Lesson 7

Comparing Voting Systems

Goals

- Choose and justify (orally) which voting system seems the fairest for dealing with more than two choices.
- Compare and contrast (orally and in writing) different voting systems for dealing with more than two choices, i.e., plurality, runoff, and instant runoff.
- Interpret (orally and in writing) voting situations involving two choices, through the use of ratios and percentages.

Lesson Narrative

This optional lesson continues to explore the effects of different voting systems when there are more than two options. Students explore an instant runoff voting system and compare it to the plurality, runoff, and ranked voting systems from the earlier lesson. While comparing and analyzing the results of each system, students reason abstractly and quantitatively. Students also analyze the effectiveness of the election models.

Student Learning Goal

Let's compare voting systems.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Lesson Timeline

Warm-up	Activity 1	Activity 2
5 min	25 min	10 min

Warm-up

Rank Your Vote

5 min

Activity Narrative

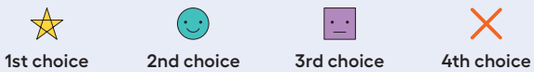
In this activity we return to in-class voting. Students will rank the options for the field trip to show their personal preference. These rankings will be used in the next activity.

Launch

Remind students of the meaning of the symbols used to rank the choices for the field trip. Give students 2 minutes to make their choices.

Student Task Statement

Use these symbols to rank your choices for the field trip.



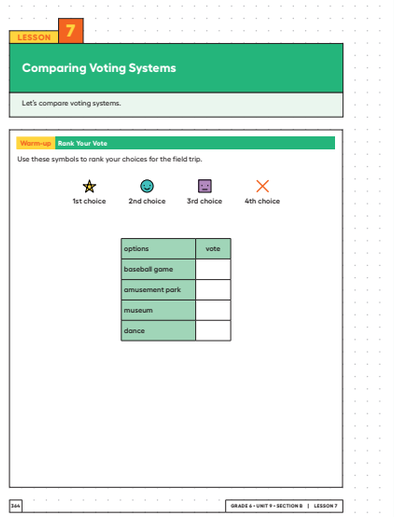
options	vote
baseball game	
amusement park	
museum	
dance	

Answers vary.

Activity Synthesis

The goal of this activity is to make sure that all students have ranked their choices. Check in with students to make sure that no symbols were repeated and that everyone has locked in their decision. The choices students make in this activity will be used in the next activity.

Student Workbook


LESSON 7

Comparing Voting Systems

Let's compare voting systems.

Warm-up Rank Your Vote

Use these symbols to rank your choices for the field trip.

1st choice 2nd choice 3rd choice 4th choice

options	vote
baseball game	
amusement park	
museum	
dance	

Activity 1
Just Vote Once

25 min

Activity Narrative

This activity presents another method for choosing among 3 or more choices when none wins a majority: instant runoff voting. In this voting system, voters again rank their choices. Each choice is given points, with 0 for the last choice, 1 for the next to last, and so on. The choice with the most total points wins, and no runoff elections are needed. Students use quantitative reasoning when analyzing the number of people satisfied by each voting method. They model with mathematics by comparing the voting methods and deciding what quantities are important.

Launch

Arrange students in groups of 2–4.

Tell students that we are now looking at another voting system: instant runoff. In this system, each voter ranks their choices and each rank is given a number of points: 0 for the last choice, 1 for the next to last, and so on. The choice with the most points wins, so only one round of voting is needed.

Display the table and ask students to raise their hand for each of their votes, using their choices from the previous activity. Each student should raise their hand 4 times total, once for each trip choice. Record the votes in a table like the one shown here:

option	number of votes for top choice (star)	number of votes for second choice (smiley)	number of votes for third choice (square face)	number of votes for last choice (X)	total points
baseball game					
amusement park					
museum					
dance					

After the results are recorded for all to see and students understand the presented information, students work in groups and answer the questions in the *Student Task Statement*.

Student Task Statement

Your class just voted using the *instant runoff* system. Use the class data for the following questions.

1. For our class, which choice received the most points? Answers vary.
2. Does this result agree with that from the runoff election in an earlier activity? Answers vary.
3. In the runoff voting system, if there is not a winner from everyone's first choices another vote is held and voters can change their choice because the least-voted option is eliminated. How does the instant runoff voting system also show voters' other choices?

   Sample response: The instant runoff method includes the information about people's choices in the number of points that are assigned to each vote.
4. Complete the table to see voters' satisfaction with the instant runoff results. After comparing the satisfaction for the 3 voting rules our class used (plurality, runoff, instant runoff), which method do you think is fairest? Explain your reasoning.

what points did you give the winner?	number of people	% of people
top choice (3)	13	26%
second choice (2)	21	42%
third choice (1)	16	32%
last choice (0)	0	0%

Sample response: In the plurality method, most people did not get their first choice. In the runoff method, nobody got their last choice. In the instant runoff method, more people got a higher choice, even if it was not their first one. Instant runoff is the most fair because 68% of people got their first or second choice, compared with 32% in runoff and 42% in plurality.

Are You Ready for More?

Numbering your choices 0 through 3 might not really describe your opinions. For example, what if you really liked A and C a lot, and you really hated B and D? You might want to give A and C both a 3, and B and D both a 0.

1. Design a numbering system where the size of the number shows how much you like a choice. Some ideas:
   - The same 0 to 3 scale, but you can choose more than one of each number, or even decimals between 0 and 3.
   - A scale of 1 to 10, with 10 for the best and 1 for the worst.

   Sample response: A scale of 1 to 10, with 10 for the best and 1 for the worst.
2. Try out your system with the people in your group, using the same field trip options for the election.

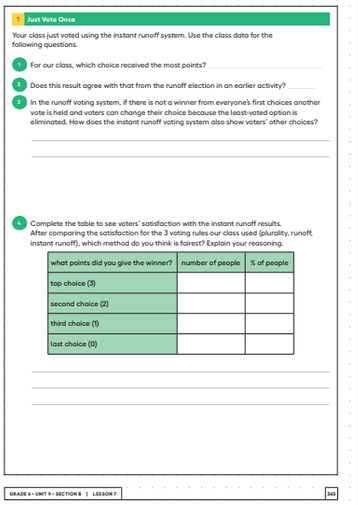
   Answers vary.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, after students have solved the first 2–3 problems, check in with either select groups of students or the whole class. Invite students to share the strategies they have used so far, as well as inviting them to ask any questions they have before continuing.

Supports accessibility for: Organization, Attention

Student Workbook


1 Just Vote Once

Your class just voted using the *instant runoff* system. Use the class data for the following questions.

1. For our class, which choice received the most points?
2. Does this result agree with that from the runoff election in an earlier activity?
3. In the runoff voting system, if there is not a winner from everyone's first choices another vote is held and voters can change their choice because the least-voted option is eliminated. How does the instant runoff voting system also show voters' other choices?

4. Complete the table to see voters' satisfaction with the instant runoff results. After comparing the satisfaction for the 3 voting rules our class used (plurality, runoff, instant runoff), which method do you think is fairest? Explain your reasoning.

what points did you give the winner?	number of people	% of people
top choice (3)		
second choice (2)		
third choice (1)		
last choice (0)		

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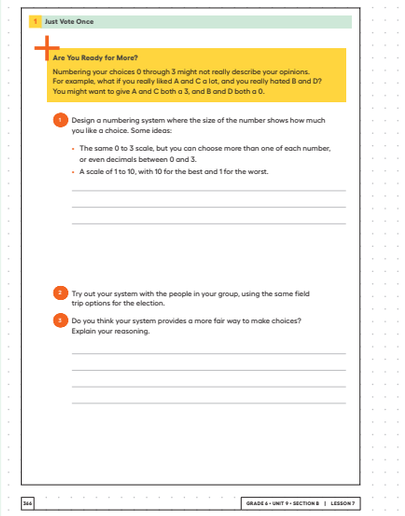
Student Workbook


1 Just Vote Once

Are You Ready for More?

Numbering your choices 0 through 3 might not really describe your opinions. For example, what if you really liked A and C a lot, and you really hated B and D? You might want to give A and C both a 3, and B and D both a 0.

1. Design a numbering system where the size of the number shows how much you like a choice. Some ideas:
   - The same 0 to 3 scale, but you can choose more than one of each number, or even decimals between 0 and 3.
   - A scale of 1 to 10, with 10 for the best and 1 for the worst.

2. Try out your system with the people in your group, using the same field trip options for the election.
3. Do you think your system provides a more fair way to make choices? Explain your reasoning.

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Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support small-group discussions, such as "I think the _____ method is most fair because ..." "I agree/disagree because ..." "_____'s idea reminds me of ..." "How can we justify that more students were represented in the final results?"

Advances: Speaking, Representing

3. Do you think your system provides a more fair way to make choices? Explain your reasoning.

Sample response: A wider range of numbers can give more opportunities for voters to show their level of support. For voters who are happy with all of the options, all of their numbers will be higher. For voters who really support one option over the others, they can make one number much larger than the others while still showing their order of preference. For voters who are not enthusiastic about any of the options, they can show that with lower numbers while still showing their order of preference. This system better represents voter's views, making it a more fair election system.

Activity Synthesis

The purpose of this discussion is to compare instant runoff voting with plurality, runoff, and ranked voting. Here are some questions for discussion:

- *"How do the 4 voting methods we have seen compare?"*

  All 4 methods let voters show their top choice. Runoff, ranked, and instant runoff also allow voters to consider choices beyond their top choice.

- *"Which voting method would be best for a large election, for example, at the state or country level?"*

  Plurality or instant runoff would be best for a large election because they take only one round of voting.

- *"Which method should we use the next time our class has to make a decision? Why?"*

  We should use instant runoff to make sure everyone's preferences are included.

We have seen several methods for fairly deciding between more than two choices. There is no single fairest method. Different voting systems are used by different local, state, and federal governments as well as different countries.

Activity 2
Weekend Choices

10 min

Activity Narrative

In this activity, students make sense of how 5 friends voted on 3 choices of weekend activities. They also compare the results of some of the voting systems from the previous activities: plurality, runoff, and instant runoff. Students reason abstractly and quantitatively when calculating and interpreting the results of each voting method. They model with mathematics by evaluating the effectiveness of each voting method.

Launch

Arrange students in groups of 2–4.

Student Task Statement

Clare, Han, Mai, Tyler, and Noah are deciding what to do on the weekend. Their options are cooking, hiking, and bowling. Here are the points for their instant runoff vote. Each first choice gets 2 points, the second choice gets 1 point, and the last choice gets 0 points.

	cooking	hiking	bowling
Clare	2	1	0
Han	2	1	0
Mai	2	1	0
Tyler	0	2	1
Noah	0	2	1

1. Which activity won using the instant runoff method? Show your calculations, and use expressions or equations.

   Hiking

   Sample response: Cooking got three first place votes. Hiking got two first place votes and three second place votes. Bowling got two second place votes.

   The calculation of points for cooking is $3 \cdot 2 = 6$.

   The calculation of points for hiking is $2 \cdot 2 + 3 \cdot 1 = 7$.

   The calculation of points for bowling is $2 \cdot 1 = 2$.

2. Which activity would have won if there were just a vote for their top choice, with a majority or plurality winning?

   Cooking

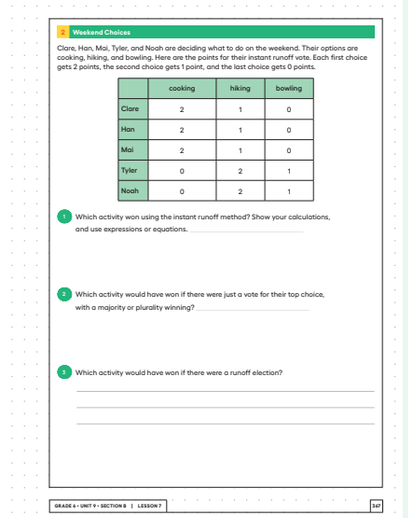
   Sample response: With a plurality wins system, cooking would win since it got a majority of the first place votes.

3. Which activity would have won if there were a runoff election?

   Cooking

   Sample response: With a runoff system, cooking would win since it already got the majority of first place votes in one round of voting.

Student Workbook



2 Weekend Choices

Clare, Han, Mai, Tyler, and Noah are deciding what to do on the weekend. Their options are cooking, hiking, and bowling. Here are the points for their instant runoff vote. Each first choice gets 2 points, the second choice gets 1 point, and the last choice gets 0 points.

	cooking	hiking	bowling
Clare	2	1	0
Han	2	1	0
Mai	2	1	0
Tyler	0	2	1
Noah	0	2	1

1. Which activity won using the instant runoff method? Show your calculations, and use expressions or equations.

2. Which activity would have won if there were just a vote for their top choice, with a majority or plurality winning?

3. Which activity would have won if there were a runoff election?

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Activity Synthesis

The purpose of this discussion is for students to see another example of each voting method and how they are used. Here are some questions for discussion:

- *"Based on their votes, which activity should they do this weekend?"*

  They should go hiking because everyone was at least a little interested in that.

- *"Why are the results the same for a runoff or a plurality?"*

  Both runoff and plurality start with everyone voting for their first choice. In a plurality, the choice with the most votes wins. In a runoff, if an option gets a majority of the votes, then there is only one round of voting. In this case, cooking has a majority of the first choice votes, so only one round happens.

- *"Why are the results different for an instant runoff and a runoff?"*

  Instant runoff takes everyone's rankings into account, not just their first choice. This means the fact that some people did not want cooking but everyone had some interest in hiking changed the total points and the winner.