Using Base-Ten Diagrams to Divide (Optional)

Goals

Explain (orally, in writing, and using other representations) division of whole numbers and decimals in terms of equal-size groups and decomposition of a larger base-ten unit into 10 units of a lower place value..

Interpret and explain

 (orally) base-ten diagrams
 that represent the quotient
 of a whole-number or
 decimal dividend by a
 whole-number divisor.

Learning Target

I can use base-ten diagrams to represent division of whole numbers and division of a decimal by a whole number.

Lesson Narrative

This lesson is optional because it revisits grade 5 standards on division of multi-digit numbers. Students recall ways to divide two numbers using strategies based on place value, properties of operations, or the relationship between multiplication and division. Base-ten diagrams, the main representation used here, remind students that a base-ten unit can be decomposed into 10 of a smaller unit in order to divide. The insights from this lesson prepare students to reason more abstractly about division of decimals in upcoming lessons.

Students begin by interpreting base-ten diagrams that represent division of whole numbers in which the quotient is also a whole number. They recall that dividing 372 by 3, for instance, can mean finding the amount in one group if 372 is put into 3 equal-size groups.

Next, students analyze division of whole numbers that leave a wholenumber remainder. To put the remainder into equal-size groups requires decomposing it into tenths or hundredths, giving a quotient that is no longer a whole number.

Finally, students use base-ten diagrams to represent division of a decimal by a whole number, which also yields a decimal quotient.

Access for Students with Diverse Abilities

• Representation (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

• MLR3: Critique, Correct, Clarify

Required Materials

Materials to Gather

· Base-ten blocks: Activity 1

Materials to Copy

 Squares and Rectangles Cutouts (1 copy for every 1 student): Activity 1

Required Preparation

Activity 1:

Prepare either physical base-ten blocks or paper cutouts of base-ten representations from the blackline master.

Lesson Timeline



Warm-up

20 min

Activity 1

15 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Using Base-Ten Diagrams to Divide (Optional)

Lesson Narrative (continued)

In all activities in the lesson, students may choose to represent decomposition and grouping of base-ten units visually. This can be done by drawing base-ten diagrams, using physical base-ten blocks (if available), or using paper cutouts of base-ten representations from the blackline master. Consider providing access to these tools as alternatives to drawing.

Student Learning Goal

Let's use base-ten diagrams to find quotients.

Inspire Math Gold Mine video

Go Online

Before the lesson, show this video to reinforce the real-world connection.

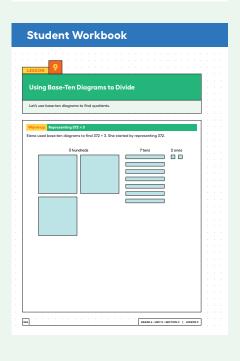
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Building on Student Thinking

If students have difficulty making sense of Elena's method, consider providing students with actual base-ten blocks or paper cutouts and asking them to use them to represent 372 ÷ 3.



Warm-up

Representing 372 ÷ 3



Activity Narrative

This *Warm-up* prompts students to divide two whole numbers by reasoning about place value and using base-ten diagrams. The work here builds on students' prior experience with base-ten representations and on their understanding that division can be interpreted in terms of creating equal-size groups.

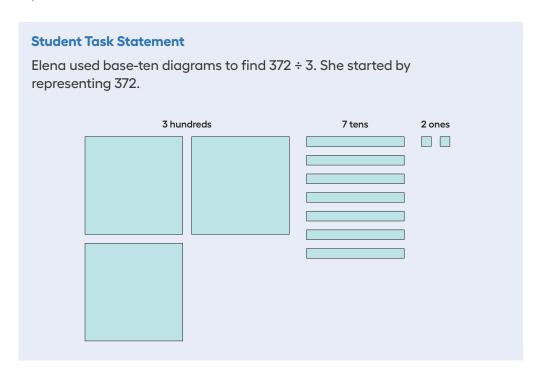
The divisor and dividend are chosen so that the hundreds in the dividend can be partitioned into equal groups of whole numbers without a remainder but the tens cannot. The quotient, however, is a whole number. The key ideas that would enable students to ultimately divide a decimal by a decimal are present in this example:

- A number can be decomposed to make the division convenient. For example, 372 can be viewed as 300 + 60 + 12.
- Place value, expressed in the form of base-ten diagrams, plays a very important role in division.

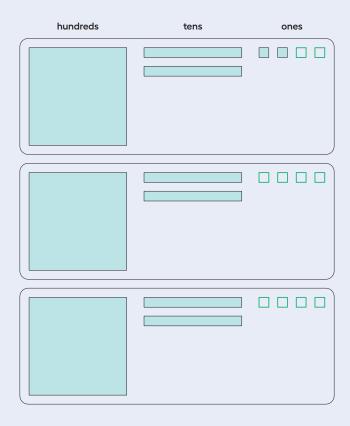
Launch

Arrange students in groups of 2. Display the diagrams showing Elena's method, and read aloud the accompanying paragraphs.

Give students 1 minute of quiet think time and another minute to discuss with a partner. Follow with a whole-class discussion.



She made 3 groups, each with 1 hundred. Then, she put the tens and ones in each of the 3 groups. Here is her diagram for $372 \div 3$.



Discuss with a partner:

- Elena's diagram for 372 has 7 tens. The one for 372 \div 3 has only 6 tens. Why?
- · Where did the extra ones (small squares) come from?

Sample reasoning: Elena first put the 3 hundreds into 3 groups, placing I hundred in each group. Then she put 6 of the 7 tens into 3 groups, giving 2 tens to each group. She traded the remaining ten for IO ones. Combining these IO ones with the original 2 ones, she then has I2 ones. Elena put the I2 ones into 3 groups, putting 4 ones in each group. Each group then has I24, so $372 \div 3 = 124$.

Activity Synthesis

Highlight Elena's process of separating base-ten units into equal groups. Discuss questions such as:

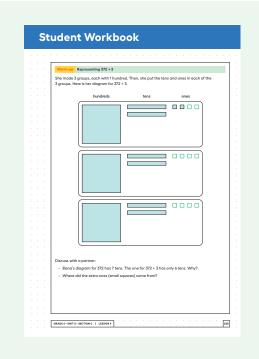
- "Which base-ten unit(s) did Elena decompose or break up?"
 She decomposed a tens unit.
- "What did that accomplish?"

She had only I ten left and there were 3 equal groups. Breaking up the ten into smaller units made it possible to place parts of the ten in the 3 groups.

"How might one find 378 ÷ 3 using Elena's method?"

By thinking of 378 as 3 hundreds, 6 tens, and 18 ones and placing them into 3 equal groups.

Tell students that they will use base-ten representations to explore division of other numbers.



Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.

Invite students to act out each step of Elena's process of representing 53.8 ÷ 4 by using base-ten blocks and drawings.

Supports accessibility for: Language, Conceptual Processing

Activity 1: Optional

Keep Dividing



Activity Narrative

In this activity, students use base-ten diagrams to divide two whole numbers that result in terminating decimal quotients. In the given problems, the ones cannot be placed into equal groups of whole numbers without a remainder and must be decomposed into tenths. Students see that this process is conceptually no different than decomposing hundreds into tens or tens into ones so that all the pieces can form equal-size groups.

Launch



Arrange students in groups of 2. Display Mai's diagrams and read aloud the problem stems.

Give students 2 minutes of quiet time to analyze Mai's work and think about the first set of questions, followed by 2–3 minutes to discuss their observations with their partner. Pause for a whole-class discussion, making sure that all students understand how Mai dealt with the remainder.

Give students 5–7 minutes to complete the final two questions. Provide access to base-ten blocks or paper cutouts of base-ten representations (from the blackline master). Follow with a whole-class discussion.

Student Task Statement

Mai used base-ten diagrams to calculate 62 \div 5. She started by representing 62.

6 tens	2 ones

She then made 5 groups, each with 1 ten. There was 1 ten left. She decomposed it into 10 ones and distributed the ones across the 5 groups.

Here is Mai's diagram for $62 \div 5$.



- 1. Discuss these questions with a partner:
 - **a.** Mai should have a total of 12 ones, but her diagram shows only 10. Why?

Mai decomposed 2 ones to make 20 tenths. So instead of 12 ones, her diagram has 10 ones and 20 tenths.

b. She did not originally have tenths, but in her diagram each group has 4 tenths. Why?

Mai took the 20 tenths (decomposed from 2 ones) and placed 4 tenths in each of the equal groups.

c. What value has Mai found for 62 ÷ 5?

12.4 because each group has I ten, 2 ones, and 4 tenths, so $62 \div 5 = 12.4$.

2. Find the quotient of $511 \div 5$. Show your reasoning. If you get stuck, try drawing a base-ten diagram or using base-ten representations.

102.2

Sample reasoning:

- 511 can be represented with 5 large squares (5 hundreds), I rectangle (1 ten), and I small square (I one).
 - The 5 hundreds can go into 5 groups (I hundred in each group).
 - The I ten can be decomposed into IO ones and distributed into 5 groups (2 ones in each).
 - The I one can be decomposed into IO tenths and distributed into 5 groups (2 tenths in each).
- 500 ÷ 5 = 100, and 10 ÷ 5 = 2, so 510 ÷ 5 = 102. There is 1 left. 1 ÷ 5 = 0.2, so 511 ÷ 5 is 102.2.
- **3.** Four students share a \$271 prize from a science competition. How much does each student get if the prize is shared equally? Show your reasoning.

\$67.75

Sample reasoning: There are 4 groups of \$67 in \$268, so each student gets \$67 dollars, and the group must split the remaining \$3 evenly. Because 300 pennies can be divided into 4 groups of 75 pennies, each student receives \$0.75. This means that each student gets \$67.75.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect.

Invite students to prepare a visual display that shows the strategy they used to find the value of 511 ÷ 5. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students:

"How are the strategies or diagrams the same?", "How are they different?" "How is the remaining 1 one dealt with in each strategy?" "Where do we see the 1 one decomposed into 10 tenths in each

Advances: Representing, Conversing

Student Workbook

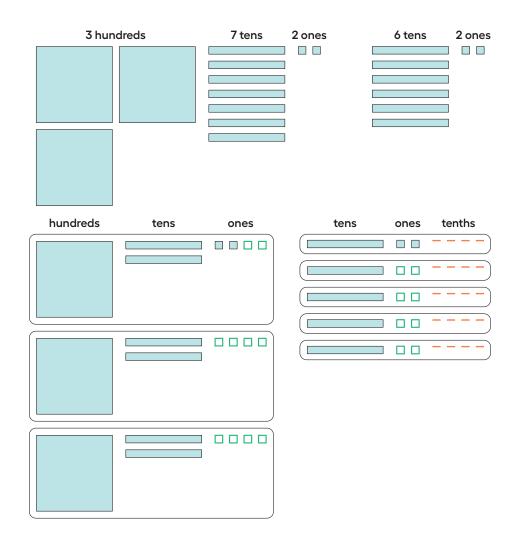
strategy?"



Activity Synthesis

The goal of the discussion is to highlight that the same reasoning process is involved when we divide two numbers, regardless of whether the quotient is or is not a whole number.

Consider displaying Elena's diagram for $372 \div 3$ and Mai's diagram for $62 \div 5$.



Ask questions such as:

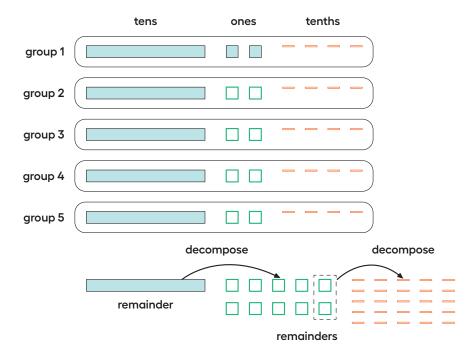
 \bigcirc "What's the same about Mai's reasoning for finding 62 ÷ 5 and Elena's process for finding 372 ÷ 3?"

They both represented the pieces by place value and put them into equalsize groups. They both had to decompose one or more pieces into IO smaller units.

"What's different?"

Elena decomposed a ten into 10 ones. Mai decomposed a one into 10 tenths. In Mai's division, the quotient is not a whole number.

The big new idea here is that sometimes division of whole numbers does not end when we get to the ones place. When the whole-number amount can no longer be distributed into equal groups, we can first decompose them into tenths, hundredths, or other smaller base-ten units and then distribute them.



Activity 2

Explaining a Representation of Division

15 min

Activity Narrative

In this activity, students analyze several diagrams that represent a decimal being divided by a whole number. They interpret and explain the presence or arrangement of base-ten units in several stages of reasoning, including in a final diagram, which shows the quotient. As students decompose and distribute base-ten pieces into equal-size groups and think about the meaning of each piece, students practice reasoning abstractly and quantitatively.

Launch



Keep students in groups of 2. Tell students that in this activity they will try to follow Elena's reasoning as she used base-ten diagrams to represent a division of a decimal, 53.8, by a whole number, 4.

Give students 2–3 minutes of quiet time to read the problem stem and to analyze Elena's work, and another 2–3 minutes to discuss the questions with their partner.

Instructional Routines

MLR3: Critique, Correct, Clarify

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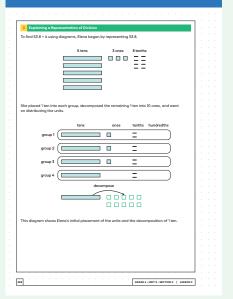
Access for Multilingual Learners (Activity 2)

MR3: Critique, Correct, Clarify
This activity uses the *Critique*, *Correct, Clarify* math language
routine to advance representing and
conversing as students critique and
revise mathematical arguments.

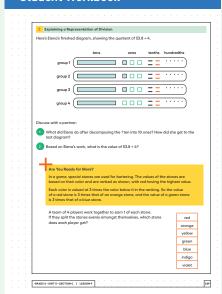
Building on Student Thinking

Some students may stop dividing when they reach a remainder rather than decomposing the remainder into smaller units. Remind them that they can continue to divide the remainder by decomposing and to refer to Elena's worked-out example or those from earlier lessons, if needed.

Student Workbook

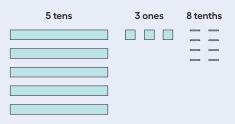


Student Workbook

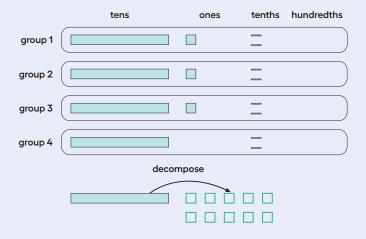


Student Task Statement

To find $53.8 \div 4$ using diagrams, Elena began by representing 53.8.

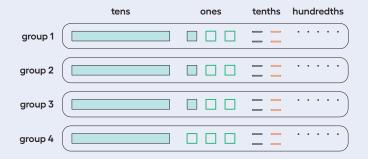


She placed 1 ten into each group, decomposed the remaining 1 ten into 10 ones, and went on distributing the units.



This diagram shows Elena's initial placement of the units and the decomposition of 1 ten.

Here's Elena's finished diagram, showing the quotient of $53.8 \div 4$.



Discuss with a partner:

1. What did Elena do after decomposing the 1 ten into 10 ones? How did she get to the last diagram?

Sample response: Elena put 9 of the 10 ones into the 4 groups so that each group now has 3 ones. There is 1 one left, which she decomposed into 10 tenths. She put 8 tenths into the 4 groups, 2 tenths in each group. There are 2 tenths left, which she decomposed into 20 hundredths. She put 5 hundredths into each of the 4 groups. There are no pieces left.

2. Based on Elena's work, what is the value of $53.8 \div 4?$

13.45, because there are I ten, 3 ones, 4 tenths, and 5 hundredths in each group.

Are You Ready for More?

In a game, special stones are used for bartering. The values of the stones are based on their color and are ranked as shown, with red having the highest value.

Each color is valued at 3 times the color below it in the ranking. So the value of a red stone is 3 times that of an orange stone, and the value of a green stone is 3 times that of a blue stone.

A team of 4 players work together to earn 1 of each stone. If they split the stones evenly amongst themselves, which stone does each player get?

Each player gets I orange stone, I green stone, and I indigo stone. Together the four players must share I violet stone.

Activity Synthesis

Invite a student or a group to share their explanation of Elena's diagrams. If possible, display and mark up the second given diagram to reflect students' explanation and to arrive at the same result as shown in the final diagram.

Emphasize that anytime Elena had base-ten units that can't be distributed equally into 4 groups, she decomposed each into 10 of the next smaller unit and distributed those. She kept going until there were no pieces remaining.

If time permits, use *Critique*, *Correct*, *Clarify* to give students an opportunity to improve a sample written prediction about the value of $53.8 \div 4$, by correcting errors, clarifying meaning, and adding details.

Display this first draft:

- "The value of 53.8 ÷ 4 would be a decimal with a whole number and 2 tenths because the 8 tenths in 53.8 are divided into 4 groups, which means 2 tenths in each group."
- · Ask,
- "What parts of this response are unclear, incorrect, or incomplete?"
 As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

 Select 1–2 students or groups to slowly read aloud their draft. Record for all to see as each draft is shared. Then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Lesson Synthesis

The key takeaways from this lesson are:

- We can think of division by a whole-number divisor in terms of splitting an amount into equal-size groups. Base-ten representations can help to show the groups and what is in each group.
- We divide whole numbers and decimals by place value, usually starting with the greatest. When there isn't enough of one place-value unit, we can decompose it into 10 of the next smaller unit and divide.

To highlight these ideas, present an example, such as $456 \div 4$, and ask questions such as:

 \bigcirc "How can we represent the dividend with base-ten diagrams?"

Use 4 large squares for hundreds, 5 rectangles for tens, and 6 small squares for ones.

"How can we represent the division by 4?"

Put the pieces in 4 equal groups.

"How might we start distributing the pieces?"

Start with the 4 hundreds, then the 5 tens, and then the 6 ones.

"What happens if there is a remainder? For example, after putting 1 ten into each group, there is still 1 ten left."

Decompose the I ten into 10 ones.

"What does that accomplish?"

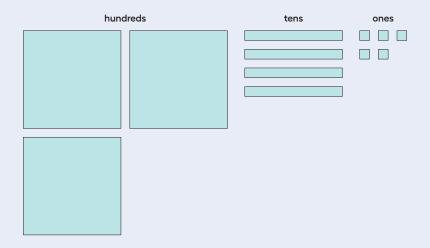
It allows us to have smaller units to divide. Combined with the 6 ones from the dividend, we now have 16 ones, which means 4 ones in each group.

"What is the value of the quotient?"

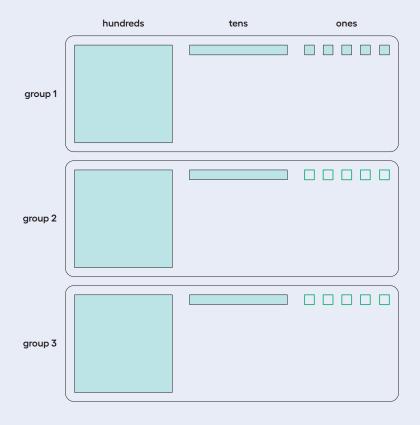
The amount in one group, 114

Lesson Summary

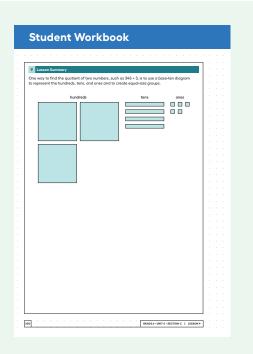
One way to find the quotient of two numbers, such as $345 \div 3$, is to use a base-ten diagram to represent the hundreds, tens, and ones and to create equal-size groups.

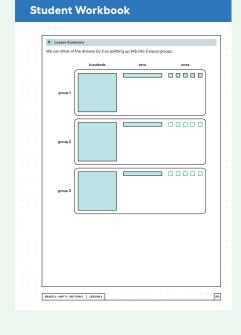


We can think of the division by 3 as splitting up 345 into 3 equal groups.



Each group has 1 hundred, 1 ten, and 5 ones, so $345 \div 3 = 115$. Notice that in order to split 345 into 3 equal groups, one of the tens had to be decomposed into 10 ones.



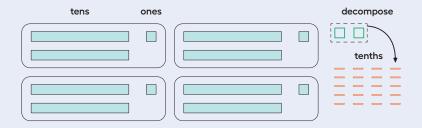


Responding To Student Thinking

Points to Emphasize

If most students struggled to explain the process of decomposing baseten units so that there would be enough pieces to create equal-size units, focus on articulating this idea as opportunities arise in the next few lessons. For example, after discussing Kiran's equations in the Warm-up referred to here, discuss how to distribute 6 hundreds, 5 tens, and 7 ones into 3 equal-size groups if we were to use base-ten blocks or diagrams to represent $657 \div 3$. Highlight that 2 of the tens need to be decomposed into 20 ones and combined with the 7 ones. The 27 ones can then be placed into 3 groups.

Grade 6, Unit 5, Lesson 10, Warm-up Notice and Wonder: Kiran's Calculations Base-ten diagrams can also help us think about division when the result is not a whole number. Let's look at $86 \div 4$, which we can think of as dividing 86 into 4 equal groups.



We can see that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we first need to decompose the 2 ones into 20 tenths and then put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.

For some division problems, such as $1,248 \div 36$ or $9.65 \div 1.5$, it is not convenient to draw and reason with base-ten diagrams. We will look at other strategies in upcoming lessons.

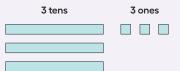
Cool-down

Putting 33 into 4 Groups

5 mir

Student Task Statement

To find $33 \div 4$, Clare drew a diagram and thought about how to put the tens and ones into 4 equal-size groups.



1. There aren't enough tens or ones to put into 4 groups. What can Clare do to find the quotient? Explain or show your reasoning.

Sample response: The 3 tens can be decomposed into 30 ones, making a total of 33 ones. Of these, 32 ones can be distributed into 4 groups, 8 ones in each. There is I one left. This can be decomposed into IO tenths and distributed into 4 groups, 2 tenths in each group. There are 2 tenths left. These can be decomposed into 20 hundredths and then distributed into 4 groups, 5 hundredths in each group.

2. What is the value of $33 \div 4$?

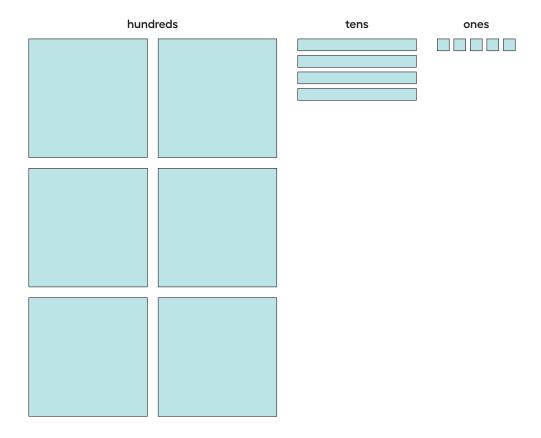
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Practice Problems

5 Problems

Problem 1

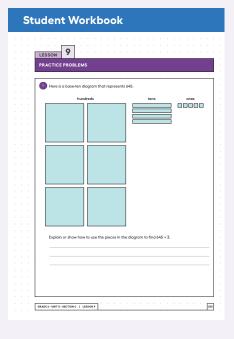
Here is a base-ten diagram that represents 645.

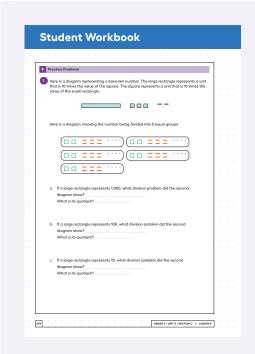


Explain or show how to use the pieces in the diagram to find $645 \div 3$.

Sample response: The blocks can be divided into 3 groups so that each group has 2 hundreds, I ten, and 5 ones, or 215:

- The 6 hundreds can be divided into 3 groups, with 2 hundreds in each.
- Of the 4 tens, 3 tens can be divided into 3 groups, with I ten in each.
- The remaining I ten can be decomposed into IO ones and combined with the 5 ones.
- The 15 ones can be divided into 3 groups, with 5 ones in each.





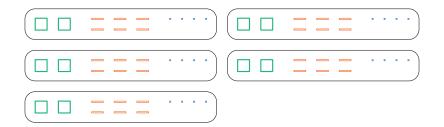
Student Workbook Procedure Procedure Procedure Show your reasoning. D. 355+4 Procedure Students sit up a lemonode stand. At the end of the day, their profit is 50752. Workmach monoring do they each flow when the profit is spit equally? Show or explain your reasoning.

Problem 2

Here is a diagram representing a base-ten number. The large rectangle represents a unit that is 10 times the value of the square. The square represents a unit that is 10 times the value of the small rectangle.



Here is a diagram showing the number being divided into 5 equal groups.



a. If a large rectangle represents 1,000, what division problem did the second diagram show? What is its quotient?

1,320 ÷ 5

The quotient is 264.

b. If a large rectangle represents 100, what division problem did the second diagram show? What is its quotient?

132 ÷ 5

The quotient is 26.4.

c. If a large rectangle represents 10, what division problem did the second diagram show? What is its quotient?

13.2 ÷ 5

The quotient is 2.64.

Problem 3

Find each quotient. Show your reasoning.

 $a. 195 \div 3$

65

Sample reasoning: $180 \div 3 = 60$, and $15 \div 3 = 5$, so the quotient is 60 + 5, which is 65.

b. 356 ÷ 4

89

Sample reasoning: 4.90 = 360, and 356 is 4 less than 360, so it is 4.89.

Problem 4

Four students set up a lemonade stand. At the end of the day, their profit is \$17.52. How much money do they each have when the profit is split equally? Show or explain your reasoning.

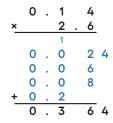
\$4.38

Sample reasoning: Four people are sharing \$17.52 equally, so each person gets \$17.52 \div 4. Each person can be given \$4, and then \$1.52 remains. Each person can be given \$0.30, and then \$0.32 remains. So they each get \$0.08 more. That means each person gets a total of 4 + 0.30 + 0.08 or \$4.38.

Problem 5

from Unit 5, Lesson 8

- **a.** Jada calculates (2.6) · (0.14) by calculating 26 · 14, then moving the digits three places to the right. Explain why Jada's method makes sense, and use it to calculate (2.6) · (0.14).
 - 2.6 is $26 \cdot \frac{1}{10}$, and 0.14 is (14) $\cdot \frac{1}{100}$, so (2.6) \cdot (0.14) = (26 \cdot 14) $\cdot \frac{1}{1,000}$. This is the same as (26 \cdot 14) with its digits moved three places to the right. 26 \cdot 14 = 364, so (2.6) \cdot (0.14) = 0.364.
- **b.** Diego calculates $(2.6) \cdot (0.14)$ using partial products.



Explain how Diego computed the four partial products.

- The 0.024 is (0.04) · (0.6)
- The 0.08 is (0.04) · 2
- \circ The 0.06 is 0.1 \cdot (0.6), and the 0.2 is (0.1) \cdot 2
- Their sum is 0.364

