Homework 10: More integer programs

Due date: **Wednesday** Mai 4, 2022
See the course website for instructions and submission details.

A Note on Course Evaluation: The course evaluations are ongoing. Everyone in the class will receive an additional bonus homework point (not belonging to any specific homework) if more than 70% of the students in the class fill in the online evaluation.
I would like to take a moment to explain why your feedback is so important to me. I am continuously working to make the class better and my teaching more engaging, and your feedback is an essential part of this work. In this spirit, I promise that your feedback will be read carefully and taken into consideration in future iterations of this class. Thank you for helping me improving the class!

1. **Laurent goes to the gym.** Laurent has a regular weight lifiting program. Today, he will be doing deadlifts and front squats. For each exercise he does 8 sets, with several repetitions per set (i.e., x3 means 3 repetitions). In the table below, you can see the weight he lifts in each set (given in lbs) and the number of repetitions for both deadlifts and front squats.

Set	**1**		**2**		**3**		**4**		**5**		**6**		**7**		**8**	
Deadlift	310	x5	355	x3	395	x1+	375	x3	355	x3	330	x3	310	x3	290	x3
Front Squat	100	x5	130	x5	160	x3	160	x5	160	x7	160	x4	160	x6	160	x8

   For both the deadlifts and the front squats, Laurent uses a steel bar which weighs 45 lbs. There are many available bumper plates (weights which are put on each side of the bar) with a weight of 2.5 lbs, 5lbs, 10 lbs, 25 lbs, and 45 lbs. Remember that there has to be the same amount of weight and the same type of plates on each side!

   (a) Laurent would prefer to not think to much while he is at the gym (lifting takes a lot of concentration). Can you help him solve an optimization problem which takes the current weight (considering one set and one exercise at the time) and minimizes the number of plates he has to put on the barbell? Make a schedule that shows which bumper plates Laurent should use for each weight.

   (b) Because of COVID-19, Laurent can no longer go to the gym and has decided to invest in lifting equipment which he will install in his basement. He would like to continue doing the same exercises as in the table above, but now wants to minimize the total number of bumper plates that he has to order (while being able to combine them to give all of the above weights). Solve an optimization problem to determine which bumper plates Laurent should invest in.

   (c) Resolve the problem from (a), but consider that Laurent only has access to the bumper plates that he decided to buy. Print the new schedule that shows which bumper plates to put on the barbell for each exercise.

2. **Voting.** Governor Blue of the state of Berry is attempting to get the state legislator to gerrymander Berry's congressional districts. The state consists of ten cities, and the numbers of registered Republicans and Democrats (in thousands) in each city are shown below

   Berry has five congressional representatives. To form the five congressional districts, cities must be grouped together according to the following restrictions:

   - Districts cannot subdivide cities; all voters in a city must be in the same district.
   - Each district must contain between 150,000 and 250,000 voters (there are no independent voters).

   Governor Blue is a Democrat. Assume 100% voter turnout and that each voter always votes according to their registered party. Formulate and solve an optimization problem to help Governor Blue maximize the number of congressional districts that have a Democratic majority.

City	Republicans	Democrats
1	80	34
2	60	44
3	40	44
4	20	24
5	40	114
6	40	64
7	70	14
8	50	44
9	70	54
10	70	64

3. The Queens problem. You are given a standard 8×8 chess board. The following problems involve placing queens on the board such that certain constraints are satisfied. For each of the following problems, model the optimization task as an integer program, solve it, and show what an optimal placement of queens on the board looks like.

a) Find a way to place 8 queens on the board so that no two queens threaten each other. We say that two queens *threaten* each other if they occupy the same row, column, or diagonal. Show what this placement looks like.

b) Repeat part (a) but this time find a placement of the 8 queens that has point symmetry. In other words, find a placement that looks the same if you rotate the board 180°.

c) What is the smallest number of queens that we can place on the board so that each empty cell is threatened by at least one queen? Show a possible optimal placement.

d) Repeat part (c) but this time find a placement of the queens that also has point symmetry. Does the minimum number of queens required change? Show a possible optimal placement.