide access to four-function calculators. Vital information to have on hand includes:

- The distance between Los Angeles and New York City is about 3,944 km.
- An ant can run about 18 mm per second.

If conducting a *Gallery Walk* at the end, provide access to tools for making a visual display.

Student Task Statement

An ant is running from Los Angeles to New York City. How long will the journey take?

Sample response: It will take the ant about 7 years to run from Los Angeles to New York City without stopping. The ant can run 18 mm per second. There are 60 seconds per minute and 60 minutes per hour, so this is $18 \cdot 60 \cdot 60 = 64,800$, or 64,800 mm per hour. 64,800 mm per hour is 64.8 meters per hour, or 0.0648 km per hour. A speed of 0.0648 km per hour translates to a pace of $1 \div 0.0648$ hours per km. So to travel 3944 km, it would take $3944 \div 0.0648$ or approximately 60,864 hours. There are 24 hours per day and 365 days per year, so dividing by 24 and then by 365 tells us that it will take the ant about 7 years to make this trip without stopping.

Activity Synthesis

Invite students or groups to share different solution approaches. Alternatively, consider asking students to create a visual display and conducting a *Gallery Walk*. Highlight explanations or visual displays that include keeping careful track of information such as:

- How far the ant has to travel.
- How fast the ant travels.
- A step-by-step analysis changing mm per second to km per day and eventually km per year.

The goal of this discussion is to make sure students understand that many different estimates are expected and correct. Here are some questions for discussion:

"Why did groups end up with different answers? Is that expected?"

It is expected because any approach involves different assumptions. Even a small difference in ant speed, when applied over such a long distance, makes a large difference in the final answer.

"Did you calculate only the time the ant is moving? Or did you include time for the ant to rest?"

Our calculations include only the time the ant is moving. For a real ant, it would take longer to allow things like eating and resting.

"Did you round any numbers while working on the problem? Which quantities did you end up rounding and why?"

We rounded the distance between the cities, the number of hours, and the number of years. Rounding some numbers to powers of 10 or to larger units made calculation and estimation easier. Larger units could also be more informative. For example, expressing travel time in days, weeks, or months might be more helpful than in minutes or seconds.

Also of interest is the fact that ants do not live long enough to complete this trip. Many ants live for only a couple months. If students realize this, ask them how many ant lifetimes it would take for an ant to make this journey.

Activity 2

Stacks and Stacks of Cereal Boxes

20 min

Activity Narrative

In this activity, students solve a Fermi problem involving geometry: estimating the total volume occupied by all of the breakfast cereal purchased in a year in the United States. To do so, students need to gather information about the number of households, estimate the size of a typical cereal box, and make assumptions about cereal consumption. This work engages them in aspects of mathematical modeling. As students make estimates and work with large numbers, they reason abstractly and quantitatively.

Monitor for different approaches to solving the problem, and select students to share during the discussion. In particular, look for:

- The assumptions students make about the size of the boxes.
- The units of measurement students choose for the boxes and the warehouse.
- Whether or how measurements or estimates are rounded and converted in the process of problem solving.

Launch

If available, display a cereal box for all to see. Tell students that they are going to solve a Fermi problem about cereal boxes. Ask students to estimate how many boxes of that size could fill their locker, a storage cabinet in the classroom, or the classroom itself. Give students a minute of quiet think time and invite students to share their responses.

Next, display and read aloud the first paragraph of the task. If needed, clarify that a warehouse is a large building used to store goods or equipment. Give students a minute of quiet think time to make a guess. Poll the class on their responses and record all guesses.

Ask students to brainstorm the information they need to answer the question. Give the information provided when students ask for it, or provide access to internet-enabled devices so that students can find the information they need.

Vital information to have on hand includes:

- Every year, people in the U.S. buy 2.7 billion boxes of breakfast cereal.
- A "typical" cereal box has dimensions of 2.5 inches by 7.75 inches by 11.75 inches.

Arrange students in groups of 2–4. Provide access to four-function calculators. If conducting a *Gallery Walk* at the end, provide access to tools for making a visual display.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

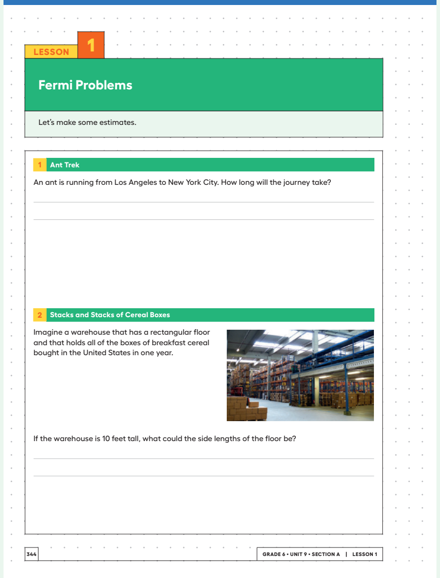
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students find it challenging to conceptualize a number as large as 2.7 billion, consider asking:

"Tell me more about a billion."
"How could you write 2.7 billion in a different way?"

Student Workbook


LESSON 1

Fermi Problems

Let's make some estimates.

1 Ant Trek

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Imagine a warehouse that has a rectangular floor and that holds all of the boxes of breakfast cereal bought in the United States in one year.

If the warehouse is 10 feet tall, what could the side lengths of the floor be?

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Student Task Statement

Imagine a warehouse that has a rectangular floor and that holds all of the boxes of breakfast cereal bought in the United States in one year.

If the warehouse is 10 feet tall, what could the side lengths of the floor be?

Sample response:

The warehouse would be a little more than one mile wide on each side. A typical cereal box measures 2.5 inches by 7.75 inches by 11.75 inches. So the volume of a single box is about 228 cubic inches. There are 12^3 or 1728 cubic inches per cubic foot, so a box of cereal is about 0.13 cubic feet. Since there are 2.7 billion boxes, the total volume is about 360,000,000 cubic feet. If the warehouse is 10 feet tall, then the area of the floor needs to be at least 360,000,000 ÷ 10 or 36,000,000 square feet. So if the warehouse floor were a square with side length 6,000 feet, it could hold all of the boxes. There are 5,280 feet in a mile, so the warehouse would be a little more than one mile wide on each side.

Activity Synthesis

Invite students or groups to share different solution approaches. Consider asking students to create a visual display, and consider conducting a *Gallery Walk*. Highlight explanations or visual displays that include keeping careful track of information such as:

- The side lengths and volume of each box.
- How the boxes are stacked or arranged.
- The side lengths and volume of the warehouse.
- Conversions from the smaller units used for the cereal box to larger units used for the building (or vice versa).

The goal of this discussion is to make sure students understand that there are many possible ways to approach the problem, and that different considerations will yield different estimates. Here are some questions for discussion:

"Why did groups end up with different answers? Is that expected?"

It is expected because any approach will have different assumptions. A focus on volume will give a different answer than a focus on how many boxes fit in each direction.

"How did you begin to answer the question? What aspects of the situation did you think about first?"

We started by using one side of the box to decide how long another side of the warehouse could be. Since each box is 2.5 inches wide, we planned for 20,000 boxes to fit along one wall. That wall would need to be about 4,200 ft long. We then knew two side lengths of the warehouse, so we could solve for the third.

"Did you round any numbers while working on the problem? Which quantities did you end up rounding and why?"

We rounded when converting between units, for example when changing from inches to feet to miles. Rounding numbers to powers of 10 or to larger units made calculations and estimation easier. Larger units could also be more informative. For example, expressing the side length in miles might be more understandable than thousands of feet or inches.

There are an infinite number of rectangles with suitable area to serve as the floor of the warehouse. Note, however, that having an appropriate area (for example, 36,000,000 square feet) is *not* sufficient. For example, we could not have a warehouse that is 1 inch wide and 432,000,000 feet long. In general, if one of the dimensions is too small, it increases the amount of wasted space.

Activity 3

Covering the Washington Monument

20 min

Activity Narrative

In this activity, students estimate the total number of tiles needed to cover the Washington Monument. To do so, they apply what they know about units of measurement, tiling, and ways to find the area of two-dimensional figures. Students' answers will vary depending on, among others, the assumptions and estimates made about the situation, the size of tiles used, and the precision of rounding. Estimation may also come into play as students consider the height of each triangular face, how to tile the edges of each trapezoidal face, the thickness of any joints between the tiles, and so on.

Monitor for groups who use different approaches to determine the area of the faces of the monument, such as:

- Composing two (or all four) trapezoidal faces to make a parallelogram and then finding its area.
- Decomposing each trapezoid face into a rectangle and two triangles and then finding the sum of their areas.

As students apply the mathematics they know and make assumptions, decisions, and approximations to solve a practical problem, they engage in aspects of mathematical modeling. In communicating their assumptions, selecting the units to use, and deciding on the level of precision in reporting results, students practice attending to precision.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

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Access for Multilingual Learners (Activity 3)

MLR7: Compare and Connect. This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

Please log in to the site before using the QR code or URL.

Launch

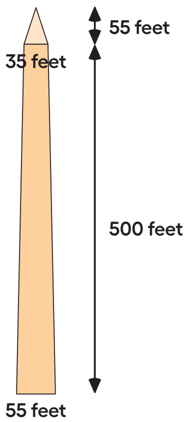
Show a picture of the Washington Monument.

Ask students if they know the monument or have visited it. Invite students who are familiar with the monument to share what they know, such as its location and purpose, when and how it was constructed, and so on. Offer additional background information about the monument as needed to orient students to the context.

Next, invite students to guess the height of the monument. Poll the class on their responses, and record all guesses, including the units of measurement in each guess. Then, read the question that students are to answer. Give students a minute to brainstorm the information they need to answer the question. Give the information provided when students ask for it, or provide access to internet-enabled devices so that students can find the information they need.

Information students may need includes:

- Length measurements of the monument:



- Standard sizes for square tiles: side lengths of 1 inch, 6 inches, 8 inches, 1 foot, and $1\frac{1}{2}$ feet.

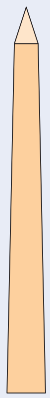
Note that 55 feet is the height of the pyramid at the top, not the height of each triangular face. Students would need the Pythagorean Theorem to find the height of these faces, so either tell them that it is close to 57.5 feet, or they can use the 55 feet for their estimate. This does not have a significant effect on the area calculation, because the pyramid at the top does not account for most of the surface area.

Arrange students in groups of 2–4. Provide access to four-function calculators. If conducting a *Gallery Walk* at the end, provide access to tools for making a visual display.

Select work from students with different strategies, such as those described in the *Activity Narrative,* to share later.

Student Task Statement

How many tiles would it take to cover the Washington Monument?



Sample response: It would take at least 94,025 1-foot square tiles or about 13,540,000 1-inch square tiles to cover the Washington Monument.

- Two of the trapezoidal faces make a parallelogram with a base of 90 feet and height of 500 feet. The area of that parallelogram is 45,000 square feet.
- There are two other trapezoidal faces that also have a combined area of 45,000 square feet.
- The four triangular faces have a height of 57.5 feet and a base of 35 feet. Their total area is $4 \cdot \frac{1}{2} \cdot 35 \cdot (57.5)$ or 4,025 square feet.
- The total area of all faces is 45,000 + 45,000 + 4,025, which is 94,025 square feet.
- If using tiles with 1-foot side lengths, 94,025 tiles are needed. If using tiles with 1-inch side-lengths, each square foot requires 144 tiles, so 94,025 square feet requires 94,025 · 144, or 13,539,600 tiles.
- Because the faces of the monument have non-right angles, some tiles will need to be cut, which means extra tiles will be needed.

Activity Synthesis

The goal of this discussion is to share a variety of approaches to the same situation. Invite students or groups to share different solution approaches. Consider asking students to create a visual display, and consider conducting a *Gallery Walk*.

After all strategies have been presented, display 2–3 approaches from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the approaches have in common? How are they different?"

"Did anyone solve the problem the same way, but would explain it differently?"

"How do these different representations show the same information?"

Building on Student Thinking

If students do not account for the faces they do not see in the picture, consider asking:

"Explain the shapes you used in your calculations."
"How could you be sure that you've included all faces of the monument?"

Student Workbook