Lesson 1

Introducing Ratios and Ratio Language

Goals

- Comprehend the word "ratio" (in written and spoken language) and the notation $a:b$ (in written language) to refer to an association between quantities.
- Describe (orally and in writing) associations between quantities using the language "For every a of these, there are b of those" and "The ratio of these to those is $a:b$ (or a to b)."

Learning Targets

- I can write or say a sentence that describes a ratio.
- I know how to say words and numbers in the correct order to accurately describe the ratio.

Lesson Narrative

In this lesson, students use collections of objects to make sense of and use **ratio** language. They see that there are several different ways to describe a situation using ratio language. For example, if we have 12 squares and 4 circles, we can say the ratio of squares to circles is 12:4 and the ratio of circles to squares is 4 to 12. We may also see a structure that prompts us to regroup them and say that there are 6 squares for every 2 circles, or 3 squares for every one circle.

Expressing associations of quantities in a context—as students will be doing in this lesson—requires students to use ratio language with care. Making groups of physical objects that correspond with "for every" language is a concrete way for students to make sense of the problem.

It is important that in this first lesson students work with objects that can be physically rearranged. This experience can help students make sense of the increasingly abstract representations that they will encounter as the unit progresses.

Student Learning Goal

Let's describe two quantities at the same time.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Teacher's collection of objects: Activity 1, Activity 2
- Tools for creating a visual display: Activity 1, Activity 2
- Students' collections of objects: Activity 2

Required Preparation

Lesson:

A few days before this lesson, ask students to bring a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Bring in your personal collection, and display it ahead of time. Think of possible ways to sort your collection. (See the *Launch* of "The Teacher's Collection" for details.) Prepare a few extra collections for students who don't bring one.

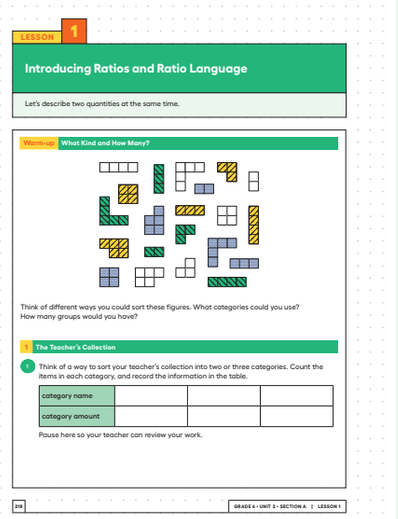
Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	20 min	10 min	5 min

Building on Student Thinking

If students struggle to create their own categories, prompt them to consider a specific attribute of the figures, such as the size, color, or shape.

Student Workbook


LESSON 1

Introducing Ratios and Ratio Language

Let's describe two quantities at the same time.

Warm-up What Kind and How Many?

Think of different ways you could sort these figures. What categories could you use? How many groups would you have?

1 The Teacher's Collection

1. Think of a way to sort your teacher's collection into two or three categories. Count the items in each category, and record the information in the table.

category name			
category amount			

Pause here so your teacher can review your work.

216 GRADE 6 • UNIT 2 • SECTION A | LESSON 1

Warm-up

What Kind and How Many?

5 min

Activity Narrative

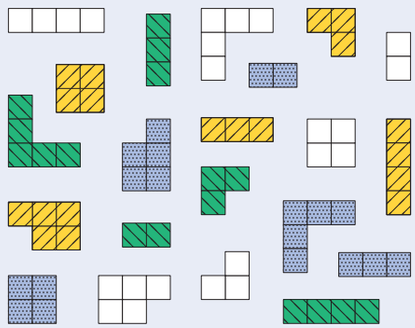
In this *Warm-up*, students sort figures into categories based on the attributes that they see. The work here encourages students to describe quantities in different ways, preparing them to make sense of ratios later in the lesson.

Launch

Display the image for all to see.

Give students 1 minute of quiet think time followed by 2 minutes of partner discussion.

Student Task Statement



Think of different ways you could sort these figures. What categories could you use? How many groups would you have?

Sample responses:

- By area: Four groups: 2, 3, 4, and 5 square units
- By color: Four groups: blue, green, yellow, and white
- By pattern: Four groups: striped, dotted, cross-hatch, and blank
- By shape:
  - Two groups: Rectangles and non-rectangles.
  - Three groups: Rectangles, two different squares glued together, and L-shapes.
  - Four groups: Squares, rectangles, two different squares glued together, and L-shapes.
  - Seven groups: Small, medium, and large rectangles, 2 by 2 squares, small L, big L, and a small and a big square glued together.

Activity Synthesis

Invite students to share their categories and record their responses for all to see. After students explain how they sorted the figures, solicit additional ways of seeing the collection until several different ways of seeing the figures have been shared.

Emphasize that the important thing is to describe the way they sorted the figures clearly enough that everyone agrees that it is a reasonable way to sort them. Tell students we will be looking at different ways of seeing the same set of objects in the next activity.

Activity 1
The Teacher's Collection

10 min

Activity Narrative

This activity introduces students to ratio language and notation through examples based on a collection of everyday objects. Students learn that a ratio is an association between quantities, and that this association can be expressed in multiple ways.

After discussing examples of ratio language and notation for one way of categorizing the objects in the collection, students write ratios to describe the quantities for another way of categorizing objects in the collection.

As students work, circulate and monitor for those who:

- Create different categories from the given collection.
- Create categories whose quantities can be rearranged into smaller groups (for instance, 6 A's and 4 B's can be expressed as "for every 3 A's there are 2 B's").
- Express the same ratio in opposite order or by using different words (for instance, "the ratio of A to B is 7 to 3," and "for every 7 A's there are 3 B's").

Have a collection of objects ready to display for the *Launch*. Make sure that there are different ways in which the collection can be sorted.

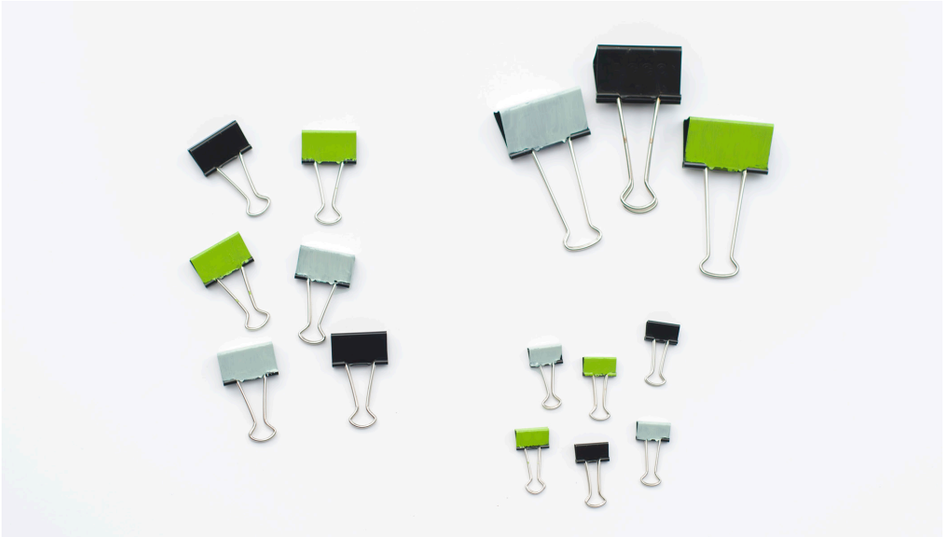
For example, the dinosaurs in this picture can be categorized by color (green, orange, and purple), by the number of legs they stand on (standing on 4 legs or on 2 legs), or by the features along their backs (crest, white stripe, or nothing).

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.
Create a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of the words "ratio" and "category."

Supports accessibility for: Language, Memory

Familiar classroom objects such as binder clips or pattern blocks can also be used to form collections. This picture shows a collection of binder clips that could be categorized by size (small, medium, and large) or by color (black, green, and blue).



Launch

Display a collection of objects for all to see.

Give students 2 minutes of quiet time to think of as many different categories for sorting the collection as they can.

Ask students to share their categories. Record the categories for all to see. Sort the collection into one of the student-suggested categories and count the number of items in each. Record the number of objects in each category and display for all to see. For example:

category A: green	category B: orange	category C: purple
3	2	4

Explain that we can talk about the quantities in the different categories using something called **ratios**. Tell students:

"A ratio is an association between two or more quantities."

We use a colon, or the word "to," between two values that we are associating.

Share the following examples (adapt them to suit your collection) and display them for all to see. Keep the examples visible for the duration of the lesson.

- The ratio of purple to orange dinosaurs is 4 to 2.
- The ratio of purple to orange dinosaurs is 4:2.
- The ratio of orange to purple dinosaurs is 2 to 4.
- The ratio of orange to purple dinosaurs is 2:4

Explain that we can also associate two quantities using the phrase "for every a of these, there are b of those." Add the following examples to the display.

- For every 3 green dinosaurs there are 4 purple dinosaurs.
- There are 4 purple dinosaurs for every 2 orange dinosaurs.

Finally, find two categories whose items can be rearranged into smaller groups, for example, 4 purple dinosaurs to 2 orange dinosaurs. Point out that in some cases we can associate the same categories using different numbers. Share the following example and add it to the display.

- For every 2 purple dinosaurs, there is 1 orange dinosaur.

Have students write two or three sentences to describe ratios between the categories they suggested.

Student Task Statement

1. Think of a way to sort your teacher's collection into two or three categories. Count the items in each category, and record the information in the table.

category name			
category amount			

Pause here so your teacher can review your work.

2. Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:
   - The ratio of *one category* to *another category* is _____ to _____.
   - The ratio of *one category* to *another category* is _____ : _____.
   - There are _____ of *one category* for every _____ of *another category*.

Answers vary depending on the particulars of the teacher's collection and the choices made by students.

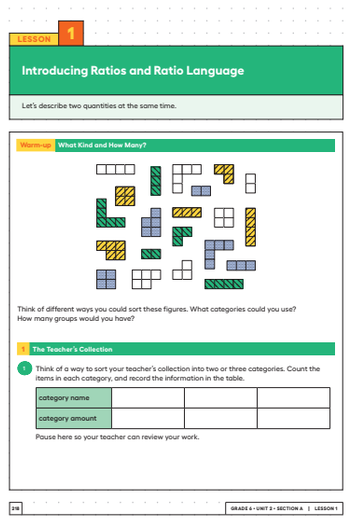
Activity Synthesis

Invite several students to share their categories and sentences. Display them for all to see, attending to correct ratio language. Be sure to include students who express the same categories in reverse order, in different words, or with a different set of numbers (which students will later call an equivalent ratio). Leave several sentences displayed for students to see and use as a reference while working on the next task.

Building on Student Thinking

Students may write ratios with no descriptive words. Although 8:2 is a good start, part of writing a ratio is stating what those numbers mean. Draw students' attention to the sentence stems in the *Task Statement* and encourage them to use those words.

Student Workbook


LESSON 1

Introducing Ratios and Ratio Language

Let's describe two quantities at the same time.

Warm-up What Kind and How Many?

Think of different ways you could sort these figures. What categories could you use? How many groups would you have?

1 The Teacher's Collection

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category name			
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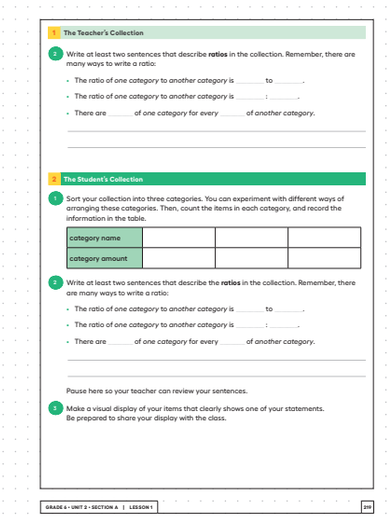
Pause here so your teacher can review your work.

216 GRADE 6 • UNIT 2 • SECTION A | LESSON 1

Student Workbook


1 The Teacher's Collection

2 Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:

- The ratio of one category to another category is _____ to _____.
- The ratio of one category to another category is _____ : _____.
- There are _____ of one category for every _____ of another category.

2 The Student's Collection

1 Sort your collection into three categories. You can experiment with different ways of arranging these categories. Then, count the items in each category, and record the information in the table.

category name			
category amount			

2 Write at least two sentences that describe the **ratios** in the collection. Remember, there are many ways to write a ratio:

- The ratio of one category to another category is _____ to _____.
- The ratio of one category to another category is _____ : _____.
- There are _____ of one category for every _____ of another category.

Pause here so your teacher can review your sentences.

3 Make a visual display of your items that clearly shows one of your statements. Be prepared to share your display with the class.

GRADE 6 • UNIT 2 • SECTION A | LESSON 1 217

Activity 2

The Student's Collection

20 min

Activity Narrative

In this activity, students write ratios to describe objects in their own collection. They create a display of objects and circulate to look at their classmates' work. Students see that there are several ways to write ratios to describe the same situation.

This activity uses the *Compare and Connect* math language routine to support mathematically precise language in discussion.

Launch

Invite students to share what types of items are in their personal collections. If students did not bring in a collection, pair them with another student, or provide them with an extra collection that you have brought in for that purpose.

Provide access to tools for creating a visual display. Tell students that they will pause their work to get their sentences approved before they create their visual display.

Student Task Statement

1. Sort your collection into three categories. You can experiment with different ways of arranging these categories. Then, count the items in each category, and record the information in the table.

category name			
category amount			

2. Write at least two sentences that describe the **ratios** in the collection. Remember, there are many ways to write a ratio:
   - The ratio of *one category* to *another category* is _____ to _____.
   - The ratio of *one category* to *another category* is _____ : _____.
   - There are _____ of *one category* for every _____ of *another category*.

   Pause here so your teacher can review your sentences.
3. Make a visual display of your items that clearly shows one of your statements. Be prepared to share your display with the class.

Sample responses for a collection of pens:

1. Gel pens (8), brush pens (6), ballpoint pens (5).
2. ◦ The ratio of gel pens to brush pens is 8 to 6.
   ◦ The ratio of gel pens to ballpoint pens is 8 to 5.
   ◦ There are 6 brush pens for every 5 ballpoint pens.
3. Answers vary.

Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2, Launch)

MLR7: Compare and Connect. Invite students to consider their audience when preparing to display their work. Encourage them to include details that will help others interpret their thinking. Ask students,

"What could you do or include to help someone understand your display and the ratios of objects in your collection?"

Examples might include labeling each category of objects, using the same terms in the labels and in the sentences, and listing the categories to match the order in which the objects are arranged. If time allows, ask students to describe additional details that other groups used that helped them understand the displays.

Advances: Representing, Conversing

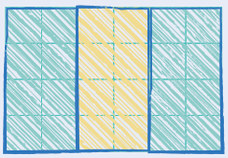
Building on Student Thinking

Watch for students who simply write writing a numerical ratio, such as 3:7, without any descriptive words. Draw their attention to the sentence stems in the *Task Statement*.

Are You Ready for More?

1. Use two colors to shade the rectangle so that there are 2 square units of one color for every 1 square unit of the other color.

   A correct answer will have 16 square units of one color and 8 square units of the other color. Sample response:



2. The rectangle you just colored has an area of 24 square units. Draw a different shape that does *not* have an area of 24 square units, but that can also be shaded with two colors in a 2:1 ratio. Shade your new shape using two colors.

   Answers vary.

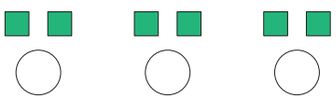
Activity Synthesis

After students have had enough time to create their displays, circulate amount the displays and listen to how students describe their ratios.

As students present their displays, point out the various ways that students chose to showcase their work, including different ways to say the same ratio. Ask students who used two sets of numbers to describe the same categories (for example, 8 to 2 and "4 for every 1") to demonstrate the two ways of grouping the objects.

Lesson Synthesis

This lesson is all about how to use ratio language and notation to describe an association between two or more quantities. Wrap up the lesson by drawing a diagram for all to see of, for example, 6 squares and 3 circles.



Say,

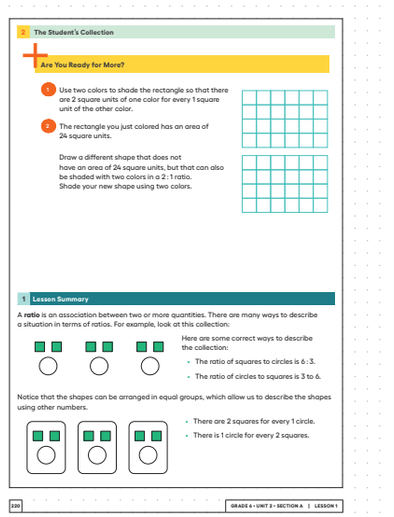
> *"One way to write this ratio is, there are 6 squares for every 3 circles. What are some other ways to write this ratio?"*

Some correct options might be:

- The ratio of squares to circles is 6:3.
- The ratio of circles to squares is 3 to 6.
- There are 2 squares for every 1 circle.

Display, somewhere in the classroom, this diagram and the associated sentences that the class comes up with so students can refer back to the correct ratio and rate language during subsequent lessons.

Student Workbook


2 The Student's Collection

Are You Ready for More?

1. Use two colors to shade the rectangle so that there are 2 square units of one color for every 1 square unit of the other color.

2. The rectangle you just colored has an area of 24 square units.

Draw a different shape that does not have an area of 24 square units, but that can also be shaded with two colors in a 2 : 1 ratio. Shade your new shape using two colors.

1 Lesson Summary

A **ratio** is an association between two or more quantities. There are many ways to describe a situation in terms of ratios. For example, look at this collection:

Here are some correct ways to describe the collection:

- The ratio of squares to circles is 6 : 3.
- The ratio of circles to squares is 3 to 6.

Notice that the shapes can be arranged in equal groups, which allow us to describe the shapes using other numbers.

- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

230 GRADE 6 • UNIT 2 • SECTION A | LESSON 1

Responding To Student Thinking

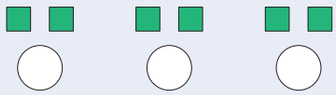
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Consider posing some more general questions, such as:

- *"In your own words, explain what a ratio is."*
- *"What things must you pay attention to when writing a ratio?"*
- *"What are some words and phrases that are used to write a ratio?"*

Lesson Summary

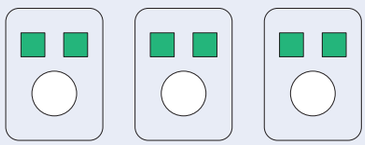
A **ratio** is an association between two or more quantities. There are many ways to describe a situation in terms of ratios. For example, look at this collection:



Here are some correct ways to describe the collection:

- The ratio of squares to circles is 6:3.
- The ratio of circles to squares is 3 to 6.

Notice that the shapes can be arranged in equal groups, which allow us to describe the shapes using other numbers.



- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

Cool-down

5 min

A Collection of Animals

Student Task Statement

Here is a collection of dogs, mice, and cats:

Write *two* sentences that describe a ratio of types of animals in this collection.

Sample responses:

- The ratio of dogs to cats is 6:4.
- There are 3 dogs for every 2 cats.
- There is 1 mouse for every 2 cats.
- The ratio of cats to mice is 4:2.

Lesson 1

Practice Problems

6 Problems

Problem 1

In a fruit basket there are 9 bananas, 4 apples, and 3 plums.

a. The ratio of bananas to apples is 9 : 4.

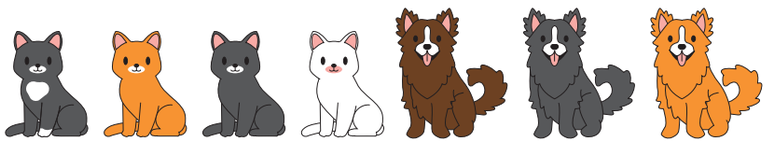
b. The ratio of plums to apples is 3 to 4.

c. For every 4 apples, there are 3 plums.

d. For every 3 bananas there is one plum.

Problem 2

Complete the sentences to describe this picture.

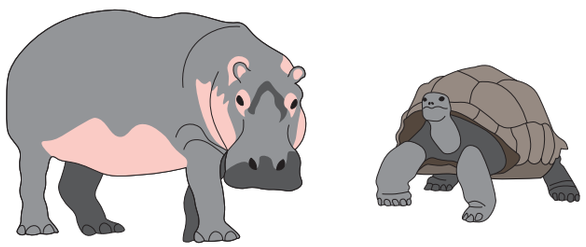


a. The ratio of dogs to cats is 3 to 4.

b. For every 3 dogs, there are 4 cats.

Problem 3

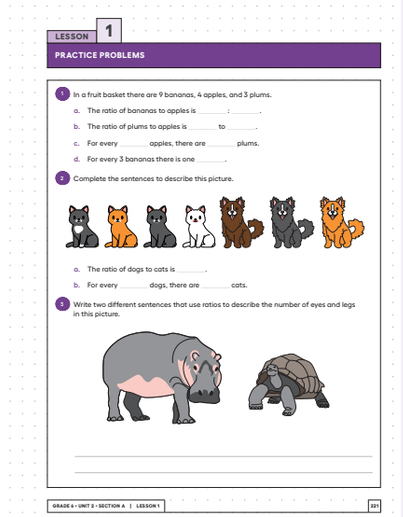
Write two different sentences that use ratios to describe the number of eyes and legs in this picture.



Sample responses:

- The ratio of legs to eyes is 8 to 4.
- The ratio of eyes to legs is 4:8.
- There are 2 legs for every eye.
- There are 4 legs for every 2 eyes.

Student Workbook



LESSON 1

PRACTICE PROBLEMS

1 In a fruit basket there are 9 bananas, 4 apples, and 3 plums.

a. The ratio of bananas to apples is _____ : _____.

b. The ratio of plums to apples is _____ to _____.

c. For every _____ apples, there are _____ plums.

d. For every 3 bananas there is one _____.

2 Complete the sentences to describe this picture.

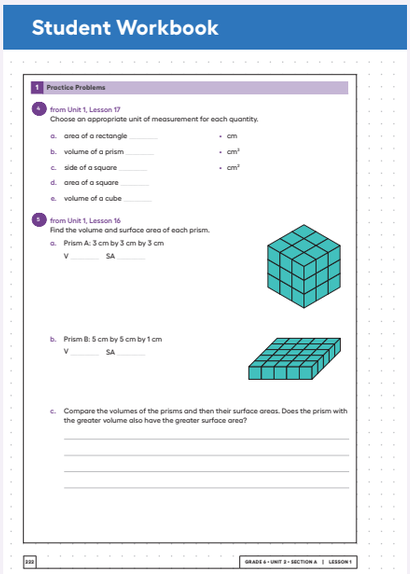
a. The ratio of dogs to cats is _____.

b. For every _____ dogs, there are _____ cats.

3 Write two different sentences that use ratios to describe the number of eyes and legs in this picture.

GRADE 6 • UNIT 2 • SECTION A | LESSON 1

Student Workbook


1 Practice Problems

4 from Unit 1, Lesson 17
Choose an appropriate unit of measurement for each quantity.

a. area of a rectangle _____ • cm
b. volume of a prism _____ • cm^3
c. side of a square _____ • cm^2
d. area of a square _____
e. volume of a cube _____

5 from Unit 1, Lesson 16
Find the volume and surface area of each prism.

a. Prism A: 3 cm by 3 cm by 3 cm
V _____ SA _____

b. Prism B: 5 cm by 5 cm by 1 cm
V _____ SA _____

c. Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?

223 GRADE 6 • UNIT 2 • SECTION A | LESSON 1

Problem 4

from Unit 1, Lesson 17

Choose an appropriate unit of measurement for each quantity.

a. area of a rectangle

cm^2

b. volume of a prism

cm^3

c. side of a square

cm

d. area of a square

cm^2

e. volume of a cube

cm^3

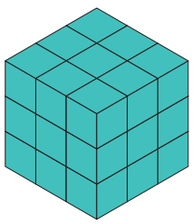
- cm
- cm^3
- cm^2

Problem 5

from Unit 1, Lesson 16

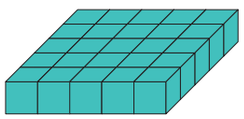
Find the volume and surface area of each prism.

a. Prism A: 3 cm by 3 cm by 3 cm



V 27^3 SA 54^2

b. Prism B: 5 cm by 5 cm by 1 cm



V 25^3 SA 70^2

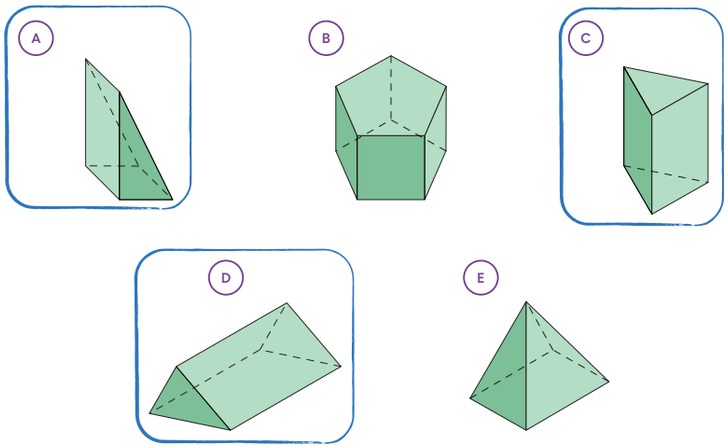
c. Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?

Prism A has a greater volume, but Prism B has a greater surface area.

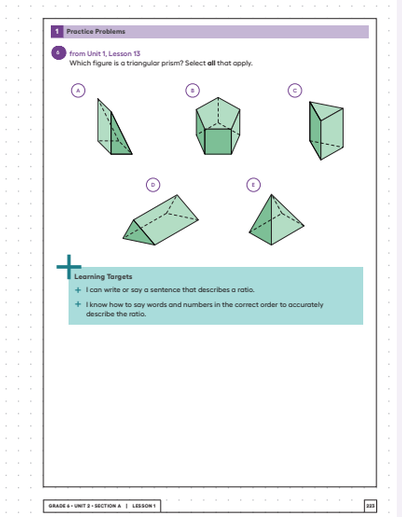
Problem 6

from Unit 1, Lesson 13

Which figure is a triangular prism? Select **all** that apply.



Student Workbook


1 Practice Problems

6 from Unit 1, Lesson 13
Which figure is a triangular prism? Select **all** that apply.

A B C D E

Learning Targets
- I can write or say a sentence that describes a ratio.
- I know how to say words and numbers in the correct order to accurately describe the ratio.

GRADE 6 • UNIT 2 • SECTION A | LESSON 1