



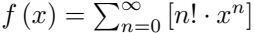
$$f\left(j\right) =\sum_{i=0}^{\infty }\left[C_n\left(z-a\right) ^n\right] ,\quad \exists z\in \mathbb{C}$$

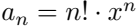
$$f\left(z\right)=\sum_{i=0}^{\infty}\left[C_n\left(z-a\right)^n\right],\quad\exists z\in\mathbb{C}$$











$$\begin{aligned}
\frac{\lim_{n \rightarrow \infty} |a_{n+1}|}{\lim_{n \rightarrow \infty} |a_n|} &= \lim_{n \rightarrow \infty} \left| \frac{(n+1)! \cdot x^n \cdot x}{n! \cdot x^n} \right| \\
&= (n+1) \cdot |x| \\
&= 0 \iff x = 0
\end{aligned}$$





$$\begin{aligned}
S_n &= \sum_{k=0}^n r^k \\
&= 1 + r + r^2 + r^3 \dots + r^{n-1} + r^n \\
\implies r \cdot S_n &= r + r^2 + r^3 + r^4 \dots r^n + r^{n+1} \\
\implies S_n - r \cdot S_n &= 1 + r^{n+1} \\
\implies S_n &= \frac{1 + r^{n+1}}{1 - r}
\end{aligned}$$

$$\begin{aligned}
\sum_{k=0}^{\infty} [x^k] &= \lim_{n \rightarrow \infty} \left[\sum_{k=0}^n x^k \right] \\
&= \lim_{n \rightarrow \infty} \left[\frac{1 + x^{n+1}}{1 - x} \right] \\
&= \frac{1 + \lim_{n \rightarrow \infty} [x^{n+1}]}{1 - x} \\
&= \frac{1 + 0}{1 - x} \\
&= \frac{1}{1 - x}
\end{aligned}$$

$$g\left(x\right)=\frac{1}{1+x^2}$$

$$\frac{1}{1-\#_1}=\sum_{n=0}^{\infty}[\#_1^n]$$



$$\frac{1}{1-(-x^2)}=\sum_{n=0}^{\infty}\left[(-x^2)^n\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sum_{n=1}^{\infty}c_n\left(z-a\right)^n\right)=\sum_{n=1}^{\infty}\left[\frac{\mathrm{d}}{\mathrm{d}x}\left(c_n\left(z-a\right)^n\right)\right]$$

$$\int \left(\sum_{n=1}^{\infty} c_n (z-a)^n \right) dx = \sum_{n=1}^{\infty} [c_n (z-a)^n]$$

for $\pi = 1$ on \mathbb{R}^2

$$f(z) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

$$\implies f(a) = c_0$$

$$f'(z) = c_1 + 2c_2(z-a) + 3c_3(z-a)^2 + 4c_4(z-a)^3$$

$$\implies f'(a) = c_1$$

$$f''(z) = 2c_2 + 3 \times 2 \times c_3(z-a) + 4 \times 3c_4(z-a)^2 + \dots$$

$$\implies f''(a) = 2 \cdot c_2$$

$$f'''(z) = 3 \times 2 \times 1 \cdot c_3 + 4 \times 3 \times 2c_4(z-a) + \dots$$

$$\implies f'''(a) = 3!c_3$$

$$f^{(n)}(a) = n! \cdot c_n$$

$$\Rightarrow c_n = \frac{f^{(n)}(a)}{n!}$$



$$f(z) = \sum_{n=0}^{\infty} \left[\frac{f^{(n)}(a)}{n!} (x-a)^n \right]$$



$$f\left(z \right) = e^z = \sum_{n=0}^{\infty} \left[\frac{f^{(n)}\left(0 \right)}{n!} \cdot x^n \right]$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^0}{n!} x^n \right]$$

$$= \sum_{n=0}^{\infty} \left[\frac{x^n}{n!} \right]$$

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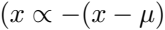
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$$\frac{\mathrm{d} f}{\mathrm{d} x} \propto -\left(x-\mu\right)$$

$$\frac{\mathrm{d} f}{\mathrm{d} x}=-k\left(x-\mu\right), \quad \exists k \in \mathbb{R}$$

$$\int \frac{\mathrm{d} f}{\mathrm{d} x} \mathrm{d} x=-\int\left(x-\mu\right) \mathrm{d} x$$

$$\text{let: } v = x - \mu$$

$$\Rightarrow \frac{dv}{dx} = 1$$

$$\Rightarrow dv = dx$$

$$\int \frac{\mathrm{d}f}{\mathrm{d}x} \mathrm{d}x = - \int (x - \mu) \mathrm{d}x$$

$$\implies \int \mathrm{d}p = - \int v \mathrm{d}v$$

$$p = -\frac{1}{2} v^2 \cdot k + C$$

$$p = -\frac{1}{2} (x - \mu)^2 \cdot k + C$$





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dx





$$\frac{\mathrm{d}f}{\mathrm{d}x} \propto f$$

$$\int \frac{1}{f} \cdot \frac{\mathrm{d}f}{\mathrm{d}x} \mathrm{d}x = k \cdot \int \mathrm{d}x$$

$$\ln |f| = k \cdot x$$

$$f = C \cdot e^{\pm x}$$

$$f \propto e^{\pm x}$$

$$f \propto f \wedge f \propto -(x - \mu)$$

$$\implies \frac{\mathrm{d}f}{\mathrm{d}x} \propto -f (x - \mu)$$

$$\int \frac{1}{f} \mathrm{d}f = -k \cdot \int (x - \mu) \mathrm{d}x$$

$$\ln |f| = -k \int (x - \mu) \mathrm{d}x$$



$$\ln (f) = -k \cdot \frac{1}{2} (x - \mu)^2 + C$$

$$f \propto e^{\frac{(x-\mu)^2}{2}}$$

$$1 = \int_{-\infty}^{\infty} f \mathrm{d} f$$

$$1 = -C \int_{-\infty}^{\infty} e^{\frac{k}{2} (x-\mu)^2} \mathrm{d} f$$

$$\text{let: } u^2 = \frac{k}{2} (x - \mu)^2$$

$$u = \sqrt{\frac{k}{2}} (x - \mu)$$

$$\frac{du}{dx} = \sqrt{\frac{k}{2}}$$

$$1 = -C \int_{-\infty}^{\infty} e^{\frac{k}{2}(x-\mu)^2}$$

$$1 = \sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u$$

$$1^2 = \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right)^2$$

$$1^2 = \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right) \times \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right)$$







$$1^2 = \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-x^2} dx \right) \times \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-y^2} dy \right)$$



$$\begin{aligned}
 1 &= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} e^{-y^2} \mathrm{d}y \times \beta \\
 &= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \beta \cdot e^{-y^2} \mathrm{d}y \\
 &= \frac{2}{k} \cdot C^2 \cdot \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x \right) e^{-y^2} \mathrm{d}y \\
 &= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \mathrm{d}x \mathrm{d}y
 \end{aligned}$$

$$\iint_D f(x,y) \, dA = \int_{\alpha}^{\beta} \int_{h_1(\phi)}^{h_2(\phi)} f(r \cdot \cos(\phi), r \cdot \sin(\phi)) \, dr d\phi$$

$$1 = \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{(r \cdot \cos \theta)^2 + (r \cdot \sin \theta)^2} dr d\theta$$

$$1 = \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{r^2} dr d\theta$$

$$\text{let: } u = -r^2$$

$$\frac{du}{dr} = -2r$$

$$dr = -\frac{1}{2r} du$$

$$1 = \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{r^2} \mathrm{d}r \mathrm{d}\theta$$

$$\implies 1 = -\frac{2}{k} c^2 \int_0^{2\pi} \int_0^\infty r \cdot e^{r^2} \mathrm{d}r \mathrm{d}\theta$$

$$1 = \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{r^2} dr d\theta$$

$$\implies 1 = -\frac{2}{k} c^2 \int_0^{2\pi} \int_0^\infty -\frac{1}{2} e^{-u} du d\theta$$

$$= \frac{1}{k} c^2 \int_0^{2\pi} \int_0^\infty e^{-u} du d\theta$$

$$= \frac{1}{k} c^2 \int_0^{2\pi} [-e^{-u}]_0^\infty d\theta$$

$$1 = \frac{1}{k} c^2 2\pi$$

$$\implies C^2 = \frac{k}{2\pi}$$

$$f = -C \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}$$

$$= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}$$



$$\mu = E(x) = \int_a^b x \cdot f(x) \, dx$$

$$V(x)=\int_a^b(x-\mu)^2f(x)\,\mathrm{d}x$$





$$\text{let: } v = x - \mu$$

$$\Rightarrow dv = dx$$



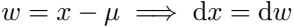
$$\begin{aligned}
E(v) &= \int_{-\infty}^{\infty} v \times f(v) \, dv \\
&= k \cdot \int_{-\infty}^{\infty} v \cdot e^{v^2} \, dv \\
&= \frac{1}{2} \left[e^{x^2} \right]_{\infty}^{\infty} \\
&= \frac{1}{2} \lim_{b \rightarrow \infty} \left[\left[e^{x^2} \right]_{-b}^b \right] \\
&= \frac{1}{2} \lim_{b \rightarrow \infty} \left[e^{b^2} - e^{(-b)^2} \right] \\
&= \lim_{b \rightarrow \infty} [0] \times \frac{1}{2} \\
&= \frac{1}{2} \times 0 \\
&= 0
\end{aligned}$$





$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \times f(x) \, dx$$





$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2} w^2} dw$$



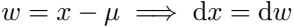
$$\int u dv = u \cdot v - \int v du$$





$$\begin{aligned}
 \sigma^2 &= \int_{-\infty}^{\infty} (x - \mu)^2 \times f(x) \, dx \\
 &= \int_{-\infty}^{\infty} (x - \mu)^2 \times \left(\sqrt{\frac{k}{2\pi}} e^{-\frac{k}{2}(x-\mu)^2} \right) \, dx \\
 &= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} (x - \mu)^2 \times \left(e^{-\frac{k}{2}(x-\mu)^2} \right) \, dx
 \end{aligned}$$





$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2} w^2} dw$$



$$\int u dv = u \cdot v - \int v du$$





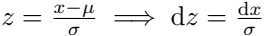
$$\begin{aligned}
\sigma^2 &= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2}w^2} \mathrm{d}w \\
&= \sqrt{\frac{k}{2\pi}} \left[u \cdot v - \int v \mathrm{d}u \right]_{\infty}^{\infty} \\
&= \sqrt{\frac{k}{2\pi}} \left(\left[\frac{-w}{k} \cdot e^{-\frac{k}{2}w^2} \right]_{\infty}^{\infty} - \frac{1}{k} \int_{-\infty}^{\infty} e^{\frac{k}{2}w^2} \mathrm{d}w \right) \\
&= \sqrt{\frac{k}{2\pi}} \left[\frac{-w}{k} \cdot e^{-\frac{k}{2}w^2} \right]_{\infty}^{\infty} - \frac{1}{k} \left(\sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} e^{\frac{k}{2}w^2} \mathrm{d}w \right)
\end{aligned}$$

$$\sigma^2 = 0 - \frac{1}{k}$$

$$\Rightarrow k = \frac{1}{\sigma^2}$$

$$= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}$$

$$= \sqrt{\frac{1}{2\pi\sigma^2}} \cdot e^{\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}}$$



$$f(x) = \sqrt{\frac{1}{2\pi}} \cdot e^{-\frac{1}{2}x^2}$$

$$e^{-\frac{1}{2}z^2} = \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2}z^2\right)^n}{n!} \right]$$



$$\begin{aligned}
 f(x) &= \sqrt{\frac{1}{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2}z^2\right)^n}{n!} \right] \\
 \int f(x) \, dx &= \frac{1}{\sqrt{2\pi}} \int \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2}z^2\right)^n}{n!} \right] \, dz \\
 &= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\int \frac{(-1)^{-1} z^{2n}}{2^n \cdot n!} \, dz \right] \\
 &= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{(-1)^n \cdot z^{2n+1}}{2^n (2n+1) n!} \right]
 \end{aligned}$$









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$$\mathbb{E}(\text{FPR}) = \alpha;$$

$$\text{FPR} = \frac{FP}{N}$$

$$= \frac{FP}{FN + TP}$$

$$= \frac{8}{8 + 72}$$

$$= 9\%$$





















FOR = TFP + FP









$$PPV = \frac{TP}{TP + FP}$$

$$FDR = \frac{FP}{TP + FN}$$

$$\alpha = \frac{FP}{N} = \frac{TP}{TN + FP}$$

$$\beta = \frac{TP}{P} = \frac{TP}{TP + FN}$$

























$$\overline{X} \sim \mathcal{N}\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$$\Rightarrow \bar{x}_{crit} = \mu + z_{\alpha} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\bar{x}_{crit} = \mu + z_{0.05} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\begin{aligned}\bar{x}_{crit} &= \mu + 1.645 \cdot \left(\frac{5.2}{\sqrt{100}}\right) \\ &= 10.8554\end{aligned}$$





$$\begin{aligned}
 z &= \frac{\overline{x}_{crit} - \mu_{true}}{\left(\frac{\sigma}{\sqrt{n}}\right)} \\
 &= \frac{10.86 - 12}{\frac{5.2}{10}} = -2.2
 \end{aligned}$$



$$\begin{aligned}
 \beta &= \text{P (Type II Error)} \\
 &= \text{P} (H_0 \text{ is not rejected} \mid H_0 \text{ is false}) \\
 &= \text{P} \left(\mu_{\bar{X}_{\text{crit}}} < \bar{x}_{\text{crit}} \mid \mu = 12 \right) \\
 &= 0.014
 \end{aligned}$$

$$\text{Power} = (H_0 \text{ is not rejected} \mid H_0 \text{ is false})$$

$$= P\left(\mu_{\bar{X}_{\text{crit}}} < \bar{x}_{\text{crit}}\right)$$

$$= 1 - \beta$$

$$= 1 - 0.14$$

$$= 98.6\%$$





$$s_p$$

$$\cdot$$

$$\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$



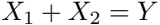












$$\text{var} \left(y \right) = \text{var} \left(X_1 \right) + \text{var} \left(X_2 \right)$$

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2$$



$$s_p = 505.09$$

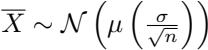
$$\frac{3515.64 - 3260.29}{505.05 \times \sqrt{\frac{1}{742} + \frac{1}{484}}} = 8.653$$

$$t = \frac{162.7 - 154.04}{22.39 \times 22.39 \times \sqrt{\frac{1}{52} + \frac{1}{48}}}$$













$$\sigma_x = \left(\frac{\sigma}{\sqrt{n}} \right)$$

