Lesson 16

Finding the Percentage (Optional)

Goals

- Calculate the percentage one value is of another value and explain (orally) the solution method.
- Generalize a process for finding the percentage that C is of B and justify (orally) why this can be expressed as $\frac{C}{B} \cdot 100$.

Learning Target

I can solve different problems like “60 is what percentage of 40?” by dividing and multiplying.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- Math Talk
- MLR7: Compare and Connect

Lesson Narrative

This lesson is optional because it goes beyond the expectations of grade 6 standards. Students learn to generalize the process for expressing a number as a percentage of another number, or finding what percentage C is of B in a situation where A% of B is C. While students have reasoned about such problems using strategies learned in this unit, developing a general structure that will work with any pair of B and C is not required in this course.

Students begin by revisiting fraction and decimal equivalence in the *Warm-up*. Next, they reason repeatedly about what percentage and what fraction one number is of another number in the context of a situation. They write each fraction in decimal form and then look for regularity in the way the decimals relate to the percentages.

Then, students generalize the process of expressing one number, C, as a fraction of another, B, as $\frac{C}{B}$. They see that the decimal equivalent of $\frac{C}{B}$ is related to the numeric value of the percentage (the P in P%) by a factor of 100. This means that the percentage C is of B can be found by computing: $\frac{A}{B} \cdot 100$.

The last activity allows students to apply their generalized process to solve similar problems in a new context.

Student Learning Goal

Let’s find percentages in general.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	20 min	15 min	10 min	5 min

Warm-up

Math Talk: Fractions and Decimals

10 min

Activity Narrative

This *Math Talk* focuses on fraction and decimal equivalence. It encourages students to think about equivalent fractions and to rely on what they know about the relationship between fractions and decimals to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students generalize the process of finding what percentage a number is of another number and express it with one or more expressions.

To determine whether two fractions (or a fraction and a decimal) are equivalent, students need to look for and make use of structure. In explaining their reasoning, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally if each equation is true or false.

A. $\frac{1}{5} = \frac{2}{10}$

True

Sample reasoning:

- They are equivalent fractions. Multiplying the numerator and denominator of $\frac{1}{5}$ by 2 gives $\frac{2}{10}$.
- Both are equivalent to 0.2
- Both are in the same location on the number line.

B. $\frac{3}{5} = 0.35$

False

Sample reasoning:

- $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ or 0.6
- $\frac{3}{5} = \frac{60}{100}$, which is 60 hundredths and 0.35 is 35 hundredths.

Instructional Routines

Math Talk

ilclass.com/r/10694967

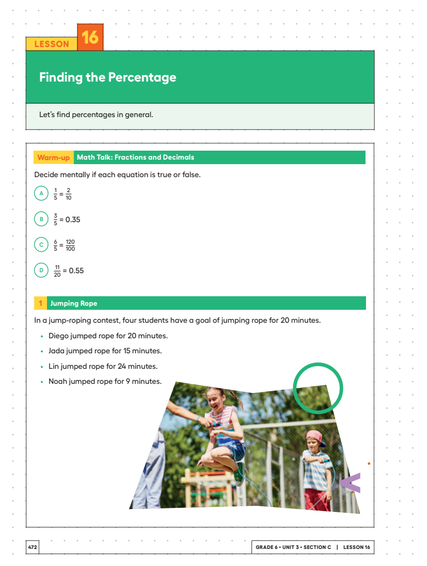
Please log in to the site before using the QR code or URL

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _________ because ..." or "I noticed _________ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

C. $\frac{6}{5} = \frac{120}{100}$

True

Sample reasoning:

- $\frac{6}{5} = \frac{12}{10}$, which is equivalent to $\frac{120}{100}$
- $6 \div 5 = 1.2$ and $120 \div 10$ is also 1.2

D. $\frac{11}{20} = 0.55$

True

Sample reasoning:

- $\frac{11}{20}$ is equivalent to $\frac{55}{100}$, which is 0.55
- $110 \div 20 = 5.5$, so $11 \div 20 = 0.55$

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___________'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ___________'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

If not brought up in students' explanations, emphasize the following points:

- Two fractions are equivalent if multiplying the numerator and denominator of one fraction by the same number gives the numerator and denominator of the other fraction. (For example, $\frac{11}{20} = \frac{55}{100}$ because multiplying 11 and 20 each by 5 gives 55 and 100.)
- Dividing the numerator of a fraction by the denominator gives an equivalent number in decimal form. (For example, $11 \div 20 = 0.55$)
- Writing an equivalent fraction with a power of 10 in the denominator is another way to express a fraction in decimal form. (For example, $\frac{11}{20} = \frac{55}{100}$, which is 0.55.)

Activity 1
Jumping Rope

20 min

Activity Narrative

This activity prompts students to generalize the process of determining what percentage one number is of another number (or finding C as a percentage of B, where $A\%$ of B is C).

Students begin by using familiar strategies to reason about what percentages some numbers are of 20. They record their responses in a table. Then, they express the numbers as fractions of 20 and write the decimal equivalents in the same table.

Next, students analyze the relationships between these numbers. They observe that the percentages and decimals are related by a factor of 100: multiplying the decimals by 100 gives the percentages, and dividing the percentages by 100 gives the decimals. Students then think about how to determine the percentage that any number, t, is of 20 and write an expression to describe it.

As students work, monitor for the ways in which they complete the last column of the table. Select students who divide each jumping duration by 20 to share later.

As they analyze the values in the table and describe the relationships they observe in general terms, students practice looking for and making use of structure.

Launch

Ask students if they have jumped rope or seen it done. Invite students who are familiar with the activity to introduce it or to briefly share their experience. (When did they learn it? Do they jump to a rope spun by two other people or do they spin their own rope? What kinds of skips do they do?) Ask them if they have timed their jumping sessions and, if so, what their longest duration was.

Tell students that this activity is about the lengths of time that some students jumped rope in a contest.

Arrange students in groups of 2–4. Provide access to calculators. Give students 3–4 minutes of quiet work time and then time to discuss their thinking with their group. Leave at least 5 minutes for discussion.

Student Task Statement

In a jump-roping contest, four students have a goal of jumping rope for 20 minutes.

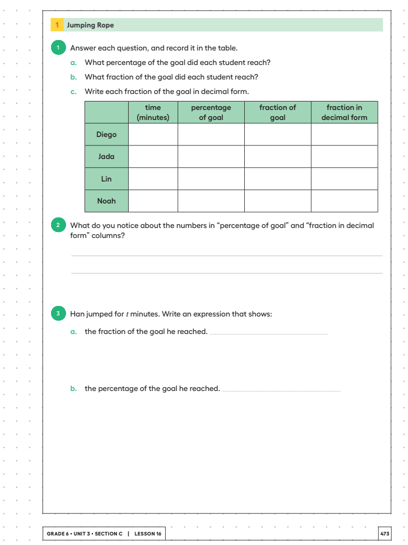
- Diego jumped rope for 20 minutes.
- Jada jumped rope for 15 minutes.
- Lin jumped rope for 24 minutes.
- Noah jumped rope for 9 minutes.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.
Activate or supply background knowledge. To help students recall the terms “fraction” and “decimal” and the ways in which they are related, ask, “When do we use fractions and decimals?”, “How do we write fractions and decimals?” and “How can we write fractions like $\frac{6}{8}$ and $\frac{3}{5}$ in decimal form?”

Supports accessibility for: Memory, Language

Student Workbook


1 Jumping Rope

1 Answer each question, and record it in the table.

a. What percentage of the goal did each student reach?

b. What fraction of the goal did each student reach?

c. Write each fraction of the goal in decimal form.

	time (minutes)	percentage of goal	fraction of goal	fraction in decimal form
Diego				
Jada				
Lin				
Noah				

2 What do you notice about the numbers in "percentage of goal" and "fraction in decimal form" columns?

3 Han jumped for t minutes. Write an expression that shows:

a. the fraction of the goal he reached.

b. the percentage of the goal he reached.

GRADE 6 • UNIT 3 • SECTION C | LESSON 16

1. Answer each question, and record it in the table.
   - **a.** What percentage of the goal did each student reach?
   - **b.** What fraction of the goal did each student reach?
   - **c.** Write each fraction of the goal in decimal form.

	time (minutes)	percentage of goal	fraction of goal	fraction in decimal form
Diego	20	100	$\frac{20}{20}$	1
Jada	15	75	$\frac{15}{20}$	0.75
Lin	24	120	$\frac{24}{20}$	1.2
Noah	9	45	$\frac{9}{20}$	0.45

2. What do you notice about the numbers in "percentage of goal" and "fraction in decimal form" columns?

   Sample responses:
   - The percentages are 100 times the decimals in the last column.
   - The decimals are the percentages divided by 100.

3. Han jumped for t minutes. Write an expression that shows:
   - **a.** the fraction of the goal he reached.

     $\frac{t}{20}$
   - **b.** the percentage of the goal he reached.

     $\frac{t}{20} \cdot 100$

Activity Synthesis

A key takeaway from the discussion is that we can determine what percentage one number is of another number by dividing the former by the latter and multiplying the result by 100.

First, invite students to share how they completed the last column, ending with students who divide time in minutes by 20. If no students found the decimal equivalents by dividing, remind students that a fraction can also be interpreted as division. For example, $\frac{24}{20}$ can be seen as $24 \div 20$, which is 1.2.

Next, display the completed table for all to see. Ask students the following questions, and record the responses for all to see:

- *"What relationship do you see between the percentages and the decimals?"*

  The percentages are each 100 times the decimals.

- *"Suppose a student jumped for 1 minute. What fraction of the goal is that?"*

  $\frac{1}{20}$

💬 *"How can this fraction help us find what percentage of 20 that is?"*

Divide 1 by 20, which gives 0.05, and multiply 0.05 by 100, which gives 5%

💬 *"Someone else jumped for t minutes. How can we write that amount as a fraction of the goal?"*

$\frac{t}{20}$

💬 *"How can we find out what percentage of 20 that is?"*

Divide t by 20 and multiply the result by 100, or calculate $\frac{t}{20} \cdot 100$.

💬 *"If the goal is 30 minutes instead of 20, how would we write each jumping time as a fraction of the goal?"*

The time would be the numerator and 30 the denominator.

💬 *"How would we write each fraction in decimal form?"*

Divide each numerator by 30.

💬 *"How would we calculate the percentages?"*

Multiply either the fractions or the decimals by 100.

💬 *"Suppose the goal is G minutes instead of 20 minutes. How do we calculate what percentage of the goal is 7 minutes?"*

Divide 7 by G and multiply by 100, or $\frac{7}{G} \cdot 100$.

💬 *"What percentage of the goal is t minutes?"*

$\frac{t}{G} \cdot 100$

Activity 2: Optional
Restaurant Capacity

15 min

Activity Narrative

This activity gives students a chance to apply the general process for calculating a value as a percentage of another value in a new context. It also offers students another opportunity to look for and make use of structure in the process of describing C as a percentage of B, allowing them to better make sense of the expression $\frac{C}{B} \cdot 100$.

Note that at this point it is expected and perfectly acceptable for students to use strategies or representations that may be more familiar but less efficient. Monitor for the different methods being used so that connections can be made between the methods students know well and those that are newer but more efficient. For instance, monitor for students who reason about what percentage 9 is of 75 in the following ways:

- Using a table to find what percentage 3 people or 1 person is of 75, and then find the percentage that corresponds to 9 people.

number of people	percentage
75	100
3	4
9	12

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL

Access for Multilingual Learners (Activity 2)

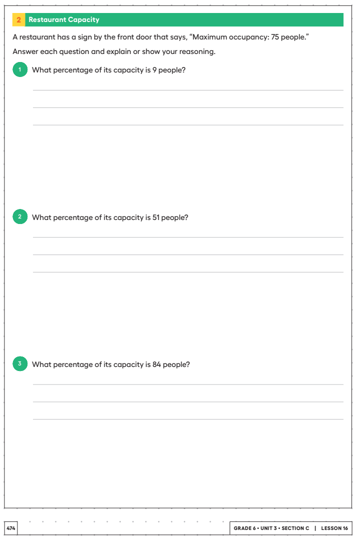
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Provide students with a graphic organizer, such as a problem-solving template to record their problem-solving process. The graphic organizer should ask students to identify what they need to find out, what information is provided, how they solved the problem, and why their answer is correct.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook


2 Restaurant Capacity

A restaurant has a sign by the front door that says, "Maximum occupancy: 75 people."

Answer each question and explain or show your reasoning.

1. What percentage of its capacity is 9 people?

2. What percentage of its capacity is 51 people?

3. What percentage of its capacity is 84 people?

474 | GRADE 6 • UNIT 3 • SECTION C | LESSON 16

number of people	percentage
75	100
1	$\frac{100}{75}$ or $\frac{4}{3}$
9	12

- Expressing 9 as a fraction of 75, $\frac{9}{75}$, rewriting it as $\frac{3}{25}$ and $\frac{12}{100}$, and multiplying $\frac{12}{100}$ by 100 to get 12.
- Dividing 9 by 75 to get 0.12, and then multiplying 0.12 by 100 to get 12.

Launch

Read the first sentence of the activity statement. Clarify that "maximum occupancy" is the greatest number of people that are permitted in a space. It is based on how many people can quickly exit the space in an emergency given the number of doors. Restaurants, schools, theaters, and other public buildings all have occupancy limits.

Keep students in groups of 2–4. Provide continued access to calculators. Give students 3–4 minutes of quiet work time and then time to discuss their thinking with a partner.

Select work from students with different strategies or representations, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

A restaurant has a sign by the front door that says, "Maximum occupancy: 75 people."

Answer each question and explain or show your reasoning.

1. What percentage of its capacity is 9 people?

   12%

   Sample reasoning:

   - $\frac{9}{75}$ is equivalent to $\frac{3}{25}$, which is equivalent to $\frac{12}{100}$.
   - $\frac{9}{75} \cdot 100 = \frac{900}{75}$, which is 12.

2. What percentage of its capacity is 51 people?

   68%

   Sample reasoning:

   - $\frac{51}{75}$ is equivalent to $\frac{17}{25}$, which is equivalent to $\frac{68}{100}$.
   - $\frac{51}{75} \cdot 100 = \frac{5{,}100}{75}$, which is 68.
   - If 9 people is 12% of 75, then 45 people, which is $5 \cdot 9$, is $5 \cdot 12$ or 60%. 6 more people is $\frac{6}{9} \cdot 12$ or 8%. That means 51 people is $(60 + 8)$ (or 68%).

3. What percentage of its capacity is 84 people?

112%.

Sample reasoning:

- 84 is 9 more than 75. We already know that 9 people is 12% and 75 people is 100%, so 84 people is (12 + 100) or 112%.
- $\frac{84}{75} \cdot 100 = \frac{8{,}400}{75}$, which is 112.

Are You Ready for More?

Of all the Earth's water, 96% is salt water in oceans and 4% is fresh water in lakes, rivers, glaciers, and polar ice caps.

The total volume of water on Earth is 1,386 million km^3.

1. What is the volume of salt water?

1,330.56 million or 1,330,560,000 km^3

2. What is the volume of fresh water?

55.44 million or 55,440,000 km^3

Activity Synthesis

The goal of this discussion is to highlight connections between various strategies for finding what percentage a number is of 75, allowing students to further make sense of the general method from an earlier activity.

Display 2–3 previously selected strategies for finding what percentage 9 is of 75. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the strategies have in common?"

They either involve dividing 100 by 25 and multiplying the result by 3, which is multiplying 100 by $\frac{3}{25}$, or involve dividing 100 by 75 and multiplying the result by 9, which is multiplying by $\frac{9}{75}$.

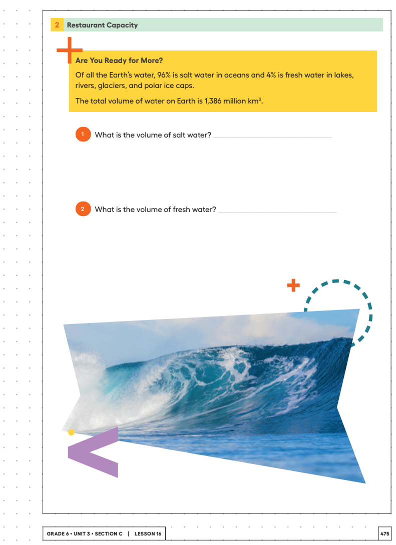
"How are they different?"

Some involve only computations and no tables or diagrams. Some ways are quicker than others.

"If some strategies involve multiplying 100 by $\frac{3}{25}$ and others involve multiplying 100 by $\frac{9}{75}$, why are the results the same?"

$\frac{3}{25}$ is equivalent to $\frac{9}{75}$.

Student Workbook


2 Restaurant Capacity

Are You Ready for More?

Of all the Earth's water, 96% is salt water in oceans and 4% is fresh water in lakes, rivers, glaciers, and polar ice caps.

The total volume of water on Earth is 1,386 million km^3.

1 What is the volume of salt water?

2 What is the volume of fresh water?

GRADE 6 • UNIT 3 • SECTION C | LESSON 16

To highlight the structure that leads to the generalized expression, consider using tables to illustrate how the same reasoning is applied to all three questions, as shown. Point out that in each case, the calculations involve dividing the occupancy by 75 and multiplying the result by 100.

occupancy	percentage
75	100
1	$\frac{1}{75} \cdot 100$
9	$\frac{9}{75} \cdot 100$

occupancy	percentage
75	100
1	$\frac{1}{75} \cdot 100$
51	$\frac{51}{75} \cdot 100$

occupancy	percentage
75	100
1	$\frac{1}{75} \cdot 100$
84	$\frac{84}{75} \cdot 100$

Lesson Synthesis

The main idea in this lesson is that to find what percentage C is of B, we can calculate: $\frac{C}{B} \cdot 100$.

Revisit a question seen in the lesson, such as: "What percent of 20 is 9?" Remind students that sometimes we can express one value as a benchmark fraction of the other value (such as finding 5 as a percent of 20), but this is not always the case. One value also may not be a factor or a multiple of the other. In this example, it can be helpful to first find what percent of 20 is 1, and then multiply that percentage by 9. Consider using a table to illustrate this reasoning:

value	percentage
20	100
1	$\frac{1}{20} \cdot 100$
9	$\frac{9}{20} \cdot 100$

$\frac{1}{20} = \frac{5}{100}$ or 0.05, and $(0.05) \cdot 100 = 5$, so 1 is 5% of 20. This means that 9 is $(9 \cdot 5)$%, or 45%, of 20.

We can also see that $\frac{9}{20} = \frac{45}{100}$ or 0.45, and $(0.45) \cdot 100 = 45$.

The same reasoning can also help us find the answer to the question:

"What percent of 20 is a number, C?"

value	percentage
20	100
1	$\frac{1}{20} \cdot 100$
C	$\frac{C}{20} \cdot 100$

It also helps us see how to answer a more general question:

"What percent of a number, B, is C?"

We can divide C by B, and then multiply the result by 100.

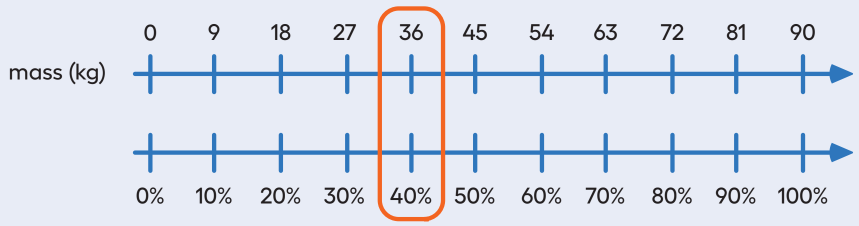
	value	percentage	
	B	100	$\cdot \frac{1}{B}$
$\cdot C$	1	$\frac{1}{B} \cdot 100$	
	C	$\frac{C}{B} \cdot 100$	$\cdot C$

Lesson Summary

What percentage of 90 kg is 36 kg? One way to solve this problem is to first find what percentage 1 kg is of 90, and then multiply by 36.

	mass (kg)	percentage	
$\cdot \frac{1}{90}$	90	100	$\cdot \frac{1}{90}$
$\cdot 36$	1	$\frac{1}{90} \cdot 100$	$\cdot 36$
	36	$\frac{36}{90} \cdot 100$	

From the table we can see that 1 kg is $\left(\frac{1}{90} \cdot 100\right)$%, so 36 kg is $\left(\frac{36}{90} \cdot 100\right)$% or 40% of 90. We can confirm this on a double number line diagram:

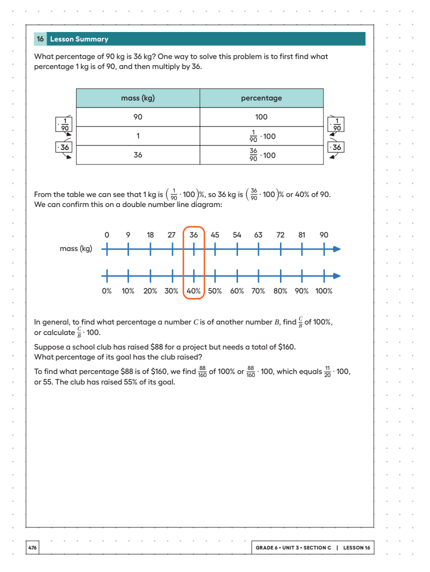


In general, to find what percentage a number C is of another number B, find $\frac{C}{B}$ of 100%, or calculate $\frac{C}{B} \cdot 100$.

Suppose a school club has raised $88 for a project but needs a total of $160. What percentage of its goal has the club raised?

To find what percentage $88 is of $160, we find $\frac{88}{160}$ of 100% or $\frac{88}{160} \cdot 100$, which equals $\frac{11}{20} \cdot 100$, or 55. The club has raised 55% of its goal.

Student Workbook



Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Library and Cafeteria

5 min

Launch

Provide access to calculators.

Student Task Statement

There are 50 people in a school.

1. The library has a capacity of 23 people. What percentage of the school population is that? Show your reasoning.

   46%

   Sample reasoning: $\frac{23}{50} = \frac{46}{100} = 0.46$ and $(0.46) \cdot 100 = 46$.

2. The cafeteria has a capacity of 65 people. What percentage of the school population is that? Show your reasoning.

   130%

   Sample reasoning:

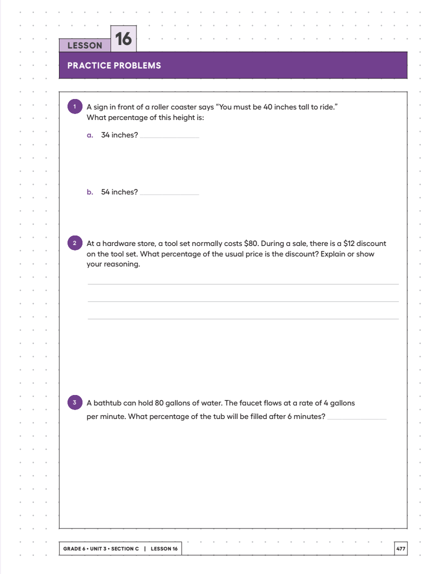
   - $\frac{65}{50} = \frac{130}{100}$ or 1.3. Multiplying 1.3 by 100 gives 130.
   - If 50 people is 100%, then 1 person is 2% of the school population and 65 people is $65 \cdot 2$ or 130% of the school population.

Lesson 16

Practice Problems

6 Problems

Student Workbook


LESSON 16

PRACTICE PROBLEMS

1 A sign in front of a roller coaster says "You must be 40 inches tall to ride." What percentage of this height is:

a. 34 inches?

b. 54 inches?

2 At a hardware store, a tool set normally costs $80. During a sale, there is a $12 discount on the tool set. What percentage of the usual price is the discount? Explain or show your reasoning.

3 A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percentage of the tub will be filled after 6 minutes?

Problem 1

A sign in front of a roller coaster says "You must be 40 inches tall to ride." What percentage of this height is:

a. 34 inches?

85%

b. 54 inches?

135%

Problem 2

At a hardware store, a tool set normally costs $80. During a sale, there is a $12 discount on the tool set. What percentage of the usual price is the discount? Explain or show your reasoning.

15%

Sample reasoning: $\frac{12}{80} = \frac{3}{20} = \frac{15}{100}$ and $\frac{15}{100} \cdot 100 = 15$.

Problem 3

A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percentage of the tub will be filled after 6 minutes?

30%

Sample reasoning: The tub will hold 24 gallons after 6 minutes and 24 is 30% of 80.

Problem 4

from Unit 3, Lesson 15

The sale price of every item in a store is 85% of its usual price.

Here are the usual prices of some items. Calculate the sale price of each.

a. backpack: $30

$25.50

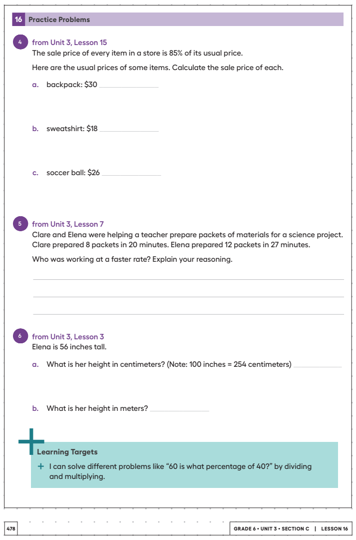
b. sweatshirt: $18

$15.30

c. soccer ball: $26

$22.10

Student Workbook



16 Practice Problems

4 from Unit 3, Lesson 15

The sale price of every item in a store is 85% of its usual price.

Here are the usual prices of some items. Calculate the sale price of each.

a. backpack: $30

b. sweatshirt: $18

c. soccer ball: $26

5 from Unit 3, Lesson 7

Clare and Elena were helping a teacher prepare packets of materials for a science project. Clare prepared 8 packets in 20 minutes. Elena prepared 12 packets in 27 minutes.

Who was working at a faster rate? Explain your reasoning.

6 from Unit 3, Lesson 3

Elena is 56 inches tall.

a. What is her height in centimeters? (Note: 100 inches = 254 centimeters)

b. What is her height in meters?

Learning Targets

+ I can solve different problems like “60 is what percentage of 40?” by dividing and multiplying.

478 GRADE 6 • UNIT 3 • SECTION C | LESSON 16

Problem 5

from Unit 3, Lesson 7

Clare and Elena were helping a teacher prepare packets of materials for a science project. Clare prepared 8 packets in 20 minutes. Elena prepared 12 packets in 27 minutes.

Who was working at a faster rate? Explain your reasoning.

Elena was working at a faster rate

Sample reasoning: $20 \div 8 = 2\frac{4}{8} = 2\frac{1}{2}$ and $27 \div 12 = 2\frac{3}{12} = 2\frac{1}{4}$. It took Clare $2\frac{1}{2}$ minutes and Elena $2\frac{1}{4}$ minutes to prepare 1 packet. Elena prepared each packet in less time than Clare did.

Problem 6

from Unit 3, Lesson 3

Elena is 56 inches tall.

a. What is her height in centimeters? (Note: 100 inches = 254 centimeters)

142.24 centimeters

b. What is her height in meters?

1.42 meters