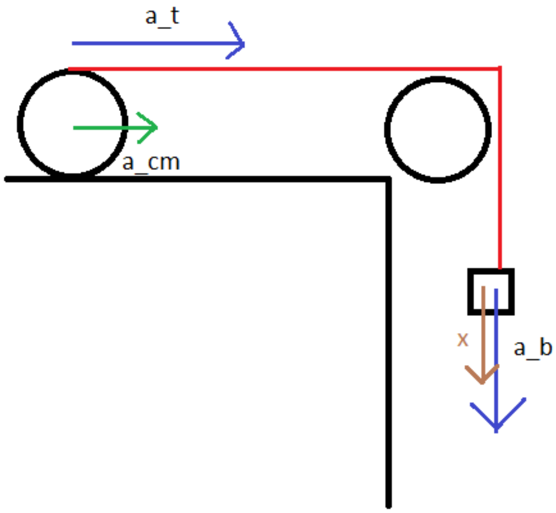
# Falling Box with Cylinder Proofs

**Ryan Ellin**

Method 1:




$$a_t = a_b$$

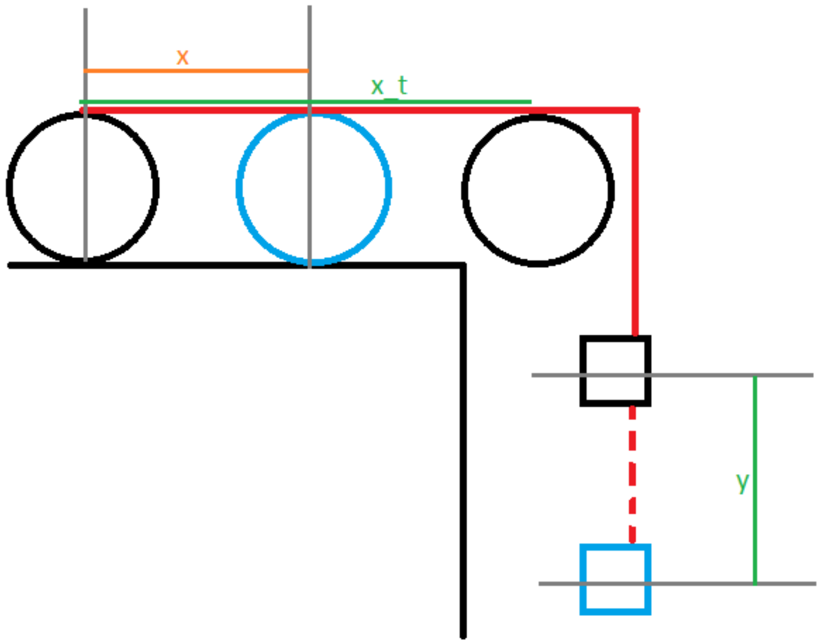
Since it is smooth rolling, and both objects begin from rest:

$$\begin{aligned}
a_t &= 2a_{cm} \\
a_b &= 2a_{cm} \\
a_{cm} &= R\alpha \\
a_b &= 2R\alpha \\
\alpha &= \frac{a_b}{2R} \\
\int \alpha dt &= \int \frac{a_b}{2R} dt \\
\omega &= \frac{v}{2R} + c \\
\omega(0) &= 0 \\
&= \frac{v(0)}{2R} + c \\
&= c
\end{aligned}$$

$$
\begin{aligned}
\therefore c &= 0 \\
\omega &= \frac{v}{2R} \\
\int \omega dt &= \int \frac{v}{2R} dt \\
\theta &= \frac{x}{2R} + c_2 \\
\theta(0) &= \theta_i \\
&= \frac{x(0)}{2R} + c_2 \\
&= c_2 \\
\therefore c_2 &= \theta_i \\
\theta &= \frac{x}{2R} + \theta_i \\
\theta - \theta_i &= \frac{x}{2R} \\
&\boxed{\Delta\theta = \frac{x}{2R}}
\end{aligned}
$$

Second method on next page.

Method 2:




First, we define the difference in the distance travelled by the box center of mass and the cylinder:

$$d = y - x$$

Now, since the rope is unable to slip on the cylinder, this difference in distance travelled must be accounted for by rotation of the cylinder. Thus, we can find the number of revolutions which the cylinder travels through::

$$\text{Revs} = \frac{d}{2\pi R}$$
$$\text{Revs} = \frac{y - x}{2\pi R}$$

Which can then be converted into theta:

$$\text{Revs} = \frac{\Delta\theta}{2\pi}$$
$$\therefore \frac{\Delta\theta}{2\pi} = \frac{y - x}{2\pi R}$$
$$\Delta\theta = \frac{y - x}{R}$$

Next, since the cylinder is smooth rolling, we'll have the following relationship relating the distance that the cm of the cylinder travels, and the distance the top travels:

$$x_t = 2x$$

And the following relationship between the distance travelled by a point on the top of the cylinder, and the distance travelled by the falling box:

$$x_t = y$$

Thus:

$$\begin{aligned}
2x &= y \\
x &= \frac{y}{2} \\
\Delta\theta &= \frac{y-x}{R} \\
&= \frac{y-\frac{y}{2}}{R} \\
&\boxed{\Delta\theta = \frac{y}{2R}}
\end{aligned}$$

Thus it has been proven by two methods (with slightly differently named variables).

Note: if the cylinder is held fixed and only allowed to rotate in place, $x = 0$, and we'll find the more familiar:

$$\begin{aligned}
\Delta\theta &= \frac{y-x}{R} \\
\Delta\theta &= \frac{y-0}{R} \\
&\boxed{\Delta\theta = \frac{y}{R}}
\end{aligned}$$