CMSC 5718 Introduction to Computational Finance

Assignment 2: Monte Carlo method in derivative pricing (20% of total grade)

Instructions

1) Submit a copy of your report together with supporting programs and/or data files (as a zipped file) by **uploading to Blackboard on or before March 27, 2023, 11:59pm.** The **file name** of the zipped file or your report should include your surname and have the following format, **e.g. ChauKL_Assign2**. [If uploading to Blackboard is not successful, you may consider sending a email to kalokchau@cuhk.edu.hk, but submission through Blackboard is preferred.]
2) Late submission of one week or less will attract a penalty of 20% of the assignment mark. ***Submission will not be accepted after April 3, 2023, 11:59pm.***
3) You can either submit your work **individually** or work together in a **group of two or three students**. Students in the same group will get the same assignment grade. Please state the name(s) and student number(s) clearly in the report. However, if you are submitting as a group, you only need to submit one copy of the report with one of the names in the file name (described in (1) above).
4) ***Please observe the university's plagiarism guidelines.***

Introduction

In this assignment, we test the implementation of an option pricing model using the Monte Carlo method. It will be applied to the pricing of an exotic option and a popular equity structured product.

Choose the stock that you have to work on

Use your student number to decide which stocks you have to use to perform the analysis. Take the last digit of your student number to obtain the order number, and look up the stock codes from the given data sheet. For example, if your student number ends with 7, the order number is 7, and the stocks are Xiaomi Corporation (stock code 1810) and HSBC (stock code 5). If you work in a group, select the order number based on one of your student numbers. In Part 1, use the two stocks S_1 and S_2. In Part 2, the calculation should be based on S_1 only.

Part I: Exotic Option Pricing (40%)

1. Volatility and correlation calculation from historical data (9%)

i) Calculate the realized volatilities of stock S_1 and stock S_2 with the following formula, using the daily data of June 30, 2022 to December 30, 2022.

$$\sigma_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(u_{i,j} - \overline{u_i})^2} \times \sqrt{252}, \qquad i = 1,2$$

$u_{i,j} = ln\left(\frac{S_{i,j}}{S_{i,j-1}}\right)$, $S_{i,j}$ is the stock price at time j, $\overline{u_i}$ is the mean of the $u_{i,j}$'s; $S_{i,0}$ is the price of the stock on June 30, 2022; 252 represents the number of trading days per year

ii) Calculate the correlation coefficient ρ_{12} of stock S_1 and stock S_2 with the following formula, using the daily data of June 30, 2022 to December 30, 2022.

$$\sigma_{12} = \sum_{j=1}^{n}\left[(u_{1,j} - \overline{u_1})(u_{2,j} - \overline{u_2})\right] \times \frac{252}{n-1}$$

$$\rho_{12} = {}^{\sigma_{12}}/_{\sigma_1\sigma_2}$$

[c.f. The formulas are slightly different from the ones given in Topic 1-1, slide 97; ln(return) is used here]

2. Option pricing with Monte Carlo simulation (31%)

The payoff of an *Average Best-of Option* with two stocks S_1 and S_2 is based on the following formula: max(A – 100%, 0) payable at maturity (T=1 year from start date).

where
$S_{1,0}$, $S_{2,0}$ = stock prices at time t=0 (December 30, 2022);
$S_{1,1}$, $S_{2,1}$ = stock prices at time t=0.5 year;
$S_{1,2}$, $S_{2,2}$ = stock prices at time t=1 year;
$A = (B_1 + B_2)/2$;
$B_1 = \max(S_{1,1} / S_{1,0},\ S_{2,1} / S_{2,0})$;
$B_2 = \max(S_{1,2} / S_{1,0},\ S_{2,2} / S_{2,0})$;

Calculate the option price with the realized volatilities and correlation coefficient from your answers in question 1, interest rate *r* = 3.75% p.a..

i) Use a Monte Carlo scheme with time steps N = 180, i.e. $\Delta t = T/N = 1/180$ (refer to the discretization scheme in Topic 2-2, slides 38 and 42). Give the answers with: (a) 1000 paths; (b) 10000 paths; (c) 100000 paths; (d) 500000 paths.

ii) Use a Monte Carlo scheme with two time steps N = 2, $\Delta t_1 = \Delta t_2 = ½$ (refer to the discretization scheme in Topic 2-2, slides 40, 42 and 43). Give the answers with: (a) 1000 paths; (b) 10000 paths; (c) 100000 paths; (d) 500000 paths.

Part II: Pricing of a structured product (60%)

Product description

Nominal amount (NOM):	$100,000
Price paid by investor (PP):	NOM * P%
Start date (D_s):	t
Initial stock price:	S_0
Expiry date (T):	t+ 7/12 year
Expiry date stock closing price:	S_M
Strike (K):	S_0 * K_0%
Auto-call price (P_c):	S_0 * AC%
First auto-call date (D_c):	t+ 1/12 year
Knock-in price (P_k):	S_0 * KI%
First knock-in date (D_k):	t
Coupon per month:	CP%
Coupon dates:	t+ 1/12 year, t+ 1/6 year, t+ 1/4 year, t+ 1/3 year, t+ 5/12 year, t+ 1/2 year, t+ 7/12 year

Coupons to be paid to investor:

i. NOM*CP% at t+ 1/12 year; plus
ii. NOM*CP% at t+ 1/6 year if product has not terminated already; plus
iii. NOM*CP% at t+ 1/4 year if product has not terminated already; plus
iv. NOM*CP% at t+ 1/3 year if product has not terminated already; plus
v. NOM*CP% at t+ 5/12 year if product has not terminated already; plus
vi. NOM*CP% at t+ 1/2 year if product has not terminated already; plus
vii. NOM*CP% at t+ 7/12 year if product has not terminated already.

Early termination condition:

i. If the stock price is greater than or equal to P_c at any date on or after D_c, then investor receives NOM + accrued interest, and the product is terminated early. Accrued interest = NOM*CP% * num_days / total days, where num_days is the number of days between the call date and the coupon date immediately preceding the call date, and total_days is the number of days between the coupon dates immediately preceding and following the call date.

Payoff at expiry if the product has not terminated early:

i. If a *knock-in event* has not occurred, then investor receives NOM; or
ii. If the expiry date stock closing price S_M is greater than or equal to the strike price K, then the investor receives NOM; or
iii. The investor receives NOM * S_M / K.

Knock-in event

i. If the stock price at any date after the start date D_s is below the knock-in price P_k, then a knock-in event has occurred.
ii. If the stock price from the start date D_s to the expiry date T has never dropped below the knock-in price P_k, then a knock-in event has not occurred.

[Note that this structure is slightly different from the one described in the lecture powerpoint and the first given example. (i) Knock-in condition can be triggered from the start date up to the expiry date, and not just observed on the expiry date; (ii) the calculation of the number of days and total days is simplified and make use of calendar days instead of trading days.]

[*A recommended pricing procedure is given in the course notes, Appendix of Chapter 10. Use a minimum of 210 time steps.*]

Questions

1. [30 marks] For stock S_1, look up the values of K_0, KI, AC, and the coupon per month CP **according to the given group**. Based on these parameters, coupon per month CP set 1, and the given ***implied volatility*** of stock S_1, calculate the fair price of the product F%. Interest rate r = 3.75% p.a. Give the answers based on (a) 1000; (b) 10000; (c) 100000; and (d) 500000 Monte Carlo paths. In each case, record the number of times the auto-call condition is triggered. If the product is sold at an initial price of P=100%, what is the profit of the investment bank in each case?
2. [16 marks] Repeat the above calculations with coupon per month CP set 2. What is the additional profit of the investment bank in each case?
3. [14 marks] Instead of using the given CP, find the level of CP such that the initial profit margin of the investment bank is 2.50%. You only need to generate one result based on 500000 Monte Carlo paths. [Hint: obtain the answer by trial and error]
4. [Bonus 2 marks] The above pricing procedure is an approximation (refer to the comments in the course notes). In real life, weekends and holidays need to be considered and these are non-business days. If the above procedure is modified by taking these into account, do you think that the coupon calculated to be higher or lower than CP calculated in question 3 above? Give a brief reason to explain your answer.