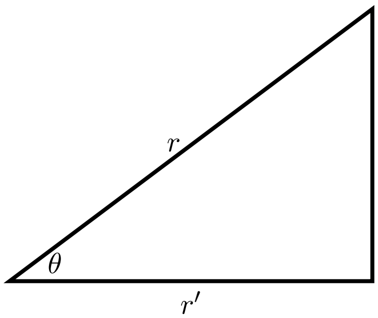
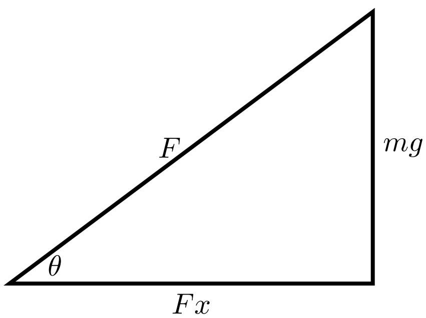
We are trying to find an expression for r', ideally one that does not use any assumptions as to the magnitude of the tension force, as that will be prone to errors.

We will begin by drawing the triangle whose sides represent the measured length of string r and the actual radius of rotation r':



Similarly, we can draw a triangle whose sides represent the forces acting on the rotating object:



Since these 2 triangles are similar, there must exist some scaling factor n such that $F_x = nr'$ and $F = nr$:

$$\sum F_{centripetal} = ma_c$$
$$F_x = ma_c$$
$$F_x = \frac{mv^2}{r'}$$
$$\boxed{v = \frac{2\pi r'}{T}}$$
$$F_x = \frac{4\pi^2 r' m}{T^2}$$
$$F_x = nr'$$
$$nr' = \frac{4\pi^2 r' m}{T^2}$$
$$n = \frac{4\pi^2 m}{T^2}$$

Thus:

$$F = nr$$

$$\boxed{F = \frac{4\pi^2 m}{T^2} r}$$

$$F^2 = F_x^2 + (mg)^2 \quad \text{by Pythagorean theorem}$$

$$\left[\frac{4\pi^2 m}{T^2} r\right]^2 = \left[\frac{4\pi^2 m}{T^2} r'\right]^2 + (mg)^2$$

$$r' = \frac{T^2\sqrt{\left[\frac{4\pi^2 mr}{T^2}\right]^2 - (mg)^2}}{4\pi^2 m}$$

$$r' = \frac{\sqrt{16\pi^4 r^2 - T^4 g^2}}{4\pi^2}$$

$$r' = \sqrt{r^2 - \frac{T^4 g^2}{16\pi^4}}$$

$$\boxed{r' = \sqrt{r^2 - \left(\frac{T^2 g}{4\pi^2}\right)^2}}$$