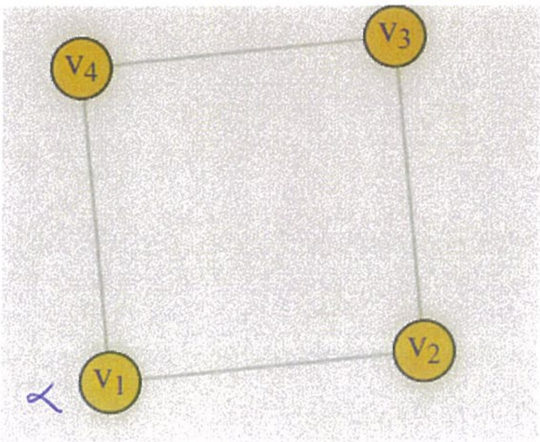
Slide 11



State Distribution after length $n = 1, 2, 3$ random walk

$$
\begin{array}{c}
\\ \vec{P_0} \\ \vec{P_1} \\ \vec{P_2} \\ \vec{P_3}
\end{array}
\begin{array}{c}
\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 1/2 \\
1/2 & 0 & 1/2 & 0 \\
0 & 1/2 & 0 & 1/2
\end{bmatrix}
\end{array}
$$

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$$
T = \begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4
\end{array}
\begin{array}{c}
\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\
\begin{bmatrix}
0 & 1/2 & 0 & 1/2 \\
1/2 & 0 & 1/2 & 0 \\
0 & 1/2 & 0 & 1/2 \\
1/2 & 0 & 1/2 & 0
\end{bmatrix}
\end{array}
\overset{\vec{P_0}}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}
= \overset{P_1}{\begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1/2 \end{bmatrix}}
$$

$$
\vec{P_2} = T\,\vec{P_1} = \begin{bmatrix}
0 & 1/2 & 0 & 1/2 \\
1/2 & 0 & 1/2 & 0 \\
0 & 1/2 & 0 & 1/2 \\
1/2 & 0 & 1/2 & 0
\end{bmatrix}
\begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1/2 \end{bmatrix}
= \begin{bmatrix}
1/4 + 1/4 = 1/2 \\
0 = 0 \\
1/4 + 1/4 = 1/2 \\
0 = 0
\end{bmatrix}
$$

$$\vec{P_3} = T\,\vec{P_2}$$

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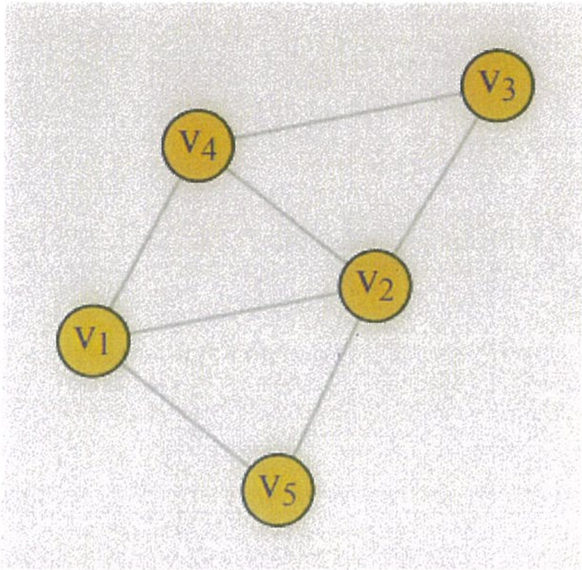

Transition Probability Matrix

Similar to Adjacency matrix, but all columns are normalized.

what is the probability I can go from v_1 to v_2. There are 3 edges out of 3 from v_1.

$$T = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1/4 & 0 & 1/3 & 1/2 \\ v_2 & 1/3 & 0 & 1/2 & 1/3 & 1/2 \\ v_3 & 0 & 1/4 & 0 & 1/3 & 0 \\ v_4 & 1/3 & 1/4 & 1/2 & 0 & 0 \\ v_5 & 1/3 & 1/4 & 0 & 0 & 0 \\ \hline + & 1 & 1 & 1 & 1 & 1 \end{array}$$

Columnwise!

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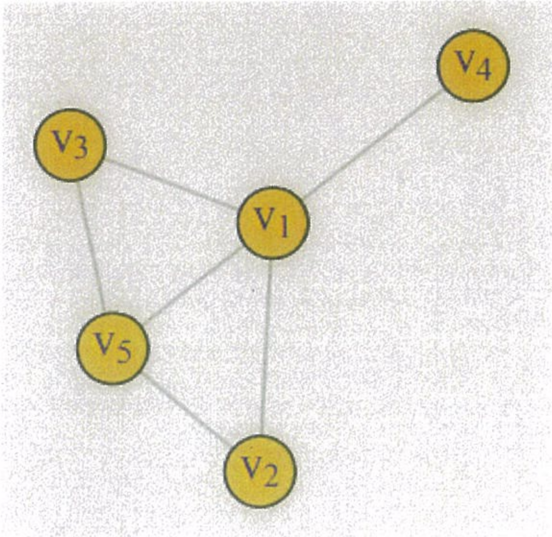

Transition Probability Matrix

$$
T = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\
\begin{bmatrix}
0 & 1/2 & 1/2 & 1 & 1/3 \\
1/4 & 0 & 0 & 0 & 1/3 \\
1/4 & 0 & 0 & 0 & 1/3 \\
1/4 & 0 & 0 & 0 & 0 \\
1/4 & 1/2 & 1/2 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\Downarrow$

all add up to 1.

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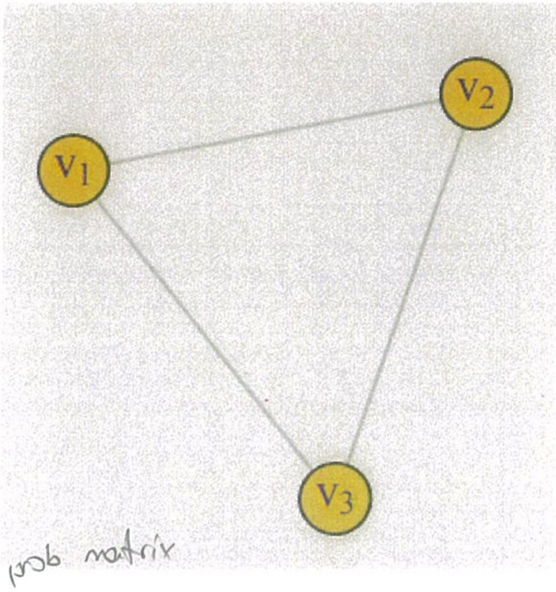
Stationary Distribution



transition prob matrix

$$T = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$$\vec{p} = T\vec{p}$$

let say $p = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$

$$T\vec{p} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 1/6 + 1/6 = 1/3 \\ 1/6 + 1/6 = 1/3 \\ 1/6 + 1/6 = 1/3 \end{bmatrix}$$

↳ that's the probability of where you are in the graph.

Take a step. Still 1/3 chance to be in the others.

$Vol(G) = 2+2+2 = 6$

$$\vec{p} = \left[\frac{2}{6} \ \frac{2}{6} \ \frac{2}{6}\right]$$

$$= \left[\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right]$$

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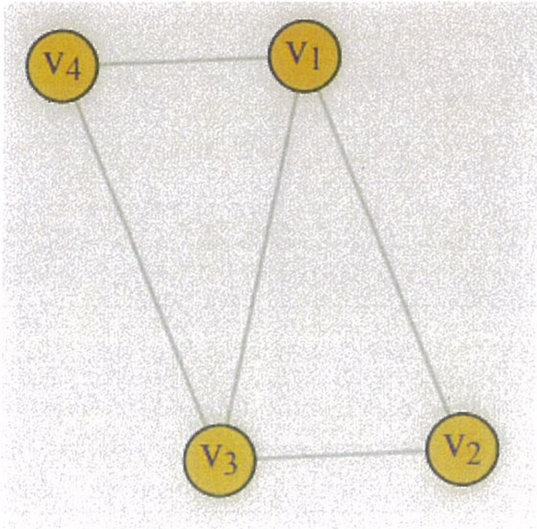

$3+2+3+2=10$

$$T = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1/2 & 1/3 & 1/2 \\ v_2 & 1/3 & 0 & 1/3 & 0 \\ v_3 & 1/3 & 1/2 & 0 & 1/2 \\ v_4 & 1/3 & 0 & 1/3 & 0 \end{array}$$

$$\vec{p} = \begin{bmatrix} \frac{3}{10} & \frac{2}{10} & \frac{3}{10} & \frac{2}{10} \end{bmatrix}$$

$$T\vec{p} = \begin{bmatrix} 0 & 1/2 & 1/3 & 1/2 \\ 1/3 & 0 & 1/3 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 0 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 3/10 \\ 2/10 \\ 3/10 \\ 2/10 \end{bmatrix} = \begin{bmatrix} 0+\frac{1}{10}+\frac{1}{10}+\frac{1}{10} \\ \frac{1}{10}+0+\frac{1}{10}+0 \\ \frac{1}{10}+\frac{1}{10}+0+\frac{1}{10} \\ \frac{1}{10}+0+\frac{1}{10}+0 \end{bmatrix} = \underset{\vec{p}}{\begin{bmatrix} 3/10 \\ 2/10 \\ 3/10 \\ 2/10 \end{bmatrix}}$$

we got the same thing back!

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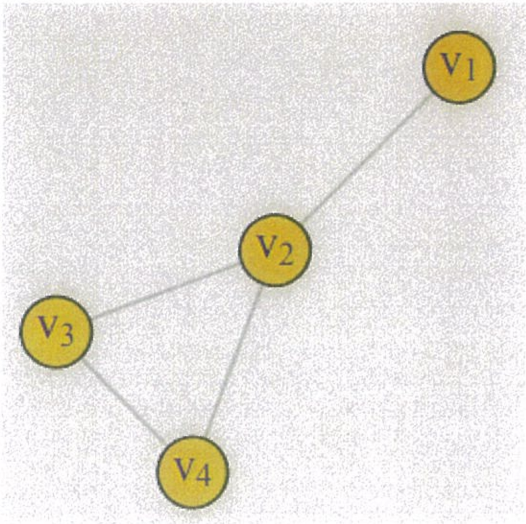

$$
T = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}
\begin{array}{c}
\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\
\begin{bmatrix} 0 & 1/3 & 0 & 0 \\ 1 & 0 & 1/2 & 1/2 \\ 0 & 1/3 & 0 & 1/2 \\ 0 & 1/3 & 1/2 & 0 \end{bmatrix}
\end{array}
\begin{bmatrix} 1/8 \\ 3/8 \\ 2/8 \\ 2/8 \end{bmatrix}
= \begin{bmatrix} 1/8 \\ 1/8 + 0 + 1/8 + 1/8 \\ 1/8 + 1/8 \\ 1/8 + 1/8 \end{bmatrix}
= \begin{bmatrix} 1/8 \\ 3/8 \\ 2/8 \\ 2/8 \end{bmatrix}
$$

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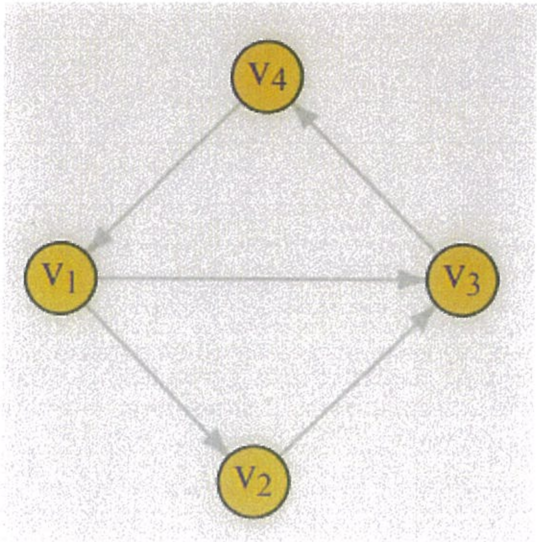

$$T = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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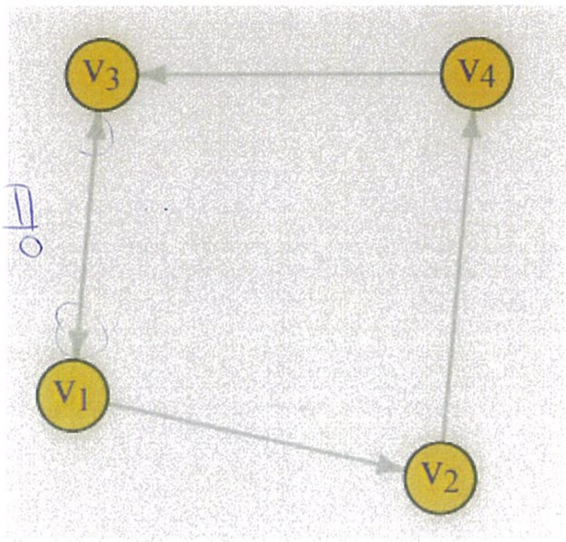
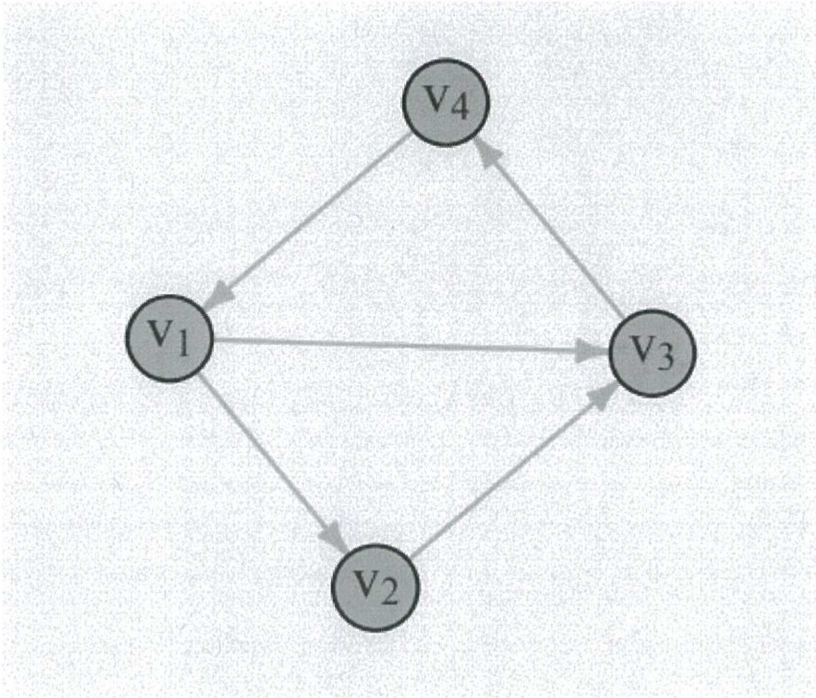
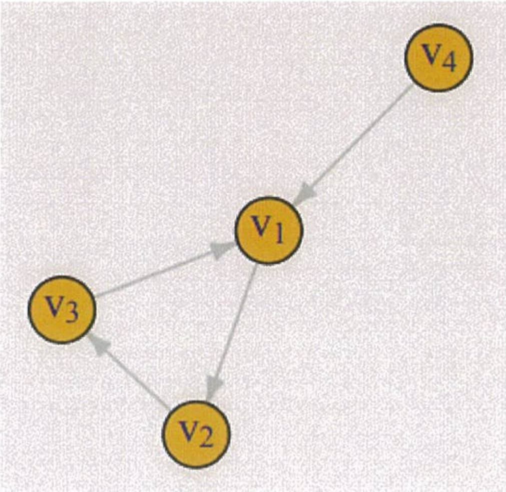


$$\begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 0 & 1 & 0 \\ v_2 & 1/2 & 0 & 0 & 0 \\ v_3 & 1/2 & 0 & 0 & 1 \\ v_4 & 0 & 1 & 0 & 0 \end{array}$$

35)



$$T = \begin{array}{c|cccc|} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 0 & 0 & 1 \\ v_2 & 1/2 & 0 & 0 & 0 \\ v_3 & 1/2 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 1 & 0 \end{array}$$



$$T = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 \end{array}$$

Random Jump Matrix (Random Surfer) $B = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$

$\lambda = 0.8$

mix then together

$0.8(1) + 0.2\left(\frac{1}{4}\right) = 0.85$

$0.8(0) + 0.2\left(\frac{1}{4}\right) = 0.05$

$$S = \lambda T + (1 - \lambda) B$$

$$= 0.8T + 0.2B$$

$$= \begin{bmatrix} 0.05 & 0.05 & 0.85 & 0.85 \\ 0.85 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 \end{bmatrix}$$

Page Rank ?

1. 2. 3. 4

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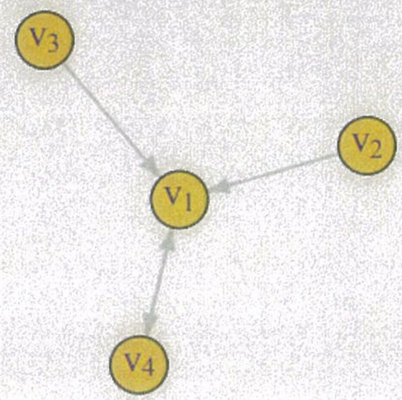
1. Compute the random surfer probability transition matrix for the following graph, using $\lambda = 0.8$:
2. Verify that the stationary distribution is $\vec{p} = [0.47\ 0.05\ 0.05\ 0.43]$



$$T = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 1 & 1 \\ v_2 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 0 & 0 \\ v_4 & 1 & 0 & 0 & 0 \end{array} \qquad B = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

$$S = \lambda T + (1-\lambda)B = 0.8T + 0.2B$$

$$S = \begin{bmatrix} 0.05 & 0.85 & 0.85 & 0.85 \\ 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 \\ 0.85 & 0.05 & 0.05 & 0.05 \end{bmatrix} \overbrace{\begin{bmatrix} 0.47 \\ 0.05 \\ 0.05 \\ 0.43 \end{bmatrix}}^{\vec{p}}$$

$$\vec{p} = S\vec{p} = \begin{bmatrix} 0.05 \times 0.47 + 0.85 \times 0.05 + 0.85 \times 0.05 + 0.85 \times 0.43 \\ 0.05 \times 0.47 + 0.05 \times 0.05 + 0.05 \times 0.05 + 0.05 \times 0.43 \\ 0.05 \times 0.47 + 0.05 \times 0.05 + 0.05 \times 0.05 + 0.05 \times 0.43 \\ 0.085 \times 0.47 + 0.05 \times 0.05 \quad 0.05 \times 0.05 + 0.05 \times 0.43 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.47 \\ 0.05 \\ 0.05 \\ 0.43 \end{bmatrix}}_{\vec{p}}$$