Creating Double Number Line Diagrams

Goals

- Comprehend and use the word "per" (orally and in writing) to mean "for each."
- Draw and label a double number line diagram from scratch, with parallel lines and equally-spaced tick marks.
- Use double number line diagrams to find a wider range of equivalent ratios.

Learning Targets

- I can create a double number line diagram and correctly place and label tick marks to represent equivalent ratios.
- I can explain what the word "per" means.

Lesson Narrative

In this lesson, students create double number line diagrams from scratch. They see that it is important to use parallel lines, equally-spaced tick marks, and descriptive labels. They are also introduced to using the word **per** to refer to how much of one quantity there is for every one unit of the other quantity.

As students progress through the activities, they learn to draw double number line diagrams with decreasing levels of scaffolding. In a preceding lesson, students put the numbers on pre-drawn lines with tick marks and extrapolated the values up the number line. Here, students start by placing their own tick marks and interpolating the given values down to smaller numbers. Next, they draw a double number line diagram from scratch but by following instructions. Finally, in an optional activity, they create a double number line diagram with no guidance.

In the optional activity, students revisit a situation previously represented with a discrete diagram. They examine how the new representation can help to answer questions about the ratios in the situation.

In later lessons, students make their own strategic choice of an appropriate representation to support their reasoning. Regardless of method, students practice attending to precision as they indicate the units that go with the numbers in a ratio in both verbal statements and diagrams.

Note that students are not expected to use or understand the term "unit rate" in this lesson.

Student Learning Goal

Let's draw double number line diagrams like a pro.

Lesson Timeline

10 mins

Warm-up

10 mins

Activity 1

15 mins

Activity 2

10 mins

Activity 3

10 min

Lesson Synthesis

Access for Students with Diverse Abilities

• Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Warm-up, Activity 2)
- MLR8: Discussion Supports (Warmup, Activity 2)

Instructional Routines

 MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

• Rulers: Activity 2

Required Preparation

Activity 1:

Consider bringing back the blue and yellow water mixtures or the applet from an earlier activity to remind students of the context and to support their reasoning.

Assessment

5 mins

Cool-down

Warm-up

Ordering on a Number Line



Activity Narrative

In this Warm-up, students partition a number line and locate fraction and decimal equivalents in preparation for working with double number lines in this unit. Students are purposely not asked to locate 1 on the number line to see how they reason about locating the $\frac{1}{2}$ and $\frac{1}{4}$. It is important for students to be able to identify the fractions or decimals and label tick marks correctly, interpreting the distance between tick marks rather than the number of tick marks, as the fractional size.

As students discuss with their partner, select students to share their answers to the first question during the whole-class discussion.

Launch



Arrange students in groups of 2. Display the number line for all to see.

Give students 2 minutes of quiet think time.

Ask students to compare their number line with a partner's number line and to share the fractions or decimals they chose to place on the number line for the second question.

Student Task Statement

Locate and label each number on the number line:



Activity Synthesis

Select students to explain how they reasoned about the location of each number on the number line. After each number, ask the class whether they agree or disagree, and if anyone else had a different way of thinking about that number.

If time permits, ask students to locate and label a few more fractions or decimals on the number line.

Access for Multilingual Learners (Warm-up, Launch)

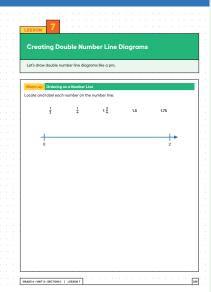
MLR8: Discussion Supports.

Briefly review the meaning of the terms "label" and "tick marks" as you or a student points to these features in the Student Task Statement. Review the meaning of the term "locate" by demonstrating and thinking aloud.

Advances: Reading, Writing

Building on Student Thinking

Students may place $\frac{1}{2}$ in the center of the number line, reasoning that it is half of the number line. Explain to the students that they are placing the number $\frac{1}{2}$, which has a specific value and location on the number line.



Instructional Routines

MLR1: Stronger and Clearer Each Time







Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time

This activity uses the Stronger and Clearer Each Time math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Student with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.

Represent the same information through different modalities by using physical objects and kinesthetic experience to access abstract concepts. For example, some students may benefit by representing the number line on a clothesline. They can place and adjust numbers on folded paper or cardstock on the clothesline.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Activity 1

Just a Little Green



Activity Narrative

In this activity, students continue to use double number lines to reason about equivalent ratios. Students revisit the green water recipe from an earlier activity. This time, their attention is directed to amounts of blue water and yellow water that are less than the 5:15 ratio given in the original recipe but would produce the same shade of green. Students begin to find and use ratios containing a 1.

One key idea to convey here is that finding a ratio associated with 1 unit of a quantity can be very helpful because it can help us determine any equivalent ratio. Another key idea is that the intervals on double number lines can be partitioned to help us find such ratios.

As students work, monitor for those who use division to determine the amount of yellow water for 1 ml of blue water, and then use multiplication to determine the ratios for 8 ml and 11 ml of blue water. This is a key insight for a type of reasoning that is broadly useful and will be developed further.

This is the first time that Math Language Routine 1: Stronger and Clearer Each Time is suggested in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. It is not necessary that students finish this draft before moving to the structured pair meetings step. Students then meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, "What did you mean by ...?" and "Can you say that another way?" Finally, students write a second draft of their response, reflecting ideas from partners and improvements on their initial ideas. Students should be encouraged to incorporate any good ideas and words that they got from their partners to make their second draft stronger and clearer.

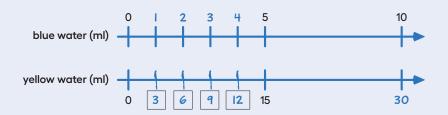
Launch

Ask students to recall what double number lines are and how they can be used to represent problems that involve equivalent ratios. Explain that they are going to investigate the structure of double number lines in more detail.

Give students 5 minutes of quiet think time, and then time to discuss their responses with a partner.

Student Task Statement

The other day, we mixed 5 ml of blue water with 15 ml of yellow water to make green water. Now we want to make a very small batch of the same shade of green water.



- 1. On the number line for blue water, label the four tick marks shown.
 - The tick marks should be labeled 1, 2, 3, and 4 because the fifth tick mark is 5.
- **2.** On the number line for yellow water, draw and label tick marks to show the amount of yellow water needed for each amount of blue water.
 - The tick marks should be labeled 3, 6, 9, and 12 because the fifth tick mark is 15.
- **3.** How much yellow water should be used for 1 ml of blue water? Circle where you can see this on the double number line.
 - 3 ml of yellow water is needed.
- 4. How much yellow water should be used for 11 ml of blue water?
 - 33 ml of yellow water is needed, because 33 = 3 · II.
- 5. How much yellow water should be used for 8 ml of blue water?
 - 24 ml of yellow water is needed, because 24 = 3 · 8.
- **6.** Why is it useful to know how much yellow water should be used with 1 ml of blue water?
 - Sample response: We can use that amount and multiplication to figure out the amount of yellow water needed for any given amount of blue water.

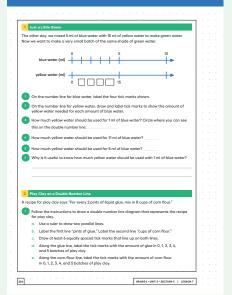
Activity Synthesis

Focus discussions on how students determined the amount of yellow water for 1 ml, 8 ml, and 11 ml of blue water. Ask previously selected students to share their reasoning.

Use Stronger and Clearer Each Time to give students an opportunity to revise and refine their response to the last question on why it is useful to know the amount of yellow water to be used with 1 ml of blue water. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

Building on Student Thinking

Students may have trouble figuring out that the length of a segment between consecutive tick marks is $\frac{1}{5}$ of the interval from 0 to 5, especially because there are four tick marks (not five). When focusing on blue, students' first guess about the tick marks is generally correct. For yellow, remind them that the numbers on the tick marks are made by skip counting; they are then likely to try 3's and 5's because both can make 15. Students who label the spaces between tick marks rather than the tick marks themselves may need additional work with important measurement conventions.



Consider displaying these prompts for feedback:

"The part I understood best was ..."

"What do you mean when you say ...?"

"Can you say more about ...?"

"Can you describe that another way?"

"How do you know ...?"

Close the partner conversations, and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

Provide these sentence frames to help students organize their thoughts in a clear, precise way:

"It is helpful to know how much yellow water to use with 1 ml of blue water because ..."

"If I know how much yellow water to use with 1 ml of blue water, then I can ..."

If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After Stronger and Clearer Each Time, highlight the relationship of blue to yellow using phrases such as "for every 1 ml of ..." or "per milliliter of ..."

There are 3 milliliters of yellow water for every 1 milliliter of blue water."

"There are 3 milliliters of yellow water per milliliter of blue water."

The word **per** means "for every." Ask students to think of any other situation in which they may use the word "per" as it is used here (for example, price per bottle of water, cost per ticket, and so on). Discuss why knowing the value of one item would be helpful.

Activity 2

Play Clay on a Double Number Line

15 min

Activity Narrative

Prior to this point, students were given blank double number line diagrams and were responsible only for labeling them to match the situation. In this activity, students follow instructions for drawing a double number line diagram from scratch. Then, they use their diagram to answer questions about the ratios in a recipe that they encountered in an earlier lesson. During discussion, students reflect on elements that are important when creating a useful double number line diagram.



Tell students that they will now create a double number line diagram from scratch and use their diagram to answer questions about equivalent ratios.

Arrange students in groups of 2. Ensure that each student has access to a ruler. Ask students to check with a partner and come to an agreement about how to draw the diagrams before moving on to Question 3.

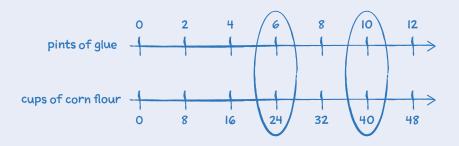
If needed, remind students that the word "per" means "for every."

Student Task Statement

A recipe for play clay says "For every 2 pints of liquid glue, mix in 8 cups of corn flour."

- **1.** Follow the instructions to draw a double number line diagram that represents the recipe for play clay.
 - a. Use a ruler to draw two parallel lines.
 - **b.** Label the first line "pints of glue." Label the second line "cups of corn flour."
 - c. Draw at least 6 equally spaced tick marks that line up on both lines.
 - **d.** Along the glue line, label the tick marks with the amount of glue in 0, 1, 2, 3, 4, and 5 batches of play clay.
 - **e.** Along the corn-flour line, label the tick marks with the amount of corn flour in 0, 1, 2, 3, 4, and 5 batches of play clay.

Sample response:



(Students may circle certain pairs of numbers when answering subsequent questions.)

2. Compare your double number line diagram with your partner's. Discuss your thinking. If needed, revise your diagram.

No response required.

- **3.** Next, use your double number line to answer these questions:
 - a. How much corn flour should be used with 10 pints of glue?

40 cups of corn flour for 10 pints of glue

- b. How much glue should be used with 24 cups of corn flour?
 - 6 pints of glue for 24 cups of corn flour
- c. How much corn flour per pint of glue does this recipe use?

4 cups of corn flour per pint of glue

Access for Multilingual Learners (Activity 2, Launch)

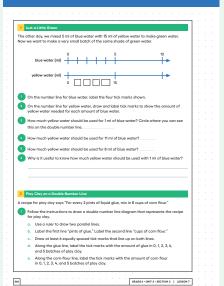
MLR8: Discussion Supports.

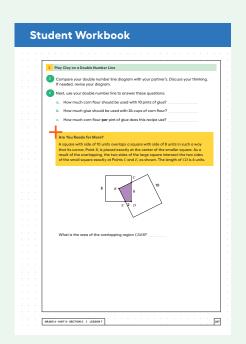
During the *Launch*, review the terms that students will need to access for this activity: "double number line diagram," "parallel lines," "tick marks," "equal increments," "equivalent ratios," and "line up." Use visuals to support understanding of these terms in the context of this problem.

Advances: Representing

Building on Student Thinking

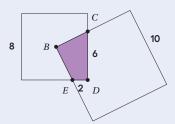
Students may not label tick marks with equal increments or may not align the tick marks.





Are You Ready for More?

A square with side of 10 units overlaps a square with side of 8 units in such a way that its corner, Point B, is placed exactly at the center of the smaller square. As a result of the overlapping, the two sides of the large square intersect the two sides of the small square exactly at Points C and E, as shown. The length of CD is 6 units.



What is the area of the overlapping region CDEB?

16 square units

Sample reasoning: If we extend *BC* and *BE*, the smaller square is partitioned into four regions of equal area.

(A rigorous argument can be made using symmetry, or side lengths and angles, for why these four regions are congruent, but at this stage of learning, students could simply reason that these appear to be four identical copies.)

Since the area of the smaller square is 64 square units, the area of the shaded region is 16 square units, because 64 ÷ 4 = 16.

Activity Synthesis

Select students to explain how they used their double number line diagram to answer the last question. Ask students how they can indicate the number of cups of corn flour per pint of glue on the diagram.

If incorrect or imprecise diagrams are observed in students' work, consider inviting students to take a closer look at diagram construction. Ask questions such as:

"Do the tick marks on each line need to be equally spaced? Why or why not?"

"Do the tick marks on the top line need to match those on the bottom line? Why or why not?"

"Does it matter what number we use to start each line? Why or why not?"

Activity 3: Optional

Revisiting Radish Cake



Activity Narrative

In this activity, students revisit equivalent ratios in a previously introduced context. They see how double number line diagrams are helpful for answering more questions about quantities in a recipe.

Students also see that a double number line diagram can be adapted to include a third number line that represents another quantity. This third quantity forms a ratio with each of the other two quantities in the recipe, so the diagram can be used to find equivalent ratios involving any two quantities or all three of them.

Launch

Remind students that they looked at a recipe for radish cake in an earlier activity. Tell students that they will create a double number line diagram to represent the amounts of ingredients in the recipe and use it to answer questions.

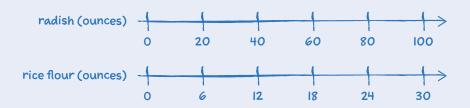
Arrange students in groups of 2.

Give students 3–4 minutes of quiet work time and time to discuss their responses with their partner.

Student Task Statement

A recipe for radish cake calls for 20 ounces of radish for every 6 ounces of rice flour.

1. Draw a double number line diagram that represents the amounts of radish and rice flour in different-size batches of this recipe.

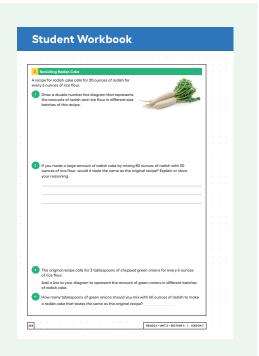


2. If you made a large amount of radish cake by mixing 80 ounces of radish with 30 ounces of rice flour, would it taste the same as the original recipe? Explain or show your reasoning.

No, it would not taste the same.

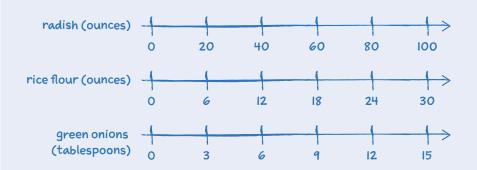
Sample reasoning:

- If we use 80 ounces of radish, we should use 24 ounces of rice flour because 24 is directly below 80 on the double number line.
- If we use 30 ounces of rice flour, we should use 100 ounces of radish because 100 is directly above the 30 on the double number line.



3. The original recipe calls for 3 tablespoons of chopped green onions for every 6 ounces of rice flour.

Add a line to your diagram to represent the amount of green onions in different batches of radish cake.



- **4.** How many tablespoons of green onions should you mix with 60 ounces of radish to make a radish cake that tastes the same as the original recipe?
 - 9 tablespoons of green onions, because the point on the bottom line that lines up with 60 on the top line of the diagram is 9.

Activity Synthesis

Invite students to share their responses and display their diagrams. Discuss how using a double (or triple) number line diagram helped them find equivalent ratios in this situation. Consider asking students:

"How is reasoning with three number lines like reasoning with two number lines?"

We're still looking for values that are in a ratio and that line up vertically. The tick marks still represent different numbers of batches.

"How is it different?"

We can use the same diagram to reason about the ratios of different pairs of quantities, such as the ratio of radish to rice flour, rice flour to green onions, or radish to green onions. We can also use it to reason about the ratio of all three quantities.

"Can we use the double (or triple) number line diagram to find the amounts of ingredients for a fraction of a batch of radish cake? How?"

Yes, we can partition the first interval of each number line into equal parts.

If time permits, solicit new questions that could be asked about the ingredients or the number of batches and could be answered by using their double (or triple) number line diagram.

Lesson Synthesis

To emphasize some important things to attend to when creating a double number line diagram, create a double number line with the help of the class. Choose a situation that students have already encountered in this lesson or in an earlier lesson.

Then, ask students:

"What are some important things to pay attention to when you create a double number line?"

Write down anything mentioned that it is important to pay attention to. For example:

- The two lines should be drawn parallel to each other. One practice is to use both edges of a ruler to create two parallel lines.
- Each line should be labeled with what it represents and should include units of measure.
- Tick marks should be evenly spaced, and the two sets of tick marks should be lined up vertically in pairs.

Consider doing something wrong intentionally and asking students how to fix the error. For example, draw tick marks that are very obviously not evenly spaced, or neglect to include units of measure in the labels.

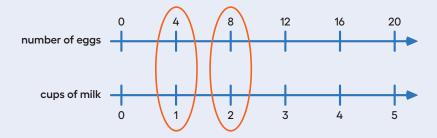
Clarify that while it is important to attend to these aspects of a diagram, double number line diagrams are tools for reasoning, so they don't have to be perfect.

Lesson Summary

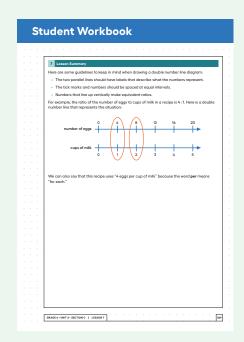
Here are some guidelines to keep in mind when drawing a double number line diagram:

- The two parallel lines should have labels that describe what the numbers represent.
- The tick marks and numbers should be spaced at equal intervals.
- Numbers that line up vertically make equivalent ratios.

For example, the ratio of the number of eggs to cups of milk in a recipe is 4:1. Here is a double number line that represents the situation:



We can also say that this recipe uses "4 eggs per cup of milk" because the word **per** means "for each."



Responding To Student Thinking

Points to Emphasize

If students struggle with creating a double number line diagram or with spacing tick marks evenly, focus on using this representation when opportunities arise over the next several lessons. For example, in this activity, encourage students to create double number line diagrams—with evenly spaced tick marks that line up vertically—to represent the ratios. Suggest using a simple reference for spacing, such as one finger-width between tick marks.

Unit 2, Lesson 8, Activity 2 Grocery Shopping

Cool-down

Representing Paws, Ears, and Tails

5 min

Student Task Statement

Each of these cats has 2 ears, 4 paws, and 1 tail.

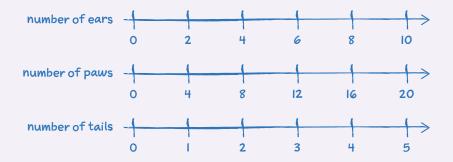






1. Draw a double number line diagram that represents a ratio in the situation.

Students may draw any 2 of the 3 number lines shown.



2. Write a sentence that describes this situation and that uses the word *per*.

Sample responses:

- There are 2 ears per tail.
- There are 4 paws per tail.
- · There are 2 paws per ear.
- There is $\frac{1}{2}$ tail per ear.

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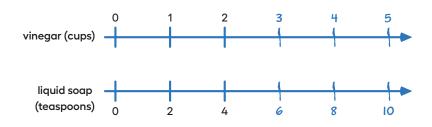
Practice Problems

6 Problems

Problem 1

A liquid for cleaning carpet stains uses 2 teaspoons of liquid soap per cup of vinegar.

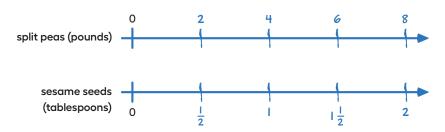
Complete the double number line diagram to show the amount of vinegar and liquid soap in 3, 4, and 5 batches of cleaning solution.



Problem 2

One batch of homemade chicken feed contains 2 pounds of split peas and $\frac{1}{2}$ tablespoon of sesame seeds.

Complete the double number line diagram to show the amounts of split peas and sesame seeds needed for 1, 2, 3, and 4 batches of chicken feed.



Problem 3

A recipe for tropical fruit punch says, "Combine 4 cups of pineapple juice with 5 cups of orange juice."

a. Create a double number showing the amount of each type of juice in 1, 2, 3, 4, and 5 batches of the recipe.

A correct double number line will have equally spaced tick marks. A line labeled "cups of pineapple juice" is labeled 0, 4, 8, 12, 16, 20 and a line labeled "cups of orange juice" is labeled 0, 5, 10, 15, 20, 25.

b. If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

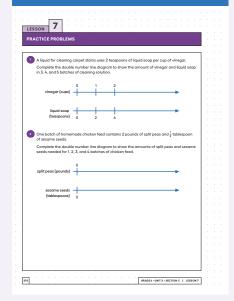
No, it will not taste the same.

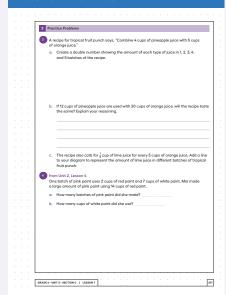
12 cups of pineapple juice should be mixed with 15 cups of orange juice.

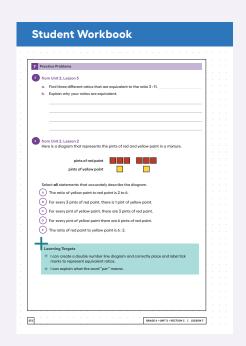
c. The recipe also calls for $\frac{1}{3}$ cup of lime juice for every 5 cups of orange juice. Add a line to your diagram to represent the amount of lime juice in different batches of tropical fruit punch.

A line labeled "cups of lime juice" is labeled $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$.

Student Workbook







Problem 4

from Unit 2, Lesson 4

One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of pink paint using 14 cups of red paint.

a. How many batches of pink paint did she make?

7 batches, because 14 is 7 · 2

b. How many cups of white paint did she use?

49 cups, because 7 · 7 = 49

Problem 5

from Unit 2, Lesson 5

a. Find three different ratios that are equivalent to the ratio 3:11.

Sample response:

- 0 6:22
- 0 9:33
- 0 12:44
- **b.** Explain why your ratios are equivalent.

Sample response: These ratios come from 3:11 by multiplying both numbers in the ratio by 2, 3, and 4 respectively.

Problem 6

from Unit 2, Lesson 2

Here is a diagram that represents the pints of red and yellow paint in a mixture.



Select all statements that accurately describe the diagram.

- **A.** The ratio of yellow paint to red paint is 2 to 6.
- **B.** For every 3 pints of red paint, there is 1 pint of yellow paint.
- **C.** For every pint of yellow paint, there are 3 pints of red paint.
- **D.** For every pint of yellow paint there are 6 pints of red paint.
- **E.** The ratio of red paint to yellow paint is 6:2.