Lesson 10

The Distributive Property, Part 2

Goals

- Generate algebraic expressions that represent the area of a rectangle with an unknown length.
- Justify (orally and using other representations) that algebraic expressions that are related by the distributive property are equivalent.

Learning Target

I can use a diagram of a split rectangle to write different expressions with variables representing its area.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR6: Three Reads (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- MLR6: Three Reads

Lesson Narrative

This lesson is to extend the work students did with the distributive property in a previous lesson to situations in which one of the quantities is represented by a variable, as in $2(x + 3) = 2x + 2 \cdot 3$. As they did before, students begin by using rectangular diagrams to represent these relationships, reinforcing the idea that the work they do with expressions with variables is an extension of the work they did with numbers.

Students see that the distributive property can arise when we represent the area of a rectangle in two different ways: as a product of its length and width or as the sum of the areas of two smaller rectangles that form the larger one. In making connections between expressions and quantities in a diagram, students engage in abstract and quantitative reasoning. This dual representation emphasizes the idea of equivalent expressions as two different ways of writing the same quantity.

Student Learning Goal

Let's use rectangles to understand the distributive property with variables.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	20 min	10 min	5 min

Warm-up
Possible Areas

5 min

Activity Narrative

Students reason about the area of a rectangle with a variable side length. They express the area for different values of the variable and when the value is unknown. Students review symbolic notation for showing multiplication as they express the product of a number and a variable. The reasoning here will be helpful later in the lesson when students apply the distributive property in the context of finding the areas of rectangles whose side lengths are expressions with variables.

Launch

Allow students 2–3 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

1. A rectangle has a length of 4 units and a width of m units. Write an expression for the area of this rectangle.

   $4m$ (or equivalent)

2. What is the area of the rectangle if m is:

   3 units?

   12 square units

   2.2 units?

   8.8 square units

   $\frac{1}{5}$ unit?

   $\frac{4}{5}$ square unit

3. Could the area of this rectangle be 11 square units? Explain your reasoning.

   Yes, the area could be 11 square units.

   Sample reasoning: m would have to be $\frac{11}{4}$ units, since $4 \cdot \frac{11}{4} = 11$.

Activity Synthesis

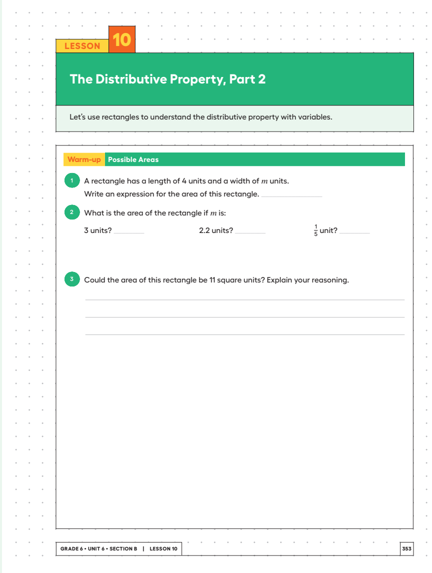
Select students to share their response to each question. Consider displaying a diagram of a rectangle and annotating it to illustrate students' responses or explanations. Highlight the following points:

- Rectangle areas can be found by multiplying length by width.
- Both $4m$ and $m \cdot 4$ are expressions for the area of this rectangle. These are equivalent expressions.
- Lengths don't have to be whole numbers. Neither do areas.

Building on Student Thinking

If students indicate they are not sure how to start and haven't drawn a diagram of a rectangle, suggest that they do so.

Student Workbook



Activity 1
Partitioned Rectangles When Lengths Are Unknown

10 min

Activity Narrative

In this activity, students use expressions with variables to represent lengths of sides and areas of rectangles. These expressions are used to help students understand the distributive property and its use in creating equivalent expressions.

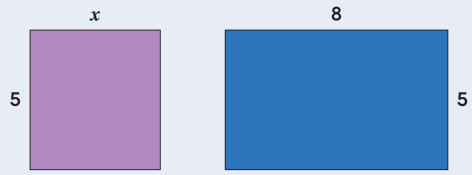
Launch

Arrange students in groups of 2–3.

Give students 3–4 minutes of group work time, followed by a quick whole-class discussion.

Student Task Statement

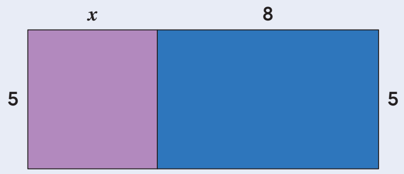
1. Here are two rectangles. The length and width of one rectangle are 8 and 5 units. The width of the other rectangle is 5 units, but its length is unknown so we labeled it x.



Write an expression for the sum of the areas of the two rectangles.

$5x + 40$ or $5x + 5 \cdot 8$

2. The two rectangles can be composed into one larger rectangle, as shown.



What are the length and width of the new, larger rectangle?

The length is $x + 8$ units and the width is 5 units (or vice versa).

3. Write an expression for the total area of the new, larger rectangle as the product of its width and its length.

$5(x + 8)$ or $(x + 8) \cdot 5$ (or equivalent)

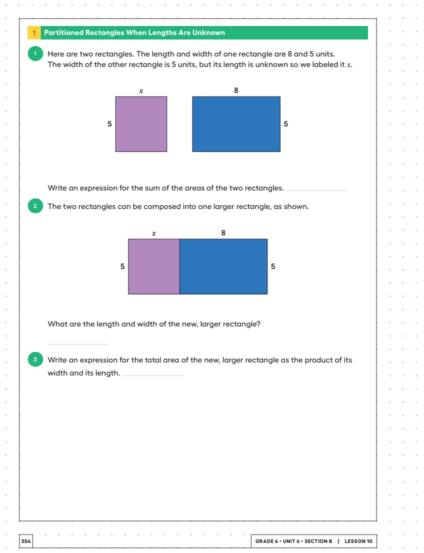
Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads
Keep books or devices closed. Display only the description of the two rectangles, without revealing the instructions to write expressions. Say,

"We are going to read this question 3 times."

- After the 1st read:
  "Tell your partner what this situation is about."
  The problem is about two rectangles with some side lengths given.
- After the 2nd read:
  "List the quantities. What can be counted or measured?"
  The width of both rectangles is 5. The length of one rectangle is 8, and the other rectangle's length is x.
- For the 3rd read: Reveal and read the instructions to write expressions. Ask,
  "What are some ways we might get started on this?"
  Advances: Reading, Representing

Student Workbook


1 Partitioned Rectangles When Lengths Are Unknown

1. Here are two rectangles. The length and width of one rectangle are 8 and 5 units. The width of the other rectangle is 5 units, but its length is unknown so we labeled it x.

Write an expression for the sum of the areas of the two rectangles. ____

2. The two rectangles can be composed into one larger rectangle, as shown.

What are the length and width of the new, larger rectangle?

____

3. Write an expression for the total area of the new, larger rectangle as the product of its width and its length. ____

Activity Synthesis

Solicit students' responses to the first and third questions. Display two of the expressions, as shown. (Expressions that are equivalent to these are fine.) Ensure that everyone agrees that one expression is an acceptable response to the first question and the other is an acceptable response to the third question.

$$5 \cdot x + 5 \cdot 8$$
$$5(x + 8)$$

Ask students to look at the two expressions and invite them to share something they notice and something they wonder. Here are some things that students might notice.

- The 5 appears twice in one expression and only once in the other.
- These look like an example of the distributive property, but with a variable.
- These expressions must be equivalent to each other, because they each represent the area of the same rectangle.

If no students mention the last point—that the expressions are equivalent—ask them to discuss this idea.

Activity 2
Areas of Partitioned Rectangles

20 min

Activity Narrative

In this activity, students are presented with several partitioned rectangles. They identify the length and width for each rectangle, and then write expressions for the area in two different ways, as:

- The product of the length and the width, one of which is expressed as a sum of partial lengths.
- The sum of the areas of the smaller rectangles that make up the large rectangle.

Students reason that these two expressions must be equal since they both represent the total area of the partitioned rectangle. In this way, students see several examples of the distributive property. Students may choose to assign values to the variable in each rectangle to check that their expressions for area are equal.

Launch

Keep students in groups of 2–3.

Give students 10 minutes of group work time, followed by a whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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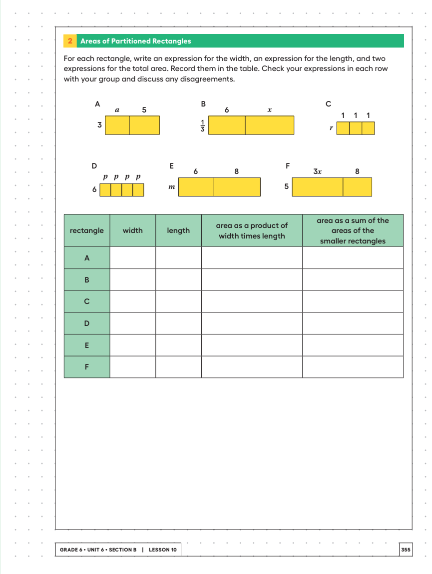
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Internalize Executive Functions.
To remind students how to write expressions for the length, width, and total area of a rectangle, begin with a small-group or whole-class demonstration and think-aloud that shows how to complete the first row of the table. Keep the worked-out calculations on display for students to reference as they work.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


2 Areas of Partitioned Rectangles

For each rectangle, write an expression for the width, an expression for the length, and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.

rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A				
B				
C				
D				
E				
F				

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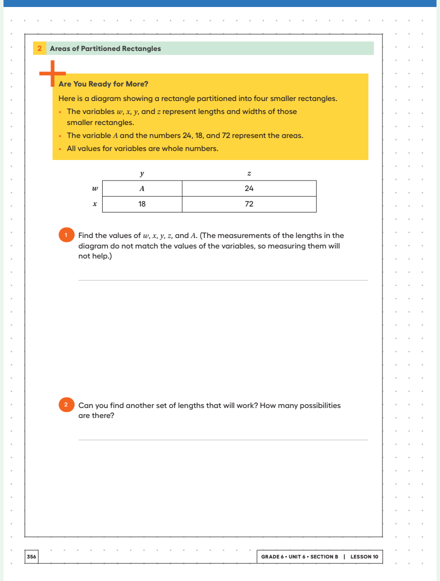
Student Workbook


2 Areas of Partitioned Rectangles

Are You Ready for More?

Here is a diagram showing a rectangle partitioned into four smaller rectangles.

- The variables w, x, y, and z represent lengths and widths of those smaller rectangles.
- The variable A and the numbers 24, 18, and 72 represent the areas.
- All values for variables are whole numbers.

1 Find the values of w, x, y, z, and A. (The measurements of the lengths in the diagram do not match the values of the variables, so measuring them will not help.)

2 Can you find another set of lengths that will work? How many possibilities are there?

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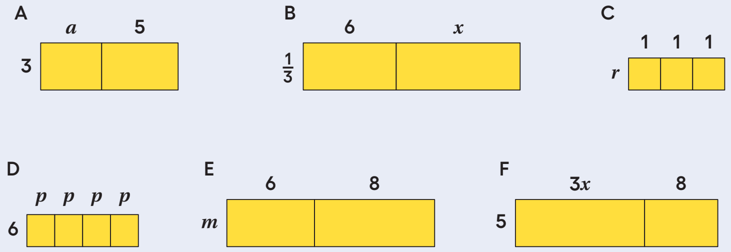
Student Task Statement

For each rectangle, write an expression for the width, an expression for the length, and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.



Width and length can be interchanged. Sample responses are shown for expressions. Accept all equivalent forms.

rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A	3	$a+5$	$3(a+5)$	$3a+15$ or $3a+3\cdot5$
B	$\frac{1}{3}$	$6+x$ or $x+6$	$\frac{1}{3}(6+x)$	$2+\frac{1}{3}x$
C	r	3 or $1+1+1$	$r(1+1+1)$ or $3r$	$r+r+r$ or $1r+1r+1r$
D	6	$p+p+p+p$ or $4p$	$6(p+p+p+p)$ or $6(4p)$ or $24p$	$6p+6p+6p+6p$
E	m	$6+8$ or 14	$m(6+8)$ or $14m$	$6m+8m$
F	5	$3x+8$	$5(3x+8)$	$15x+40$ or $5\cdot3x+5\cdot8$

Are You Ready for More?

Here is a diagram showing a rectangle partitioned into four smaller rectangles.

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- The variable A and the numbers 24, 18, and 72 represent the areas.
- All values for variables are whole numbers.

	y	z
w	A	24
x	18	72

1. Find the values of w, x, y, z, and A. (The measurements of the lengths in the diagram do not match the values of the variables, so measuring them will not help.)
2. Can you find another set of lengths that will work? How many possibilities are there?

   There are four solutions to this problem. The values given for w, x, y, and z are in units. The value of A is always 6 square units.

   - $w = 1, x = 3, y = 6, z = 24, A = 6$
   - $w = 2, x = 6, y = 3, z = 12, A = 6$
   - $w = 3, x = 9, y = 2, z = 8, A = 6$
   - $w = 6, x = 18, y = 1, z = 4, A = 6$

Activity Synthesis

Select students to share their expressions for the areas of Rectangles D and F. Ask them to explain how each expression relates to the diagram. Display and annotate the diagrams, if possible. Ask students:

- *"How do you know if the pair of expressions for the area of each rectangle are equivalent?"*

  They represent the area of the same figure. Applying the distributive property in one expression gives the other expression, so we know they are equivalent.

- *"Suppose we know the value of the variable p. What can you predict about the values of the two expressions?"*

  They would be equal.

- *"How would you test your prediction?"*

  Substitute the value for the variable and do the computation.

Display the expressions from the rows for Rectangles D and F:

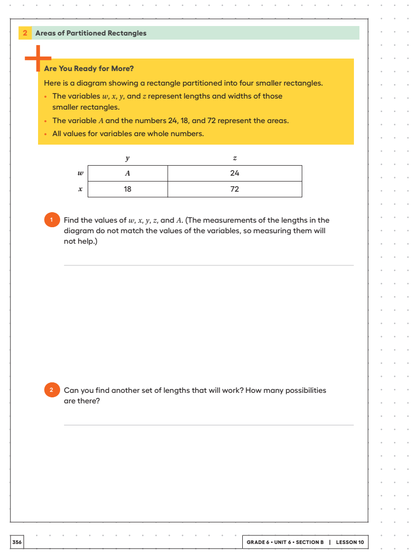
D	6	$p + p + p + p$	$6 \cdot (p + p + p + p)$	$6p + 6p + 6p + 6p$
F	5	$3x + 8$	$5 \cdot (3x + 8)$	$15x + 40$

Tell students that as they work with a greater variety of expressions, it is helpful to be able to refer to the parts in the expression (just as students learned to use "factors" and "product" to refer to the parts in a multiplication, and "dividend," "divisor," and "quotient" for division.)

Introduce the word "term" to students. Explain that a **term** is a part of an expression, separated from other terms by a + or − symbol. A term can be a single number, a single variable, or a product of numbers and variables. In the displayed expressions, 5, $3x$, p, and $6p$ are terms. Invite students to identify a few other terms in their completed table.

Add "term" to the display of other vocabulary words from the unit (or revise similar words or phrases students used previously to name the same concept).

Student Workbook



2 Areas of Partitioned Rectangles

Are You Ready for More?

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	y	z
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1. Find the values of w, x, y, z, and A. (The measurements of the lengths in the diagram do not match the values of the variables, so measuring them will not help.)

2. Can you find another set of lengths that will work? How many possibilities are there?

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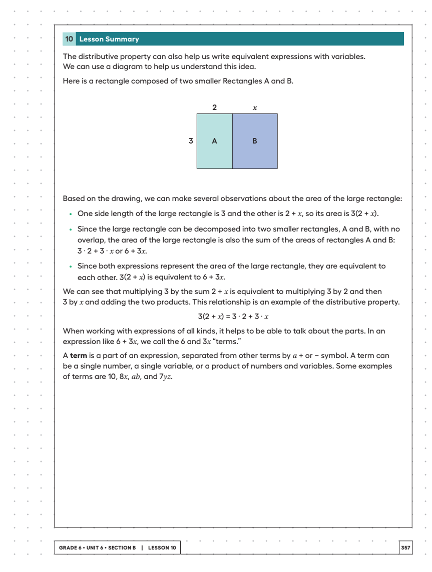
Student Workbook


10 Lesson Summary

The distributive property can also help us write equivalent expressions with variables. We can use a diagram to help us understand this idea.

Here is a rectangle composed of two smaller Rectangles A and B.

Based on the drawing, we can make several observations about the area of the large rectangle:

- One side length of the large rectangle is 3 and the other is 2 + x, so its area is 3(2 + x).
- Since the large rectangle can be decomposed into two smaller rectangles, A and B, with no overlap, the area of the large rectangle is also the sum of the areas of rectangles A and B: $3 \cdot 2 + 3 \cdot x$ or $6 + 3x$.
- Since both expressions represent the area of the large rectangle, they are equivalent to each other. $3(2 + x)$ is equivalent to $6 + 3x$.

We can see that multiplying 3 by the sum 2 + x is equivalent to multiplying 3 by 2 and then 3 by x and adding the two products. This relationship is an example of the distributive property.

$3(2 + x) = 3 \cdot 2 + 3 \cdot x$

When working with expressions of all kinds, it helps to be able to talk about the parts. In an expression like $6 + 3x$, we call the 6 and $3x$ "terms."

A **term** is a part of an expression, separated from other terms by a + or − symbol. A term can be a single number, a single variable, or a product of numbers and variables. Some examples of terms are 10, $8x$, ab, and $7yz$.

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Lesson Synthesis

Display a pair of equivalent algebraic expressions from this lesson (such as $3(a + 5)$ and $3a + 15$) and a pair of equivalent numerical expressions from a previous lesson (such as, $33(10 + 2)$ and $33 \cdot 10 + 33 \cdot 2$).

Ask students to compare the expressions. Discuss questions such as:

"How are they alike? How are they different?"

"How can you use a rectangular diagram to show that the distributive property applies to both expressions with numbers and those with variables?"

Students should see that their work with expressions containing variables is an extension of the work they did with expressions with numbers.

Lesson Summary

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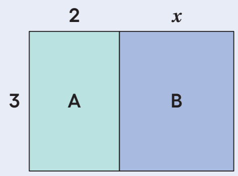
Here is a rectangle composed of two smaller Rectangles A and B.



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When working with expressions of all kinds, it helps to be able to talk about the parts. In an expression like $6 + 3x$, we call the 6 and $3x$ "terms."

A **term** is a part of an expression, separated from other terms by a + or − symbol. A term can be a single number, a single variable, or a product of numbers and variables. Some examples of terms are 10, $8x$, ab, and $7yz$.

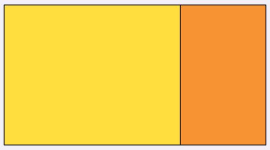
Cool-down

Which Expressions Represent Area?

5 min

Student Task Statement

Select **all** the expressions that represent the large rectangle's total area.



- $3(5 + b)$
- $5(b + 3)$
- $5b + 15$
- $15 + 5b$
- $3 \cdot 5 + 3b$

Responding To Student Thinking

Press Pause
If most students struggle with using the distributive property to identify equivalent expressions that represent the area of a rectangle, make time to offer additional practice. For example, complete the optional lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

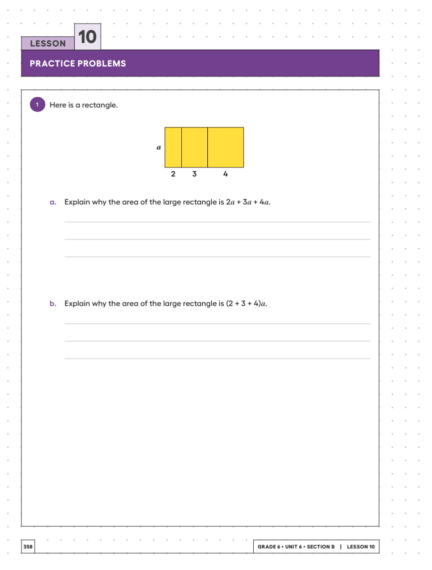
Grade 6, Unit 6, Lesson 11 The Distributive Property, Part 3

Lesson **10**

Practice Problems

7 Problems

Student Workbook


LESSON 10

PRACTICE PROBLEMS

1 Here is a rectangle.

a. Explain why the area of the large rectangle is $2a + 3a + 4a$.

b. Explain why the area of the large rectangle is $(2 + 3 + 4)a$.

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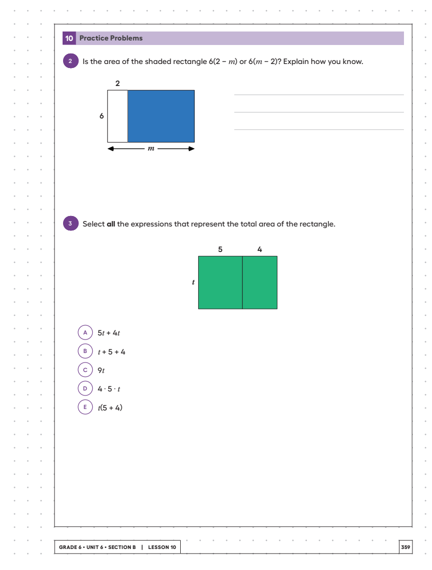
Student Workbook


10 Practice Problems

2 Is the area of the shaded rectangle $6(2 - m)$ or $6(m - 2)$? Explain how you know.

3 Select **all** the expressions that represent the total area of the rectangle.

A $5t + 4t$

B $t + 5 + 4$

C $9t$

D $4 \cdot 5 \cdot t$

E $t(5 + 4)$

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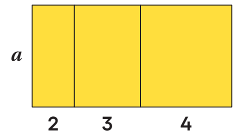
Problem 1

Here is a rectangle.



a. Explain why the area of the large rectangle is $2a + 3a + 4a$.

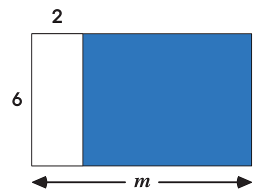
The large rectangle is made up of three smaller rectangles whose areas are $2a$, $3a$, and $4a$.

b. Explain why the area of the large rectangle is $(2 + 3 + 4)a$.

The large rectangle has width a and length $2 + 3 + 4$, so its area is $(2 + 3 + 4)a$.

Problem 2

Is the area of the shaded rectangle $6(2 - m)$ or $6(m - 2)$? Explain how you know.

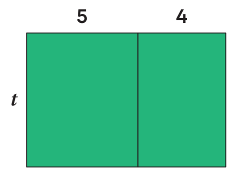


$6(m - 2)$

Sample reasoning: The width of the shaded rectangle is 6. The length is what is left over if 2 is removed from m, so $m - 2$. So, the area of the rectangle is $6(m - 2)$.

Problem 3

Select **all** the expressions that represent the total area of the rectangle.



A. $5t + 4t$

B. $t + 5 + 4$

C. $9t$

D. $4 \cdot 5 \cdot t$

E. $t(5 + 4)$

Problem 4

from Unit 6, Lesson 9

Use the distributive property to write an equivalent expression.

a. $35 \cdot 91 - 35 \cdot 89$

$35 \cdot (91 - 89)$ or $35 \cdot 2$ (or equivalent)

b. $22 \cdot 87 + 22 \cdot 13$

$22 \cdot (87 + 13)$ or $22 \cdot 100$ (or equivalent)

c. $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10}$

$\frac{9}{11}\left(\frac{7}{10} - \frac{3}{10}\right)$ or $\frac{9}{11} \cdot \frac{4}{10}$ (or equivalent)

Problem 5

from Unit 6, Lesson 8

Select **all** the expressions that are equivalent to $4b$.

A. $b + b + b + b$

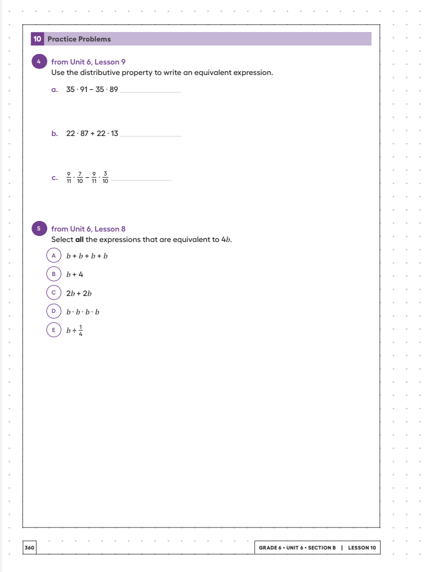
B. $b + 4$

C. $2b + 2b$

D. $b \cdot b \cdot b \cdot b$

E. $b \div \frac{1}{4}$

Student Workbook


10 Practice Problems

4 from Unit 6, Lesson 9

Use the distributive property to write an equivalent expression.

a. $35 \cdot 91 - 35 \cdot 89$

b. $22 \cdot 87 + 22 \cdot 13$

c. $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10}$

5 from Unit 6, Lesson 8

Select **all** the expressions that are equivalent to $4b$.

A $b + b + b + b$

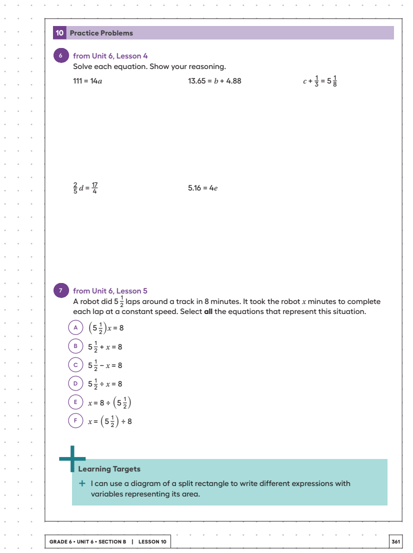
B $b + 4$

C $2b + 2b$

D $b \cdot b \cdot b \cdot b$

E $b \div \frac{1}{4}$

Student Workbook


10 Practice Problems

6 from Unit 6, Lesson 4

Solve each equation. Show your reasoning.

$111 = 14a$ $13.65 = b + 4.88$ $c + \frac{1}{3} = 5\frac{1}{8}$

$\frac{2}{5}d = \frac{17}{4}$ $5.16 = 4e$

7 from Unit 6, Lesson 5

A robot did $5\frac{1}{2}$ laps around a track in 8 minutes. It took the robot x minutes to complete each lap at a constant speed. Select **all** the equations that represent this situation.

A $\left(5\frac{1}{2}\right)x = 8$

B $5\frac{1}{2} + x = 8$

C $5\frac{1}{2} - x = 8$

D $5\frac{1}{2} \div x = 8$

E $x = 8 \div \left(5\frac{1}{2}\right)$

F $x = \left(5\frac{1}{2}\right) \div 8$

Learning Targets

+ I can use a diagram of a split rectangle to write different expressions with variables representing its area.

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Problem 6

from Unit 6, Lesson 4

Solve each equation. Show your reasoning.

$111 = 14a$

$a = \frac{111}{14}$ (or equivalent)

$13.65 = b + 4.88$

$b = 8.77$

$c + \frac{1}{3} = 5\frac{1}{8}$

$c = 4\frac{19}{24}$ (or equivalent)

$\frac{2}{5}d = \frac{17}{4}$

$d = \frac{85}{8}$ (or equivalent)

$5.16 = 4e$

$e = 1.29$ (or equivalent)

Problem 7

from Unit 6, Lesson 5

A robot did $5\frac{1}{2}$ laps around a track in 8 minutes. It took the robot x minutes to complete each lap at a constant speed. Select **all** the equations that represent this situation.

A. $\left(5\frac{1}{2}\right)x = 8$

B. $5\frac{1}{2} + x = 8$

C. $5\frac{1}{2} - x = 8$

D. $5\frac{1}{2} \div x = 8$

E. $x = 8 \div \left(5\frac{1}{2}\right)$

F. $x = \left(5\frac{1}{2}\right) \div 8$