Lesson 12

Constructing the Coordinate Plane

Goals

- Choose and label appropriate scales for the axes of the coordinate plane, according to the coordinates to be plotted, and explain (orally and in writing) the choice.
- Compare and contrast different scales for the axes of the coordinate plane.

Learning Target

When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes.

Lesson Narrative

In this lesson, students explore the idea of scaling axes appropriately to accommodate data where coordinates are rational numbers. Students attend to precision as they plan where to place axes on a grid and how to scale them to represent data clearly. In an optional activity, students practice working with coordinates in all four quadrants as they navigate a maze on a coordinate grid. This lesson gives students the opportunity to develop fluency with plotting coordinates in all four quadrants and scaling axes to fit data that is essential for the context-driven work over the next few lessons.

Student Learning Goal

Let's investigate different ways of creating a coordinate plane.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)

Instructional Routines

- 5 Practices

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	25 min	15 min	10 min	5 min

Warm-up

English Winter

5 min

Activity Narrative

In this *Warm-up*, students reason about the need for quadrants beyond the first quadrant in the coordinate plane when representing data within a situation's context. When choosing an appropriate set of axes, students should also notice that the scale of the axes is important for the given data. Both of these ideas will be important for students' reasoning in upcoming activities.

Launch

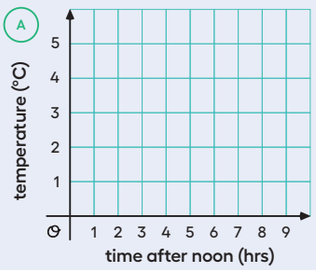
Give students 2 minutes of quiet work time, and follow with a whole-class discussion. If needed, clarify that the term "noon" refers to 12 p.m.

Student Task Statement

The following data were collected over one December afternoon in England.

time after noon (hours)	temperature (°C)
0	5
1	3
2	4
3	2
4	1
5	-2
6	-3
7	-4

A



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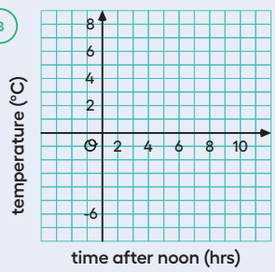

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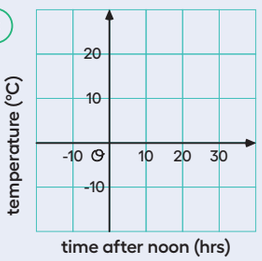
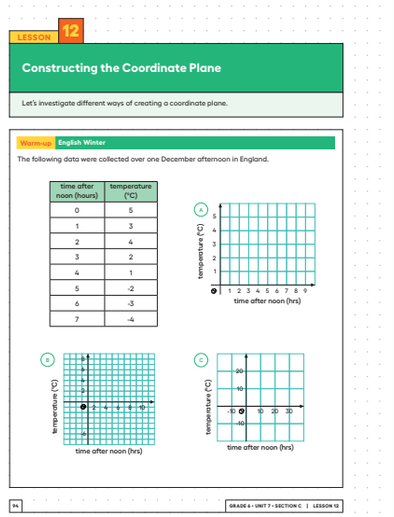


Student Workbook


LESSON 12

Constructing the Coordinate Plane

Let's investigate different ways of creating a coordinate plane.

Warm-up English Winter

The following data were collected over one December afternoon in England.

time after noon (hours)	temperature (°C)
0	5
1	3
2	4
3	2
4	1
5	-2
6	-3
7	-4

1. Which set of axes would you choose to represent these data? Explain your reasoning.

   Set B

   Sample reasoning: All of the data points will fit on this set of axes, and it will be easy to tell exactly where each data point goes because of the spacing of the grid lines.

2. Explain why the other two sets of axes did not seem as appropriate as the one you chose.

   Sample explanation: Set A does not include any negative numbers, so all the data would not get represented. Set C will fit all the data, but since the grid lines are spaced 10 units apart, it would be hard to plot the data accurately.

Activity Synthesis

The goal of this discussion is for students to share their responses and reasoning. Begin by asking the class which set of axes they chose to represent the data, and record their responses for all to see. Invite students to share their reasoning.

If time allows, ask students what kind of data would make the other sets of axes appropriate choices. For example, Set A would be appropriate if the temperatures were all positive, and Set C would be appropriate if the data were collected at 10-hour intervals and happened to be close to multiples of 10.

Activity 1
Axes Drawing Decisions

25 min

Activity Narrative

In this activity students draw their own axes for different sets of coordinates. They must decide which of the four quadrants they need to use and how to scale the axes.

Monitor for groups who scale the axes in these different ways for the first set of coordinates, ordered from more common to less common.

- Divide the grid into 4 equal-size quadrants. Scale both the x- and y-axes using the same scale.
- Divide the grid into 4 equal-size quadrants. Scale the x-axis and the y-axis using different scales.
- Divide the grid into 4 unequal-size quadrants.

Launch

Arrange students in groups of 2.

Give students 10 minutes to construct their graphs and discuss with their partners.

Follow with a whole-class discussion. Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Answers vary.

1. For each set of coordinates, draw and label an appropriate pair of axes, and plot the points.
   - **a.** (1, 2),(3, -4),(-5, -2),(0, 2.5)
   - **b.** (50, 50),(0, 0),(-10, -30),(-35, 40)
   - **c.** $\left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{-5}{4}, \frac{1}{2}\right), \left(-1\frac{1}{4}, \frac{-3}{4}\right), \left(\frac{1}{4}, \frac{-1}{2}\right)$
2. Discuss with a partner:
   - How are the axes and labels of your three drawings different?
   - How did the coordinates affect the way you drew the axes and label the numbers?

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time or after the first problem. Check in with students to provide feedback and encouragement after each chunk.

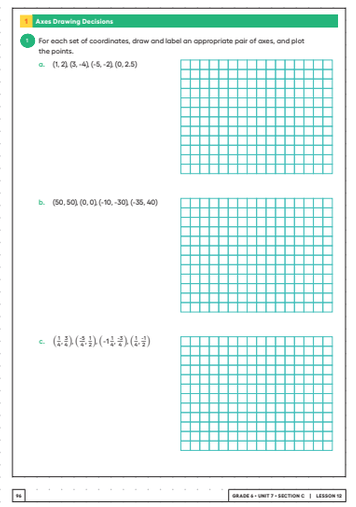
Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

If some students label their axes inconsistently, consider asking:

"What is the value of the interval between 0 and your first tick mark on the x- (or y-) axis?"
"What is the value of the interval between the first and second tick marks?" Does it match the first interval?"

Student Workbook



Activity Synthesis

The key takeaway from this discussion is that defining axes and scale is a process of reasoning, and that details such as the amount of empty space and the size of the numbers being plotted need to be considered.

Invite previously selected students to share their scaled axes for the first set of coordinates. Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions, such as:

- *"Does this set of axes allow for all the coordinates to be plotted?"*

  Yes

- *"What is different from previous approaches?"*

  The x- or y-axis counts by a different amount. The x- or y-axis is in a different location [higher, lower, farther left or right].

- *"How does this change show the coordinates differently?"*

  The locations of points are more accurate because each point can be plotted at the intersection of grid lines. There is less empty space. Points are located more in the center of the space instead of at the edges.

- *"What strategies can we come up with to help when drawing and labeling axes in the future?"*

  Look at the minimum and maximum values that need to be displayed, and count how many grid lines there are available.

Activity 2: Optional

Positively "a-MAZE-ing"

15 min

Activity Narrative

In this activity students locate and express coordinates in all four quadrants as they navigate through a maze. Students plan their route through the maze and strategically choose coordinates to correctly execute their plans. Consider using this activity if students would benefit from additional practice naming coordinates in all four quadrants.

Launch

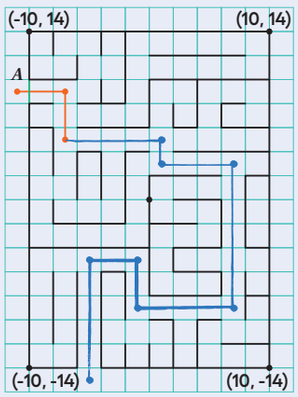
Arrange students in groups of 2.

Give students 8 minutes of quiet work time followed by a brief partner discussion.

Then hold a whole-class discussion.

Student Task Statement

Here is a maze in a coordinate plane.
The black point in the center is (0, 0).
The side of each grid square is 2 units long.



The starting point of the maze is located at the top left of the maze and labeled as point A. Draw line segments to show your way through and out of the maze. Label each turning point with a letter. Then, list all the letters, and write their coordinates.

- Start
- (-11, 9)
- $A(-7, 9)$
- $B(-7, 5)$
- $C(1, 5)$
- $D(1, 3)$
- $E(7, 3)$
- $F(7, -9)$
- $G(-1, -9)$
- $H(-1, -5)$
- $I(-5, -5)$
- $J(-5, -15)$

Access for Multilingual Learners (Activity 2, Launch)

MLR5: Co-Craft Questions
Keep books or devices closed. Display only the maze from the *Task Statement*, without revealing the question, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task, and invite additional connections.

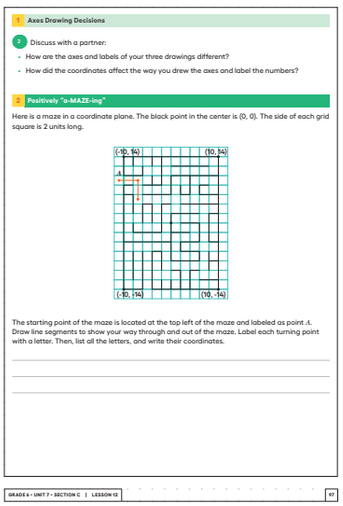
Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Provide students with access to mazes superimposed over a set of axes with grid lines labeled by one unit instead of two.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook


1 Axes Drawing Decisions

2 Discuss with a partner:

- How are the axes and labels of your three drawings different?
- How did the coordinates affect the way you drew the axes and label the numbers?

2 Positively "a-MAZE-ing"

Here is a maze in a coordinate plane. The black point in the center is (0, 0). The side of each grid square is 2 units long.

(-10, 14) (10, 14) A (-10, -14) (10, -14)

The starting point of the maze is located at the top left of the maze and labeled as point A. Draw line segments to show your way through and out of the maze. Label each turning point with a letter. Then, list all the letters, and write their coordinates.

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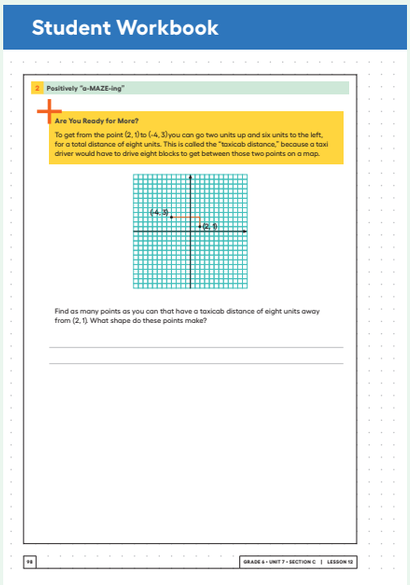
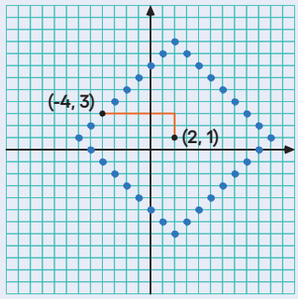
Student Workbook



Are You Ready for More?

To get from the point (2, 1) to (-4, 3) you can go two units up and six units to the left, for a total distance of eight units. This is called the "taxicab distance," because a taxi driver would have to drive eight blocks to get between those two points on a map.



Find as many points as you can that have a taxicab distance of eight units away from (2, 1). What shape do these points make?

The points form a square with vertices (10, 1), (2, 9), (-6, 1), and (2, -7).

Activity Synthesis

The goal of this discussion is for students to share how they determined the coordinates for each point. Tell students to compare their coordinates for each point in the maze with their partner. Consider discussing the following questions:

- *"Were there any disagreements between you and your partner? How did you resolve them?"*

  Answers vary.

- *"Did you notice anything about the coordinates of points at the ends of the same line segment?"*

  They always had either the x- or y-coordinate the same.

- *"Why do the two points at the end of a line segment always share a common x- or y-coordinate?"*

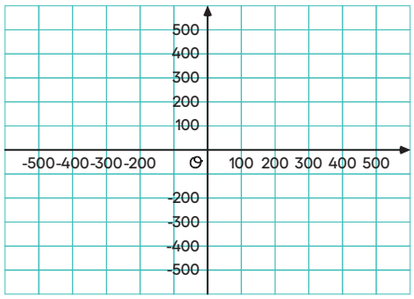
  The line segment is part of a horizontal or vertical line.

- *"Can you think of any other situations where coordinates are used to communicate location?"*

  In modern navigation systems, directions are precisely given in terms of coordinates and are then translated into a visual display for the driver. Precise coordinates are also used to navigate through virtual space in computer simulations.

Lesson Synthesis

The purpose of this discussion is to emphasize the importance of proper scaling. Begin by displaying this coordinate plane:



Tell students we want to plot the points (-2, 3) and (5, 7), and ask students why using this coordinate plane would not work well. (The x- and y-coordinates are very small in comparison with the scale, so it would be hard to accurately place the points and differentiate between them because they would be too close together.)

Here are some additional questions for discussion. It may be helpful to display an empty grid to place and label the axes and plot the points for each example.

- *"What are some points that would make sense to plot in this coordinate plane?"*

  (100, 200), (-300, 400)

- *"When plotting (-2, 3) and (5, 7), how many units across would you make the x- and y-axes? How would you label the axes?"*

  The x-axis needs to be at least 7 units across to go from -2 to 5. The y-axis needs to include values from 3 to 7, but in order to meet the x-axis, it should go at least from 0 to 7. It might look nicer to give some space in either direction, for example, going from -4 to 7 in the x-direction and -1 to 8 in the y-direction. In this case, the grid lines could be labeled by 1 unit.

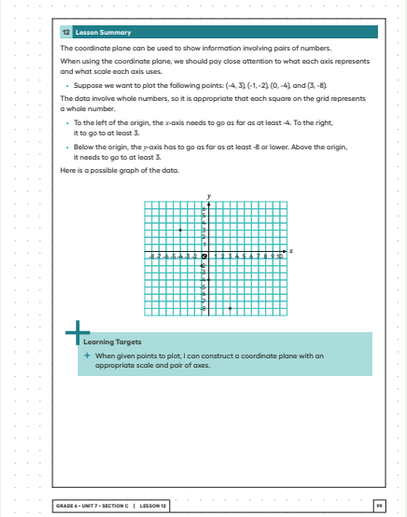
- *"When plotting (1.75, -0.5) and (-2.25, 1.5), how many units across would you make the x- and y-axes? How would you label the axes?"*

  The coordinates all look like multiples of 0.25, so the grid lines could be labeled by multiples of 0.25 units. The x-axis could go from -2.5 to 2, and the y-axis could go from -0.75 to 1.75.

- *"When plotting (-3, 40) and (4, -60), how many units across would you make the x- and y-axes? How would you label the axes?"*

  The x-axis could be labeled by 1 unit and the y-axis could be labeled by 10 units. The x-axis could go from -5 to 5 and the y-axis could go from -70 to 50.

Student Workbook


12 Lesson Summary

The coordinate plane can be used to show information involving pairs of numbers.

When using the coordinate plane, we should pay close attention to what each axis represents and what scale each axis uses.

- Suppose we want to plot the following points: (-4, 3), (-1, -2), (0, -4), and (3, -8).

The data involve whole numbers, so it is appropriate that each square on the grid represents a whole number.

- To the left of the origin, the x-axis needs to go as far as at least -4. To the right, it to go to at least 3.
- Below the origin, the y-axis has to go as far as at least -8 or lower. Above the origin, it needs to go to at least 3.

Here is a possible graph of the data.

Learning Targets

+ When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes.

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Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

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When using the coordinate plane, we should pay close attention to what each axis represents and what scale each axis uses.

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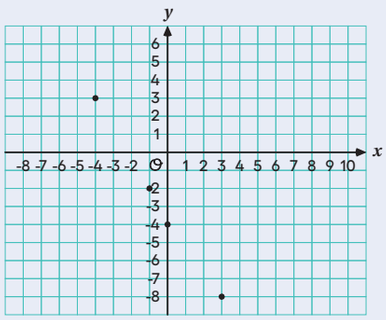


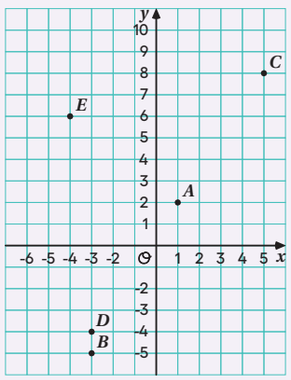
Cool-down

5 min

What Went Wrong: Graphing Edition

Student Task Statement

Lin labeled this set of axes and plotted the points $A(1, 2)$, $B(-3, -5)$, $C(5, 7)$, $D(-4, -3)$, and $E(-4, 6)$ in the coordinate plane.



Identify as many mistakes as you notice in Lin's graph.

- Point C is plotted at $(5, 8)$ instead of $(5, 7)$
- Point D is plotted at $(-3, -4)$ instead of $(-4, -3)$

Lesson 12

Practice Problems

6 Problems

Problem 1

Draw and label an appropriate pair of axes, and plot the points.

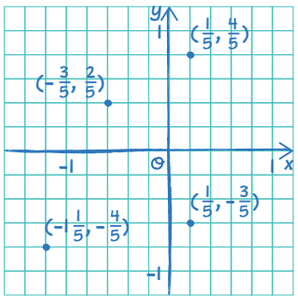
$\left(\frac{1}{5}, \frac{4}{5}\right)$

$\left(-\frac{3}{5}, \frac{2}{5}\right)$

$\left(-1\frac{1}{5}, -\frac{4}{5}\right)$

$\left(\frac{1}{5}, -\frac{3}{5}\right)$

Sample response:



Problem 2

Diego was asked to plot these points: (-50, 0), (150, 100), (200, -100), (350, 50), (-250, 0). What interval could he use for each axis? Explain your reasoning.

Sample response: Use an interval of 50, because all the coordinates involve points that are greater than 50 and multiples of 50.

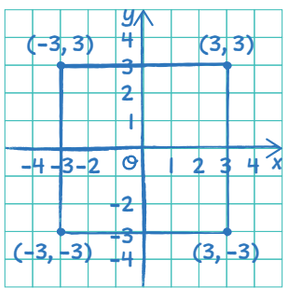
Problem 3

a. Name 4 points that would form a square with the origin at its center.

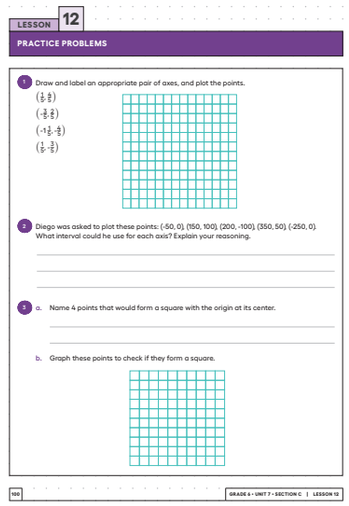
Sample response:

- A (3, 3)
- B (3, -3)
- C (-3, 3)
- D (-3, -3)

b. Graph these points to check if they form a square.



Student Workbook


LESSON 12

PRACTICE PROBLEMS

1 Draw and label an appropriate pair of axes, and plot the points.

$\left(\frac{1}{5}, \frac{4}{5}\right)$

$\left(-\frac{3}{5}, \frac{2}{5}\right)$

$\left(-1\frac{1}{5}, -\frac{4}{5}\right)$

$\left(\frac{1}{5}, -\frac{3}{5}\right)$

2 Diego was asked to plot these points: (-50, 0), (150, 100), (200, -100), (350, 50), (-250, 0). What interval could he use for each axis? Explain your reasoning.

3 a. Name 4 points that would form a square with the origin at its center.

b. Graph these points to check if they form a square.

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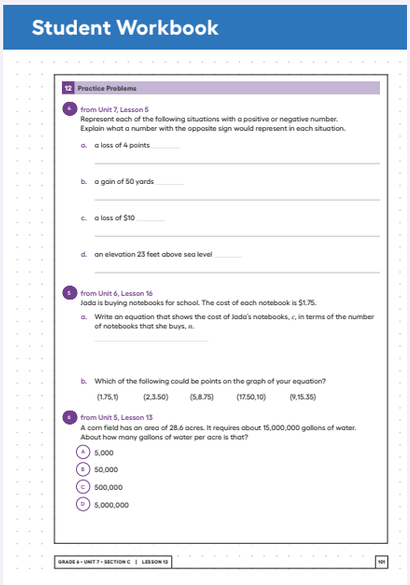
Student Workbook



12 Practice Problems

A from Unit 7, Lesson 5
Represent each of the following situations with a positive or negative number. Explain what a number with the opposite sign would represent in each situation.

a. a loss of 4 points _____

b. a gain of 50 yards _____

c. a loss of $10 _____

d. an elevation 23 feet above sea level _____

5 from Unit 6, Lesson 16
Jada is buying notebooks for school. The cost of each notebook is $1.75.

a. Write an equation that shows the cost of Jada's notebooks, c, in terms of the number of notebooks that she buys, n.

b. Which of the following could be points on the graph of your equation?

(1.75, 1) (2, 3.50) (5, 8.75) (17.50, 10) (9, 15.35)

6 from Unit 5, Lesson 13
A corn field has an area of 28.6 acres. It requires about 15,000,000 gallons of water. About how many gallons of water per acre is that?

A 5,000
B 50,000
C 500,000
D 5,000,000

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Problem 4

from Unit 7, Lesson 5

Represent each of the following situations with a positive or negative number. Explain what a number with the opposite sign would represent in each situation.

Sample responses:

a. a loss of 4 points

-4

Sample response: A positive number would represent a gain of points.

b. a gain of 50 yards

50

Sample response: A negative number would represent a loss of yards.

c. a loss of $10

-10

Sample response: A positive number would represent earning money.

d. an elevation 23 feet above sea level

23

Sample response: A negative number would represent an elevation below sea level.

Problem 5

from Unit 6, Lesson 16

Jada is buying notebooks for school. The cost of each notebook is $1.75.

a. Write an equation that shows the cost of Jada's notebooks, c, in terms of the number of notebooks that she buys, n.

$c = 1.75n$

b. Which of the following could be points on the graph of your equation?

(1.75, 1) (2, 3.50) (5, 8.75) (17.50, 10) (9, 15.35)

Problem 6

from Unit 5, Lesson 13

A corn field has an area of 28.6 acres. It requires about 15,000,000 gallons of water. About how many gallons of water per acre is that?

A. 5,000

B. 50,000

C. 500,000

D. 5,000,000