Relationship of Model Parameters and Solution Stability in the Power Walk Page Rank Method.

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Contents

Ok. The discovery project is meant to be more of a data analysis project, so say that you will be investigating the relationship between the second eigenvalue and the model and graph parameters.

1 TODO Introduction

> Your title should give a clear indication of your proposed research approach or key question

Much information is interconnected, [2]
another test reference [1]

2 TODO Background and Rationale

2.1 Background of issue and proposed research

2.2 Indentify discipline

2.3 Summary of Developments in the field

3 TODO Literature Review

3.1 Introduction

3.2 Body

Structure the literature in a logical way

3.2.1 Different Sources

4 TODO Research Question

You should formulate these clearly, giving an explanation as to what problems and issues are to be explored and why they are worth exploring* TODO Research Methodology

5 Research Methodology

5.1 Theoretical Resources to be drawn on

5.2 Research Approach

5.3 Advantages and disadvantages

6 Plan of work

- Regular consultations

7 Notes

7.1 Question

Can we determine the second eigenvalue from the method parameters? For PageRank, the second eigenvalue is equal to the smoothing parameter α

> Yes. An open question for the Power Walk method is, can we determine the second eigenvalue from the method parameters? For PageRank, the second eigenvalue is equal to the smoothing parameter α. The second eigenvalue determines how long the algorithm takes to converge and how stable the solution is. To begin, implement the method for computing PageRank and then the Power Walk. It can all be done using sparse matrices, so it only requires a fraction of the memory and is each iteration is quick.

7.2 Working

Take the exemplar Graph from Figure 1:

Listing 1: Code to Generate DOT Graph

$$\Gamma = I - nD_B^{-1}$$

Where we have the following:

$$\beta \;\&{=}\; 10$$

$$B\& = \beta^{\mathrm{A}}$$

$$A\& = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\implies B = \begin{bmatrix} 10 & 1 & 1 & 1 \\ 1 & 10 & 1 & 1 \\ 1 & 1 & 10 & 1 \\ 1 & 1 & 1 & 10 \end{bmatrix}$$

D_B is a diagonal matrix of the column sums:

$$D = \begin{bmatrix} 13 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 \\ 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 13 \end{bmatrix}$$

\$\$ Hence the Inverse is: \$\$

$$D_B^{-1} = \frac{I}{13}$$

Putting it all together:

$$\begin{aligned} \Gamma &= I - nD_B^{-1} \\ &= I - \frac{4 \cdot I}{13} \\ &= \frac{9}{13} \cdot I \\ &= \begin{bmatrix} \frac{9}{13} & 0 & 0 & 0 \\ 0 & \frac{9}{13} & 0 & 0 \\ 0 & 0 & \frac{9}{13} & 0 \\ 0 & 0 & 0 & \frac{9}{13} \end{bmatrix} \\ &\approx \begin{bmatrix} 0.6923 & 0 & 0 & 0 \\ 0 & 0.6923 & 0 & 0 \\ 0 & 0 & 0.6923 & 0 \\ 0 & 0 & 0 & 0.6923 \end{bmatrix} \end{aligned}$$

References

[1] Sepandar Kamvar, Taher Haveliwala, and Gene Golub. "Adaptive Methods for the Computation of PageRank". In: *Linear Algebra and its*

Applications. Special Issue on the Conference on the Numerical Solution of Markov Chains 2003 386 (July 15, 2004), pp. 51–65. ISSN: 0024-3795. DOI: 10.1016/j.laa.2003.12.008. URL: http://www.sciencedirect.com/science/article/pii/S0024379504000023 (visited on 07/31/2020) (cit. on p. 2).

[2] Stan Development Team. *6 Sparse Matrix Operations | Stan Functions Reference*. URL: https://mc-stan.org/docs/2_22/functions-reference/sparse-matrices.html (visited on 08/07/2020) (cit. on p. 2).