Polygons

Goals

- Compare and contrast (orally) different strategies for finding the area of a polygon.
- Describe (orally and in writing) the defining characteristics of polygons.
- Find the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using words and other representations).

Learning Targets

- I can describe the characteristics of a polygon using mathematical vocabulary.
- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.

In this lesson, students explore the defining characteristics of polygons. Then, they find the areas of polygons by decomposing the regions into triangles or parallelograms.

Students have worked with polygons in earlier grades and throughout this unit without having a formal term for this category of shapes. This lesson prompts them to examine examples and non-examples of polygons and write a definition for a polygon. There are many different accurate definitions for a polygon. The goal is not to find the most succinct definition possible, but to articulate the defining characteristics of a polygon in a way that makes sense to students.

Then, students reason about the areas of quadrilaterals on a grid. The lesson also includes an optional activity that involves finding the area of a polygon in the shape of a pinwheel. The activity is an opportunity for students to apply familiar reasoning strategies to find the area of a more complex figure—a polygon with 8 sides.

The work here allows students to see that the area of a polygon can be found by decomposing it into triangles. In observing and using this fact, students look for and make use of structure.

Access for Students with Diverse Abilities

• Engagement (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 3)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- · Which Three Go Together?

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Geometry toolkits: Activity 2, Activity 3

Required Preparation

Activity 3:

For this optional activity, if larger paper (and a photocopier that can accommodate it) is available, it would be helpful to have larger-format copies of the blackline master.

Lesson Timeline







Activity 1



Activity 2



Activity 3



Lesson Synthesis



Cool-down

Polygons

Math Community

Today's math community building time has two goals. The first is for students to make a personal connection to the math actions chart and to share on their *Cool-down* the math action that is most important to them. The second is to introduce the idea that the math actions that students have identified will be used to create norms for their mathematical community in upcoming lessons.

Student Learning Goal

Let's investigate polygons and their areas.

Warm-up

Which Three Go Together: Triangles



Activity Narrative

This Warm-up prompts students to carefully analyze and compare features of triangles. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear how students talk about characteristics of triangles and their area.

Students may describe the differences in the triangles in terms of:

- The angles (acute, right, or obtuse).
- The orientation of sides (vertical, horizontal).
- The side likely to be chosen as a base.
- The length of base or height.
- The area.

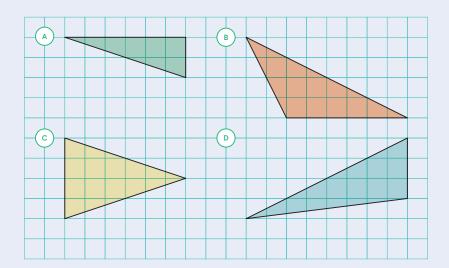
Launch



Arrange students in groups of 2–4. Display the triangles for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three triangles that go together and can explain why. Next, tell each student to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- They have a base or a height that is 6 units long.
- They have a side that slants down from left to right.

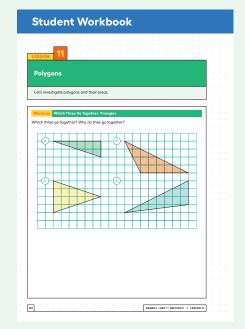
Instructional Routines

Which Three Go Together?

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A, B, and D go together because:

- · All their sides have different lengths.
- · All their angles have different measures.

A, C, and D go together because:

- They all have a side that is vertical.
- We could find their area by choosing the vertical side as a base.

B, C, and D go together because:

- They are not right triangles (or have no right angle).
- · They all have an area of 12 square units.
- They have just one side that would be easy to use as the base (where we can tell the corresponding height from the grid).

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use (such as "vertical," "horizontal," "right angle," "base," and "height") and to clarify their reasoning. Consider asking:

○ "How do you know ...?"

"What do you mean by ...?"

"Can you say that in another way?"

Math Community

After the Warm-up, display the revisions to the class Math Community Chart that were made from student suggestions in an earlier exercise. Tell students that over the next few exercises, this chart will help the class decide on community norms—how they as a class hope to work and interact together over the year. To get ready for making those decisions, students are invited at the end of today's lesson to share which "Doing Math" action on the chart is most important to them personally.

Activity 1

What Are Polygons?

15 min

Activity Narrative

In this activity, students examine examples and non-examples of polygons and identify the defining characteristics of a polygon.

Developing a useful and complete definition of a polygon is harder than it seems. A formal definition is often very wordy or hard to parse. Polygons are often referred to as "closed" figures, but if used, this term needs to be defined, as the everyday meaning of "closed" is different from its meaning in a geometric context.

This activity prompts students to develop a working definition of polygon that makes sense to them, but that also captures all of the necessary aspects that makes a figure a polygon. Here are some important characteristics of a polygon.

- It is composed of line segments. Line segments are always straight.
- Each line segment meets one and only one other line segment at each end.
- The line segments never cross each other except at the end points.
- It is two-dimensional.

One consequence of the definition of a polygon is that there are always as many vertices as edges. Students may observe this and want to include it in their definition, although technically it is a result of the definition rather than a defining feature.

As students work, monitor for both correct and incorrect definitions of a polygon. Listen for clear and correct descriptions as well as common but inaccurate descriptions (so they can be discussed and refined later).

Launch



Arrange students in groups of 2–4. Give students 3–4 minutes of quiet think time. Afterward, ask them to share their responses with their group and complete the second question together. If there is a disagreement about whether a figure is a polygon, ask them to discuss each point of view and try to come to an agreement. Follow with a whole-class discussion.

Student Task Statement

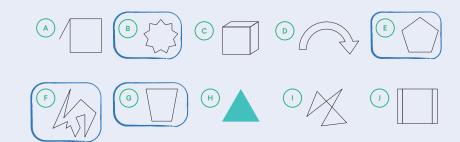
These five figures are **polygons**.



The next six figures are *not* polygons.



1. Circle the figures that are polygons.



Access for Multilingual Learners (Activity 1, Student Task)

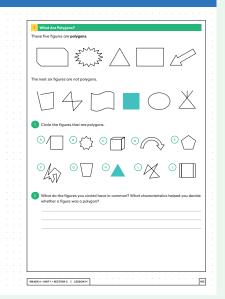
MLR2: Collect and Display.

Circulate, and listen for and collect the language that students use as they discuss characteristics of polygons. On a visible display, record words and phrases such as "straight sides," "edges," "vertex (vertices)," or "intersect." Invite students to borrow language from the display as needed and update it throughout the lesson. Advances: Conversing, Reading

Building on Student Thinking

Students may think that Figures C and I are polygons because they can see several triangles or quadrilaterals in each figure. Ask students to look closely at the examples and non-examples and see if there is a figure composed of multiple triangles or quadrilaterals, and if so, to see in which group it belongs.

Student Workbook



2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

Answers vary. Characteristics that the polygons have in common:

They are two-dimensional, composed of line segments that never cross each other, and each line segment meets one and only one other line segment at each end.

Activity Synthesis

Display the figures in the first question for all to see. For each figure, ask at least one student to explain why they think it is or is not a polygon. (It is fine if students' explanations are not precise at this point.) Then, circle the figures that are polygons on the visual display.

Next, ask students to share their ideas about the characteristics of polygons. Record them for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate on to make the description clearer or more precise. If they disagree, ask for an explanation.

Make the key characteristics of a polygon explicit: It is a two-dimensional figure, it is composed of line segments, the line segments meet only one other line segment at each end, and the line segments cross another line segment only at the endpoints. If one of these key characteristics is not mentioned by students, bring it up and revisit it at the end of the lesson.

Tell students we call the line segments in a polygon the "edges" or "sides," and we call the points where the edges meet the "vertices." Point to the sides and vertices in a few of the identified polygons.

If time permits, point out that polygons always enclose a region, but the region is not technically part of the polygon. When we talk about finding the area of a polygon, we are in fact finding the area of the region it encloses. So "the area of a triangle," for example, is really shorthand for "area of the region enclosed by the triangle."

Activity 2

Quadrilateral Strategies

15 min

Activity Narrative

This activity has several aims. It prompts students to apply what they learned to find the area of quadrilaterals that are not parallelograms, encourages them to plan before jumping into a problem, and urges them to reflect on the merits of different methods.

Activity 1

Warm-up

Note that it is unnecessary for students to take the most efficient path. It is more important that they choose an approach that makes sense to them but have the chance to see the pros and cons of various approaches.

Launch

Ask students to recall the definition of "quadrilateral" from earlier grades, or tell students that a quadrilateral is a polygon with 4 sides. Tell students that we will now think about how to find the area of quadrilaterals.

Arrange students in groups of 4. Display the image of Quadrilaterals A-F for all to see. Direct their attention to Quadrilateral D.

Give students a minute of quiet time to think about the first 2–3 moves they would make to find the area of Quadrilateral D. Offer some sentence starters: "First, I would ... Next I would ..., and then I would ..." Encourage them to show their moves on the diagram in their material. Emphasize that we are interested only in the plan for finding area and not in the area itself, so no calculation is expected. Then, give them 1–2 minutes to share their moves with their group.

Ask students to indicate what their first planned move is. Does their very first move involve:

- Decomposing the quadrilateral?
- Enclosing the quadrilateral?
- · Another move?

Ask the students whose first move is to decompose the figure:

☐ "How many pieces will result from the decomposition? 2 pieces? 3 pieces? 4 pieces? More?"

"What is the next move? Rearrange? Duplicate a piece? Calculate the area of a piece? Something else?"

Ask the students whose first move is to enclose the figure:

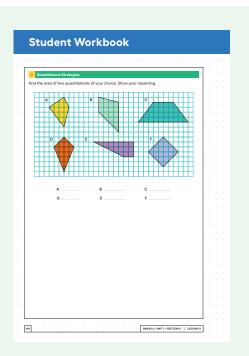
"How many rectangles will you create? 1 rectangle? 2 rectangles? More?"

"What is the next move? Rearrange the extra pieces? Calculate the area of an extra piece? Something else?"

For each sequence that students mentioned, draw a quick diagram to illustrate it for all to see.

Once students have a chance to see a variety of approaches, ask students to revisit their sequence of moves. Give students 1–2 minutes to think about the pros and cons of their original plan, and if there was another strategy that they found productive. Invite a few students to share their reflections.

Then, give students access to their geometry toolkits and quiet time to complete the activity. Ask students to keep in mind the merits of the different strategies they have seen as they plan their work.



Student Task Statement

Find the area of two quadrilaterals of your choice. Show your reasoning.

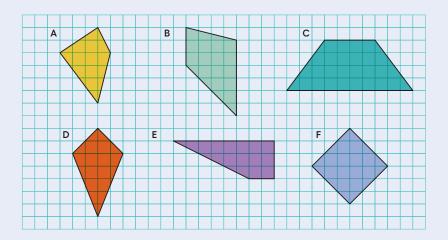


Figure A: 12 square units

Figure B: 18 square units

Figure C: 28 square units

Figure D: 14 square units

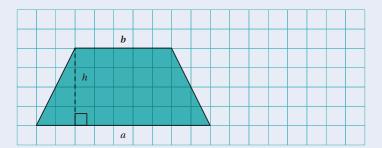
Figure E: 15 square units

Figure F: 18 square units

Reasoning varies. Students could decompose the quadrilateral into parallelograms and triangles to find the area, decompose and rearrange the pieces into a shape of which they can easily find the area, or enclose the figure in a rectangle and subtract the area of the extra pieces.

Are You Ready for More?

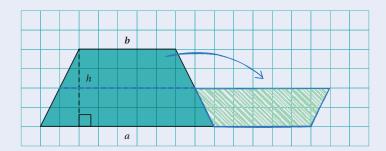
Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.



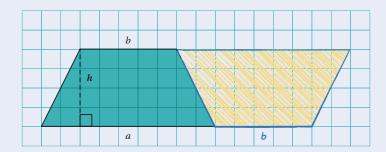
Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—to the trapezoid so that you have one or more shapes with areas that you already know how to find. Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

Sample responses:

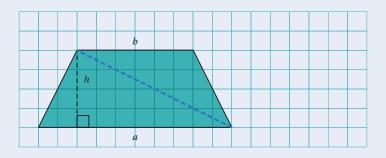
• Cut the trapezoid in half horizontally, rotate the top piece, and attach it to the bottom piece. Add the top and bottom side lengths and multiply that by half of the original height. The result is a parallelogram. $\frac{1}{2} \cdot h \cdot (a+b)$



• Place an identical but rotated copy of the same trapezoid next to the original to make a parallelogram. Then, find the area of the parallelogram and divide that by 2. $(a+b) \cdot h \div 2$



• Draw a diagonal and add the areas of the two resulting triangles. $a \cdot h \div 2 + b \cdot h \div 2$



Activity Synthesis

To conclude the activity, ask students to choose one quadrilateral they worked on (other than D) and tell their group the first couple of moves they made for finding its area and why. Encourage other group members to listen carefully, check that the reasoning is valid, and offer feedback.

Students may have noticed that all the approaches involved decomposing one or more regions into triangles, rectangles, or both. If not mentioned by students, point this out. Emphasize that we can decompose any polygon into triangles and rectangles to find its area.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Students who are working toward verbal output may benefit from access to mini-whiteboards, sticky notes, or spare paper to write down and show to their group the first couple of moves they made to find the area of their quadrilateral.

Advances: Writing, Representing

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 3)

MLR7: Compare and Connect

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

Connect a new concept to one with which students have experienced success. For example, remind students that triangles can be decomposed, rearranged, enclosed, or duplicated to determine area. Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Activity 3: Optional

Pinwheel



Activity Narrative

In this activity, students determine the area of an unfamiliar polygon and think about various ways for doing so. The task prepares students to find the areas of other unfamiliar shapes in real-world contexts. It also reinforces the practice of sense-making, planning, and persevering when solving a problem. Students reason independently before discussing and recording their strategies in groups.

Because the shape of the polygon is more complex than what students may have seen so far, expect students to experiment with one or more strategies. Consider preparing extra copies of the blackline master for students to use, if needed.

As students work, monitor for those who:

- Decompose the pinwheel into triangles and find the areas.
- Decompose the pinwheel into rectangles and triangles, rearrange the pieces into parallelograms (right or non-right), and find the areas.
- Enclose the pinwheel with one large square or several smaller rectangles, decompose the extra regions into triangles and rectangles, find the areas of the extra pieces, and subtract them from the area(s) of the enclosing rectangle(s).

Make note of the variations and complexities in how students obtain shapes whose areas can be found. If there is limited variation in strategies, look for different ways of recording the same strategy.

Students have opportunities to notice and make use of the structure of the pinwheel in their reasoning. For instance, the pinwheel can be decomposed into four identical pieces (or sets of pieces). Also, enclosing the pinwheel with a square creates four extra regions that are identical.

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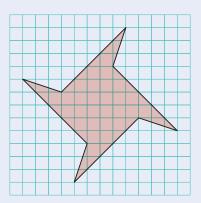
Arrange students in groups of 4. Give students access to their geometry toolkits and 5 minutes of quiet time to plan an approach for finding the area of the pinwheel. Then, ask them to share their plan with their group.

The group then decides on one or more strategies to pursue, works together to find the area, and creates a visual display of the strategy (or strategies) used. Make sure each group has access to one or more copies of the pinwheel for the visual display. Encourage students to include details that will help others interpret their thinking. For example, specific language, use of different colors, shading, arrows, labels, notes, diagrams, or drawings.

Student Task Statement

Find the area of the shaded region in square units. Show your reasoning.

Warm-up

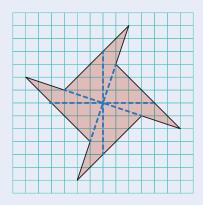


40 square units

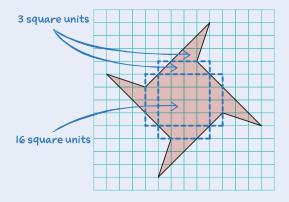
Sample reasoning:

• There are 4 identical sets of two different triangles. The area of each triangle can be found by enclosing it in a rectangle and subtracting the areas of right triangles, or by using the formula for the area of a triangle.

$$4 \cdot \frac{1}{2}(3 \cdot 4 + 2 \cdot 4) = 40$$



• The middle of the pinwheel is a square with the area of 16 square units. Around the square are 4 identical trapezoids. The area of each trapezoid can be found by enclosing it with a rectangle and subtracting the area of the extra pieces, or by decomposing and rearranging the pieces into a rectangle with an area of 3 square units. The pointy parts of the pinwheel are triangles, each with a base of 2 units and a height of 3 units, so the area of each is 3 square units.

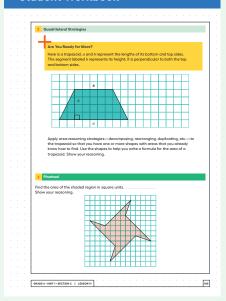


Building on Student Thinking

Lesson Synthesis

Students who overlay a rotated square over the figure such that the four pins are shown as four right triangles may use incorrect side lengths for the square or the triangles (for instance, assuming that one of the side lengths is 2 units instead of a little less than 3 units). Help them see, by measuring one, that the diagonal of a unit square is longer than its side length.

Student Workbook



Activity Synthesis

Use Compare and Connect to help students compare, contrast, and connect the different approaches. Give students time to visit one another's visual display. Consider displaying the following questions for students to discuss as they investigate others' work:

 \bigcirc "Did this group find the same area as our group? If not, why?"

"How is their strategy like our strategy?"

"How is their strategy different from ours?"

During the whole-class discussion, ask students:

☐ "What did the strategies have in common? How were they different?"

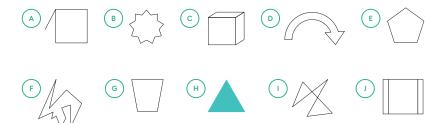
"Are there any benefits or drawbacks to one representation compared to another?"

Highlight similarities in students' work in broader terms, as outlined in the activity narrative. Reinforce that all approaches involve decomposing a polygon into triangles and rectangles to find area.

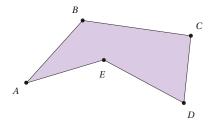
Lesson Synthesis

To review the defining characteristics of a polygon, return to the image in the "What are Polygons?" activity and display the list of defining features generated by students in that activity.

Revisit each figure that is *not* a polygon and ask students to explain why it is not a polygon. Encourage students to use their list to support their explanations, as well as to suggest revisions to their working definition.



Here is a polygon with 5 sides.



Ask students:

 \bigcirc "How do we know this figure is a polygon?"

It is composed of line segments. Each segment meets only one other segment at each end. The segments do not cross one another. It is two-dimensional.

"What does it mean to find the area of this polygon?"

It means finding the area of the region inside it.

 \bigcirc "How can we find the area of this polygon?"

We can decompose the region inside it into triangles and rectangles.

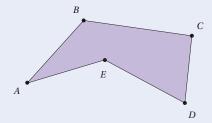
Lesson Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

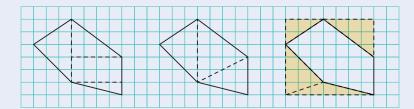
- Each end of a line segment connects to one other line segment. The point where two segments connect is a vertex. The plural of vertex is vertices.
- The segments are called the edges or sides of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled A, B, C, D, and E.

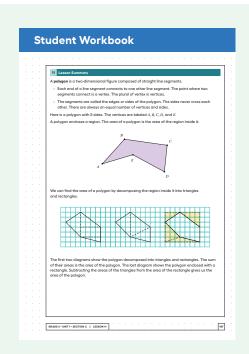
A polygon encloses a region. The area of a polygon is the area of the region inside it.



We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles. The sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle. Subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.



Responding To Student Thinking

Points to Emphasize

If students struggle with decomposing a polygon into parallelograms and triangles whose areas can be calculated, integrate discussions about different ways to find the area of a polygon. For example, ask students to analyze Lin's and Andre's ways of decomposing the hexagon in this practice problem and explain how its area can be found using each method:

Unit 1, Lesson 11, Practice Problem 4

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community-building question "Which 'Doing Math' action is most important to you, and why?" Ask students to respond to the question after completing the *Cool-down*.

After collecting the *Cool-downs*, review student responses to the community-building question. Use the responses to draft a student norm and a teacher norm to use as an example in Exercise 6. For example, if "sharing ideas" is a common choice for students, a possible norm is "We listen as others share their ideas."

For the teacher norms section, if "questioning vs. telling" from the "Doing Math" section is key for your teaching practice, then one way to express that as a norm is "Ask questions first to make sure I understand how someone is thinking."

Give students access to their geometry toolkits. Tell students that they need to show only how the area could be found; they do not have to actually calculate the area.

Cool-down

Triangulation

This *Cool-down* assesses students' understanding of the defining characteristics of a polygon and the ways it can be decomposed.

Student Task Statement

1. Here are two five-pointed stars. A student said, "Both figures A and B are polygons. They are both composed of line segments and are two-dimensional. Neither have curves." Do you agree with the statement? Explain your reasoning.

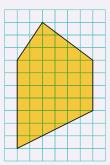




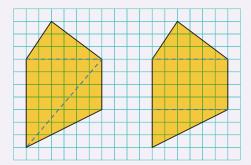
Disagree. Only Figure B is a polygon.

Sample reasoning: Every segment in Figure A meets or crosses more than two segments at its ends, so it is not a polygon. Each segment in Figure B meets only one other segment at each end.

2. Here is a five-sided polygon. Describe or show the strategy you would use to find its area. Mark up and label the diagram to show your reasoning so that it can be followed by others. (It is not necessary to actually calculate the area.)

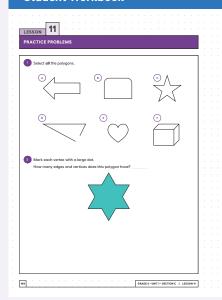


Sample responses:

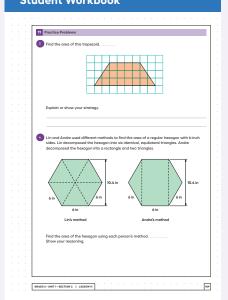


- The polygon can be decomposed into three triangles: one with a base of 6 units and a height of 3, a second one with a base of 7 and a height of 6, and a third with a base of 4 and a height of 6. All areas can be calculated using the area formula.
- The polygon can be decomposed into two triangles and a rectangle. One triangle has a base of 6 and a height 3, and the second has a base of 6 and a height of 3. Their areas can be calculated with the area formula. The rectangle is 6 by 4, so its area is the product of 6 and 4.

Student Workbook Practice Prol



Student Workbook

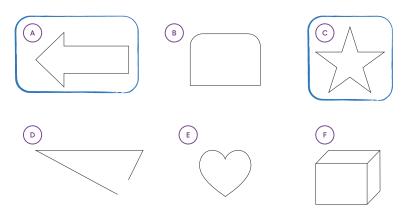


Practice Problems

6 Problems

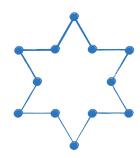
Problem 1

Select **all** the polygons.



Problem 2

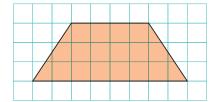
Mark each vertex with a large dot. How many edges and vertices does this polygon have?



12 edges and 12 vertices

Problem 3

Find the area of this trapezoid. Explain or show your strategy.



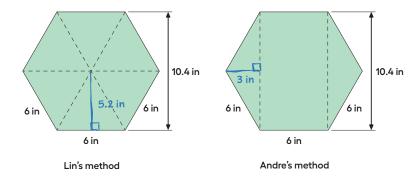
18 square units

Sample reasoning: Enclose the trapezoid inside a

3-unit-by-8-unit rectangle. The area of the rectangle is 24 square units because $8 \cdot 3 = 24$. The area of each unshaded triangle within the rectangle is 3 square units because $(2 \cdot 3) \div 2 = 3$. The sum of areas of the two triangles is 6 square units. 24 - 6 = 18, so the area of the trapezoid is 18 square units.

Problem 4

Lin and Andre used different methods to find the area of a regular hexagon with 6-inch sides. Lin decomposed the hexagon into six identical, equilateral triangles. Andre decomposed the hexagon into a rectangle and two triangles.

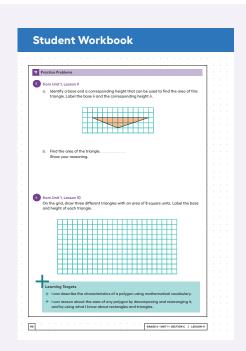


Find the area of the hexagon using each person's method. Show your reasoning.

93.6 square inches

Sample reasoning:

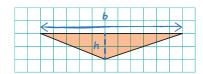
- The height of each triangle in Lin's diagram is half of 10.4 inches or 5.2 inches. The area of each triangle is 15.6 square inches. $\frac{1}{2} \cdot 6 \cdot (5.2) = 15.6$ The hexagon is composed of 6 triangles, so its area is $6 \cdot (15.6)$ or 93.6 square inches.
- The rectangle in Andre's diagram is $(10.4) \cdot 6$ or 62.4 square inches. Each triangle has a base of 10.4 inches and a height of 3 inches. (The horizontal distance across the middle of the hexagon is composed of two 6-inch segments. The vertical line that Andre drew cuts one 6-inch segment in half, so the segment on one side is 3 inches long.) The area of each triangle is $\frac{1}{2} \cdot 10.4 \cdot 3$ or 15.6 square inches. The area of the hexagon is therefore 62.4 + 15.6 + 15.6 or 93.6 square inches.



Problem 5

from Unit 1, Lesson 9

a. Identify a base and a corresponding height that can be used to find the area of this triangle. Label the base b and the corresponding height h.



b. Find the area of the triangle. Show your reasoning.

Il square units

Sample reasoning: $\frac{1}{2} \cdot II \cdot 2 = II$

Problem 6

from Unit 1, Lesson 10

On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.

Drawings should show triangles with a base and a height that multiply to be 24 square units (that is, each triangle is half of a parallelogram with an area of 24 square units).

Sample responses:

