Lesson 17

Using Box Plots

Goals

- Compare and contrast (orally and in writing) box plots that represent different data sets, including ones with the same median but very different IQRs and vice versa.
- Determine what information is needed to solve problems about comparing box plots. Ask questions to elicit that information.
- Interpret a box plot to answer (orally) statistical questions about a data set.

Learning Targets

- I can use a box plot to answer questions about a data set.
- I can use medians and IQRs to compare groups.

Lesson Narrative

In this lesson, students use box plots to make sense of data in context, compare distributions, and answer statistical questions about a situation.

Students compare box plots for distributions that have the same median but different IQRs, as well as box plots with the same IQRs but different medians. They recognize and articulate that the centers are the same but the spreads are different in the first case, and that the centers are different but the spreads are the same in the second case. They use this understanding to compare typical members of different groups in terms of the context of the problem.

Student Learning Goal

Let's use box plots to make comparisons.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

- Math Community Chart: Activity 1
- Straightedges: Activity 2

Materials To Copy

- Sea Turtles Cards (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:
Provide access to straightedges for drawing box plots. Consider creating a few paper planes of different sizes or styles to fly for the "Paper Planes" activity.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	20 min	20 min	10 min	5 min

Warm-up

Hours of Slumber

10 min

Activity Narrative

This *Warm-up* allows students to practice creating a box plot from a five-number summary and to think about the types of questions that can be answered using the box plot. To develop questions based on the box plot prompts students to put the numbers of the five-number summary into context.

As students work, identify a student who has clearly and correctly drawn the box plot. Ask that student to share during the whole-class discussion.

For the second question, some students may write decontextualized questions that are simply about parts of the box plot like "What is the IQR?" or "What is the range?" Others might write contextualized questions that the box plot could help to answer like "What is the least amount of sleep in this data set?" or "What is the median number of hours of sleep for this group?" Identify a few students from each group so that they can share later.

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time, followed by a whole-class discussion.

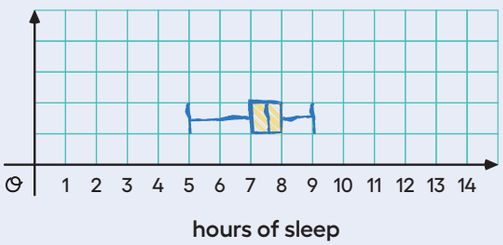
Select students with types of questions, such as those suggested in the activity narrative, to share later.

Student Task Statement

Ten sixth-grade students are asked how much sleep, in hours, they usually get on a school night. Here is the five-number summary of their responses.

- Minimum: 5 hours
- First quartile: 7 hours
- Median: 7.5 hours
- Third quartile: 8 hours
- Maximum: 9 hours

1. On the grid, draw a box plot for this five-number summary.

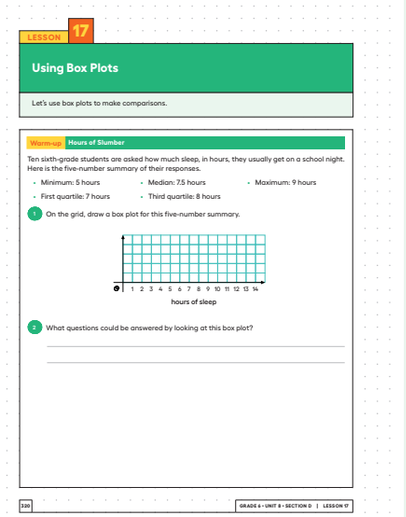


2. What questions could be answered by looking at this box plot?

Sample responses:

- What is the least amount of sleep a student usually gets on a school night?
- How many students usually get at least 7 hours of sleep on a school night?

Student Workbook


LESSON 17

Using Box Plots

Let's use box plots to make comparisons.

Warm-up Hours of Slumber

Ten sixth-grade students are asked how much sleep, in hours, they usually get on a school night. Here is the five-number summary of their responses.

- Minimum: 5 hours
- Median: 7.5 hours
- Maximum: 9 hours
- First quartile: 7 hours
- Third quartile: 8 hours

1. On the grid, draw a box plot for this five-number summary.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

hours of sleep

2. What questions could be answered by looking at this box plot?

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards. This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity Synthesis

Select the previously identified student with a correct box plot to display it for all to see. If that is not possible, ask the student to share how they drew the box plot, and record and display the drawing based on the student's directions.

Select other previously identified students to share questions that could be answered by looking at the box plot. First, ask the questions that can be answered without the context, and then ask the questions that rely on the context. Record and display these questions for all to see. After each question, ask the rest of the class if they agree or disagree that the answer can be found using the box plot. If time permits, ask students for the answer to each shared question.

Point out questions that are contextualized versus those that are not. Explain that a box plot can help us to make sense of a data set in context and can help answer questions about a group or a characteristic of a group in which we are interested. The different measures that we learned to identify or calculate help to make sense of a data distribution in context.

Activity 1

Info Gap: Sea Turtles

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to create and use box plots to get information about distributions of data.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information that they need to solve the problems. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and to ask increasingly more precise questions until they get the information that they need.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose, focus on, and practice a norm that they think will help themselves and their group during the activity. At the end of the activity, students can share what norms they choose and how the norms did or did not support their group.

Tell students that they will practice using box plots to answer questions about populations of sea turtles. Display, for all to see, the Info Gap graphic that illustrates a framework for the routine.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

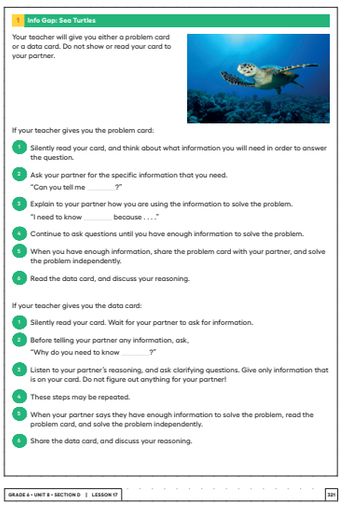
1. Silently read your card, and think about what information you will need in order to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. When you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the Info Gap graphic visible throughout the activity, or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Workbook



1 Info Gap: Sea Turtles

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you will need in order to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."
4. Continue to ask questions until you have enough information to solve the problem.
5. When you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. When your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

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If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. When your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

Problem Card 1: Box Plot 1 represents the data for the olive ridley sea turtles, and Box Plot 2 represents the data for the hawksbill.

Problem Card 2:

1. The green sea turtles have a heavier typical weight because their median weight is higher.
2. The loggerhead sea turtles show greater variability. Their IQRs are about the same (50 pounds for the green sea turtles and 45 for the loggerhead), but the range of weights for the loggerhead sea turtles is much larger (145 pounds, compared to 85 pounds for the green sea turtles).

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"What information was helpful to ask for when determining which box plot might match the species of sea turtle?"*

  Anything from the five-number summary is helpful for narrowing down the choices of box plot.

- *"When you have a box plot, what is the easiest way to describe a typical value or variability?"*

  From a box plot, the median is the easiest measure of center to find, so that can be used for typical values. The IQR or range are both easy to find from a box plot to describe variability.

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide a sentence frame to help students organize their thoughts in a clear, precise way:

- "I picked the norm '_____.' It really helped me/my group because _____."
- "I picked the norm '_____.' During the activity, I realized that the norm '_____' would be a better focus because _____."

Activity 2: Optional

Paper Planes

20 min

Activity Narrative

The optional lesson provides students with a chance to practice finding a five-number summary from data, drawing a box plot, and then comparing distributions.

As students work, make sure that they correctly identify the five-number summary of each data set. If students have trouble making comparisons, prompt them to study the medians, IQRs, and ranges of the data sets. Then, notice how they compare the box plots and whether they interpret the different measures in the context of the given situation. If they make comparisons only in abstract terms (for example, "The median for both data sets are the same"), push them to specify what the comparisons mean in this situation (for example, "What does the equal median tell us in this context?"). Identify students who made sense of these numbers in terms of typical distances and consistency of the flights of each person's plane. Ask them to share later.

Launch

Tell students that they will analyze data sets about flight distances of paper airplanes. To familiarize students with the context of this activity, consider preparing a few different styles or sizes of paper airplanes. Before students begin working, fly each paper plane a couple of times and ask students to observe their flight distances.

Arrange students in groups of 3–4. Provide access to straightedges. Give groups 8–10 minutes to complete the activity. Ask each group member to find the five-number summary and draw the box plot for one student (Andre, Lin, or Noah) and then to share their summaries and drawings. Ask them to pause and have their summaries and drawings reviewed before answering the last two questions. Consider posting somewhere in the classroom the five-number summaries and the box plots so that students can check their answers. Ask students to be prepared to explain how Andre, Lin, and Noah's flight distances are alike or different.

Student Task Statement

Andre, Lin, and Noah each design and build a paper airplane. They launch each plane several times and record the distance of each flight in yards.

Andre

25	26	27	27	27	28	28	28	29	30	30

Lin

20	20	21	24	26	28	28	29	29	30	32

Noah

13	14	15	18	19	20	21	23	23	24	25

Work with your group to summarize the data sets with numbers and box plots.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.
Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

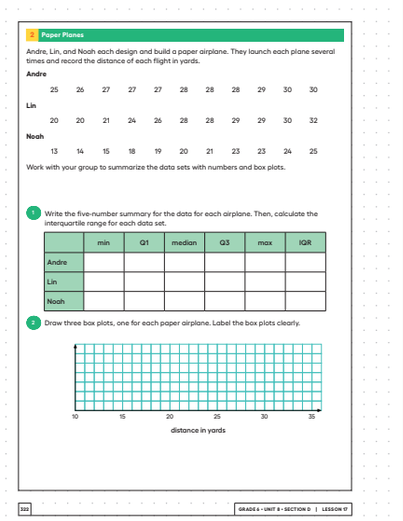
Supports accessibility for: Language

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.
Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Student Workbook


2 Paper Planes

Andre, Lin, and Noah each design and build a paper airplane. They launch each plane several times and record the distance of each flight in yards.

Andre
25 26 27 27 27 28 28 28 29 30 30

Lin
20 20 21 24 26 28 28 29 29 30 32

Noah
13 14 15 18 19 20 21 23 23 24 25

Work with your group to summarize the data sets with numbers and box plots.

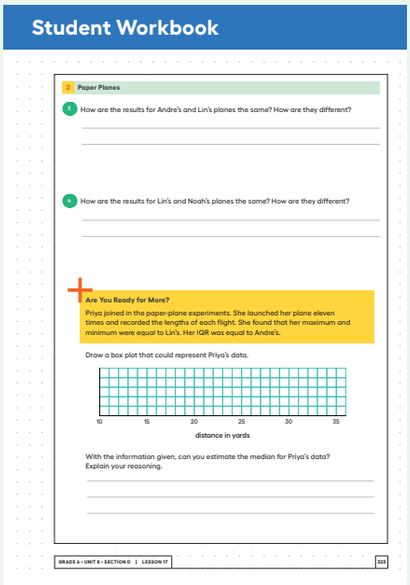
1. Write the five-number summary for the data for each airplane. Then, calculate the interquartile range for each data set.

	min	Q1	median	Q3	max	IQR
Andre						
Lin						
Noah						

2. Draw three box plots, one for each paper airplane. Label the box plots clearly.

10 15 20 25 30 35
distance in yards

Student Workbook


2 Paper Planes

3 How are the results for Andre's and Lin's planes the same? How are they different?

4 How are the results for Lin's and Noah's planes the same? How are they different?

Are You Ready for More?

Priya joined in the paper-plane experiments. She launched her plane eleven times and recorded the lengths of each flight. She found that her maximum and minimum were equal to Lin's. Her IQR was equal to Andre's.

Draw a box plot that could represent Priya's data.

10 15 20 25 30 35

distance in yards

With the information given, can you estimate the median for Priya's data? Explain your reasoning.

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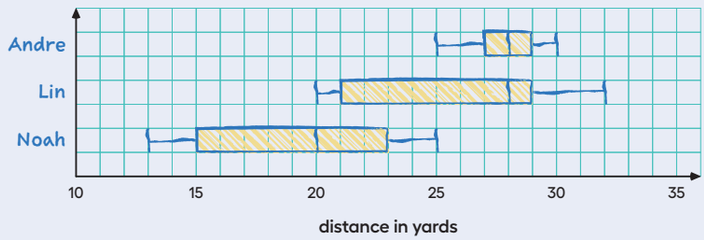
1. Write the five-number summary for the data for each airplane. Then, calculate the interquartile range for each data set.

	min	Q1	median	Q3	max	IQR
Andre	25	27	28	29	30	2
Lin	20	21	28	29	32	8
Noah	13	15	20	23	25	8

2. Draw three box plots, one for each paper airplane. Label the box plots clearly.



3. How are the results for Andre's and Lin's planes the same? How are they different?

Sample response: Andre's and Lin's results have the same median flight length of 28 yards. Andre's plane traveled more consistent distances than Lin's plane, which traveled more variable distances, as shown by her plane's larger IQR.

4. How are the results for Lin's and Noah's planes the same? How are they different?

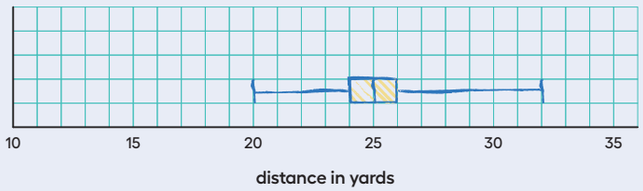
Sample response: Lin's and Noah's planes have the same IQR of 8 yards, meaning that their planes have a similar amount of variability in flight distance. Noah's median is much lower than Lin's, however, so generally speaking, his planes fly a much shorter distance.

Are You Ready for More?

Priya joined in the paper-plane experiments. She launched her plane eleven times and recorded the lengths of each flight. She found that her maximum and minimum were equal to Lin's. Her IQR was equal to Andre's.

Draw a box plot that could represent Priya's data.

Sample box plot:



With the information given, can you estimate the median for Priya's data? Explain your reasoning.

Sample response: No, I cannot estimate the median. I know the IQR, but I don't have enough information to tell where the first, second, or third quartiles are. The box could be anywhere between 20 and 32, and the median could be anywhere in that box.

Activity Synthesis

Focus the whole-class discussion on students' analyses and interpretations of the box plots. Display the box plots for all to see.

Select a few students or groups to share their responses comparing Andre's and Lin's data. Be sure to discuss what it means when two data sets have the same median but different IQRs, as in Andre's and Lin's cases. If no students connect these values to the center and spread and data, ask them to do so.

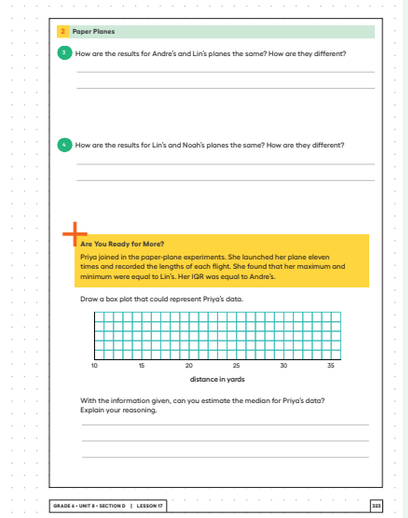
- *"What can you say about the center of Andre's data and that of Lin's data?"*

  They have the same median of 28 yards.

- *"What does the same center tell us in this context?"*

  The same number could be used to describe a typical flight distance for both Andre's and Lin's flight distances.

Student Workbook


2 Paper Planes

3. How are the results for Andre's and Lin's planes the same? How are they different?

4. How are the results for Lin's and Noah's planes the same? How are they different?

Are You Ready for More?

Priya joined in the paper-plane experiments. She launched her plane eleven times and recorded the lengths of each flight. She found that her maximum and minimum were equal to Lin's. Her IQR was equal to Andre's.

Draw a box plot that could represent Priya's data.

10 15 20 25 30 35

distance in yards

With the information given, can you estimate the median for Priya's data? Explain your reasoning.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

- *"What can you say about the spread of Andre's data and that of Lin's data?"*

  Andre's data are far more concentrated than Lin's, which are quite spread out. The range of her data is 12 yards, which tells us there is much more variability in her flight distances. For Andre the range is only 5 yards, less than half of Lin's, and his IQR is a quarter of Lin's. Overall, there is much less variability in his data.

- *"Which of the two planes—Andre's or Lin's—flies a more consistent distance? How do you know?"*

  Andre's, because the spread of his data is much smaller.

Then, select a few other students or groups to compare Lin's and Noah's data. Be sure to discuss what it means when two data sets have the same spread (IQR in this case) but different medians.

- *"What do the two very different centers tell us in this context?"*

  Generally speaking, a typical flight distance for Lin's plane is quite different from that for Noah's.

- *"What can you say about the spreads of Lin's and Noah's data?"*

  Both sets have the same range and the same IQR, though the values of the quartiles are different for the two sets of data.

- *"What does the same range tell us in this case?"*

  The difference between the shortest flight distance and the longest one is the same for both data sets.

- *"What does the same IQR tell us in this case?"*

  The middle half of the two sets of data cover the same distance.

- *"Whose plane—Lin's or Noah's—flies a more consistent distance?"*

  Their planes fly with very similar consistency, or inconsistency. The identical IQR and range tell us that their data have very similar variability.

Lesson Synthesis

In this lesson, we see that box plots can tell us stories about the center and spread of data sets.

- *"What are some questions you can ask to match box plots to data?"*

  Questions about the 3 quartiles, maximum, or minimum will help distinguish the different box plots.

- *"What does it mean when two box plots show the same median but different IQRs?"*

  The data they summarize have the same center but different spreads.

- *"How can we see this in a box plot?"*

  The lines inside the box will be at the same place, but the widths of the boxes will be different.

- *"What does 'the same median, different IQRs' mean in context?"*

  It means that we can use the same number to describe what is typical for each group, but the variability in the data is different.

"What does it mean when two box plots show the same IQR but different medians?"

The data they summarize have different centers but the same spread.

"How can we see this in a box plot?"

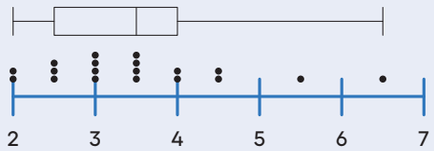
The Q2 segments in the boxes are located in different positions along the number line, but the boxes have the same width.

"What does 'the same IQR, different medians' tell us in context?"

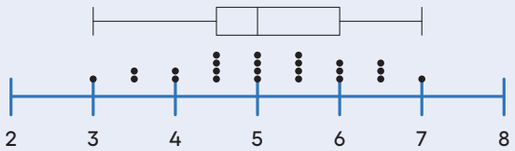
It means that what is typical for each group is different, but the variability is similar for the two groups.

Lesson Summary

Box plots are useful for comparing different groups. Here are two sets of plots that show the weights of some berries and some grapes.



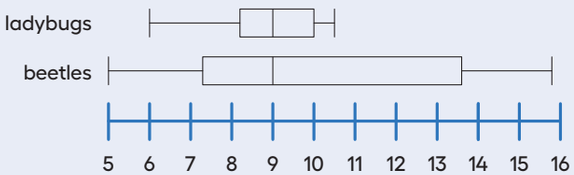
berry weights in grams



grape weights in grams

Notice that the median berry weight is 3.5 grams and the median grape weight is 5 grams. In both cases, the IQR is 1.5 grams. Because the grapes in this group have a higher median weight than the berries, we can say a grape in the group is typically heavier than a berry. Because both groups have the same IQR, we can say that they have a similar variability in their weights.

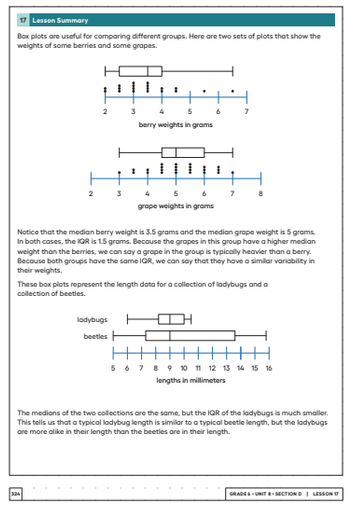
These box plots represent the length data for a collection of ladybugs and a collection of beetles.



lengths in millimeters

The medians of the two collections are the same, but the IQR of the ladybugs is much smaller. This tells us that a typical ladybug length is similar to a typical beetle length, but the ladybugs are more alike in their length than the beetles are in their length.

Student Workbook



17 Lesson Summary

Box plots are useful for comparing different groups. Here are two sets of plots that show the weights of some berries and some grapes.

2 3 4 5 6 7

berry weights in grams

2 3 4 5 6 7 8

grape weights in grams

Notice that the median berry weight is 3.5 grams and the median grape weight is 5 grams. In both cases, the IQR is 1.5 grams. Because the grapes in this group have a higher median weight than the berries, we can say a grape in the group is typically heavier than a berry. Because both groups have the same IQR, we can say that they have a similar variability in their weights.

These box plots represent the length data for a collection of ladybugs and a collection of beetles.

ladybugs
beetles
5 6 7 8 9 10 11 12 13 14 15 16

lengths in millimeters

The medians of the two collections are the same, but the IQR of the ladybugs is much smaller. This tells us that a typical ladybug length is similar to a typical beetle length, but the ladybugs are more alike in their length than the beetles are in their length.

Responding To Student Thinking

Press Pause
If students struggle to interpret box plots, leave time for students to revisit this *Cool-down* and use strategic pairings for the activity: Unit 8, Lesson 18, Warm-up Wild Bears

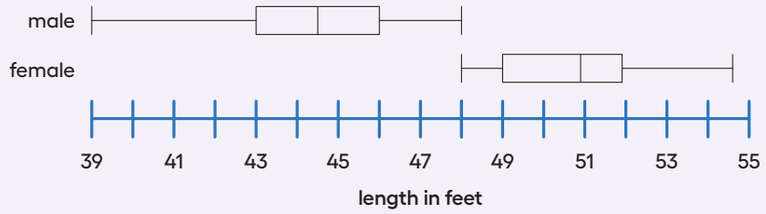
Cool-down

Humpback Whales

5 min

Student Task Statement

Researchers measures the lengths, in feet, of 20 male humpback whales and 20 female humpback whales. Here are two box plots that summarize their data.



1. How long is the longest whale measured? Is this whale male or female?

   The longest whale is about 55 feet long and is a female.

2. What is a typical length for the male humpback whales in this study?

   A typical male humpback whale is about 44.5 feet long.

3. Do you agree with each of these statements about the whales? Explain your reasoning.

   a. More than half of male humpback whales measured are longer than 46 feet.

      Disagree

      Sample explanation: The upper quartile of the data for the male humpbacks is 46 feet, which means a quarter of the whales are longer than 46 feet.

   b. The male humpback whales tend to be longer than female humpback whales.

      Disagree

      Sample explanation: The entire distribution for the lengths of female humpbacks is greater than that for male humpbacks, so female humpbacks tend to be longer than their male counterparts.

   c. The lengths of the male humpback whales tend to vary more than the lengths of the female humpback whales.

      Agree

      Sample explanation: The IQR of the data for male humpbacks is slightly greater than that for female humpbacks, and the range of the data for the males is larger than that for females, so the lengths of male humpbacks tend to vary more.

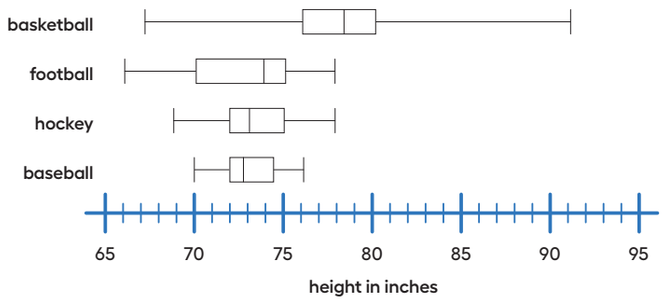
Lesson 17

Practice Problems

4 Problems

Problem 1

Here are box plots that summarize the heights of 20 professional athletes in basketball, football, hockey, and baseball.



a. In which two sports are the players' height distributions most alike? Explain your reasoning.

Hockey and baseball players are most alike.

Sample explanation: The two medians are very close (around 73 inches each), their IQRs differ by only about $\frac{1}{2}$ inch.

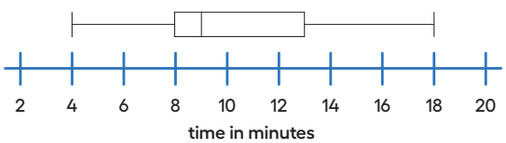
b. Which sport shows the greatest variability in players' heights? Which sport shows the least variability?

Overall, basketball players show the greatest variability in height (indicated by the largest range). Variability for the middle half of data is the greatest for football players (shown by the largest IQR). Baseball players show the least variability in height (shown by the smallest range and IQR).

Problem 2

Here is a box plot that summarizes data for the time, in minutes, that a fire department takes to respond to 100 emergency calls.

Select **all** the statements that are true, according to the box plot.



A. Most of the response times are under 13 minutes.

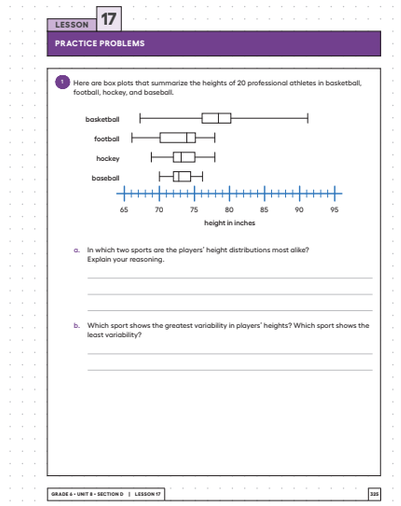
B. Fewer than 30 of the response times are over 13 minutes.

C. More than half of the response times are 11 minutes or greater.

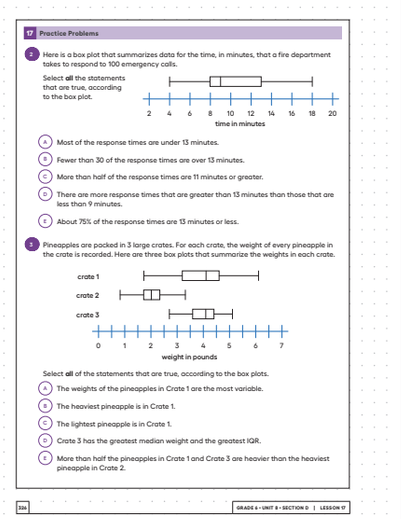
D. There are more response times that are greater than 13 minutes than those that are less than 9 minutes.

E. About 75% of the response times are 13 minutes or less.

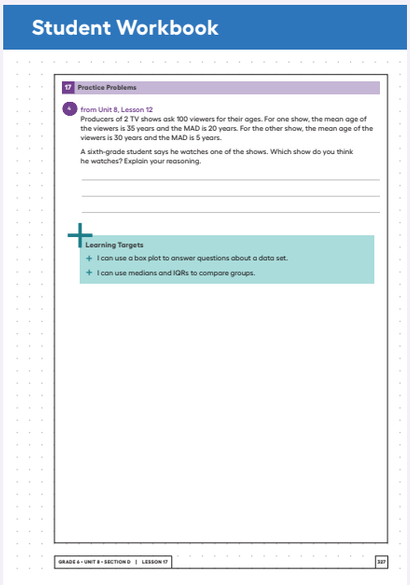
Student Workbook



Student Workbook



Student Workbook


17 Practice Problems

A from Unit 8, Lesson 12

Producers of 2 TV shows ask 100 viewers for their ages. For one show, the mean age of the viewers is 35 years and the MAD is 20 years. For the other show, the mean age of the viewers is 30 years and the MAD is 5 years.

A sixth-grade student says he watches one of the shows. Which show do you think he watches? Explain your reasoning.

Learning Targets

+ I can use a box plot to answer questions about a data set.
+ I can use medians and IQRs to compare groups.

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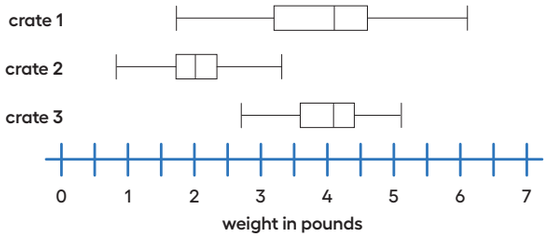
Problem 3

Pineapples are packed in 3 large crates. For each crate, the weight of every pineapple in the crate is recorded. Here are three box plots that summarize the weights in each crate.



Select **all** of the statements that are true, according to the box plots.

A. The weights of the pineapples in Crate 1 are the most variable.

B. The heaviest pineapple is in Crate 1.

C. The lightest pineapple is in Crate 1.

D. Crate 3 has the greatest median weight and the greatest IQR.

E. More than half the pineapples in Crate 1 and Crate 3 are heavier than the heaviest pineapple in Crate 2.

Problem 4

from Unit 8, Lesson 12

Producers of 2 TV shows ask 100 viewers for their ages. For one show, the mean age of the viewers is 35 years and the MAD is 20 years. For the other show, the mean age of the viewers is 30 years and the MAD is 5 years.

A sixth-grade student says he watches one of the shows. Which show do you think he watches? Explain your reasoning.

The first show

Sample reasoning: Even though the second show has a mean closer to the age of a sixth-grade student, the MAD indicates that most viewers are between 25 and 35. The much higher MAD of the first show means its viewers have a wider age range that is more likely to include the age of a sixth-grade student.