

Common Factors

Goals

- Comprehend (orally and in writing) the terms “factor,” “common factor,” and “greatest common factor.”
- Explain (orally and in writing) how to determine the greatest common factor of two whole numbers less than 100.
- List the factors of a number and identify common factors for two numbers in a real-world situation.

Learning Targets

- I can explain what a common factor is.
- I can explain what the greatest common factor is.
- I can find the greatest common factor of two whole numbers.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR6: Three Reads (Activity 1)

Lesson Narrative

In this lesson, students are introduced to **common factors** of two whole numbers as factors that both numbers share and the **greatest common factor** as the largest of these shared factors. Students reason abstractly and quantitatively as they see factors as representing the different ways to divide a certain number of books evenly. They make use of structure when they notice how factor pairs help ensure that all possible combinations have been found. Common factors represent the ways to combine two different types of books in one box, and the greatest common factor represents the largest number of combination boxes possible. Students also relate the greatest common factor to the size of the largest square that can tile a rectangle with given dimensions without gaps or overlaps.

Student Learning Goal

Let's use factors to solve problems.

Lesson Timeline

5
min

Warm-up

15
min

Activity 1

15
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Warm-up

Figures Made of Squares

5 min

Activity Narrative

In this *Warm-up*, students make observations about pairs of figures that each contain 6 and 10 squares. The height of each pair of figures changes, representing factors and common factors of 6 and 10, though students may not make this connection.

Launch

Display the image of the four pairs of figures from the *Task Statement* for all to see. Tell students to give a signal when they have at least one thing that is similar and one thing that is different.

Give students 1 minute of quiet think time, and follow with a whole-class discussion.

Inspire Math

Search and Rescue video

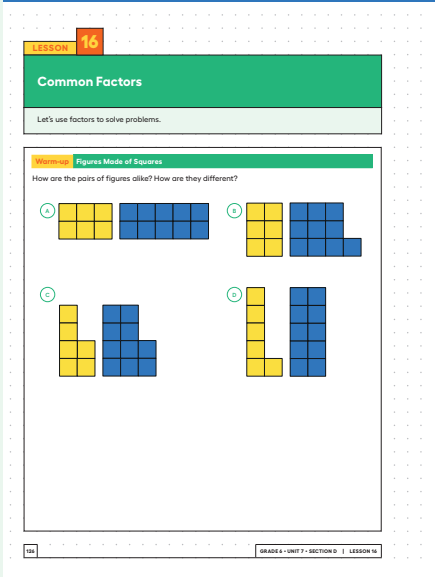


Go Online:
Before the lesson, show this video to review the real-world connection.

ilclass.com/1/614245

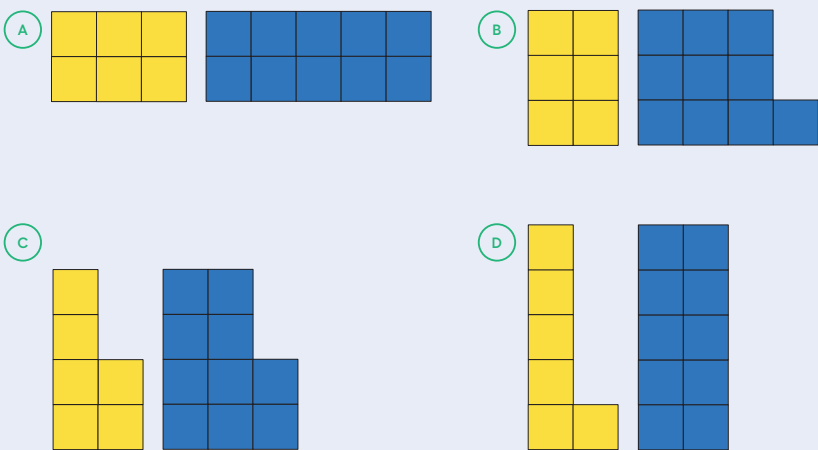
Please log in to the site before using the QR code or URL.

Student Workbook



Student Task Statement

How are the pairs of figures alike? How are they different?



Sample responses:

Similarities:

- Each pair has a blue figure and a yellow figure.
- Each figure is made of small squares.
- Each yellow figure is made up of 6 squares. Each blue figure is made up of 10 squares.

Differences:

- Some pairs have rectangles, and some do not.
- Some pairs have L-shaped figures and some do not.
- The heights of each pair is going up by 1 every time: 2, 3, 4, 5
- The first pair with a height of 2 is the only pair with two rectangles.
- The pair with a height of 4 is the only pair without at least one rectangle.

Instructional Routines

MLR6: Three Reads

ilclass.com/r/10695568

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners
(Activity 1, Launch)

MLR6: Three Reads

Keep books or devices closed.
Display only the problem stem,
without revealing the questions.
“We are going to read this problem
stem 3 times.”

After the 1st read:

*“Tell your partner what this
situation is about.”*

After the 2nd read:

*“List the quantities. What can be
counted or measured?”*

For the 3rd read: Reveal and read the
questions. Ask,

*“What are some ways we might
get started on this?”*

Advances: Reading, Representing

Activity Synthesis

The purpose of this discussion is to connect the pairs of figures to factors of 6 and 10. Ask students to share the things that are alike and different among the pairs of images. Record and display their responses for all to see. If possible, record their responses on the images where appropriate.

If not mentioned by students, discuss the following questions:

“2 and 3 are both factors of 6. How is this reflected in the diagram?”

“2 is a factor of both 6 and 10. How is this reflected in the diagram?”

“4 is not a factor of either 6 or 10. How is this reflected in the diagram?”

Remind students that a factor is one of two or more numbers that when multiplied together result in a given product. In this particular case, a factor is the height that will make a rectangle have a given area.

Activity 1

Diego's Book Drive

15
min

Activity Narrative

In this activity, students reason abstractly about common multiples and least common multiple to solve problems in context. Students find all the common factors of two whole numbers, one representing the number of fiction books and another representing nonfiction books. Then they compare the factors of each whole number to determine the greatest common factor.

Monitor for strategies and representations students use to make sure they account for all possible combinations. Some students may organize their work by number of boxes, checking each time if the total number can be divided into those boxes evenly, without a remainder. Other students may notice that combinations come in pairs. For example, 4 boxes of 12 fiction books can be paired with 12 boxes of 4 fiction books.

Launch



Arrange students in groups of 2.

Give students 10 minutes of work time, and follow with a whole-class discussion.

Encourage students to check in with their partner after each question to make sure they get every possible combination of boxes.

Student Task Statement

Diego is organizing fiction and nonfiction books into boxes to deliver to different classrooms. He has 48 fiction books and 64 nonfiction books. He would like each classroom to receive the same number of each type of book. Organize your answer to each question so that it can be followed by others.

1. How many different ways can Diego box the 48 fiction books? List all the different ways including the number of boxes and the number of fiction books in each box.

Diego can box the 48 fiction books in 10 different ways:

- 1 box of 48 fiction books and 48 boxes containing 1 fiction book
- 2 boxes with 24 fiction books and 24 boxes with 2 fiction books
- 3 boxes with 16 fiction books and 16 boxes with 3 fiction books
- 4 boxes with 12 fiction books and 12 boxes with 4 fiction books
- 6 boxes with 8 fiction books and 8 boxes with 6 fiction books

2. How many different ways can Diego box the 64 nonfiction books? List all the different ways including the number of boxes and the number of nonfiction books in each box.

Diego can box the 64 nonfiction books in 7 different ways:

- 1 box of 64 nonfiction books and 64 boxes containing 1 nonfiction book
- 2 boxes with 32 nonfiction books and 32 boxes with 2 nonfiction books
- 4 boxes with 16 nonfiction books and 16 boxes with 4 nonfiction books
- 8 boxes with 8 nonfiction books

3. How many different ways can Diego package all the 48 fiction and 64 nonfiction books so that each box has the same combination of items? List the number of boxes he can make and the number of each type of book that will be in each box.

Diego can make 5 combination boxes with both fiction and nonfiction books:

- 1 box with 48 fiction books and 64 non-fiction books
- 2 boxes with 24 fiction books and 32 non-fiction books
- 4 boxes with 12 fiction books and 16 non-fiction books
- 8 boxes with 6 fiction books and 8 non-fiction books
- 16 boxes with 3 fiction books and 4 non-fiction books

4. What is the largest number of combination boxes that Diego can make with no left over books? Explain to your partner how you know that it is the largest possible number of boxes.

The largest number of combination boxes that Diego can make is 16 boxes with 3 fiction books and 4 non-fiction books.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes and containers to model fiction and nonfiction book combinations.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Building on Student Thinking

Students might not find all combinations of factor pairs for each number. If this is the case, ask them to use snap cubes and prompt them to find more combinations. For example,

“Is there a way to place 64 snap cubes into 4 groups with no snap cubes left over? How many are in each group?”

Student Workbook

Diego's Book Drive

Diego is organizing fiction and nonfiction books into boxes to deliver to different classrooms. He has 48 fiction books and 64 nonfiction books. He would like each classroom to receive the same number of each type of book. Organize your answer to each question so that it can be followed by others.

1 How many different ways can Diego box the 48 fiction books? List all the different ways including the number of boxes and the number of fiction books in each box.

2 How many different ways can Diego box the 64 nonfiction books? List all the different ways including the number of boxes and the number of nonfiction books in each box.

3 How many different ways can Diego package all the 48 fiction and 64 nonfiction books so that each box has the same combination of items? List the number of boxes he can make and the number of each type of book that will be in each box.

Student Workbook

1

Diego's Book Drive

What is the largest number of combination boxes that Diego can make with no left-over books? Explain to your partner how you know that it is the largest possible number of boxes.

2

Greatest Common Factor

The **greatest common factor** of 30 and 18 is 6. What do you think the term "greatest common factor" means?

3

Find all of the factors of 21 and 6. Then, identify the greatest common factor of 21 and 6.

4

Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

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GRADE 6 • UNIT 7 • SECTION D | LESSON 16

Activity Synthesis

The purpose of this activity is for students to share how they organized information when finding different ways of sorting books. Invite students to share their strategies for the first two questions. Display or record their responses for all to see. Here are some strategies students may have used:

- Draw a picture to represent boxes and books.
- Make an organized list or table.
- Recognize that factors come in pairs and can always be reversed.

Ask students how they know they have found all possible combinations of boxes, and if necessary, confirm that there are 10 different possibilities for the boxes of fiction books, 7 different possibilities for the boxes of nonfiction books, and 5 different possibilities for the combination boxes.

Activity 2

Greatest Common Factor

15 min

Activity Narrative

In this activity, students are introduced to the terms “**common factor**” and “**greatest common factor**.” The final question connects the concept of greatest common factor to geometry when students describe what the greatest common factor is and how it applies to a geometric context.

Launch

Arrange students in groups of 2. Ask students to list the factors of 6 and 10 (6: 1, 2, 3, and 6; 10: 1, 2, 5, and 10). Then ask students to discuss with a partner what they think a common factor of two numbers is. Select 1–2 groups to share their thinking. If not brought up in students’ explanations, tell students that a **common factor** is a number that divides evenly into both numbers, or a factor that two numbers have in common. For example, 1 and 2 are common factors of 6 and 10.

Then give students 8 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. The **greatest common factor** of 30 and 18 is 6. What do you think the term “greatest common factor” means?

Sample response: The greatest common factor is the largest factor that numbers share.

2. Find all of the factors of 21 and 6. Then, identify the greatest common factor of 21 and 6.

The factors of 21 are 1, 3, 7, 21. The factors of 6 are 1, 2, 3, and 6.
The greatest common factor is 3.

3. Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

The factors of 28 are 1, 2, 4, 7, 14, 28. The factors of 12 are 1, 2, 3, 4, 6, and 12.
The greatest common factor is 4.

4. A rectangular bulletin board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes, but all of them have whole-number inches for their side lengths.

a. What is the side length of the largest square that Elena could use to completely cover the bulletin board without gaps or overlaps? Explain or show your reasoning.

3 inches
She could fit 4 squares by 9 squares within the rectangle.

b. How is the solution to this problem related to greatest common factor?

Sample reasoning: Since a square has the same length on all sides, its side length must be a number that goes evenly into both 12 and 27. The numbers 1 and 3 are both common factors of 12 and 27, but 3 is the largest common factor.

Are You Ready for More?

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then ...

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker: If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

The lockers that are open are 1, 4, 9, 16, 25, ... (all of the square numbers up to 1,000). This is because most numbers have factor pairs and therefore have an even number of factors. For example, $6 = 1 \cdot 6$ and $2 \cdot 3$. So the locker is opened twice and shut twice, meaning that it is closed at the end of the process. The exceptions are the square numbers, which have an odd number of factors. For example, $25 = 1 \cdot 25$ and $5 \cdot 5$, which means that it is only touched three times: opened, closed, and then opened again.

Student Workbook

1

Diego's Book Drive

2

What is the largest number of combination boxes that Diego can make with no left over books? Explain to your partner how you know that it is the largest possible number of boxes.

3

Greatest Common Factor

4

The greatest common factor of 30 and 18 is 6. What do you think the term "greatest common factor" means?

5

Find all of the factors of 21 and 6. Then, identify the greatest common factor of 21 and 6.

6

Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

Student Workbook

7

Greatest Common Factor

8

A rectangular bulletin board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes, but all of them have whole-number inches for their side lengths.
a. What is the side length of the largest square that Elena could use to completely cover the bulletin board without gaps or overlaps? Explain or show your reasoning.
b. How is the solution to this problem related to greatest common factor?

9

Are You Ready for More?

10

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then ...
• One student goes down the hall and opens each locker.
• A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
• A third student goes down the hall and changes every third locker: If a locker is open, he closes it. If a locker is closed, he opens it.
• A fourth student goes down the hall and changes every fourth locker.
This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to “What do you think the term ‘greatest common factor’ means?” Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Activity Synthesis

The purpose of this discussion is to formally introduce the term greatest common factor as the largest factor that two numbers share and to help students extend the connection between factors and the area model to include common factors and greatest common factor. Begin by inviting students to share their thinking on the last question. Record and display their responses for all to see. Here are some questions to discuss:

- “Consider a new bulletin board that is 18 inches tall and 63 inches wide. What are the side lengths of some squares that could completely cover the area with no gaps or overlaps?”

1-inch squares, 3-inch squares, 9-inch squares

- “If we wanted to use the largest size paper, which size squares should we use?”

the 9-inch squares

Explain how the 1-, 3-, and 9-inch squares represent the common factors of 18 and 63. The largest square that can tile the area represents the **greatest common factor** because it is the largest of the common factors and divides evenly into both 18 and 63.

Lesson Synthesis

The purpose of this discussion is for students to summarize the difference between factors, common factors, and the greatest common factor of two whole numbers. Here are some questions for discussion:

- “What is the factor of a number?”

Factors are numbers that can be multiplied together to result in a given product.

- “What are all the factors of 12? What are all the factors of 20?”

The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 20 are 1, 2, 4, 5, 10, and 20.

- “What are common factors of two numbers? What are the common factors of 12 and 20?”

Common factors are numbers that are factors for both of the given numbers. The common factors of 12 and 20 are 1, 2, and 4.

- “What is the greatest common factor of two numbers? What is the greatest common factor of 12 and 20?”

The greatest common factor of 2 numbers is the largest factor that both numbers share. The greatest common factor of 12 and 20 is 4.

- “What are some situations in which finding the greatest common factor is useful?”

It is useful when forming the largest amount of equal mixed groups with no items left over, or when determining the largest side length of a square that can be used to tile a rectangle.

Lesson Summary

A factor of a whole number is a whole number that divides evenly into that number, without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly, without a remainder.

A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45. They are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **greatest common factor** (sometimes written as GCF) of two whole numbers is the greatest of all the common factors. For example, 15 is the greatest common factor for 45 and 60.

One way to find the greatest common factor of two whole numbers is to list all of the factors for each and then look for the greatest factor they have in common. To find the greatest common factor of 18 and 24, first list all the factors of each number.

- Factors of 18: **1, 2, 3, 6, 9, 18**
- Factors of 24: **1, 2, 3, 4, 6, 8, 12, 24**

The common factors are 1, 2, 3, and 6. Of these common factors, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook

16 Lesson Summary

A factor of a whole number is a whole number that divides evenly into that number, without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly, without a remainder.

A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45. They are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **greatest common factor** (sometimes written as GCF) of two whole numbers is the greatest of all the common factors. For example, 15 is the greatest common factor for 45 and 60.

One way to find the greatest common factor of two whole numbers is to list all of the factors for each and then look for the greatest factor they have in common. To find the greatest common factor of 18 and 24, first list all the factors of each number.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these common factors, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

+

Learning Targets

- + I can explain what a common factor is.
- + I can explain what the greatest common factor is.
- + I can find the greatest common factor of two whole numbers.

60

GRADE 6 • UNIT 7 • SECTION D | LESSON 16

Cool-down

In Your Own Words

5 min

Student Task Statement

- What is the greatest common factor of 24 and 64? Explain or show your reasoning.
8
Sample reasoning: The common factors of 24 and 64 are 1, 2, 4, and 8, and 8 is the greatest.
- In your own words, what is the greatest common factor of two whole numbers? How can you find it?
Sample response: The greatest common factor of two whole numbers is the largest number that divides evenly into both numbers. You can find the greatest common factor by listing the factors of each number and then finding the greatest one that both numbers share.

Practice Problems

6 Problems

Student Workbook

LESSON 16
PRACTICE PROBLEMS

1. A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers in each bag.

What are the other possibilities? Explain or show your reasoning.

2. a. List all the factors of 42.

b. What is the greatest common factor of 42 and 15?

c. What is the greatest common factor of 42 and 50?

GRADE 4 • UNIT 7 • SECTION D | LESSON 16

Student Workbook

Practice Problems

3. A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

a. What is the largest number of groups that could be formed? Explain or show your reasoning.

b. If that many groups are formed, how many students of each grade level would be in each group?

4. From Unit 7, Lesson 15

Here are some bank transactions from a bank account last week. Which days' transactions represent negative values?

Monday: \$650 paycheck deposited

Tuesday: \$40 withdrawal from the ATM at the gas pump

Wednesday: \$20 credit for returned merchandise

Thursday: \$125 deducted for cell phone charges

Friday: \$45 check written to pay for book order

Saturday: \$80 withdrawal for weekend spending money

Sunday: \$10 cash-back reward deposited from a credit card company

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GRADE 4 • UNIT 7 • SECTION D | LESSON 16

Problem 1

A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers in each bag.

What are the other possibilities? Explain or show your reasoning.

Sample responses:

- 3 bags with 8 pencils and 12 stickers
 $3 \cdot 8 = 24$ and $3 \cdot 12 = 36$
- 4 bags with 6 pencils and 9 stickers
 $4 \cdot 6 = 24$ and $4 \cdot 9 = 36$
- 6 bags with 4 pencils and 6 stickers
 $6 \cdot 4 = 24$ and $6 \cdot 6 = 36$
- 12 bags with 2 pencils and 3 stickers
 $12 \cdot 2 = 24$ and $12 \cdot 3 = 36$

Problem 2

a. List all the factors of 42.

1, 2, 3, 6, 7, 14, 21, 42

b. What is the greatest common factor of 42 and 15?

3

c. What is the greatest common factor of 42 and 50?

2

Problem 3

A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

a. What is the largest number of groups that could be formed? Explain or show your reasoning.

15 groups

Sample reasoning: The greatest common factor of 75 and 90 is 15.

b. If that many groups are formed, how many students of each grade level would be in each group?

6 sixth-grade students and 5 seventh-grade students

Sample reasoning: $6 \cdot 15 = 90$ and $5 \cdot 15 = 75$

Problem 4

from Unit 7, Lesson 13

Here are some bank transactions from a bank account last week. Which days' transactions represent negative values?

Monday: \$650 paycheck deposited

Tuesday: \$40 withdrawal from the ATM at the gas pump

Wednesday: \$20 credit for returned merchandise

Thursday: \$125 deducted for cell phone charges

Friday: \$45 check written to pay for book order

Saturday: \$80 withdrawal for weekend spending money

Sunday: \$10 cash-back reward deposited from a credit card company

Problem 5

from Unit 4, Lesson 11

Find the quotients.

a. $\frac{1}{7} \div \frac{1}{8}$
 $\frac{8}{7}$

b. $\frac{12}{5} \div \frac{6}{5}$
 2

c. $\frac{1}{10} \div 10$
 $\frac{1}{100}$

d. $\frac{9}{10} \div \frac{10}{9}$
 $\frac{81}{100}$

Problem 6

from Unit 2, Lesson 9

An elephant can walk at a constant speed of up to 4 miles per hour, while a giraffe can walk at a constant speed of 3 miles in 20 minutes.

a. Which animal walks faster? Explain your reasoning.

The giraffe walks faster

Sample reasoning: There are 60 minutes in 1 hour, so the giraffe can walk 9 miles in 1 hour, while the elephant can only walk 4 miles.

b. How far can each animal walk in 3 hours?

Elephant: 12 miles; Giraffe: 27 miles

Student Workbook

16 Practice Problems

1 A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

a. What is the largest number of groups that could be formed? Explain or show your reasoning.

b. If that many groups are formed, how many students of each grade level would be in each group?

From Unit 7, Lesson 13
Here are some bank transactions from a bank account last week. Which days' transactions represent negative values?

Monday: \$650 paycheck deposited _____

Tuesday: \$40 withdrawal from the ATM at the gas pump _____

Wednesday: \$20 credit for returned merchandise _____

Thursday: \$125 deducted for cell phone charges _____

Friday: \$45 check written to pay for book order _____

Saturday: \$80 withdrawal for weekend spending money _____

Sunday: \$10 cash-back reward deposited from a credit card company _____

Student Workbook

16 Practice Problems

1 From Unit 4, Lesson 11
Find the quotients.

a. $\frac{1}{7} \div \frac{1}{8}$ _____

b. $\frac{12}{5} \div \frac{6}{5}$ _____

c. $\frac{1}{10} \div 10$ _____

d. $\frac{9}{10} \div \frac{10}{9}$ _____

From Unit 2, Lesson 9
An elephant can walk at a constant speed of up to 4 miles per hour, while a giraffe can walk at a constant speed of 3 miles in 20 minutes.

a. Which animal walks faster? Explain your reasoning.

b. How far can each animal walk in 3 hours?