Speed of Sound Derivation

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\begin{aligned}
v &= \sqrt{\frac{\gamma RT}{M}} \\
v &= \sqrt{\frac{\gamma R}{M}}\sqrt{T} \\
T_o &= 273.15\text{K} \\
v_o &= v(T_o) \\
\Delta T &= T - T_o \\
v' &= \frac{dv}{dT} \\
v &\approx v_o + v'(T_o)\Delta T \\
v_o &= \sqrt{\frac{(1.4)(8.31\text{J/mol}\cdot\text{K})}{.029\text{kg/ mol}}}\sqrt{273.15\text{K}} \\
v_o &= 331\text{m/sec} \\
v' &= \sqrt{\frac{\gamma R}{M}}\left(\frac{1}{2\sqrt{T}}\right) \\
v'(T_o) &= \sqrt{\frac{(1.4)(8.31\text{J/mol}\cdot\text{K})}{.029\text{kg/ mol}}}\left(\frac{1}{2\sqrt{273.15\text{K}}}\right) \\
v'(T_o) &= .606\text{m/sec}\cdot\text{K} \\
v &\approx 331\text{m/sec} + .606(\text{m/sec}\cdot\text{K})\Delta T \\
&\approx 331\text{m/sec} + .606(\text{m/sec}\cdot\text{K})(T - T_o) \\
&\approx 331\text{m/sec} + .606(\text{m/sec}\cdot\text{K})(T - 273.15\text{K}) \\
&\boxed{v \approx 331\text{m/sec} + .606(\text{m/sec}\cdot\text{C}^\circ)T_C}
\end{aligned}
$$