Lesson 13

# Tables and Double Number Line Diagrams

## Goals

- Compare and contrast (orally) double number line diagrams and tables representing the same situation.
- Draw and label a table of equivalent ratios from scratch to solve problems about constant speed.

## Learning Targets

- I can create a table that represents a set of equivalent ratios.
- I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios.
- I include column labels when I create a table, so that the meaning of the numbers is clear.

### Access for Students with Diverse Abilities

- Representation (Activity 2)

### Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

### Required Materials

**Materials to Copy**

- International Space Station Handout (1 copy for every 4 students): Activity 2

## Lesson Narrative

In this lesson, students explicitly connect and contrast double number lines and tables. Students consider the ways in which the two representations are alike (for instance, labels are needed and numbers that form equivalent ratios line up) and how they are different. Students examine the relationship between values in different rows in a table. Two features of tables make them more flexible than double number lines:

On a double number line, differences between values are represented by lengths. This feature can help support reasoning about relative sizes, but it can also be a limitation when large or small numbers are involved. A table removes this limitation because relative sizes of values are no longer represented by the geometry of a number line.

On a number line, values are listed in the order of their size. In a table, pairs of values can be written in any order, allowing for more nimble problem solving. Suppose 5 pounds of coffee cost $40. How much does 8.5 pounds cost? The table here shows that we can scale up and down as needed to find the cost of 8.5 pounds.

| weight of coffee (pounds) | cost (dollars) |
|---|---|
| 5 | 40 |
| 1 | 8 |
| 8.5 | 68 |

## Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Cool-down (Assessment) |
|---|---|---|---|---|
| 10 min | 10 min | 15 min | 10 min | 5 min |

Lesson 13

# Tables and Double Number Line Diagrams

## Lesson Narrative (continued)

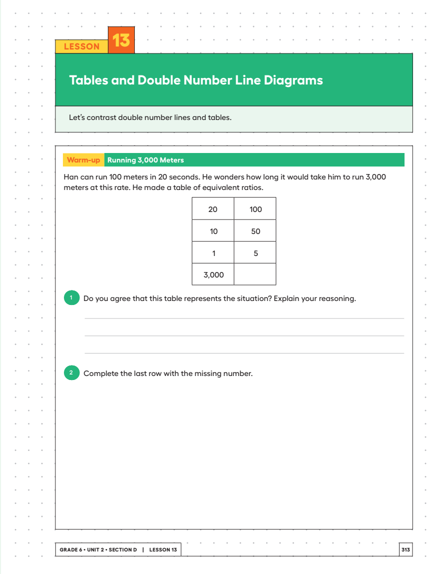
At this point in the unit, students should have a strong sense of what it means for two ratios to be equivalent, so they can fill in a table of equivalent ratios with understanding instead of just by following a procedure (such as dividing and multiplying the values in the table in a particular order). Students can also use other representations to support or check their reasoning, if needed.

As students analyze different representations and identify and explain correspondences between them, they practice making sense of problems.

## Student Learning Goal

Let's contrast double number lines and tables.

Student Workbook


LESSON 13

Tables and Double Number Line Diagrams

Let's contrast double number lines and tables.

Warm-up Running 3,000 Meters

Han can run 100 meters in 20 seconds. He wonders how long it would take him to run 3,000 meters at this rate. He made a table of equivalent ratios.

| | |
|---|---|
| 20 | 100 |
| 10 | 50 |
| 1 | 5 |
| 3,000 | |

1 Do you agree that this table represents the situation? Explain your reasoning.

2 Complete the last row with the missing number.

GRADE 6 • UNIT 2 • SECTION D | LESSON 13

## Warm-up

## Running 3,000 Meters

10 min

### Activity Narrative

In this activity, students examine a given table of equivalent ratios to reason about a situation involving distance and time. To answer questions, students need to interpret the quantities in the situation and the values in a table that is set up to conflict with what is described. The table has no headers, which makes it less obvious that the value 3,000 is in the wrong place.

The work here prompts students to reason quantitatively and abstractly and to attend to precision. In examining the table, students notice that labels or descriptions of the quantities are important when using a table of equivalent ratios to solve problems.

### Launch

Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time and 1–2 minutes to discuss with their partner.

### Student Task Statement

Han can run 100 meters in 20 seconds. He wonders how long it would take him to run 3,000 meters at this rate. He made a table of equivalent ratios.

| | |
|---|---|
| 20 | 100 |
| 10 | 50 |
| 1 | 5 |
| 3,000 | 15,000 |

1. Do you agree that this table represents the situation? Explain your reasoning.

   Sample response: I agree with the first three rows, but the last row would be for 3,000 seconds instead of 3,000 meters, so it wouldn't help Han answer the question.

2. Complete the last row with the missing number.

## Activity Synthesis

Invite students to share their response to the first question. Discuss how they knew that the values in the first three rows represent the times and distances of Han's run. Consider displaying the table and annotating it to illustrate students' thinking, especially multiplicative reasoning.

Next, discuss how students interpreted the 3,000 in the last row.

If students saw the 3,000 as time in seconds—matching the meaning of the other values in that column—and wrote 15,000 in the blank cell, discuss what the 15,000 represents. Then, ask students:

*"Does this value tell Han what he wanted to know? If not, what does it tell Han?"*

No. It tells Han how far he would go in 3,000 seconds.

If students thought of the 3,000 as the distance in meters and wrote 600 in the empty cell, discuss how they arrived at the 600 and what it represents. Then, ask students to consider how the 3,000 and 600 relate to the values in the preceding rows.

Make sure students see that the values in a column are meant to represent the same quantity. While Han would run 3,000 meters in 600 seconds, the 3,000 here represents time in seconds and the missing value is the distance in meters.

Finally, ask students:

*"What could Han do to improve the table?"*

He could label the quantity that each column represents.

## Activity 1

## Biking 3,000 Meters

10 min

### Activity Narrative

In this activity, students continue to use tables of equivalent ratios to solve problems but with decreasing scaffolding. For the first problem, there is an empty table students can use to reason about the quantities. In the second problem, no scaffolding is provided to allow students to create a table from scratch.

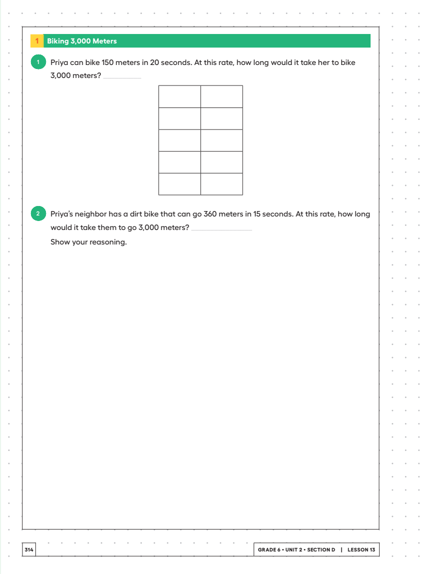
Monitor for the different ways students solve the problems, especially the last one. Select students who use different strategies to share during discussion later.

### Launch

Arrange students in groups of 2.

Give 3 minutes of quiet work time and then have students work with their partner.

**Student Workbook**




**Access for Multilingual Learners (Activity 1, Synthesis)**

**MLR8: Discussion Supports.**
For each method that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language. Consider selecting 1–2 students to share with the class and discussing words or phrases that might help to clarify the original statement or the revoiced statement.

*Advances: Listening, Speaking*

### Student Task Statement

1. Priya can bike 150 meters in 20 seconds. At this rate, how long would it take her to bike 3,000 meters?

   400 seconds

   Sample reasoning:

| | time (seconds) | distance (meters) | |
|---|---|---|---|
| | 20 | 150 | |
| · 2 | 40 | 300 | · 2 |
| · 10 | 400 | 3,000 | · 10 |

2. Priya's neighbor has a dirt bike that can go 360 meters in 15 seconds. At this rate, how long would it take them to go 3,000 meters? Show your reasoning.

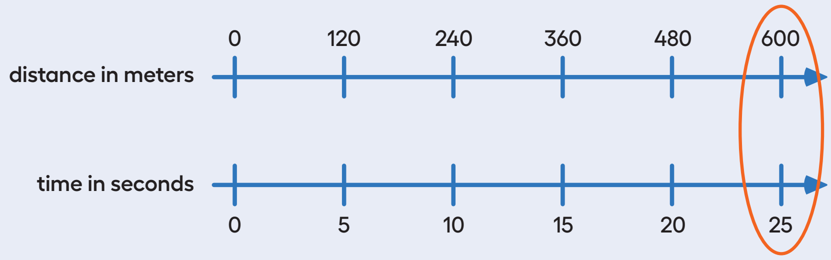
   125 seconds

   Sample reasoning:

| | distance (meters) | time (seconds) | |
|---|---|---|---|
| | 360 | 15 | |
| $\cdot \frac{1}{6}$ | 60 | 2.5 | $\cdot \frac{1}{6}$ |
| · 50 | 3,000 | 125 | · 50 |

| | distance (meters) | time (seconds) | |
|---|---|---|---|
| | 360 | 15 | |
| $\cdot \frac{1}{3}$ | 120 | 5 | $\cdot \frac{1}{3}$ |
| · 5 | 600 | 25 | · 5 |
| · 5 | 3,000 | 125 | · 5 |

- It takes 25 seconds to bike 600 meters. Since 3,000 is 5 times 600, it would take 5 times 25 or 125 seconds to go 3,000 meters.




### Activity Synthesis

Invite 1–2 previously selected students to display and briefly explain their solutions to each problem. Highlight the connections between different strategies, including different ways of using the same representation (such as different ways of reasoning with a table). If time permits, discuss connections across representations (such as between tables and double number line diagrams).

**Activity 2**

## The International Space Station

**15 min**

### Activity Narrative

This activity prompts students to compare and contrast two representations of equivalent ratios. Students work collaboratively to observe similarities and differences of using a double number line diagram and using a table to express the same situation. Here are some key distinctions:

| double number line | table |
|---|---|
| Distances between numbers and lengths of lines matter. | Distances and lengths do not matter because there are no lines. |
| The numbers on each line must be in order. | Rows of ratios can be out of order; within a column, numbers can go in any order that is convenient. |
| Each value of a ratio is shown on a line. | Each value of a ratio is shown in a column. |
| Pairs of values of a ratio are aligned vertically. | Pairs of values of a ratio appear in the same row. |

### Launch

To help students build some intuition about kilometers, begin by connecting it with contexts that are familiar to them. Tell students that "kilometer" is a unit used in the problem. Then ask a few guiding questions.

*"Can you name two things in our town (or city) that are about 1 kilometer apart?"*

Consider finding some examples of 1-kilometer distances near your school ahead of time.

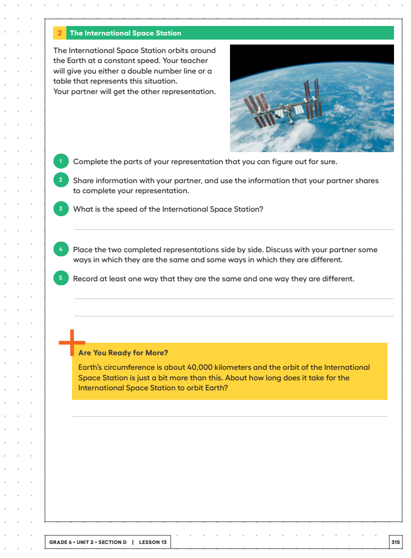
*"How long do you think it would take you to walk 1 kilometer?"*

Typical human walking speed is about 5 kilometers per hour, so it takes a person about 12 minutes to walk 1 kilometer.

*"What might be a typical speed limit on a highway, in kilometers per hour?"*

100 kilometers per hour is a typical highway speed limit.
Students might be more familiar with a speed limit such as 65 miles per hour. Since there are about 1.6 kilometers in every mile, the same speed will be expressed as a higher number in kilometers per hour than in miles per hour.

### Student Workbook


2 The International Space Station

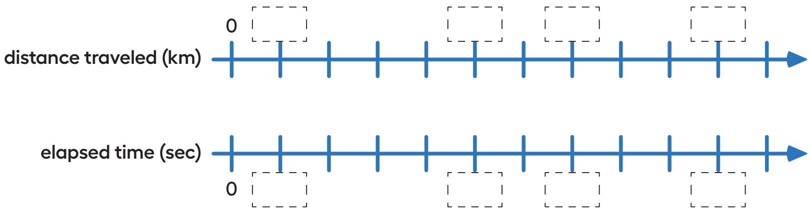
The International Space Station orbits around the Earth at a constant speed. Your teacher will give you either a double number line or a table that represents this situation. Your partner will get the other representation.

1. Complete the parts of your representation that you can figure out for sure.
2. Share information with your partner, and use the information that your partner shares to complete your representation.
3. What is the speed of the International Space Station?
4. Place the two completed representations side by side. Discuss with your partner some ways in which they are the same and some ways in which they are different.
5. Record at least one way that they are the same and one way they are different.

**Are You Ready for More?**
Earth's circumference is about 40,000 kilometers and the orbit of the International Space Station is just a bit more than this. About how long does it take for the International Space Station to orbit Earth?

Arrange students in groups of 2. Give one person a slip with the table and the other a slip with a double number line (shown below). Ask students to first do what they can independently, and then to obtain information from their partners to fill in all the blanks. Explain that when the blanks are filled, the two representations will show the same information.

| distance traveled (km) | elapsed time (sec) |
|---|---|
| 0 | 0 |
| 80 | 10 |
| | 1 |
| | |
| | |

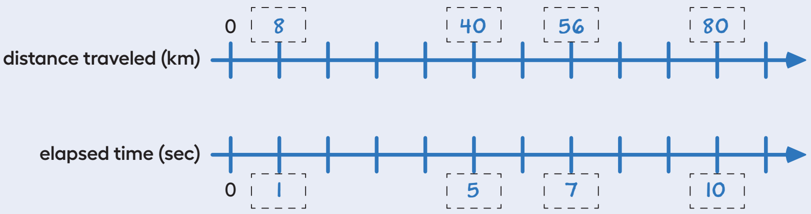



### Student Task Statement

The International Space Station orbits around the Earth at a constant speed. Your teacher will give you either a double number line or a table that represents this situation. Your partner will get the other representation.

**1.** Complete the parts of your representation that you can figure out for sure.

| distance traveled (km) | elapsed time (sec) |
|---|---|
| 0 | 0 |
| 80 | 10 |
| 8 | 1 |
| 40 | 5 |
| 56 | 7 |




See table and double number line.

**2.** Share information with your partner, and use the information that your partner shares to complete your representation.

See table and double number line.

**3.** What is the speed of the International Space Station?

The ISS is traveling in its orbit at a speed of 8 kilometers per second. We can see from both representations, which show the distance traveled in 1 second.

**4.** Place the two completed representations side by side. Discuss with your partner some ways in which they are the same and some ways in which they are different.

No written response required.

**5.** Record at least one way that they are the same and one way they are different.

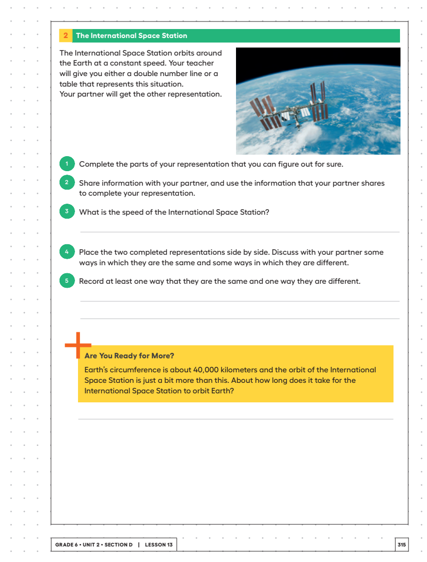
Sample response:

- Alike: Both representations show some equivalent ratios in the situation and have labels that show the quantities.
- Different: The table shows equivalent ratios in rows. The double number line diagram shows equivalent ratios as locations that line up vertically on the number lines.

**Building on Student Thinking**

Students with the double number line representation may decide to label every tick mark instead of just the ones indicated with dotted rectangles. This is fine. Make sure they understand that the tick marks with dotted rectangles are the ones they are supposed to record in the table.

**Student Workbook**



**Access for Students with Diverse Abilities (Activity 2, Synthesis)**

**Representation: Internalize Comprehension.**
Use color coding and annotations to highlight connections between representations in a problem. For example, to draw attention to common features, color code corresponding values in the table and double number line diagram. To highlight distinctions in the orientation of the representations and the order in which the values are listed, use arrows or other annotations.

*Supports accessibility for: Visual-Spatial Processing*

### Are You Ready for More?

Earth's circumference is about 40,000 kilometers and the orbit of the International Space Station is just a bit more than this. About how long does it take for the International Space Station to orbit Earth?

Between 80 and 100 minutes.

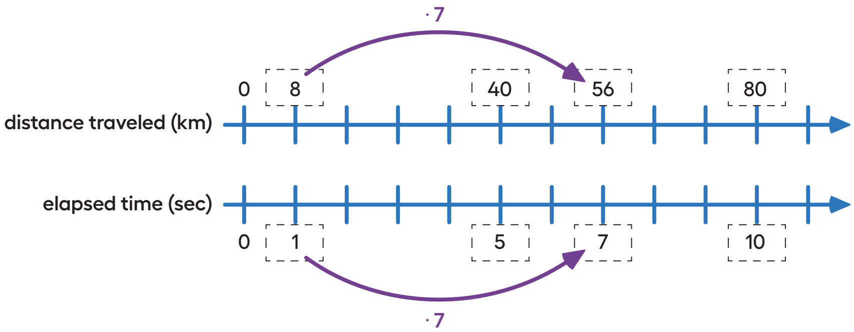
Sample reasoning: Earth's circumference is about 40,000 kilometers. The orbit of the International Space Station is longer than this, but not a lot longer. The orbit will take a little more than 40,000 ÷ 8 = 5,000 seconds. 80 minutes is 4,800 seconds, and 100 minutes is 6,000 seconds.

## Activity Synthesis

Display completed versions of both representations for all to see. Invite students to share the ways the representations are alike and different. Consider writing some of these on the board, or this could just be a verbal discussion. Highlight the distinctions in terms of distances between numbers, order of numbers, and the vertical or horizontal orientations of the representations.

Although it is not a structural distinction, students might describe the direction in which multiplying happens as a difference between the two representations. They might say that we "multiply up or down" to find equivalent ratios in a table, and we "multiply across" to do the same on a double number line. You could draw arrows to illustrate this fact:

| distance traveled (km) | elapsed time (sec) |
|---|---|
| 0 | 0 |
| 80 | 10 |
| 8 | 1 |
| 40 | 5 |
| 56 | 7 |




The vertical orientation of tables and the horizontal orientation of double number lines are conventions we decided to consistently use in these materials. Mathematically, there is nothing wrong with orienting each representation the other way. Students may encounter tables oriented horizontally in a later course. Later in this course, they will encounter number lines oriented vertically.

## Lesson Synthesis

Briefly revisit the two tasks, displaying the representations for all to see, and pointing out ways in which tables and double number lines are alike and different.

*"Why is it important to include descriptive column headers on tables?"*

Without headers, it's unclear which quantity each column represents, whether the values we put in the cells correctly represent the situation, or whether they show equivalent ratios.

*"What are the main differences between using a double number line and using a table in solving problems that involve equivalent ratios?"*

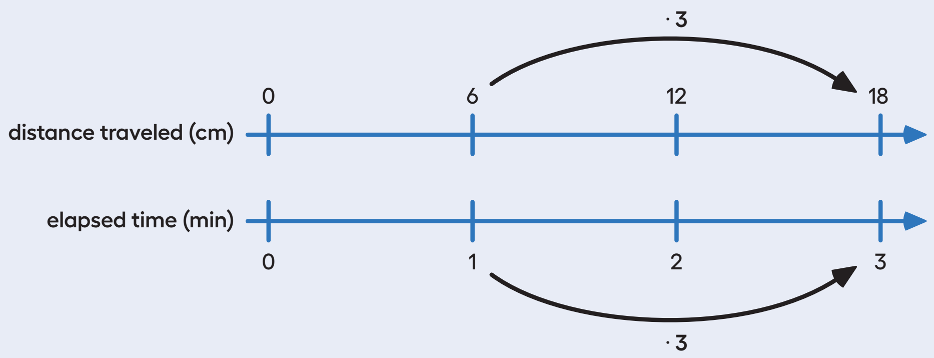
Tables can be created fairly quickly and there is flexibility in what numbers to use to help solve a problem. Double number lines may be more involved to draw, may need to be pretty long to accommodate the numbers, or may require careful thinking about the scale being used. The numbers need to be listed in order of size—from smaller to larger.

Emphasize that because of their flexibility, tables are sometimes easier to work with.

### Lesson Summary

On a double number line diagram, we put labels in front of each line to tell what the numbers represent. On a table, we put labels at the top of each column to tell what the numbers represent.

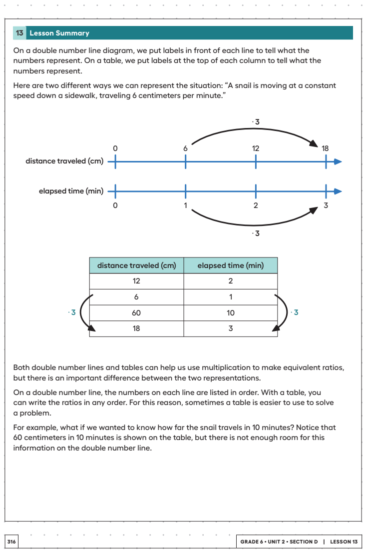
Here are two different ways we can represent the situation: "A snail is moving at a constant speed down a sidewalk, traveling 6 centimeters per minute."




| distance traveled (cm) | elapsed time (min) |
|---|---|
| 12 | 2 |
| 6 | 1 |
| 60 | 10 |
| 18 | 3 |

· 3 · 3

**Student Workbook**



13 Lesson Summary

On a double number line diagram, we put labels in front of each line to tell what the numbers represent. On a table, we put labels at the top of each column to tell what the numbers represent.

Here are two different ways we can represent the situation: "A snail is moving at a constant speed down a sidewalk, traveling 6 centimeters per minute."

| distance traveled (cm) | elapsed time (min) |
|---|---|
| 12 | 2 |
| 6 | 1 |
| 60 | 10 |
| 18 | 3 |

Both double number lines and tables can help us use multiplication to make equivalent ratios, but there is an important difference between the two representations.

On a double number line, the numbers on each line are listed in order. With a table, you can write the ratios in any order. For this reason, sometimes a table is easier to use to solve a problem.

For example, what if we wanted to know how far the snail travels in 10 minutes? Notice that 60 centimeters in 10 minutes is shown on the table, but there is not enough room for this information on the double number line.

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**Responding To Student Thinking**

**Points to Emphasize**
If students struggle with making sense of rates represented in tables, revisit the use of tables to solve rate problems when opportunities arise over the next several lessons. For example, continue to compare and interpret tables and double number lines by finding unit rates in the first three practice problems of this lesson:

Unit 2, Lesson 13 Tables and Double Number Line Diagrams

Both double number lines and tables can help us use multiplication to make equivalent ratios, but there is an important difference between the two representations.

On a double number line, the numbers on each line are listed in order. With a table, you can write the ratios in any order. For this reason, sometimes a table is easier to use to solve a problem.

For example, what if we wanted to know how far the snail travels in 10 minutes? Notice that 60 centimeters in 10 minutes is shown on the table, but there is not enough room for this information on the double number line.

## Cool-down

## Bicycle Sprint

5 min

### Student Task Statement

In a sprint to the finish line, a professional cyclist travels 380 meters in 20 seconds. At that rate, how far does the cyclist travel in 3 seconds? You can use a table if it is helpful.

They travel 57 meters in 3 seconds

Sample reasoning:

| distance traveled (meters) | elapsed time (seconds) |
|---|---|
| 380 | 20 |
| 19 | 1 |
| 57 | 3 |

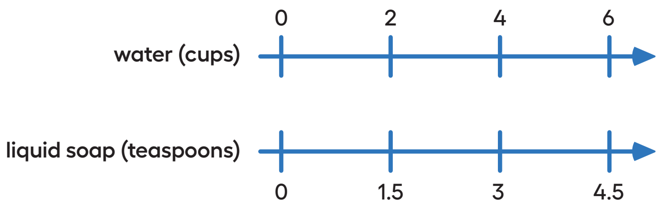
**Lesson 13**

## Practice Problems

6 Problems

### Problem 1

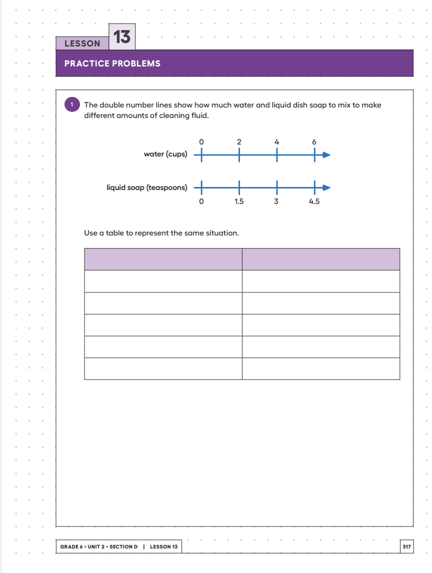
The double number lines show how much water and liquid dish soap to mix to make different amounts of cleaning fluid.




Use a table to represent the same situation.

| water (cups) | liquid soap (teaspoons) |
|---|---|
| 0 | 0 |
| 2 | 1.5 |
| 4 | 3 |
| 6 | 4.5 |

**Student Workbook**



LESSON 13

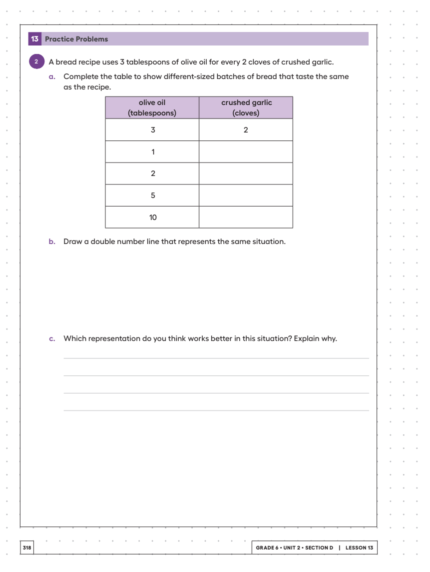
PRACTICE PROBLEMS

1 The double number lines show how much water and liquid dish soap to mix to make different amounts of cleaning fluid.

Use a table to represent the same situation.

| | |
|---|---|
| | |
| | |
| | |
| | |
| | |

**Student Workbook**


13 Practice Problems

2 A bread recipe uses 3 tablespoons of olive oil for every 2 cloves of crushed garlic.

a. Complete the table to show different-sized batches of bread that taste the same as the recipe.

| olive oil (tablespoons) | crushed garlic (cloves) |
|---|---|
| 3 | 2 |
| 1 | |
| 2 | |
| 5 | |
| 10 | |

b. Draw a double number line that represents the same situation.

c. Which representation do you think works better in this situation? Explain why.

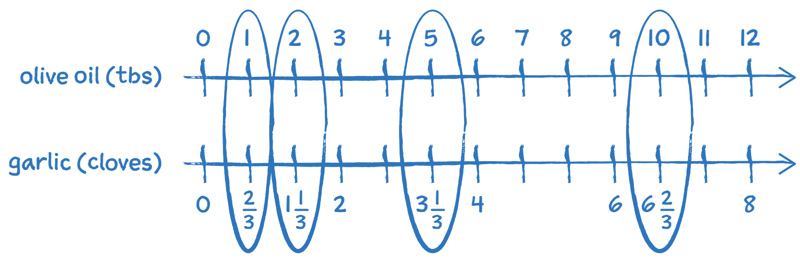
## Problem 2

A bread recipe uses 3 tablespoons of olive oil for every 2 cloves of crushed garlic.

**a.** Complete the table to show different-sized batches of bread that taste the same as the recipe.

| olive oil (tablespoons) | crushed garlic (cloves) |
|---|---|
| 3 | 2 |
| 1 | $\frac{2}{3}$ |
| 2 | $1\frac{1}{3}$ |
| 5 | $3\frac{1}{3}$ |
| 10 | $6\frac{2}{3}$ |

**b.** Draw a double number line that represents the same situation.

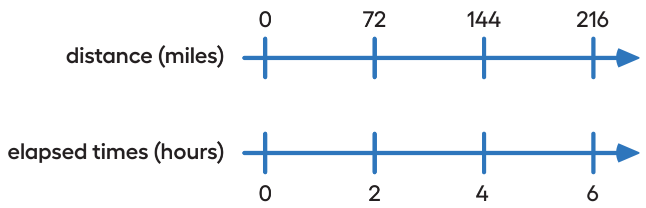



**c.** Which representation do you think works better in this situation? Explain why.

Sample response: The table is more convenient because the rows of the table can be listed in any order and without worrying about placing numbers accurately on the number line.

## Problem 3

Clare travels at a constant speed, as shown on the double number line.




At this rate, how far does she travel in each of these intervals of time? Explain or show your reasoning. If you get stuck, consider using a table.

Sample responses:

**a.** 1 hour

36 miles

One hour is half of 2 hours, and half of 72 is 36.

**b.** 3 hours

108 miles

Since the rate is 36 miles per hour, to find her distance in 3 hours, multiply 36 by 3.

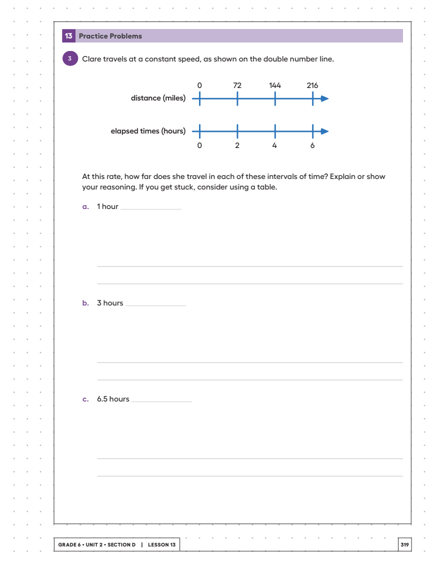
**c.** 6.5 hours

234 miles

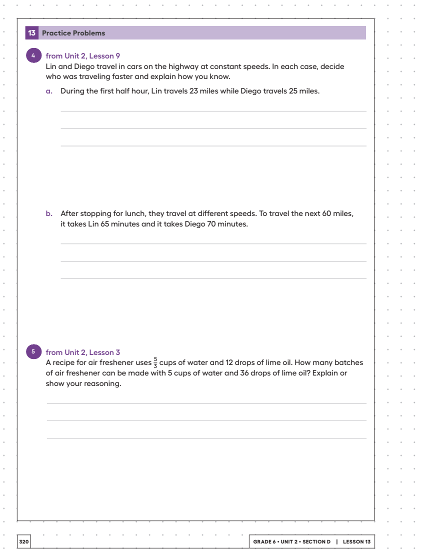
Multiply the distance per hour by 6.5.

| distance (miles) | elapsed time (hours) |
|---|---|
| 72 | 2 |
| 36 | 1 |
| 108 | 3 |
| 234 | 6.5 |

Student Workbook



**Student Workbook**


13 Practice Problems

4 from Unit 2, Lesson 9
Lin and Diego travel in cars on the highway at constant speeds. In each case, decide who was traveling faster and explain how you know.

a. During the first half hour, Lin travels 23 miles while Diego travels 25 miles.

b. After stopping for lunch, they travel at different speeds. To travel the next 60 miles, it takes Lin 65 minutes and it takes Diego 70 minutes.

5 from Unit 2, Lesson 3
A recipe for air freshener uses $\frac{5}{3}$ cups of water and 12 drops of lime oil. How many batches of air freshener can be made with 5 cups of water and 36 drops of lime oil? Explain or show your reasoning.

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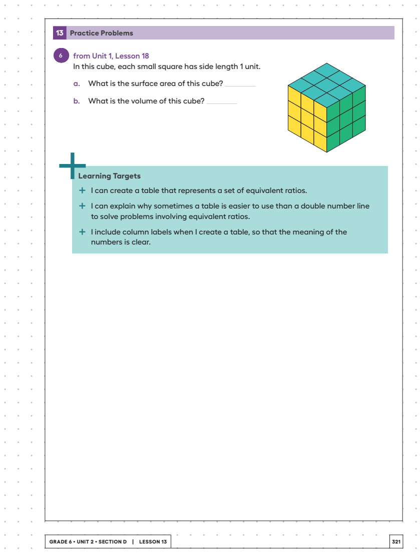
**Student Workbook**


13 Practice Problems

6 from Unit 1, Lesson 18
In this cube, each small square has side length 1 unit.

a. What is the surface area of this cube? ______

b. What is the volume of this cube? ______

Learning Targets
+ I can create a table that represents a set of equivalent ratios.
+ I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios.
+ I include column labels when I create a table, so that the meaning of the numbers is clear.

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## Problem 4

from Unit 2, Lesson 9

Lin and Diego travel in cars on the highway at constant speeds. In each case, decide who was traveling faster and explain how you know.

Sample responses:

**a.** During the first half hour, Lin travels 23 miles while Diego travels 25 miles.

Diego traveled faster because he covered more distance than Lin in the same amount of time.

**b.** After stopping for lunch, they travel at different speeds. To travel the next 60 miles, it takes Lin 65 minutes and it takes Diego 70 minutes.

Lin traveled faster because she covered the same distance as Diego but in less time.

## Problem 5

from Unit 2, Lesson 3

A recipe for air freshener uses $\frac{5}{3}$ cups of water and 12 drops of lime oil. How many batches of air freshener can be made with 5 cups of water and 36 drops of lime oil? Explain or show your reasoning.

3 batches

Sample reasoning: There are $\frac{15}{3}$ in 5, so that's enough water for 3 batches ($3 \cdot \frac{5}{3} = \frac{15}{3}$). Likewise, $3 \cdot 12 = 36$, so 36 drops are enough for 3 batches of air freshener.

## Problem 6

from Unit 1, Lesson 18

In this cube, each small square has side length 1 unit.

**a.** What is the surface area of this cube?

54 square units

**b.** What is the volume of this cube?

27 cubic units