Lesson 18

More Relationships (Optional)

Goals

- Coordinate (orally and in writing) graphs, tables, and equations that represent the same relationship.
- Create an equation and a graph to represent the relationship between two variables.
- Describe and interpret (orally and in writing) a graph that represents a nonlinear relationship between independent and dependent variables.

Learning Targets

- I can create tables and graphs that show different kinds of relationships between amounts.
- I can write equations that describe relationships with area and volume.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)
- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Support (Activity 3)

Instructional Routines

- Which Three Go Together?

Lesson Narrative

This lesson is optional. It offers additional opportunities to look at multiple representations (equations, graphs, and tables) of relationships between two quantities in situations that involve constant area, constant volume, and a doubling relationship. Students have an opportunity to look for and make use of the similar structure in the geometric relationships the first two activities. as well as to connect the reasoning in the activity about mosquitos to prior work with exponents earlier in the unit. Students may use those observations and knowledge to more easily solve the problems in the activities.

Consider offering students a choice about which problem they work on. Then in the *Lesson Synthesis*, invite students to share their work with the class and compare and contrast the representations of the different contexts.

Student Learning Goal

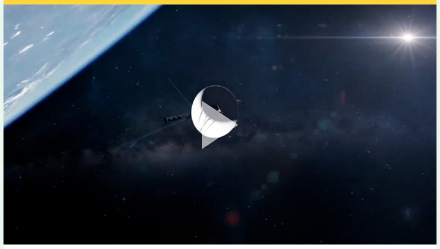
Let's use graphs and equations to show relationships involving area, volume, and exponents.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	10 min	5 min

Inspire Math

Voyager video



Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614238

Please log in to the site before using the QR code or URL.

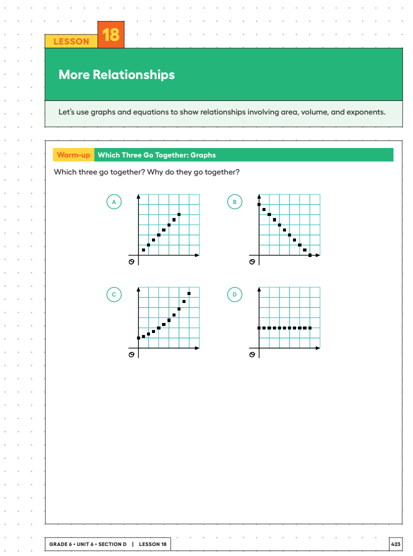
Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL

Student Workbook


LESSON 18
More Relationships
Let's use graphs and equations to show relationships involving area, volume, and exponents.
Warm-up Which Three Go Together: Graphs
Which three go together? Why do they go together?

Warm-up
Which Three Go Together: Graphs

5 min

Activity Narrative

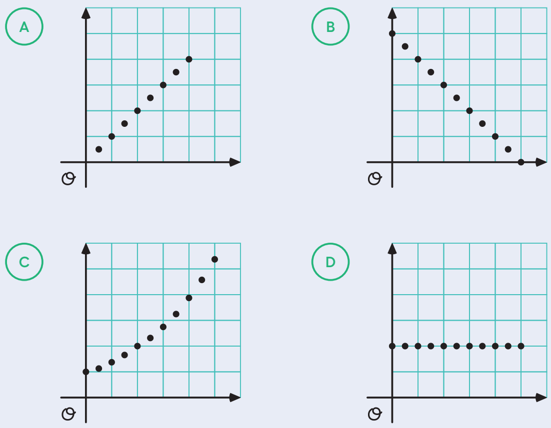
This *Warm-up* prompts students to compare four graphs of points on a coordinate grid. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the graphs for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three graphs that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

- A, B, and C go together because the vertical values change, either increase or decrease, as the graph goes from the left to the right.
- A, B, and D go together because the points look like they are in a straight line.
- A, C, and D go together because they all have a point at $(2, 2)$.
- B, C, and D go together because the first point on the left is above the origin $(0, 0)$.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "points," "coordinate," "increasing," and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Activity 1: Optional

Making a Banner

15 min

Activity Narrative

In this activity, students consider the relationship between length and width for different rectangles with the same given area and are asked to compare strategies for finding various lengths and widths. Students make sense of how the graph shows how the width changes when the length changes and what the plotted points on the graph mean in the context of the problem.

Launch

Give students 10 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Mai is creating a rectangular banner to advertise the school play. The material for the banner is sold by the square foot. Mai has enough money to buy 36 square feet of material. She is trying to decide on the length and width of the banner.

1. What is the width of the banner if the length is:

   a. 6 feet? 6 feet

   b. 4 feet? 9 feet

   c. 9 feet? 4 feet

2. To find different combinations of length and width that give an area of 36 square feet, Mai uses the equation $w = \frac{36}{\ell}$, where w is the width and ℓ is the length. Compare your strategy and Mai's method for finding the width. How were they the same or different?

   Sample response: I looked for a number to multiply the given length by to get 36. Mai's equation is different because it divides 36 by the given length.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Activate or supply background knowledge. Remind students that they can draw rectangle diagrams to help them determine the missing values.

Supports accessibility for: Social-emotional skills, Conceptual Processing

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.
Keep books or devices closed. Display only the problem stem, without revealing the questions. Say,

"We are going to read this problem 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

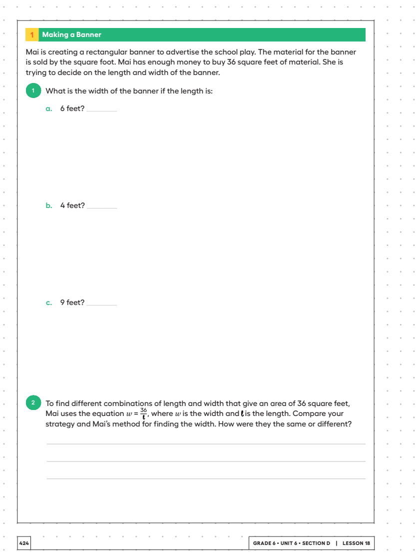
"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the question(s). Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Student Workbook


1 Making a Banner

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1. What is the width of the banner if the length is:

   a. 6 feet? ______

   b. 4 feet? ______

   c. 9 feet? ______

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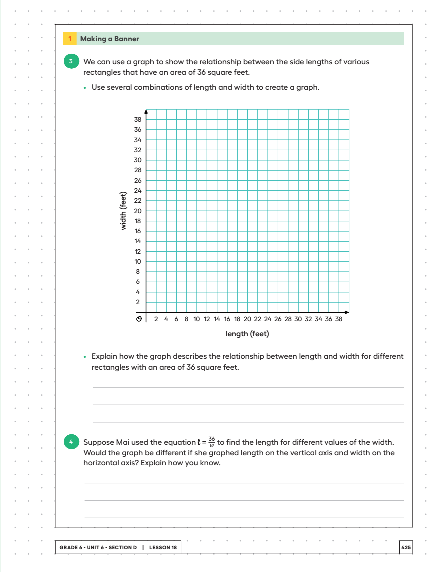
Student Workbook


1 Making a Banner

3 We can use a graph to show the relationship between the side lengths of various rectangles that have an area of 36 square feet.

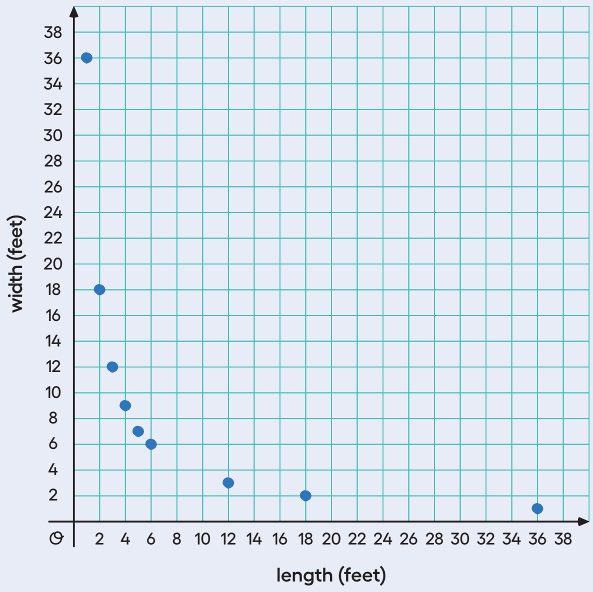
- Use several combinations of length and width to create a graph.
- Explain how the graph describes the relationship between length and width for different rectangles with an area of 36 square feet.

4 Suppose Mai used the equation $\ell = \frac{36}{w}$ to find the length for different values of the width. Would the graph be different if she graphed length on the vertical axis and width on the horizontal axis? Explain how you know.

GRADE 6 • UNIT 6 • SECTION D | LESSON 18

3. We can use a graph to show the relationship between the side lengths of various rectangles that have an area of 36 square feet.

- Use several combinations of length and width to create a graph.
- Explain how the graph describes the relationship between length and width for different rectangles with an area of 36 square feet.



Sample response: As the length increases, the width decreases, but the values always multiply to be 36.

4. Suppose Mai used the equation $\ell = \frac{36}{w}$ to find the length for different values of the width. Would the graph be different if she graphed length on the vertical axis and width on the horizontal axis? Explain how you know.

The graph would look the same.

Sample reasoning:

The values for length and width would just switch. It would just show that as the width increases, the length decreases, but the product is still 36.

Activity Synthesis

The discussion should focus on the connection between the situation, the equation (or another strategy) for finding combinations that make the area 36 square feet, and the graph that represents the relationship between length and width in the different rectangles.

Invite students to share their responses and reasoning to the last two questions. Discuss questions such as:

- *"What does the point (2, 18) in the graph mean? In general, what does each point represent?"*

  The point (2, 18) represents a rectangle with length 2 feet and height 18 feet. Each point represents a rectangle with an area of 36 square feet.

- *"Why does it make sense that the graph falls as you move to the right?"*

  The length and width are factors of the same product, so if one increases the other has to decrease.

"Where do you see the area 36 square feet in each equation or strategy for finding combinations, and the graph that shows those combinations?"

In the equations, 36 is the product of the length and width. In the graph, the coordinates of each point multiply to 36.

If time permits, consider asking students:

"What would the graph look like if it were to extend more to the right? Can you name some points on the graph and describe their coordinates?"

Points could be $\left(\frac{9}{10}, 40\right)$, $\left(\frac{18}{25}, 50\right)$, $\left(\frac{1}{2}, 72\right)$. w would be a fraction less than 1 as ℓ gets larger than 36 because w and ℓ have to multiply to 36. The points will keep getting closer to the x-axis as w gets smaller.

Activity 2: Optional

Cereal Boxes

15 min

Activity Narrative

This activity presents a situation with a similar structure to the area situation in the activity about making a banner. Students consider different combinations of base areas and heights that keep the volume of a rectangular box at 225 cubic inches. They complete a table for given values of area and height, write an equation relating the area and height, and graph the relationship.

If students have completed the activity about making a banner, prompt them to think about similarities and differences they noticed between the activities when synthesizing the lesson. In comparing the two situations and then using that insight to solve the problems in this activity, students practice looking for and making use of structure.

Launch

Give students 10 minutes of quiet work time and follow with a whole-class discussion.

Student Task Statement

A cereal manufacturer needs to design a cereal box that has a volume of 225 cubic inches and a height that is no more than 15 inches.

1. The designers know that the volume of a rectangular prism can be calculated by multiplying the area of its base and its height. Complete the table with pairs of values that will make the volume 225 in^3.

height (in)	3	5	9	12	15	$7\frac{1}{2}$
area of base (in^2)	75	45	25	18.75	15	30

2. Describe how you found the missing values for the table.

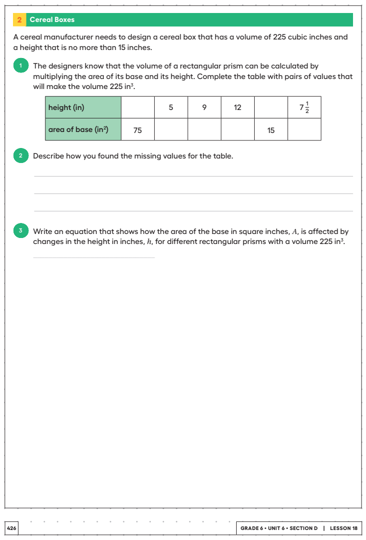
Sample response: Divide 225 by either the area or height.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies. Allow students to use the applet for this activity to facilitate plotting the ordered pairs from the table on a graph.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Student Workbook


2 Cereal Boxes

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1 The designers know that the volume of a rectangular prism can be calculated by multiplying the area of its base and its height. Complete the table with pairs of values that will make the volume 225 in^3.

height (in)		5	9	12		$7\frac{1}{2}$
area of base (in^2)	75				15	

2 Describe how you found the missing values for the table.

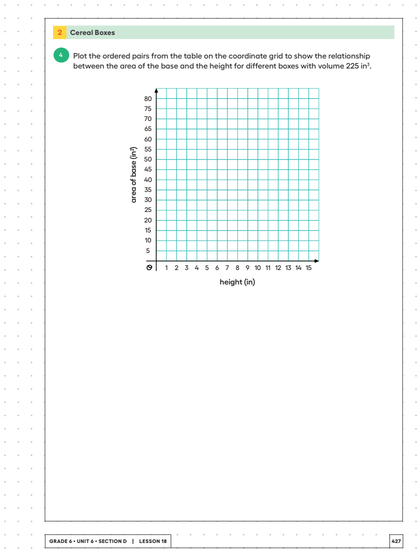
3 Write an equation that shows how the area of the base in square inches, A, is affected by changes in the height in inches, h, for different rectangular prisms with a volume 225 in^3.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect. Invite students to prepare a visual display that shows their table, graph, and equation that represent the relationship between the area of the base and height of the cereal box. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each others' work. During the whole-class discussion, listen for and amplify the language students use to describe how the area of the base decreases as the height of the cereal boxes increases. Invite students to identify where they see the volume of 225 cubic inches in each representation. Listen for and amplify the language students use to describe how the coordinates of each point on the graph multiply to 225.

Advances: Representing, Conversing

Student Workbook

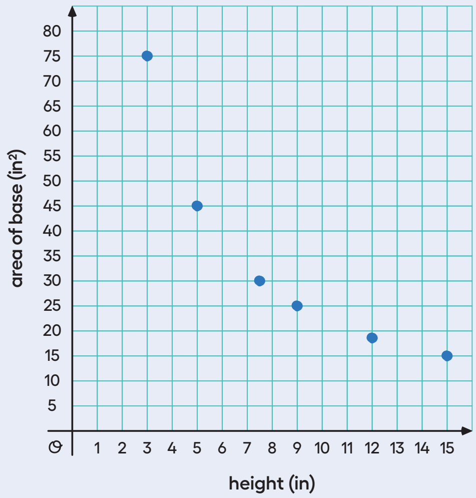


3. Write an equation that shows how the area of the base in square inches, A, is affected by changes in the height in inches, h, for different rectangular prisms with a volume 225 in^3.

   Sample response: $A = \frac{225}{h}$ (or equivalent)

4. Plot the ordered pairs from the table on the coordinate grid to show the relationship between the area of the base and the height for different boxes with volume 225 in^3.

   See graph.



Activity Synthesis

Invite students to share how they went about finding the missing values in the table, writing an equation that represents the relationship of the two quantities, and deciding on which variable depends on the other. Then discuss questions such as:

"Is one equation more helpful than the other when figuring out the design for the cereal box? If so, which one, and why is it more helpful? If not, why not?"

"What might be some benefits of using a table to represent the relationship between the two quantities in this relationship?"

"What might be some benefits of writing an equation or creating a graph?"

Consider asking the following questions:

"How does this situation compare to the one in the 'Making a Banner' activity?"

It involved measurements. We knew the product, but one or both of the two factors were unknown. The structure of the equations was similar: they both had the dependent variable equal to a number divided by the independent variable.

"In each situation, are there any values of the variables that do not make sense? Explain."

For the banner, having a very small width and very long length does not make sense, and vice versa. For example, a banner that $\frac{1}{8}$ foot wide and 288 feet long is not useful. For the cereal box, a prism that is very short with a very large base area does not make sense, and vice versa. For example, a cereal box that is 225 inches tall and has a base area of 1 in^2 is not useful.

Activity 3: Optional
Multiplying Mosquitoes

10 min

Activity Narrative

In this activity, students consider a relationship in which one quantity doubles each time the other increases by 1. the exponent is a variable. Students interpret a given equation with an exponent that is a variable and explain what it means in the context of the situation.

Monitor for students who connect this activity to the lessons on exponents, or who recognize that the quantities in this relationship are changing with respect to each other in a different manner than previous examples they have seen. Ask these students to share during the discussion.

Launch

Give students 5–7 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

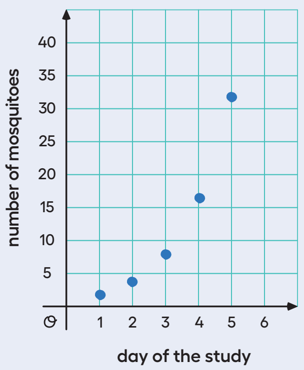
A researcher who is studying mosquito populations collects the following data:

day in the study (d)	number of mosquitoes (n)
1	2
2	4
3	8
4	16
5	32

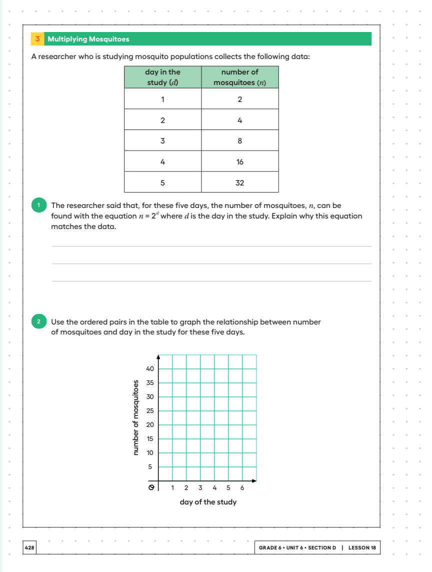
1. The researcher said that, for these five days, the number of mosquitoes, n, can be found with the equation $n = 2^d$ where d is the day in the study. Explain why this equation matches the data.

   Sample response: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and $2^5 = 32$, so the equation matches the data. All the data values in the table make the equation true.

2. Use the ordered pairs in the table to graph the relationship between number of mosquitoes and day in the study for these five days.



Student Workbook



Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support whole-class discussion. Examples:

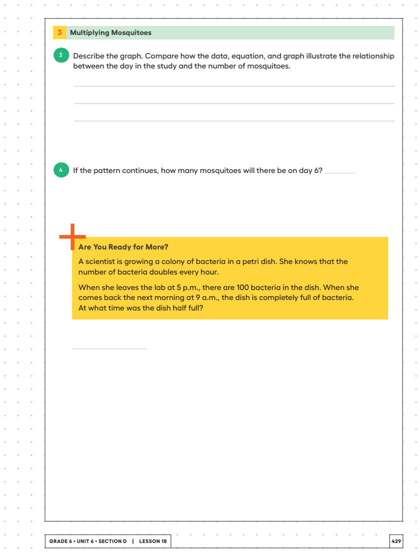
"The dot pattern is similar to/ different from the mosquito situation because ..."

"The rice situation is similar to/ different from the mosquito situation because ..."

This routine will support rich and inclusive discussion about the similarities and differences between all of the relationships students have seen in the last few lessons.

Advances: Speaking, Listening

Student Workbook



3 Multiplying Mosquitoes

3 Describe the graph. Compare how the data, equation, and graph illustrate the relationship between the day in the study and the number of mosquitoes.

4 If the pattern continues, how many mosquitoes will there be on day 6?

Are You Ready for More?

A scientist is growing a colony of bacteria in a petri dish. She knows that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

3. Describe the graph. Compare how the data, equation, and graph illustrate the relationship between the day in the study and the number of mosquitoes.

Sample response: The graph shows that as the number of days increases, the number of mosquitoes increases very quickly. We see bigger and bigger jumps from day to day in the data. In the equation, the exponent means that the numbers are doubling, or that there are twice as many mosquitos, each day.

4. If the pattern continues, how many mosquitoes will there be on day 6?

64 mosquitoes

Sample reasoning: $2^6 = 64$

Are You Ready for More?

A scientist is growing a colony of bacteria in a petri dish. She knows that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

8 a.m.

Sample reasoning: A half-full dish will take one hour to become a full dish.

Activity Synthesis

Focus the discussion on the connections between the table, the graph, and the equation that describe this situation. Invite students to share their responses. Consider asking questions such as:

- *"Which gives you a better idea of how the mosquito population changes with the day in the study—the data table, the equation, or the graph? How is that so?"*

  "How is the relationship between the quantities in this situation like the ones in an earlier activity?"

  When the quantities are graphed as points, they form a curve. There is one operation that relates one quantity to the other.

- *"How is it different?"*

  In this case, we can double the number of mosquitos each day to get the number for the next day. In earlier activities, it was necessary to divide a number, either 36 or 225, by one of the values each time.

- *"Think back to the activities about a dot pattern and about rice grains on a chessboard. How is this situation about mosquitos similar to those situations?"*

  They are all about multiplying by the same factor repeatedly.

- *"How would tables of data, graphs, and equations compare?"*

  The values for one quantity in the table increase much faster than the values of the other. The equations would all involve an exponent. The vertical values on the graph all increase very quickly.

Lesson Synthesis

In this lesson we looked at three situations: rectangles with the same area, rectangular prisms with the same volume, and one quantity that doubles repeatedly each time another quantity is increased by 1. In each situation, we examined the relationship between two quantities: length and width of the rectangle; area of the base and height of the prism; and number of mosquitos and number of days.

Invite students to compare and connect the relationships they saw. If students completed both the activity about making a banner and the one about cereal boxes, ask questions such as:

- *"How are the relationships in the first two activities—about making a banner and about cereal boxes—alike?"*

  They both involved measurements. In each situation, the product of the two quantities is known, but one or both of the two factors were unknown. The structure of the equations was similar: the value of the dependent variable can be calculated by dividing a number by the independent variable.

- *"How are they different?"*

  In the banner activity , the relationship is between length and width. In the cereal box activity, the relationship is between area of the base and height.

- *"In each situation, were there any values of the variables that would not make sense? If so, what kinds of values wouldn't make sense?"*

  For the banner, having a very small width and very long length does not make sense, and vice versa. For example, a banner that $\frac{1}{8}$-foot wide and 288 feet long is not useful. For the cereal box, a prism that is very short with a very large base area does not make sense, and vice versa. For example, a cereal box that is 225 inches tall and has a base area of 1 in^2 is not useful.

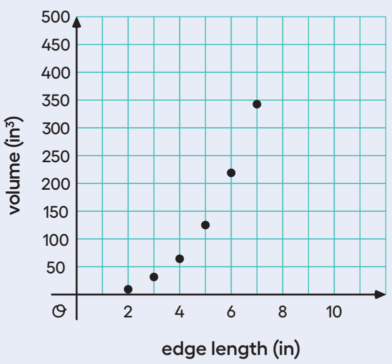
Lesson Summary

Equations can represent relationships between geometric quantities. Examples:

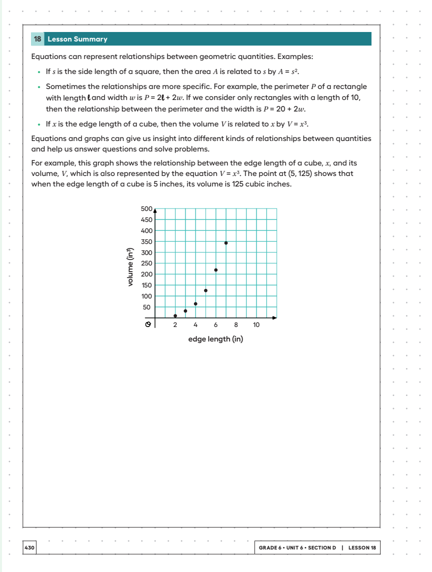
- If s is the side length of a square, then the area A is related to s by $A = s^2$.
- Sometimes the relationships are more specific. For example, the perimeter P of a rectangle with length ℓ and width w is $P = 2\ell + 2w$. If we consider only rectangles with a length of 10, then the relationship between the perimeter and the width is $P = 20 + 2w$.
- If x is the edge length of a cube, then the volume V is related to x by $V = x^3$.

Equations and graphs can give us insight into different kinds of relationships between quantities and help us answer questions and solve problems.

For example, this graph shows the relationship between the edge length of a cube, x, and its volume, V, which is also represented by the equation $V = x^3$. The point at (5, 125) shows that when the edge length of a cube is 5 inches, its volume is 125 cubic inches.



Student Workbook



Responding To Student Thinking

More Chances

The content of this *Cool-down* is beyond grade level expectations. Students will have more opportunities to develop this understanding in later courses. There is no need to slow down or add additional work to review this concept at this time.

Cool-down

Table, Equation, and Graph

5 min

Student Task Statement

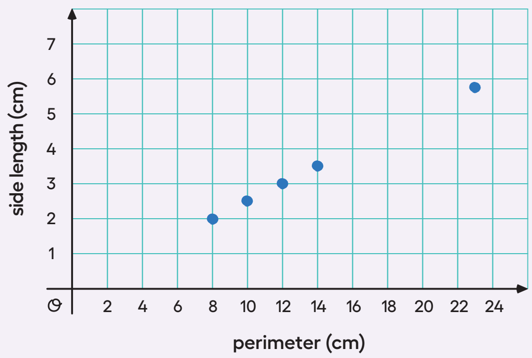
The table shows the relationship between the perimeter of a square and its side length.

perimeter (cm)	8	10	12	14	23
side length (cm)	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	$5\frac{3}{4}$

1. Write an equation that shows how the side length of a square in cm, s, is affected by changes in the perimeter of the square in cm, P.

   $\frac{1}{4}P = s$ (or equivalent)

2. Plot the ordered pairs from the table on the graph to show the relationship.



3. What does the point (12, 3) represent in this situation?

   Sample response: When the perimeter of square is 12 cm, its side length is 3 cm.

Lesson 18

Practice Problems

5 Problems

Problem 1

Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches. She draws bases of different lengths and tries to compute the height for each.

a. Write an equation Elena can use to find the height in inches, h, for each value of the base in inches, b.

$h = \frac{12}{b}$ (or equivalent)

b. Use your equation to find the height of a parallelogram with a base of 1.5 inches.

8 inches

Problem 2

Han is planning to ride his bike 24 miles.

a. How long will it take if he rides at a constant rate of:

3 miles per hour?

8 hours

4 miles per hour?

6 hours

6 miles per hour?

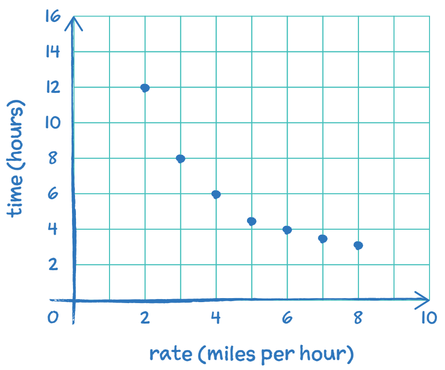
4 hours

b. Write an equation that Han can use to find t, the time in hours it will take to ride 24 miles, if his constant rate in miles per hour is represented by r.

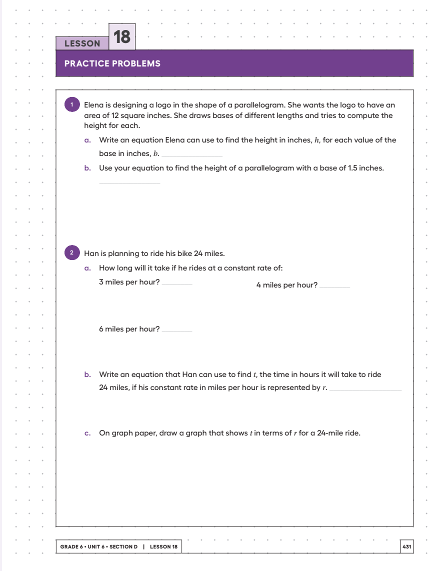
$t = 24 \div r$ or $t = \frac{24}{r}$

c. On graph paper, draw a graph that shows t in terms of r for a 24-mile ride.

Sample response:



Student Workbook


LESSON 18

PRACTICE PROBLEMS

1 Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches. She draws bases of different lengths and tries to compute the height for each.

a. Write an equation Elena can use to find the height in inches, h, for each value of the base in inches, b.

b. Use your equation to find the height of a parallelogram with a base of 1.5 inches.

2 Han is planning to ride his bike 24 miles.

a. How long will it take if he rides at a constant rate of:

3 miles per hour?

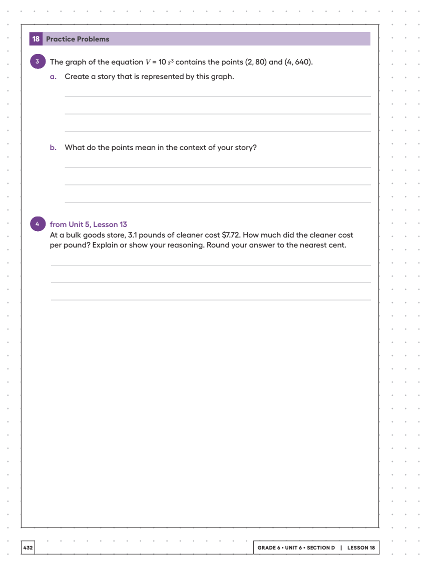
4 miles per hour?

6 miles per hour?

b. Write an equation that Han can use to find t, the time in hours it will take to ride 24 miles, if his constant rate in miles per hour is represented by r.

c. On graph paper, draw a graph that shows t in terms of r for a 24-mile ride.

Student Workbook


18 Practice Problems

3 The graph of the equation $V = 10s^3$ contains the points (2, 80) and (4, 640).

a. Create a story that is represented by this graph.

b. What do the points mean in the context of your story?

4 from Unit 5, Lesson 13
At a bulk goods store, 3.1 pounds of cleaner cost $7.72. How much did the cleaner cost per pound? Explain or show your reasoning. Round your answer to the nearest cent.

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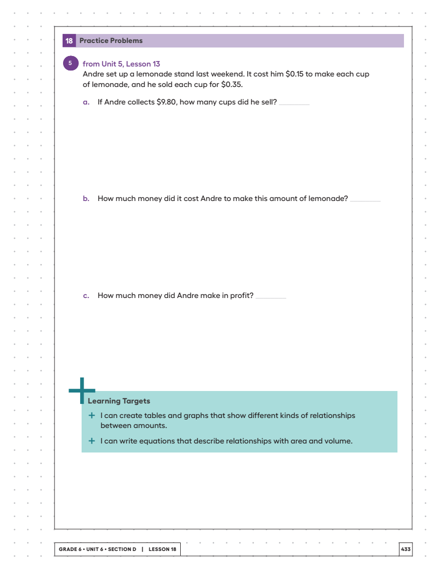
Student Workbook


18 Practice Problems

5 from Unit 5, Lesson 13
Andre set up a lemonade stand last weekend. It cost him $0.15 to make each cup of lemonade, and he sold each cup for $0.35.

a. If Andre collects $9.80, how many cups did he sell?

b. How much money did it cost Andre to make this amount of lemonade?

c. How much money did Andre make in profit?

Learning Targets
+ I can create tables and graphs that show different kinds of relationships between amounts.
+ I can write equations that describe relationships with area and volume.

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Problem 3

The graph of the equation $V = 10\,s^3$ contains the points (2, 80) and (4, 640).

a. Create a story that is represented by this graph.

Sample response:

Lin wants to build a tower of 10 identical cubes. She wants to know how the volume of the tower is affected by the edge length of the cube.

b. What do the points mean in the context of your story?

The point (2, 80) means that when the edge length of the cubes is 2 units, the volume of the cube tower is 80 cubic units. The point (4, 640) means that when the edge length of the cubes is 4 units, the volume of the cube tower is 640 cubic units.

Problem 4

from Unit 5, Lesson 13

At a bulk goods store, 3.1 pounds of cleaner cost $7.72. How much did the cleaner cost per pound? Explain or show your reasoning. Round your answer to the nearest cent.

$2.49 per pound

Sample reasoning:

Divide 7.72 by 3.1 by calculating $772 \div 310$ using long division. The quotient is a little more than 2.490.

Problem 5

from Unit 5, Lesson 13

Andre set up a lemonade stand last weekend. It cost him $0.15 to make each cup of lemonade, and he sold each cup for $0.35.

a. If Andre collects $9.80, how many cups did he sell?

28

$9.80 \div 0.35 = 28$

b. How much money did it cost Andre to make this amount of lemonade?

$4.20

$28 \cdot (0.15) = 4.20$

c. How much money did Andre make in profit?

$5.60

$9.80 - 4.20 = 5.60$