$$
\begin{aligned}
n &= \sin\theta + b\cos\theta \\
x &= \cos\theta \\
y &= \sin\theta \\
n &= y + bx \\
1 &= \sin^2\theta + \cos^2\theta \\
1 &= y^2 + x^2 \\
1 &= (n - bx)^2 + x^2 \\
1 &= n^2 - 2nbx + b^2x^2 + x^2 \\
0 &= x^2(b^2+1) - 2nbx + (n^2 - 1) \\
x &= \frac{2nb \pm \sqrt{4n^2b^2 - 4b^2n^2 + 4b^2 - 4n^2 + 4}}{2(b^2+1)} \\
x &= \frac{nb \pm \sqrt{b^2 - n^2 + 1}}{(b^2+1)} \\
\cos\theta &= \frac{nb \pm \sqrt{b^2 - n^2 + 1}}{(b^2+1)} \\
\theta &= \cos^{-1}\left[\frac{nb \pm \sqrt{b^2 - n^2 + 1}}{(b^2+1)}\right]
\end{aligned}
$$

To find maximum n / minimum b:

$$
\begin{aligned}
b^2 - n^2 + 1 &\geq 0 \\
b^2 + 1 &\geq n^2
\end{aligned}
$$

At $n^2 = b^2 + 1$:

$$
\begin{aligned}
\theta &= \cos^{-1}\left[\frac{nb \pm \sqrt{b^2 - n^2 + 1}}{(b^2+1)}\right] \\
\theta &= \cos^{-1}\left[\frac{nb \pm \sqrt{b^2 - b^2 - 1 + 1}}{n^2}\right] \\
\theta &= \cos^{-1}\left(\frac{b}{n}\right)
\end{aligned}
$$