f

$$f\left(j\right)=\sum_{i=0}^{\infty}\left[C_n\left(z-a\right)^n\right],\quad \exists z\in\mathbb{C}$$

$$f\left(z\right)=\sum_{i=0}^{\infty}\left[C_n\left(z-a\right)^n\right],\quad \exists z\in\mathbb{C}$$

$$x = a$$

$\forall x$

$$|z - a| < R$$

$$f\left(x\right)=\sum_{n=0}^{\infty}\left[n!\cdot x^{n}\right]$$

$$a_n = n! \cdot x^n$$

$$
\begin{aligned}
\frac{\lim_{n\to\infty}|a_{n+1}|}{\lim_{n\to\infty}|a_n|} &= \lim_{n\to\infty}\left|\frac{(n+1)!\cdot x^n\cdot x}{n!\cdot x^n}\right| \\
&= (n+1)\cdot|x| \\
&= 0 \iff x = 0
\end{aligned}
$$

$\therefore$

$$x = 0$$

$$
\begin{aligned}
S_n &= \sum_{n=0}^{n} r^k \\
&= 1 + r + r^2 + r^3 \ldots + r^{n-1} + r^n \\
\implies r \cdot S_n &= r + r^2 + r^3 + r^4 \ldots r^n + r^{n+1} \\
\implies S_n - r \cdot C_n &= 1 + r^{n+1} \\
\implies S_n &= \frac{1 + r^{n+1}}{1 - r}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=0}^{\infty}\left[x^k\right] &= \lim_{n\to\infty}\left[\sum_{k=0}^{n} x^k\right] \\
&= \lim_{n\to\infty}\left[\frac{1+x^{n+1}}{1-x}\right] \\
&= \frac{1+\lim_{n\to\infty}\left[x^{n+1}\right]}{1-x} \\
&= \frac{1+0}{1-x} \\
&= \frac{1}{1-x}
\end{aligned}
$$

$$g\left(x\right)=\frac{1}{1+x^{2}}$$

$$\frac{1}{1 - \#_1} = \sum_{n=0}^{\infty} [\#_1^n]$$

$$\#_1 = \left(-x^2\right)$$

$$\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} \left[\left(-x^2 \right)^n \right]$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sum_{n=1}^{\infty} c_n\left(z-a\right)^n\right)=\sum_{n=1}^{\infty}\left[\frac{\mathrm{d}}{\mathrm{d}x}\left(c_n\left(z-a\right)^n\right)\right]$$

$$\int \left(\sum_{n=1}^{\infty} c_n \left(z-a \right)^n \right) \mathrm{d}x = \sum_{n=1}^{\infty} \left[c_n \left(z-a \right)^n \right]$$

$$f\left(z\right)=\sum_{n=1}^{\infty}c_{n}\left(z-a\right)^{n}$$

$$\begin{aligned}
f(z) &= c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots \\
&\implies f(a) = c_0 \\
f'(z) &= c_1 + 2c_2(z-a) + 3c_3(z-a)^2 + 4c_4(z-a)^3 \\
&\implies f'(a) = c_1 \\
f''(z) &= 2c_2 + 3 \times 2 \times c_3(z-a) + 4 \times 3c_4(z-a)^2 + \ldots \\
&\implies f''(a) = 2 \cdot c_2 \\
f'''(z) &= 3 \times 2 \times 1 \cdot c_3 + 4 \times 3 \times 2c_4(z-a) + \ldots \\
&\implies f'''(a) = 3!c_3
\end{aligned}$$

$$f^{(n)}(a) = n! \cdot c_n$$

$$\implies c_n = \frac{f^{(n)}(a)}{n!}$$

f

$$f\left(z\right)=\sum_{n=0}^{\infty}\left[\frac{f^{(n)}\left(a\right)}{n!}\left(x-a\right)^{n}\right]$$

e

$$
\begin{aligned}
f(z) = e^z &= \sum_{n=0}^{\infty} \left[\frac{f^{(n)}(0)}{n!} \cdot x^n \right] \\
&= \sum_{n=0}^{\infty} \left[\frac{e^0}{n!} x^n \right] \\
&= \sum_{n=0}^{\infty} \left[\frac{x^n}{n!} \right]
\end{aligned}
$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f\right) \propto -\left(x-\mu\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \propto f$$

$(x \propto -(x - \mu)$

$$y \propto y$$

f

x

f

$$
\begin{aligned}
\frac{\mathrm{d}f}{\mathrm{d}x} &\propto -\left(x-\mu\right) \\
\frac{\mathrm{d}f}{\mathrm{d}x} &= -k\left(x-\mu\right), \quad \exists k \in \mathbb{R} \\
\int \frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x &= -\int \left(x-\mu\right)\mathrm{d}x
\end{aligned}
$$

$$\begin{aligned}
\text{let:} \quad v &= x - \mu \\
\implies \frac{\mathrm{d}v}{\mathrm{d}x} &= 1 \\
\implies \mathrm{d}v &= \mathrm{d}x
\end{aligned}$$

$$
\begin{aligned}
\int \frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x &= -\int (x-\mu)\,\mathrm{d}x \\
\implies \int \mathrm{d}p &= -\int v\mathrm{d}v \\
p &= -\frac{1}{2}v^2 \cdot k + C \\
p &= -\frac{1}{2}\left(x-\mu\right)^2 \cdot k + C
\end{aligned}
$$

f

f

$$\frac{\mathrm{d}f}{\mathrm{d}x} \propto -(x-\mu)$$

f

$$\frac{\mathrm{d}f}{\mathrm{d}x} \propto f$$

$$\begin{aligned}
\frac{\mathrm{d}f}{\mathrm{d}x} &\propto f \\
\int \frac{1}{f} \cdot \frac{\mathrm{d}f}{\mathrm{d}x} \mathrm{d}x &= k \cdot \int \mathrm{d}x \\
\ln|f| &= k \cdot x \\
f &= C \cdot e^{\pm x} \\
f &\propto e^{\pm x}
\end{aligned}$$

$$
\begin{aligned}
f \propto f \wedge f &\propto -(x-\mu) \\
\implies \frac{\mathrm{d}f}{\mathrm{d}x} &\propto -f\,(x-\mu) \\
\int \frac{1}{f}\mathrm{d}f &= -k \cdot \int (x-\mu)\,\mathrm{d}x \\
\ln|f| &= -k \int (x-\mu)\,\mathrm{d}x
\end{aligned}
$$

$f > 0$

$$\ln(f) = -k \cdot \frac{1}{2}(x-\mu)^2 + C$$

$$f \propto e^{\frac{(x-\mu)^2}{2}}$$

$$1 = \int_{-\infty}^{\infty} f \mathrm{d} f$$

$$1 = -C \int_{-\infty}^{\infty} e^{\frac{k}{2}(x-\mu)^2} \mathrm{d} f$$

$$
\begin{aligned}
\text{let:} \quad u^2 &= \frac{k}{2}\,(x-\mu)^2 \\
u &= \sqrt{\frac{k}{2}}\,(x-\mu) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= \sqrt{\frac{k}{2}}
\end{aligned}
$$

$$
\begin{aligned}
1 &= -C \int_{-\infty}^{\infty} e^{\frac{k}{2}(x-\mu)^2} \\
1 &= \sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \\
1^2 &= \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right)^2 \\
1^2 &= \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right) \times \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-u^2} \mathrm{d}u \right)
\end{aligned}
$$

u

x

y

$$1^2 = \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x \right) \times \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-y^2} \mathrm{d}y \right)$$

$\beta \in \mathbb{R}$

$$
\begin{aligned}
1 &= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} e^{-y^2} \mathrm{d}y \times \beta \\
&= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \beta \cdot e^{-y^2} \mathrm{d}y \\
&= \frac{2}{k} \cdot C^2 \cdot \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x \right) e^{-y^2} \mathrm{d}y \\
&= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\left(x^2+y^2\right)} \mathrm{d}x \mathrm{d}y
\end{aligned}
$$

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\phi)}^{h_2(\phi)} f\left(r\cdot\cos(\phi), r\cdot\sin(\phi)\right) \mathrm{d}r \mathrm{d}\phi$$

$$1 = \frac{2}{k}c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{(r \cdot \cos\theta)^2 + (r \cdot \sin\theta)^2} \mathrm{d}r\mathrm{d}\theta$$

$$1 = \frac{2}{k}c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta$$

$$f'(x) \times g(f(x))$$

$$\begin{aligned} \text{let:} \quad u &= -r^2 \\ \frac{\mathrm{d}u}{\mathrm{d}r} &= -2r \\ \mathrm{d}r &= -\frac{1}{2r}\mathrm{d}u \end{aligned}$$

$$
\begin{aligned}
& 1 = \frac{2}{k}c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta \\
\implies & 1 = -\frac{2}{k}c^2 \int_0^{2\pi} \int_0^{\infty} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta
\end{aligned}
$$

$$
\begin{aligned}
1 &= \frac{2}{k} c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{r^2} \mathrm{d}r \mathrm{d}\theta \\
\implies 1 &= -\frac{2}{k} c^2 \int_0^{2\pi} \int_0^{\infty} -\frac{1}{2} e^{-u} \mathrm{d}u \mathrm{d}\theta \\
&= \frac{1}{k} c^2 \int_0^{2\pi} \int_0^{\infty} e^{-u} \mathrm{d}u \mathrm{d}\theta \\
&= \frac{1}{k} c^2 \int_0^{2\pi} \left[-e^{-u} \right]_0^{\infty} \mathrm{d}\theta \\
1 &= \frac{1}{k} c^2 2\pi \\
\implies C^2 &= \frac{k}{2\pi}
\end{aligned}
$$

$$
\begin{aligned}
f &= -C \cdot e^{k \cdot \frac{(x-\mu)^2}{2}} \\
&= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}
\end{aligned}
$$

$f(x)$

$$\mu = E(x) = \int_a^b x \cdot f\,(x)\,\mathrm{d}x$$

$$V\left(x\right)=\int_{a}^{b}\left(x-\mu\right)^{2}f\left(x\right)\mathrm{d}x$$

$$\sigma = \sqrt{V(x)}$$

$\boldsymbol{\mu}$

$$\begin{aligned} &\text{let:} \quad v = x - \mu \\ \implies &\mathrm{d}v = \mathrm{d}x \end{aligned}$$

$\pm\infty$

$$
\begin{aligned}
E(v) &= \int_{-\infty}^{\infty} v \times f(v) \, \mathrm{d}v \\
&= k \cdot \int_{-\infty}^{\infty} v \cdot e^{v^2} \mathrm{d}v \\
&= \frac{1}{2} \left[e^{x^2} \right]_{\infty}^{\infty} \\
&= \frac{1}{2} \lim_{b \to \infty} \left[\left[e^{x^2} \right]_{-b}^{b} \right] \\
&= \frac{1}{2} \lim_{b \to \infty} \left[e^{b^2} - e^{(-b)^2} \right] \\
&= \lim_{b \to \infty} [0] \times \frac{1}{2} \\
&= \frac{1}{2} \times 0 \\
&= 0
\end{aligned}
$$

$$0 = x - \mu$$

$$E(x) = \mu$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \times f(x) \, \mathrm{d}x$$

$$(x - \mu)$$

$$w = x - \mu \implies \mathrm{d}x = \mathrm{d}w$$

$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2} w^2} \mathrm{d}w$$

$$f(x) \times g(x)$$

$$\int u\mathrm{d}v = u \cdot v - \int v\mathrm{d}u$$

u

$\mathrm{d}v$

$$
\begin{aligned}
\sigma^2 &= \int_{-\infty}^{\infty} (x-\mu)^2 \times f(x)\,\mathrm{d}x \\
&= \int_{-\infty}^{\infty} (x-\mu)^2 \times \left(\sqrt{\frac{k}{2\pi}} e^{-\frac{k}{2}(x-\mu)^2} \right) \mathrm{d}x \\
&= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 \times \left(e^{-\frac{k}{2}(x-\mu)^2} \right) \mathrm{d}x
\end{aligned}
$$

$$(x - \mu)$$

$$w = x - \mu \implies \mathrm{d}x = \mathrm{d}w$$

$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2} w^2} \mathrm{d}w$$

$$f(x) \times g(x)$$

$$\int u \mathrm{d}v = u \cdot v - \int v \mathrm{d}u$$

u

$$\mathrm{d}v$$

$$
\begin{aligned}
\sigma^2 &= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2} w^2} \mathrm{d}w \\
&= \sqrt{\frac{k}{2\pi}} \left[u \cdot v - \int v \mathrm{d}u \right]_{\infty}^{\infty} \\
&= \sqrt{\frac{k}{2\pi}} \left(\left[\frac{-w}{k} \cdot e^{-\frac{k}{2} w^2} \right]_{\infty}^{\infty} - \frac{1}{k} \int_{-\infty}^{\infty} e^{\frac{k}{2} w^2} \mathrm{d}w \right) \\
&= \sqrt{\frac{k}{2\pi}} \left[\frac{-w}{k} \cdot e^{-\frac{k}{2} w^2} \right]_{\infty}^{\infty} - \frac{1}{k} \left(\sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} e^{\frac{k}{2} w^2} \mathrm{d}w \right)
\end{aligned}
$$

$$\begin{aligned} \sigma^2 &= 0 - \frac{1}{k} \\ \implies k &= \frac{1}{\sigma^2} \end{aligned}$$

$$= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}$$

$$= \sqrt{\frac{1}{2\pi\sigma^2}} \cdot e^{\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}}$$

$$z = \frac{x-\mu}{\sigma} \implies \mathrm{d}z = \frac{\mathrm{d}x}{\sigma}$$

$$f\left(x\right)=\sqrt{\frac{1}{2\pi}}\cdot e^{-\frac{1}{2}z^{2}}$$

$$e^{-\frac{1}{2}z^2} = \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2}z^2\right)^n}{n!} \right]$$

$$f\left(x\right)$$

$$
\begin{aligned}
f(x) &= \sqrt{\frac{1}{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2} z^2\right)^n}{n!} \right] \\
\int f(x)\,\mathrm{d}x &= \frac{1}{\sqrt{2\pi}} \int \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2} z^2\right)^n}{n!} \right] \mathrm{d}z \\
&= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\int \frac{(-1)^{-1}\, z^{2n}}{2^n \cdot n!} \mathrm{d}z \right] \\
&= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{(-1)^n \cdot z^{2n+1}}{2^n (2n+1)\, n!} \right]
\end{aligned}
$$

$$\alpha \times 90 \approx 8$$

$$\beta \times 10 = 3$$

$$1 - \alpha$$

$$\beta \times 10 = 3$$

$$\mathsf{FPR} = \alpha \times 90 \approx 8$$

$$1-\beta$$

$$\frac{8}{90} \approx 0.09$$

$$\frac{7}{7+6} \approx 0.5$$

α

$$
\begin{aligned}
\mathrm{E}\left(\mathsf{FPR}\right) &= \alpha; \\
\mathsf{FPR} &= \frac{FP}{N} \\
&= \frac{FP}{FN + TP} \\
&= \frac{8}{8 + 72} \\
&= 9\%
\end{aligned}
$$

α

α

p

α

α

p

α

p

$\geq$

α

$$\mathrm{FDR} = \frac{\mathrm{FP}}{\mathrm{TP}+\mathrm{FP}}$$

α

H_0

p

α

$$
\begin{aligned}
PPV &= \frac{TP}{TP + FP} \\
FDR &= \frac{FP}{TP + FN} \\
\alpha = \frac{FP}{N} &= \frac{TP}{TN + FP} \\
\beta = \frac{TP}{P} &= \frac{TP}{TP + FN}
\end{aligned}
$$

$n = 100$

$\bar{x}$

$H_0 \quad :$

H_a :

$n = 100$

$\sigma = 5.2$

$\mu = 10$

$$\bar{x} = 11$$

$$\mu_T rue = 12$$

$\alpha = 0.05$

$\beta = ??$

$\bar{x}_{\mathsf{crit}}$

$$\overline{X}$$

$$\overline{X} \sim \mathcal{N}\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$
\begin{aligned}
Z &= \frac{\overline{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \\
\implies \overline{x}_{crit} &= \mu + z_{\alpha} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) \\
\overline{x}_{crit} &= \mu + z_{0.05} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) \\
\overline{x}_{crit} &= \mu + 1.645 \cdot \left(\frac{5.2}{\sqrt{100}}\right) \\
&= 10.8554
\end{aligned}
$$

H_0

$$\alpha = 0.05$$

$$
\begin{aligned}
z &= \frac{\overline{x}_{crit} - \mu_{true}}{\left(\frac{\sigma}{\sqrt{n}}\right)} \\
&= \frac{10.86 - 12}{\frac{5.2}{10}} \qquad = -2.2
\end{aligned}
$$

β

$$
\begin{aligned}
\beta &= \mathrm{P}\left(\text{Type II Error}\right) \\
&= \mathrm{P}\left(H_0 \text{ is not rejected} \mid H_0 \text{ is false}\right) \\
&= \mathrm{P}\left(\mu_{\overline{X}_{\textbf{Crit}}} < \overline{x}_{\text{crit}} \mid \mu = 12\right) \\
&= 0.014
\end{aligned}
$$

$$\begin{aligned}
\text{Power} &= (H_0 \text{ is not rejected} \mid H_0 \text{ is false}) \\
&= \mathrm{P}\left(\mu_{\overline{X}_{\textbf{Crit}}} < \overline{x}_{\text{Crit}}\right) \\
&= 1 - \beta \\
&= 1 - 0.14 \\
&= 98.6\%
\end{aligned}$$

t

σ

$$s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

X_1;

μ_1

σ_1

$X_2;$

μ_2

σ_2

$$X_1 + X_2 = Y$$

$$\begin{aligned} \operatorname{var}(y) &= \operatorname{var}(X_1) + \operatorname{var}(X_2) \\ \sigma_y^2 &= \sigma_1^2 + \sigma_2^2 \end{aligned}$$

t

$$s_p = 505.09$$

$$\frac{3515.64 - 3260.29}{505.05 \times \sqrt{\frac{1}{742} + \frac{1}{484}}} = 8.653$$

$$t = \frac{162.7 - 154.04}{22.39 \times 22.39 \times \sqrt{\frac{1}{52} + \frac{1}{48}}}$$

t

$\overline{X}$

X

$$\overline{X} \sim \mathcal{N}\left(\mu\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

n

μ

$$\sigma_{\overline{x}} = \left(\frac{\sigma}{\sqrt{n}}\right)$$

$n \geq 30$