Lesson 18

Using Common Multiples and Common Factors

Goals

- Choose to calculate the greatest common factor or least common multiple to solve a problem about a real-world situation, and justify (orally) the choice.
- Present (orally, in writing, and using other representations) the solution method for a problem involving greatest common factor or least common multiple.

Learning Target

I can solve problems using common factors and multiples.

Lesson Narrative

In this lesson, students apply what they have learned about factors and multiples to solve a variety of problems. Students make sense of problems by determining whether they involve finding factors or multiples before working with a partner to solve them.

One optional activity allows students to go beyond the standards addressed at this level by exploring common factors and common multiples of three whole numbers. A second optional activity gives students an opportunity for additional practice finding multiples and factors in a game-like setting.

Student Learning Goal

Let's use common factors and common multiples to solve problems.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 2
- Bingo chips: Activity 3

Materials to Copy

- Factors and Multiples Bingo Cards (1 copy for every 18 students): Activity 3

Required Preparation

Activity 3:
Preview the calling cards and decide if students are familiar with all the vocabulary used in the statements. Some of the words may need to be reviewed ahead of time.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	25 min	40 min	20 min	10 min	5 min
					Cool-down

Warm-up

Keeping a Steady Beat

5 min

Activity Narrative

In this *Warm-up*, students explore the concept of least common multiples using rhythm.

Launch

Tell students they will be establishing a steady beat as a class. Use a visual hand motion or display a visual timer to set the beat. Tell one half of the class to clap on every other beat, and tell the other half to stomp their feet on every third beat. If time allows, repeat this activity for the following intervals: every 3rd and 4th beat, every other and every 4th beat, every 4th and 6th beat. Follow with a whole-class discussion.

Student Task Statement

Your teacher will give you instructions to create a class rhythm. As you participate, think about these questions:

- When will the two sounds happen at the same time?

  Sample response: When claps happen every 2 beats and stomps happen every 3 beats, they happen at the same time after 6, 12, 18, and 24 beats.

- How does this game relate to common factors or common multiples?

  Sample response: These are all common multiples of 2 and 3. The first time it happens is the least common multiple, which is after 6 beats.

Activity Synthesis

The purpose of this discussion is for students to share their thinking on how this activity is related to common multiples or common factors. Invite students to explain what they noticed. If not mentioned by students, make sure they understand that students clapping on every other beat is like finding multiples of 2, and students stomping on every 3rd beat is like finding multiples of 3. When the two things happen at the same time, it is like a common multiple.

Activity 1

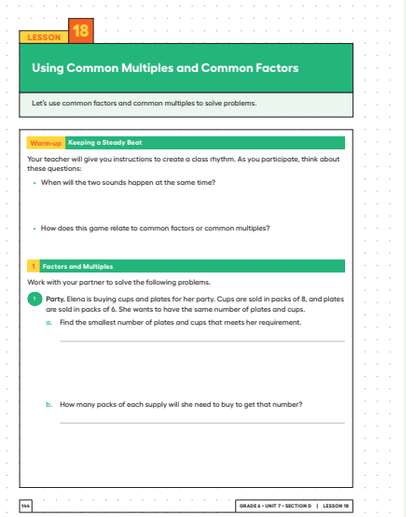
Factors and Multiples

25 min

Activity Narrative

In this activity, students work in pairs to solve problems that involve thinking about factors and multiples, including the greatest common factor and least common multiple. Students reflect on what type of mathematical work was used in each problem and begin to notice similarities in problems that involve factors and those that involve multiples. Students must make sense and persevere as they decide how the problems relate to common factors and common multiples.

Student Workbook



LESSON 18

Using Common Multiples and Common Factors

Let's use common factors and common multiples to solve problems.

Warm-up Keeping a Steady Beat

Your teacher will give you instructions to create a class rhythm. As you participate, think about these questions:

- When will the two sounds happen at the same time?
- How does this game relate to common factors or common multiples?

1 Factors and Multiples

Work with your partner to solve the following problems.

1. Party. Elena is buying cups and plates for her party. Cups are sold in packs of 8, and plates are sold in packs of 6. She wants to have the same number of plates and cups.
   a. Find the smallest number of plates and cups that meets her requirement.
   b. How many packs of each supply will she need to buy to get that number?

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Access for Multilingual Learners (Activity 1, Student Task)

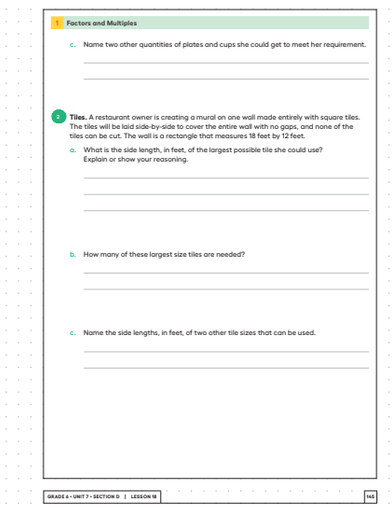
MLR2: Collect and Display
Collect the language that students use to solve problems involving common multiples and common factors. Display words and phrases, such as "multiples," "factors," and "multiply," or "divide evenly." During the *Activity Synthesis*, invite students to suggest ways to update the display:

> *"What are some other words or phrases we should include?"*

Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student Workbook



1 Factors and Multiples

c. Name two other quantities of plates and cups she could get to meet her requirement.

2 **Tiles.** A restaurant owner is creating a mural on one wall made entirely with square tiles. The tiles will be laid side-by-side to cover the entire wall with no gaps, and none of the tiles can be cut. The wall is a rectangle that measures 18 feet by 12 feet.

a. What is the side length, in feet, of the largest possible tile she could use? Explain or show your reasoning.

b. How many of these largest size tiles are needed?

c. Name the side lengths, in feet, of two other tile sizes that can be used.

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Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms, or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Arrange students in groups of 2.

Give students 15–20 minutes of work time, and follow with a whole-class discussion.

Student Task Statement

Work with your partner to solve the following problems.

1. **Party.** Elena is buying cups and plates for her party. Cups are sold in packs of 8, and plates are sold in packs of 6. She wants to have the same number of plates and cups.

   a. Find the smallest number of plates and cups that meets her requirement.

   24 cups and 24 plates

   b. How many packs of each supply will she need to buy to get that number?

   3 packages of cups and 4 packages of plates to get 24 of each

   c. Name two other quantities of plates and cups she could get to meet her requirement.

   Sample responses: She could get 48 of each if she gets 6 packages of cups and 8 packages of plates. She could also get 72 of each if she gets 9 packages of cups and 12 packages of plates.

2. **Tiles.** A restaurant owner is creating a mural on one wall made entirely with square tiles. The tiles will be laid side-by-side to cover the entire wall with no gaps, and none of the tiles can be cut. The wall is a rectangle that measures 18 feet by 12 feet.

   a. What is the side length, in feet, of the largest possible tile she could use? Explain or show your reasoning.

   The largest possible tile has a side length of 6 feet.

   Sample reasoning: Three 6-foot-long tiles can be laid to cover the length of the wall, which is 18 feet, and two 6-foot-long tiles can be laid to cover the width of the wall, which is 12 feet.

   b. How many of these largest size tiles are needed?

   6 tiles are needed.

   c. Name the side lengths, in feet, of two other tile sizes that can be used.

   Sample responses: She could also use tiles of length 1, 2, or 3 feet.

3. Stickers. To celebrate the first day of spring, Lin is putting stickers on some of the 100 pages in her math notebook. She puts a skateboard sticker on every 4th page, and a kite sticker on every 5th page.

a. Find three page numbers that will get both stickers.

Sample responses: pages 20, 40, 60, 80, and 100

b. Will the 30th page have no stickers, 1 sticker, or 2 stickers? Explain or show your reasoning.

1 sticker (kite)

Sample reasoning: Since 30 is a multiple of 5, page 30 will get a kite sticker. Since 30 is not a multiple of 4, page 30 will not get a flower sticker.

4. Kits. The school nurse is assembling first-aid kits for the teachers. She has 75 small bandages and 90 large bandages. All the kits must have the same number of each size bandage, and all bandages must be used.

a. What is the largest number of kits the nurse can make?

The greatest amount of kits that can be made is 15.

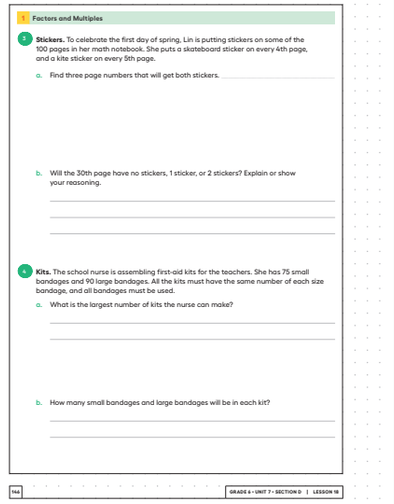
b. How many small bandages and large bandages will be in each kit?

Each kit will have 5 small bandages and 6 large bandages.

5. Put a checkmark in the appropriate column to show what kind of mathematical work was involved in each problem situation.

problem	finding multiples	finding least common multiple	finding factors	finding greatest common factor
Party	x	x		
Tiles			x	x
Stickers	x			
Kits			x	x

Student Workbook


1 Factors and Multiples

3 **Stickers.** To celebrate the first day of spring, Lin is putting stickers on some of the 100 pages in her math notebook. She puts a skateboard sticker on every 4th page, and a kite sticker on every 5th page.

a. Find three page numbers that will get both stickers.

b. Will the 30th page have no stickers, 1 sticker, or 2 stickers? Explain or show your reasoning.

4 **Kits.** The school nurse is assembling first-aid kits for the teachers. She has 75 small bandages and 90 large bandages. All the kits must have the same number of each size bandage, and all bandages must be used.

a. What is the largest number of kits the nurse can make?

b. How many small bandages and large bandages will be in each kit?

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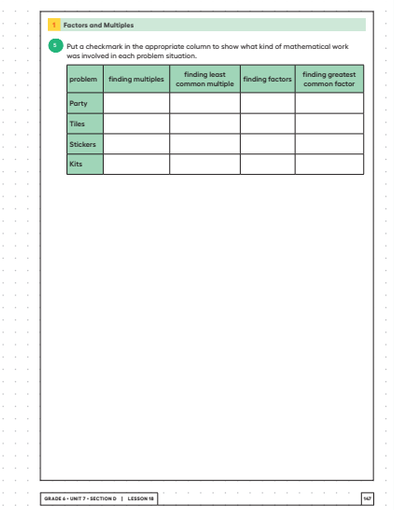
Student Workbook


1 Factors and Multiples

5 Put a checkmark in the appropriate column to show what kind of mathematical work was involved in each problem situation.

problem	finding multiples	finding least common multiple	finding factors	finding greatest common factor
Party				
Tiles				
Stickers				
Kits				

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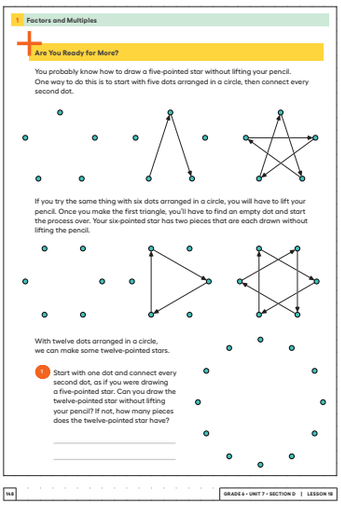
Student Workbook


1 Factors and Multiples

Are You Ready for More?

You probably know how to draw a five-pointed star without lifting your pencil. One way to do this is to start with five dots arranged in a circle, then connect every second dot.

If you try the same thing with six dots arranged in a circle, you will have to lift your pencil. Once you make the first triangle, you'll have to find an empty dot and start the process over. Your six-pointed star has two pieces that are each drawn without lifting the pencil.

With twelve dots arranged in a circle, we can make some twelve-pointed stars.

1. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?

Are You Ready for More?

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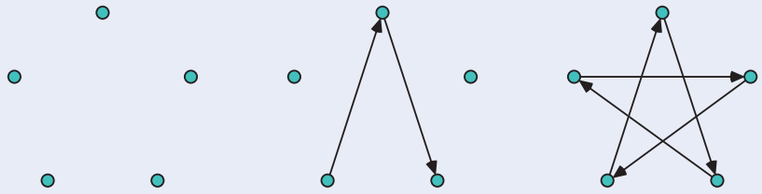

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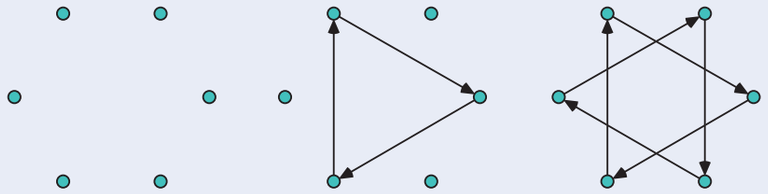
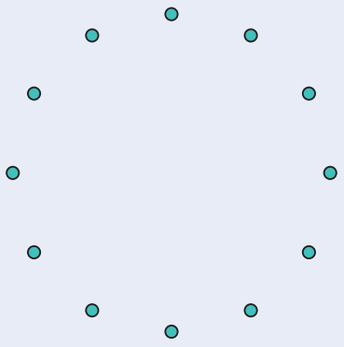


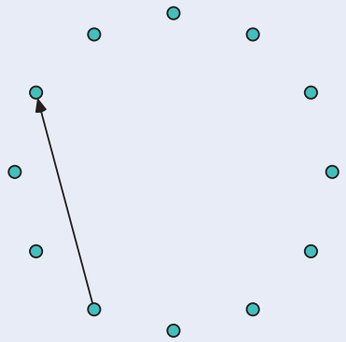
With twelve dots arranged in a circle, we can make some twelve-pointed stars.

1. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?



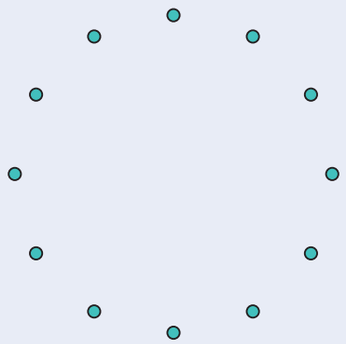
I have to lift up my pencil. There are two pieces.

2. This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?



No, I have to lift up my pencil here, too. This time, there are three pieces.

3. What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?



There are four pieces.

4. Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.

Yes. If I connect every fifth dot, I can draw the star in one motion. This also works if I connect every seventh dot (or every "first" or 11th dot).

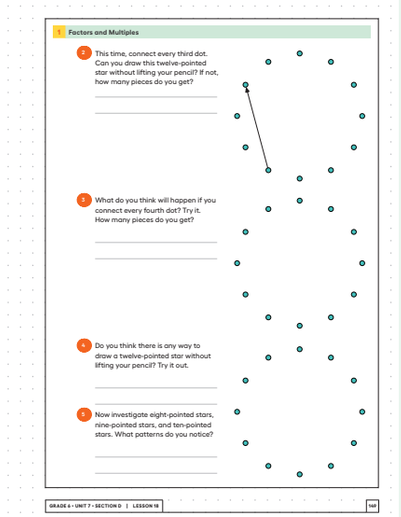
5. Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

Sample response: I can only draw the star without picking up my pencil if the total number of dots and the "skip number" have no factors in common.

Activity Synthesis

The purpose of this discussion is for students to express what kinds of problems have to do with least common multiples and what kinds have to do with greatest common factors. For each problem, ask students to indicate whether they think the problem had to do with common multiples or common factors, and invite a few to share their reasoning. Select students to explain their reasoning about how they solved the problems as time allows.

Student Workbook


1 Factors and Multiples

2 This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?

3 What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?

4 Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.

5 Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

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Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Provide Access by Recruiting Interest.
Invite students to generate a list of additional examples of situations that involve greatest common factor or least common multiple that connect to their personal backgrounds and interests.

Supports accessibility for: Conceptual Processing, Attention

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- "I noticed our norm '_____' in action today, and it really helped me/my group because _____."

Activity 2: Optional

More Factors and Multiples

40 min

Activity Narrative

In this activity, students work in groups to predict whether a problem involves common factors or common multiples. Groups then solve one assigned problem, create a visual display to represent their work, and prepare a brief presentation. While analyzing only pairs of numbers is the expectation for this course, this activity asks students to think about the greatest common factor and least common multiple for sets of three whole numbers.

Launch

Arrange students in groups of 4. Provide access to tools for creating a visual display.

Give students 5 minutes to read through each problem and discuss whether its solution has to do with finding common factors or common multiples.

Then assign each group one of the problems.

Give students 15–20 minutes of work time to solve their problem, create their visual display, and prepare a short presentation.

Student Task Statement

Read and discuss each problem with your group. *Without solving*, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

Then your teacher will assign one problem to your group. Work with your group to solve the problem. Then create a visual display that explains your group's mathematical thinking while solving the problem. Your display may include a diagram, lists, tables, equations, descriptions, and math vocabulary.

1. **Soccer.** Diego and Andre are both in a summer soccer league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.

   - common multiples
   - common factors
   - least common multiple
   - greatest common factor

   a. What is the first date that both Diego and Andre will have a game?

   August 12th

   b. How many of their games fall on the same date in the month of August?

   Two games: August 12th and 24th.

   The next common multiple is 36, and there are only 31 days in August.

2. **Performances.** During a performing arts festival, students from elementary and middle schools will be grouped together for various performances. There are 32 elementary students and 40 middle-school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student must be in a group and can be a part of only one group.

   - common multiples
   - common factors
   - least common multiple
   - greatest common factor

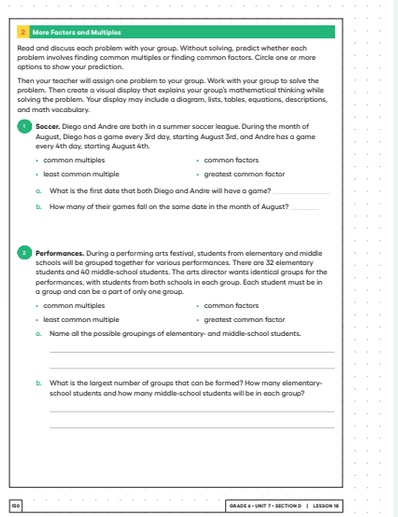
   a. Name all the possible groupings of elementary- and middle-school students.

   - There could be 1 group with 32 elementary-school students and 40 middle-school students
   - 2 groups with 16 elementary-school students and 20 middle-school students each
   - 4 groups with 8 elementary-school students and 10 middle-school students each
   - 8 groups with 4 elementary-school students and 5 middle-school students each.

   b. What is the largest number of groups that can be formed? How many elementary-school students and how many middle-school students will be in each group?

   8 groups with 4 elementary-school students and 5 middle-school students in each group

Student Workbook



2 More Factors and Multiples

Read and discuss each problem with your group. Without solving, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

Then your teacher will assign one problem to your group. Work with your group to solve the problem. Then create a visual display that explains your group's mathematical thinking while solving the problem. Your display may include a diagram, lists, tables, equations, descriptions, and math vocabulary.

1. **Soccer.** Diego and Andre are both in a summer soccer league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.
   - common multiples
   - common factors
   - least common multiple
   - greatest common factor

   a. What is the first date that both Diego and Andre will have a game? _____

   b. How many of their games fall on the same date in the month of August? _____

2. **Performances.** During a performing arts festival, students from elementary and middle schools will be grouped together for various performances. There are 32 elementary students and 40 middle-school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student must be in a group and can be a part of only one group.
   - common multiples
   - common factors
   - least common multiple
   - greatest common factor

   a. Name all the possible groupings of elementary- and middle-school students.

   b. What is the largest number of groups that can be formed? How many elementary-school students and how many middle-school students will be in each group?

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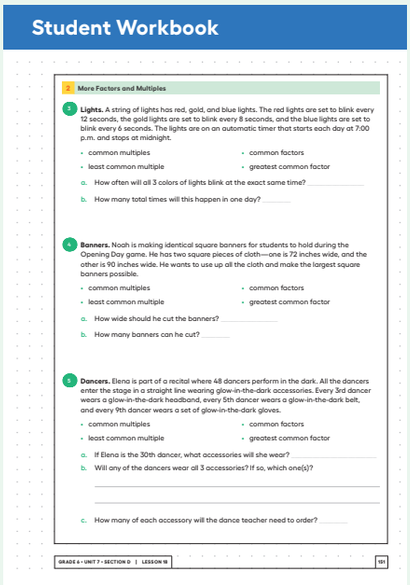
Student Workbook



2 More Factors and Multiples

3 **Lights.** A string of lights has red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midnight.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. How often will all 3 colors of lights blink at the exact same time? _____

b. How many total times will this happen in one day? _____

4 **Banners.** Noah is making identical square banners for students to hold during the Opening Day game. He has two square pieces of cloth—one is 72 inches wide, and the other is 90 inches wide. He wants to use up all the cloth and make the largest square banners possible.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. How wide should he cut the banners? _____

b. How many banners can he cut? _____

5 **Dancers.** Elena is part of a recital where 48 dancers perform in the dark. All the dancers enter the stage in a straight line wearing glow-in-the-dark accessories. Every 3rd dancer wears a glow-in-the-dark headband, every 5th dancer wears a glow-in-the-dark belt, and every 9th dancer wears a set of glow-in-the-dark gloves.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. If Elena is the 30th dancer, what accessories will she wear? _____

b. Will any of the dancers wear all 3 accessories? If so, which one(s)?

_____

_____

c. How many of each accessory will the dance teacher need to order? _____

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3. Lights. A string of lights has red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midniaht.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. How often will all 3 colors of lights blink at the exact same time?

every 24 seconds

b. How many total times will this happen in one day?

The 3 lights will blink together 75 times each day.

In 5 hours, there are 300 minutes or 1,800 seconds, and 1,800 ÷ 24 = 75.

4. Banners. Noah is making identical square banners for students to hold during the Opening Day game. He has two square pieces of cloth—one is 72 inches wide, and the other is 90 inches wide. He wants to use up all the cloth and make the laraest sauare banners possible.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. How wide should he cut the banners?

18 inches wide

b. How many banners can he cut?

41 banners

Since 72 ÷ 18 = 4, Noah will be able to cut 4 banners across in both directions for a total of 16 banners. Since 90 ÷ 18 = 5, Noah will be able to cut 5 banners across in both directions for a total of 25 banners, and 16 + 25 = 41.

5. Dancers. Elena is part of a recital where 48 dancers perform in the dark. All the dancers enter the stage in a straight line wearing glow-in-the-dark accessories. Every 3rd dancer wears a glow-in-the-dark headband, every 5th dancer wears a glow-in-the-dark belt, and every 9th dancer wears a set of alow-in-the-dark aloves.

- common multiples
- common factors
- least common multiple
- greatest common factor

a. If Elena is the 30th dancer, what accessories will she wear?

Elena will wear a headband and belt.

30 is a multiple of 3 and 5 but not of 9.

b. Will any of the dancers wear all 3 accessories? If so, which one(s)?

The 45th dancer will wear all 3 accessories because it is the least common multiple of 3, 5, and 9.

c. How many of each accessory will the dance teacher need to order?

- 116 headbands; there are 16 multiples of 3 up to 48, and 48 ÷ 3 = 16
- 9 belts; there are 9 multiples of 5 up to 48
- 5 sets of gloves; there are 5 multiples of 9 up to 48

Activity Synthesis

The purpose of this discussion is for groups to share their visual displays and explain their reasoning. Give each group the opportunity to briefly present their visual display and approach to solving their problem. If time allows, highlight the different ways in which students used diagrams, equations, and vocabulary to represent their work.

Activity 3: Optional

Factors and Multiples Bingo

20 min

Activity Narrative

In this activity, students play bingo to review factors and multiples. There are two versions of the game:

Version A: 10 Anywhere

- The calling cards are mixed, and one is randomly selected.
- The statement on the card is read out loud and recorded for all to see.
- Students cover *all* numbers that fit the statement. If a number has already been covered, it gets a second chip stacked on top.
- 10 markers anywhere on the board make a bingo.

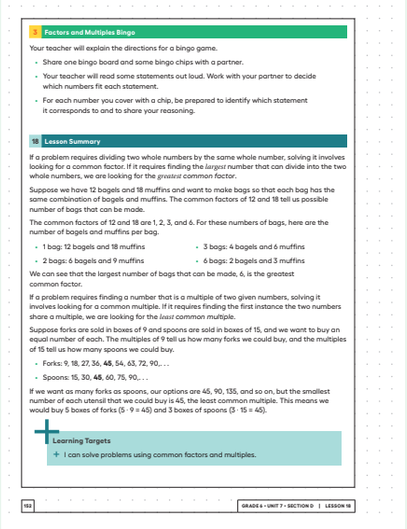
Version B: 4 in a Row

- The calling cards are mixed, and one is randomly selected.
- The statement on the card is read out loud and recorded for all to see.
- Students cover *all* numbers that fit the statement. If a number has already been covered, it does *not* get a second chip stacked on top.
- 4 markers in a row make a bingo.

Launch

Arrange students in groups of 2. Distribute bingo chips and 1 bingo board to each group. Explain the rules for the version of bingo that will be played. Play as many rounds as time allows before a whole-class discussion.

Student Workbook



7 Factors and Multiples Bingo

Your teacher will explain the directions for a bingo game.

- Share one bingo board and some bingo chips with a partner.
- Your teacher will read some statements out loud. Work with your partner to decide which numbers fit each statement.
- For each number you cover with a chip, be prepared to identify which statement it corresponds to and to share your reasoning.

12 Lesson Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *greatest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so that each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 12 and 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

- 1 bag: 12 bagels and 18 muffins
- 2 bags: 6 bagels and 9 muffins
- 3 bags: 4 bagels and 6 muffins
- 6 bags: 2 bagels and 3 muffins

We can see that the largest number of bags that can be made, 6, is the greatest common factor.

If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the first instance the two numbers share a multiple, we are looking for the *least common multiple*.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy.

- Forks: 9, 18, 27, 36, **45**, 54, 63, 72, 90, . . .
- Spoons: 15, 30, **45**, 60, 75, 90, . . .

If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of each utensil that we could buy is 45, the least common multiple. This means we would buy 5 boxes of forks ($5 \cdot 9 = 45$) and 3 boxes of spoons ($3 \cdot 15 = 45$).

Learning Targets

+ I can solve problems using common factors and multiples.

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Student Task Statement

Your teacher will explain the directions for a bingo game.

- Share one bingo board and some bingo chips with a partner.
- Your teacher will read some statements out loud. Work with your partner to decide which numbers fit each statement.
- For each number you cover with a chip, be prepared to identify which statement it corresponds to and to share your reasoning.

No response needed.

Activity Synthesis

The purpose of this discussion is to reflect on any strategies used during the game. After playing a few rounds, discuss:

- *"Is this a game of luck, strategy, or a combination of both? Explain your reasoning."*

  Luck is part of the game, but students must also be familiar with the math vocabulary words and must also be accurate in identifying all the factors and multiples of various numbers.

- *"Are some numbers better to have on a game board than others? Explain your reasoning."*

  Yes, the number zero is not a good number to have because not many statements fit this value. Larger composite numbers are more likely to get covered because they have many factors and could also be multiples of the smaller numbers.

Lesson Synthesis

The goal of this discussion is for students to summarize their understanding of factors and multiples. Here are some questions for discussion:

- *"What is the greatest common factor of two whole numbers? How can it be determined?"*

  The greatest common factor of 2 whole numbers is the greatest number that evenly divides both numbers, without a remainder. One way to find the greatest common factor is to list all factors for both numbers and choose the largest number that appears in both lists.

- *"What types of situations involve finding the greatest common factor?"*

  situations that involve having to divide two different numbers into equal groups, with no remainders

- *"What is the least common multiple of two whole numbers? How can it be determined?"*

  The least common multiple of two whole numbers is the smallest number that is a multiple of both numbers. One way to find the least common multiple is to list multiples of each number in order and choose the first number that appears on both lists.

- *"What types of situations involve finding the least common multiple?"*

  situations that involve different numbers that need to be multiplied to make the same number

Lesson Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *greatest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so that each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 12 and 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

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We can see that the largest number of bags that can be made, 6, is the greatest common factor.

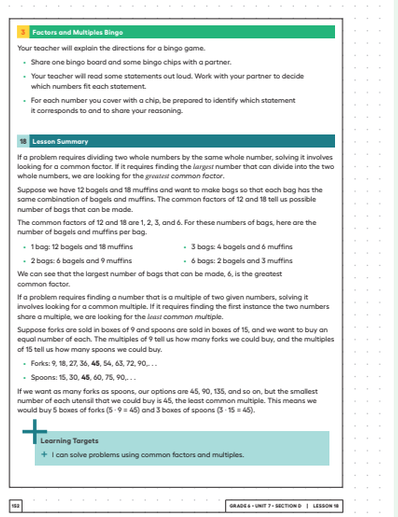
If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the *first* instance the two numbers share a multiple, we are looking for the *least common multiple*.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy.

- Forks: 9, 18, 27, 36, **45**, 54, 63, 72, 90, ...
- Spoons: 15, 30, **45**, 60, 75, 90, ...

If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of each utensil that we could buy is 45, the least common multiple. This means we would buy 5 boxes of forks ($5 \cdot 9 = 45$) and 3 boxes of spoons ($3 \cdot 15 = 45$).

Student Workbook



3 Factors and Multiples Bingo

Your teacher will explain the directions for a bingo game.

- Share one bingo board and some bingo chips with a partner.
- Your teacher will read some statements out loud. Work with your partner to decide which numbers fit each statement.
- For each number you cover with a chip, be prepared to identify which statement it corresponds to and to share your reasoning.

18 Lesson Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *greatest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so that each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

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Learning Targets

+ I can solve problems using common factors and multiples.

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Responding To Student Thinking

Next Day Supports
If most students struggle with understanding the difference between common multiples and common factors in context, revisit this *Cool-down* before the end of the unit. Ask students to find either the least common multiple or greatest common factor and to use the value in the situation to determine if it makes sense.

Cool-down

What Kind of Problem?

5 min

Student Task Statement

1. For each problem, tell whether finding the answer requires finding a greatest common factor or a least common multiple. You do not need to solve the problems.

   a. Elena has 20 apples and 35 crackers. She wants to make as many snack bags as possible that each contain the same combination of apples and crackers. What is the largest number of snack bags she can make?

   Greatest common factor

   b. A string of lights flashes two colors—red every 5 seconds and blue every 3 seconds. How long before both lights flash at the same time?

   Least common multiple

   c. A florist orders sunflowers every 6 days and daisies every 4 days. On which day will she order both kinds of flowers on the same day?

   Least common multiple

Lesson 18

Practice Problems

5 Problems

Problem 1

Mai, Clare, and Noah are making signs to advertise the school dance. It takes Mai 6 minutes to complete a sign, it takes Clare 8 minutes to complete a sign, and it takes Noah 5 minutes to complete a sign. They keep working at the same rate for a half hour.

a. Will Mai and Clare complete a sign at the same time? Explain your reasoning.

Yes

Sample reasoning: They will both finish at 24 minutes, because 24 is a common multiple of 6 and 8.

b. Will Mai and Noah complete a sign at the same time? Explain your reasoning.

Yes

Sample reasoning: They will both finish at 30 minutes, because 30 is a common multiple of 6 and 5.

c. Will Clare and Noah complete a sign at the same time? Explain your reasoning.

No

Sample reasoning: The first common multiple of 8 and 5 is 40, and 40 minutes is longer than a half hour.

d. Will all three students complete a sign at the same time? Explain your reasoning.

No

Sample reasoning: If Clare and Noah will not finish together, then all three won't either.

Problem 2

from Unit 7, Lesson 16

Diego has 48 purple beads, 64 blue beads, and 100 orange beads to make a bracelet. He wants to make bracelets that have all three colors and the same number of each color per bracelet.

a. How many bracelets can he make without having any beads left over?

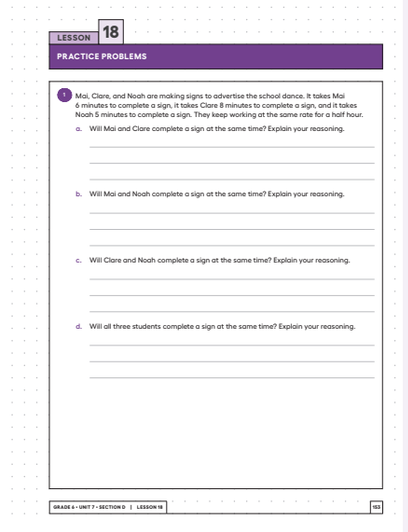
4 bracelets with 12 purple beads, 16 blue beads, and 25 orange beads.

b. Find another solution to this problem.

2 bracelets with 24 purple beads, 32 blue beads, and 50 orange beads.

The two solutions could swap places.

Student Workbook


LESSON 18

PRACTICE PROBLEMS

1. Mai, Clare, and Noah are making signs to advertise the school dance. It takes Mai 6 minutes to complete a sign, it takes Clare 8 minutes to complete a sign, and it takes Noah 5 minutes to complete a sign. They keep working at the same rate for a half hour.
   a. Will Mai and Clare complete a sign at the same time? Explain your reasoning.
   b. Will Mai and Noah complete a sign at the same time? Explain your reasoning.
   c. Will Clare and Noah complete a sign at the same time? Explain your reasoning.
   d. Will all three students complete a sign at the same time? Explain your reasoning.

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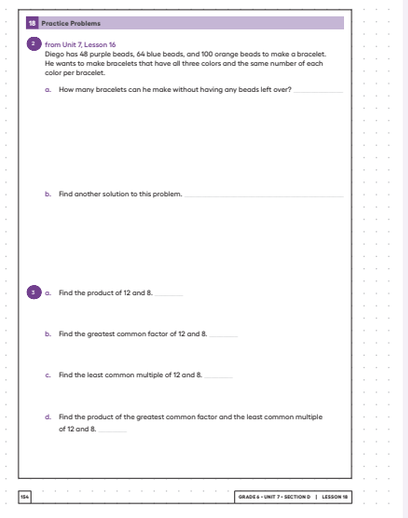
Student Workbook


18 Practice Problems

2. from Unit 7, Lesson 16
   Diego has 48 purple beads, 64 blue beads, and 100 orange beads to make a bracelet. He wants to make bracelets that have all three colors and the same number of each color per bracelet.
   a. How many bracelets can he make without having any beads left over?
   b. Find another solution to this problem.
3. a. Find the product of 12 and 8.
   b. Find the greatest common factor of 12 and 8.
   c. Find the least common multiple of 12 and 8.
   d. Find the product of the greatest common factor and the least common multiple of 12 and 8.

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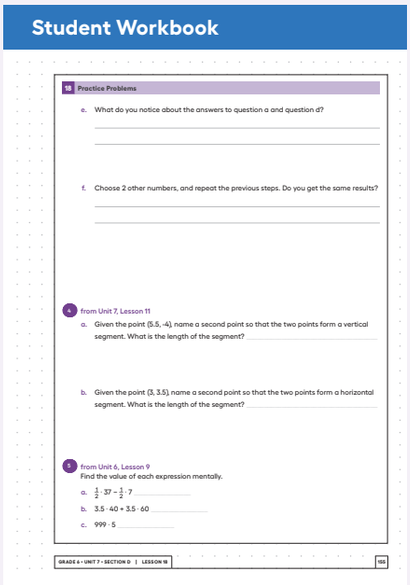
Student Workbook

18 Practice Problems

e. What do you notice about the answers to question a and question d?

f. Choose 2 other numbers, and repeat the previous steps. Do you get the same results?

4 from Unit 7, Lesson 11

a. Given the point (5.5, -4), name a second point so that the two points form a vertical segment. What is the length of the segment?

b. Given the point (3, 3.5), name a second point so that the two points form a horizontal segment. What is the length of the segment?

5 from Unit 6, Lesson 9
Find the value of each expression mentally.

a. $\frac{1}{2} \cdot 37 - \frac{1}{2} \cdot 7$

b. $3.5 \cdot 40 + 3.5 \cdot 60$

c. $999 \cdot 5$

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Problem 3

a. Find the product of 12 and 8.

$12 \cdot 8 = 96$

b. Find the greatest common factor of 12 and 8.

4 is the greatest common factor.

c. Find the least common multiple of 12 and 8.

24 is the least common multiple.

d. Find the product of the greatest common factor and the least common multiple of 12 and 8.

$4 \cdot 24 = 96$

e. What do you notice about the answers to question 1 and question 4?

The answers are the same.

f. Choose 2 other numbers, and repeat the previous steps. Do you get the same results?

Yes, the results are the same.

Problem 4

from Unit 7, Lesson 11

a. Given the point (5.5, -4), name a second point so that the two points form a vertical segment. What is the length of the segment?

Sample response: (5.5, -7)

The length is 3 because $-4 - (-7) = 3$

b. Given the point (3, 3.5), name a second point so that the two points form a horizontal segment. What is the length of the segment?

Sample response: (0, 3.5)

The length is 3 because $3 - 0 = 3$

Problem 5

from Unit 6, Lesson 9

Find the value of each expression mentally.

a. $\frac{1}{2} \cdot 37 - \frac{1}{2} \cdot 7$

15

b. $3.5 \cdot 40 + 3.5 \cdot 60$

350

c. $999 \cdot 5$

4,995