

$$\frac{\mathrm{d}}{\mathrm{d} x} (u \cdot v) = \frac{\mathrm{d} u}{\mathrm{d} v} \cdot v + u \cdot \frac{\mathrm{d} v}{\mathrm{d} x} \quad (1.1)$$

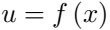
$$\frac{\mathrm{d}}{\mathrm{d} x} (f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad (1.2)$$

$$\frac{\mathrm{d} y}{\mathrm{d} x} = \frac{\mathrm{d} y}{\mathrm{d} u} \cdot \frac{\mathrm{d} u}{\mathrm{d} x} \quad (1.3)$$

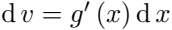
$$\frac{\mathrm{d}}{\mathrm{d} x} [f(g(x))] = f'(g(x)) \cdot g'(x) \quad (1.4)$$

$$\int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du \quad (1.5)$$

$$\int f(u) \cdot \frac{du}{dx} \, dx = \int f(u) \, du \quad (1.6)$$



dx/dx = dx/dx





$$\int u \, dv = u \cdot v - \int v \, du \qquad (1.7)$$

$$\begin{aligned} u &= g(x) \\ \frac{du}{dx} &= g'(x) \end{aligned} \quad F(x) : F'(x) = f(x) = y \quad (1.8)$$

$$\begin{aligned}
& \frac{\mathrm{d}}{\mathrm{d} x} [F' (u)] = F' (g (x)) \cdot g' (x) \\
& \qquad \qquad \qquad = f (g (x)) \cdot g' (x) \\
\Rightarrow & \quad f (g (x)) \cdot g' (x) = \frac{\mathrm{d}}{\mathrm{d} x} [F (u)] \\
& \quad f (g (x)) \cdot g' (x) = \frac{\mathrm{d}}{\mathrm{d} x} [F (u) + C]
\end{aligned}
\tag{1.9}$$

$$\begin{aligned}
 \int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d} x &= \int \frac{\mathrm{d}}{\mathrm{d} x}\left[F\left(u\right)+C\right] \mathrm{d} x \\
 &= F\left(u\right)+C \\
 &= \int f\left(u\right) \mathrm{d} u
 \end{aligned}$$

$$\int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du$$

$$\int f(u) \cdot \frac{du}{dx} \, dx = \int f(u) \, du$$

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$$\frac{\mathrm{d}}{\mathrm{d} x} [f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x) \tag{1.10}$$

$$\begin{aligned} u &= f(x) & v &= g(x) \\ \frac{du}{dx} &= f'(x) & \frac{dv}{dx} &= g'(x) \end{aligned} \tag{1.11}$$

$$\int \left(\frac{\mathrm{d} u}{\mathrm{d} x} \cdot v + u \cdot \frac{\mathrm{d} v}{\mathrm{d} x} \right) \mathrm{d} x = u \cdot v$$

$$\int \left(v \cdot \frac{\mathrm{d} u}{\mathrm{d} x} \right) \mathrm{d} x + \int \left(u \cdot \frac{\mathrm{d} v}{\mathrm{d} x} \right) \mathrm{d} x = u \cdot v$$

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$$\int v \, \mathrm{d} u + \int u \, \mathrm{d} v = u \cdot v$$

$$\int u \, \mathrm{d} v = u \cdot v - \int v \, \mathrm{d} u$$

l i d x

$$\left[f(x) \cdot \frac{dx}{dx} \right] = \left[f(x) \cdot 1 \right]$$

$$\left[f(x) \cdot \frac{d}{dx} \right]$$

$$= \left[f(x) \cdot \right]$$





dy

dy

aw

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$$\sum_0^n \left[a_0(x) \cdot \left(\frac{d^n y}{d x^n} \right) \right] \tag{1.12}$$





A differential equation of the form:

$$g(y) \cdot \frac{dy}{dx} = f(x) \quad (1.13)$$

Is a seperable Ordinary Differential Equation and has a solution:

$$\int g(y) dy = \int f(x) dx \quad (1.14)$$

$$\begin{aligned}
 & g(y) \cdot \frac{\mathrm{d} y}{\mathrm{d} x} = f(x) \\
 \Rightarrow & \int g(y) \frac{\mathrm{d} y}{\mathrm{d} x} \mathrm{d} x = \int f(x) \mathrm{d} x
 \end{aligned}
 \tag{1.15}$$



$$\int g(y) \, \mathrm{d}y = \int f(x) \, \mathrm{d}x \qquad (1.16)$$



d

v



d

x

=

f

(

x

v

)

$$u = \frac{y}{x}$$

$$\implies y = u \cdot x$$

$$\implies \frac{\mathrm{d} y}{\mathrm{d} x} = \frac{\mathrm{d} u}{\mathrm{d} x} \cdot x + (1) \cdot u$$

$$\frac{\mathrm{d} y}{\mathrm{d} x}=f\left(\frac{y}{x}\right)$$

$$\frac{\mathrm{d} u}{\mathrm{d} x} \cdot x+u=f(u)$$

$$\frac{\mathrm{d} u}{\mathrm{d} x} \cdot x=f(u)-u$$

$$\frac{1}{f(u)-u} \cdot \frac{\mathrm{d} u}{\mathrm{d} x} \cdot x=1$$

$$\frac{1}{f(u)-u} \cdot \frac{\mathrm{d} u}{\mathrm{d} x}=\int \frac{1}{x} \mathrm{d} x$$

$$\int \frac{1}{f(u)-u} \cdot \frac{\mathrm{d} u}{\mathrm{d} x} \mathrm{d} x=\int \frac{1}{x} \mathrm{d} x$$

$$\int \frac{1}{f(u)-u} \mathrm{d} u=\ln |x|+c \quad (1.17)$$

$$E_G(v) : G(v) = \sqrt{\frac{1}{f(v) - v}}$$

$$G(u) = \ln |x| + c$$

$$G\left(\frac{y}{x}\right) = \ln |x| + c$$

$$G\left(\frac{y}{x}\right) + \ln |x| + c = 0 \tag{1.18}$$

$$\sum_0^n \left[a_n (x) \cdot f^{(n)} (x) \right] = g (x)$$

If $g (x) = 0$ it is said to be homogenous

$$a_1(x) \cdot \frac{dy}{dx} + a_0(x) \cdot y = g(x)$$

Where $a(x)$ is a function

(1.19)

Linear First Order ODE:

$$\frac{dy}{dx} + p(x) \cdot y = f(x) \quad (1.20)$$

if $f(x) = 0$ the equation is said to be homogenous





$$v_n : \frac{dv_n}{dx} + p(x) \cdot v_n = 0$$

$$v_p = \frac{dv_p}{dx} + p(x) \cdot v_p = f(x)$$

- Rewrite the Equation in the standard form:

$$\frac{dy}{dx} + p(x) \cdot y = f(x)$$

- Identify $p(x)$ and find the integrating factor:

$$e^{\int p(x) dx}$$

- Multiply through by the integrating factor:

$$e^{\int p(x) dx} \left(\frac{dy}{dx} + p(x) \cdot y \right) = e^{\int p(x) dx} f(x)$$

It may be concluded:

$$\frac{d}{dx} \left[e^{\int p(x) dx} \cdot y \right] = e^{\int p(x) dx} \cdot f(x)$$

- Integrate both sides in order to solve:

$$\frac{dy}{dx} + p(x) \cdot y = f(x) \qquad (1.21)$$

$$\frac{\mathrm{d} y}{\mathrm{d} x} + p(x) \cdot y = 0 \quad \Rightarrow \quad y = y_h \quad (1.22)$$

$$\frac{\mathrm{d} y}{\mathrm{d} x} + p(x) \cdot y = f(x) \Rightarrow y = y_p \quad (1.23)$$

$$\begin{aligned}
& \frac{d}{dx} (y_h + y_p) + p(x) \cdot (y_h + y_p) = f(x) \\
& \frac{dy_h}{dx} + \frac{dy_p}{dx} + p(x) \cdot y_h + p(x) \cdot y_p = f(x) \\
& \frac{dy_h}{dx} + p(x) \cdot y_h + \frac{dy_p}{dx} + p(x) \cdot y_p = f(x) \\
& 0 + f(x) = f(x)
\end{aligned} \tag{1.24}$$

$$\frac{\mathrm{d} y}{\mathrm{d} x} + p(x) \cdot y = 0$$

$$\frac{1}{y} \cdot \frac{\mathrm{d} y}{\mathrm{d} x} = -p(x)$$

$$\ln |y| = \int -p(x) \mathrm{d} x + c$$

$$|y| = e^{\int -p(x) \mathrm{d} x} \cdot e^c \tag{1.25}$$



$$\Rightarrow y_n = e^{-\int p(x) dx} \cdot c$$

$$v_1 = e^{-\int p(x) dx}$$

$$y_n = y_1(x) \cdot c \quad (1.26)$$











$$\begin{aligned}
 y_p &= u(x) \times y_h(x) \\
 &= e^{-\int p(x) \, dx} \cdot u(x)
 \end{aligned}
 \tag{1.27}$$

$$y_p = e^{-\int p(x) \, dx} \cdot u(x)$$

$$\frac{dy_p}{dx} + p(x) \cdot y_p = f(x)$$

$$\frac{d}{dx} (u(x) \cdot y_1(x)) + p(x) u(x) y_1(x) = f(x)$$

$$\frac{du}{dx} \cdot y_1(x) + \frac{dy_1}{dx} \cdot u(x) + p(x) \cdot u(x) \cdot y_1(x) = f(x)$$

$$u(x) \left(\frac{dy_1}{dx} + p(x) y_1 \right) + \frac{dy}{dx} \cdot y_1(x) = f(x)$$

$$0 + \frac{dy}{dx} \cdot y_1(x) = f(x)$$

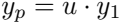
$$\frac{dy}{dx} = f(x) / y_1(x)$$

$$\int \frac{du}{dx} \, dx = \int f(x) / y_1(x) \, dx$$

$$\int du = \int f(x) / y_1(x) \, dx$$

$$u = \int f(x) / y_1(x) \, dx \quad (1.28)$$

$$u = \int f(x) \cdot e^{\int p(x) dx} dx \quad (1.29)$$



$$y_p = \frac{1}{y_1} \cdot \int f(x) \cdot e^{\int p(x) dx}$$

$$y_p = e^{-\int p(x) dx} \int f(x) \cdot e^{\int p(x) dx} \quad (1.30)$$

2025

Project 1

$$e^{\int p(x) \, dx} \cdot y_p = e^{\int p(x) \, dx} \cdot e^{-\int p(x) \, dx} \int f(x) \cdot e^{\int p(x) \, dx}$$

$$e^{\int p(x) \, dx} \cdot y_p = \int f(x) \cdot e^{\int p(x) \, dx}$$

$$\frac{d}{dx} \left(e^{\int p(x) \, dx} \cdot y_p \right) = \frac{d}{dx} \left(\int f(x) \cdot e^{\int p(x) \, dx} \right)$$

$$= f(x) \cdot e^{\int p(x) \, dx}$$

$$e^{\int p(x) \, dx} \frac{dy}{dx} + p(x) \cdot e^{\int p(x) \, dx} \cdot y = e^{\int p(x) \, dx} \cdot f(x)$$

$$\implies \frac{dy}{dx} + p(x) \cdot y = f(x)$$

$$(x+1) \cdot \frac{dy}{dx} + y = \ln(x) \ ; \qquad y(1) = 10 \qquad (1.31)$$

$$\frac{dy}{dx} + \frac{y}{x+1} = \frac{\ln(x)}{x+1} \quad : \quad (x \in \mathbb{R} \setminus \{-1, 0\}) \quad (1.32)$$

$$\begin{aligned}
 u &= e^{\int \frac{1}{x+1} \, dx} \\
 &= e^{\int \ln |x+1| \, dx} \\
 &= |x+1|
 \end{aligned}
 \tag{1.33}$$



$$\begin{aligned}
 & (x+1) \cdot \frac{dy}{dx} + y = \ln(x) \\
 \Rightarrow & \frac{d}{dx} ((x+1) \cdot y) = \ln(x)
 \end{aligned}
 \tag{1.34}$$

$$\int \frac{d}{dx} [(x+1) \cdot y] dx = \int \ln(x) dx$$

$$(x+1) \cdot y = \int \ln(x) \, dx$$

(1.35)

$$u = \ln(x) \qquad \mathrm{d}v = \mathrm{d}x$$

$$\mathrm{d}u = \frac{1}{x} \mathrm{d}x \qquad v = x$$

$$\Rightarrow \int u \mathrm{d}v = u \cdot v + \int v \mathrm{d}u$$

$$\begin{aligned}
 (x+1) \cdot y &= \ln(x) \cdot x - \int \mathrm{d}x \\
 &= x \cdot (\ln(x) - 1) + c \\
 \Rightarrow y &= \frac{x \cdot (\ln(x) - 1 + c)}{x+1}
 \end{aligned}$$

101 = 10

$$10 = \frac{1 (\ln (1) - 1 + c)}{2}$$

$$20 = 1 (0 - 1) + c$$

$$c = 19$$

(1.36)

$$v = x(10^x - 1 + 10) \frac{1}{x+1}, \quad v \in (-1, 0]$$