6.1.7

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$$\begin{aligned}
K_{false} &= K_{true} \\
\frac{1}{2}mv_f^2 &= mc^2(\gamma - 1) \\
\frac{1}{2}v_f^2 &= c^2(\gamma - 1) \\
\gamma &= \frac{1}{\sqrt{1 - \frac{v_t^2}{c^2}}} \\
\frac{1}{2}v_f^2 &= c^2\left(\frac{1}{\sqrt{1 - \frac{v_t^2}{c^2}}} - 1\right) \\
\frac{c^2}{\sqrt{1 - \frac{v_t^2}{c^2}}} &= \frac{v_f^2 + 2c^2}{2} \\
\sqrt{1 - \frac{v_t^2}{c^2}} &= \frac{2c^2}{v_f^2 + 2c^2} \\
1 - \frac{v_t^2}{c^2} &= \left(\frac{2c^2}{v_f^2 + 2c^2}\right)^2 \\
\frac{v_t^2}{c^2} &= 1 - \left(\frac{2c^2}{v_f^2 + 2c^2}\right)^2 \\
&\boxed{v_t = c\sqrt{1 - \left(\frac{2c^2}{v_f^2 + 2c^2}\right)^2}}
\end{aligned}$$

$$\left|\frac{v_f - v_t}{v_t}\right| \times 100 = E$$

$$v_f > v_t$$

$$\frac{v_f - v_t}{v_t} \times 100 = E$$

$$\frac{v_f}{v_t} - 1 = \frac{E}{100}$$

$$\frac{v_f}{v_t} = \frac{E}{100} + 1$$

$$\frac{v_f}{c\sqrt{1 - \left(\frac{2c^2}{v_f^2 + 2c^2}\right)^2}} = \frac{E}{100} + 1$$

$$\frac{v_f^2}{c^2\left(\frac{v_f^4 + 4v_f^2c^2 + 4c^4}{v_f^4 + 4v_f^2c^2 + 4c^4} - \frac{4c^4}{v_f^4 + 4v_f^2c^2 + 4c^4}\right)} = \left(\frac{E}{100} + 1\right)^2$$

$$\frac{v_f^6 + 4v_f^4c^2 + 4v_f^2c^4}{v_f^4 + 4v_f^2c^2} = c^2\left(\frac{E}{100} + 1\right)^2$$

$$\frac{v_f^4 + 4v_f^2c^2 + 4c^4}{v_f^2 + 4c^2} = c^2\left(\frac{E}{100} + 1\right)^2$$

$$\text{Define: } n = \left(\frac{E}{100} + 1\right)^2$$

$$\frac{v_f^4 + 4v_f^2c^2 + 4c^4}{v_f^2 + 4c^2} = c^2 n$$

$$v_f^4 + 4v_f^2c^2 + 4c^4 = v_f^2c^2n + 4c^4n$$

$$v_f^4 + v_f^2c^2(4 - n) = 4c^4(n - 1)$$

$$v_f^4 + v_f^2c^2(4 - n) + \frac{1}{4}c^4(4 - n)^2 = 4c^4(n - 1) + \frac{1}{4}c^4(4 - n)^2$$

$$(v_f^2 + \frac{1}{2}c^2[4 - n])^2 = c^4(4n - 4 + (4 - 2n + \frac{n^2}{4}))$$

$$(v_f^2 + \frac{1}{2}c^2[4 - n])^2 = c^4\left(\frac{n^2}{4} + 2n\right)$$

$$v_f^2 + \frac{1}{2}c^2(4 - n) = c^2\left(\frac{n^2}{4} + 2n\right)^{1/2}$$

$$v_f^2 = c^2\left(\frac{n^2}{4} + 2n\right)^{1/2} - \frac{1}{2}c^2(4 - n)$$

$$v_f = c\left[\left(\frac{n^2}{4} + 2n\right)^{1/2} + \frac{n}{2} - 2\right]^{1/2}$$

$$v_f = \frac{c}{\sqrt{2}}\left[(n^2 + 8n)^{1/2} + n - 4\right]^{1/2}$$