$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(u\cdot v\right)=\frac{\mathrm{d}\,u}{\mathrm{d}\,v}\cdot v+u\cdot\frac{\mathrm{d}\,v}{\mathrm{d}\,x} \tag{1.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(f\left(x\right)\cdot g\left(x\right)\right)=f'\left(x\right)\cdot g\left(x\right)+f\left(x\right)\cdot g'\left(x\right) \tag{1.2}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,y}{\mathrm{d}\,u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \tag{1.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(g\left(x\right)\right)\right] = f'\left(g\left(x\right)\right) \cdot g\left(x\right) \tag{1.4}$$

$$\int f\left(g\left(x\right)\right)\cdot g'\left(x\right)\mathrm{d}\,x=\int f\left(u\right)\mathrm{d}\,u \quad (1.5)$$

$$\int f\left(u\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\mathrm{d}\,x=\int f\left(u\right)\mathrm{d}\,u \quad (1.6)$$

$$u = f(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[\sin\left(x\right)\right] = \cos\left(x\right)$$

$$\mathrm{d}\,v = g'(x)\,\mathrm{d}\,x$$

v

$$\int u \,\mathrm{d}\, v = u \cdot v - \int v \,\mathrm{d}\, u \tag{1.7}$$

$$\begin{aligned} u &= g(x) \qquad F(x): \ \ F'(x) = f(x) = y \\ \frac{du}{dx} &= g'(x) \end{aligned} \tag{1.8}$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\,x}\left[F'\left(u\right)\right] &= F'\left(g\left(x\right)\right)\cdot g'\left(x\right) \\
&= f\left(g\left(x\right)\right)\cdot g'\left(x\right) \\
\implies f\left(g\left(x\right)\right)\cdot g'\left(x\right) &= \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)\right] \\
f\left(g\left(x\right)\right)\cdot g'\left(x\right) &= \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)+C\right]
\end{aligned}
\tag{1.9}
$$

$$
\begin{aligned}
\int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d}\, x &= \int \frac{\mathrm{d}}{\mathrm{d}\, x}\left[F\left(u\right) + C\right] \mathrm{d}\, x \\
&= F\left(u\right) + C \\
&= \int f\left(u\right) \mathrm{d}\, u
\end{aligned}
$$

$$\int f\left(g\left(x\right)\right)\cdot g'\left(x\right)\mathrm{d}\,x = \int f\left(u\right)\mathrm{d}\,u$$
$$\int f\left(u\right)\cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int f\left(u\right)\mathrm{d}\,u$$

(1.2)

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(x\right)\cdot g\left(x\right)\right]=f'\left(x\right)\cdot g\left(x\right)+f\left(x\right)\cdot g'\left(x\right) \tag{1.10}$$

$$\begin{array}{ll} u = f(x) & v = g(x) \\ \frac{\mathrm{d}\,u}{\mathrm{d}\,x} = f'(x) & \frac{\mathrm{d}\,v}{\mathrm{d}\,x} = g'(x) \end{array} \tag{1.11}$$

$$\int \left(\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot v + u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

$$\int \left(v \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right) \mathrm{d}\,x + \int \left(u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

(1.6)

$$\int v \,\mathrm{d}\, u + \int u \,\mathrm{d}\, v = u \cdot v$$

$$\int u \,\mathrm{d}\, v = u \cdot v - \int v \,\mathrm{d}\, u$$

$$\int [\ ]\, \mathrm{d}x$$

$$\left[f\left(u\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\right]=\left[f\left(g\left(x\right)\right)\cdot g'\left(x\right)\right]$$

$$\left[f\left(x\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\right]=\left[f\left(x\right)\cdot g'\left(x\right)\right]$$

$$\mathrm{d}\, y$$

$$\mathrm{d}\,x$$

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x}$$

$$\frac{\partial u}{\partial x}$$

$$\sum_{0}^{n}\left[a_0\left(x\right)\cdot\left(\frac{\mathrm{d}^n y}{\mathrm{d}\, x^n}\right)\right] \tag{1.12}$$

y

x

A differential equation of the form:

$$g(y) \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} = f(x) \tag{1.13}$$

Is a seperable Ordinary Differential Equation and has a solution:

$$\int g(y) \, \mathrm{d}\, y = \int f(x) \, \mathrm{d}\, x \tag{1.14}$$

$$
\begin{aligned}
& g\,(y) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\,(x) \\
\implies & \int g\,(y)\,\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int f\,(x)\,\mathrm{d}\,x
\end{aligned}
\tag{1.15}
$$

1.6

$$\int g(y)\,\mathrm{d}\,y = \int f(x)\,\mathrm{d}\,x \tag{1.16}$$

u

$$\mathrm{d}\,y_{\overline{\mathrm{d}\,x=f\left(\frac{x}{y}\right)}}$$

$$
\begin{aligned}
u &= \frac{y}{x} \\
\implies \quad y &= u \cdot x \\
\implies \quad \frac{\mathrm{d}\, y}{\mathrm{d}\, x} &= \frac{\mathrm{d}\, u}{\mathrm{d}\, x} \cdot x + (1) \cdot u
\end{aligned}
$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(\frac{y}{x}\right)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x + u = f\left(u\right)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = f\left(u\right) - u$$

$$\frac{1}{f\left(u\right) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = 1$$

$$\frac{1}{f\left(u\right) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f\left(u\right) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f\left(u\right) - u}\,\mathrm{d}\,u = \ln|x| + c \quad (1.17)$$

$$\exists G\left(u\right):G\left(u\right)=\int\frac{1}{f(u)-u}\,\mathrm{d}\,u$$

$$G(u) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) + \ln|x| + c = 0 \qquad (1.18)$$

$$\sum_{0}^{n} \left[a_n(x) \cdot f^{(n)}(x)\right] = g(x)$$

If $g(x) = 0$ it is said to be homogenous

$$a_1(x) \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} + a_0(x) \cdot y = g(x)$$

Where $a(x)$ is a function (1.19)

Linear First Order ODE:

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x) \tag{1.20}$$

if $f\,(x) = 0$ the equation is said to be homogenous

$$y_h$$

y_p

$$y_h : \qquad \frac{\mathrm{d}\, y_h}{\mathrm{d}\, x} + p\left(x\right) \cdot y_h = 0$$

$$y_p : \quad \frac{\mathrm{d}\, y_p}{\mathrm{d}\, x} + p\,(x) \cdot y_p = f\,(x)$$

1. Rewrite the Equation in the standard form:
$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+p\left(x\right)\cdot y=f\left(x\right)$$
2. Identify $p\left(x\right)$ and find the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}$$
3. Multiply through by the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}\left(\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+p\left(x\right)\cdot y\right)=e^{\int p(x)\,\mathrm{d}\,x}f$$
It may be concluded:
$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[e^{\int p(x)\,\mathrm{d}\,x\cdot}\cdot y\right]=e^{\int p(x)\,\mathrm{d}\,x}\cdot f\,($$
4. Integrate both sides in order to solve:

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = f\left(x\right) \tag{1.21}$$

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\,(x) \cdot y = 0 \qquad \Longrightarrow \; y = y_h \tag{1.22}$$

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\,(x) \cdot y = f(x) \Longrightarrow \; y = y_p \tag{1.23}$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\,x}\left(y_h + y_p\right) + p\left(x\right)\cdot\left(y_h + y_p\right) &= f\left(x\right) \\
\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right)\cdot y_h + p\left(x\right)\cdot y_p &= f\left(x\right) \\
\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\left(x\right)\cdot y_h + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right)\cdot y_p &= f\left(x\right) \\
0 + f\left(x\right) &= f\left(x\right)
\end{aligned}
\tag{1.24}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\,(x) \cdot y &= 0 \\
\frac{1}{y} \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} &= -p\,(x) \\
\ln |y| &= \int -p\,(x)\,\mathrm{d} + c \\
|y| &= e^{\int -p(x)x\,\mathrm{d}\, x} \cdot e^{c} \qquad (1.25)
\end{aligned}
$$

$$y > 0$$

$$\implies y_h = e^{-\int p(x)\,\mathrm{d}\,x} \cdot c$$

$$y_1 = e^{-\int p(x)\,\mathrm{d}\,x}$$

$$y_h = y_1(x) \cdot c \tag{1.26}$$

$$p\left(x\right)$$

$\mathcal{C}$

$$f\left(x\right)$$

$$c = u\left(x\right)$$

$$u(x)$$

$$
\begin{aligned}
y_p &= u(x) \times y_h(x) \\
&= e^{-\int p(x)\,\mathrm{d}\,x} \cdot u(x) \qquad (1.27)
\end{aligned}
$$

$$
\begin{aligned}
y_p &= e^{-\int p(x)\,\mathrm{d}\,x} \cdot u\left(x\right) \\
\frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right) \cdot &= f\left(x\right) \\
\frac{\mathrm{d}}{\mathrm{d}\,x}\left(u\left(x\right) \cdot y_1\left(x\right)\right) + p\left(x\right) u\left(x\right) y_1\left(x\right) &= f\left(x\right) \\
\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot y_1\left(x\right) + \frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} \cdot u\left(x\right) + p\left(x\right) \cdot u\left(x\right) \cdot y_1\left(x\right) &= f\left(x\right) \\
u\left(x\right)\left(\frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} + p\left(x\right) y_1\right) + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} \cdot y_1\left(x\right) &= f\left(x\right) \\
0 + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} - y_1\left(x\right) &= f\left(x\right) \\
\frac{\mathrm{d}\,y}{\mathrm{d}\,x} &= f\left(x\right)/y_1\left(x\right) \\
\int \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x &= \int f\left(x\right)/y_1\left(x\right)\mathrm{d}\,x \\
\int \mathrm{d}\,u &= \int f\left(x\right)/y_1\left(x\right)\mathrm{d}\,x \\
u &= \int f\left(x\right)/y_1\left(x\right)\mathrm{d}\,x
\end{aligned}
\tag{1.28}
$$

$$u = \int f(x) \cdot e^{\int p(x)\,\mathrm{d}\,x}\,\mathrm{d}\,x \tag{1.29}$$

$$y_p = u \cdot y_1$$

$$y_p = \frac{1}{y_1} \cdot \int f(x) \cdot e^{\int p(x)\,\mathrm{d}\,x}$$

$$y_p = e^{-\int p(x)\,\mathrm{d}\,x} \int f(x) \cdot e^{\int p(x)\,\mathrm{d}\,x} \quad (1.30)$$

$$e^{\int p(x)\,\mathrm{d}\,x}$$

$$e^{\int p(x)\,\mathrm{d}\,x} \cdot y_p = e^{\int p(x)\,\mathrm{d}\,x} \cdot e^{-\int p(x)\,\mathrm{d}\,x} \int f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}$$

$$e^{\int p(x)\,\mathrm{d}\,x} \cdot y_p = \int f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(e^{\int p(x)\,\mathrm{d}\,x} \cdot y_p\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left(\int f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}\right) = f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}$$

$$e^{\int p(x)\,\mathrm{d}\,x}\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x} \cdot y = e^{\int p(x)\,\mathrm{d}\,x} \cdot f\left(x\right)$$

$$\implies \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = f\left(x\right)$$

$$(x+1)\cdot\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+y=\ln\left(x\right)\ ;\qquad y(1)=10 \tag{1.31}$$

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + \frac{y}{x+1} = \frac{\ln(x)}{x+1} \quad : \qquad (x \in \mathbb{R} \setminus \{-1 \tag{1.32}$$

$$
\begin{aligned}
u &= e^{\int \frac{1}{x+1}\,\mathrm{d}\,x} \\
&= e^{\int \ln|x+1|\,\mathrm{d}\,x} \\
&= |x+1| \qquad (1.33)
\end{aligned}
$$

$$x > 0$$

$$
\begin{aligned}
&(x+1)\cdot\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+y=\ln(x)\\
\Longrightarrow\;\;&\frac{\mathrm{d}}{\mathrm{d}\,x}\left((x+1)\cdot y\right)=\ln(x)
\end{aligned}
\tag{1.34}
$$

$$\int \frac{\mathrm{d}}{\mathrm{d}\,x}\left[(x+1)\cdot y\right]\mathrm{d}\,x = \int \ln\left(x\right)\mathrm{d}\,x$$

$$(x+1)\cdot y=\int \ln{(x)}\,\mathrm{d}\,x \tag{1.35}$$

$$
\begin{aligned}
u &= \ln(x) & \mathrm{d}\,v &= \mathrm{d}\,x \\
\mathrm{d}\,u &= \frac{1}{x}\,\mathrm{d}\,x & v &= x
\end{aligned}
$$

$$
\implies \int u\,\mathrm{d}\,v = u \cdot v + \int v\,\mathrm{d}\,u
$$

$$
\begin{aligned}
(x+1)\cdot y &= \ln(x)\cdot x - \int \mathrm{d}\,x \\
&= x\cdot(\ln(x)-1)+c \\
\implies y &= \frac{x\cdot(\ln(x)-1+c)}{x+1}
\end{aligned}
$$

$$y(1) = 10$$

$$
\begin{aligned}
10 &= \frac{1\left(\ln\left(1\right) - 1 + c\right)}{2} \\
20 &= 1\left(0 - 1\right) + c \\
c &= 19
\end{aligned} \tag{1.36}
$$

$$\text{y}= \text{x}(\ln(x) - 1 + 19)\frac{}{x+1 \quad ; \qquad \forall x \in \mathbb{C} \setminus \{-1{,}0\}}$$