

Subgraphs

300958 Social Web Analytics

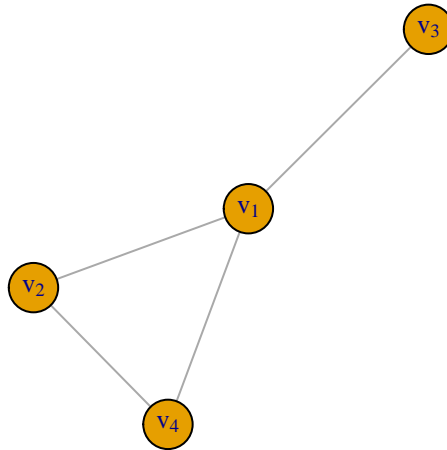
Week 5 Supplementary Material

Subgraph Definition

A subgraph $H = (V_H, E_H)$ of a graph $G = (V_G, E_G)$ contains a subset of the vertices of G (that is $V_H \subset V_G$) and a subset of the edges of G ($E_H \subset E_G$) with the requirement that the end vertices of any edge must be included in V_H . If the subgraph contains all edges for the given vertex set it is known as an induced subgraph.

Example

Consider the following graph:



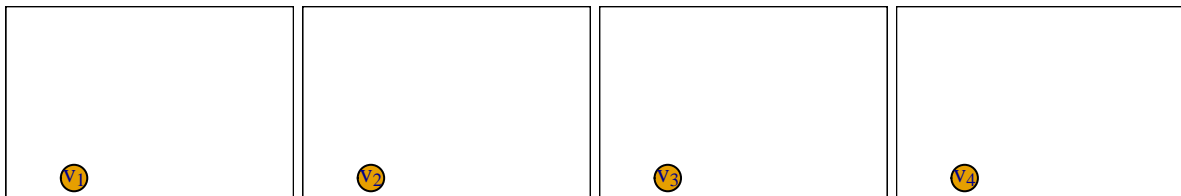
This graph has a total of 49 subgraphs which we can classify by the number of vertices they contain.

0 Vertices

The empty graph (containing no vertices and hence no edges) is a subgraph of every graph.

1 Vertex

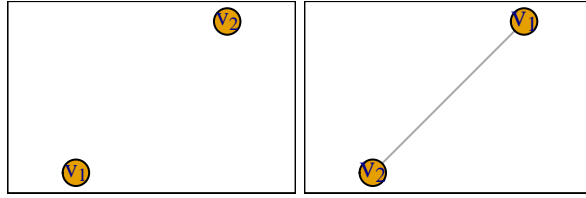
There are 4 possible subgraphs with one vertex



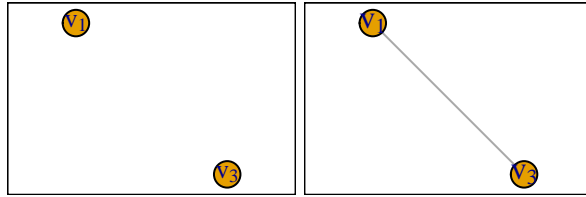
2 Vertices

There are 10 possible subgraphs which include two of the vertices. These are:

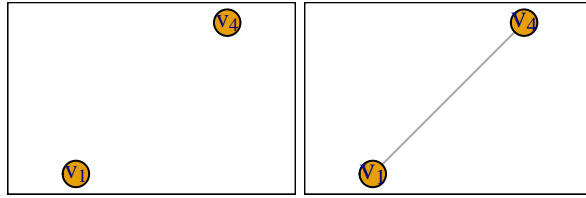
- Using V_1 and V_2



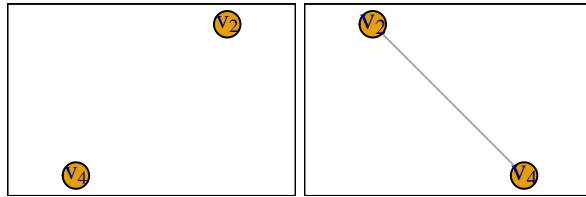
- Using V_1 and V_3



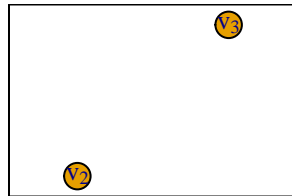
- Using V_1 and V_4



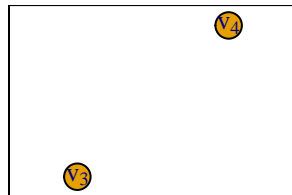
- Using V_2 and V_4



- Using V_2 and V_3



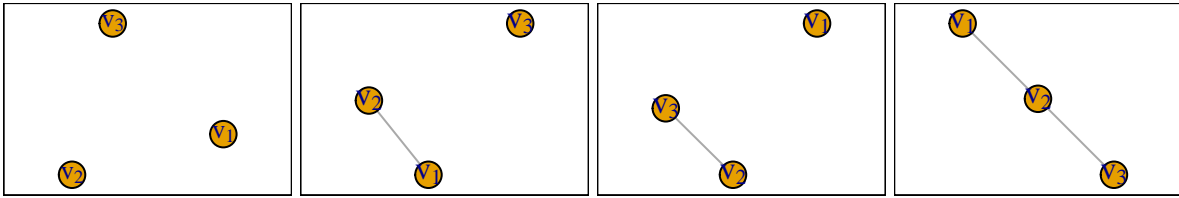
- Using V_3 and V_4



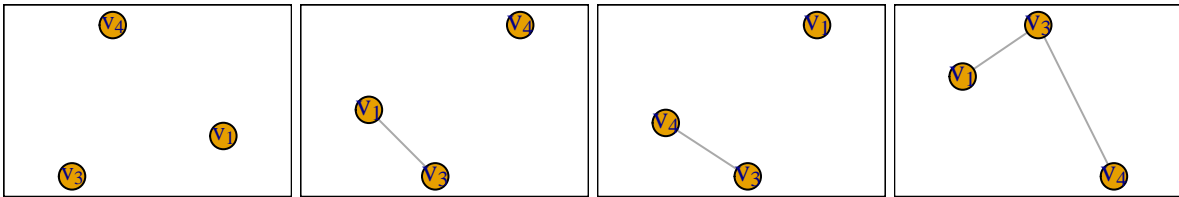
3 Vertices

There are 18 possibilities using three vertices. These are:

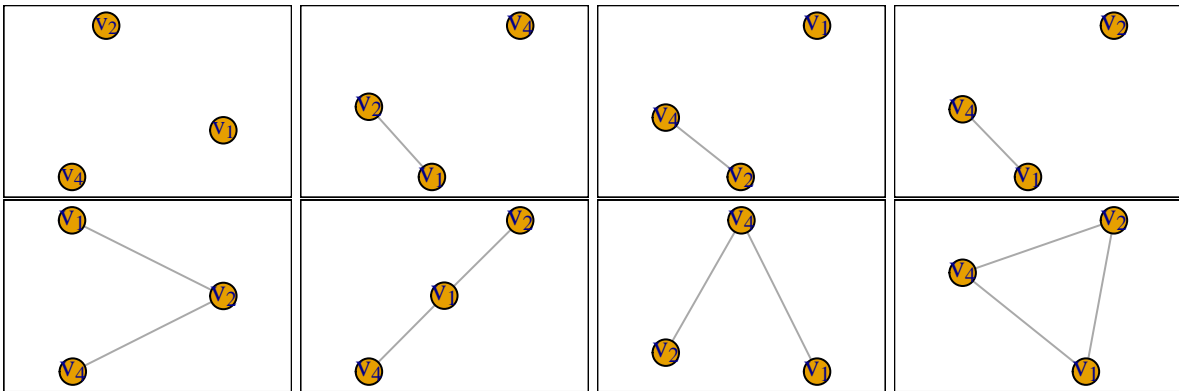
- Using V_1 , V_2 , and V_3



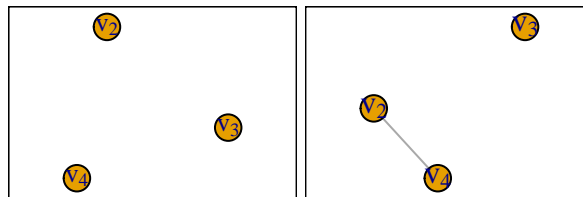
- Using V_1 , V_3 , and V_4



- Using V_1 , V_2 , and V_4

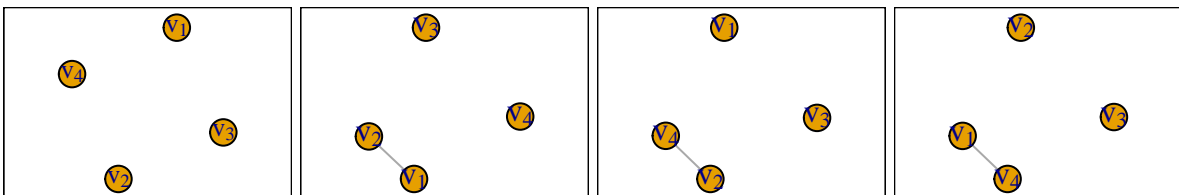


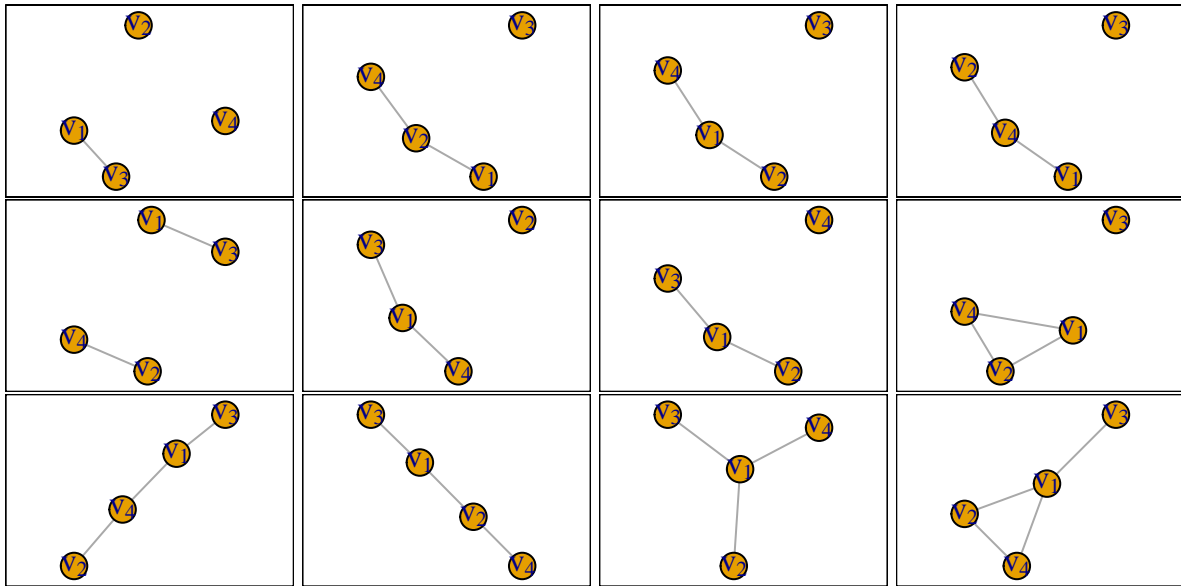
- Using V_2 , V_3 , and V_4



4 Vertices

Here we need to consider all possible subsets of the four edges, so there are a total of 16 possible subgraphs:





Of these 49 subgraphs, 16 are induced subgraphs. Can you identify them?

Whilst there are formulas for the number of subgraphs for graphs of particular types (complete graphs for example) the number of subgraphs of a random graph depends on the structure of the graph.