Lesson 8

Solving Rate Problems

Goals

- Choose a strategy to solve problems involving rates and explain (orally, in writing, and through other representations) the solution method.
- Evaluate (orally) the usefulness of calculating a rate per 1 when solving problems involving unfamiliar rates.

Learning Target

I can see that thinking about "how much for 1" is useful for solving different types of problems.

Lesson Narrative

In this lesson, students solve problems that reward finding and making sense of unit rates. All problems use the Burj Khalifa—the tallest building in the world as of 2023—as a context. Solving them requires students to make sense of and talk about rates in terms of the situation, giving students opportunities to apply and deepen their understanding from earlier lessons.

Students reason about two rates—a rate for climbing the building and a rate for washing all of its windows. In the activity about climbing, students work with smaller values that are related by familiar whole-number factors or by benchmark fractions. The problems are well defined. In the activity about window washing, students need to request one of the values, the number of windows on the Burj Khalifa, which is a large number and not a multiple of either value in the given ratio. Possible solution paths may not be as apparent—an opportunity for students to make sense of problems and persevere in solving them.

Student Learning Goal

Let's investigate the Burj Khalifa building.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Instructional Routines

- 5 Practices
- Notice and Wonder

Required Materials

Materials to Gather

- Four-function calculators: Activity 2

Required Preparation

Lesson:

All computations in this lesson can be done with methods that students learned up through grade 5. However, you may wish to provide access to calculators to deemphasize computation and allow students to focus on reasoning about the context.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Notice and Wonder: Heights

5 min

Activity Narrative

The purpose of this *Warm-up* is to elicit observations about tall objects and their measurements, which will be useful when students reason about rate problems in a context involving a skyscraper later in the lesson. Although students may notice and wonder many things about the objects in the image, ways to estimate the heights of the tall objects are the important discussion points.

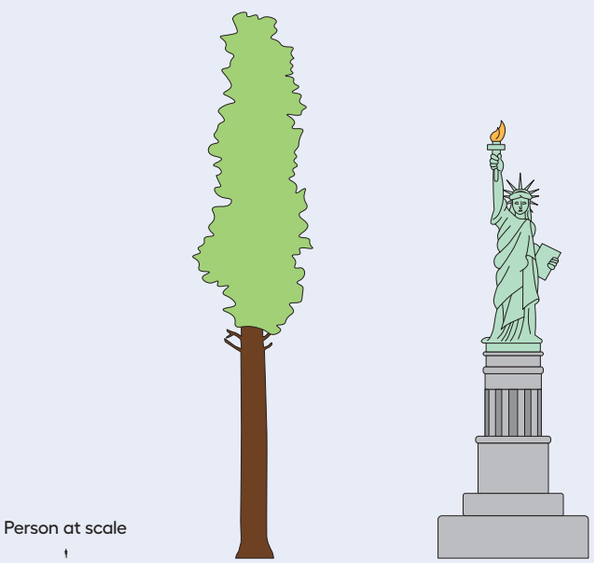
When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language that they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch

Arrange students in groups of 2. Display the image for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing that they notice and one thing that they wonder about. Give students another minute to discuss their observations and questions with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- There is a person, a tree, and the Statue of Liberty.
- The tree is taller than the statue. The tree and the statue are both much taller than the person.
- The person is tiny compared to the other two objects.
- The pedestal on which the statue sits is about half the height of the entire structure.

Instructional Routines

Notice and Wonder

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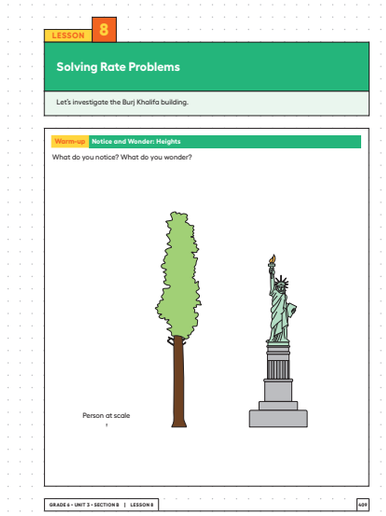
Student Workbook


LESSON 8

Solving Rate Problems

Let's investigate the Burj Khalifa building.

Warm-up Notice and Wonder: Heights

What do you notice? What do you wonder?

Person at scale

Students may wonder:

- What kind of tree is it? Do trees that tall really exist?
- How tall are the tree and the statue?
- How long would it take a tree to grow to that size?
- How was a statue of that size built?

Activity Synthesis

Ask students to share the things that they noticed and wondered. Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the image.

Tell students that the image shows the tallest tree in the world, the Hyperion. If questions about the height of the tree do not come up during the conversation, ask students how it could be estimated. Students may suggest referring to different heights in the image to estimate the height of the tree. Those who are familiar with the Statue of Liberty or have an idea of its approximate height might use the base of the pedestal, the statue without the pedestal, or the full statue to estimate the height of the Hyperion. Others may do so using the human figure.

If time permits, ask students to make an estimate for the height of the Hyperion. (The tree measures approximately 380 feet or 116 meters. The Statue of Liberty measures approximately 305 feet or 93 meters from the ground to the torch.) If there is a tall structure in the community that students might be familiar with, consider sharing how its height compares to the Hyperion.

Activity 1
Climbing the Burj Khalifa

15 min

Activity Narrative

In this activity, students learn about the Burj Khalifa, the tallest high-rise in the world, and calculate the distance scaled by a climber given a rate (828 meters in 6 hours) and some amounts of time. Students may approach each question in different ways, with or without using a unit rate.

- To find the distance climbed in 2 hours, students are likely to divide both the total distance climbed and the total number of hours by 3.
- To find the distance climbed in 5 hours, they are likely to divide the total distance by 6 to find the rate per hour and multiply it by 5.
- To find the distance climbed in the last 15 minutes, students are likely to multiply the rate per hour by $\frac{1}{4}$ (or divide it by 4), recognizing that to climb "at the same rate" means the distance covered in any 1 hour or any 15-minute period is the same throughout the climb. Though less likely, some students may find the distance climbed in $5\frac{3}{4}$ hours and subtract it from 828 meters, the distance climbed in 6 hours.

Monitor for students who approach the task in different ways or use different representations to reason, and select them to share later.

Launch

Tell students to close their workbooks or devices. Invite students to think of some tall buildings that they have seen or heard of. Ask them to identify the tallest building they know of and to recall information about its height. If no students mention the Burj Khalifa, ask if they know about it. Allow students who are familiar with the building to share what they know.

Then, display a picture of the building and a map of its location. Explain that the Burj Khalifa is located in Dubai, a city in the United Arab Emirates and on the coast of the Arabian Gulf, surrounded by desert. As of the end of 2022, the Burj Khalifa is the tallest manmade structure in the world. Consider sharing some other facts about the Burj Khalifa:

- Its height is 2,722 ft (829.8 meters).
- It has 154 usable floors, 9 maintenance floors, 46 spire levels, and 2 below-ground parking lots.
- It has 57 elevators.

Tell students that now they are to solve some problems about a climber who scaled the Burj Khalifa. Explain that to "scale" in this context means to climb up a surface.

Give students 4 minutes of quiet think time and then a minute to share their responses with a partner. Ask students to be prepared to explain their thinking.

Student Task Statement

In 2011, a professional climber, Alain Robert, scaled the outside of the Burj Khalifa, making it all the way to 828 meters (the highest point on which a person can stand) in 6 hours.

Assuming that he climbed at the same rate the whole way:

1. How far did he climb in the first 2 hours?

   276 meters

   Sample reasoning: $\frac{1}{3} \cdot 6$ is 2 and $\frac{1}{3} \cdot 828$, or $828 \div 3$, is 276.

Access for Multilingual Learners (Activity 1, Student Task)

MLR6: Three Reads.
Keep books or devices closed. Display only the first sentence of the problem, without revealing the questions. "We are going to read this problem 3 times." After the first read: "Tell your partner what this situation is about." After the second read: "List the quantities. What can be counted or measured?" Reveal the questions. After the third read: "What are some ways we might get started on this?"

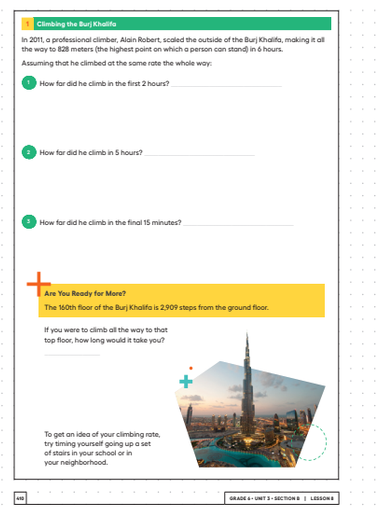
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Provide Access for Physical Action.
Activate or supply background knowledge. Provide students with access to a blank double number line diagram to support information processing.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook



1 Climbing the Burj Khalifa

In 2011, a professional climber, Alain Robert, scaled the outside of the Burj Khalifa, making it all the way to 828 meters (the highest point on which a person can stand) in 6 hours.

Assuming that he climbed at the same rate the whole way:

1. How far did he climb in the first 2 hours?
2. How far did he climb in 5 hours?
3. How far did he climb in the final 15 minutes?

Are You Ready for More?

The 160th floor of the Burj Khalifa is 2,909 steps from the ground floor.

If you were to climb all the way to that top floor, how long would it take you?

To get an idea of your climbing rate, try timing yourself going up a set of stairs in your school or in your neighborhood.

Building on Student Thinking

If students are unsure how to approach the last problem, ask them to explain their understanding of the situation. (If needed, clarify that "the final 15 minutes" means the last 15 minutes of the climb.) Encourage them to create a sketch, a timeline, or another representation to help them make sense of the situation and what the question might be asking.

2. How far did he climb in 5 hours?

690 meters

Sample reasoning: The climber climbed 828 ÷ 6 or 138 meters in 1 hour, so he climbed 5 • 138 or 690 in 5 hours.

3. How far did he climb in the final 15 minutes?

34.5 meters

Sample reasoning: The climber climbed 138 meters in 1 hour, so that means 138 ÷ 4 or 34.5 in 15 minutes, including the last 15 minutes.

time (hours)	height (meters)
6	828
2	276
1	138
$\frac{1}{2}$	69
$\frac{1}{4}$	34.5

Are You Ready for More?

The 160th floor of the Burj Khalifa is 2,909 steps from the ground floor. If you were to climb all the way to that top floor, how long would it take you?

To get an idea of your climbing rate, try timing yourself going up a set of stairs in your school or in your neighborhood.

Answers vary.

Activity Synthesis

Invite students who use different strategies to briefly share their work. Focus the discussion on when it might be helpful to find the unit rate (the number of meters climbed per hour). Because an amount of time is specified in each question, we can multiply the given amount of time by the number of meters climbed per hour to find the heights climbed.

Activity 2
Window Washing

15 min

Activity Narrative

In this activity, students determine how long it would take to wash all of the 24,348 windows of the Burj Khalifa given a specific rate of work by a window-washing crew.

The total number of windows is not a multiple of the number of windows in the given ratio (15 windows in 18 minutes). This may motivate some students to identify the rate for washing 1 window and scale that number. It may prompt other students to reason less directly. Monitor for students who apply their understanding of equivalent ratios in different ways. Here are some likely approaches, from less direct to more direct:

- Use the equivalent ratio of 10 windows in 12 minutes to find the time for washing 24,350 windows (the nearest multiple of 10) and then subtract the time for washing 2 windows.
- Find the number of windows that can be washed in 1 minute and divide 24,348 by that number.
- Find how many groups of 15 windows are in 24,348 and multiply that number by 18.
- Find the time for washing 1 window and then multiply it by 24,348.

Regardless of the strategy used, students should be encouraged to explain the quantity they are calculating—such as windows washed per minute, minutes to wash 1 window, minutes to wash 10 windows, and so on—and how they plan to use it.

Because not all the necessary information is given up front and no solution pathway is suggested, students have an opportunity to make sense of the problem and persevere in solving it. Students who find the amount of time for washing the nearest multiple of 10 or 5 windows also practice considering the level of precision that is appropriate in the context of the problem.

Launch

Keep the picture of the Burj Khalifa displayed. Draw students' attention to the windows of the building. Ask students: "How long do you think it would take to wash all the windows on the Burj Khalifa?"

A video of a window washing crew on the Burj Khalifa can be found online. If time permits, consider showing how the crew used brushes and squeegees to clean the windows while being harnessed to ropes.

Tell students that they will now try to answer a question about the window washing. Read aloud the task statement and question. Ask students what information they would need to answer the question. Upon students' request, share that the Burj Khalifa has 24,348 windows.

Keep students in groups of 2. Give students 3–4 minutes of quiet work time and then time to discuss their responses with their partner. Provide access to calculators.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
Before they begin, invite students to verbalize their strategy for finding how long it would take to wash all the windows. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Building on Student Thinking

If students are unsure where to start, ask:

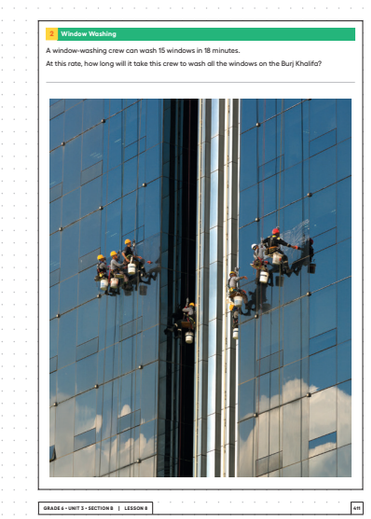
> *"At this rate, how long will it take the crew to wash 300 windows? 6,000 windows? 10,000 windows?"*

Encourage them to create a table, or provide a blank table that they can use to organize their work.

Students may get stuck when they try to write the value of minutes per window because 15 does not divide evenly into 18. Encourage them to write one or more equivalent ratios with smaller values and see if those would help find the number of minutes for 1 window. For instance, ask:

> *"Can you first find the number of minutes to wash 5 windows or 10 windows?"*

Student Workbook


Window Washing

A window-washing crew can wash 15 windows in 18 minutes.

At this rate, how long will it take this crew to wash all the windows on the Burj Khalifa?

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Student Task Statement

A window-washing crew can wash 15 windows in 18 minutes.

At this rate, how long will it take this crew to wash all the windows on the Burj Khalifa?

Sample responses:

- It will take 29,217.6 minutes.

number of windows	time (minutes)
15	18
1	1.2 or $1\frac{1}{5}$
24,348	29,217.6

- It will take a little less than 29,220 minutes (or 29,217.6 minutes).
  - Washing 15 windows in 18 minutes means washing 5 windows in 6 minutes and 10 windows in 12 minutes.
  - We can round the number of windows to the nearest multiple of 10, which is 24,350. There are 2,435 groups of 10 windows in 24,350, so it takes $2{,}435 \cdot 12$, or 29,220, to wash 24,350 windows.
  - If 10 windows take 12 minutes to wash, then 2 windows take $\frac{12}{5}$ or 2.4 minutes to wash. Subtracting 2.4 from 29,220 gives 29,217.6.

Activity Synthesis

Invite previously selected students to share their responses and reasoning. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*.

Connect the different responses to the learning goals by asking questions such as:

- *"One strategy we saw involves dividing 24,238 by 15. What does the result represent in this situation?"*

  how many groups of 15 windows are on the Burj Khalifa

- *"How does it help us?"*

  We know it takes 18 minutes to wash 15 windows, so we can multiply the number of groups of 15 windows by 18 to answer the question.

- *"Dividing 15 by 18 gives $\frac{5}{6}$. What does that value represent in this situation?"*

  windows washed in 1 minute

- *"Dividing 18 by 15 gives 1.2 or $1\frac{1}{5}$. What does that value represent?"*

  minutes needed to wash 1 window

- *"To find the amount of time for washing a given number of windows, which unit rate is more convenient or useful to know: windows washed in 1 minute or minutes needed to wash 1 window?"*

Highlight that the $\frac{5}{6}$ window per minute and $1\frac{1}{5}$ minutes per window are the two unit rates associated with the ratio 15 windows to 18 minutes.

If time permits, ask students to consider what 29,217.6 minutes mean in terms of normal work days. Students may point out that there are 1,440 minutes in a 24-hour day but dividing the minutes of window washing by 1,440 would not represent the number of normal work days. When students interpret the result of their calculations in context, they practice reasoning quantitatively and abstractly.

Lesson Synthesis

To highlight the efficiency of using unit rates in solving problems, display the two main unit rates from the activities: 138 meters climbed per hour and $1\frac{1}{5}$ or 1.2 minutes per window. Reinforce their usefulness by asking questions such as:

- *"How high is the climber after 2.5 hours?"*

  345 meters

- *"2.25 hours?"*

  310.5 meters

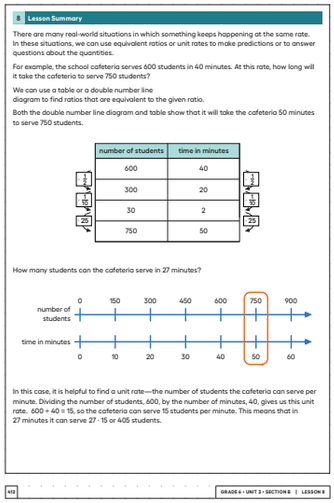
- *"2.2 hours?"*

  303.6 meters

- *"If the Burj Khalifa had 10,000 windows, how many minutes would it take the washing crew to clean all of them?"*

  12,000 minutes or 200 hours

Student Workbook



8 Lesson Summary

There are many real-world situations in which something keeps happening at the same rate. In these situations, we can use equivalent ratios or unit rates to make predictions or to answer questions about the quantities.

For example, the school cafeteria serves 600 students in 40 minutes. At this rate, how long will it take the cafeteria to serve 750 students?

We can use a table or a double number line diagram to find ratios that are equivalent to the given ratio.

Both the double number line diagram and table show that it will take the cafeteria 50 minutes to serve 750 students.

number of students	time in minutes
600	40
300	20
30	2
750	50

How many students can the cafeteria serve in 27 minutes?

number of students: 0, 150, 300, 450, 600, 750, 900

time in minutes: 0, 10, 20, 30, 40, 50, 60

In this case, it is helpful to find a unit rate—the number of students the cafeteria can serve per minute. Dividing the number of students, 600, by the number of minutes, 40, gives us this unit rate. 600 ÷ 40 = 15, so the cafeteria can serve 15 students per minute. This means that in 27 minutes it can serve 27 · 15 or 405 students.

💬 *"What about 100,000 windows?"*

120,000 minutes or 2,000 hours

If time permits, consider asking:

💬 *"Suppose the window washing crew plans to work 8 hours a day. Can we still use the $1\frac{1}{5}$ or 1.2 unit rate to find how many windows the crew could wash in a day?"*

Yes, we can divide 480, the number of minutes in 8 hours, by 1.2 to find the number of windows.

💬 *"Is there another way to find the number of windows that could be washed in 8 hours?"*

We can use the other unit rate, $\frac{5}{6}$ window per minute, and multiply it by 480.

Lesson Summary

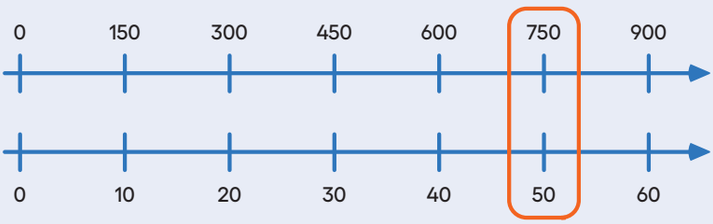
There are many real-world situations in which something keeps happening at the same rate. In these situations, we can use equivalent ratios or unit rates to make predictions or to answer questions about the quantities.

For example, the school cafeteria serves 600 students in 40 minutes. At this rate, how long will it take the cafeteria to serve 750 students?

We can use a table or a double number line diagram to find ratios that are equivalent to the given ratio.

Both the double number line diagram and table show that it will take the cafeteria 50 minutes to serve 750 students.

number of students	time in minutes
600	40
300	20
30	2
750	50



How many students can the cafeteria serve in 27 minutes?

In this case, it is helpful to find a unit rate—the number of students the cafeteria can serve per minute. Dividing the number of students, 600, by the number of minutes, 40, gives us this unit rate. 600 ÷ 40 = 15, so the cafeteria can serve 15 students per minute. This means that in 27 minutes it can serve 27 · 15 or 405 students.

Cool-down

Going Up?

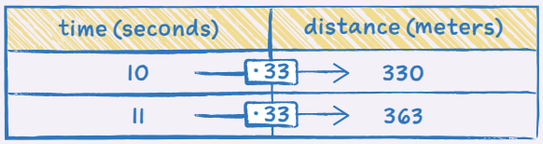
5 min

Student Task Statement

The fastest elevators in the Burj Khalifa can travel 330 feet in just 10 seconds. How far does the elevator travel in 11 seconds? Explain or show your reasoning.

363 feet

Sample reasoning:



If the elevator travels 330 feet in 10 seconds, then it is traveling 33 feet per second. Adding 33 feet per second to 330 feet in 10 seconds gives 363 feet in 11 seconds.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of using unit rates in solving problems. If students struggle with using unit rates in context, make time to examine related work in previous lessons. For example, highlight finding the unit rate as a key part of making sense of the problems in the activities referred to here. The Course Guide provides additional ideas for revisiting earlier work.

Unit 3, Lesson 6, Activity 3 How Much Applesauce?

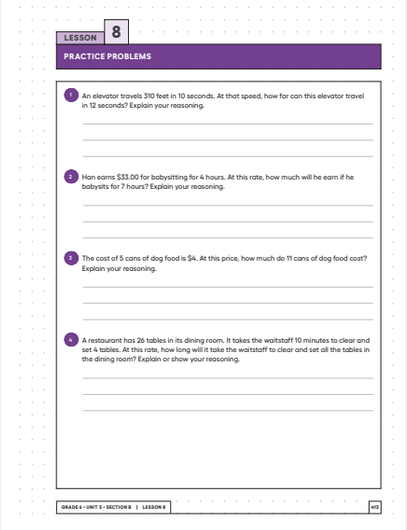
Unit 3, Lesson 7, Activity 2 The Fastest of All

Lesson 8

Practice Problems

7 Problems

Student Workbook



LESSON 8

PRACTICE PROBLEMS

1. An elevator travels 310 feet in 10 seconds. At that speed, how far can this elevator travel in 12 seconds? Explain your reasoning.

2. Han earns $33.00 for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your reasoning.

3. The cost of 5 cans of dog food is $4. At this price, how much do 11 cans of dog food cost? Explain your reasoning.

4. A restaurant has 26 tables in its dining room. It takes the waitstaff 10 minutes to clear and set 4 tables. At this rate, how long will it take the waitstaff to clear and set all the tables in the dining room? Explain or show your reasoning.

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Problem 1

An elevator travels 310 feet in 10 seconds. At that speed, how far can this elevator travel in 12 seconds? Explain your reasoning.

372 feet

Sample reasoning: $310 \div 10 = 31$, so the elevator travels 31 feet per second, and $31 \cdot 12 = 372$.

Problem 2

Han earns $33.00 for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your reasoning.

$57.75

Sample reasoning: $33 \div 4 = 8.25$, so the hourly rate is $8.25. In 7 hours, he will earn $7 \cdot (8.25)$ or $57.75.

Problem 3

The cost of 5 cans of dog food is $4. At this price, how much do 11 cans of dog food cost? Explain your reasoning.

$8.80

Sample reasoning: $4 \div 5 = 0.8$, so each can costs $0.80 and 11 times that amount is $8.80.

Problem 4

A restaurant has 26 tables in its dining room. It takes the waitstaff 10 minutes to clear and set 4 tables. At this rate, how long will it take the waitstaff to clear and set all the tables in the dining room? Explain or show your reasoning.

65 minutes or 1 hour and 5 minutes

Sample reasoning:

number of tables	time in minutes
4	10
1	2.5
26	65

Problem 5

from Unit 2, Lesson 16

For an activity, a teacher gave each student in the class 4 purple cards and 3 white cards. In total, the teacher gave out 91 cards.

a. How many students are in the class?

13 students

b. How many purple cards were given out?

52 purple cards

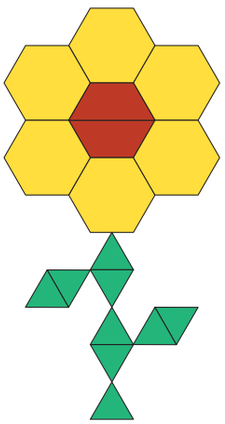
c. How many white cards were given out?

39 white cards

Problem 6

from Unit 2, Lesson 14

Here is a flower made up of yellow hexagons, red trapezoids, and green triangles.



a. How many copies of this flower pattern could you build if you had 30 yellow hexagons, 50 red trapezoids, and 60 green triangles?

5 copies

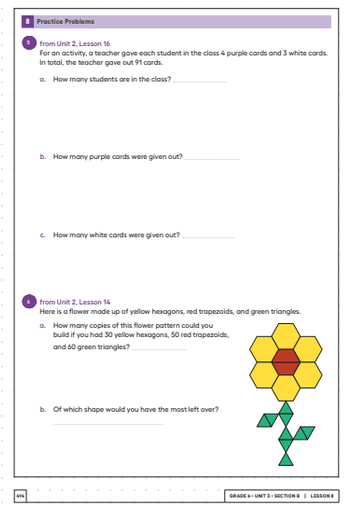
Making 5 copies would use up all 30 of the yellow hexagons, 10 red trapezoids, and 45 green triangles.

b. Of which shape would you have the most left over?

Red trapezoids

There will be 40 red trapezoids and 15 green triangles left over.

Student Workbook



8 Practice Problems

5 from Unit 2, Lesson 16
For an activity, a teacher gave each student in the class 4 purple cards and 3 white cards. In total, the teacher gave out 91 cards.

a. How many students are in the class? _____

b. How many purple cards were given out? _____

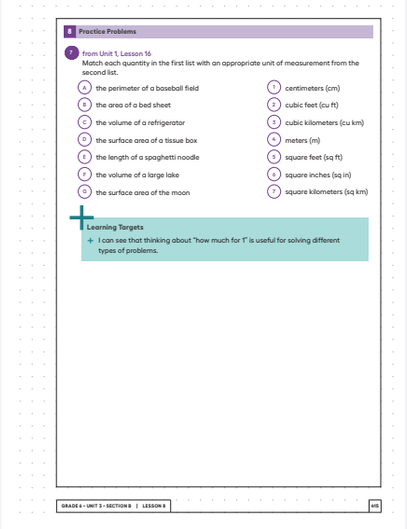
c. How many white cards were given out? _____

6 from Unit 2, Lesson 14
Here is a flower made up of yellow hexagons, red trapezoids, and green triangles.

a. How many copies of this flower pattern could you build if you had 30 yellow hexagons, 50 red trapezoids, and 60 green triangles? _____

b. Of which shape would you have the most left over?

Student Workbook



8 Practice Problems

7 from Unit 1, Lesson 16
Match each quantity in the first list with an appropriate unit of measurement from the second list.

A the perimeter of a baseball field	1 centimeters (cm)
B the area of a bed sheet	2 cubic feet (cu ft)
C the volume of a refrigerator	3 cubic kilometers (cu km)
D the surface area of a tissue box	4 meters (m)
E the length of a spaghetti noodle	5 square feet (sq ft)
F the volume of a large lake	6 square inches (sq in)
G the surface area of the moon	7 square kilometers (sq km)

Learning Targets
+ I can see that thinking about "how much for 1" is useful for solving different types of problems.

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Problem 7

from Unit 1, Lesson 16

Match each quantity in the first list with an appropriate unit of measurement from the second list.

A. the perimeter of a baseball field
B. the area of a bed sheet
C. the volume of a refrigerator
D. the surface area of a tissue box
E. the length of a spaghetti noodle
F. the volume of a large lake
G. the surface area of the moon

1. centimeters (cm)
2. cubic feet (cu ft)
3. cubic kilometers (cu km)
4. meters (m)
5. square feet (sq ft)
6. square inches (sq in)
7. square kilometers (sq km)