$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(u\cdot v\right)=\frac{\mathrm{d}\,u}{\mathrm{d}\,v}\cdot v+u\cdot\frac{\mathrm{d}\,v}{\mathrm{d}\,x} \tag{1.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(f\left(x\right)\cdot g\left(x\right)\right)=f'\left(x\right)\cdot g\left(x\right)+f\left(x\right)\cdot g'\left(x\right) \tag{1.2}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,y}{\mathrm{d}\,u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \tag{1.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(g\left(x\right)\right)\right] = f'\left(g\left(x\right)\right) \cdot g\left(x\right) \tag{1.4}$$

$$
\begin{aligned}
u &= g(x) & F(x): \quad F'(x) = f(x) = y \\
\frac{du}{dx} &= g'(x)
\end{aligned}
\tag{1.5}
$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[F'\left(u\right)\right] = F'\left(g\left(x\right)\right) \cdot g'\left(x\right)$$

$$= f\left(g\left(x\right)\right) \cdot g'\left(x\right)$$

$$\implies f\left(g\left(x\right)\right) \cdot g'\left(x\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)\right]$$

$$f\left(g\left(x\right)\right) \cdot g'\left(x\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right) + C\right]$$

$$(1.6)$$

$$\int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d}\, x = \int \frac{\mathrm{d}}{\mathrm{d}\, x}\left[F\left(u\right) + C\right] \mathrm{d}\, x$$
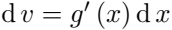$$= F\left(u\right) + C$$
$$= \int f\left(u\right) \mathrm{d}\, u$$

$$\int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u \tag{1.7}$$

$$\int f\left(u\right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u \tag{1.8}$$

$$u = f(x)$$

$$\frac{d}{dx}\left[\sin\left(x\right)\right]=\cos\left(x\right)$$

$$\mathrm{d}\,v = g'\,(x)\,\mathrm{d}\,x$$

(1.2)
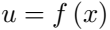
$$\frac{\mathrm{d}}{\mathrm{d}\,x} \left[f\left(x\right) \cdot g\left(x\right) \right] = f'\left(x\right) \cdot g\left(x\right) + f\left(x\right) \cdot g'\left(x\right)$$

(1.9)

$$u = f(x) \qquad v = g(x)$$
$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} = f'(x) \qquad \frac{\mathrm{d}\,v}{\mathrm{d}\,x} = g'(x) \tag{1.10}$$

$$\int \left(\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot v + u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

$$\int \left(v \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right) \mathrm{d}\,x + \int \left(u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

$$\int v \, \mathrm{d}\, u + \int u \, \mathrm{d}\, v = u \cdot v$$
$$\int u \, \mathrm{d}\, v = u \cdot v - \int v \, \mathrm{d}\, u \tag{1.11}$$

$$\int [\quad] \, dx$$

$$\left[f\left(u \right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right] = \left[f\left(g\left(x \right) \right) \cdot g'\left(x \right) \right]$$

$$\left[f\left(x \right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right] = \left[f\left(x \right) \cdot g'\left(x \right) \right]$$

$$\mathrm{d}\,y$$

$$\mathrm{d}\,x$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x}$$

$$\frac{\partial u}{\partial x}$$

$$\sum_0^n \left[a_0\left(x\right) \cdot \left(\frac{\mathrm{d}^n y}{\mathrm{d}\,x^n}\right) \right] \tag{1.12}$$

y

x

A differential equation of the form:

$$g\left(y\right) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(x\right) \tag{1.13}$$

Is a seperable Ordinary Differential Equation and has a solution:

$$\int g\left(y\right) \mathrm{d}\,y = \int f\left(x\right) \mathrm{d}\,x \tag{1.14}$$

$$g\left(y\right) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(x\right)$$

$$\implies \int g\left(y\right) \frac{\mathrm{d}\,y}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int f\left(x\right)\mathrm{d}\,x$$

<div align="right">(1.15)</div>

1.8

$$\int g\left(y\right) \mathrm{d}\, y = \int f\left(x\right) \mathrm{d}\, x \tag{1.16}$$

$$\mathrm{d}\, y \, \overline{\Big|_{\mathrm{d}\, x = f\left(\frac{x}{y}\right)}}$$

$$u = \frac{y}{x}$$

$$\implies y = u \cdot x$$

$$\implies \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x + (1) \cdot u$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(\frac{y}{x}\right)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x + u = f(u)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = f(u) - u$$

$$\frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = 1$$

$$\frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f(u) - u}\,\mathrm{d}\,u = \ln|x| + c \tag{1.17}$$

$$\exists G(u) : G(u) = \int \frac{1}{f(u) - u} \, du$$

$$G\left(u\right) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) + \ln|x| + c = 0 \tag{1.18}$$

$$\sum_{0}^{n} \left[a_n(x) \cdot f^{(n)}(x) \right] = g(x)$$

If $g(x) = 0$ it is said to be homogenous

$$a_1\left(x\right) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + a_0\left(x\right) \cdot y = g\left(x\right)$$

Where $a\left(x\right)$ is a function
(1.19)

Linear First Order ODE:
$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x) \qquad (1.20)$$
if $f\,(x) = 0$ the equation is said to be homogenous

y_h

y_p

$y_h:$ $\dfrac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\,(x) \cdot y_h = 0$

$$y_p: \quad \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\,(x) \cdot y_p = f\,(x)$$

- Rewrite the Equation in the standard form:
$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x)$$
- Identify $p\,(x)$ and find the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}$$
- Multiply through by the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}\left(\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y\right) = e^{\int p(x)\,\mathrm{d}\,x}f\,(x)$$
  It may be concluded:
$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[e^{\int p(x)\,\mathrm{d}\,x\cdot} \cdot y\right] = e^{\int p(x)\,\mathrm{d}\,x} \cdot f\,(x)$$
- Integrate both sides in order to solve:

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = f\left(x\right) \tag{1.21}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = 0 \qquad \Longrightarrow \quad y = y_h \tag{1.22}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = f(x) \Longrightarrow y = y_p \tag{1.23}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(y_h + y_p\right) + p\left(x\right) \cdot \left(y_h + y_p\right) = f\left(x\right)$$

$$\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right) \cdot y_h + p\left(x\right) \cdot y_p = f\left(x\right)$$

$$\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\left(x\right) \cdot y_h + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right) \cdot y_p = f\left(x\right)$$

$$0 + f\left(x\right) = f\left(x\right) \tag{1.24}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = 0$$

$$\frac{1}{y} \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = -p\,(x)$$

$$\ln|y| = \int -p\,(x)\,\mathrm{d} + c$$

$$|y| = e^{\int -p(x)x\,\mathrm{d}\,x} \cdot e^c \tag{1.25}$$

$y > 0$

$$\implies y_h = e^{-\int p(x)\, \mathrm{d}x} \cdot c$$

$$y_1 = e^{-\int p(x)\,\mathrm{d}x}$$

$$y_h = y_1(x) \cdot c \tag{1.26}$$

$p(x)$

c

$f(x)$

$$c = u\left(x\right)$$

$u(x)$

$$y_p = u\left(x\right) \times y_h\left(x\right)$$
$$= e^{-\int p(x)\,\mathrm{d}x} \cdot u\left(x\right) \tag{1.27}$$

$$y_p = e^{-\int p(x) \, dx} \cdot u(x)$$

$$\frac{d\,y_p}{d\,x} + p(x) \cdot = f(x)$$

$$\frac{d}{d\,x}(u(x) \cdot y_1(x)) + p(x)\,u(x)\,y_1(x) = f(x)$$

$$\frac{d\,u}{d\,x} \cdot y_1(x) + \frac{d\,y_1}{d\,x} \cdot u(x) + p(x) \cdot u(x) \cdot y_1(x) = f(x)$$

$$u(x)\left(\frac{d\,y_1}{d\,x} + p(x)\,y_1\right) + \frac{d\,y}{d\,x} \cdot y_1(x) = f(x)$$

$$0 + \frac{d\,y}{d\,x} - y_1(x) = f(x)$$

$$\frac{d\,y}{d\,x} = f(x)/y_1(x)$$

$$\int \frac{d\,u}{d\,x}\,d\,x = \int f(x)/y_1(x)\,d\,x$$

$$\int d\,u = \int f(x)/y_1(x)\,d\,x$$

$$u = \int f(x)/y_1(x)\,d\,x \qquad (1.28)$$

$$u = \int f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}\,\mathrm{d}\,x \tag{1.29}$$

$$y_p = u \cdot y_1$$

$$y_p = \frac{1}{y_1} \cdot \int f(x) \cdot e^{\int p(x) \, \mathrm{d}x}$$

$$y_p = e^{-\int p(x) \, \mathrm{d}x} \int f(x) \cdot e^{\int p(x) \, \mathrm{d}x} \tag{1.30}$$

$$e^{\int p(x)\,\mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x} \cdot y_p = e^{\int p(x)\,\mathrm{d}x} \cdot e^{-\int p(x)\,\mathrm{d}x} \int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x} \cdot y_p = \int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\int p(x)\,\mathrm{d}x} \cdot y_p\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}\right)$$

$$= f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}x} + p(x) \cdot e^{\int p(x)\,\mathrm{d}x} \cdot y = e^{\int p(x)\,\mathrm{d}x} \cdot f(x)$$

$$\implies \frac{\mathrm{d}y}{\mathrm{d}x} + p(x) \cdot y = f(x)$$

$$(x + 1) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + y = \ln(x) \; ; \qquad y(1) = 10 \qquad (1.31)$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + \frac{y}{x+1} = \frac{\ln(x)}{x+1} \quad : \qquad (x \in \mathbb{R} \setminus \{-1, 0\}) \tag{1.32}$$

$$u = e^{\int \frac{1}{x+1} \, \mathrm{d}x}$$
$$= e^{\int \ln|x+1| \, \mathrm{d}x}$$
$$= |x+1| \tag{1.33}$$

$x > 0$

$$(x + 1) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + y = \ln{(x)}$$

$$\implies \frac{\mathrm{d}}{\mathrm{d}\,x}\left((x + 1) \cdot y\right) = \ln{(x)}$$

<div align="right">(1.34)</div>

$$\int \frac{\mathrm{d}}{\mathrm{d}\,x}\left[(x+1)\cdot y\right]\mathrm{d}\,x = \int \ln\left(x\right)\mathrm{d}\,x$$

$$(x + 1) \cdot y = \int \ln (x) \, \mathrm{d} x$$

$$u = \ln(x) \qquad \mathrm{d}\,v = \mathrm{d}\,x$$
$$\mathrm{d}\,u = \frac{1}{x}\,\mathrm{d}\,x \qquad v = x$$

$$\implies \int u\,\mathrm{d}\,v = u \cdot v + \int v\,\mathrm{d}\,u$$

$$(x + 1) \cdot y = \ln(x) \cdot x - \int \mathrm{d}x$$
$$= x \cdot (\ln(x) - 1) + c$$
$$\implies y = \frac{x \cdot (\ln(x) - 1 + c)}{x + 1}$$

$y(1) = 10$

$$10 = \frac{1\left(\ln\left(1\right) - 1 + c\right)}{2}$$
$$20 = 1\left(0 - 1\right) + c$$
$$c = 19 \tag{1.36}$$

$$y = x(\ln(x) - 1 + 19)\frac{}{x+1} \quad ; \qquad \forall x \in \mathbb{C} \setminus \{-1, 0\}$$