$$\frac{\mathrm{d}}{\mathrm{d}x}(u \cdot v) = \frac{\mathrm{d}u}{\mathrm{d}v} \cdot v + u \cdot \frac{\mathrm{d}v}{\mathrm{d}x} \qquad (1.1)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \qquad (1.2)$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,y}{\mathrm{d}\,u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \qquad (1.3)$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[f\left(g\left(x\right)\right)\right] = f'\left(g\left(x\right)\right) \cdot g\left(x\right) \qquad (1.4)$$

$$\int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u \qquad \textbf{(1.5)}$$

$$\int f\left(u\right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u \qquad \textbf{(1.6)}$$

$$u = f\left(x\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin\left(x\right)\right] = \cos\left(x\right)$$

$$\mathrm{d}\,v = g'\,(x)\,\mathrm{d}\,x$$

$$\int u \, \mathrm{d}\, v = u \cdot v - \int v \, \mathrm{d}\, u \qquad (1.7)$$

$$
\begin{aligned}
u &= g(x) \qquad F(x): \ F'(x) = f(x) = y \\
\frac{du}{dx} &= g'(x)
\end{aligned}
\tag{1.8}
$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[F'\left(u\right)\right] = F'\left(g\left(x\right)\right) \cdot g'\left(x\right)$$

$$= f\left(g\left(x\right)\right) \cdot g'\left(x\right)$$

$$\implies f\left(g\left(x\right)\right) \cdot g'\left(x\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)\right]$$

$$f\left(g\left(x\right)\right) \cdot g'\left(x\right) = \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right) + C\right]$$

$$(1.9)$$

$$\int f\left(g\left(x\right)\right)\cdot g'\left(x\right)\mathrm{d}\,x = \int \frac{\mathrm{d}}{\mathrm{d}\,x}\left[F\left(u\right)+C\right]\mathrm{d}\,x$$
$$= F\left(u\right)+C$$
$$= \int f\left(u\right)\mathrm{d}\,u$$

$$\int f\left(g\left(x\right)\right) \cdot g'\left(x\right) \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u$$
$$\int f\left(u\right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \mathrm{d}\,x = \int f\left(u\right) \mathrm{d}\,u$$

(1.2)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left[f(x) \cdot g(x) \right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

(1.10)

$$u = f(x) \qquad v = g(x)$$
$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} = f'(x) \qquad \frac{\mathrm{d}\,v}{\mathrm{d}\,x} = g'(x)$$

(1.11)

$$\int \left(\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot v + u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

$$\int \left(v \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right) \mathrm{d}\,x + \int \left(u \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,x} \right) \mathrm{d}\,x = u \cdot v$$

$$(1.6)$$

$$\int v \, \mathrm{d}\, u + \int u \, \mathrm{d}\, v = u \cdot v$$
$$\int u \, \mathrm{d}\, v = u \cdot v - \int v \, \mathrm{d}\, u$$

$$\int [\quad] \, dx$$

$$\left[f\left(u \right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right] = \left[f\left(g\left(x \right) \right) \cdot g'\left(x \right) \right]$$

$$\left[f\left(x\right) \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \right] = \left[f\left(x\right) \cdot g'\left(x\right) \right]$$

$\mathrm{d}\,y$

$$\mathrm{d}\,x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\frac{\partial u}{\partial x}$$

$$\sum_0^n \left[a_0\left(x\right) \cdot \left(\frac{\mathrm{d}^n y}{\mathrm{d}\, x^n} \right) \right] \tag{1.12}$$

y

x

A differential equation of the form:

$$g(y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \tag{1.13}$$

Is a seperable Ordinary Differential Equation and has a solution:

$$\int g(y)\,\mathrm{d}y = \int f(x)\,\mathrm{d}x \tag{1.14}$$

$$g\left(y\right) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(x\right)$$

$$\implies \int g\left(y\right) \frac{\mathrm{d}\,y}{\mathrm{d}\,x} \,\mathrm{d}\,x = \int f\left(x\right) \mathrm{d}\,x$$

$$(1.15)$$

1.6

$$\int g\left(y\right)\mathrm{d}\,y = \int f\left(x\right)\mathrm{d}\,x \qquad (1.16)$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\Big|_{x=f\left(\frac{x}{y}\right)}$$

$$u = \frac{y}{x}$$
$$\implies y = u \cdot x$$
$$\implies \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x + (1) \cdot u$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f\left(\frac{y}{x}\right)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x + u = f(u)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = f(u) - u$$

$$\frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot x = 1$$

$$\frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x} = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f(u) - u} \cdot \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int \frac{1}{x}\,\mathrm{d}\,x$$

$$\int \frac{1}{f(u) - u}\,\mathrm{d}\,u = \ln|x| + c \qquad (1.17)$$

$$\exists G\left(u\right) : G\left(u\right) = \int \frac{1}{f(u)-u} \, \mathrm{d}u$$

$$G(u) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) = \ln|x| + c$$

$$G\left(\frac{y}{x}\right) + \ln|x| + c = 0 \qquad (1.18)$$

$$\sum_{0}^{n} \left[a_n \left(x \right) \cdot f^{(n)} \left(x \right) \right] = g \left(x \right)$$

If $g \left(x \right) = 0$ it is said to be homogenous

$$a_1\left(x\right) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + a_0\left(x\right) \cdot y = g\left(x\right)$$

Where $a\left(x\right)$ is a function $\qquad (1.19)$

Linear First Order ODE:
$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x) \qquad (1.20)$$
if $f\,(x) = 0$ the equation is said to be homogenous

y_h

y_p

$$y_h: \qquad \frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\left(x\right) \cdot y_h = 0$$

$y_p:$ $\quad \dfrac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\,(x) \cdot y_p = f\,(x)$

1. Rewrite the Equation in the standard form:
$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f\,(x)$$
2. Identify $p\,(x)$ and find the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}$$
3. Multiply through by the integrating factor:
$$e^{\int p(x)\,\mathrm{d}\,x}\left(\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x)\cdot y\right) = e^{\int p(x)\,\mathrm{d}\,x}f\,(x)$$
   It may be concluded:
$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left[e^{\int p(x)\,\mathrm{d}\,x\cdot}\cdot y\right] = e^{\int p(x)\,\mathrm{d}\,x}\cdot f\,(x)$$
4. Integrate both sides in order to solve:

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, x} + p\left(x\right) \cdot y = f\left(x\right) \qquad (1.21)$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = 0 \quad \implies \quad y = y_h \qquad \text{(1.22)}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\,(x) \cdot y = f(x) \implies y = y_p \qquad \text{(1.23)}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(y_h + y_p\right) + p\left(x\right)\cdot\left(y_h + y_p\right) = f\left(x\right)$$

$$\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right)\cdot y_h + p\left(x\right)\cdot y_p = f\left(x\right)$$

$$\frac{\mathrm{d}\,y_h}{\mathrm{d}\,x} + p\left(x\right)\cdot y_h + \frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p\left(x\right)\cdot y_p = f\left(x\right)$$

$$0 + f\left(x\right) = f\left(x\right) \tag{1.24}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + p\left(x\right) \cdot y = 0$$

$$\frac{1}{y} \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = -p\left(x\right)$$

$$\ln|y| = \int -p\left(x\right)\mathrm{d} + c$$

$$|y| = e^{\int -p(x)x\,\mathrm{d}\,x} \cdot e^{c} \tag{1.25}$$

$$y > 0$$

$$\implies y_h = e^{-\int p(x)\, \mathrm{d}x} \cdot c$$

$$y_1 = e^{-\int p(x)\,\mathrm{d}x}$$

$$y_h = y_1(x) \cdot c \qquad (1.26)$$

$p(x)$

c

$f\left(x\right)$

$$c = u\left(x\right)$$

$u\left(x\right)$

$$y_p = u(x) \times y_h(x)$$
$$= e^{-\int p(x)\,\mathrm{d}x} \cdot u(x) \qquad (1.27)$$

$$y_p = e^{-\int p(x)\,\mathrm{d}x} \cdot u(x)$$

$$\frac{\mathrm{d}\,y_p}{\mathrm{d}\,x} + p(x) \cdot = f(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\left(u(x) \cdot y_1(x)\right) + p(x)\,u(x)\,y_1(x) = f(x)$$

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,x} \cdot y_1(x) + \frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} \cdot u(x) + p(x) \cdot u(x) \cdot y_1(x) = f(x)$$

$$u(x)\left(\frac{\mathrm{d}\,y_1}{\mathrm{d}\,x} + p(x)\,y_1\right) + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} \cdot y_1(x) = f(x)$$

$$0 + \frac{\mathrm{d}\,y}{\mathrm{d}\,x} - y_1(x) = f(x)$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = f(x)\,/y_1(x)$$

$$\int \frac{\mathrm{d}\,u}{\mathrm{d}\,x}\,\mathrm{d}\,x = \int f(x)\,/y_1(x)\,\mathrm{d}\,x$$

$$\int \mathrm{d}\,u = \int f(x)\,/y_1(x)\,\mathrm{d}\,x$$

$$u = \int f(x)\,/y_1(x)\,\mathrm{d}\,x$$

$$(1.28)$$

$$u = \int f\left(x\right) \cdot e^{\int p(x)\,\mathrm{d}\,x}\,\mathrm{d}\,x \qquad (1.29)$$

$$y_p = u \cdot y_1$$

$$y_p = \frac{1}{y_1} \cdot \int f(x) \cdot e^{\int p(x) \, dx}$$

$$y_p = e^{-\int p(x) \, dx} \int f(x) \cdot e^{\int p(x) \, dx} \tag{1.30}$$

$$e^{\int p(x)\, \mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x} \cdot y_p = e^{\int p(x)\,\mathrm{d}x} \cdot e^{-\int p(x)\,\mathrm{d}x} \int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x} \cdot y_p = \int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\int p(x)\,\mathrm{d}x} \cdot y_p\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\int f(x) \cdot e^{\int p(x)\,\mathrm{d}x}\right)$$

$$= f(x) \cdot e^{\int p(x)\,\mathrm{d}x}$$

$$e^{\int p(x)\,\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}x} + p(x) \cdot e^{\int p(x)\,\mathrm{d}x} \cdot y = e^{\int p(x)\,\mathrm{d}x} \cdot f(x)$$

$$\implies \frac{\mathrm{d}y}{\mathrm{d}x} + p(x) \cdot y = f(x)$$

$$(x + 1) \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,x} + y = \ln{(x)} \;; \qquad y(1) = 10 \qquad \text{(1.31)}$$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} + \frac{y}{x+1} = \frac{\ln(x)}{x+1} \quad : \qquad (x \in \mathbb{R} \setminus \{-1, 0\}) \quad \text{(1.32)}$$

$$u = e^{\int \frac{1}{x+1} \, \mathrm{d}x}$$
$$= e^{\int \ln|x+1| \, \mathrm{d}x}$$
$$= |x+1| \qquad (1.33)$$

$x > 0$

$$(x + 1) \cdot \frac{\mathrm{d}\, y}{\mathrm{d}\, x} + y = \ln(x)$$

$$\implies \frac{\mathrm{d}}{\mathrm{d}\, x}\left((x + 1) \cdot y\right) = \ln(x)$$

<div align="right">(1.34)</div>

$$\int \frac{\mathrm{d}}{\mathrm{d}\,x}\left[(x+1)\cdot y\right]\mathrm{d}\,x = \int \ln\left(x\right)\mathrm{d}\,x$$

$$(x + 1) \cdot y = \int \ln (x) \, \mathrm{d} x$$

$$(1.35)$$

$$
\begin{aligned}
u &= \ln(x) & \mathrm{d}\,v &= \mathrm{d}\,x \\
\mathrm{d}\,u &= \frac{1}{x}\,\mathrm{d}\,x & v &= x
\end{aligned}
$$

$$
\implies \int u\,\mathrm{d}\,v = u \cdot v + \int v\,\mathrm{d}\,u
$$

$$(x + 1) \cdot y = \ln (x) \cdot x - \int \mathrm{d} x$$
$$= x \cdot (\ln (x) - 1) + c$$
$$\implies y = \frac{x \cdot (\ln (x) - 1 + c)}{x + 1}$$

$y(1) = 10$

$$10 = \frac{1\left(\ln\left(1\right) - 1 + c\right)}{2}$$
$$20 = 1\left(0 - 1\right) + c$$
$$c = 19 \qquad (1.36)$$

$$y = x(\ln(x) - 1 + 19)\frac{}{x+1} \quad ; \qquad \forall x \in \mathbb{C} \setminus \{-1, 0\}$$