Lesson 15

Finding This Percent of That

Goals

- Calculate any percentage of a value and explain (orally) the solution method.
- Generalize a process for finding A% of B and justify (orally) why this can be expressed as $\frac{A}{100} \cdot B$.

Learning Target

I can solve different problems like "What is 40% of 60?" by dividing and multiplying.

Lesson Narrative

In this lesson, students work toward generalizing the process for finding a percentage of a value and for calculating "A% of B" efficiently. In previous lessons, the numbers were chosen so that they could be calculated mentally or easily represented on a diagram. In this lesson, the numbers are intentionally chosen to be difficult to work with mentally or to represent on a diagram. This is to motivate students to find a simpler way to do the calculation by hand.

Students solve a series of problems that involve finding percentages that are not factors of 100. Through repeated reasoning, they notice regularity: to find P% of x, it is helpful to find 1% of x and then multiply that amount by P. In general, it can be done by computing $\frac{P}{100} \cdot x$.

Student Learning Goal

Let's solve percentage problems like a pro.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR5: Co-Craft Questions

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up

Math Talk: Multiply and Divide

5 min

Activity Narrative

This *Math Talk* focuses on the relationship between division by 100 and multiplication by $\frac{1}{100}$. It encourages students to rely on what they know about place value, fractions, decimals, and the relationship between multiplication and division to mentally solve problems. The reasoning elicited here will be helpful when students work with percentages later in the lesson.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A. $70 \div 100$

0.7 (or equivalent)

Sample reasoning:

- $700 \div 100$ is 7, so $70 \div 100$ is one tenth of 7, which is 0.7
- $70 \div 100$ is $\frac{70}{100}$, or $\frac{7}{10}$, which is 0.7
- $70 \div 100$ is equivalent to $70 \cdot \frac{1}{100}$, which is $\frac{70}{100}$ or 0.70, or 0.7

B. $35 \cdot \frac{1}{100}$

0.35

Sample reasoning:

- $35 \cdot \frac{1}{100}$ is 35 groups of one-hundredth, which is 35-hundredths.
- $35 \cdot \frac{1}{100}$ is the same as $35 \div 100$, which is 0.35
- $70 \div 100$ or $70 \cdot \frac{1}{100}$ is 0.7. Because 35 is half of 70, and $35 \cdot \frac{1}{100}$ is half of 0.7, which is 0.35

C. $(0.35) \cdot 100$

35

Sample reasoning:

- $35 \div 100$ is 0.35, so $(0.35) \cdot 100$ is 35.
- $(0.35) \cdot 10$ is 3.5, so $(0.35) \cdot 100$ is 10 times 3.5 or 35.
- 0.35 is $\frac{35}{100}$, and $\frac{35}{100} \cdot 100 = \frac{3{,}500}{100}$, which is 35.

Instructional Routines

Math Talk

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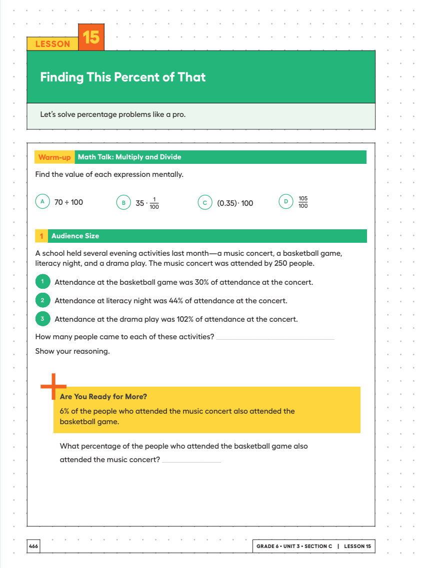
Please log in to the site before using the QR code or URL

Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because ..." or "I noticed ________ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

D. $\frac{105}{100}$

1.05 (or equivalent)

Sample reasoning:

- $\frac{100}{100}$ is 1 and $\frac{5}{100}$ is 0.05, so $\frac{105}{100}$ is 1.05
- $\frac{105}{100}$ is $105 \div 100$, which is 1.05
- $(1.05) \cdot 100$ is 105

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ________'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ________'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Highlight two points: that dividing a number by 100 gives the same result as multiplying the number by $\frac{1}{100}$, and that a fraction can be interpreted as division ($\frac{105}{100}$ can be understood as $105 \div 100$).

Activity 1

Audience Size

15 min

Activity Narrative

In this activity, students encounter percentage problems that are inconvenient to solve by drawing double number line diagrams or by using benchmark percentages, encouraging them to reason differently and look for a more practical approach.

To find 30% of a value in the first question, students can use a familiar percentage, 10%, as a stepping stone. For instance, they may find 10% or $\frac{1}{10}$ of the value and then multiply the result by 3. The next two questions ask students to find 44% and 102% of some values, respectively, prompting students to find different intermediate steps or use alternative strategies.

Some students may opt to use 4% and 2% as stepping stones to answer the two questions. Others may choose to find the value for 1% in each situation and multiply the result by the targeted percentage. Monitor for students who take this path and select them to share later.

Launch

Ask students if they have attended evening activities at their school or at another school. Invite them to share some evening activities they know about or have attended.

Display the first paragraph of the task statement for all to see, and read it aloud. Tell students that the activity has to do with the number of people attending different evening events. Invite students to ask clarifying questions about any unfamiliar event.

Arrange students in groups of 2. Give students 5 minutes of quiet work time and then time to share their explanation with a partner.

Student Task Statement

A school held several evening activities last month—a music concert, a basketball game, literacy night, and a drama play. The music concert was attended by 250 people.

1. Attendance at the basketball game was 30% of attendance at the concert.

   75 people attended the basketball game. Sample reasoning: 10% of 250 is 25, so 30 % is $3 \cdot 25$, which is 75.

2. Attendance at literacy night was 44% of attendance at the concert.

   110 people attended literacy night. Sample reasoning:

   - Since 10% of 250 is 25, 40% of 250 is $4 \cdot 25$ or 100, and 4% of 250 is 10, so 44% of 250 is 100 + 10 or 110.

number of people	percentage
250	100
$\cdot \frac{1}{25}$ 10	$\cdot \frac{1}{25}$ 4
$\cdot 11$ 110	$\cdot 11$ 44

3. Attendance at the drama play was 102% of attendance at the concert.

   How many people came to each of these activities? Show your reasoning.

   255 people attended the drama play. Sample reasoning:

   - If 100% is 250 people, then 20% of 250 is 50 people and 2% of 250 is 5 people. This means 102% of 250 is 250 + 5, or 255 people.
   - Dividing 100 by 50 gives 2 and dividing 250 by 50 gives 5, so 2% of 250 is 5 people, and 102% is 255 people.

number of people	percentage
250	100
$\cdot \frac{1}{100}$ 2.5	$\cdot \frac{1}{100}$ 1
$\cdot 102$ 255	$\cdot 102$ 102

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the first paragraph of the activity, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

> *"What do these questions have in common? How are they different?"*

Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Are You Ready for More?

6% of the people who attended the music concert also attended the basketball game.

What percentage of the people who attended the basketball game also attended the music concert?

20%. Sample reasoning: 60% of 250 is 150 and 6% of 250 is a tenth of 150, which is 15. So 15 people who attended the music concert also attended the basketball game. This number, 15, is $\frac{1}{5}$ of 75, which is 20% of 75, the number of people at the game.

Activity Synthesis

The aim of the discussion is to draw attention to the benefits of using 1% as an intermediate step for finding percentages of a number.

Invite a couple of students to share their work on the first question. Then, focus the discussion on the last two questions. Select several students who effectively found 44% and 102% of 250 to share their strategies, saving the strategy involving 1% for last. If no one used this method, ask whether finding 1% of 250 would be a helpful stepping stone for finding each of the three percentages.

Highlight that this method of first finding 1% of a quantity can be applied regardless of the percentage involved, much like finding a unit rate is an effective way to solve any ratio problem.

Consider using a table to illustrate this point. Start with the first two rows showing 100% and 1% of 250. Add a new row to show how each percentage can be found using the value for 1%, ending with a table such as shown:

number of people	percentage
250	100
2.5	1
75	30
110	44
255	102

Students may note that it is not possible to have 2.5 people or a non-whole number of people at an event, which is the case here if the percentage is an odd number. Acknowledge that this is true. Clarify that finding 1% of a quantity is still a valid intermediate step even in situations where only even-numbered percentages make sense.

Activity 2

Some Percentage for Charity

15 min

Activity Narrative

This activity prompts students to generalize the process of finding any percentage of a number. It makes it explicit that, in general, P% of a number is $\frac{P}{100}$ times that number.

Students are asked to find 1% of a set of values (1, 40, 100, 3,200, and x) and repeat the process to find 15% and 67% of the same set of values. Through repeated reasoning, students notice regularity in their calculations, enabling them to write expressions for finding P% of any value.

To find 1% of a number, students may opt to divide the number by 100, multiply it by $\frac{1}{100}$, or multiply it by 0.01, all of which are valid. When finding P% of a number, students who divide by 100 or multiply by a fraction can multiply $(P \div 100)$ or $\frac{P}{100}$ by the number, respectively. Students who multiply by a decimal, however, may find it more challenging to write a general expression. See Building on Student Thinking for a way to support these students.

As students calculate percentages of different numerical values and describe the process for finding any percentage, P, of any value, x, they practice expressing regularity through repeated reasoning.

Launch

Ask students if they have seen shirts, hats, or other accessories that bear the name of a music artist, a band, a sports team, or an online “influencer.” Explain that the term “merchandise” or “merch” is used to describe such products. Fans may buy merchandise that shows the name of a person or a brand they admire. Consider showing images of merchandise of a celebrity or a local sports team.

Tell students that celebrities and companies would sometimes donate to charity a percentage of their income from selling merchandise. In this activity, they will calculate how much money three artists would donate to charity based on the percentage they decide to give and the amount of money from merchandise sales.

Read the first two sentences of the first question, and display a table for all to see.

sales ($)	1	40	100	3,200	x
donation ($)					

Explain to students that they are to find the donation amount for each sales amount and based on the percentage that each artist is giving to charity (1% for Artist A, 15% for Artist B, and 67% for Artist C).

Arrange students in groups of 2–4. Provide access to calculators. Give students time to complete the activity, allowing at least 5 minutes for a whole-class discussion.

If time is limited, consider asking students to complete only two of the tables—for Artist A and one other artist—in the first question and then proceed to the last question.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Instruct students to complete one table at a time and then pause for a check-in. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

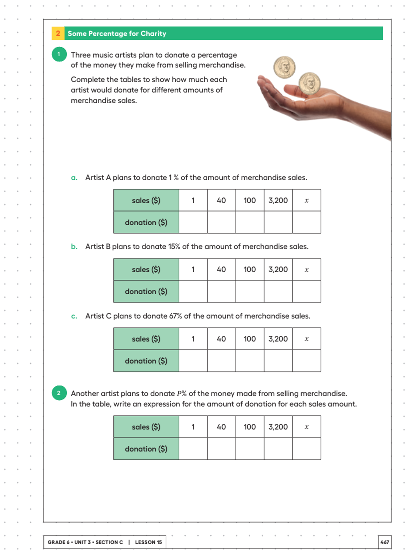
Building on Student Thinking

Students who use 0.01, 0.15, and 0.67 to describe how to find 1%, 15%, and 67% of a number, respectively, may be unsure how to express P% of a number in decimal form. Encourage students to look at the values in the first column of each table. Consider asking:

> *"Look at the values for 1% of \$1 and 15% of \$1. What is the relationship between the two values?"*
>
> 0.15 is 15 times 0.01.
>
> *"What is the relationship between 1% of 1 and 67% of \$1?"*
>
> 0.67 is 67 times 0.01.
>
> *"What might be the relationship between "1% of \$1 and P% of \$1?"*
>
> The amount for P% of \$1 would be P times 0.01.
>
> *"What expression can you write to describe that relationship?"*
>
> $P \cdot (0.01)$

Then, ask students how that expression could help them write an expression for finding P% of any amount, x.

Student Workbook


Some Percentage for Charity

1. Three music artists plan to donate a percentage of the money they make from selling merchandise.
Complete the tables to show how much each artist would donate for different amounts of merchandise sales.

a. Artist A plans to donate 1 % of the amount of merchandise sales.

b. Artist B plans to donate 15% of the amount of merchandise sales.

c. Artist C plans to donate 67% of the amount of merchandise sales.

2. Another artist plans to donate P% of the money made from selling merchandise. In the table, write an expression for the amount of donation for each sales amount.

Student Task Statement

1. Three music artists plan to donate a percentage of the money they make from selling merchandise.

   Complete the tables to show how much each artist would donate for different amounts of merchandise sales.

   a. Artist A plans to donate 1 % of the amount of merchandise sales.

sales (\$)	1	40	100	3,200	x
donation (\$)	0.01	0.40	1	32	$(0.01) \cdot x$ or $\frac{1}{100} \cdot x$

   b. Artist B plans to donate 15% of the amount of merchandise sales.

sales (\$)	1	40	100	3,200	x
donation (\$)	0.15	6	15	480	$(0.15) \cdot x$ or $\frac{15}{100} \cdot x$

   c. Artist C plans to donate 67% of the amount of merchandise sales.

sales (\$)	1	40	100	3,200	x
donation (\$)	0.67	26.80	67	2,144	$(0.67) \cdot x$ or $\frac{67}{100} \cdot x$

2. Another artist plans to donate P% of the money made from selling merchandise. In the table, write an expression for the amount of donation for each sales amount.

sales (\$)	1	40	100	3,200	x
donation (\$)	$\frac{P}{100} \cdot 1$	$\frac{P}{100} \cdot 40$	$\frac{P}{100} \cdot 100$	$\frac{P}{100} \cdot 3{,}200$	$\frac{P}{100} \cdot x$ or $(P \div 100) \cdot x$

Activity Synthesis

The goal of the discussion is to highlight the structure in the calculation to find a percentage of a number.

Invite students to share how they found 1% of the different sales amounts in the first table ($1, $40, $100, and $3,200). Record each unique strategy, and ask the students using that strategy for the expression that they wrote to find 1% of x. Record the corresponding expression next to each strategy. For instance:

Dividing each sales amount by 100.	$\frac{x}{100}$
Multiplying each sales amount by $\frac{1}{100}$.	$\frac{1}{100} \cdot x$
Finding 1% of $1, which is $0.01, and multiplying it by 40, 100, and 3,200 to get 1% of those amounts.	$(0.01) \cdot x$

Next, discuss how students completed the table for finding 15% of the same sales amounts. Record each unique approach and the corresponding expression for finding 15% of x.
For instance:

Finding 15% of $1, which is $0.15, and multiplying that number by 40, 100, and 3,200.	$(0.15) \cdot x$
Multiplying each donation amount in the first table (for 1% donation) by 15 to get the corresponding amount in the second table (for 15% donation).	$15 \cdot \frac{x}{100}$
	$15 \cdot \frac{1}{100} \cdot x$
	$15 \cdot (0.01) \cdot x$

Discuss with students:

If different expressions were written for 1% of x:

"Are the expressions for finding 1% of x equivalent?"

Yes

"How do you know?"

They are all equivalent to $\frac{1}{100}$ times x.

If different expressions were written for 15% of x:

"Are the expressions for finding 15% of x equivalent?"

Yes

"How do you know?"

They are all equivalent to 15-hundredths times x or $\frac{15}{100} \cdot x$.

"How can we find 67% of x dollars?"

Use the same strategies for finding 15% but with 67 in place of 15. Multiply 67-hundredths by x.

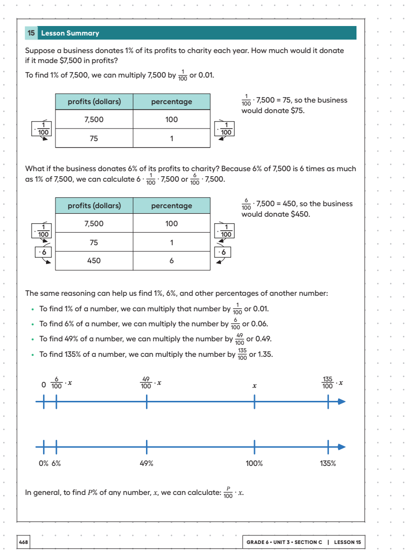
"What expressions can we write?"

$\frac{67}{100} \cdot x$ or equivalent

"How can we find P% or any percentage of x dollars?"

Multiply P-hundredths by x or calculate $\frac{P}{100} \cdot x$ or $P \cdot (0.01) \cdot x$.

Student Workbook



Lesson Synthesis

The main idea in this lesson is that to find P% of x, we can calculate $\frac{P}{100} \cdot x$ (or equivalent).

To highlight the reasoning that leads to this general expression, consider asking the following questions and recording the responses for all to see:

- *"Suppose we want to find 17% of 500 and 93% of 500. We saw that it can be helpful to find 1% of 500 first. Why is that?"*

  We can then multiply that value by 17 and 93.

- *"What is 1% of 500? How can we find it?"*

  5. Divide 500 by 100, or multiply 500 by $\frac{1}{100}$.

- *"What calculations can we make now to find 17% of 500 and 93% of 500?"*

  $17 \cdot 5$ and $93 \cdot 5$

- *"Suppose we need to find the same percentages—17% and 93%—but of some value x instead of 500. How can we find 1% of x?"*

  Divide x by 100 or multiply x by $\frac{1}{100}$, or find $\frac{1}{100} \cdot x$ or $\frac{x}{100}$

- *"What calculations can we make now to find 17% of x and 93% of x?"*

  $\frac{17}{100} \cdot x$ and $\frac{93}{100} \cdot x$

- *"In general, how can we find P% of x?"*

  Find 1% of x or $\frac{1}{100} \cdot x$, and then multiply it by P, or compute $\frac{P}{100} \cdot x$.

Lesson Summary

Suppose a business donates 1% of its profits to charity each year. How much would it donate if it made $7,500 in profits?

To find 1% of 7,500, we can multiply 7,500 by $\frac{1}{100}$ or 0.01.

profits (dollars)	percentage
7,500	100
75	1

$\cdot \frac{1}{100}$ $\cdot \frac{1}{100}$

$\frac{1}{100} \cdot 7{,}500 = 75$, so the business would donate $75.

What if the business donates 6% of its profits to charity? Because 6% of 7,500 is 6 times as much as 1% of 7,500, we can calculate $6 \cdot \frac{1}{100} \cdot 7{,}500$ or $\frac{6}{100} \cdot 7{,}500$.

profits (dollars)	percentage
7,500	100
75	1
450	6

$\cdot \frac{1}{100}$ $\cdot 6$ $\cdot \frac{1}{100}$ $\cdot 6$

$\frac{6}{100} \cdot 7{,}500 = 450$, so the business would donate $450.

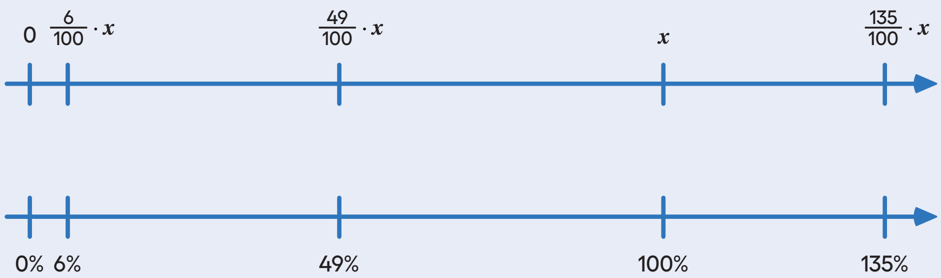
The same reasoning can help us find 1%, 6%, and other percentages of another number:

To find 1% of a number, we can multiply that number by $\frac{1}{100}$ or 0.01.

To find 6% of a number, we can multiply the number by $\frac{6}{100}$ or 0.06.

To find 49% of a number, we can multiply the number by $\frac{49}{100}$ or 0.49.

To find 135% of a number, we can multiply the number by $\frac{135}{100}$ or 1.35.



In general, to find P% of any number, x, we can calculate: $\frac{P}{100} \cdot x$.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery in finding percentages of a number. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 3, Section C Percentages

Cool-down

Percentages of Different Numbers

5 min

Student Task Statement

Find each percentage. Explain or show your reasoning.

1. 170% of 30

   51

   Sample reasoning:

   - $\frac{170}{100} \cdot 30 = 51$
   - $(1.7) \cdot 30 = 51$

2. 6% of 110

   6.6

   Sample reasoning:

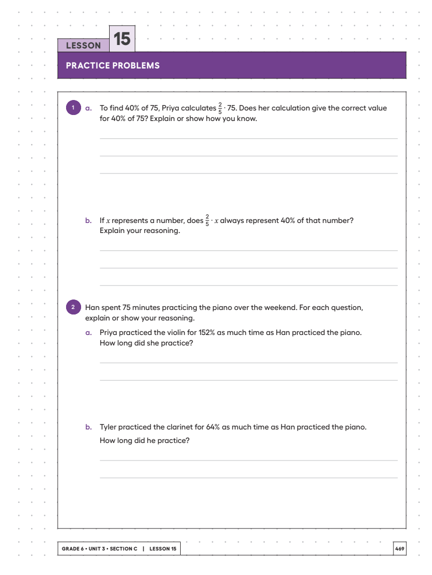
   - $\frac{6}{100} \cdot 110 = 6.6$
   - $(0.06) \cdot 110 = 6.6$

Lesson **15**

Practice Problems

7 Problems

Student Workbook


LESSON 15
PRACTICE PROBLEMS

1 a. To find 40% of 75, Priya calculates $\frac{2}{5} \cdot 75$. Does her calculation give the correct value for 40% of 75? Explain or show how you know.

b. If x represents a number, does $\frac{2}{5} \cdot x$ always represent 40% of that number? Explain your reasoning.

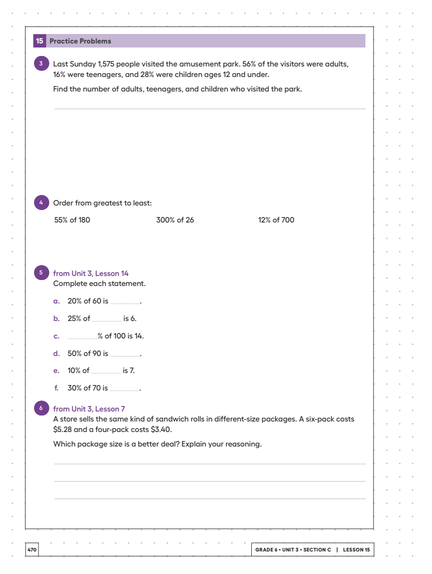
2 Han spent 75 minutes practicing the piano over the weekend. For each question, explain or show your reasoning.

a. Priya practiced the violin for 152% as much time as Han practiced the piano. How long did she practice?

b. Tyler practiced the clarinet for 64% as much time as Han practiced the piano. How long did he practice?

GRADE 6 • UNIT 3 • SECTION C | LESSON 15

Student Workbook


15 Practice Problems

3 Last Sunday 1,575 people visited the amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children ages 12 and under.

Find the number of adults, teenagers, and children who visited the park.

4 Order from greatest to least:

55% of 180 300% of 26 12% of 700

5 from Unit 3, Lesson 14
Complete each statement.

a. 20% of 60 is ____.
b. 25% of ____ is 6.
c. ____% of 100 is 14.
d. 50% of 90 is ____.
e. 10% of ____ is 7.
f. 30% of 70 is ____.

6 from Unit 3, Lesson 7
A store sells the same kind of sandwich rolls in different-size packages. A six-pack costs \$5.28 and a four-pack costs \$3.40.

Which package size is a better deal? Explain your reasoning.

GRADE 6 • UNIT 3 • SECTION C | LESSON 15

Problem 1

a. To find 40% of 75, Priya calculates $\frac{2}{5} \cdot 75$. Does her calculation give the correct value for 40% of 75? Explain or show how you know.

Yes

Sample reasoning: 40% of a number is 0.4 of the number and $(0.4) \cdot 75 = 30$. Using Priya's method: $\frac{2}{5} \cdot 75 = 30$.

b. If x represents a number, does $\frac{2}{5} \cdot x$ always represent 40% of that number? Explain your reasoning.

Yes

Sample reasoning: 40% of x is $\frac{40}{100} \cdot x$. This is the same as $\frac{2}{5} \cdot x$, since $\frac{40}{100}$ and $\frac{2}{5}$ are equivalent fractions.

Problem 2

Han spent 75 minutes practicing the piano over the weekend. For each question, explain or show your reasoning.

a. Priya practiced the violin for 152% as much time as Han practiced the piano. How long did she practice?

114 minutes

Sample reasoning: 152% of 75 minutes is $\frac{152}{100} \cdot 75 = 114$.

b. Tyler practiced the clarinet for 64% as much time as Han practiced the piano. How long did he practice?

48 minutes

Sample reasoning: 64% of 75 minutes is $\frac{64}{100} \cdot 75 = 48$.

Problem 3

Last Sunday 1,575 people visited the amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children ages 12 and under.

Find the number of adults, teenagers, and children who visited the park.

- 882 adults
- 252 teenagers
- 441 children

Problem 4

Order from greatest to least:

55% of 180	300% of 26	12% of 700
55% of 180	12% of 700	300% of 26

Problem 5 from Unit 3, Lesson 14

Complete each statement.

a. 20% of 60 is 12.

b. 25% of 24 is 6.

c.f 14 % of 100 is 14.

d. 50% of 90 isf 45.

e. 10% off 70 is 7.

f. 30% of 70 isf 21.

Problem 6 from Unit 3, Lesson 7

A store sells the same kind of sandwich rolls in different-size packages. A six-pack costs $5.28 and a four-pack costs $3.40.

Which package size is a better deal? Explain your reasoning.

The four-pack is a better deal. Sample reasoning: In the four-pack, each roll is 85 cents since 3.40 ÷ 4 = 0.85. In the six-pack, each roll is 88 cents since 5.28 ÷ 6 = 0.88. The unit price is lower when buying the four-pack.

Problem 7 from Unit 2, Lesson 15

On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip.

a. How many chaperones are there?

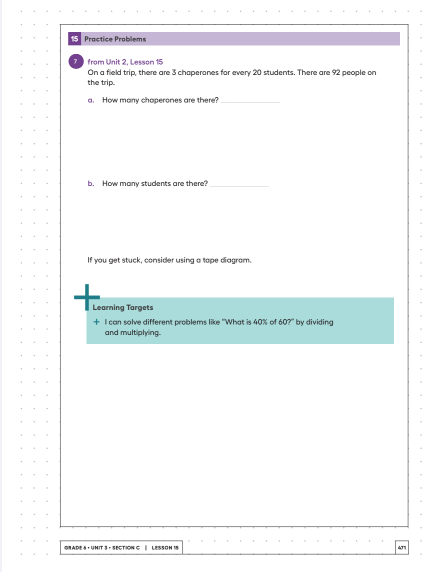
12 chaperones

b. How many students are there?

80 children

If you get stuck, consider using a tape diagram.

Student Workbook


15 Practice Problems

7 from Unit 2, Lesson 15

On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip.

a. How many chaperones are there?

b. How many students are there?

If you get stuck, consider using a tape diagram.

Learning Targets

+ I can solve different problems like "What is 40% of 60?" by dividing and multiplying.