Level-Set Shape Optimization: 2D Contour Vertices Distance Variance Minimization

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Shape optimization is a developing field for many disciplines. The Level-Set Method in the context of shape optimization, uses an evolving boundary to express the shape to be optimized. One such requirement for a well-posed boundary is that the discretization of the boundary is computed accurately and robustly, especially in regards to complex shapes. In this paper, an implementation of an optimizer solves the problem of generating accurate discretized contours for shape optimization in 2D. This implementation increases accuracy for complex shapes and the contours as they evolve through an optimization.

Nomenclature

φ	level-set function
Γ	level-set boundary
Ω	level-set interior
d_i	distance between vertices for the i^{th} line
$\boldsymbol{B}_{i,k}$	B-spline basis function of direction i and order k
$\boldsymbol{C}_{ij}$	control point at location (i, j)
$\boldsymbol{P}_{xyz}$	discrete set of vertices defining a level-set boundary

I. Introduction

Shape optimization is used in a variety of disciplines. Shape optimization can be used in structural topology optimizations [8-9], image segmentation [5], and visualize morphing three dimensional objects [6-7]. These optimizations represent solid shapes in two or three spatial dimensions using computerized models of functions. The functions to define these shapes are, in general, categorized to implicit and explicit definitions [2]. The implicit representation of shapes in these models is defined by a Level-Set Function (hereon referred to as LSF), and is optimized by the Level-Set Method. The Level-Set Method (hereon referred to as LSM) was first introduced by Osher and Sethian [1]. The LSM is described by a LSF, that evolves with respect to the Hamilton-Jacobi equation. For a wide range of implementations and solvers of the Hamilton-Jacobi equation, the reader is encouraged to view reference [3]. The other

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representation of shapes is by a linear combination of explicit basis functions. One such combination of functions, called B-splines, represent an object with continuous and differentiable splines. B-splines define a unique function with minimal support [10]. Shape optimization problems are commonly solved with the usage of either B-splines or LSFs. In this implementation, the LSM will be utilized.

One such shape optimization problem stems from a volume reachability optimization for a concentric tube robot (hereon referred to as CTR). The CTR is designed to reach through complex 3D shapes with maximal volume coverage. To appropriately tackle this problem, a simplified problem is being posed in this paper. Instead of a volume maximization problem, the simplified 2D area maximization problem is posed. In the process of optimizing this newly defined 2D shape, the precision of calculating its shape becomes a priority. Each iteration of this optimization must define a zero-contour in a discrete way and must readily calculate the area (2D) or volume (3D) of that zero-contour. Although the points along a zero-contour are trivial to calculate, the computational model requires that a discrete set of countour vertices (hereon referred to as CVs) be defined. Because the CVs are defined along each iteration of the optimization, it is imperative that the accuracy, distribution, and number of CVs is exact and subject to minimal error.

This paper presents the implementation of an optimizer that generates a zero-contour with equally spaced CVs. This optimizer solves a distance variance minimization problem for an arbitrarily defined LSF. Section 2 describes background on LSM and B-spline surfaces. Section 3 discusses the method of solving the distance variance minimization. Section 4 shows the result of this implementation with two case-studies. Section 5 discusses the results and significance to this contribution. The work is then summarized in Section 6.

II. Background

The Level-Set Function (LSF) is most commonly defined as a linear combination of basis functions [2]. These basis functions can be manipulated to define a new LSF, extending or reducing the boundaries of the LSF's zero-contour in different directions. This definition ensures that the Level-Set Method (LSM) can handle complex shapes and topological changes. Henceforth, the LSM is an excellent tool for representing evolving contour shapes, and the shape optimization problem posed by the CTR is ideally computed with the LSM [3]. The representation of the LSF's contour boundaries and interior shape are defined as

$$\Gamma := \{(x, y) \mid \varphi(x, y) = 0\} \tag{1}$$

$$\Omega := \{(x, y) \mid \varphi(x, y) > 0\}. \tag{2}$$

Through these equations, the LSM can easily calculate shape parameters [3].

III. Methods

To generate a well-defined zero-contour, the task is given to an optimizer. OpenMDAO is a robust, open-source framework that can solve multidisciplinary optimization problems [11]. OpenMDAO is the tool used to solve the posed problem of this paper. OpenMDAO uses the exact derivatives from the B-spline LSF representation [10] and converges on a solution that equates the distances between each CV to generates a well-defined contour.

As defined by Gray, Hwang, and Martins [11], a design optimization problem has three main components: the objective, design variables, and constraints. To generate equally spaced CVs, the objective to be minimized is selected to be the variance of all distances. This objective function is analogous to the least-squares problem. Conclusively, the optimization problem is defined as

minimize	$\sum_i d_i^2 - d_{mean}^2$
with respect to	u, v
subject to	$P_z = 0$

To represent the LSF, a B-spline representation of the 3D surface is chosen. B-splines are defined by

$$\boldsymbol{P}_{xyz}(u,v) \;=\; \sum_i \boldsymbol{B}_{i,k}(u)\boldsymbol{B}_{j,k}(v)\boldsymbol{C}_{ij}. \tag{3}$$

The set of control points are defined as a 10x10 grid of equally spaced points with one degree-of-freedom to control the z-coordinate of each point. Varying the z-coordinate positions of these control points allow the user to manually shape the

IV. Results

This section will discuss the results of three contours generated from the optimizer.

The first case-study was on an 'X' contour. This case was generated with a 10x10 grid of control points, 12x12 grid of sampling points to get an initial contour, and 120 contour vertices (CVs). Figure 1 visualizes the 3D B-spline surface and the corresponding contour. Figure 2 displays the solution pathway. The reader should note that the initialization of this contour is not the same as iteration 1.

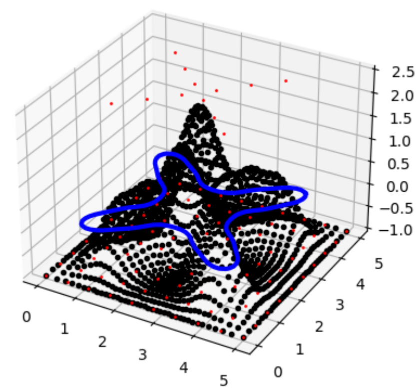
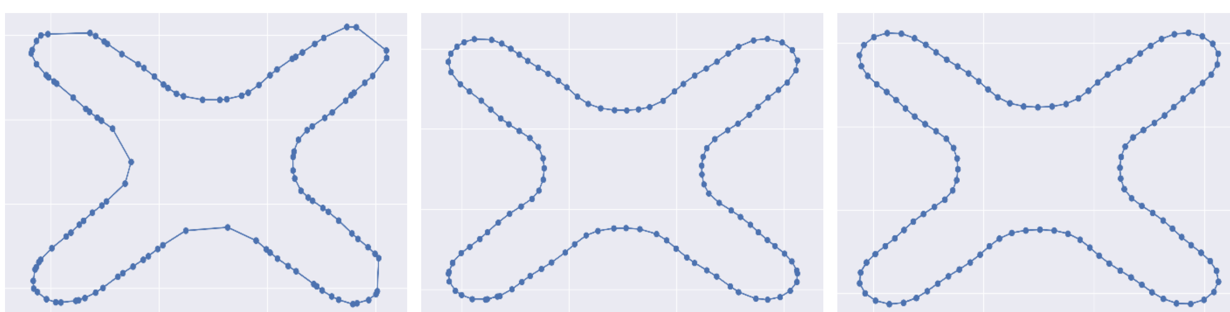


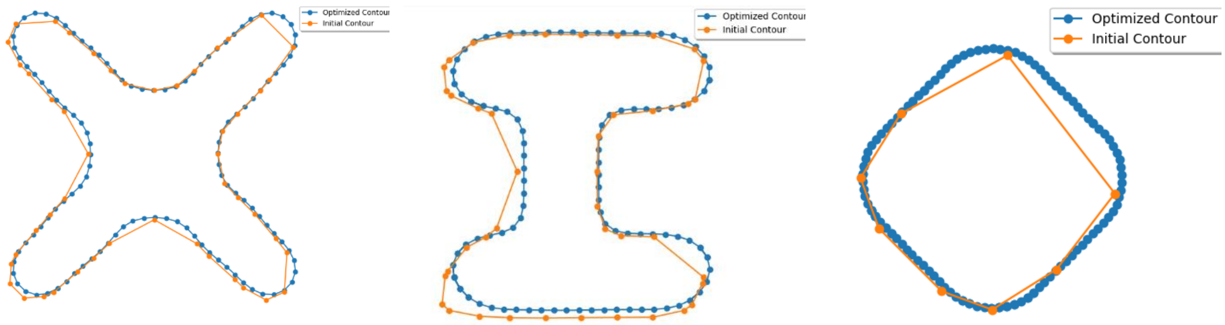
Figure 1 – 3D representation of the LSF (black), control points (red), and the zero-contour (blue)



a) Iteration 1 *b) Iteration 30* *c) Iteration 217*

Figure 2 – The X-contour solution pathway at Iternations 1, 30, 217

Figure 3 displays the results of all three case-studies. As can be seen, the optimization is robust enough to handle poor initial contours, as well as accurate enough to actualize the zero-contour more accurately than the initial zero-contour.



a) X – Contour *b) I – Contour* *c) O – Contour*

Figure 3 – The initial and final results for the X, I, and O contour optimizations

In addition, video representations of these contours have been linked below, as the file size was prohibitive to the submission of this project.
https://1drv.ms/u/s!AuehtV_4owE-g7hJJzlXTr3vuSWRGA?e=lqMtbl

V. Discussion

The implementation of this algorithm is shown to actualize the exact contour from an inaccurate initial guess. In addition, the algorithm will always produces a contour with the same number of vertices. The shortcomings of this implementation is that it cannot handle multiple disjoint contours, holes, or progression through the edges of the design domain.

VI.Conclusion

The field of shape optimization is applied in a variety of fields. Many fields utilize 2D and 3D optimizations represented by B-splines and Level-Set Functions (LSF). In this paper, the discussion of a 2D optimization was expanded to the accuracy of LSF zero-contour actualization. Further implementation of this algorithm hope to span three spatial dimensions and eventually solve the original volume maximization problem of the concentric tube robot.

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References

[1] S. Osher and J. A. Sethian, "Fronts Propogating with Curvature Dependent Speed: Algorithms Based on Hamilton Jacobi Formulations," Journal of Computational Physics, 1988.

[2] R. T. Whitaker, "Isosurfaces and Level-Set Surface Models," IEEE Visualization Technical Report UUCS-02-010. Apr. 2002.

[3] S. Osher and R. P. Fedkiw, "A Survey on Level Set Methods for Inverse Problems and Optimal Design," University of Californa Los Angeles and Stanford University. Apr. 2005.

[4] G. R. Anderson, "Shape Optimization in Adaptive Search Spaces," Stanford University. Dec 2015. pp. 9,11

[5] C. Li, C. Xu, C. Gui, M. D. Fox, "Distance Regularized Level Set Evolution and Its Application to Image Segmentation," IEEE Transaction on Image Processing, VOL 19, NO. 12. Dec. 2010. pp. 54

[6] D. E. Breen and R. T. Whitaker, "A Level-Set Approach for the Metamorphosis of Solid Models," California Institute of Technology and University of Utah. Mar. 2001.

[7] P. Burchard, L. T. Cheng, B. Merriman, S. Osher, "Motion of Curves in Three Spatial Dimensions Using a Level Set Approach," Journal of Computational Physics. July 2001.

[8] G. Allaire, F. Jouve, A. M. Toader, "Structural optimization using sensitivity analysis and a level-set method," Journal of Computational Physics. Feb 2004.

[9] M. Y. Wang, X. Wang, D. Guo, "A level set method for structural topology optimization," Computer Methods in Applied Mechanics and Engineering. Jan 2003.

[10] J. T. Hwang, J. R. R. A. Martins, "A Dynamic Parameterization Scheme for Shape Optimization Using Quasi-Newton Methods," American Institute of Aeronautics and Astronautics, Inc. Jan 2012.

[11] J. S. Gray, J. T. Hwang, J. R. R. A. Martins, K. T. Moore, and B. A. Naylor, "OpenMDAO: An Open-Source Framework for Multidisciplinary Design, Analysis, and Optimization," Structural and Multidisciplinary Optimization, 2019.