Lesson 11

The Distributive Property, Part 3 (Optional)

Goals

- Explain (orally) how to use the distributive property to identify or generate equivalent algebraic expressions.
- Use the distributive property to write equivalent algebraic expressions, including where the common factor is a variable.

Learning Target

I can use the distributive property to write equivalent expressions with variables.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Warm-up)
- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Take Turns

Lesson Narrative

This lesson is optional because it includes opportunities to practice identifying and writing equivalent expressions that not all classes may need. Students look for structure in expressions to match equivalent expressions based on the distributive property. They also use the distributive property to generate equivalent sums or differences when given a product, and to generate an equivalent product when given a sum or difference (such as $4x + 7x$). The latter includes opportunities for students to look for a common factor that is not the variable (such as in $10x - 5$).

Student Learning Goal

Let's practice writing equivalent expressions by using the distributive property.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	15 min	15 min	10 min	5 min

Access for Multilingual Learners (Warm-up)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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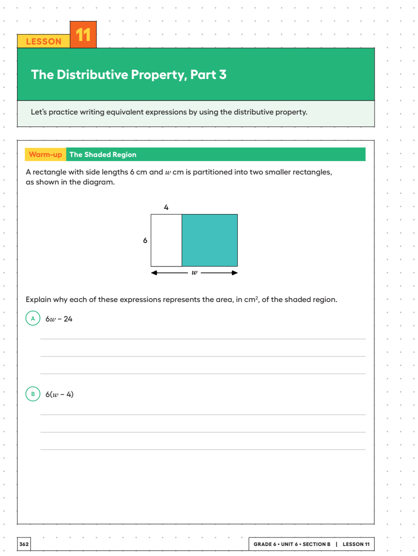
Student Workbook



Warm-up
The Shaded Region

10 min

Activity Narrative

In this activity, students are prompted to explain why two expressions with a variable represent the area of a shaded rectangle that is part of a larger rectangle of unknown width.

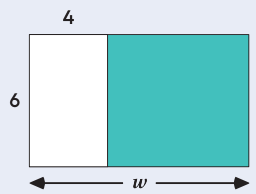
In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language they use to describe their thinking.

Launch

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

A rectangle with side lengths 6 cm and w cm is partitioned into two smaller rectangles, as shown in the diagram.



Explain why each of these expressions represents the area, in cm^2, of the shaded region.

A. $6w - 24$

Sample response:

The area, in cm^2, of the entire rectangle is $6w$. The area of the unshaded rectangle is $6 \cdot 4$ or 24 cm^2. Subtracting the area of the unshaded rectangle from the area of the entire rectangle, $6w - 24$, gives the area of the shaded rectangle.

B. $6(w - 4)$

Sample response:

The length of the shaded rectangle is $w - 4$ cm. Its width is 6 cm, so its area, in cm^2, is $6(w - 4)$.

Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the *Warm-up*. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

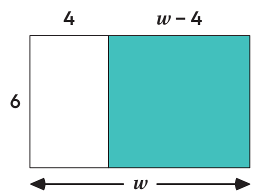
"_________ makes sense, but what do you mean when you say …?"

"Can you describe that another way?"

"How do you know …? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

After *Stronger and Clearer Each Time*, invite students to share their second draft explanation for $6(w - 4)$. Highlight the ways students connect the terms 6 and $w - 4$ to the length and width of the shaded rectangle.



Activity 1: Optional
Matching to Practice Distributive Property

15 min

Activity Narrative

In this partner activity, students take turns matching expressions that are equivalent because of the distributive property. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Arrange students in groups of 2. Display the task for all to see. Tell students that for each expression in Column A, there is an equivalent expression in Column B. If time allows, choose a student to be your partner and demonstrate how to set up and do the activity, otherwise share these steps:

- One partner picks an expression from column A.
- They identify an equivalent expression in column B and explain why they think it is equivalent.
- The other partner listens and makes sure they agree with the match and the reasoning.
- If they don't agree, the partners discuss until they come to an agreement.
- For the next expression in column A, the students swap roles.

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Activate or supply background knowledge. Remind students that they can use rectangle diagrams to help them match expressions.

Supports accessibility for: Social-emotional skills, Conceptual Processing

Student Workbook

1 Matching to Practice Distributive Property

Match each expression in Column A to an equivalent expression in Column B. If you get stuck, consider drawing a diagram. Be prepared to explain your reasoning.

Column A	Column B
1 ____ $a(1 + 2 + 3)$	$3(4a + b)$
2 ____ $2(12 - 4)$	$12 \cdot 2 - 4 \cdot 2$
3 ____ $12a + 3b$	$2(3a + 5b)$
4 ____ $\frac{2}{3}(15a - 18)$	$(2 + 3)a$
5 ____ $6a + 10b$	$a + 2a + 3a$
6 ____ $0.4(5 - 2.5a)$	$10a - 12$
7 ____ $2a + 3a$	$2 - a$

As students work, look for expressions that give students more and less of a challenge to discuss during the *Activity Synthesis*. Look for the ways students use diagrams to make sense of the expressions or to explain their reasoning. Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Student Task Statement

Match each expression in Column A to an equivalent expression in Column B. If you get stuck, consider drawing a diagram. Be prepared to explain your reasoning.

Column A	Column B
1. $a(1 + 2 + 3)$	3 $3(4a + b)$
2. $2(12 - 4)$	2 $12 \cdot 2 - 4 \cdot 2$
3. $12a + 3b$	5 $2(3a + 5b)$
4. $\frac{2}{3}(15a - 18)$	7 $(2 + 3)a$
5. $6a + 10b$	1 $a + 2a + 3a$
6. $0.4(5 - 2.5a)$	4 $10a - 12$
7. $2a + 3a$	6 $2 - a$

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their strategy for picking an expression in Column B and finding the match in Column B. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

Once all groups have completed the matching, discuss questions such as:

"Which matches were tricky? What makes them so?"

"Did you need to make adjustments in your matches? What adjustments were made?"

"If you made an error, what might have caused it?"

Activity 2: Optional
Writing Equivalent Expressions Using the Distributive Property

15 min

Activity Narrative

In this activity, students generate equivalent algebraic expressions using the distributive property. They practice writing a sum or difference when given a product, and a product when given a sum or difference. Although most expressions will be familiar to students from previous lessons, they also include some unfamiliar variants. For example, $(9-5)x$ has the expression in parentheses first, and products vary in whether the variable is the common factor or included in the expression in parentheses. The sums and differences include variables as the common factor and others include more than two terms.

Launch

Give students 10 minutes of quiet work time, followed by a whole-class discussion.

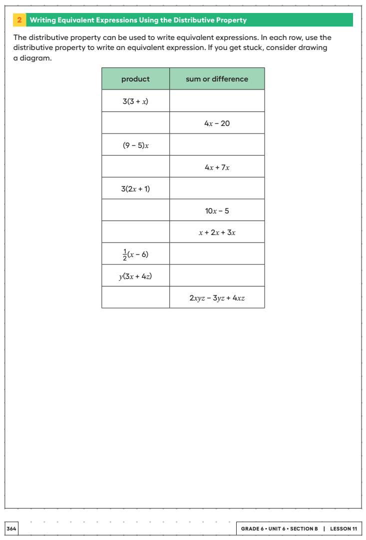
Student Task Statement

The distributive property can be used to write equivalent expressions. In each row, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

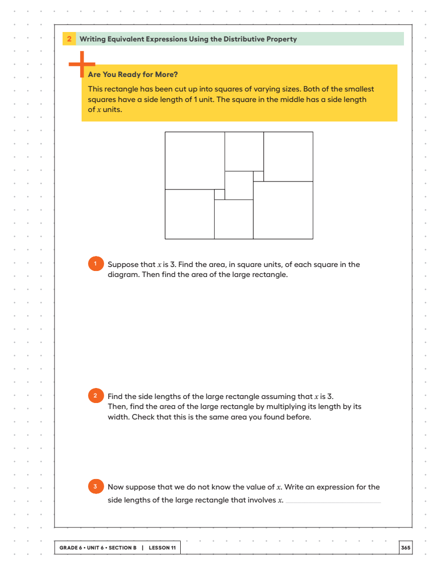
product	sum or difference
$3(3+x)$	$9+3x$
$4(x-5)$	$4x-20$
$(9-5)x$	$9x-5x$
$(4+7)x$	$4x+7x$
$3(2x+1)$	$6x+3$
$5(2x-1)$	$10x-5$
$x(1+2+3)$ or $6x$	$x+2x+3x$
$\frac{1}{2}(x-6)$	$\frac{1}{2}x-3$
$y(3x+4z)$	$3xy+4zy$
$z(2xy-3y+4x)$	$2xyz-3yz+4xz$

Sample responses are shown. Accept any order of factors or addends. Note that in cases where factoring happens, expressions equivalent to these are also acceptable. For example, for $4x-20$, equivalent expressions are $2(2x-10)$ and $20\left(\frac{1}{5}x-1\right)$ in addition to $4x-5$.

Student Workbook



Student Workbook


2 Writing Equivalent Expressions Using the Distributive Property

Are You Ready for More?

This rectangle has been cut up into squares of varying sizes. Both of the smallest squares have a side length of 1 unit. The square in the middle has a side length of x units.

1 Suppose that x is 3. Find the area, in square units, of each square in the diagram. Then find the area of the large rectangle.

2 Find the side lengths of the large rectangle assuming that x is 3. Then, find the area of the large rectangle by multiplying its length by its width. Check that this is the same area you found before.

3 Now suppose that we do not know the value of x. Write an expression for the side lengths of the large rectangle that involves x.

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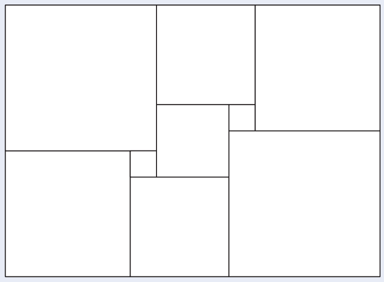
Are You Ready for More?

This rectangle has been cut up into squares of varying sizes. Both of the smallest squares have a side length of 1 unit. The square in the middle has a side length of x units.



1. Suppose that x is 3. Find the area, in square units, of each square in the diagram. Then find the area of the large rectangle.

   Answers are given in a sequence in which they may be derived:
   - Small squares: 1 square unit each
   - Center square: 9 square units
   - Top center: 16 square units
   - Top right: 25 square units
   - Bottom right: 36 square units
   - Bottom center: 16 square units
   - Bottom left: 25 square units
   - Top left: 36 square units

   The area of the large rectangle is the sum of all these areas: 165 square units.

2. Find the side lengths of the large rectangle assuming that x is 3. Then, find the area of the large rectangle by multiplying its length by its width. Check that this is the same area you found before.

   11 units by 15 units, $11 \cdot 15 = 165$, so the area is 165 square units.

3. Now suppose that we do not know the value of x. Write an expression for the side lengths of the large rectangle that involves x.

   Sample response: the width is $2x + (2x - 1)$, or $4x - 1$ units, and the length is $2x + (x + 1) + (2x - 1)$ or $5x$ units.

Activity Synthesis

Ask students to review the rows where they had to generate an equivalent expression for a given sum or difference. Invite students to explain how they decided what term to use as the common factor, and what terms to use in the expression inside the parentheses.

Highlight that for some expressions, there are multiple equivalent expressions that can be written. For example, the terms in the expression $4x - 20$ has 2 and 4 as common factors, so we can write $4(x - 5)$ and $2(2x - 10)$.

Lesson Synthesis

Invite students to summarize what they have learned about equivalent expressions in this section. Consider displaying some of the equivalent expressions from this lesson, especially those that students identified as more challenging. Encourage students to continue to use the vocabulary introduced in this unit, such as “coefficient,” “term,” “variable,” and “equivalent expressions.”

Consider asking some of the following questions:

- *“What does it mean for expressions to be equivalent?”*

  They are always equal. If they have variables, they are equal for all values of that variable.

- *“What are some of the ways you can show that two expressions with a variable are equivalent?”*

  Substitute different values for the variables. Use the distributive property or other properties of operations. Draw diagrams.

- *“Is using the distributive property to generate an equivalent expression for $4x + 7x$ more or less challenging than generating an equivalent expression for $4x - 20$? Why or why not?”*

  “Which diagrams have helped you make sense of and create equivalent expressions? How did you use them?”

Lesson Summary

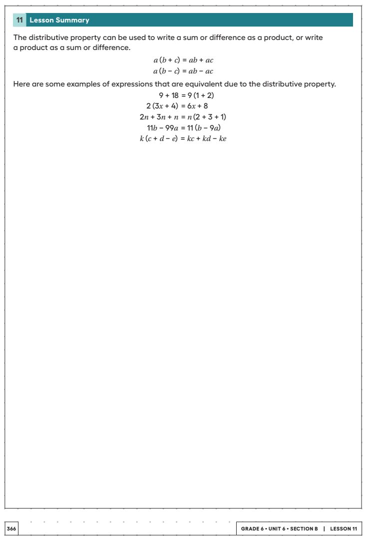
The distributive property can be used to write a sum or difference as a product, or write a product as a sum or difference.

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Here are some examples of expressions that are equivalent due to the distributive property.

$$9 + 18 = 9(1 + 2)$$
$$2(3x + 4) = 6x + 8$$
$$2n + 3n + n = n(2 + 3 + 1)$$
$$11b - 99a = 11(b - 9a)$$
$$k(c + d - e) = kc + kd - ke$$

Student Workbook


11 Lesson Summary

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$k(c + d - e) = kc + kd - ke$

Responding To Student Thinking

Points to Emphasize
If students struggle with determining if expressions are equivalent, focus on this idea when opportunities arise over the next several lessons. For example, consider inviting students to reflect on the reasoning behind these practice problems:

Grade 6, Unit 6, Lesson 12, Practice Problem 7

Grade 6, Unit 6, Lesson 13, Practice Problem 5

Cool-down
Writing Equivalent Expressions

5 min

Student Task Statement

Use the distributive property to write an expression that is equivalent to each expression. If you get stuck, consider drawing a diagram.

1. $(3r - 1)\,8$

   $24r - 8$

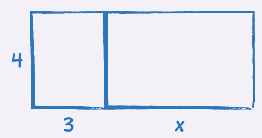
2. $p\,(6 + 2t + 5y)$

   $6p + 2pt + 10py$

3. $12 + 4x$

   Sample responses:

   - $4(3 + x)$
   - $2(6 + 2x)$
   - $\frac{1}{2}(24 + 8x)$

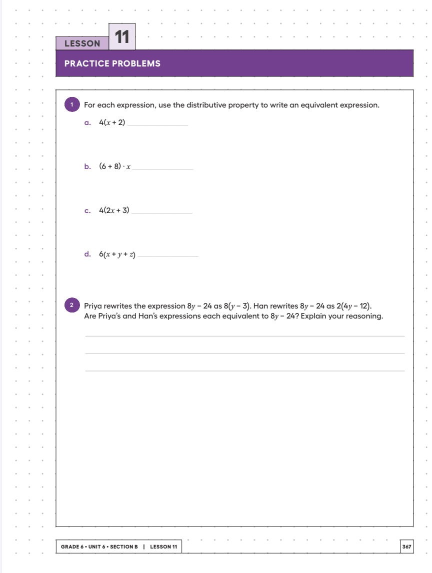


Lesson 11

Practice Problems

7 Problems

Student Workbook


LESSON 11

PRACTICE PROBLEMS

1 For each expression, use the distributive property to write an equivalent expression.

a. $4(x + 2)$

b. $(6 + 8) \cdot x$

c. $4(2x + 3)$

d. $6(x + y + z)$

2 Priya rewrites the expression $8y - 24$ as $8(y - 3)$. Han rewrites $8y - 24$ as $2(4y - 12)$. Are Priya's and Han's expressions each equivalent to $8y - 24$? Explain your reasoning.

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Problem 1

For each expression, use the distributive property to write an equivalent expression.

a. $4(x + 2)$

Sample response: $4x + 4 \cdot 2$

b. $(6 + 8) \cdot x$

Sample response: $6x + 8x$

c. $4(2x + 3)$

Sample response: $8x + 4 \cdot 3$

d. $6(x + y + z)$

Sample response: $6x + 6y + 6z$

Expressions that are equivalent to these are also acceptable, for example, $4x + 8$ for the first expression.

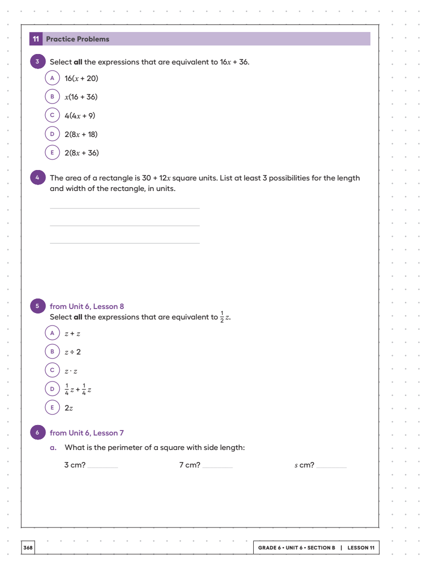
Problem 2

Priya rewrites the expression $8y - 24$ as $8(y - 3)$. Han rewrites $8y - 24$ as $2(4y - 12)$. Are Priya's and Han's expressions each equivalent to $8y - 24$? Explain your reasoning.

Yes

Sample reasoning: The distributive property shows that each expression is equivalent to $8y - 24$.

Student Workbook


11 Practice Problems

3 Select **all** the expressions that are equivalent to $16x + 36$.

- A $16(x + 20)$
- B $x(16 + 36)$
- C $4(4x + 9)$
- D $2(8x + 18)$
- E $2(8x + 36)$

4 The area of a rectangle is $30 + 12x$ square units. List at least 3 possibilities for the length and width of the rectangle, in units.

5 from Unit 6, Lesson 8
Select **all** the expressions that are equivalent to $\frac{1}{2}z$.

- A $z + z$
- B $z \div 2$
- C $z \cdot z$
- D $\frac{1}{4}z + \frac{1}{4}z$
- E $2z$

6 from Unit 6, Lesson 7
a. What is the perimeter of a square with side length:

3 cm? ______ 7 cm? ______ s cm? ______

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Problem 3

Select **all** the expressions that are equivalent to $16x + 36$.

A. $16(x + 20)$

B. $x(16 + 36)$

C. $4(4x + 9)$

D. $2(8x + 18)$

E. $2(8x + 36)$

Problem 4

The area of a rectangle is $30 + 12x$ square units. List at least 3 possibilities for the length and width of the rectangle, in units.

Sample responses:

length	width
$10 + 4x$	3
$5 + 2x$	6
$15 + 6x$	2
$60 + 24x$	$\frac{1}{2}$
$3 + 1.2x$	10

Problem 5

from Unit 6, Lesson 8

Select **all** the expressions that are equivalent to $\frac{1}{2}z$.

A. $z + z$

B. $z \div 2$

C. $z \cdot z$

D. $\frac{1}{4}z + \frac{1}{4}z$

E. $2z$

Problem 6

from Unit 6, Lesson 7

a. What is the perimeter of a square with side length:

3 cm?

12 cm ($3 \cdot 4 = 12$)

7 cm?

28 cm ($7 \cdot 4 = 28$)

s cm?

$4s$ cm (or equivalent)

b. If the perimeter of a square is 360 cm, what is its side length?

90 cm ($4s = 360$, $s = 360 \div 4 = 90$)

c. What is the area of a square with side length:

3 cm?

9 cm^2 ($3 \cdot 3 = 9$)

7 cm?

49 cm^2 ($7 \cdot 7 = 49$)

s cm?

$s \cdot s$ cm^2 (or equivalent)

d. If the area of a square is 121 cm^2, what is its side length?

11 cm ($s \cdot s = 121$, $11 \cdot 11 = 121$)

Problem 7

from Unit 6, Lesson 4

Solve each equation.

$10 = 4a$

$a = 2.5$ (or equivalent)

$5b = 17.5$

$b = 3.5$ (or equivalent)

$1.036 = 10c$

$c = 0.1036$

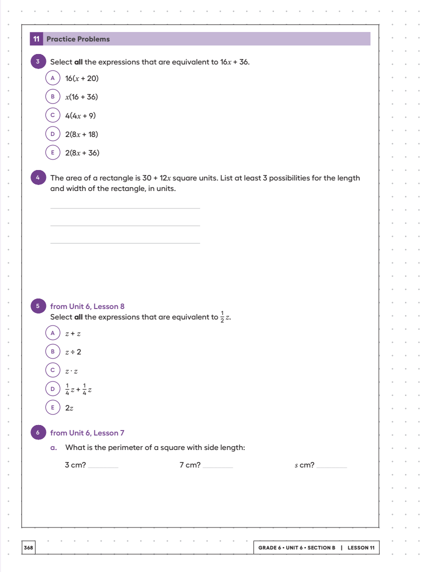
$0.6d = 1.8$

$d = 3$

$15 = 0.1e$

$e = 150$

Student Workbook


11 Practice Problems

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B. $x(16 + 36)$

C. $4(4x + 9)$

D. $2(8x + 18)$

E. $2(8x + 36)$

4 The area of a rectangle is $30 + 12x$ square units. List at least 3 possibilities for the length and width of the rectangle, in units.

5 from Unit 6, Lesson 8

Select **all** the expressions that are equivalent to $\frac{1}{2}z$.

A. $z + z$

B. $z \div 2$

C. $z \cdot z$

D. $\frac{1}{4}z + \frac{1}{4}z$

E. $2z$

6 from Unit 6, Lesson 7

a. What is the perimeter of a square with side length:

3 cm? ______ 7 cm? ______ s cm? ______

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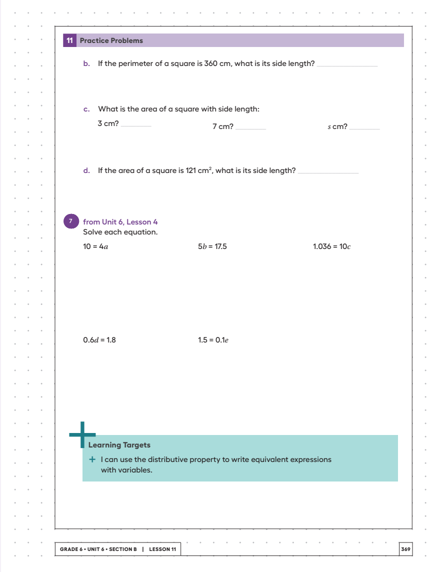
Student Workbook


11 Practice Problems

b. If the perimeter of a square is 360 cm, what is its side length? ______

c. What is the area of a square with side length:

3 cm? ______ 7 cm? ______ s cm? ______

d. If the area of a square is 121 cm^2, what is its side length? ______

7 from Unit 6, Lesson 4

Solve each equation.

$10 = 4a$ $5b = 17.5$ $1.036 = 10c$

$0.6d = 1.8$ $1.5 = 0.1e$

Learning Targets

+ I can use the distributive property to write equivalent expressions with variables.

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