45 Degree Destructive Interference

Ryan Ellin

Given a thin film such that perpendicularly incident light has totally destructive interference for a given wavelength:

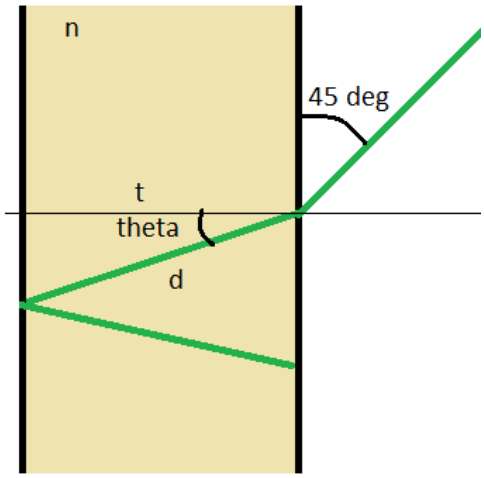
$$\begin{aligned}\phi_{tot1} &= \phi_{Refl} + \phi_{p1} \\ &= (1+2m)\pi,\ m \in \mathbb{Z} \text{ s.t. } m \geq 0\end{aligned}$$

Next, we shall consider the phase shift for light at a 45 degree incident angle:

$$\begin{aligned}\text{Let } \phi_{p_2} - \phi_{p_1} &= 2\pi z,\ z \in \mathbb{Z} \text{ s.t. } z > 0 \\ \phi_{p_2} &= \phi_{p_1} + 2\pi z \\ \phi_{tot2} &= \phi_{Refl} + \phi_{p_2} \\ &= \phi_{Refl} + (\phi_{p_1} + 2\pi z) \\ &= (\phi_{Refl} + \phi_{p_1}) + 2\pi z \\ &= \phi_{tot1} + 2\pi z \\ &= (1+2m)\pi + 2\pi z \\ &\boxed{\phi_{tot2} = (1+2m+2z)\pi}\end{aligned}$$

Thus, if the phase shift due to path length difference satisfies the above assumption, the interference will be fully destructive.

Next, we will find an expression for the path length of the light incident at 45 degrees in terms of other variables:



$$n \sin\theta = \sin 45^\circ$$

$$\sin\theta = \frac{1}{n\sqrt{2}}$$

$$\therefore \cos\theta = \frac{\sqrt{2n^2-1}}{n\sqrt{2}}$$

$$\cos\theta = \frac{t}{d}$$

$$d = \frac{t}{\cos\theta}$$

$$\boxed{d = \frac{tn\sqrt{2}}{\sqrt{2n^2-1}}}$$

Next, will define expressions for the phase shifts due to path length difference:

$$\boxed{\phi_{p_1} = \frac{4\pi tn}{\lambda_o}}$$

$$\phi_{p_2} = \frac{4\pi nd}{\lambda_o}$$

$$\boxed{\phi_{p_2} = \left(\frac{4\pi tn}{\lambda_o}\right)\left(\frac{n\sqrt{2}}{\sqrt{2n^2-1}}\right)}$$

Finally, we will use these expressions in conjunction with our initial relationship between the two path length phase shifts:

$$\begin{aligned}
\phi_{p_2} - \phi_{p_1} &= 2\pi z \\
\left(\frac{4\pi tn}{\lambda_o}\right)\left(\frac{n\sqrt{2}}{\sqrt{2n^2-1}}\right) - \frac{4\pi tn}{\lambda_o} &= 2\pi z \\
\left(\frac{4\pi tn}{\lambda_o}\right)\left(\frac{n\sqrt{2}}{\sqrt{2n^2-1}} - 1\right) &= 2\pi z \\
\boxed{\left(\frac{2tn}{\lambda_o}\right)\left(\frac{n\sqrt{2}-\sqrt{2n^2-1}}{\sqrt{2n^2-1}}\right) = z}
\end{aligned}$$

Thus, any combination of film thickness, index of refraction, and wavelength of light which satisfies this equation will have fully destructive interference at an incident angle of 45 degrees.