Lesson 10

Bases and Heights of Triangles

Goals

- Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.
- Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.

Learning Targets

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson Narrative

In this lesson, students further their ability to identify and work with bases and heights in a triangle by:

- Drawing a triangle with a given area (reversing the reasoning process they have done so far).
- Learning to draw (not just to recognize) a segment to show the corresponding height for any given base.

The lesson also includes an optional activity, in which students learn to choose appropriate base-height pairs to enable area calculations. Because there are three possible pairs of bases and heights in any triangle, some care is needed in identifying the right combination of measurements. Some base-height pairs may be more practical or efficient to use than others, so it helps to be strategic in choosing a side to use as a base.

Student Learning Goal

Let's use different base-height pairs to find the area of a triangle.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 2
- Index cards: Activity 1, Activity 2

Required Preparation

Warm-up:
For the digital version of the activity, acquire devices that can run the applet.

Activity 1:
Each student especially needs an index card.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	25 min	15 min	10 min	5 min

Warm-up
An Area of 12

10 min

Activity Narrative

There is a digital version of this activity.

In this *Warm-up*, students are given an area measure and are asked to create several triangles with that area. This work involves reversing the reasoning process used in previous lessons, in which students were given triangles with measurements and asked to find the area.

Students are likely to gravitate toward right triangles first (or to halve rectangles that have factors of 12 as their side lengths). This is a natural and productive starting point. Prompting students to create non-right triangles encourages them to apply their understanding of the area of non-right parallelograms.

As students work alone and discuss with partners, notice the strategies they use to draw their triangles and to verify their areas. Identify a few students with different strategies and, later, ask them to share.

In the digital version of the activity, students use an applet to draw triangles on a grid and reason about their area. The applet allows students to adjust the vertices of line segments, measure lengths, and make annotations.

Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Give students 2–3 minutes of quiet think time and 2 minutes to share their drawings with their partner afterwards. Encourage students to refer to previous work as needed. If students finish their first drawing early, tell them to draw a different triangle with the same area.

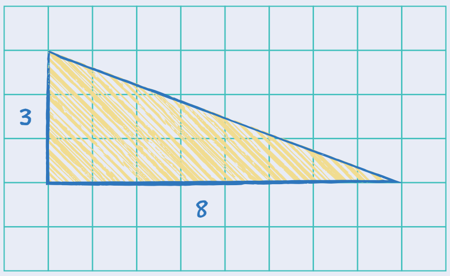
During partner discussion, each partner should convince the other that the triangle drawn is indeed 12 square units.

Student Task Statement

On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.

Sample responses:

- This right triangle has a base of 8 units and a height of 3 units. The area is half of $3 \cdot 8$ or half of 24, which is 12.



Building on Student Thinking

If students have trouble getting started, ask:

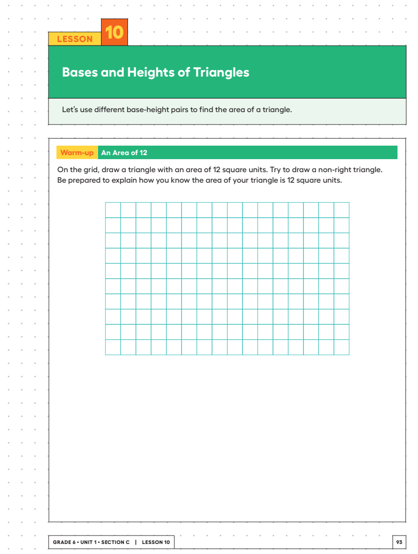
"Can you draw a quadrilateral with an area of 12?"

"Can you use what you know about parallelograms to help you?"

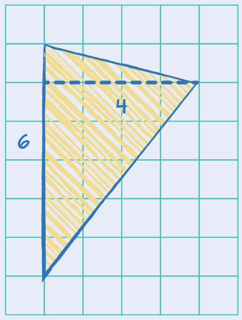
"Can you use any of the area strategies—decomposing, rearranging, enclosing, subtracting—to arrive at an area of 12?"

Students who start by drawing rectangles and other parallelograms may use factors of 12, instead of factors of 24, for the base and height. If this happens, ask them what the area of the their quadrilateral is and how it relates to the triangle they are trying to draw.

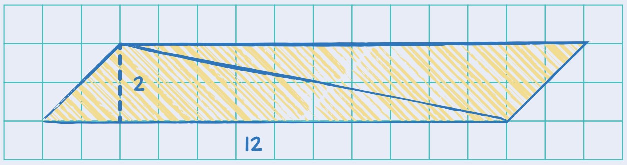
Student Workbook



- This triangle has a side of 6 units. This can be the base. Draw a height segment that is perpendicular to the base and is 4 units long. The area of the triangle is $b \cdot h \div 2$, so it is $6 \cdot 4 \div 2$, which is 12



- Draw a parallelogram with a base of 12 and a height of 2, and then draw a diagonal line to create two identical triangles. Each of the triangles has an area of 12 because it is half of a parallelogram with an area of 24



Activity Synthesis

Invite a few students to share their drawings and ways of reasoning with the class. For each drawing shared, ask the creator for the base and height and record them for all to see. Ask the class:

"Did anyone else draw an identical triangle?"

"Did anyone draw a different triangle but with the same base and height measurements?"

To reinforce the relationship between base, height, and area, discuss:

"Which might be a better way to draw a triangle: by starting with the base measurement or with the height? Why?"

"Can you name other base-height pairs that would produce an area of 12 square units without drawing? How?"

Activity 1

Hunting for Heights

25 min

Activity Narrative

Students may be able to recognize a measurement that can be used for height when they see it, but identifying and drawing an appropriate segment is more challenging. This activity, and the demonstration needed to launch it, gives students a concrete strategy for identifying a height accurately. When students use a strategy of drawing an auxiliary line to solve problems, they are looking for and making use of structure. Explicit instruction, as in this activity, is often needed before students can be expected to use this strategy spontaneously.

This is the first time Math Language Routine 3: *Critique, Correct, Clarify* is suggested in this course. In this routine, students are given a "first draft" statement or response to a question that is intentionally unclear, incorrect, or incomplete. Students analyze and improve the written work by first identifying what parts of the writing need clarification, correction, or details, and then writing a second draft (individually or with a partner). Finally, the teacher scribes as a selected second draft is read aloud by its author(s), and the whole class is invited to help edit this "third draft" by clarifying meaning and adding details to make the writing as convincing as possible to everyone in the room. Typical prompts are: "Is anything unclear?" and "Are there any reasoning errors?". The purpose of this routine is to engage students in analyzing mathematical writing and reasoning that is not their own, and to solidify their knowledge and use of language.

Launch

Explain to students that they will try to draw a height that corresponds to each side of a triangle. Arrange students in groups of 2. Give each student an index card and 1–2 minutes to complete the first question. Remind them that there is more than one correct way to draw the corresponding height for a base. Ask them to pause after the first question. As students work, notice how students are using the index cards (if at all).

Afterward, solicit a few quick comments on the exploration. Ask questions such as:

"How did you know where to draw the segments?"

"How did you draw them?"

"Why were you given index cards? How might they help?"

Explain that you will now demonstrate a way to draw heights effectively. (If any students used the index card correctly, acknowledge that they were on the right track.)

Remind students that any line we draw to show the height of a triangle must be drawn *perpendicular* to the base. Having a tool with a right angle and with straight edges can help us make sure the line we draw is both straight and perpendicular to the base. This is what the index card is for.

Ask:

"How do we know where to stop this line we are drawing? How long should it be?"

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 1, Narrative)

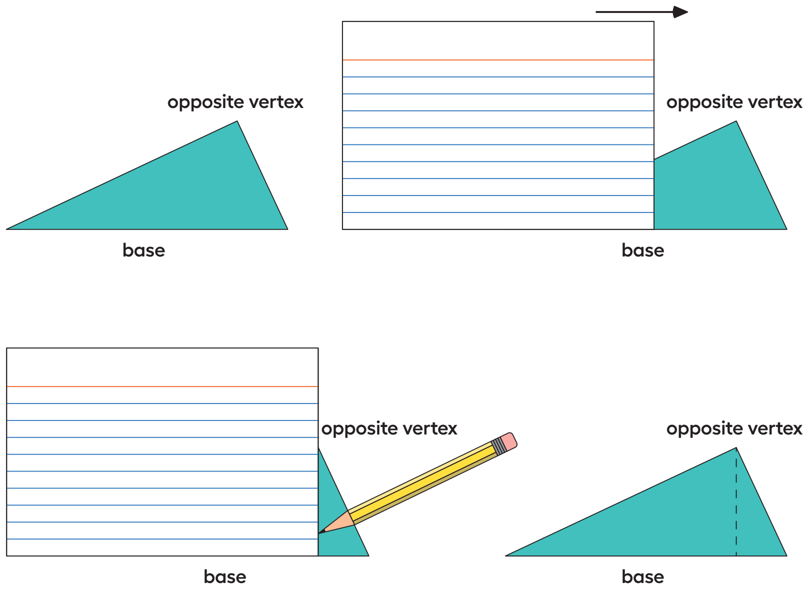
MLR3: Critique, Correct, Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Access for Perception.
Students may benefit from watching the demonstration more than once. Consider pulling a small group of students aside for additional demonstration and clarification.
Supports accessibility for: Language, Attention

Explain that the easiest way is to draw the line so it would pass through the **vertex** opposite the chosen base. Draw or display a triangle for all to see. Demonstrate the following.

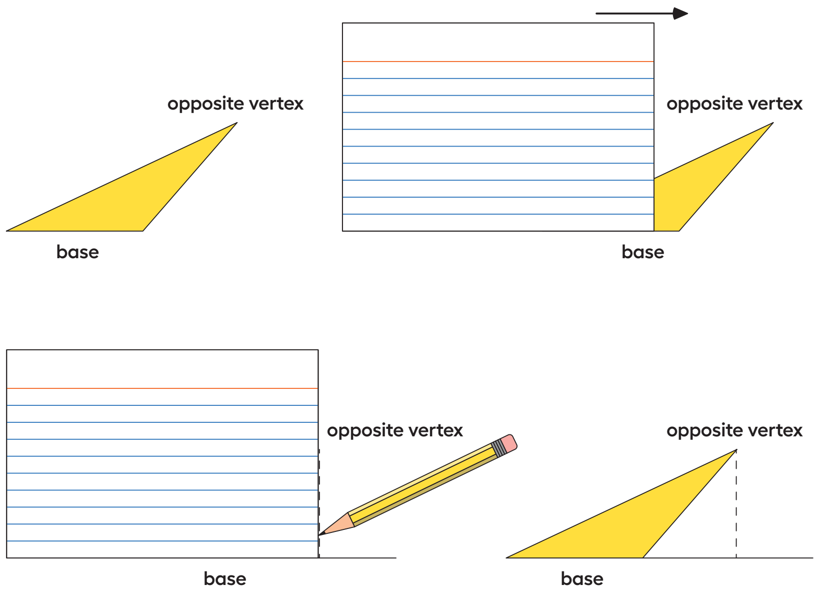
- Choose one side of the triangle as the base. Identify the opposite vertex.
- Line up one **edge** of the index card with that base.
- Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
- Use that edge to draw a line segment from that vertex to the base. The measure of that segment is the height.



Ask:

🗨 *"What if the opposite vertex is not directly over the base?"*

Explain that sometimes we need to extend the line of the base. Demonstrate the process.



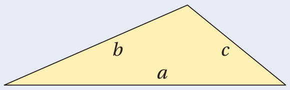
Demonstrate the process with another example in which the card needs to slide from right to left (for example, by rotating the obtuse triangle above clockwise). Left-handed students may find this particularly helpful.

Prompt students to use this method to check the heights they drew in response to the first question, revise the drawings if they were incorrect, and share their revisions with their partners. Circulate, and support students as they draw. Those who finish verifying the heights in the first question can move on to complete the rest of the activity with their partners.

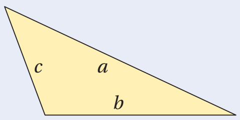
Student Task Statement

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

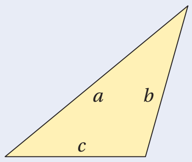
   Use Side a as the base:



   Use Side b as the base:

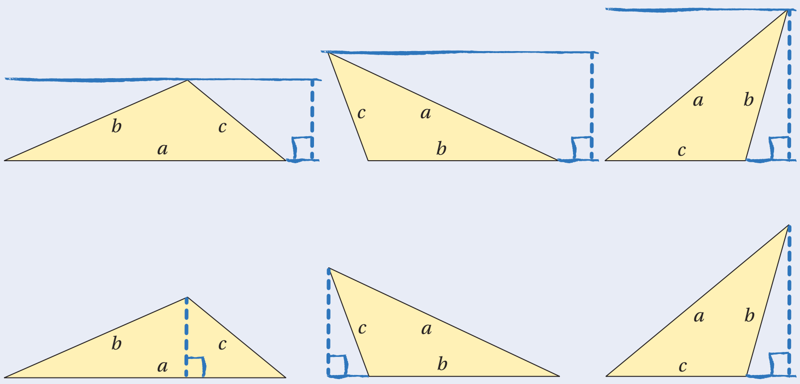


   Use Side c as the base:



   Pause for your teacher's instructions before moving to the next question.

   Sample responses:



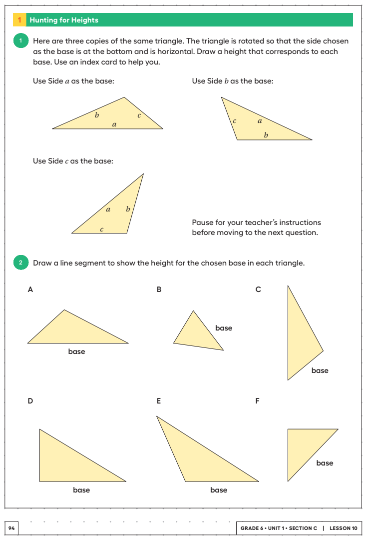
Building on Student Thinking

Some students may use the index card simply as a straightedge and therefore draw heights that are not perpendicular to the given base. Remind them that a height needs to be perpendicular (at a right angle) to the base. If necessary, demonstrate again how the corner of the index card can be used to draw the height at a right angle to the base.

Students may mistakenly think that a base must be a horizontal side of a triangle (or one closest to being horizontal) and a height must be drawn inside of the triangle. Point to some examples from earlier work to remind students that neither is true. Remind them to align their index card to the side labeled "base."

Some students may find it awkward to draw height segments when the base is not horizontal. Encourage students to rotate their paper as needed to make drawing easier.

Student Workbook


1 Hunting for Heights

1 Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

Use Side a as the base:

Use Side b as the base:

Use Side c as the base:

Pause for your teacher's instructions before moving to the next question.

2 Draw a line segment to show the height for the chosen base in each triangle.

A base

B base

C base

D base

E base

F base

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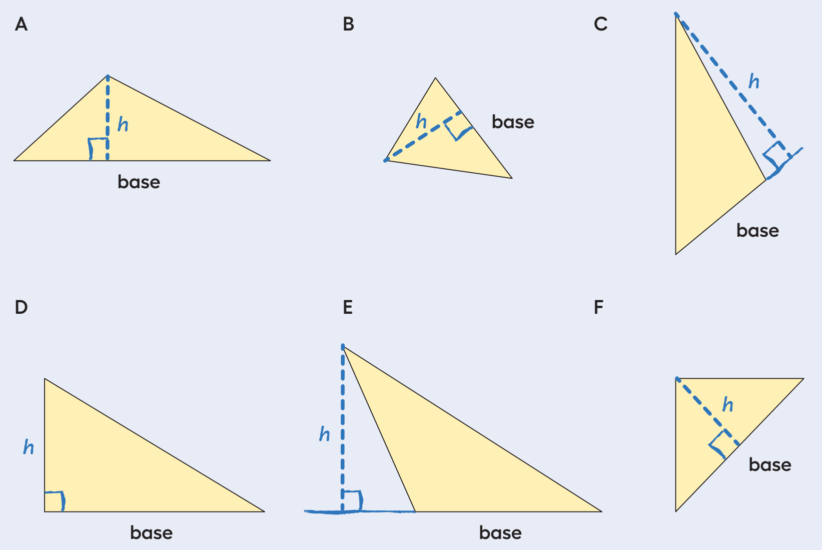
2. Draw a line segment to show the height for the chosen base in each triangle.



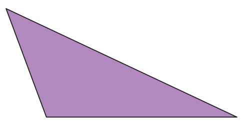
Activity Synthesis

If time permits, consider selecting one student to share the height drawing for each triangle, or display the solutions for all to see.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response about drawing a height segment of a triangle by correcting errors, clarifying meaning, and adding details.

- Display the following triangle and first draft:

"To draw a height segment for a triangle like this one, we would always need to extend the base, no matter which side it is."



- Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–3 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

The goal of the discussion is to highlight that:

- A segment that represents height needs to be drawn perpendicular to the side chosen as the base.
- The perpendicular segment needs to connect the base and the vertex opposite the base.

- If this can't be done, we can first extend the base. Then, we can draw a perpendicular segment that connects the extension and the opposite vertex.
- Alternatively, we can draw a line that is parallel to the base and goes through the opposite vertex. Then, we can draw a perpendicular segment that connects the base and that line.

Activity 2: Optional
Some Bases Are Better Than Others

15 min

Activity Narrative

This activity allows students to practice identifying the base and height of triangles and using them to find areas.

Because there are no directions on which base or height to use, and because not all sides would enable them to calculate area easily, students need to think structurally and choose strategically. All triangles in the problems have either a vertical or a horizontal side. Choosing such a side as the base makes it easier to identify the corresponding height.

In some cases, students may opt to use a combination of area-reasoning strategies rather than finding the base and height of the shaded triangles and applying the formula. For instance, they may enclose a shaded triangle with a rectangle and subtract the areas of extra triangles (with or without using the formula on those extra triangles). Notice students who use such strategies so they can share later.

Launch

Keep students in groups of 2. Explain that they will now practice locating or drawing heights and using them to find the area of triangles. Give students 8–10 minutes of quiet think time and time to share their responses with a partner afterward. Provide access to their geometry toolkits (especially index cards).

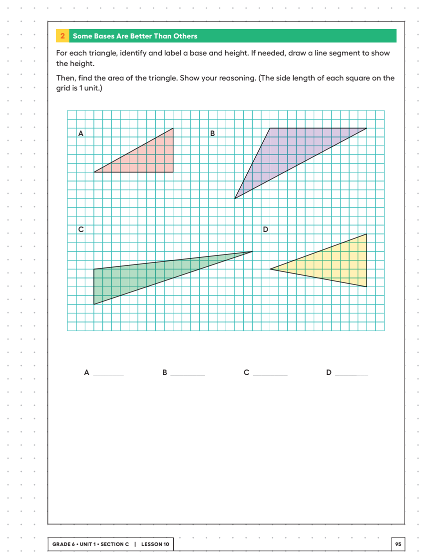
If time is limited, consider asking students to find the area of two or three triangles instead of all four.

Building on Student Thinking

Students may think that a vertical side of a triangle is the height regardless of the segment used as the base. If this happens, have them use an index card as a straightedge to check if the two segments they are using as base and height are perpendicular.

Some students may not immediately see that choosing a side that is either vertical or horizontal would enable them to find the corresponding height very easily. They may choose a non-vertical or non-horizontal side and not take advantage of the grid. Ask if a different side might make it easier to determine the base-height lengths without having to measure.

Student Workbook


2 Some Bases Are Better Than Others

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)

A ______ B ______ C ______ D ______

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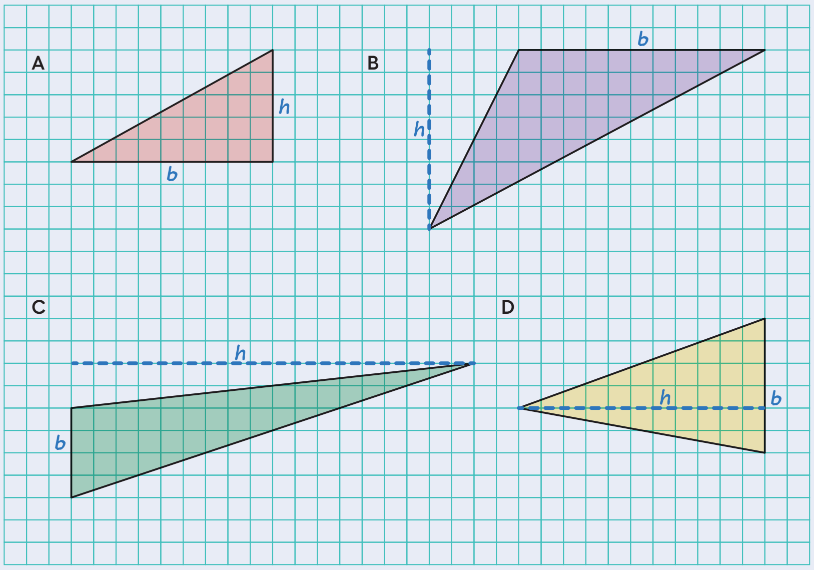
Student Task Statement

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



Triangle A area: 22.5 square units

$b = 9$ and $h = 5$, $5 \div 2 = 22.5$

Triangle B area: 44 square units

$b = 11$ and $h = 8$, $11 \cdot 8 \div 2 = 44$

Triangle C area: 36 square units

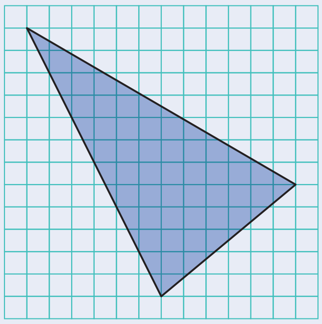
$b = 4$ and $h = 18$, $4 \cdot 18 \div 2 = 36$

Triangle D area: 33 square units

$b = 6$ and $h = 11$, $6 \cdot 11 \div 2 = 33$

Are You Ready for More?

Find the area of this triangle. Show your reasoning.



51, since we can enclose the given triangle in a square that has an area of 144 ($12 \cdot 12 = 144$), then subtract away the area from right triangles in each corner.

Activity Synthesis

Focus the whole-class discussion on how students went about identifying bases and heights. Discuss:

"Which side did you choose as the base for Triangle B? Triangle C? Why?"

"Aside from choosing a vertical or horizontal side as the base, is there another way to find the area of the shaded triangles without using their bases and heights?"

Invite a couple of students who use the enclose-and-subtract method to find the area of Triangles B, C, or D to share.

"Which strategy do you prefer or do you think is more efficient?"

"Can you think of an example where it might be preferable to find the base and height of the triangle of interest?"

Students may point to any of the triangles in the task.

"Can you think of an example where it might be preferable to enclose the triangle of interest and subtract other areas?"

Students may point to the triangle shown in "Are You Ready for More?", where none of the shaded triangle's sides are horizontal or vertical.

Lesson Synthesis

In this lesson, students looked closely at the heights of a triangle. They located or drew a height using any side of a triangle as the base. They also considered which pair of base and height to use to find area. Discuss with students:

"What must we remember about the relationship between a base of a triangle and its corresponding height?"

The height must be perpendicular to the base.

"What tools might help us draw a height segment? What is it about an index card or a ruler that helps us?"

A tool with straight edges and a right angle can help us draw perpendicular segments.

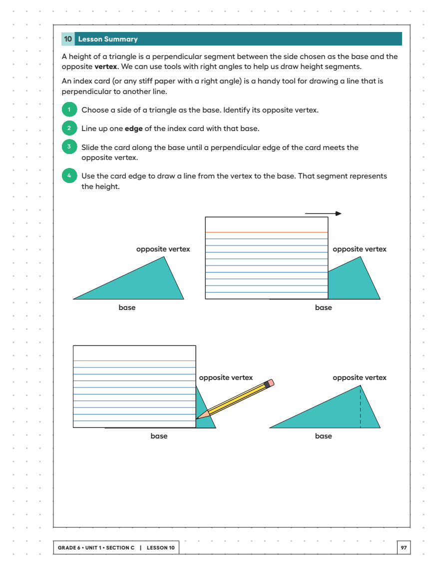
"Every triangle has multiple base-height pairs. Does it matter which side we choose as the base? How do we decide?"

For the base, we need a side with a known length. For the height, we need a segment that is perpendicular to that base and whose length we can determine.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to help students produce statements that describe the strategies they use to identify bases and heights, and to find area. For example, "For triangle ___, I chose side ___ as the base because ..." or "The next time I need to find the area of a triangle, the strategy I will use is _____, because ..."
Advances: Speaking, Conversing

Student Workbook


10 Lesson Summary
A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite **vertex**. We can use tools with right angles to help us draw height segments.
An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.
1 Choose a side of a triangle as the base. Identify its opposite vertex.
2 Line up one **edge** of the index card with that base.
3 Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4 Use the card edge to draw a line from the vertex to the base. That segment represents the height.

Lesson Summary

A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite **vertex**. We can use tools with right angles to help us draw height segments.

An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
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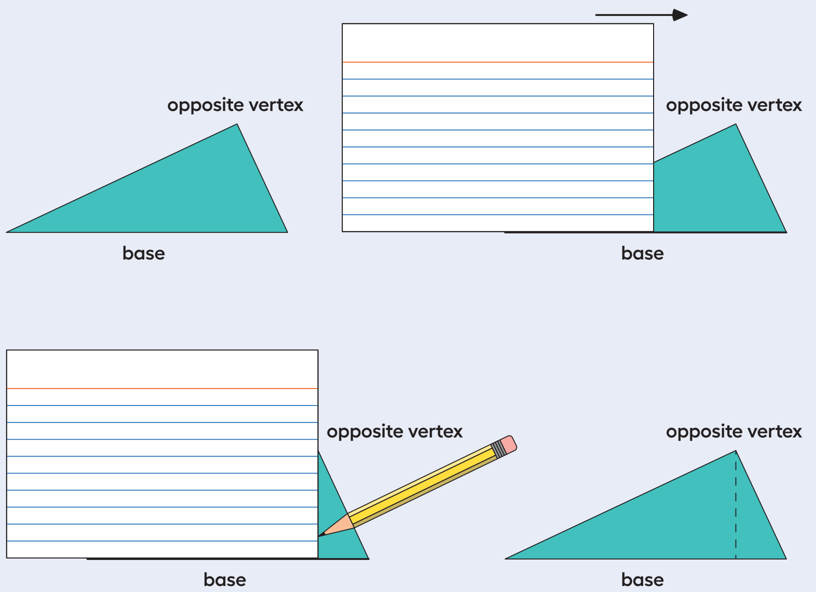
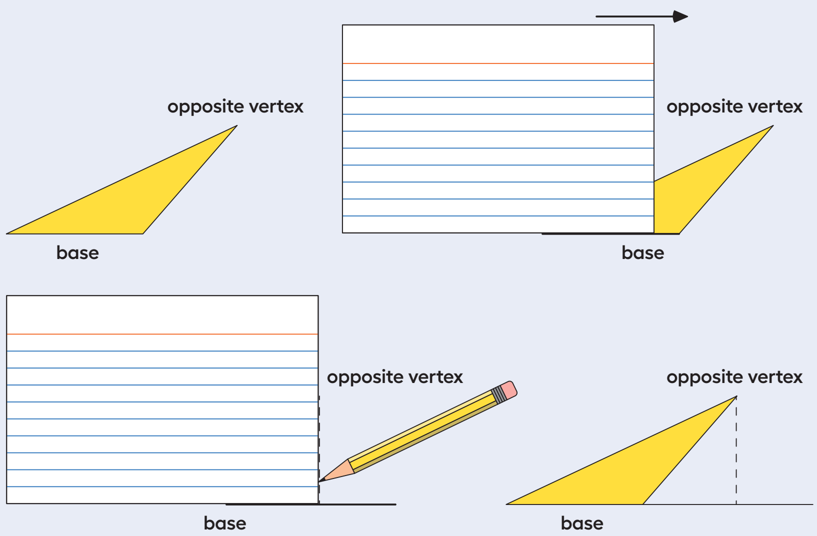


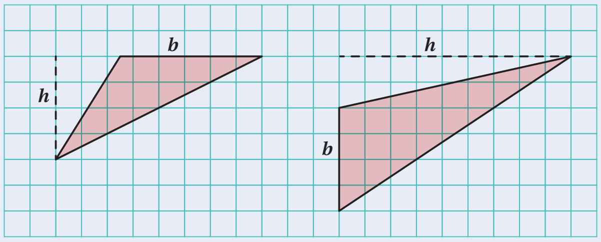
Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.



Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically.

For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



Cool-down

Stretched Sideways

5 min

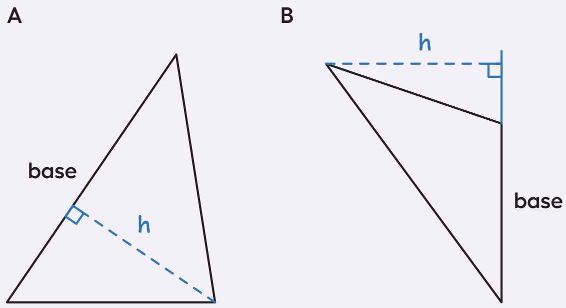
Launch

Provide access to geometry toolkits.

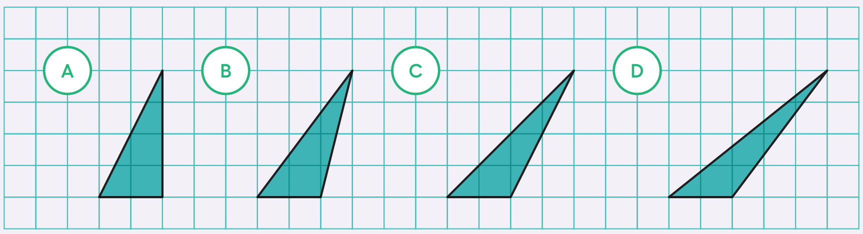
Student Task Statement

1. For each triangle, draw a height segment that corresponds to the given base, and label it h. Use an index card if needed.

   There are many possible locations for a height segment. The segments shown are the most straightforward.



2. Which triangle has the greatest area? The least area? Explain your reasoning.



All of the triangles have the same area: 4 square units

Sample reasoning: They all have a base of 2 units and a height of 4 units.

Responding To Student Thinking

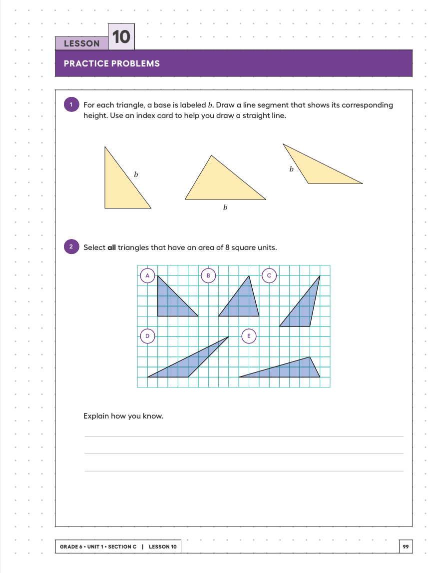
Points to Emphasize

If students struggle with identifying bases and corresponding heights or with calculating areas, revisit this idea when opportunities arise over the next several lessons. For example, these practice problems require students to identify the bases and corresponding heights of triangles and to calculate their areas:

Unit 1, Lesson 11, Practice Problem 5

Unit 1, Lesson 11, Practice Problem 6

Student Workbook


LESSON 10

PRACTICE PROBLEMS

1 For each triangle, a base is labeled b. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.

2 Select **all** triangles that have an area of 8 square units.

Explain how you know.

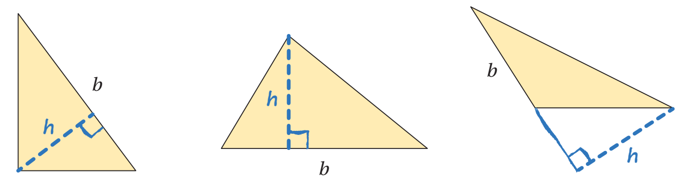
Lesson 10

Practice Problems

6 Problems

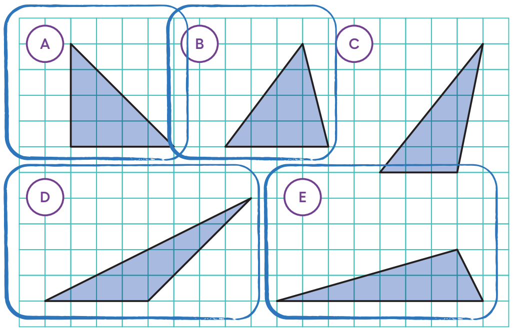
Problem 1

For each triangle, a base is labeled b. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.



Problem 2

Select **all** triangles that have an area of 8 square units. Explain how you know.



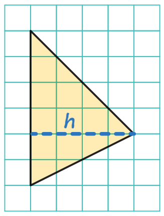
Sample reasoning:

- Triangles A, B, and D all have a horizontal base of 4 units and a height of 4 units. $\frac{4 \cdot 4}{2} = 8$, so the area of each is 8 square units.
- Triangle C has a horizontal base of 3 units and a height of 5 units, so its area is 7.5 square units.
- Triangle E has a horizontal base of 8 units and a height of 2 units, so its area is 8 square units, since $\frac{8 \cdot 2}{2} = 8$.

Problem 3

Find the area of the triangle. Show your reasoning.

If you get stuck, carefully consider which side of the triangle to use as the base.

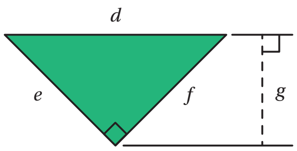


12 square units

Sample reasoning: The vertical side is 6 units long, and that side can be used as the base. The corresponding height, shown in the diagram, is 4 units. So the area is 12 square units. Another method is to surround the triangle with a rectangle then subtract the parts that are not in the triangle.

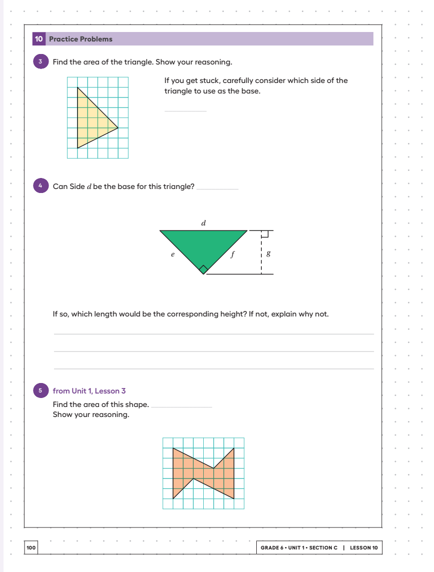
Problem 4

Can Side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



Yes, Side d can be the base, because it is a side of the triangle. The corresponding height is g.

Student Workbook


10 Practice Problems

3 Find the area of the triangle. Show your reasoning.

If you get stuck, carefully consider which side of the triangle to use as the base.

4 Can Side d be the base for this triangle?

If so, which length would be the corresponding height? If not, explain why not.

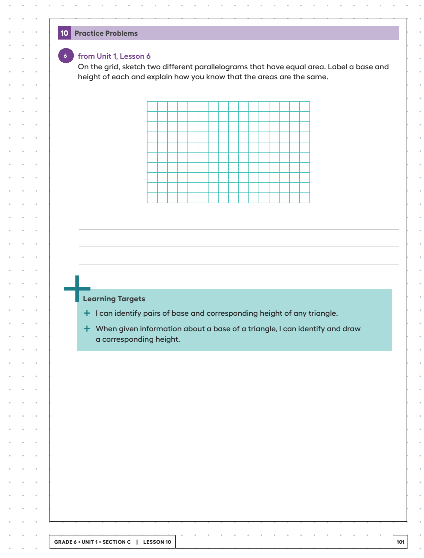
5 from Unit 1, Lesson 3

Find the area of this shape.

Show your reasoning.

100 GRADE 6 • UNIT 1 • SECTION C | LESSON 10

Student Workbook



10 Practice Problems

6 from Unit 1, Lesson 6

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know that the areas are the same.

Learning Targets

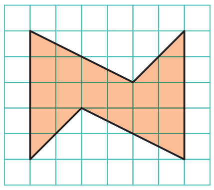
+ I can identify pairs of base and corresponding height of any triangle.
+ When given information about a base of a triangle, I can identify and draw a corresponding height.

GRADE 6 • UNIT 1 • SECTION C | LESSON 10 101

Problem 5

from Unit 1, Lesson 3

Find the area of this shape. Show your reasoning.



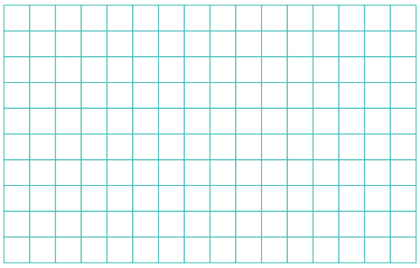
18 square units

Reasoning varies.

Problem 6

from Unit 1, Lesson 6

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know that the areas are the same.



Answers vary.