Lesson 12

Using Mean and MAD to Make Comparisons

Goals

- Compare (orally and in writing) the means and mean absolute deviations of different distributions, specifically those with the same MAD but different means.
- Interpret the mean and mean absolute deviation (MAD) in the context of the data.

Learning Targets

- I can say what the MAD tells us in a given context.
- I can use means and MADs to compare groups.

Lesson Narrative

In this lesson, students continue to develop their understanding of the mean and MAD as measures of center and spread as well as interpret these values in context. They practice computing the mean and the MAD for distributions, compare distributions with the same MAD but different means, and interpret the mean and MAD in the context of the data.

Student Learning Goal

Let's use mean and MAD to describe and compare distributions.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Warm-up)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

- Math Talk
- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Student Task)

Representation: Internalize Comprehension.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Warm-up

Math Talk: Decimal Division

5 min

Activity Narrative

This *Math Talk* focuses on division of decimal values. It encourages students to think about how to divide mentally and to rely on patterns in the problems to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students calculate means.

To divide the values, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

- 42 ÷ 12 3.5

  Possible strategy: 36 ÷ 12 + 6 ÷ 12 = 3.5
- 2.4 ÷ 12 0.2

  Possible strategy: 2.4 · 10 ÷ 12 = 2, and 2 ÷ 10 = 0.2
- 44.4 ÷ 12 3.7

  Possible strategy: 42 ÷ 12 + 2.4 ÷ 12 = 3.7
- 46.8 ÷ 12 3.9

  Possible strategy: 42 ÷ 12 + 2.4 ÷ 12 + 2.4 ÷ 12 = 3.9

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Activity 1

Which Player Would You Choose?

20 min

Activity Narrative

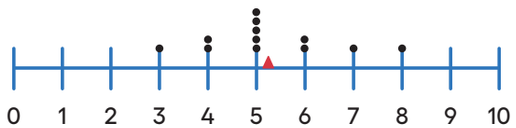
This activity allows students to practice calculating MAD and to build a better understanding of what it tells us. Students compare data sets with the same mean but different MADs and interpret what these differences imply in the context of the situation. During the discussion, they select a student to be on their team based on the comparison.

Expect students to choose different players to be on their team, but be sure they support their preferences with a reasonable explanation.

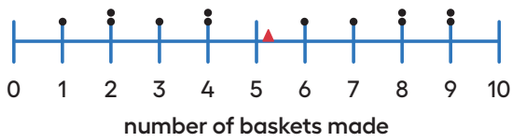
Launch

Arrange students in groups of 3–4. Before students read the task statements, display the two dot plots in the task for all to see. Give students up to 1 minute to study the dot plots and share with their group what they notice and wonder about the plots.

data set A



data set B



Next, select a few students to share what they notice and what they wonder. It is not necessary to confirm or correct students' observations or answer their questions at this point. If no one mentioned comparing the distributions, ask them to think about how they might do that. Explain to students that they will find more information in the task statement to help them compare and interpret the dot plots.

Give students 3–4 minutes of quiet work time to complete the first set of questions, and then 8–10 minutes to complete the second set with their group. Allow at least a few minutes for a whole-class discussion.

Instructional Routines

Notice and Wonder

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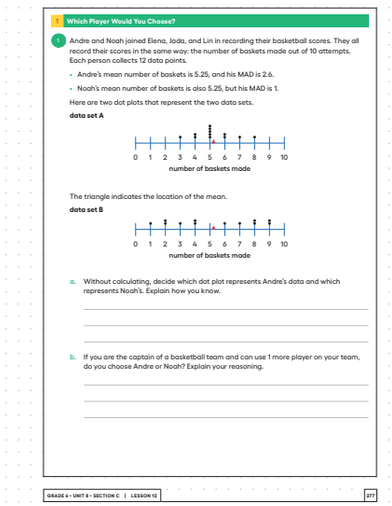
Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as a calculator or software to compute values for the MAD.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Student Workbook


1 Which Player Would You Choose?

1 Andre and Noah joined Elena, Jada, and Lin in recording their basketball scores. They all record their scores in the same way: the number of baskets made out of 10 attempts. Each person collects 12 data points.

- Andre's mean number of baskets is 5.25, and his MAD is 2.6.
- Noah's mean number of baskets is also 5.25, but his MAD is 1.

Here are two dot plots that represent the two data sets.

data set A

The triangle indicates the location of the mean.

data set B

a. Without calculating, decide which dot plot represents Andre's data and which represents Noah's. Explain how you know.

b. If you are the captain of a basketball team and can use 1 more player on your team, do you choose Andre or Noah? Explain your reasoning.

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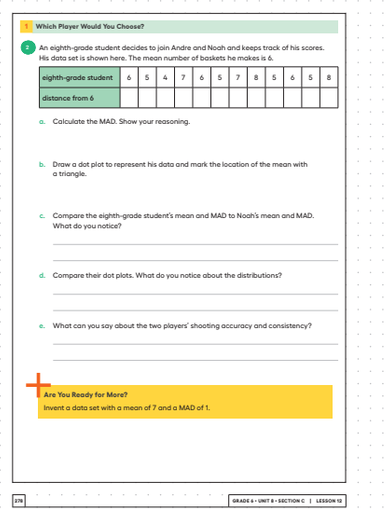
Student Workbook


1 Which Player Would You Choose?

2 An eighth-grade student decides to join Andre and Noah and keeps track of his scores. His data set is shown here. The mean number of baskets he makes is 6.

eighth-grade student	6	5	4	7	6	5	7	8	5	6	5	8
distance from 6												

a. Calculate the MAD. Show your reasoning.

b. Draw a dot plot to represent his data and mark the location of the mean with a triangle.

c. Compare the eighth-grade student's mean and MAD to Noah's mean and MAD. What do you notice?

d. Compare their dot plots. What do you notice about the distributions?

e. What can you say about the two players' shooting accuracy and consistency?

Are You Ready for More?
Invent a data set with a mean of 7 and a MAD of 1.

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Student Task Statement

1. Andre and Noah joined Elena, Jada, and Lin in recording their basketball scores. They all record their scores in the same way: the number of baskets made out of 10 attempts. Each person collects 12 data points.

- Andre's mean number of baskets is 5.25, and his MAD is 2.6.
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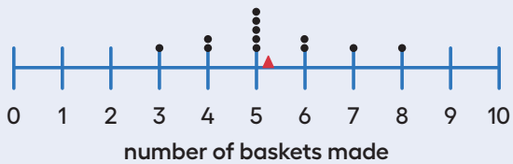
data set A



data set B

a. Without calculating, decide which dot plot represents Andre's data and which represents Noah's. Explain how you know.

Dot plot A represents Noah's data, and dot plot B represents Andre's data.

Sample explanation: I know because Andre's data has a larger MAD than Noah's, so the data are more spread out than Noah's.

b. If you are the captain of a basketball team and can use 1 more player on your team, do you choose Andre or Noah? Explain your reasoning.

Sample response: I would choose Noah. Both players make, on average, about 5 out of 10 baskets, but Noah is more consistent, so he's less likely to miss more than 6 out of 10 shots. Andre scores some high points a few times, but he also scores some very low ones.

2. An eighth-grade student decides to join Andre and Noah and keeps track of his scores. His data set is shown here. The mean number of baskets he makes is 6.

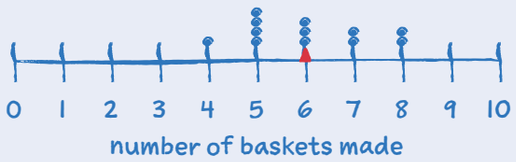
eighth-grade student	6	5	4	7	6	5	7	8	5	6	5	8
distance from 6	0	1	2	1	0	1	1	2	1	0	1	2

a. Calculate the MAD. Show your reasoning.

MAD: $\frac{0+1+2+1+0+1+1+2+1+0+1+2}{12} = 1$

b. Draw a dot plot to represent his data and mark the location of the mean with a triangle.

Dot plot for the eighth-grade student:



c. Compare the eighth-grade student's mean and MAD to Noah's mean and MAD. What do you notice?

Sample response: The eighth-grade student has a mean of 6, which is larger than Noah's mean of 5.25. Both Noah and the eighth-grade student have the same value for their MAD. Although their means are different, their data is similarly spread around their means.

d. Compare their dot plots. What do you notice about the distributions?

Sample response: Both dot plots show roughly the same spread. The points are clustered toward the middle and are within 2–3 units away from the mean.

e. What can you say about the two players' shooting accuracy and consistency?

Sample response: The eighth-grade student has a higher mean, so on average, he makes more baskets than Noah does. The same MAD value suggests that the two players are equally consistent. Each person typically scores within 1 basket of their mean, rather than scoring some very high scores and some very low ones.

Are You Ready for More?

Invent a data set with a mean of 7 and a MAD of 1.

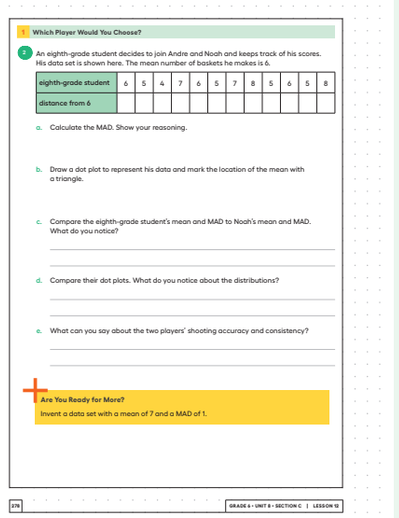
Sample response: Add 1 to each value of the eighth-grade student's baskets.

Activity Synthesis

Select a couple of students to share their responses to the first set of questions about how they matched the dot plots to the players and how they knew.

Then, display a completed table and the MAD for the second set of questions. Give students a moment to check their work. To facilitate discussion, help students connect MAD and the spread of data, and enable them to make a comparison. Consider displaying all three dot plots at the same scale and using a line segment to represent the MAD on each dot plot, as shown here.

Student Workbook



1 Which Player Would You Choose?

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distance from 6												

a. Calculate the MAD. Show your reasoning.

b. Draw a dot plot to represent his data and mark the location of the mean with a triangle.

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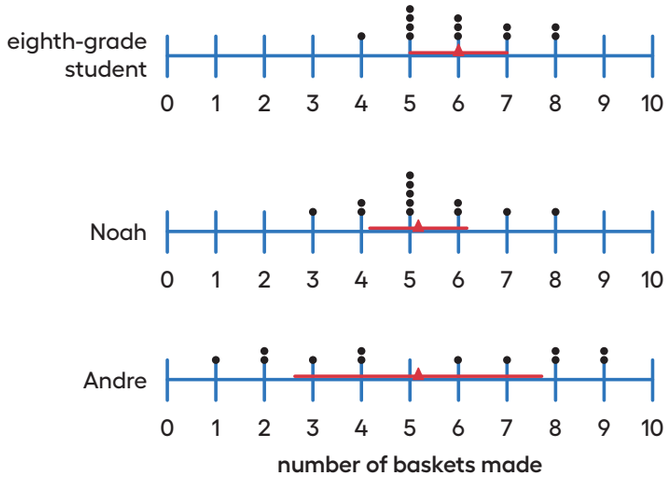
d. Compare their dot plots. What do you notice about the distributions?

e. What can you say about the two players' shooting accuracy and consistency?

Are You Ready for More?
Invent a data set with a mean of 7 and a MAD of 1.



Invite a few students to share their observations about how the means and MADs of Noah and the eighth-grade student compare. Discuss:

- *"How are the distributions of points related to the mean? How are they related to the MAD?"*

  The mean is usually near the center of the distribution and the MAD describes the spread of the distribution.

- *"Which might be more desirable for a basketball team: a lower mean or a higher mean number of baskets made? Why?"*

  A greater mean is preferable. It means that the person typically makes more baskets, which is usually good for a player on your team.

- *"Which might be more desirable: a lower MAD or a higher MAD? Why?"*

  Usually a lower MAD is better. It means that the player is more consistent, so you can more accurately estimate how the player will perform.

- *"Of the three students, which one would you want on your team? Why?"*

  I would want the eighth-grade student on my team because that player has the greatest mean and the least MAD. That player consistently scores about 6 out of 10 baskets.)Students should walk away understanding that, in this context, a higher MAD indicates more variability and less consistency in the number of shots made.

Activity 2

Swimmers Over the Years

10 min

Activity Narrative

In this activity, students continue to practice interpreting the mean and the MAD and to use them to answer statistical questions. A new context is introduced, but students should continue to consider both the center and variability of the distribution as ways of thinking about what is typical for a set of data and how consistent the data tends to be.

Launch

Give students 5–7 minutes of quiet work time. Ask students to consider drawing a triangle and a line segment on each dot plot in the last question to represent the mean and MAD for each data set (as was done in an earlier lesson).

Student Task Statement

The mean age of swimmers on a 1984 national swim team is 18.2 years and the MAD is 2.2 years. The mean age of the swimmers on the 2016 team is 22.8 years, and the MAD is 3 years.

1. How has the average age of the swimmers on the national team changed from 1984 to 2016? Explain your reasoning.

   Sample response: The average age of the team has increased by about 4.6 years over the past three decades, from 18.2 to 22.8 years old. The swimmers on the team in 2016 are older, on average, than those in 1984.

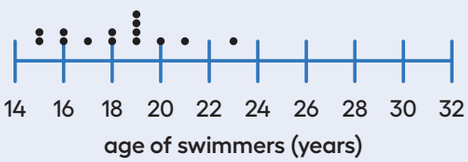
2. Are the swimmers on the 1984 team closer in age to one another than the swimmers on the 2016 team are to one another? Explain your reasoning.

   Yes

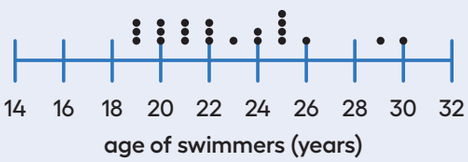
   Sample response: For the 2016 team, the average distance from the mean age is 3 years. For the 1984 team, the mean absolute deviation is 2.2 years, which means that the swimmers are closer to one another in age.

3. Here are dot plots showing the ages of the swimmers on the national swim teams in 1984 and in 2016. Use them to make two other comments about how the team has changed over the years.

   1984



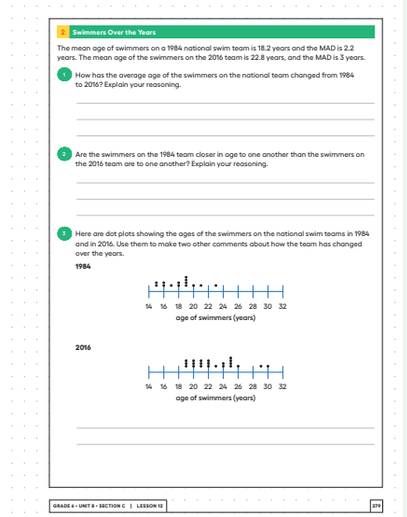
   2016



   Sample responses:

   - The 2016 team swimmers are much older than the swimmers were on the 1984 team. For the 2016 team, about half the swimmers are 22 or younger, while on the 1984 team, all but one swimmer is 22 or younger.
   - The youngest swimmers on the 2016 team are 19 years old, 4 years older than the youngest swimmers on the 1984 team.
   - The oldest swimmer on the 2016 team is 7 years older than the oldest swimmer on the 1984 team.

Student Workbook


Swimmers Over the Years

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1. How has the average age of the swimmers on the national team changed from 1984 to 2016? Explain your reasoning.

2. Are the swimmers on the 1984 team closer in age to one another than the swimmers on the 2016 team are to one another? Explain your reasoning.

3. Here are dot plots showing the ages of the swimmers on the national swim teams in 1984 and in 2016. Use them to make two other comments about how the team has changed over the years.

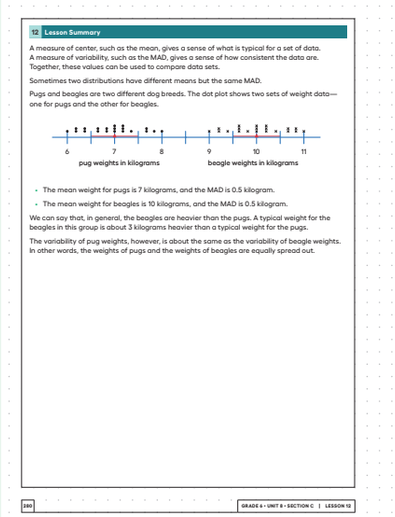
1984

14 16 18 20 22 24 26 28 30 32
age of swimmers (years)

2016

14 16 18 20 22 24 26 28 30 32
age of swimmers (years)

Student Workbook



12 Lesson Summary

A measure of center, such as the mean, gives a sense of what is typical for a set of data. A measure of variability, such as the MAD, gives a sense of how consistent the data are. Together, these values can be used to compare data sets.

Sometimes two distributions have different means but the same MAD.

Pugs and beagles are two different dog breeds. The dot plot shows two sets of weight data—one for pugs and the other for beagles.

6 7 8 9 10 11

pug weights in kilograms

beagle weights in kilograms

- The mean weight for pugs is 7 kilograms, and the MAD is 0.5 kilogram.
- The mean weight for beagles is 10 kilograms, and the MAD is 0.5 kilogram.

We can say that, in general, the beagles are heavier than the pugs. A typical weight for the beagles in this group is about 3 kilograms heavier than a typical weight for the pugs.

The variability of pug weights, however, is about the same as the variability of beagle weights. In other words, the weights of pugs and the weights of beagles are equally spread out.

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support whole-class discussion: "I know the average age changed _____ because ..." "Over the three decades, the _____ of the swimming team has changed by _____." "I know the swimmers' ages in the year _____ are closer to one another because ..."

Advances: Speaking, Listening

Activity Synthesis

Display the dot plots for all to see. Invite a student to add the means and MADs to the plots. Then invite several students to share their comparison of the distributions. Here are some discussion questions:

- *"What can you say about the size of the team? Has it changed?"*

  The team in 1986 had 14 swimmers and the team in 2016 has 22 swimmers, so there are a lot more swimmers on the later team. Maybe there are more specialty swimmers who swim in only one or two races rather than in several.

- *"Overall, has the team gotten older, younger, or stayed about the same? How do you know?"*

  Because the mean age is greater on the 2016 team, the typical swimmer is older on that team than the typical swimmer on the 1984 team was.

- *"Has the team become more diverse in ages, in general? Or have the swimmers become more alike in their age? How do you know?"*

  The swimmers on the 2016 team are more diverse in age because the MAD is greater.

Lesson Synthesis

The purpose of this discussion is to restate the importance of MAD in describing a distribution by contrasting it with the mean. Ask students,

- *"Suppose the mean height of the students in a class is 60 inches and the MAD is 2.5 inches. How do the mean and the MAD tell us about what is typical for the students' heights?"*

  It means that the typical student in that class is about 60 inches tall and most other students are, on average, 2.5 inches taller or shorter than the mean.

- *"How do two distributions compare if they have the same means but different MADs?"*

  Same center, different variability or spread.

- *"How do two distributions compare if they have the same MADs but different means?"*

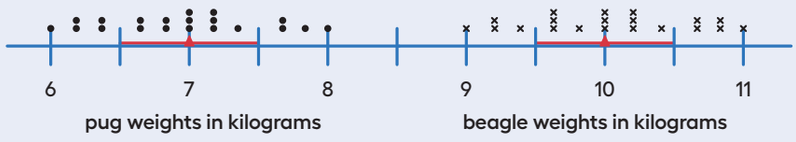
  Same variability or spread, different centers.

Lesson Summary

A measure of center, such as the mean, gives a sense of what is typical for a set of data. A measure of variability, such as the MAD, gives a sense of how consistent the data are. Together, these values can be used to compare data sets.

Sometimes two distributions have different means but the same MAD.

Pugs and beagles are two different dog breeds. The dot plot shows two sets of weight data—one for pugs and the other for beagles.



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We can say that, in general, the beagles are heavier than the pugs. A typical weight for the beagles in this group is about 3 kilograms heavier than a typical weight for the pugs.

The variability of pug weights, however, is about the same as the variability of beagle weights. In other words, the weights of pugs and the weights of beagles are equally spread out.

Cool-down

Travel Times Across the World

5 min

Student Task Statement

One hundred sixth-grade students in five different countries are asked about their travel times to school. Their responses are organized into five data sets. The mean and MAD of each data set is shown in the table.

	mean (minutes)	MAD (minutes)
United States	9	4.2
Australia	18.1	7.9
South Africa	23.5	16.2
Canada	11	8
New Zealand	12.3	5.5

1. Which group of students has the greatest variability in their travel times? Explain your reasoning.

   South Africa, because it has the largest MAD

2. Use the mean and MAD for Canada and New Zealand to compare travel times for those students.

   Sample response: The mean travel times are similar, so students in these countries typically take about the same amount of time to get to school. The MAD for New Zealand students is less than the MAD for Canadian students. This means that the travel time for students in New Zealand is more consistent than the travel times for students in Canada.

3. The data sets for Australia and Canada have very different means (18.1 and 11 minutes) but very similar MADs. What can you say about the travel times of the students in those two data sets?

   Sample response: On average, the students in Australia have a longer commute to school than students in Canada, but the travel times of students in both countries have the same variability. The data points are, on average, about 8 minutes from the mean.

Responding To Student Thinking

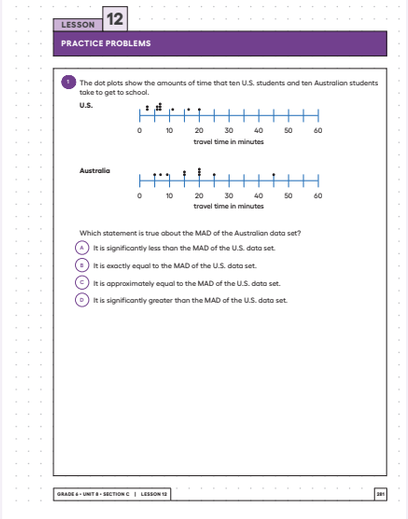
Points to Emphasize
If students struggle with appropriately using mean and MAD to discuss data sets, revisit mean and MAD in context in this activity: Unit 8, Lesson 13, Warm-up The Plot of the Story

Lesson 12

Practice Problems

6 Problems

Student Workbook


LESSON 12

PRACTICE PROBLEMS

1 The dot plots show the amounts of time that ten U.S. students and ten Australian students take to get to school.

U.S.

0 10 20 30 40 50 60

travel time in minutes

Australia

0 10 20 30 40 50 60

travel time in minutes

Which statement is true about the MAD of the Australian data set?

A It is significantly less than the MAD of the U.S. data set.

B It is exactly equal to the MAD of the U.S. data set.

C It is approximately equal to the MAD of the U.S. data set.

D It is significantly greater than the MAD of the U.S. data set.

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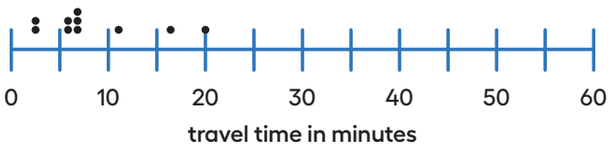
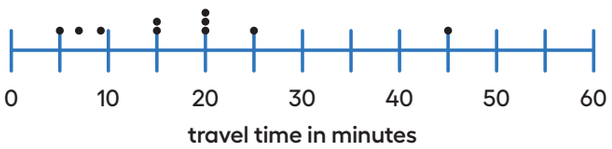
Problem 1

The dot plots show the amounts of time that ten U.S. students and ten Australian students take to get to school.

U.S.



Australia



Which statement is true about the MAD of the Australian data set?

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B. It is exactly equal to the MAD of the U.S. data set.

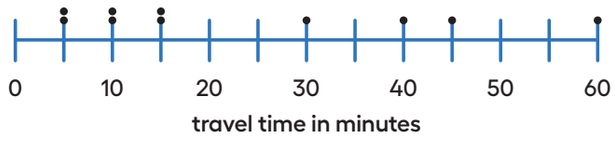
C. It is approximately equal to the MAD of the U.S. data set.

D. It is significantly greater than the MAD of the U.S. data set.

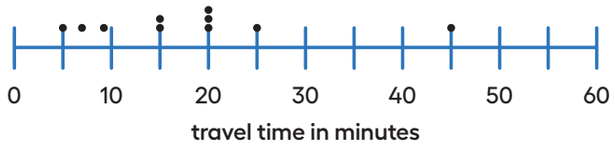
Problem 2

The dot plots show the amounts of time that ten South African students and ten Australian students took to get to school. Without calculating, answer the questions.

South Africa



Australia



a. Which data set has the smaller mean? Explain your reasoning.

Sample response: The mean of the Australian data set is smaller. The balance point of the Australian data set is less than 20 minutes. The balance point for the South African set is probably larger because four of the points are much larger than 20.

b. Which data set has the smaller MAD? Explain your reasoning.

Sample response: The MAD for the Australian data set is smaller. The data points are closer together and closer to the center of the distribution.

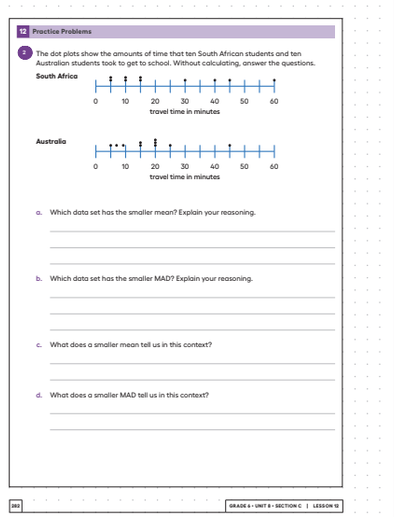
c. What does a smaller mean tell us in this context?

Sample response: A smaller mean tells us that the travel times of the students in the group are shorter overall.

d. What does a smaller MAD tell us in this context?

Sample response: A smaller MAD tells us that the travel times of the students in the group are closer to each other, closer to the mean, and more alike overall.

Student Workbook


12 Practice Problems

2 The dot plots show the amounts of time that ten South African students and ten Australian students took to get to school. Without calculating, answer the questions.

South Africa

0 10 20 30 40 50 60
travel time in minutes

Australia

0 10 20 30 40 50 60
travel time in minutes

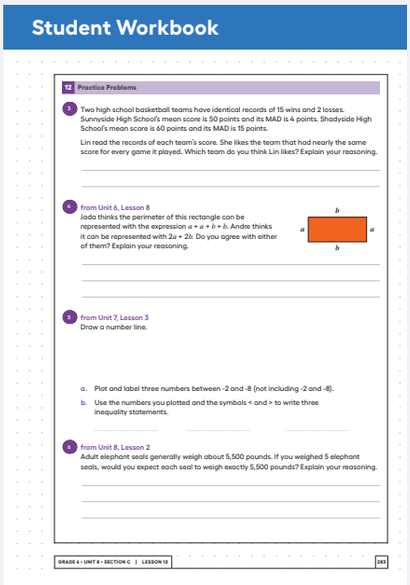
a. Which data set has the smaller mean? Explain your reasoning.

b. Which data set has the smaller MAD? Explain your reasoning.

c. What does a smaller mean tell us in this context?

d. What does a smaller MAD tell us in this context?

Student Workbook



12 Practice Problems

3 Two high school basketball teams have identical records of 15 wins and 2 losses. Sunnyside High School's mean score is 50 points and its MAD is 4 points. Shadyside High School's mean score is 60 points and its MAD is 15 points.

Lin read the records of each team's score. She likes the team that had nearly the same score for every game it played. Which team do you think Lin likes? Explain your reasoning.

4 from Unit 6, Lesson 8
Jada thinks the perimeter of this rectangle can be represented with the expression $a + a + b + b$. Andre thinks it can be represented with $2a + 2b$. Do you agree with either of them? Explain your reasoning.

5 from Unit 7, Lesson 3
Draw a number line.

a. Plot and label three numbers between -2 and -8 (not including -2 and -8).
b. Use the numbers you plotted and the symbols < and > to write three inequality statements.

6 from Unit 8, Lesson 2
Adult elephant seals generally weigh about 5,500 pounds. If you weighed 5 elephant seals, would you expect each seal to weigh exactly 5,500 pounds? Explain your reasoning.

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Problem 3

Two high school basketball teams have identical records of 15 wins and 2 losses. Sunnyside High School's mean score is 50 points and its MAD is 4 points. Shadyside High School's mean score is 60 points and its MAD is 15 points.

Lin read the records of each team's score. She likes the team that had nearly the same score for every game it played. Which team do you think Lin likes? Explain your reasoning.

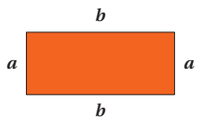
Sunnyside High School

Sample explanation: The smaller MAD indicates that most of the scores for the games are close to the mean of 50 points.

Problem 4

from Unit 6, Lesson 8

Jada thinks the perimeter of this rectangle can be represented with the expression $a + a + b + b$. Andre thinks it can be represented with $2a + 2b$. Do you agree with either of them? Explain your reasoning.



They are both correct

$2a + 2b$ and $a + a + b + b$ are equivalent expressions because $2a$ means $a + a$, and $2b$ means $b + b$.

Problem 5

from Unit 7, Lesson 3

Draw a number line.

a. Plot and label three numbers between -2 and -8 (not including -2 and -8).

Sample response: any three numbers between -2 and -8.

b. Use the numbers you plotted and the symbols < and > to write three inequality statements.

Sample response:

$-2 > -8$ $-2 > -7$ $-6 < -3$

Problem 6

from Unit 8, Lesson 2

Adult elephant seals generally weigh about 5,500 pounds. If you weighed 5 elephant seals, would you expect each seal to weigh exactly 5,500 pounds? Explain your reasoning.

No

Not every seal will weigh the same amount. There will be variability in the data.