Lesson 6

Percentages and Equations

Goals

- Use equations to solve problems involving percentages and explain (orally) the solution method.
- Write equations of the form $px = q$ or equivalent to represent situations where the amount that corresponds to 100% is unknown.

Learning Target

I can solve percent problems by writing and solving equations.

Lesson Narrative

In this lesson, students apply what they have learned about solving an equation of the form $px = q$ to situations that involve percentages. Students learned about what percentages are and how to solve percentage problems in an earlier unit. However, they did not generalize the reasoning process for finding the value that corresponds to 100% when a percentage and its value is known, or for finding B in "A% of B is C" given A and C. Students have an opportunity to generalize that process in this lesson.

Students begin the lesson by recalling that they can find A% of B by multiplying $\frac{A}{100} \cdot B$. They then use repeated calculations to extend this structure and write the equation $\frac{A}{100} \cdot B = C$ to represent percentage problems and solve for B. An optional activity is included to allow students to practice solving such problems before they move on the main activity, an *Information Gap* activity. There, students solve real-world percentage problems where B is unknown.

Throughout the lesson students have opportunities to use and compare strategies based on arithmetic and those based on algebra. As needed, encourage students to use strategies from prior units (for example, using double number line diagrams) to make sense of problems and other solution strategies.

Student Learning Goal

Let's use equations to find percentages.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
10 min	15 min	20 min	10 min	5 min

Assessment

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR4: Information Gap Cards

Required Materials

Materials to Copy

- Info Gap Staying Active Cards (1 copy for every 2 students): Activity 2

Instructional Routines

Math Talk

ilclass.com/r/10694967

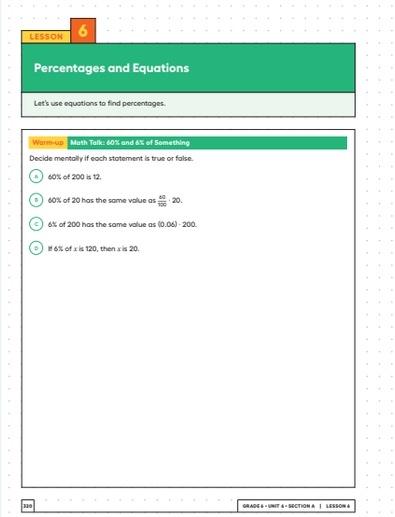
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 6

Percentages and Equations

Let's use equations to find percentages.

Warm-up Math Talk: 60% and 6% of Something

Decide mentally if each statement is true or false.

A. 60% of 200 is 12.

B. 60% of 20 has the same value as $\frac{60}{100} \cdot 20$.

C. 6% of 200 has the same value as $(0.06) \cdot 200$.

D. If 6% of x is 120, then x is 20.

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Warm-up

Math Talk: 60% and 6% of Something

10 min

Activity Narrative

This *Math Talk* focuses on percentages. It encourages students to think about the meaning of "percent" and to recall that they can find $A\%$ of B by calculating $\frac{A}{100} \cdot B$. It also encourages them to rely on what they know about fraction-decimal equivalence to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students write equations of the form $px = q$ to solve percentage problems.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally if each statement is true or false.

A. 60% of 200 is 12. False

Sample reasoning:

- 60% of 100 is 60, so 60% of 200 is twice 60, which is 120.
- 10% of 200 is 20, so 60% of 200 is $6 \cdot 20$, which is 120.

B. 60% of 20 has the same value as $\frac{60}{100} \cdot 20$. True

Sample reasoning:

- 60% of 20 is one tenth of 60% of 200, so it is one tenth of 120, which is 12. $\frac{60}{100} \cdot 20 = \frac{1,200}{100}$, which is also 12.
- 60% of 20 means $\frac{60}{100}$ of 20, which can be written as $\frac{60}{100} \cdot 20$.

C. 6% of 200 has the same value as $(0.06) \cdot 200$. True

Sample reasoning:

- 6% of 200 is $\frac{6}{100}$ of 200, and $\frac{6}{100}$ can be written as 0.06.
- 1% of 200 is 2, so 6% of 200 is $6 \cdot 2$, or 12. $(0.06) \cdot 200$ is also 12.

D. If 6% of x is 120, then x is 20. False

Sample reasoning:

- If 6% of a number is 120, that number must be much greater than 120, so it can't be 20.
- Substituting 20 for x means finding $\frac{6}{100} \cdot 20$, which gives 0.12, not 120.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

The goal of this discussion is to help students see that representing $A\%$ of B is C in terms of $px = q$ enables them to solve for x and find the value of B. Emphasize the following key points:

- "Percent" means "per hundred," so 60% means "60 per 100."
- We can find 60% of 20 by computing $\frac{60}{100} \cdot 20$ or $(0.6) \cdot 20$.
- We can represent "6% of x is 120" by writing an equation: $\frac{60}{100}x = 120$, which is equivalent to $0.06x = 120$.
- To solve $\frac{60}{100}x = 120$, we can divide each side by $\frac{60}{100}$, or multiply each side by $\frac{100}{60}$.
- To solve $0.06x = 120$, we can divide each side by 0.06.

Explain that representing situations involving percentages as equations and then finding the missing values can help us solve a wider range of percentage problems.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension. Activate or supply background knowledge about computing percentages. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory, Conceptual Processing

Activity 1

Writing Equations to Represent Percentage Problems

15 min

Activity Narrative

This activity allows students to make a stronger connection between the process of finding B in a situation in which $A\%$ of B is C and representing the situation with an equation of the form $px = q$ and finding the value of x. To reach this generalization, students perform repeated calculations, observe regularity in their calculations, and represent the regularity with algebraic expressions.

Launch

Remind students that in an earlier unit, they learned that to find P percent of a number x, they can multiply $\frac{P}{100} \cdot x$. Explain that they will now use this insight to write equations that represent relationships that involve percentages.

Arrange students in groups of 2.

Give 4–5 minutes of quiet work time and then 2–3 minutes to discuss their responses with a partner. Follow with a whole-class discussion.

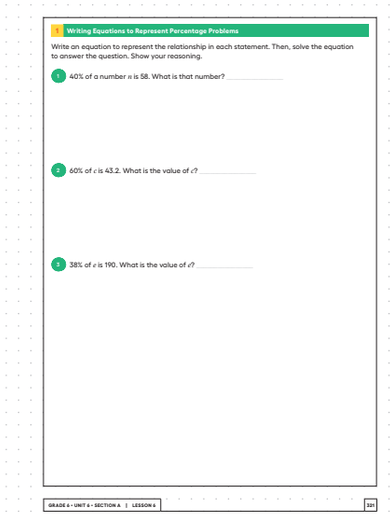
If time is limited, consider prompting students to answer one of the first two questions and then answer the last question.

Building on Student Thinking

If students set up each equation to find the percentage of the given number rather than the percentage of the variable, consider asking:

"What are the three quantities in this problem?"
"If you use a tape diagram or a double number line to represent the relationship between the three quantities, what would it look like?"
"How does the relationship described by your equation compare to that shown by your diagram?"

Student Workbook



1 Writing Equations to Represent Percentage Problems

Write an equation to represent the relationship in each statement. Then, solve the equation to answer the question. Show your reasoning.

1. 40% of a number n is 58. What is that number?
2. 60% of c is 43.2. What is the value of c?
3. 38% of e is 190. What is the value of e?

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Student Task Statement

Write an equation to represent the relationship in each statement. Then, solve the equation to answer the question. Show your reasoning.

1. 40% of a number n is 58. What is that number?

   Equation: $\frac{40}{100} \cdot n = 58$ or $0.4n = 58$

   Solution: $n = 145$

   Sample reasoning: $58 \div 0.4 = 580 \div 4 = 145$

2. 60% of c is 43.2. What is the value of c?

   Equation: $\frac{60}{100}c = 43.2$ or $0.6c = 43.2$

   Solution: $c = 72$

   Sample reasoning: $43 \div 0.6 = 432 \div 6 = 72$

3. 38% of e is 190. What is the value of e?

   Equation: $\frac{38}{100}e = 190$ or $0.38e = 190$

   Solution: $e = 500$

   Sample reasoning: $190 \cdot \frac{100}{38} = \frac{19{,}000}{38} = 500$

Activity Synthesis

The purpose of this discussion is for students to see how writing and solving an equation can be an efficient way to solve a problem about percentages.

Invite students to share their responses and strategies for the last question. If any students used tape diagrams or double number line diagrams to reason about a situation, consider displaying the diagrams alongside the equations. Doing so would help students make connections between strategies they understand well and the more abstract strategy of writing and solving an equation.

Highlight that every percentage problem in this activity is presented as "p of a number, x, equals another number q." or:

$p\%$ of $x = q$

which can be thought of as:

$\frac{p}{100} \cdot x = q$

Ask students:

- *"In all the problems in this activity, which of the three quantities corresponds to 100%?"*

  x

- *"Which quantity was unknown?"*

  the value of 100% or x

- *"Do we know the other two values?"*

  Yes

- *"How can we solve equations that have this form?"*

  Divide both sides of the equation by $\frac{p}{100}$ or its decimal equivalent.

Activity 2

Info Gap: Staying Active

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to solve percentage problems. Each problem involves finding the value that corresponds to 100% in a situation. While students have the opportunity to write equations of the form $px = q$ and then solve it to find the unknown value, they may also choose to reason in other ways.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch

Tell students they will continue to solve percentage problems in this activity, but now the problems are about everyday situations.

Invite students to share some of the ways that they stay active each day. Ask students:

- *"How do you work your muscles or get your heart rate up each day?"*

and

- *"Why might we want to incorporate movement into our days?"*

Explain to students that regular physical activity brings many benefits to our overall well-being, and that health experts have made recommendations on the amount of physical activity for children and adults to stay healthy.

Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Access for Multilingual Learners (Activity 2)

MLR4: Info Gap
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Instructional Routines

MLR4: Info Gap

ilclass.com/r/10695522

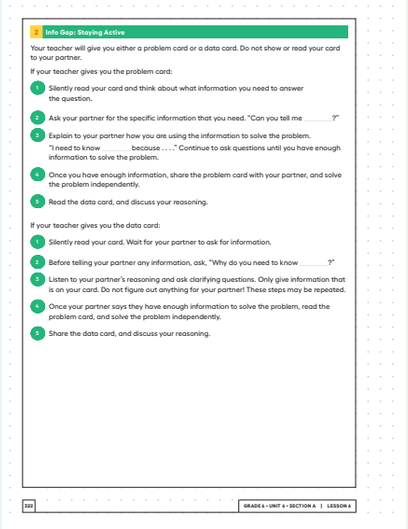
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the Info Gap graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Workbook



2 Info Gap: Staying Active

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

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3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
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2. Before telling your partner any information, ask, "Why do you need to know _____?"
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4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: 60 minutes a day

Sample reasoning:

- If 125% of the recommended time is 75 minutes, then 25% or one-fifth of that time is $75 \div 5$, or 15, minutes, and 100% of that time is $4 \cdot 15$, which is 60.
- 100 is $\frac{4}{5}$ of 125, so we can calculate $\frac{4}{5} \cdot 75$, which is $\frac{300}{5}$, or 60.
- $\frac{125}{100}x = 75$ or $1.25x = 75$, and $75 \div 1.25 = 60$.

Problem Card 2: 150 minutes a week

Sample reasoning:

- If 88% of the recommended time is 132 minutes, then 44% is 66 minutes, 4% is 6 minutes, and 100% is $25 \cdot 6$, which is 150.
- $\frac{88}{100}m = 132$ or $0.88m = 132$. Dividing 132 by 0.88 gives 150.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"For the first problem, what question did you ask first? Why?"*

  How much time did Elena spend doing something active? It's one of the two pieces of information that were needed.

- *"At what point did you realize that the recommended amount of time corresponds to 100%?"*

  From the beginning, because the problem said Elena did more than 100%.

  When hearing that Elena did physical activity for 125% of the recommended time.

- *"For each problem, what strategy did you use to solve the problem once you had all the information?"*

If writing and solving an equation is not mentioned as a strategy for answering either question, ask students to think about an equation that can be written to represent each problem.

Highlight for students that each problem can be described by an equation of the form $\frac{p}{100}x = q$, where $\frac{p}{100}$ is the percentage, x is the recommended number of minutes of physical activity, and q is the value that corresponds to the given percentage. To solve it, we can reason about what value of x makes the equation true, or divide each side of the equation by $\frac{p}{100}$ (or its decimal equivalent).

Lesson Synthesis

Students have been solving equations in the form $px = q$ with fraction and decimal coefficients in the past few lessons, so these problems involving percentages are an application of their prior work. Consider asking the following questions to help students recognize this connection and reflect on the merits and challenges of using equations:

- *"How are the equations we solved today related to the equations we solved earlier in this unit? How do solving strategies compare?"*

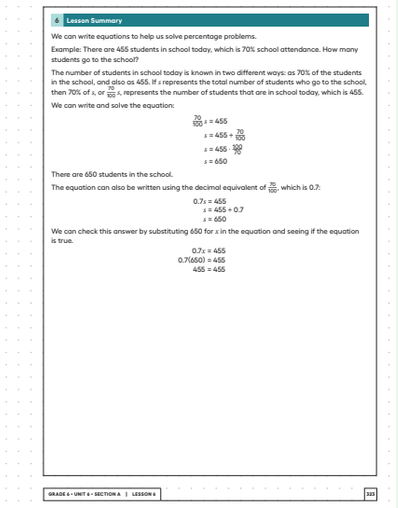
  "What are some advantages of writing an equation to solve a percentage problem rather than using a different strategy?"

  "Were there problems in this lesson for which equations were more helpful than other strategies, or less helpful than other strategies? Why might that be?"

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



6 Lesson Summary

We can write equations to help us solve percentage problems.

Example: There are 455 students in school today, which is 70% school attendance. How many students go to the school?

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s, or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

We can write and solve the equation:

$$\frac{70}{100}s = 455$$
$$s = 455 \div \frac{70}{100}$$
$$s = 455 \cdot \frac{100}{70}$$
$$s = 650$$

There are 650 students in the school.

The equation can also be written using the decimal equivalent of $\frac{70}{100}$, which is 0.7:

$$0.7s = 455$$
$$s = 455 \div 0.7$$
$$s = 650$$

We can check this answer by substituting 650 for x in the equation and seeing if the equation is true.

$$0.7x = 455$$
$$0.7(650) = 455$$
$$455 = 455$$

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Lesson Summary

We can write equations to help us solve percentage problems.

Example: There are 455 students in school today, which is 70% school attendance. How many students go to the school?

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s, or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

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We can check this answer by substituting 650 for x in the equation and seeing if the equation is true.

$$0.7x = 455$$
$$0.7(650) = 455$$
$$455 = 455$$

Cool-down

Fundraising for the Animal Shelter

5 min

Student Task Statement

Noah raised $54 to support the animal shelter, which is 60% of his fundraising goal. What is Noah's fundraising goal?

1. Write an equation with a variable to represent the situation.

   $54 = \frac{60}{100}x$ (or equivalent)

2. Answer the question. Show or explain your reasoning.

   $90

   Sample reasoning:

   - Divide both sides of the equation by $\frac{60}{100}$ or $\frac{3}{5}$ to get $x = 54 \cdot \frac{5}{3} = 90$.
   - I wrote $0.6x = 54$ and thought about six-tenths of what number would be equal to 54. $6 \cdot 9 = 54$, so 0.6 needs to be multiplied by 90 to get 54.

Lesson 6

Practice Problems

7 Problems

Problem 1

A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

$1\frac{1}{2}$ miles, because $\frac{3}{4}$ is half (or 50%) of $\frac{6}{4}$ or $1\frac{1}{2}$

Problem 2

40% of x is 35.

a. Write an equation that shows the relationship of 40%, x, and 35.

$0.4x = 35$ (or equivalent)

b. Use your equation to find the value of x. Show your reasoning.

$x = 87.5$

Sample reasoning: $35 \div 0.4 = 87.5$

Problem 3

Priya has completed 9 exam questions. This is 60% of the questions on the exam. How many questions are on the exam?

a. Write an equation with a variable to represent this situation. Explain the meaning of the variable.

Sample responses:

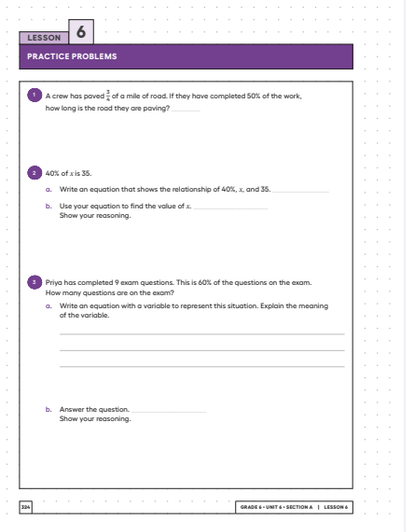
$9 = \frac{60}{100}x$ or $9 = 0.6x$, where x is the number of questions on the exam

b. Answer the question. Show your reasoning.

15 questions

Sample reasoning: $9 \div (0.6) = 15$

Student Workbook


LESSON 6

PRACTICE PROBLEMS

1. A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

2. 40% of x is 35.
   a. Write an equation that shows the relationship of 40%, x, and 35.
   b. Use your equation to find the value of x. Show your reasoning.

3. Priya has completed 9 exam questions. This is 60% of the questions on the exam. How many questions are on the exam?
   a. Write an equation with a variable to represent this situation. Explain the meaning of the variable.
   b. Answer the question. Show your reasoning.

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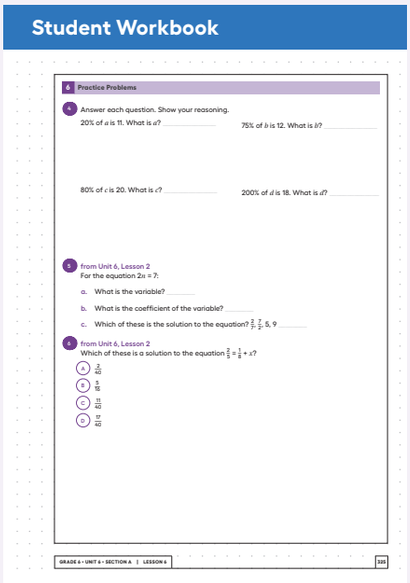
Student Workbook



6 Practice Problems

A Answer each question. Show your reasoning.

20% of a is 11. What is a? _____

75% of b is 12. What is b? _____

80% of c is 20. What is c? _____

200% of d is 18. What is d? _____

5 from Unit 6, Lesson 2
For the equation $2n = 7$:

a. What is the variable? _____

b. What is the coefficient of the variable? _____

c. Which of these is the solution to the equation? $\frac{2}{7}$, $\frac{7}{2}$, 5, 9 _____

6 from Unit 6, Lesson 2
Which of these is a solution to the equation $\frac{2}{5} = \frac{1}{8} + x$?

A. $\frac{2}{40}$
B. $\frac{5}{16}$
C. $\frac{11}{40}$
D. $\frac{17}{40}$

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Problem 4

Answer each question. Show your reasoning.

20% of a is 11. What is a?

55

75% of b is 12. What is b?

16

Sample reasoning for "75% of b is 12":

- Using an equation: $\frac{75}{100}b = 12$, so $b = 12 \div \frac{75}{100}$, so $b = 12 \cdot \frac{100}{75}$, so $b = 16$.
- Using a table: To get from the first row to the second row, divide 75 and 12 each by 3. To get from the second to the third row, multiply the 25 and 4 each by 4.

percentage	number
75	12
25	4
100	16

80% of c is 20. What is c?

25

200% of d is 18. What is d?

9

Problem 5

from Unit 6, Lesson 2

For the equation $2n = 7$:

a. What is the variable?

n

b. What is the coefficient of the variable?

2

c. Which of these is the solution to the equation? $\frac{2}{7}$, $\frac{7}{2}$, 5, 9

$\frac{7}{2}$

Problem 6

from Unit 6, Lesson 2

Which of these is a solution to the equation $\frac{2}{5} = \frac{1}{8} + x$?

A. $\frac{2}{40}$

B. $\frac{5}{16}$

C. $\frac{11}{40}$

D. $\frac{17}{40}$

Problem 7

from Unit 5, Lesson 13

Find the quotients.

a. 0.009 ÷ 0.001

9

b. 0.009 ÷ 0.002

4.5

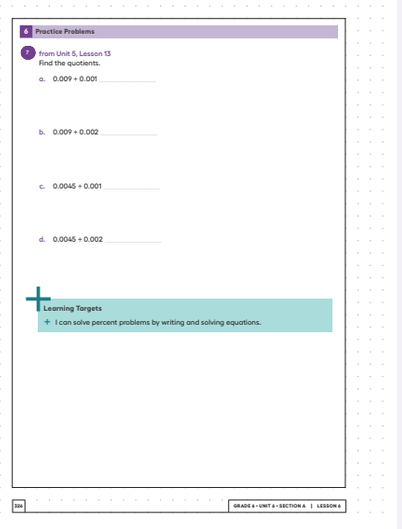
c. 0.0045 ÷ 0.001

4.5

d. 0.0045 ÷ 0.002

2.25

Student Workbook


6 Practice Problems

7 from Unit 5, Lesson 13
Find the quotients.

a. 0.009 ÷ 0.001

b. 0.009 ÷ 0.002

c. 0.0045 ÷ 0.001

d. 0.0045 ÷ 0.002

Learning Targets

+ I can solve percent problems by writing and solving equations.