Lesson 11

Representing Ratios with Tables

Goals

- Comprehend the words "row" and "column" (in written and spoken language) as they are used to describe a table of equivalent ratios.
- Explain (orally and in writing) how to find a missing value in a table of equivalent ratios.
- Interpret a table of equivalent ratios that represents different sized batches of a recipe.

Learning Targets

- If I am looking at a table of values, I know where the rows are and where the columns are.
- When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row.
- When I see a table representing a set of equivalent ratios, I can explain what the numbers mean.

Lesson Narrative

In this lesson, students learn to organize equivalent ratios in a **table**, which is a more abstract tool but also a more flexible one for solving problems.

Students first encounter a situation that involves ratios with large numbers. They see that representing the situation using a double number line diagram poses challenges and that a different representation would be helpful. Students are then introduced to a table as another way to represent and reason about equivalent ratios. Next, students work to complete a partially filled table. To do so, they need to look for and make use of structure.

Student Learning Goal

Let's use tables to represent equivalent ratios.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)

Instructional Routines

- 5 Practices
- Which Three Go Together?

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Inspire Math

Alligators video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614201

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Instructional Routines

Which Three Go Together?

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Warm-up

Which Three Go Together: Very Dairy

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four representations of situations involving ratios. In making comparisons, students have a reason to attend to quantities carefully and use language precisely. The activity also enables the teacher to hear how students use ratio language and talk about representations of ratios learned so far before working with a new representation.

Launch

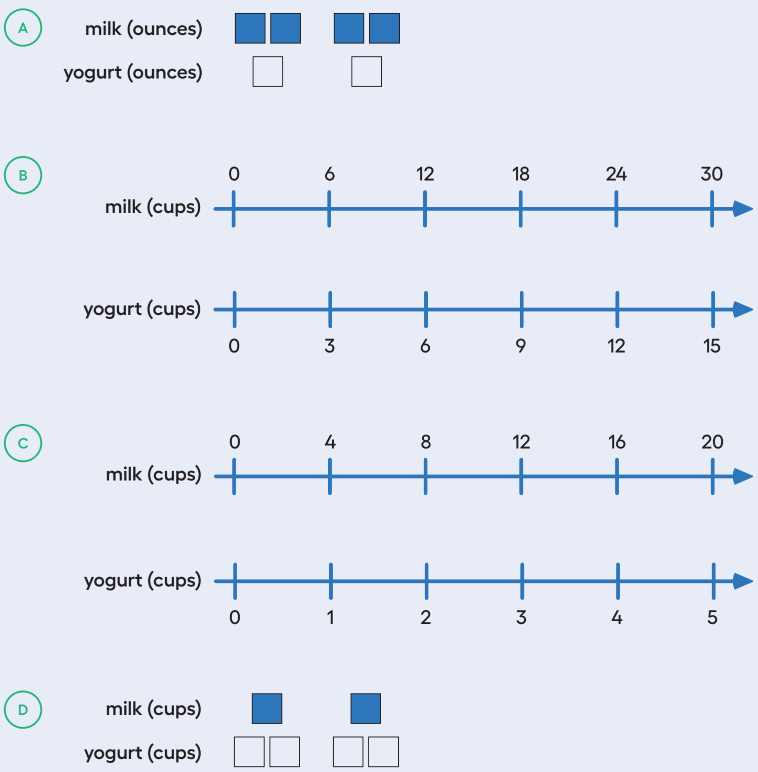
Arrange students in groups of 2–4. Display the representations for all to see.

Give students 1 minute of quiet think time and then time to share their thinking with their small group.

In their small groups, tell each student to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together?



A, B, and C go together because they represent ratios with more milk than yogurt.

A, B, and D go together because:

- They represent ratios with 2 units of something to 1 unit of something else.
- For 1 unit of one quantity there is twice the other quantity.

A, C, and D go together because they all show how much of a quantity per 1 unit of the other quantity.

B, C, and D go together because they all describe the quantities in cups.

Activity Synthesis

Invite each group to share one reason why a particular set of three goes together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, if students make broad or vague claims (for instance, "A and B show the same thing"), ask them to clarify the terms they used (for instance, "What do you mean by "the same thing?" or "How are they the same?").

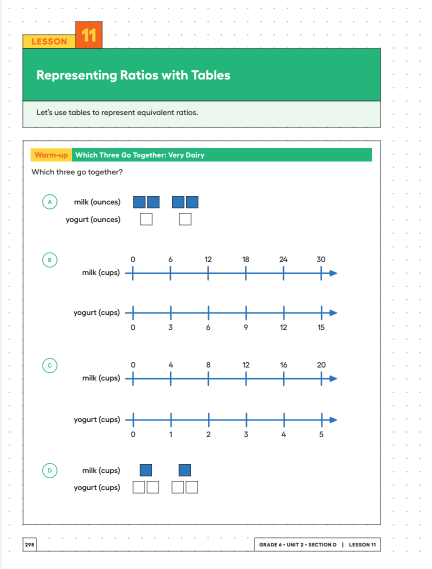
Also, prompt students to explain the meaning of any terminology they use, such as "equivalent ratios," "same rate," and "per," and to clarify their reasoning as needed. Consider asking:

"How do you know ...?"

"What do you mean by ...?"

"Can you say that in another way?"

Student Workbook



Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, ask students: "In earlier lessons, how did you represent 1, 2, 3, or more batches of color mixtures, play clay, or radish cake?" or "How did you find the amount of each ingredient when dealing with a larger number of batches?"

Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Activity 1

A Huge Amount of Sparkling Orange Juice

20 min

Activity Narrative

In this activity, students are asked to find missing values for significantly scaled-up ratios. The work serves several purposes:

- to uncover a limitation of a double number line (namely, that it is not always practical to extend it to find significantly scaled-up equivalent ratios)
- to reinforce the multiplicative reasoning needed to find equivalent ratios (especially in cases when drawing diagrams or skip counting is inefficient)
- to introduce a table as a way to represent equivalent ratios

Monitor for the different ways students find equivalent ratios involving large values. Here are some strategies they may use, from less practical or less flexible to more accommodating:

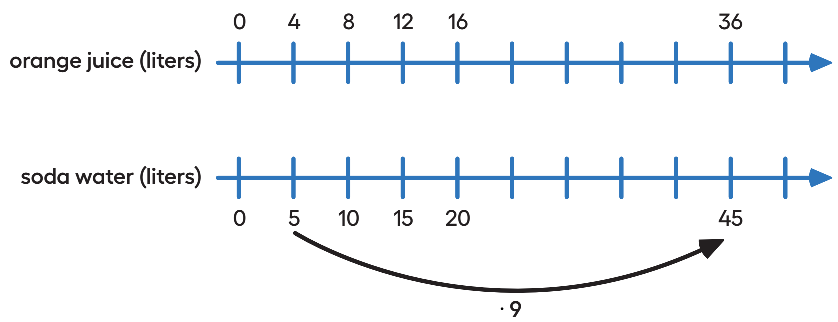
- extend the given double number line and then try to squeeze numbers on the extreme right end, ignoring the previously equal intervals
- draw a new double number line diagram that is longer or where the intervals represent multiples of 4 and 5 (rather than 4 and 5)
- use multiplication (or division) and write expressions or equations to represent the given scenarios

Select students who use different strategies to share later, including those who find the given double number line inadequate—not long enough to accommodate large numbers, requiring more marking or writing, and so on—and are consequently motivated to find a more efficient strategy.

Launch

Give students 2–3 minutes to work on the first two questions and then ask them to pause.

As a class, discuss the two approaches students are likely to take: counting multiples of 4 and 5 up to 36 and 45; and multiplicative reasoning (asking "What number times 4 equals 36?"). Also discuss how a double number line as shown here might be used to support reasoning.

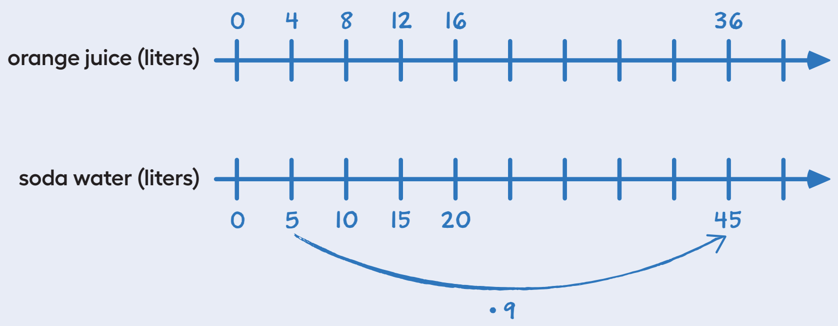


Reiterate the multiplicative relationship between equivalent ratios before students move on.

Student Task Statement

Noah's recipe for one batch of sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

1. Use the double number line to show how many liters of each ingredient to use for different-sized batches of sparkling orange juice.



Amounts for the first four batch sizes are shown.

2. If someone mixes 36 liters of orange juice and 45 liters of soda water, how many batches would they make?

   9 batches

   $4 \cdot 9 = 36$ and $5 \cdot 9 = 45$

3. If someone uses 400 liters of orange juice, how much soda water would they need?

   500 liters of soda water

   $4 \cdot 100 = 400$ and $5 \cdot 100 = 500$

4. If someone uses 455 liters of soda water, how much orange juice would they need?

   364 liters of orange juice

   $455 = 5 \cdot 91$ and $4 \cdot 91 = 364$

5. Explain the trouble with using a double number line diagram to answer the last two questions.

   Sample response: The numbers I needed to find were too big to fit on the number lines.

Building on Student Thinking

Students may become frustrated when they "run out of number line," but remind them of what they know about how to find ratios equivalent to 4:5 (they need to multiply both 4 and 5 by the same number). Consider directing their attention to a definition of equivalent ratios displayed in your room or in a previous lesson, or suggesting they reexamine some of the simpler cases (for instance, the relationship between 4:5 and 36:45). Be on the lookout for students trying to tape on more paper to extend their number lines.

Student Workbook

1 Huge Amount of Sparkling Orange Juice

Noah's recipe for one batch of sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

1. Use the double number line to show how many liters of each ingredient to use for different-sized batches of sparkling orange juice.

orange juice (liters)

soda water (liters)

2. If someone mixes 36 liters of orange juice and 45 liters of soda water, how many batches would they make?
3. If someone uses 400 liters of orange juice, how much soda water would they need?
4. If someone uses 455 liters of soda water, how much orange juice would they need?
5. Explain the trouble with using a double number line diagram to answer the last two questions.

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Activity Synthesis

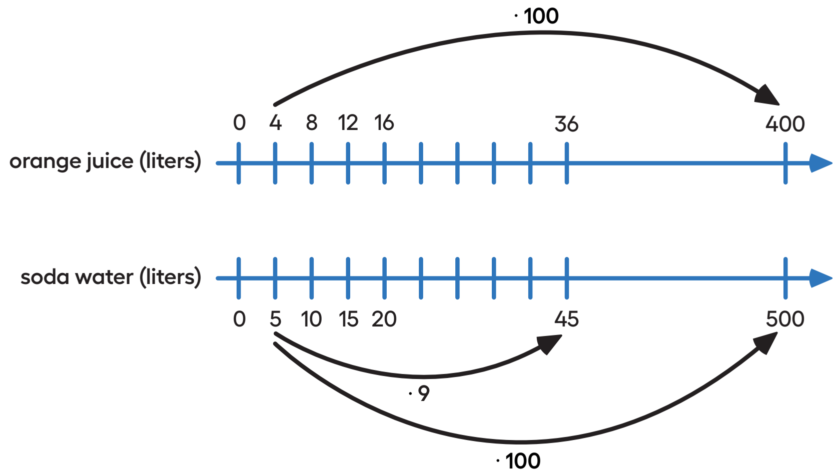
After students have a chance to share with a partner, ask previously selected students to share their reasoning for the last few questions. Sequence the discussion of students' strategies in the order listed in the *Activity Narrative*. Discuss any challenges of using the double number line and merits of alternative methods students might have used.

Explain that there is a more appropriate tool—a **table**—that can be used to represent equivalent ratios. Display for all to see a double number line representing the sparkling juice recipe and a table of equivalent ratios. Explain that even though the table is oriented vertically and the double number line is oriented horizontally, the two representations represent the same ratios. Explain what we mean by **row** and **column** and demonstrate the use of these words. Complete the table using the values from the orange-soda ratios. Along the way, connect the two representations and compare and contrast how they show the quantities in the recipe.

A few other key insights to convey:

- Just as it was important to label the double number line, it is important to label the columns of the table to indicate what the values represent.
- Each row of a table shows a pair of values from a collection of equivalent ratios. Unlike a number line, distances between values do not matter.
- On each line of a double number line, numbers are shown in order. In each column of a table, order is not important, that is, pairs of values can be placed in any order that is convenient. When complete, the display should look something like this:

orange juice (liters)	soda water (liters)
4	5
8	10
12	15
16	20
36	45
400	500
364	455



Activity 2

Batches of Nihaizu Sauce

10 min

Activity Narrative

This activity invites students to interact with a table in a way that discourages skip counting. Numbers within each column are deliberately out of order. This is intended to encourage students to multiply the pairs of values from a given ratio by the same number and to emphasize that the order in which pairs of values appear is not a necessary part of the structure of a table. (Order within rows, however, is necessary.) The last question reinforces the definition of equivalent ratios.

Students may use the given values (7 and 5) as the basis for every calculation (for instance, for every row, they think "7 times what … " or "5 times what …"). They may also reason with values from another row (for instance, they may see 250 as 10 · 25 rather than as 5 · 50). As students work, monitor for different approaches.

Launch

Ask students if they had a favorite salad dressing or dipping sauce and, if so, whether they knew the main ingredients.

Explain that there are many dressings in Japanese cooking and they often use vinegar as a base. The mixtures may also be used to marinate vegetables, meat, and seafood, or as dipping sauces. This activity looks at the ratios of vinegar and soy sauce in *nihaizu* (NEE-hye-zoo)—a sauce made only with those two ingredients—organized in a table. (The word "nihaizu" translates to "two-ingredient vinegar.")

Explain that a **table** can be used to organize equivalent ratios. In this case, one **column** contains amounts of vinegar, and the other column contains corresponding amounts of soy sauce. Each **row** shows the amount of each ingredient in a particular batch.

Reiterate that multiplying both parts of a ratio by the same non-zero number always creates a ratio that is equivalent to the original ratio.

Student Task Statement

Nihaizu is a sauce used with seafood or vegetables in Japanese cooking. A recipe for nihaizu uses 7 fluid ounces of vinegar and 5 fluid ounces of soy sauce.

The **table** shows amounts of vinegar and soy sauce that would be in different-sized batches of the recipe.

1. Complete the table so that ratios represented by each row are equivalent. Explain or show your reasoning.

   Sample reasoning: For the second row, I saw that 28 is 7 times 4, so I multiplied 5 times 4 to get 20. For the third row, I saw that 10 is twice 5, so the amount of vinegar is twice 7, which is 14.

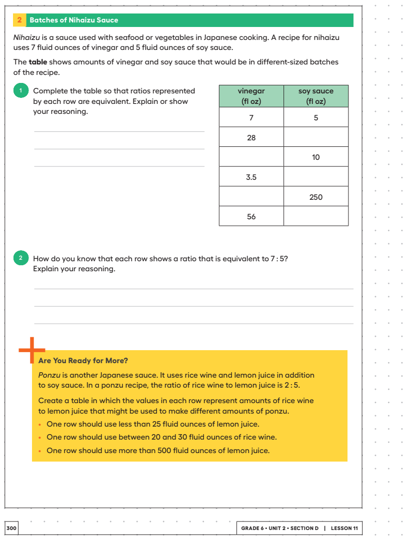
vinegar (fl oz)	soy sauce (fl oz)
7	5
28	20
14	10
3.5	2.5
350	250
56	40

Access for Multilingual Learners (Activity 2, Launch)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question on how they know that each row in the table shows a ratio that is equivalent to 7:5. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Workbook



2 Batches of Nihaizu Sauce

Nihaizu is a sauce used with seafood or vegetables in Japanese cooking. A recipe for nihaizu uses 7 fluid ounces of vinegar and 5 fluid ounces of soy sauce.

The **table** shows amounts of vinegar and soy sauce that would be in different-sized batches of the recipe.

1 Complete the table so that ratios represented by each row are equivalent. Explain or show your reasoning.

vinegar (fl oz)	soy sauce (fl oz)
7	5
28	
	10
3.5	
	250
56	

2 How do you know that each row shows a ratio that is equivalent to 7 : 5? Explain your reasoning.

Are You Ready for More?

Ponzu is another Japanese sauce. It uses rice wine and lemon juice in addition to soy sauce. In a ponzu recipe, the ratio of rice wine to lemon juice is 2 : 5.

Create a table in which the values in each row represent amounts of rice wine to lemon juice that might be used to make different amounts of ponzu.

- One row should use less than 25 fluid ounces of lemon juice.
- One row should use between 20 and 30 fluid ounces of rice wine.
- One row should use more than 500 fluid ounces of lemon juice.

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Building on Student Thinking

Students may make patterns that do not yield equivalent ratios. For example, they may think "7 minus 2 is 5, so for the next row, 28 minus 2 is 26." Or they may think "7 plus 21 is 28, so then 5 plus 21 is 26." If so, consider:

Appealing to what students know about batches of recipes.

"The second row represents how many batches of nihaizu?"

4, because 28 is $7 \cdot 4$.

"Okay, so to make 4 batches of nihaizu, how will we figure out how much soy sauce to use?"

Also multiply the 5 by 4.

Refreshing what students learned about equivalent ratios.

"We need a ratio that is equivalent to the ratio represented in row 1. So what do we need to do to the 7 and the 5?"

Multiply them by the same number.

Students may be unsure about how to find the missing value in the row with 3.5. Encourage them to reason about it the same way they reasoned about the other rows.

"We need a ratio that is equivalent to the ratio represented in row 1. So what do we need to do to the 7 and the 5?"

They may have to get there by way of division. 7 divided by 2 is 3.5, so 7 times $\frac{1}{2}$ is 3.5; this means multiplying 5 by $\frac{1}{2}$ as well.

2. How do you know that each row shows a ratio that is equivalent to 7:5? Explain your reasoning.

vinegar (fl oz)	soy sauce (fl oz)
7	5
28	20
14	10
3.5	5
350	250
56	40

Sample response: I multiplied 7 and 5 by the same number to get the values in each row, so those values have a ratio of 7 to 5.

Are You Ready for More?

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- One row should use less than 25 fluid ounces of lemon juice.
- One row should use between 20 and 30 fluid ounces of rice wine.
- One row should use more than 500 fluid ounces of lemon juice.

Sample response:

rice wine	lemon juice
2	5
8	20
26	65
240	600

Activity Synthesis

Invite one or more students who used multiplicative approaches to share their reasoning with the class. Consider displaying the table and using it to facilitate gesturing and arrow-drawing while students explain. Highlight the strategy of multiplying the 7 and 5 values by the same number.

Lesson Synthesis

In the lesson, students saw that a table can be used to represent equivalent ratios and noticed the ways in which a table and a double number line diagram represent equivalent ratios. To reinforce the key ideas from the lesson, consider asking:

- *"A table has rows and columns. Where are the rows and where are the columns?"*

  A row is a horizontal group of cells or entries. A column is a vertical group of cells or entries.

- *"How does a table represent equivalent ratios?"*

  Every row contains a ratio that is equivalent to the ratios in other rows.

- *"Compare how equivalent ratios are represented in a table and in a double number line. How are they alike or different?"*

  In both representations, the numbers in each equivalent ratio line up. In a double number line, they line up vertically. In a table, they line up horizontally. In a double number line, the numbers in the ratios are shown in order from left to right. In a table, the numbers aren't necessarily listed in order.

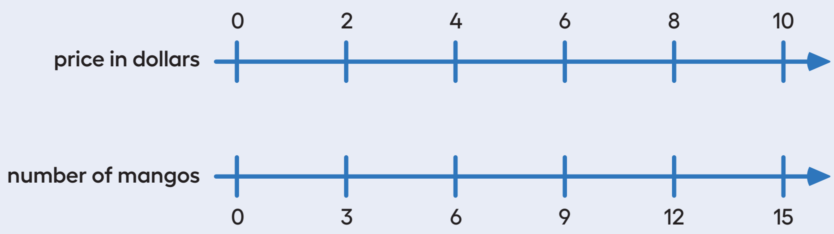
- *"How can we use a table to find equivalent ratios?"*

  We can multiply the numbers in a row by the same value and record the result in a new row.

Lesson Summary

A **table** is a way to organize information. Each horizontal set of entries is called a *row,* and each vertical set of entries is called a *column*. (The table shown has 2 columns and 5 rows.) A table can be used to represent a collection of equivalent ratios.

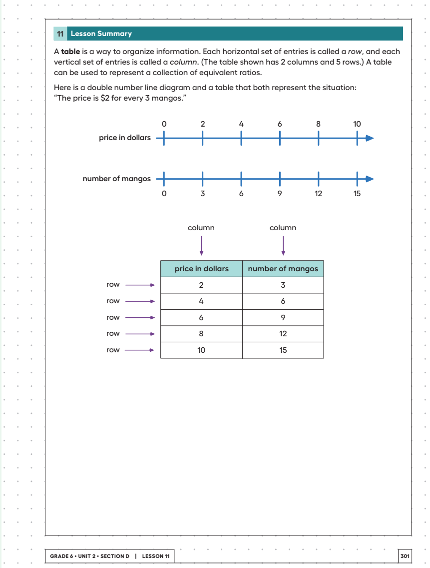
Here is a double number line diagram and a table that both represent the situation: "The price is $2 for every 3 mangos."



column | column

	price in dollars	number of mangos
row	2	3
row	4	6
row	6	9
row	8	12
row	10	15

Student Workbook



Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Batches of Cookies in a Table

5 min

Student Task Statement

Here is a table that represents a cookie recipe we saw in earlier lessons.

flour (cups)	vanilla (teaspoons)
5	2
15	6
$2\frac{1}{2}$	1

1. Write a sentence that describes a ratio shown in the table.

   Sample responses:

   - The ratio of cups of flour to teaspoons of vanilla is 5:2.
   - This recipe uses 5 cups of flour for every 2 teaspoons of vanilla.
   - This recipe uses $2\frac{1}{2}$ cups of flour per teaspoon of vanilla.

2. What does the second row of numbers represent?

   For 15 cups of flour, you need 6 teaspoons of vanilla.

3. Complete the last row for a different batch size that hasn't been used so far in the table. Explain or show your reasoning.

   Sample response: 10 cups of flour and 4 teaspoons of vanilla.

Lesson 11

Practice Problems

6 Problems

Problem 1

Complete the table to show the amounts of yellow and red paint needed for different-sized batches of the same shade of orange paint.

Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange as the mixture in the first row of the table.

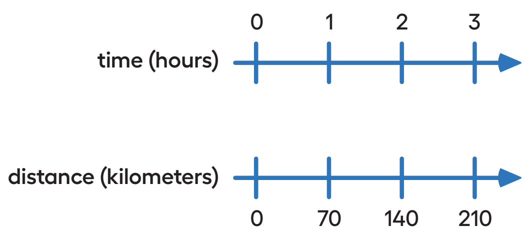
Sample response:

yellow paint (quarts)	red paint (quarts)
5	6
$\frac{5}{4}$	$\frac{3}{2}$ or equivalent
$\frac{5}{2}$	3 or equivalent
$\frac{15}{4}$	$\frac{9}{2}$ or equivalent

Each row is a multiple of the first row.

Problem 2

A car travels at a constant speed, as shown on the double number line.



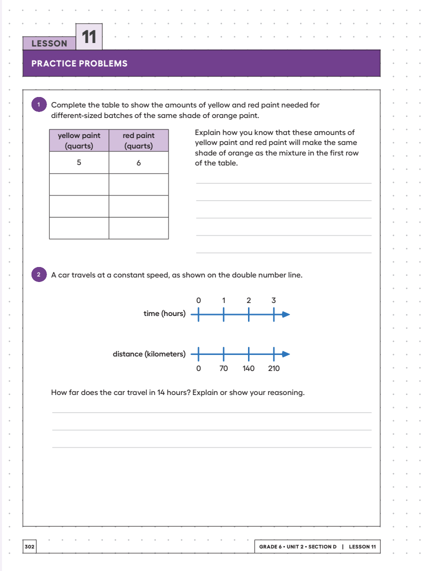
How far does the car travel in 14 hours? Explain or show your reasoning.

980 kilometers

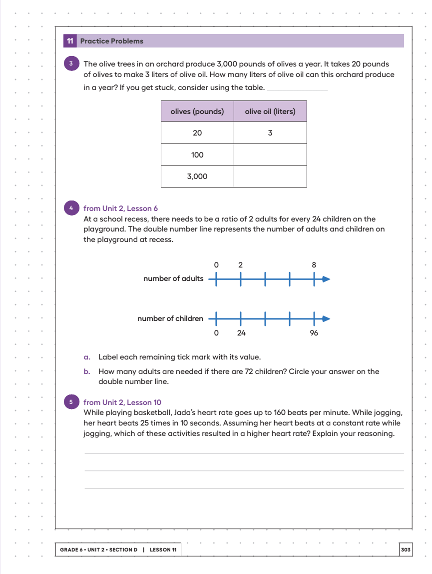
Sample reasoning: Use a table and multiply 2 by 7 to get 14 and multiply 140 by 7 to get 980.

time (hours)	distance (kilometers)
1	70
2	140
14	980

Student Workbook



Student Workbook



Problem 3

The olive trees in an orchard produce 3,000 pounds of olives a year. It takes 20 pounds of olives to make 3 liters of olive oil. How many liters of olive oil can this orchard produce in a year? If you get stuck, consider using the table.

450 liters

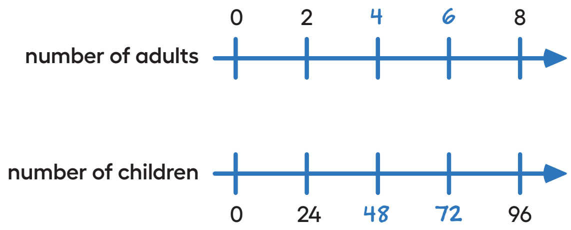
Sample reasoning:

olives (pounds)	olive oil (liters)
20	3
100	15
3,000	450

Problem 4

from Unit 2, Lesson 6

At a school recess, there needs to be a ratio of 2 adults for every 24 children on the playground. The double number line represents the number of adults and children on the playground at recess.



a. Label each remaining tick mark with its value.

b. How many adults are needed if there are 72 children? Circle your answer on the double number line.

6 adults

The portion of the double number line at 6 adults and 72 children is circled.

Problem 5

from Unit 2, Lesson 10

While playing basketball, Jada's heart rate goes up to 160 beats per minute. While jogging, her heart beats 25 times in 10 seconds. Assuming her heart beats at a constant rate while jogging, which of these activities resulted in a higher heart rate? Explain your reasoning.

Playing basketball

Sample reasoning: 25 times in 10 seconds means 150 heartbeats per minute ($25 \cdot 6 = 150$). 150 beats per minute is lower than 160 beats per minute, so Jada's heart rate is lower when she goes jogging than when she plays basketball.

Problem 6

from Unit 2, Lesson 8

A shopper bought the following items at the farmer's market:

a. 6 ears of corn for $1.80. What was the cost per ear?

$0.30

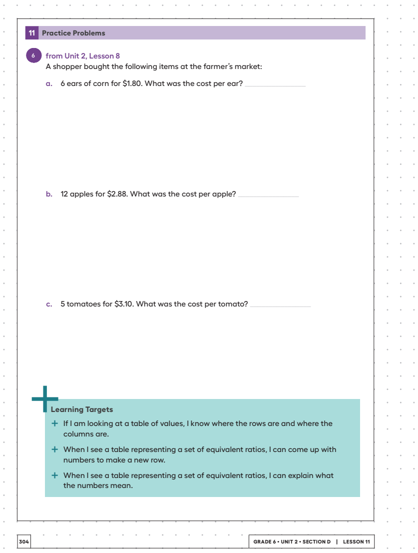
b. 12 apples for $2.88. What was the cost per apple?

$0.24

c. 5 tomatoes for $3.10. What was the cost per tomato?

$0.62

Student Workbook



11 Practice Problems

6 from Unit 2, Lesson 8

A shopper bought the following items at the farmer's market:

a. 6 ears of corn for $1.80. What was the cost per ear? ____

b. 12 apples for $2.88. What was the cost per apple? ____

c. 5 tomatoes for $3.10. What was the cost per tomato? ____

Learning Targets

+ If I am looking at a table of values, I know where the rows are and where the columns are.
+ When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row.
+ When I see a table representing a set of equivalent ratios, I can explain what the numbers mean.

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