

Solving Equivalent Ratio Problems

Goals

- Determine what information is needed to solve a problem involving equivalent ratios. Ask questions to elicit that information.
- Understand the structure of a what-why *Info Gap* activity.

Learning Targets

- I can decide what information I need to know to be able to solve problems about situations happening at the same rate.
- I can explain my reasoning using diagrams that I choose.

Lesson Narrative

The purpose of this lesson is to give students further practice in solving equivalent ratio problems and introduces them to the *Information Gap* activity structure. The *Warm-up* is designed to familiarize the class with a process that is central to *Info Gap*—asking for missing information and providing it—before students engage in the *Info Gap* activity in small groups.

The lesson includes an optional activity to further practice solving problems involving equivalent ratios, if there is time after the *Info Gap* activity.

Student Learning Goal

Let's practice getting information from our partner.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR4: Information Gap (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Copy

- Hot Chocolate and Potatoes Cards (1 copy for every 4 students): Activity 1

Lesson Timeline

10
min

Warm-up

25
min

Activity 1

10
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Student Workbook

LESSON 14

Solving Equivalent Ratio Problems

Let's practice getting information from our partner.

Warm-up: What Do You Want to Know?

A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?

What specific information do you need to be able to solve the problem?

Info Gap: Hot Chocolate and Potatoes

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
"Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem.
"I need to know _____ because ____." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

100

GRADE 6 • UNIT 2 • SECTION D | LESSON 14

Warm-up

What Do You Want to Know?

10 min

Activity Narrative

The purpose of this *Warm-up* is to prepare students for the *Info Gap* activity that follows. First, students are given a problem with incomplete information. They are prompted to brainstorm what they need to know to solve a problem that involves constant speed. Next, they practice asking for information, explaining the rationale for their request, and persevering if their initial questions are unproductive. Once students have enough information, they solve the problem.

Launch

Give students 2 minutes of quiet think time.

Student Task Statement

A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?

What specific information do you need to be able to solve the problem?

Sample responses:

- How fast is each car traveling?
- Are the cars going the same direction?
- Did the cars enter the highway at the same location?
- What is the difference between the speeds of the two cars?

Activity Synthesis

Tell students that the problem is a part of an *Info Gap* routine. In the routine, one person has a problem with incomplete information, and another person has data that can help with solving it. Explain that it is the job of the person with the problem to think about what is needed to answer the question, and then request it from the person with information.

Tell students they will try to solve the problem this way as a class to learn the routine. In this round, the students have the problem, and the teacher has the information needed to solve the problem.

Ask students,

☞ "What specific information do you need to find out how far apart the cars will be after 4 hours?"

Select students to ask their questions. Encourage students to use the format of "Can you tell me ...?" Respond to each question with,

☞ "Why do you need to know _____?"

Once students justify their question, only answer questions if they can be answered using these data:

- The red car is traveling faster than the blue car.
- One car is traveling 5 miles per hour faster than the other car.
- The slower car is traveling at 60 miles per hour.
- The blue car is traveling at 60 miles per hour.
- The faster car is traveling at 65 miles per hour.
- The red car is traveling at 65 miles per hour.
- Both cars entered the highway at the same location.
- Both cars are traveling in the same direction.

If students ask for information that is not on the data card, respond with,

“*I don’t have that information.*”

When students think they have enough information, give them **2 minutes** to solve the problem.

The cars will be 20 miles apart after 4 hours.

Tell students they will work in small groups and use the routine to solve problems in the next activity.

Activity 1

Info Gap: Hot Chocolate and Potatoes

25
min

Activity Narrative

This is the first *Info Gap* activity in the course. In this activity, students continue to solve problems involving equivalent ratios but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Students may approach the problems in various ways—some more direct or efficient than others. For instance, those creating a table may use multipliers that necessitate multiplying repeatedly over many rows, or they may choose multipliers more strategically and use fewer rows. In any case, it is important for students to choose their own method for solving them, and to explain their method so that their partner can understand.

Instructional Routines

MLR4: Info Gap

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.



Access for Multilingual Learners (Activity 1)

MLR4: Information Gap

This is the first time Math Language Routine 4: *Information Gap* is suggested in this course. This routine facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, thereby creating a need for communication. This routine supports language development by providing students with opportunities to ask for and share information, and to justify their reasoning within conversation.

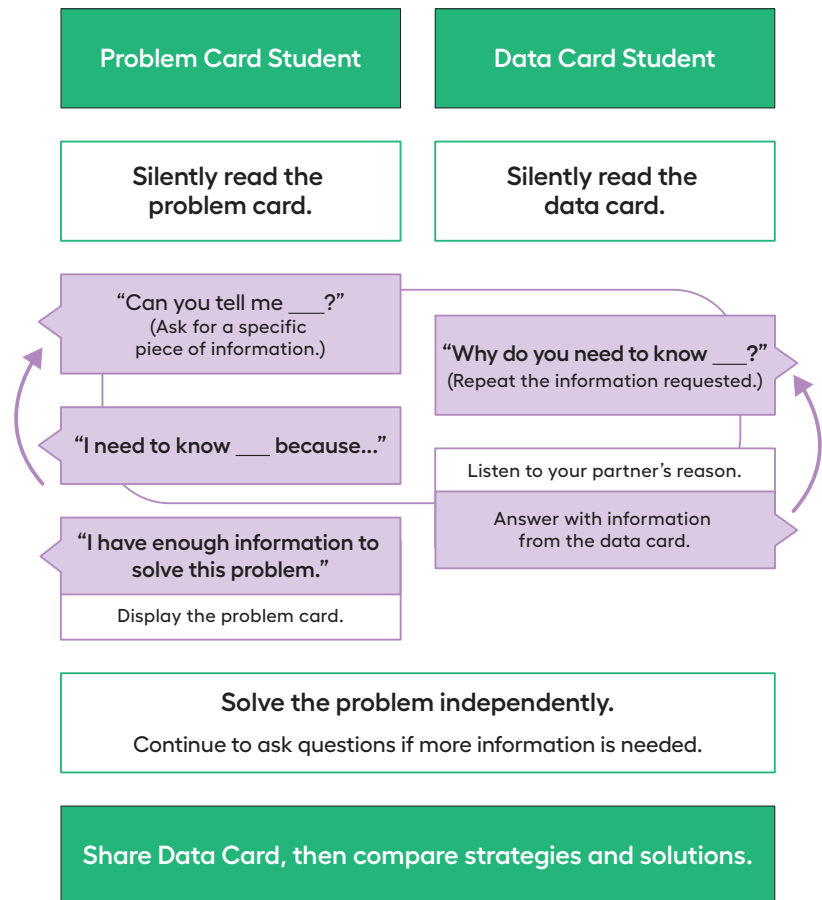
**Access for Students with Diverse Abilities
(Activity 1, Launch)**
**Action and Expression: Internalize
Executive Functions.**

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Launch


Display for all to see the graphic that illustrates a framework for the *Info Gap* routine.



Explain that in an *Info Gap* routine students work with a partner. One partner gets a problem card with a question that doesn't have enough given information, and the other partner gets a data card with information relevant to the problem card.

The person with the problem card asks questions like "Can you tell me ____?" and is expected to explain what they will do with the information. If that person asks for information that is not on the data card (including the answer!) and gives their reason, then the person with the data card must respond with, "I don't have that information." The person with the data card should just be providing information, not making assumptions. Note that it is okay to help a stuck partner by saying something like "I don't have information on how much cocoa powder Jada bought, but I do know how much she has left."

Once the partner with the problem card has enough information, both partners look at the problem card and solve the problem independently.

Arrange students in groups of 2 or 4. If students are new to the *Info Gap* routine, allowing them to work in groups of 2 for each role supports communication and understanding. In each group, distribute a problem card to one student (or group) and a data card to the other student (or group). After reviewing their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me _____?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know _____ because ...”
Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know _____?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Problem Card 1: Jada should use 13.5 cups of milk.

Sample reasoning:

- Finding a multiplier that relates 2 to 9 tablespoons of cocoa. Students may ask “2 times what is 9?” and use 4.5 as the factor to multiply by 3.
- Multiplying the number of cups of milk by 9 to correspond to 18 tablespoons of cocoa, and then dividing it by 2 for 9 tablespoons of cocoa.
- Multiplying by $\frac{1}{2}$ (or dividing by 2) to find the number of cups of milk that correspond to 1 tablespoon of cocoa, and then multiplying that number by 9 for 9 tablespoons of cocoa, as shown in the table below.

milk (cups)	cocoa (tablespoons)
3	2
$\frac{3}{2}$ or 1.5	1
$13\frac{1}{2}$ or 13.5	9

Building on Student Thinking

Students may misinterpret the meaning of the numbers or associate quantities incorrectly and multiply 8 by 6 (because $10 \cdot 6$ is 60). Encourage them to organize the given information in a table or a double number line.

Student Workbook

LESSON14

Solving Equivalent Ratio Problems

Let's practice getting information from our partner.

What Do You Want to Know?

A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?
What specific information do you need to be able to solve the problem?

Info Gap: Hot Chocolate and Potatoes

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3. Explain to your partner how you are using the information to solve the problem.
"I need to know _____ because ..." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
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Student Workbook

Info Gap: Hot Chocolate and Potatoes

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5. Share the data card, and discuss your reasoning.
Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Comparing Reading Rates

Lin read the first 54 pages from a 270-page book in the last 3 days.
Diego read the first 100 pages from a 325-page book in the last 4 days.
Elena read the first 160 pages from a 480-page book in the last 5 days.
If they continue to read every day at these rates, who will finish first, second, and third?
Explain or show your reasoning.

Are You Ready for More?

The ratio of cats to dogs in a room is 2:3. Five more cats enter the room, and then the ratio of cats to dogs is 9:11.
How many cats and dogs were in the room to begin with?

Access for Students with Diverse Abilities
(Activity 2, Student Task)

Action and Expression: Develop Expression and Communication.
Develop fluency with using representations flexibly to represent equivalent ratios. Provide access to blank or partially completed double number line diagrams or tables that students can use to represent the different reading rates.
Supports accessibility for: Visual-Spatial Processing, Attention

Access for Multilingual Learners
(Activity 2, Synthesis)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the statements about Lin’s, Diego’s and Elena’s reading rates without revealing the question. Ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions. Ask,
“What do these questions have in common? How are they different?”
Reveal the intended questions for this task and invite additional connections.
Advances: Reading, Writing

Problem Card 2: No, Noah does not have enough time. It will take him 75 minutes to finish peeling all the potatoes.

Sample reasoning:

- It will take 75 minutes to peel 60 potatoes (1.25 minutes per potato).
- He could peel 56 potatoes in 70 minutes (0.8 potatoes per minute).

elapsed time (minutes)	potatoes peeled
10	8
$\frac{10}{8}, \frac{5}{4},$ or 1.25	1
75	60
1	$\frac{8}{10}, \frac{4}{5},$ or 0.8
25	20
70	56

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Select one student to explain each distinct approach for solving the problems. Highlight how the reasoning is alike or different across strategies.

When all approaches have been discussed, ask students:

“When might it be helpful to first find the amount that corresponds to 1 unit of one quantity and scale that amount up to any value we want?”

Encourage students to refer to all examples seen in this lesson so far.

Activity 2: Optional
Comparing Reading Rates

10 min

Activity Narrative

This activity provides an opportunity for additional practice in solving equivalent ratio problems. Monitor for students solving the problems in different ways.

Launch

Give students 4 minutes of quiet work time and then have them discuss their solutions with a partner.

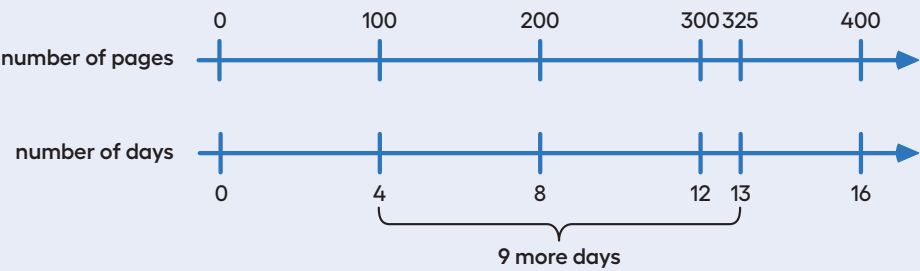
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- Lin read the first 54 pages from a 270-page book in the last 3 days.
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 - Elena read the first 160 pages from a 480-page book in the last 5 days.
- If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

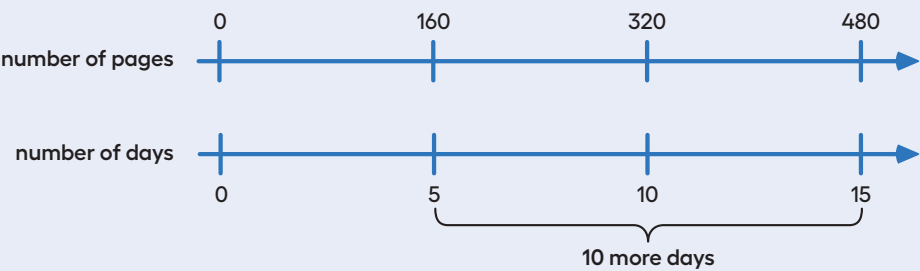
Diego will finish first, Elena will finish second, and Lin will finish third.

Sample reasoning:

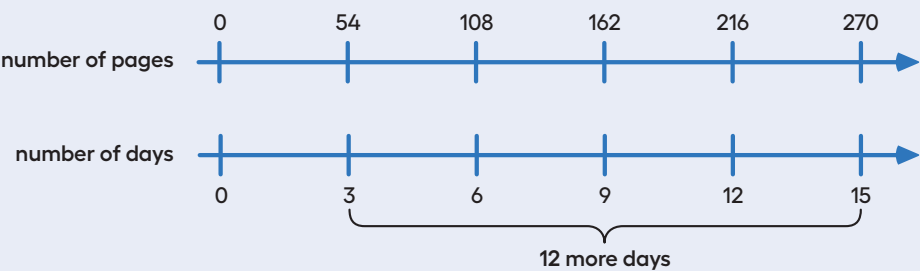
Diego



Elena



Lin



Are You Ready for More?

The ratio of cats to dogs in a room is 2:3. Five more cats enter the room, and then the ratio of cats to dogs is 9:11.

How many cats and dogs were in the room to begin with?

22 cats and 33 dogs

Student Workbook

Info Gap: Hot Chocolate and Potatoes

If your teacher gives you the data card:

- 1 Silently read your card. Wait for your partner to ask for information.
- 2 Before telling your partner any information, ask, "Why do you need to know ____?"
- 3 Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
- 4 Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
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Comparing Reading Rates

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Activity Synthesis

Select students to present their solutions. Sequence solutions with diagrams first and then tables. Make sure students see connections between the different representations and ways of solving the problems.

Lesson Synthesis

When solving problems involving equivalent ratios, we often have three pieces of information and need to find a fourth. Pose this example: If you wash 12 plates in 3 minutes, how long will it take to wash 8 plates at that rate?

Display a table showing the three known pieces of information and ask students how they might use it to find the fourth piece.

number of plates	number of minutes
12	3
8	

Sometimes we aren’t given all the information needed to solve a problem and will need to find out more. Pose another problem:

“Han jumped for 10 seconds. How many jumps can he make if he jumps at the same rate for 45 seconds?”

Ask students:

“Do you have enough information to answer the question? If not, what information would you ask for?”

No, we don’t know how many times Han jumped in 10 seconds or how long it took him to make one jump.

“How would that information help?”

Knowing how fast he was jumping would help us figure out how many he can do in 45 seconds.

Display a table that shows that Han made 8 jumps in 10 seconds and discuss how it can help in answering the question.

number of jumps	number of seconds
8	10

Lesson Summary

To solve problems about something happening at the same rate, we often need:

- Two pieces of information that allow us to write a ratio that describes the situation.
- A third piece of information that gives us one number of an equivalent ratio. Solving the problem often involves finding the other number in the equivalent ratio.

Suppose we are making a large batch of fizzy juice and the recipe says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” We know that the ratio of cranberry juice to soda water is 5:2, and that we need 2.5 cups of cranberry juice per cup of soda water.

We still need to know something about the size of the large batch. If we use 16 cups of soda water, what number goes with 16 to make a ratio that is equivalent to 5:2?

To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

cranberry juice (cups)	soda water (cups)
5	2
2.5	1
40	16

Student Workbook

14 Lesson Summary

To solve problems about something happening at the same rate, we often need:


- Two pieces of information that allow us to write a ratio that describes the situation.
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To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

cranberry juice (cups)	soda water (cups)
5	2
2.5	1
40	16



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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Sharpening Pencils

5 min

Student Task Statement

Jada is helping to sharpen colored pencils for an art class. She wants to know how much time it would take to sharpen all the pencils.

What information would she need to answer that question? How might she use that information?

Sample responses:

- Jada would need to know how many pencils there are and how quickly she can sharpen pencils. She could measure the number of pencils sharpened in 1 minute and use this ratio to find the number of minutes needed to sharpen all the pencils.
- Jada would need to know the number of pencils and her pencil-sharpening speed. She could measure the time needed to sharpen 1 pencil and multiply that by the number of pencils.

Practice Problems

6 Problems

Student Workbook

LESSON 14
PRACTICE PROBLEMS

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for \$3.00 and allows customers to buy any amount of vinegar. Decide whether each of the following ratios correctly represents the price of vinegar.

- a. 4 gallons to \$3.00 _____
- b. 1 gallon to \$1.50 _____
- c. 30 gallons to \$45.00 _____
- d. \$2.00 to 30 gallons _____
- e. \$1.00 to $\frac{2}{3}$ gallon _____

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Student Workbook

Practice Problems

2. A caterer needs to buy 21 pounds of pasta to cater a wedding. At a local store, 8 pounds of pasta cost \$12. How much will the caterer pay for the pasta there?

- a. Write a ratio for the given information about the cost of pasta.
- b. Would it be more helpful to write an equivalent ratio with 1 pound of pasta as one of the numbers, or with \$1 as one of the numbers? Explain your reasoning, and then write that equivalent ratio.

- c. Find the answer and explain or show your reasoning.



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Problem 1

A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for \$3.00 and allows customers to buy any amount of vinegar. Decide whether each of the following ratios correctly represents the price of vinegar.

- a. 4 gallons to \$3.00

No, 4 gallons of vinegar would cost \$6.

- b. 1 gallon to \$1.50

Yes

- c. 30 gallons to \$45.00

Yes

- d. \$2.00 to 30 gallons

No, \$30 would buy 20 gallons.

- e. \$1.00 to $\frac{2}{3}$ gallon

Yes

Problem 2

A caterer needs to buy 21 pounds of pasta to cater a wedding. At a local store, 8 pounds of pasta cost \$12. How much will the caterer pay for the pasta there?

- a. Write a ratio for the given information about the cost of pasta.

Sample responses: \$12 for every 8 pounds, \$12 to 8 pounds, or 8 pounds to \$12

- b. Would it be more helpful to write an equivalent ratio with 1 pound of pasta as one of the numbers, or with \$1 as one of the numbers? Explain your reasoning, and then write that equivalent ratio.

Sample response: Finding 1 pound would be easier and more helpful. The cost of 1 pound can be easily found by dividing \$12 by 8 and the result (the unit rate) can be multiplied by 21. The ratio is \$1.50 to 1 pound.

- c. Find the answer and explain or show your reasoning.

\$31.50

Sample reasoning: $21 \cdot (1.50) = 31.50$

pasta (pounds)	cost (dollars)
8	12
1	1.50
21	31.50

Problem 3

Lin is reading a 47-page book. She read the first 20 pages in 35 minutes.

- a. If she continues to read at the same rate, will she be able to complete this book in under 1 hour?

No

Sample reasoning: At her current rate, reading 40 pages will take her 70 minutes, which is already more than an hour. Her book is longer than 40 pages.

- b. If so, how much time will she have left? If not, how much more time is needed? Explain or show your reasoning.

She will need additional 22.25 or $22\frac{1}{4}$ minutes (or 22 minutes and 15 seconds).
Sample reasoning: It will take Lin 82.25 minutes to finish her book, and $82.25 - 60 = 22.25$.

number of pages	times in minutes
20	35
1	1.75 (or equivalent)
47	82.25 (or equivalent)

Problem 4

Diego can type 140 words in 4 minutes.

- a. At this rate, how long will it take him to type 385 words?

11 minutes

- b. How many words can he type in 15 minutes?

If you get stuck, consider creating a table.

525 words

Sample reasoning:

number of words	number of minutes
140	4
1	$\frac{1}{35}$
385	11
35	1
525	15

Student Workbook

14 Practice Problems

Lin is reading a 47-page book. She read the first 20 pages in 35 minutes.

a. If she continues to read at the same rate, will she be able to complete this book in under 1 hour?

b. If so, how much time will she have left? If not, how much more time is needed? Explain or show your reasoning.

Diego can type 140 words in 4 minutes.

a. At this rate, how long will it take him to type 385 words?

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If you get stuck, consider creating a table.

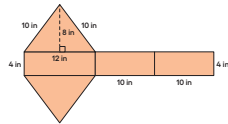
Student Workbook

Practice Problems

from Unit 2, Lesson 10
A train that travels 30 miles in $\frac{1}{3}$ hour at a constant speed is going faster than a train that travels 20 miles in $\frac{1}{2}$ hour at a constant speed. Explain or show why.

from Unit 1, Lesson 14

Find the surface area of the polyhedron that can be assembled from this net. Show your reasoning.



Learning Targets

- I can decide what information I need to know to be able to solve problems about situations happening at the same rate.
- I can explain my reasoning using diagrams that I choose.

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Problem 5

from Unit 2, Lesson 10

A train that travels 30 miles in $\frac{1}{3}$ hour at a constant speed is going faster than a train that travels 20 miles in $\frac{1}{2}$ hour at a constant speed. Explain or show why.

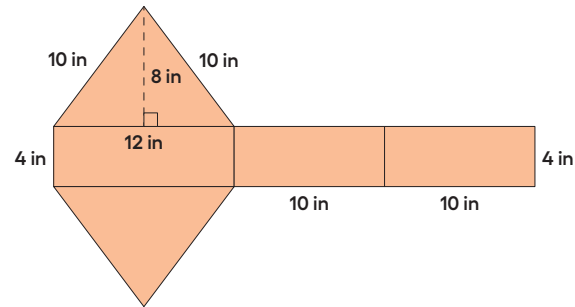
Sample responses:

- In 1 hour, the first train will travel 90 miles, while the second train only travels 40 miles. The first train is going faster.
- The train traveling 30 miles in $\frac{1}{3}$ of an hour takes $\frac{1}{9}$ of an hour to go 10 miles. The train traveling 20 miles in $\frac{1}{2}$ of an hour takes $\frac{1}{4}$ of an hour to go 10 miles. This means that the first train is traveling faster.

Problem 6

from Unit 1, Lesson 14

Find the surface area of the polyhedron that can be assembled from this net. Show your reasoning.



224 square inches

Sample reasoning: The three rectangular faces have areas 48, 40, and 40 square inches. Each triangle has a base of 12 inches and a height of 8 inches, so each triangle has an area of 48 square inches.
 $48 + 40 + 40 + 2(48) = 224$