Lesson 9

Solutions of Inequalities

Goals

- Draw and label a number line diagram to represent the solutions to an inequality.
- Recognize and explain (orally and in writing) that an inequality may have infinitely many solutions.
- Use substitution to justify (orally) whether a given value is a "solution" to a given inequality.

Learning Targets

- I can determine if a particular number is a solution to an inequality.
- I can explain what it means for a number to be a solution to an inequality.
- I can graph the solutions to an inequality on a number line.

Lesson Narrative

In this lesson, students build on their understanding of the "solution of an equation" to make sense of the **solution to an inequality** as a number that can be used in place of the variable to make the inequality true. And while the equations students solved previously generally had one solution, the inequalities in this unit have many, sometimes infinitely many, solutions.

Constraints in real-world situations reduce the range of possible solutions. Students reason abstractly by using inequalities or graphs of inequalities to represent those situations and by interpreting the solutions. Students think carefully about whether to include boundary values as solutions of inequalities in various contexts.

Student Learning Goal

Let's think about the solutions to inequalities.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Required Materials

Materials to Copy

- What Number Am I Cards (1 copy for every 2 students): Activity 2

Required Preparation

Lesson:
Colored pencils are only needed for an "Are You Ready for More" problem in Activity 1.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
10 min	25 min	15 min	10 min	5 min

Assessment

Warm-up

Notice and Wonder: Unknowns on a Number Line

10 min

Activity Narrative

The purpose of this *Warm-up* is to get students thinking about the relative values of numbers based on their location on the number line, which will be useful when students use inequality statements to compare values on the number line in a later activity. While students may notice and wonder many things about the image, the observations about which values are greater than or less than other values are the important discussion points.

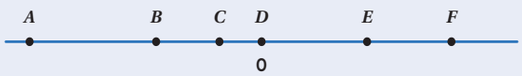
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- *A*, *B*, and *C* are negative.
- *E* and *F* are positive.
- *D* is not positive or negative.
- *A* is less than *B*.
- *E* is greater than *C*.
- The absolute value of *C* is smaller than the absolute value of *B*.

Students may wonder:

- Are *B* and *E* opposites?
- What do these points represent?
- What is the value of *A*?

Instructional Routines

Notice and Wonder

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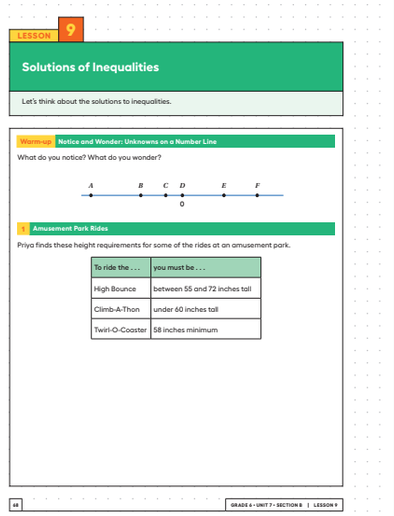
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 9

Solutions of Inequalities

Let's think about the solutions to inequalities.

Warm-up Notice and Wonder: Unknowns on a Number Line

What do you notice? What do you wonder?

A B C D E F 0

1 Amusement Park Rides

Priya finds these height requirements for some of the rides at an amusement park.

To ride the . . .	you must be . . .
High Bounce	between 55 and 72 inches tall
Climb-A-Thon	under 60 inches tall
Twirl-O-Coaster	58 inches minimum

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.
Invite students to generate a list of additional examples of inequalities that connect to their personal backgrounds and interests.

Supports accessibility for: Conceptual Processing, Attention

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Then, give students 1–2 minutes to work with a partner to determine a possible value for each letter based on its location.

Invite groups to share the responses and reasoning. A possible set of values is $A(-14)$, $B(-5)$, $C(-2)$, $D(0)$, $E(5)$, $F(9)$.

Activity 1

Amusement Park Rides

25 min

Activity Narrative

In this activity, students are formally introduced to the phrase "solution to an inequality" as a number that can be used in place of the variable to make the inequality true. Students use given height restrictions for a variety of amusement park rides to explore these ideas. They represent the height restrictions as inequality statements and graph those inequalities on the number line. Students reason abstractly when determining whether a value is a solution to one or more of the inequalities and what that means in context.

Launch

Arrange students in groups of 2. Display the inequality $x > -3$ for all to see. Ask students to determine a value for x that will make the inequality true, and record student responses for all to see.

Tell students that just as a solution to an equation is a value of the variable that makes the equation true, a **solution to an inequality** is a value of the variable that makes the inequality true. And while equations generally have one solution, inequalities have many, sometimes infinitely many, solutions.

Give students 7–8 minutes of quiet work time followed by 1–2 minutes for a partner discussion. Follow with a whole-class discussion.

Student Task Statement

Priya finds these height requirements for some of the rides at an amusement park.

To ride the …	you must be …
High Bounce	between 55 and 72 inches tall
Climb-A-Thon	under 60 inches tall
Twirl-O-Coaster	58 inches minimum

1. Write equations and/or inequalities for the height requirements of each ride. Use h for the unknown height. Then, represent each height requirement on a number line.

   High Bounce: $h > 55$ and $h < 72$

   Students may also interpret the height description to include $h = 55$ and $h = 72$. Climb-A-Thon: $h < 60$. Twirl-O-Coaster: $h > 58$ or $h = 58$.

   - High Bounce

   (Some students may opt to shade in 55 and 72.)

   - Climb-A-Thon

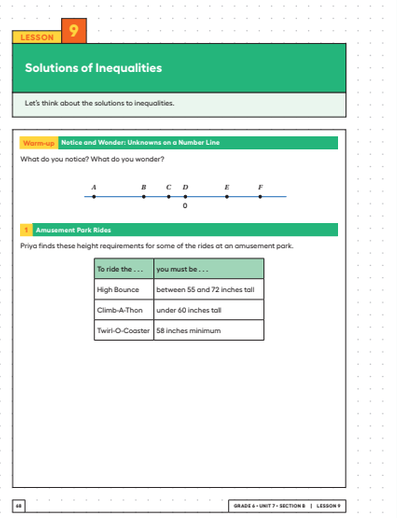
   - Twirl-O-Coaster

2. Han's cousin is 55 inches tall. Han doesn't think she is tall enough to ride the High Bounce, but Kiran believes that she is tall enough. Do you agree with Han or Kiran? Be prepared to explain your reasoning.

   Answers vary.

   If the restriction is interpreted to include the endpoints, then Han's cousin will be able to go on the High Bounce. If the restriction is interpreted not to include the endpoints, then Han's cousin cannot go on this ride.

Student Workbook


LESSON 9

Solutions of Inequalities

Let's think about the solutions to inequalities.

Warm-up: Notice and Wonder: Unknowns on a Number Line

What do you notice? What do you wonder?

A B C D E F

0

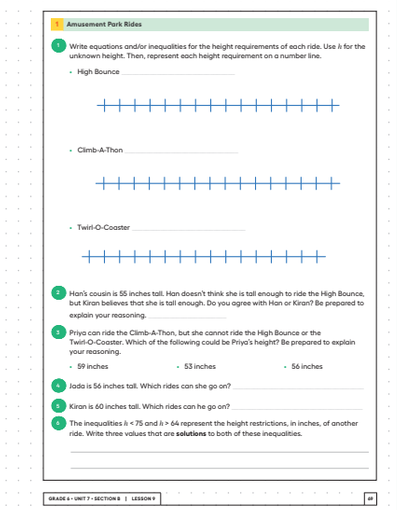
1 Amusement Park Rides

Priya finds these height requirements for some of the rides at an amusement park.

To ride the . . .	you must be . . .
High Bounce	between 55 and 72 inches tall
Climb-A-Thon	under 60 inches tall
Twirl-O-Coaster	58 inches minimum

68 GRADE 6 • UNIT 7 • SECTION 8 | LESSON 9

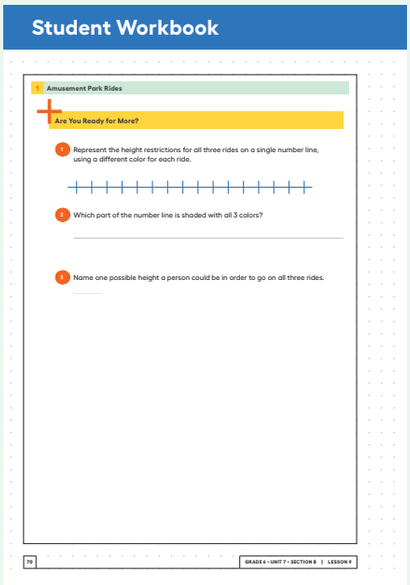
Student Workbook


1 Amusement Park Rides

1. Write equations and/or inequalities for the height requirements of each ride. Use h for the unknown height. Then, represent each height requirement on a number line.
   - High Bounce
   - Climb-A-Thon
   - Twirl-O-Coaster
2. Han's cousin is 55 inches tall. Han doesn't think she is tall enough to ride the High Bounce, but Kiran believes that she is tall enough. Do you agree with Han or Kiran? Be prepared to explain your reasoning.
3. Priya can ride the Climb-A-Thon, but she cannot ride the High Bounce or the Twirl-O-Coaster. Which of the following could be Priya's height? Be prepared to explain your reasoning.
   - 59 inches
   - 53 inches
   - 56 inches
4. Jada is 56 inches tall. Which rides can she go on?
5. Kiran is 60 inches tall. Which rides can he go on?
6. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, of another ride. Write three values that are **solutions** to both of these inequalities.

GRADE 6 • UNIT 7 • SECTION 8 | LESSON 9 69

Student Workbook


1 Amusement Park Rides

Are You Ready for More?

1. Represent the height restrictions for all three rides on a single number line, using a different color for each ride.

2. Which part of the number line is shaded with all 3 colors?

3. Name one possible height a person could be in order to go on all three rides.

3. Priya can ride the Climb-A-Thon, but she cannot ride the High Bounce or the Twirl-O-Coaster. Which of the following could be Priya's height? Be prepared to explain your reasoning.

- 59 inches
- 53 inches
- 56 inches

53 inches

Sample response: The lowest restriction is for the High Bounce, which Priya cannot ride. Therefore, Priya's height is less than 55 inches. The only choice that meets this criteria is 53 inches.

4. Jada is 56 inches tall. Which rides can she go on?

High Bounce and Climb-A-Thon

5. Kiran is 60 inches tall. Which rides can he go on?

High Bounce and Twirl-O-Coaster

6. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, of another ride. Write three values that are **solutions** to both of these inequalities.

Sample responses: 65.5, 65, and 74 inches

Any heights between 64 and 75 inches, not including 64 and 75, are possible responses.

Are You Ready for More?

1. Represent the height restrictions for all three rides on a single number line, using a different color for each ride.

High Bounce: Number line with open circles at 55 and 72. Shaded in between these circles. (Students may opt to shade in 55 and 72.)

Climb-A-Thon: Number line with open circle at 60 and shaded to the left. Bold left arrow.

Twirl-O-Coaster: Number line with closed circle at 58 and shaded to the right. Bold right arrow.

2. Which part of the number line is shaded with all 3 colors?

The space between 58 and 60, not including 58 and 60

3. Name one possible height a person could be in order to go on all three rides.

Sample response: 59 inches

This is the only whole number response possible, but other numbers, such as $59\frac{1}{4}$, are possible.

Activity Synthesis

The purpose of this discussion is for students to communicate their understanding of what it means to be a solution to an inequality.

Display the question for all to see, and give students 2–3 minutes to draft a response:

- *"What does it mean to be a solution to an inequality?"*

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- *"_____ makes sense, but what do you mean when you say … ?"*
- *"Can you describe that another way?"*
- *"How do you know … ? What else do you know is true?"*

Close the partner conversations, and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. Here is an example of a second draft:

"When a number is a solution to an inequality, that number could be used in place of the variable and the statement would be true."

If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Activity 2: Optional

What Number Am I?

15 min

Activity Narrative

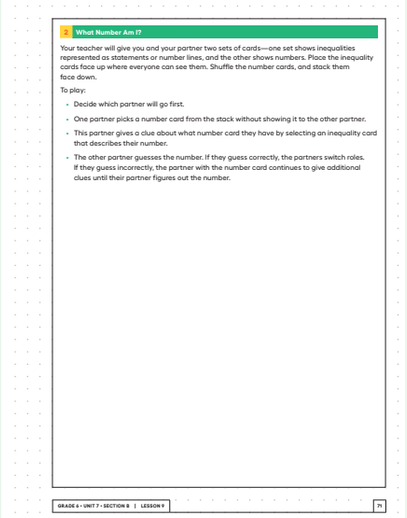
This activity provides an additional opportunity for students to play a game to practice reasoning about whether given values make an inequality true. The goal of the game is for one student to guess a mystery number using as few inequalities as possible.

Launch

Arrange students in groups of 2. Give each group 1 set of inequalities (statements and number lines) and 1 set of numbers. Review the game instructions with the class:

- Place the inequality cards (statements and number lines) face up where both partners can see them.
- Shuffle the number cards, and stack them face down.

Student Workbook


2 What Number Am I?

Your teacher will give you and your partner two sets of cards—one set shows inequalities represented as statements or number lines, and the other shows numbers. Place the inequality cards face up where everyone can see them. Shuffle the number cards, and stack them face down.

To play:

- Decide which partner will go first.
- One partner picks a number card from the stack without showing it to the other partner.
- This partner gives a clue about what number card they have by selecting an inequality card that describes their number.
- The other partner guesses the number. If they guess correctly, the partners switch roles. If they guess incorrectly, the partner with the number card continues to give additional clues until their partner figures out the number.

GRADE 6 • UNIT 7 • SECTION 8 | LESSON 9 71

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports
Display sentence frames to support whole-class discussion. Examples: "One strategy I used to choose a clue was _____" or "One way that I checked that a value was a solution to the inequality (using the number line) was to _____."

Advances: Speaking, Conversing

- One partner chooses a number card from the stack without showing it to their partner, then selects a single inequality card as a clue to determining their number.
- The other partner tries to guess the number.
- The first partner continues giving clues by selecting additional inequality cards until the correct number is guessed.
- When the correct number is guessed, partners switch roles.

Student Task Statement

Your teacher will give you and your partner two sets of cards—one set shows inequalities represented as statements or number lines, and the other shows numbers. Place the inequality cards face up where everyone can see them. Shuffle the number cards, and stack them face down.

To play:

- Decide which partner will go first.
- One partner picks a number card from the stack without showing it to the other partner.
- This partner gives a clue about what number card they have by selecting an inequality card that describes their number.
- The other partner guesses the number. If they guess correctly, the partners switch roles. If they guess incorrectly, the partner with the number card continues to give additional clues until their partner figures out the number.

No response needed

Activity Synthesis

The purpose of this discussion is for students to share their most successful strategies for guessing the mystery number. Here are some possible questions for discussion:

"When giving clues, how did you decide which inequalities would be the most helpful?"

"When receiving clues, were some inequalities more helpful than others as you tried to guess the mystery number? Why?"

"How can you check that a value is or is not a solution to an inequality statement?"

I can replace the variable in the statement with the number and see if the statement is true.

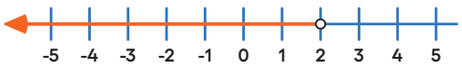
"How can you use a number line to check that a value is or is not a solution to an inequality represented?"

I can plot the value on the number line and see if it falls within a shaded region.

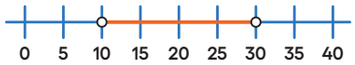
Lesson Synthesis

The purpose of this discussion is for students to practice identifying numbers that are solutions to inequalities. Display the following inequalities for all to see:

A.



B.



C. $x > -5$

D. $x < 17$

Give students one value at a time, and ask them to determine which inequalities the number is a solution to.

- 0 (A, C, D)
- 2 (C, D)
- 16 (B, C, D)
- 28 (B, C)

If time allows, ask students to come up with values that are solutions to the following combinations of inequalities.

- A and D (any value less than 2)
- B and D (any value between 10 and 17, not including 10 or 17)
- C and D (any value between -5 and 17, not including -5 or 17)
- A and B (No values are solutions to both inequalities.)

Lesson Summary

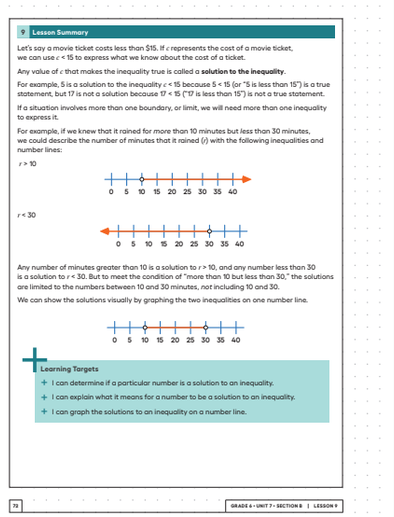
Let's say a movie ticket costs less than $15. If c represents the cost of a movie ticket, we can use $c < 15$ to express what we know about the cost of a ticket.

Any value of c that makes the inequality true is called a **solution to the inequality**.

For example, 5 is a solution to the inequality $c < 15$ because $5 < 15$ (or "5 is less than 15") is a true statement, but 17 is not a solution because $17 < 15$ ("17 is less than 15") is *not* a true statement.

If a situation involves more than one boundary, or limit, we will need more than one inequality to express it.

Student Workbook


9 Lesson Summary

Let's say a movie ticket costs less than $15. If c represents the cost of a movie ticket, we can use $c < 15$ to express what we know about the cost of a ticket.

Any value of c that makes the inequality true is called a **solution to the inequality**.

For example, 5 is a solution to the inequality $c < 15$ because $5 < 15$ (or "5 is less than 15") is a true statement, but 17 is not a solution because $17 < 15$ ("17 is less than 15") is not a true statement.

If a situation involves more than one boundary, or limit, we will need more than one inequality to express it.

For example, if we knew that it rained for more than 10 minutes but less than 30 minutes, we could describe the number of minutes that it rained (r) with the following inequalities and number lines:

$r > 10$

0 5 10 15 20 25 30 35 40

$r < 30$

0 5 10 15 20 25 30 35 40

Any number of minutes greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet the condition of "more than 10 but less than 30," the solutions are limited to the numbers between 10 and 30 minutes, *not* including 10 and 30.

We can show the solutions visually by graphing the two inequalities on one number line.

0 5 10 15 20 25 30 35 40

Learning Targets

- I can determine if a particular number is a solution to an inequality.
- I can explain what it means for a number to be a solution to an inequality.
- I can graph the solutions to an inequality on a number line.

79 GRADE 6 • UNIT 7 • SECTION 8 | LESSON 9

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

For example, if we knew that it rained for *more* than 10 minutes but *less* than 30 minutes, we could describe the number of minutes that it rained (r) with the following inequalities and number lines:

$r > 10$

$r < 30$

Any number of minutes greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet the condition of "more than 10 but less than 30," the solutions are limited to the numbers between 10 and 30 minutes, *not* including 10 and 30.

We can show the solutions visually by graphing the two inequalities on one number line.

Cool-down

Solutions of Inequalities

5 min

Student Task Statement

1. a. Select **all** numbers that are solutions to the inequality $w < 1$.

A. 5
B. -5
C. 0
D. 0.9
E. -1.3

b. Draw a number line to represent this inequality.

2. a. Write an inequality for which 3, -4, 0, and 2,300 are solutions.

Sample response: $x > -5$.

b. How many total solutions are there to your inequality?

There are infinitely many solutions.

Lesson 9

Practice Problems

6 Problems

Problem 1

a. Select **all** numbers that are solutions to the inequality $k > 5$.

4 5 (6) (5.2) (5.01) 0.5

b. Draw a number line to represent this inequality.

The number line should show an open circle above the number 5 and an arrow pointing to the right.

Problem 2

A sign on the road says, "Speed limit: 60 miles per hour."

a. Let s be the speed of a car. Write an inequality that matches the information on the sign.

$s < 60$ or $s = 60$ (or equivalent)

b. Draw a number line to represent the solutions to the inequality.

The number line should show a closed circle at 60 and an arrow pointing to the left.

c. Could 60 be a value of s? Explain your reasoning.

Yes

Sample reasoning: Drivers are allowed to travel at most 60 miles per hour.

Problem 3

One day in Boston, MA, the high temperature was 60 degrees Fahrenheit, and the low temperature was 52 degrees.

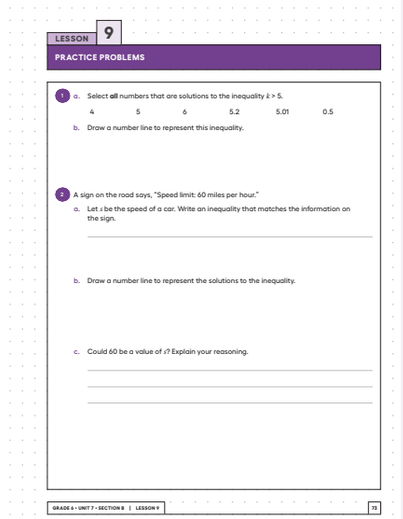
a. Write one or more inequalities to describe the temperatures T that are between the high and low temperature on that day.

$52 < T$ and $T < 60$ (or equivalent)

b. Show the possible temperatures on a number line.

A number line should show a line segment connecting empty circles at 52 and 60.

Student Workbook


LESSON 9

PRACTICE PROBLEMS

1 a. Select **all** numbers that are solutions to the inequality $k > 5$.

4 5 6 5.2 5.01 0.5

b. Draw a number line to represent this inequality.

2 A sign on the road says, "Speed limit: 60 miles per hour."

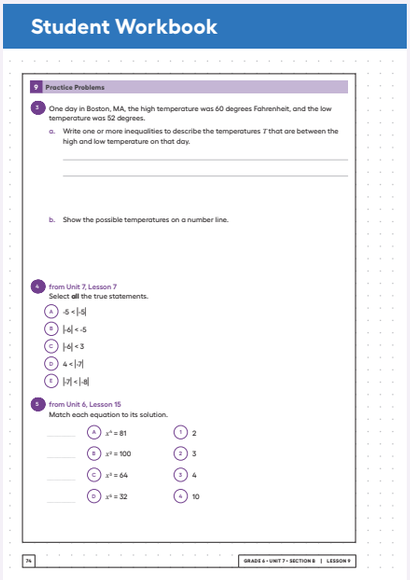
a. Let s be the speed of a car. Write an inequality that matches the information on the sign.

b. Draw a number line to represent the solutions to the inequality.

c. Could 60 be a value of s? Explain your reasoning.

GRADE 6 • UNIT 7 • SECTION B | LESSON 9 73

Student Workbook


9 Practice Problems

3 One day in Boston, MA, the high temperature was 60 degrees Fahrenheit, and the low temperature was 52 degrees.

a. Write one or more inequalities to describe the temperatures T that are between the high and low temperature on that day.

b. Show the possible temperatures on a number line.

4 from Unit 7, Lesson 7
Select **all** the true statements.

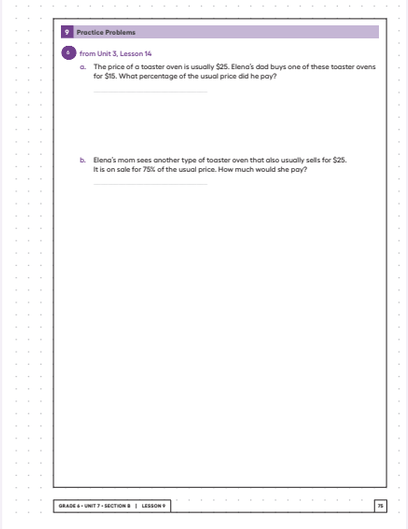
A. $-5 < |-5|$
B. $|-6| < -5$
C. $|-6| < 3$
D. $4 < |7|$
E. $|7| < |-8|$

5 from Unit 6, Lesson 15
Match each equation to its solution.

A. $x^2 = 81$ 1. 2
B. $x^2 = 100$ 2. 3
C. $x^3 = 64$ 3. 4
D. $x^5 = 32$ 4. 10

74 GRADE 6 • UNIT 7 • SECTION B | LESSON 9

Student Workbook


9 Practice Problems

A from Unit 3, Lesson 14

a. The price of a toaster oven is usually $25. Elena's dad buys one of these toaster ovens for $15. What percentage of the usual price did he pay?

b. Elena's mom sees another type of toaster oven that also usually sells for $25. It is on sale for 75% of the usual price. How much would she pay?

GRADE 6 • UNIT 7 • SECTION B | LESSON 9

75

Problem 4

from Unit 7, Lesson 7

Select **all** the true statements.

A. $-5 < |-5|$

B. $|-6| < -5$

C. $|-6| < 3$

D. $4 < |-7|$

E. $|-7| < |-8|$

Problem 5

from Unit 6, Lesson 15

Match each equation to its solution.

3	**A.** $x^4 = 81$	**1.** 2
10	**B.** $x^2 = 100$	**2.** 3
4	**C.** $x^3 = 64$	**3.** 4
2	**D.** $x^5 = 32$	**4.** 10

Problem 6

from Unit 3, Lesson 14

a. The price of a toaster oven is usually $25. Elena's dad buys one of these toaster ovens for $15. What percentage of the usual price did he pay?

60%

b. Elena's mom sees another type of toaster oven that also usually sells for $25. It is on sale for 75% of the usual price. How much would she pay?

$18.75