Parallel Rays from Distant Objects

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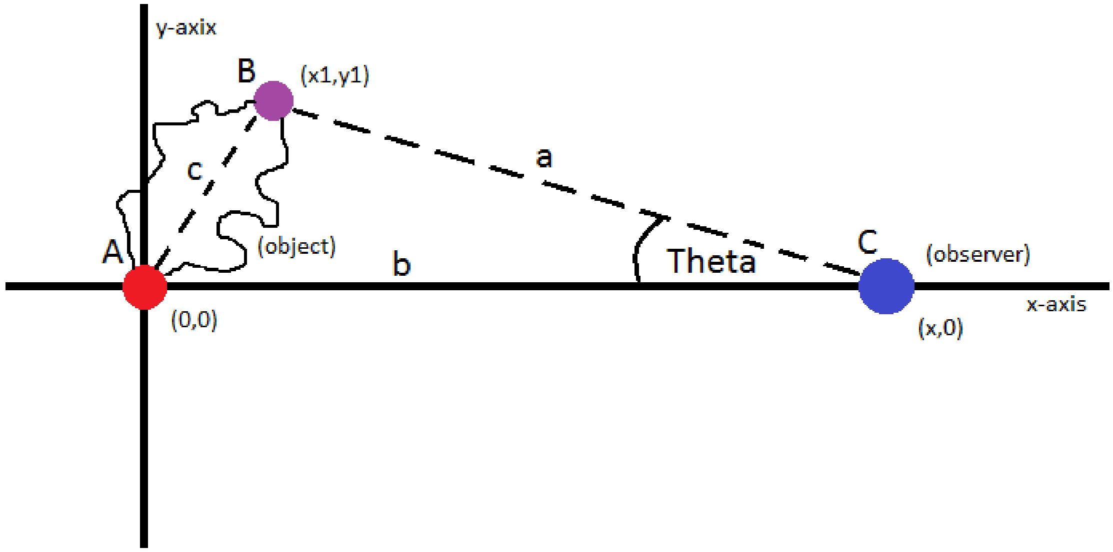

In the above image, points A and B represent the outermost edges of an object relative to the perspective of an observer at point C. For the sake of convenience, the axes have been defined such that point A lies at the origin, and the observer at point C falls on the x-axis.

The outermost edges of the object specifically are studied as if it can be demonstrated that rays from these points will approach parallel, rays from intermediate points between A and B must also approach parallel (the angle between the rays originating on any two points on the object must be less than or equal to the angle between the rays originating from point A and B).

A function for angle theta can be derived using the law of cosines:

$$c^2 = b^2 + a^2 - 2ba\cos\theta$$

$$\left(\sqrt{x_1^2 + y_1^2}\right)^2 = x^2 + \left(\sqrt{(x-x_1)^2 + y_1^2}\right)^2 - 2x\sqrt{(x-x_1)^2 + y_1^2}\cos\theta$$

$$x_1^2 + y_1^2 = x^2 + (x-x_1)^2 + y_1^2 - 2x\sqrt{(x-x_1)^2 + y_1^2}\cos\theta$$

$$x_1^2 + y_1^2 = x^2 + x_1^2 + x^2 - 2xx_1 + y_1^2 - 2x\sqrt{(x-x_1)^2 + y_1^2}\cos\theta$$

$$2x\sqrt{(x-x_1)^2 + y_1^2}\cos\theta = 2x^2 - 2xx_1$$

$$\cos\theta = \frac{2x(x-x_1)}{2x\sqrt{(x-x_1)^2 + y_1^2}}$$

$$\theta = \cos^{-1}\left(\frac{x(x-x_1)}{x\sqrt{(x-x_1)^2 + y_1^2}}\right)$$

Next, to determine what will happen to angle theta as x, the distance between point A and the observer at point C, becomes very large, a limit is applied:

Let $x \to \infty$:

$$\lim_{x\to\infty} \theta = \lim_{x\to\infty} \cos^{-1}\left(\frac{x(x-x_1)}{x\sqrt{(x-x_1)^2+y_1^2}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1} \lim_{x\to\infty}\left(\frac{x(x-x_1)}{x\sqrt{(x-x_1)^2+y_1^2}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1} \lim_{x\to\infty}\left(\frac{x-x_1}{\sqrt{(x-x_1)^2+y_1^2}} \times \frac{\frac{1}{x}}{\frac{1}{x}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1} \lim_{x\to\infty}\left(\frac{1-\frac{x_1}{x}}{\sqrt{\frac{x^2+x_1^2-2xx_1+y_1^2}{x^2}}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1} \lim_{x\to\infty}\left(\frac{1-\frac{x_1}{x}}{\sqrt{1+\frac{x_1^2}{x^2}-\frac{2x_1}{x}+\frac{y_1^2}{x^2}}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1}\left(\frac{1-0}{\sqrt{1+0-0+0}}\right)$$
$$\lim_{x\to\infty} \theta = \cos^{-1} 1$$
$$\lim_{x\to\infty} \theta = 0$$

Thus, it has been demonstrated that as the distance between an object and observer becomes very large relative to the size of the object itself, the rays from that object to the observer will approach parallel.