Volume of Prisms

Goals

- Calculate one edge length of a rectangular prism given its volume and the other two edge lengths.
- Explain (orally, in writing, and using other representations) how to solve a problem involving the volume of a rectangular prism with fractional edge lengths.
- Generalize that it takes more smaller cubes or fewer larger cubes to fill the same volume.

Learning Target

I can solve volume problems that involve fractions.

(Activity 1) • MLR8: Discussion Supports

Instructional Routines

(Activity 2)

• MLR3: Critique, Correct, Clarify

Access for Students with Diverse

Access for Multilingual Learners

• MLR3: Critique, Correct, Clarify

• Representation (Activity 1)

• Engagement (Activity 2)

Abilities

Lesson Narrative

In this lesson, students reason about the volume of various rectangular prisms, solidifying their understanding that multiplying the edge lengths, or the area of the base and the height of a prism, gives its volume. Students also have an opportunity to reason in the opposite direction: to find an unknown edge length by dividing the volume by other two edge lengths or by the area of the base.

Problems about prisms may involve three quantities—area of the base, height, and volume—or four quantities—length, width, height, and volume. To find an unknown length may involve calculating two quotients. For instance, to determine an unknown height may mean dividing the volume by the length and then by the width. It may also mean two different operations, for example, multiplying two known lengths first and then dividing the volume by that product. In tackling problems with increasing complexity, students must make sense of problems and persevere in solving them.

Student Learning Goal

Let's look at the volume of prisms that have fractional measurements.

Lesson Timeline

5 min

Warm-up

20 min

Activity 1

10 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Lesson 15 Warm-up Activity 1 Activity 2 Lesson Synthesis Cool-down

Warm-up

A Box of Cubes



Activity Narrative

This *Warm-up* reinforces the idea of using unit cubes and fractional-unit cubes as a way to measure the volume of a rectangular prism. The reasoning here prepares students to solve problems involving volumes of prisms later in the lesson.

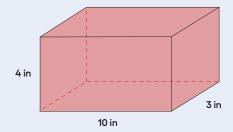
Launch

Give students 2-3 minutes of quiet work time.

Follow with a class discussion.

Student Task Statement

1. How many cubes with an edge length of 1 inch fill this box?



120 cubes.

Sample reasoning: $10 \cdot 3 \cdot 4 = 120$

2. If the cubes had an edge length of 2 inches, would you need more or fewer cubes to fill the box? Explain your reasoning.

Fewer

Sample reasoning: 8 cubes with edge lengths of I inch fit into a cube with edge lengths of 2 inches, so there will be fewer cubes.

3. If the cubes had an edge length of $\frac{1}{2}$ inch, would you need more or fewer cubes to fill the box? Explain your reasoning.

More

Sample reasoning: Each cube with edge lengths of I inch can be packed with 8 cubes with edge lengths of $\frac{1}{2}$ inch, so there will be more cubes.

Activity Synthesis

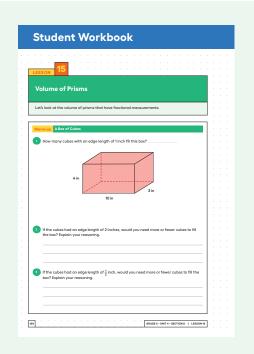
Select several students to share their responses and reasoning. After each person explains, ask students to indicate whether they agree. To involve more students in the discussion, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Does anyone want to add on to ____'s reasoning?"

"Do you agree or disagree with the reasoning? Why?"

Tell students that they will use their understanding of the volume of rectangular prisms to solve other geometric problems.



Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 1)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique*, *Correct*, *Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Access for Perception.

Use unit cubes and a sample box to demonstrate the process of packing a box with cubes to measure its volume.

Supports accessibility for: Conceptual Processing, Language, Memory

Building on Student Thinking

Some students may misinterpret the number of smaller cubes that Lin and Noah each use to pack the $\frac{1}{2}$ -inch cube as the volume measurement to be compared. Clarify that the question is about the volume of the $\frac{1}{2}$ -inch cube in cubic inches. Ask students to consider how the volume of each small cube relates to 1 cubic inch.

Activity 1

Warm-up

Cubes with Fractional Edge Lengths



Activity Narrative

In this activity, students continue the work on finding the volume of a right rectangular prism with fractional edge lengths. This time, they do so by packing it with unit cubes of different unit fractions for their edge lengths— $\frac{1}{2}$, and $\frac{1}{4}$ of an inch. They use these cubes to find the volume of the prism in cubic inches and explain whether cubes of different fractional edge lengths would lead to the same volume. In articulating their reasoning and considering others', students practice constructing a logical argument and critiquing the reasoning of others.

Launch



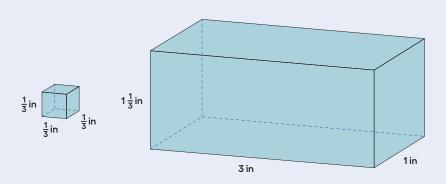
Arrange students in groups of 2.

Give them 4-5 minutes of quiet work time for the first set of questions.

Ask them to discuss their responses with their partner before moving on to the second half of the activity.

Student Task Statement

1. Diego says that 108 cubes with an edge length of $\frac{1}{3}$ inch are needed to fill a rectangular prism that is 3 inches by 1 inch by $1\frac{1}{3}$ inch.



a. Explain or show how this is true.

Sample reasoning: There are 9 groups of $\frac{1}{3}$ inch in 3 inches, 3 groups in 1 inch, and 4 groups in $\frac{4}{3}$ inches. So it would take $9 \cdot 3 \cdot 4$ (or 108 cubes) to pack the prism.

b. What is the volume, in cubic inches, of the rectangular prism? Explain or show your reasoning.

4 in³

Sample reasoning:

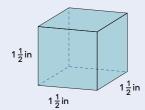
• Each cube with edge lengths of $\frac{1}{3}$ inch has a volume of $\frac{1}{27}$ in³.

$$108 \cdot \frac{1}{27} = 4$$

$$\circ 3 \cdot 1 \cdot \frac{4}{3} = 4$$

2. Lin and Noah are packing small cubes into a larger cube with an edge length of $1\frac{1}{2}$ inches. Lin is using cubes with an edge length of $\frac{1}{2}$ inch, and Noah is using cubes with an edge length of $\frac{1}{4}$ inch.

Warm-up



a. Who would need more cubes to fill the $1\frac{1}{2}$ -inch cube? Be prepared to explain your reasoning.

Noah would need more cubes, because his small cubes are smaller.

b. If Lin and Noah each use their small cubes to find the volume of the larger $1\frac{1}{2}$ -inch cube in cubic inches, will they get the same answer? Explain or show your reasoning.

Yes, they should get the same volume.

Sample reasoning:

- There are three $\frac{1}{2}$ -inches in $I\frac{1}{2}$ inches, so Lin would need $3 \cdot 3 \cdot 3$ (or 27) cubes with edge lengths of $\frac{1}{2}$ inch. The volume of each $\frac{1}{2}$ -inch cube is $\frac{1}{8}$ in³, so the volume of the $I\frac{1}{2}$ -inch cube is $27 \cdot \frac{1}{8}$ (or $\frac{27}{8}$)in³.
- There are six $\frac{1}{4}$ -inches in $I\frac{1}{2}$ inches, so Noah would need $6 \cdot 6 \cdot 6$ (or 216) cubes with edge lengths of $\frac{1}{4}$ inch. The volume of each cube with edge lengths of $\frac{1}{4}$ inch is $\frac{1}{64}$ in³, so the volume of the $I\frac{1}{2}$ -inch cube is $216 \cdot \frac{1}{64}$ (or $\frac{27}{8}$) in³.
- Both sets of cubes can be packed into the $I\frac{1}{2}$ -inch cube. In both cases, the volume can be calculated using $\left(I\frac{1}{2}\right)\cdot\left(I\frac{1}{2}\right)\cdot\left(I\frac{1}{2}\right)$, which equals $\frac{27}{8}$ (or $3\frac{3}{8}$) in³.

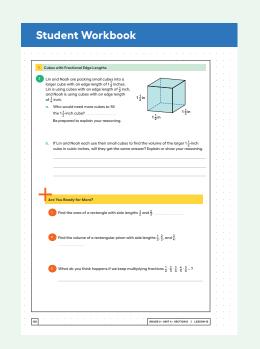
Are You Ready for More?

- **1.** Find the area of a rectangle with side lengths $\frac{1}{2}$ and $\frac{2}{3}$.
- **2.** Find the volume of a rectangular prism with side lengths $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
- **3.** What do you think happens if we keep multiplying fractions $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6}$...? The product approaches the value of 0.
- **4.** Find the area of a rectangle with side lengths $\frac{1}{1}$ and $\frac{2}{1}$.

5. Find the volume of a rectangular prism with side lengths $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{1}{3}$.

6. What do you think happens if we keep multiplying fractions $\frac{1}{1} \cdot \frac{2}{1} \cdot \frac{1}{3} \cdot \frac{4}{1} \cdot \frac{1}{5} ...?$

The product approaches the value of I.



Lesson 15 Warm-up **Activity 1** Activity 2 Lesson Synthesis Cool-down

Activity Synthesis

Select 2–3 students to explain why Diego's statement about the number of $\frac{1}{3}$ -inch cubes in the prism is correct and to share what the volume of the prism is.

Next, use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

Display this first draft:

"Lin and Noah will not find the same volume in cubic inches because Lin uses 8 cubes and Noah uses 64 cubes, which is 8 times as many cubes as Lin uses."

Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2 minutes to work with a partner to revise the first draft.

Invite students to include a labeled diagram if it could further clarify their revised explanation.

Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If time permits, ask students:

"Does it matter which fractional-unit cubes we use to find the volume? Why or why not?"

As long as the unit fraction can fit evenly into all three edge lengths of the prism, it doesn't matter what unit fraction we use.

"Is there another way of finding the volume of a rectangular prism with fractional edge lengths besides using these small cubes?"

Multiply the fractional edge lengths.

Activity 2

Fish Tank: Optional

10 min

Activity Narrative

In this activity, students solve word problems that involve finding the volume of rectangular prisms given the area of the base of the prism and its height. Students also calculate an unknown edge length in a rectangular prism given other measurements.

The question in *Are You Ready for More* requires students to interpret how the same volume of liquid would fit in two different containers in the shape of rectangular prisms. All questions offer students an opportunity to make sense of problems and persevere in solving them.

Launch



Keep students in groups of 2.

Give students 5 minutes of quiet work time and 2–3 minutes to discuss their responses with their partner.

Encourage students to draw a sketch to help with reasoning, if needed.

Student Task Statement

A nature center has a fish tank in the shape of a rectangular prism. The tank is 10 feet long, $8\frac{1}{4}$ feet wide, and 6 feet tall.

1. What is the volume of the tank in cubic feet? Show your reasoning.

495 ft³

Sample reasoning:

$$82\frac{1}{2} \cdot 6 = \frac{165}{2} \cdot 6 = \frac{990}{2} = 495$$

2. One day, a caretaker filled $\frac{4}{5}$ of the tank with water. What was the volume of the water in the tank, in cubic feet? What was the height of the water in the tank? Show your reasoning.

The volume of water is 396 ft³. The height of water is 4 \frac{4}{5} feet.

Sample reasoning:

$$\frac{4}{5} \cdot 495 = 396$$
 and $\frac{4}{5} \cdot 6 = \frac{24}{5} = 4\frac{4}{5}$

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Building on Student Thinking

Students might not recall that the volume of a rectangular prism can also be found by multiplying the area of the base of the prism by the height of the prism. If so, ask them how the area of the base is related to the edge lengths of the prism. If needed, remind students that the area of the base is the product of the length and the width of the prism.

Lesson 15 Warm-up Activity 1 Activity 2 Lesson Synthesis Cool-down

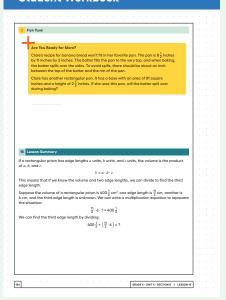
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

For each explanation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language. Remind students to use words such as "volume," "cubic feet (or inches)," "rectangular prisms," and "edge lengths."

Advances: Listening, Speaking

Student Workbook



Are You Ready for More?

Clare's recipe for banana bread won't fit in her favorite pan. The pan is $8\frac{1}{2}$ inches by 11 inches by 2 inches. The batter fills the pan to the very top, and when baking, the batter spills over the sides. To avoid spills, there should be about an inch between the top of the batter and the rim of the pan.

Clare has another rectangular pan. It has a base with an area of 81 square inches and a height of $2\frac{1}{2}$ inches. If she uses this pan, will the batter spill over during baking?

Yes, it would spill.

Sample reasoning: The volume of the batter is 187 cubic inches $(8\frac{1}{2} \cdot 11 \cdot 2 = 187)$. In the second pan, the batter will have a height of a little over 2 inches because $187 \div 81$ is about $2\frac{1}{3}$. Because the pan is only $2\frac{1}{2}$ inches deep, there wouldn't be at least 1 inch between the top of the batter and the rim of the pan.

Activity Synthesis

Focus the discussion on how students use known volume measurements to find the height of the water in the last question. Invite 1–2 students to share their solution, explanation, and drawing (if any). Record and display their reasoning for all to see.

To involve more students in the conversation, consider asking:

"Did anyone use the same strategy but would explain it differently?"
"Did anyone solve the problem in a different way?"
"Do you agree or disagree? Why?"

Lesson Synthesis

The goals of this discussion are to help students make connections between volume problems involving whole numbers and fractions, and reflect on their problem-solving process. Consider asking questions such as:

- "You have found the volume of prisms whose edge lengths are whole numbers and those whose edge lengths are fractions. How is the process the same? How is it different?"
 - "When calculating volume, did you find it harder to work with mixed numbers than with fractions less than 1? Why or why not?"
 - "Were there certain steps in calculating a volume that you found challenging or in which you were prone to make mistakes? If so, which steps?"

Lesson Summary

If a rectangular prism has edge lengths a units, b units, and c units, the volume is the product of a, b, and c.

$$V = a \cdot b \cdot c$$

This means that if we know the *volume* and *two edge lengths*, we can divide to find the *third* edge length.

Suppose the volume of a rectangular prism is $400\frac{1}{2}$ cm³, one edge length is $\frac{11}{2}$ cm, another is 6 cm, and the third edge length is unknown. We can write a multiplication equation to represent the situation:

$$\frac{11}{2} \cdot 6 \cdot ? = 400 \frac{1}{2}$$

We can find the third edge length by dividing:

$$400\frac{1}{2} \div \left(\frac{11}{2} \cdot 6\right) = ?$$

Cool-down

Storage Box

5 min

Launch

Encourage students to draw a sketch to help with reasoning, if needed.

Student Task Statement

A storage box has a base that measures 3 inches by 4 inches and a height of $1\frac{1}{2}$ inches. The box can be packed with 144 cubes with an edge length of $\frac{1}{2}$ inch.

1. Find the volume of the box in cubic inches. Show your reasoning.

18 cubic inches

Sample reasoning: $3 \cdot 4 \cdot 1\frac{1}{2} = 18$

2. Describe a different way to find the volume of the box. (It is not necessary to do the calculation.)

Sample response: Find the volume of a $\frac{1}{2}$ -inch cube and multiply it by I44. The volume of I cube is $\frac{1}{3}$ cubic inch, so the volume of the prism is I44 $\cdot \frac{1}{8}$, which is $\frac{144}{8}$ (or 18) cubic inches.

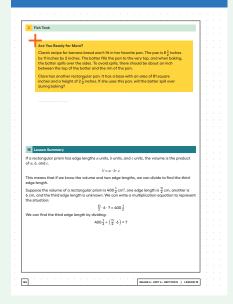
Responding To Student Thinking

Points to Emphasize

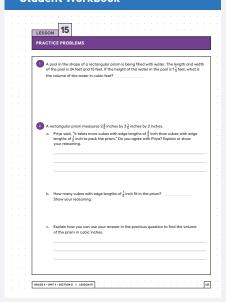
If students struggle to find the volume of the box, reinforce the idea that the volume of a rectangular prism with fractional edge lengths can be found in two ways, by packing it with unit cubes of the appropriate fractional edge lengths, or by multiplying its edge lengths. For example, ask students to show that both ways of reasoning gives the same volume for the rectangular prism in this practice problem:

Grade 6, Unit 4, Lesson 15, Practice Problem 2

Student Workbook



Student Workbook



Practice Problems

6 Problems

Problem 1

A pool in the shape of a rectangular prism is being filled with water. The length and width of the pool is 24 feet and 15 feet. If the height of the water in the pool is $1\frac{1}{3}$ feet, what is the volume of the water in cubic feet?

480 cubic feet

 $24 \cdot 15 = 360$, and $360 \cdot \frac{4}{3} = 480$.

Problem 2

A rectangular prism measures $2\frac{2}{5}$ inches by $3\frac{1}{5}$ inches by 2 inches.

a. Priya said, "It takes more cubes with edge lengths of $\frac{2}{5}$ inch than cubes with edge lengths of $\frac{1}{5}$ inch to pack the prism." Do you agree with Priya? Explain or show your reasoning.

Disagree

Sample reasoning: Cubes with side lengths $\frac{2}{5}$ inch are larger than cubes with side lengths $\frac{1}{5}$ inch, so it would take fewer of the former to pack the same prism.

b. How many cubes with edge lengths of $\frac{1}{5}$ inch fit in the prism? Show your reasoning.

1,920 cubes

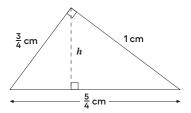
Sample reasoning: $2\frac{2}{5} \div \frac{1}{5} = 12$, $3\frac{1}{5} \div \frac{1}{5} = 16$, and $2 \div \frac{1}{5} = 10$. We can fit 12 cubes along the length of the prism, 16 cubes along the width, and 10 cubes along the height, so the number of cubes is: $12 \cdot 16 \cdot 10 = 1,920$.

c. Explain how you can use your answer in the previous question to find the volume of the prism in cubic inches.

Each unit cube (edge length $\frac{1}{5}$ inch) has a volume of $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ or $\frac{1}{125}$ cubic inch. There are 1,920 of these unit cubes, so the volume is 1,920 $\cdot \frac{1}{125}$ or 15.36 cubic inches.

Problem 3

from Unit 4, Lesson 14



a. Here is a right triangle. What is its area?

 $\frac{3}{6}$ cm²

Sample reasoning: The area of a triangle is found with $\frac{1}{2} \cdot b \cdot h$. We can use the two perpendicular sides as the base and the height. $\frac{1}{2} \cdot \frac{3}{4} \cdot I = \frac{3}{8}$

b. What is the height h for the base that is $\frac{5}{4}$ units long? Show your reasoning. $\frac{3}{5}$ cm

Sample reasoning: The area of the triangle is $\frac{3}{8}$ cm², and we can also write the area using the $\frac{5}{4}$ side and h. Because $\frac{1}{2} \cdot \frac{5}{4} h = \frac{3}{8}$, then $\frac{5}{8} h = \frac{3}{8}$. To find what value we could multiply by $\frac{5}{8}$ to get $\frac{3}{8}$, we can write $\frac{3}{8} \div \frac{5}{8}$, which is $\frac{3}{8} \cdot \frac{8}{5}$, which is $\frac{3}{5}$.

Problem 4

To give their animals essential minerals and nutrients, farmers and ranchers often have a block of salt—called a "salt lick"—available for their animals to lick.

a. A rancher is ordering a box of cube-shaped salt licks. The edge lengths of each salt lick are $\frac{5}{12}$ foot. Is the volume of one salt lick greater or less than 1 cubic foot? Explain your reasoning.

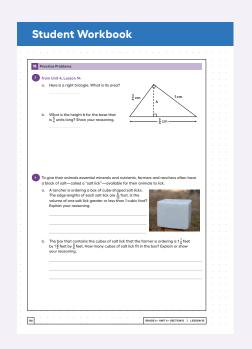
Less than I cubic foot

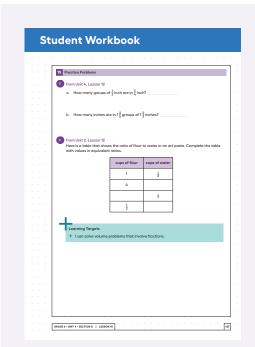
Sample reasoning: A cube with edge lengths of I foot has a volume of I cubic foot. A salt-lick cube has edge lengths of $\frac{5}{12}$ foot, which is less than I foot, so its volume $(\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12})$ is less than I cubic foot.

b. The box that contains the cubes of salt lick that the farmer is ordering is $1\frac{1}{4}$ feet by $1\frac{2}{3}$ feet by $\frac{5}{6}$ feet. How many cubes of salt lick fit in the box? Explain or show your reasoning.

24 cubes

Sample reasoning: The length of the box can fit $\frac{5}{4} \div \frac{5}{12}$ or 3 cubes. The width of the box can fit $\frac{5}{3} \div \frac{5}{12}$ or 4 cubes. The height of the box can fit $\frac{5}{6} \div \frac{5}{12}$ or 2 cubes. The box can fit $(3 \cdot 4 \cdot 2)$ or 24 cubes.





Problem 5

from Unit 4, Lesson 12

a. How many groups of $\frac{1}{3}$ inch are in $\frac{3}{4}$ inch?

2 4

Sample reasoning: To find "how many groups," compute $\frac{3}{4} \div \frac{1}{3}$, which is $\frac{3}{4} \cdot \frac{3}{1}$, which is $\frac{9}{4}$ (or $2\frac{1}{4}$).

b. How many inches are in $1\frac{2}{5}$ groups of $1\frac{2}{3}$ inches?

 $2\frac{1}{3}$

Sample reasoning: To find "how many inches," compute $l_{\frac{7}{5}} \cdot l_{\frac{3}{3}}^2$, which is $\frac{7}{5} \cdot \frac{5}{3}$, which is $\frac{7}{3}$ (or $2\frac{1}{3}$).

Problem 6

from Unit 2, Lesson 12

Here is a table that shows the ratio of flour to water in an art paste. Complete the table with values in equivalent ratios.

cups of flour	cups of water
1	1/2
4	2
6	3
1/2	14