$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(u \cdot v\right) = \frac{\mathrm{d}\,u}{\mathrm{d}\,v} \cdot v + u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \tag{1.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(f\left(x\right) \cdot g\left(x\right)\right) = f'\left(x\right) \cdot g\left(x\right) + f\left(x\right) \cdot g'\left(x\right) \tag{1.2}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,y}{\mathrm{d}\,u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \tag{1.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(g\left(x\right)\right)\right] = f'\left(g\left(x\right)\right) \cdot g\left(x\right) \tag{1.4}$$

$$f\left(g\left(x\right)\right)\cdot g'\left(x\right)\mathrm{d}\,x = f\left(u\right)\mathrm{d}\,u \tag{1.5}$$

$$f\left(u\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\mathrm{d}\,x = f\left(u\right)\mathrm{d}\,u \tag{1.6}$$

$$u = f(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\,[\sin{(x)}] = \cos{(x)}$$

$$\mathrm{d}\, v = g'(x)\, \mathrm{d}\, x$$

v

$$\int u\, \mathrm{d}\, v = u \cdot v - \int v\, \mathrm{d}\, u \tag{1.7}$$

$$
\begin{aligned}
u &= gx \qquad Fx: \quad F'x = fx = y \\
\frac{du}{dx} &= g'x
\end{aligned}
\tag{1.8}
$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\,x}\left[F'(u)\right] &= F'(g(x)) \cdot g'(x) \\
&= f(g(x)) \cdot g'(x) \\
\implies f(g(x)) \cdot g'(x) &= \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F(u)\right] \\
f(g(x)) \cdot g'(x) &= \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F(u) + C\right]
\end{aligned}
\tag{1.9}
$$

$$
\begin{aligned}
f\left(g\left(x\right)\right)\cdot g'\left(x\right)\mathrm{d}\,x &= \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)+C\right]\mathrm{d}\,x \\
&= F\left(u\right)+C \\
&= f\left(u\right)\mathrm{d}\,u
\end{aligned}
$$

$$f\ (g\ (x))\ \cdot\ g'\ (x)\,\mathrm{d}\,x\ =\ \ f\ (u)\,\mathrm{d}\,u$$
$$f\ (u)\ \cdot\ \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\ \mathrm{d}\,x\ =\ \ f\ (u)\,\mathrm{d}\,u$$

(1.2)

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(x\right)\cdot g\left(x\right)\right] = f'\left(x\right)\cdot g\left(x\right) + f\left(x\right)\cdot g'\left(x\right) \tag{1.10}$$

$$
\begin{array}{ll}
u = f\left(x\right) & v = g\left(x\right) \\
\frac{\mathrm{d}\, u}{\mathrm{d}\, x} = f'\left(x\right) & \frac{\mathrm{d}\, v}{\mathrm{d}\, x} = g'\left(x\right)
\end{array}
\tag{1.11}
$$

$$\left(\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\cdot v + u\cdot\frac{\mathrm{d}\,v}{\mathrm{d}\,x}\right)\mathrm{d}\,x = u\cdot v$$

$$\left(v\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\right)\mathrm{d}\,x + \left(u\cdot\frac{\mathrm{d}\,v}{\mathrm{d}\,x}\right)\mathrm{d}\,x = u\cdot v$$

(1.6)

$$v\,\mathrm{d}\,u + \; u\,\mathrm{d}\,v = u \cdot v$$
$$u\,\mathrm{d}\,v = u \cdot v - \; v\,\mathrm{d}\,u$$

$$[\ ]\,\mathrm{d}x$$

$$\left[f\left(u\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\right]=\left[f\left(g\left(x\right)\right)\cdot g'\left(x\right)\right]$$

$$\left[f\left(x\right)\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\right]=\left[f\left(x\right)\cdot g'\left(x\right)\right]$$

$$\mathrm{d}\,y$$

$$\mathrm{d}\,x$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x}$$

$$\frac{\partial u}{\partial x}$$

$$\sum_{0}^{n}\left[a_0(x)\cdot\left(\frac{\mathrm{d}^n y}{\mathrm{d}x^n}\right)\right] \tag{1.12}$$

y

x

A differential equation of the form:

$$g\left(y\right) \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} = f\left(x\right) \tag{1.13}$$

Is a seperable Ordinary Differential Equation and has a solution:

$$g\left(y\right) \mathrm{d}\, y = \int f\left(x\right) \mathrm{d}\, x \tag{1.14}$$

$$
\begin{aligned}
g(y) \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} &= f(x) \\
\implies \quad g(y) \frac{\mathrm{d}\, y}{\mathrm{d}\, x} \mathrm{d}\, x &= f(x) \, \mathrm{d}\, x
\end{aligned}
\tag{1.15}
$$

1.6

$$g\left(y\right)\mathrm{d}\,y = f\left(x\right)\mathrm{d}\,x \tag{1.16}$$

u

$$\mathrm{d}\,y \frac{}{\mathrm{d}\,x = f\left(\frac{x}{y}\right)}$$

$$
\begin{aligned}
u &= \frac{y}{x} \\
\implies \quad y &= u \cdot x \\
\implies \quad \frac{\mathrm{d}\, y}{\mathrm{d}\, x} &= \frac{\mathrm{d}\, u}{\mathrm{d}\, x} \cdot x + (1) \cdot u
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\,y}{\mathrm{d}\,x} &= f\left(\frac{y}{x}\right) \\
\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\cdot x + u &= f\left(u\right) \\
\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\cdot x &= f\left(u\right) - u \\
\frac{1}{f\left(u\right)-u}\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\cdot x &= 1 \\
\frac{1}{f\left(u\right)-u}\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x} &= \frac{1}{x}\,\mathrm{d}\,x \\
\frac{1}{f\left(u\right)-u}\cdot\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x &= \frac{1}{x}\,\mathrm{d}\,x \\
\frac{1}{f\left(u\right)-u}\,\mathrm{d}\,u &= \ln\left|x\right| + c
\end{aligned}
\tag{1.17}
$$

$$\exists G\left(u\right):G\left(u\right)=\frac{1}{f\left(u\right)-u}\,\mathrm{d}\,u$$

$$G\left(u\right) = \ln\left|x\right| + c$$

$$G\left(\frac{y}{x}\right) = \ln\left|x\right| + c$$

$$G\left(\frac{y}{x}\right) + \ln\left|x\right| + c = 0 \tag{1.18}$$

$$ {}_{0}^{n}\left[a_n(x)\cdot f^{(n)}(x)\right] = g(x) $$

If $g\left(x\right) = 0$ it is said to be homogenous

$$a_1(x) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + a_0(x) \cdot y = g(x)$$

$$\text{Where } a(x) \text{ is a function} \tag{1.19}$$

Linear First Order ODE:

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = f\left(x\right) \tag{1.20}$$

if $f\left(x\right) = 0$ the equation is said to be homogenous

y_h

y_p

$$y_h : \qquad \frac{\mathrm{d}\, y_h}{\mathrm{d}\, x} + p\,(x) \cdot y_h = 0$$

$$y_p : \qquad \frac{\mathrm{d}\, y_p}{\mathrm{d}\, x} + p\,(x) \cdot y_p = f\,(x)$$

1. Rewrite the Equation in the standard form:

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\,(x) \cdot y = f\,(x)$$

2. Identify $p\,(x)$ and find the integrating factor:

$$e^{\,p(x)\,\mathrm{d}\,x}$$

3. Multiply through by the integrating factor:

$$e^{\,p(x)\,\mathrm{d}\,x}\left(\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\,(x) \cdot y\right) = e^{\,p(x)\,\mathrm{d}\,x} f\,(x)$$

It may be concluded:

$$\frac{\mathrm{d}}{\mathrm{d}\, x}\left[e^{\,p(x)\,\mathrm{d}\,x \cdot} \cdot y\right] = e^{\,p(x)\,\mathrm{d}\,x} \cdot f\,(x)$$

4. Integrate both sides in order to solve:

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x) \tag{1.21}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = 0 \quad \Longrightarrow \quad y = y_h \tag{1.22}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = fx \Longrightarrow \quad y = y_p \tag{1.23}$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\,x}\left(y_h + y_p\right) + p\left(x\right) \cdot \left(y_h + y_p\right) &= f\left(x\right) \\
\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right) \cdot y_h + p\left(x\right) \cdot y_p &= f\left(x\right) \\
\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\left(x\right) \cdot y_h + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right) \cdot y_p &= f\left(x\right) \\
0 + f\left(x\right) &= f\left(x\right)
\end{aligned}
\tag{1.24}
$$

$$\begin{aligned}\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x)\cdot y &= 0\\ \frac{1}{y}\cdot\frac{\mathrm{d}\,y}{\mathrm{d}\,x} &= -p\,(x)\\ \ln|y| &= \ -p\,(x)\,\mathrm{d} + c\\ |y| &= e^{-p\left(x\right)x\ \mathrm{d}\,x}\cdot e^{c}\end{aligned} \tag{1.25}$$

$$y > 0$$

$$\Longrightarrow \quad y_h = e^{-\ \ p(x)\,\mathrm{d}\,x} \cdot c$$

$$y_1 = e^{-\int p(x)\,\mathrm{d}\,x}$$

$$y_h = y_1 x \cdot c \tag{1.26}$$

$p(x)$

c

$$f\ (x)$$

$$c = u\left(x\right)$$

$$u\left(x\right)$$

$$
\begin{aligned}
y_p &= u\left(x\right) \times y_h\left(x\right) \\
&= e^{-\ p\left(x\right)\,\mathrm{d}\,x} \cdot u\left(x\right)
\end{aligned}
\tag{1.27}
$$

$$
\begin{aligned}
y_p &= e^{-\ p(x)\,\mathrm{d}\,x} \cdot u\,(x) \\
\frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\,(x) \cdot &= f\,(x) \\
\frac{\mathrm{d}}{\mathrm{d}\,x}\,(u\,(x) \cdot y_1\,(x)) + p\,(x)\,u\,(x)\,y_1\,(x) &= f\,(x) \\
\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot y_1\,(x) + \frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} \cdot u\,(x) + p\,(x) \cdot u\,(x) \cdot y_1\,(x) &= f\,(x) \\
u\,(x)\left(\frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} + p\,(x)\,y_1\right) + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} \cdot y_1\,(x) &= f\,(x) \\
0 + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} - y_1\,(x) &= f\,(x) \\
\frac{\mathrm{d}\,y}{\mathrm{d}\,x} &= f\,(x)\,y_1\,(x) \\
\frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x &= \ f\,(x)\,y_1\,(x)\,\mathrm{d}\,x \\
\mathrm{d}\,u &= \ f\,(x)\,y_1\,(x)\,\mathrm{d}\,x \\
u &= \ f\,(x)\,y_1\,(x)\,\mathrm{d}\,x
\end{aligned}
\tag{1.28}
$$

$$u = f\,(x) \cdot e^{\,p(x)\,\mathrm{d}\,x}\,\mathrm{d}\,x \tag{1.29}$$

$$y_p = u \cdot y_1$$

$$y_p = \frac{1}{y_1} \cdot \ f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x}$$

$$y_p = e^{-\ p(x)\,\mathrm{d}\,x}\ \ f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x} \tag{1.30}$$

$$e^{p(x)\,\mathrm{d}\,x}$$

$$\mathrm{e}^{\,p(x)\,\mathrm{d}\,x} \cdot y_p = e^{\,p(x)\,\mathrm{d}\,x} \cdot e^{-\ p(x)\,\mathrm{d}\,x}\ \ f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x}$$

$$e^{\,p(x)\,\mathrm{d}\,x} \cdot y_p = \ f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(e^{\,p(x)\,\mathrm{d}\,x} \cdot y_p\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left(\ f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x}\right)$$

$$= f\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x}$$

$$e^{\,p(x)\,\mathrm{d}\,x}\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\ (x) \cdot e^{\,p(x)\,\mathrm{d}\,x} \cdot y = e^{\,p(x)\,\mathrm{d}\,x} \cdot f\ (x)$$

$$\Longrightarrow\quad \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\ (x) \cdot y = f\ (x)$$

$$(x+1)\cdot\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+y=\ln(x) \quad ; \qquad y1=10 \tag{1.31}$$

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + \frac{y}{x+1} = \frac{\ln(x)}{x+1} \quad : \qquad \Big(x \in \mathbb{R} \setminus \{-1, 0\}\Big) \tag{1.32}$$

$$
\begin{aligned}
u &= e^{\frac{1}{x+1}\,\mathrm{d}\,x} \\
&= e^{\ln|x+1|\,\mathrm{d}\,x} \\
&= |x+1|
\end{aligned}
\tag{1.33}
$$

$$x > 0$$

$$
\begin{aligned}
&(x+1)\cdot\frac{\mathrm{d}\,y}{\mathrm{d}\,x}+y=\ln(x)\\
\implies\ &\frac{\mathrm{d}}{\mathrm{d}\,x}\left((x+1)\cdot y\right)=\ln(x)
\end{aligned}
\tag{1.34}
$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[(x+1)\cdot y\right]\mathrm{d}\,x = \ln(x)\,\mathrm{d}\,x$$

$$(x + 1) \cdot y = \ln(x) \, \mathrm{d}\, x \tag{1.35}$$

$$\begin{aligned} u &= \ln(x) & \mathrm{d}\,v &= \mathrm{d}\,x \\ \mathrm{d}\,u &= \frac{1}{x}\,\mathrm{d}\,x & v &= x \end{aligned}$$

$$\Longrightarrow \quad \int u\,\mathrm{d}\,v = u \cdot v + \int v\,\mathrm{d}\,u$$

$$
\begin{aligned}
(x+1)\cdot y &= \ln(x)\cdot x - \,\mathrm{d}\,x \\
&= x\cdot(\ln(x)-1)+c \\
\implies y &= \frac{x\cdot(\ln(x)-1+c)}{x+1}
\end{aligned}
$$

$$y1 = 10$$

$$
\begin{aligned}
10 &= \frac{1\left(\ln\left(1\right) - 1 + c\right)}{2} \\
20 &= 1\left(0 - 1\right) + c \\
c &= 19
\end{aligned} \tag{1.36}
$$

$$y = x(\ln(x) - 1 + 19)\frac{}{x+1 \quad ; \qquad \forall x \in \mathbb{C}\setminus\{-1,0\}}$$