$$w : \mathbb{R} \to \mathbb{C}$$

$$w(t) = u(t) + i \cdot v(t)$$

u

v

$$w = f(z)$$

$$f'(z) = \frac{\mathrm{d}\,w}{\mathrm{d}\,z}$$

$\mathcal{Z}$

$$g\left(x,y\right)=u\left(x,y\right)+i\cdot v\left(x,y\right)$$

$$f'(z)$$

$$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}$$

$$= \frac{dw}{dz}$$

$$w = u\left(t\right) + i \cdot v\left(t\right)$$

$$w'(t) = \frac{\mathrm{d}\,u}{\mathrm{d}\,t} + i \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,t}$$

$$\int_a^b (w(t))\,\mathrm{d}\,t = \int_a^b (u)\,\mathrm{d}\,t + i \cdot \int_a^b (v)\,\mathrm{d}\,t$$

$$\int_{a}^{b} \left(w\left(t\right)\right) \mathrm{d}\, t = \left[W\left(t\right)\right]_{a}^{b}$$

$$W(t) = U(t) + i \cdot V(t)$$

$$w(t)$$

$$\int_{a}^{b} (w(t))\,\mathrm{d}\,t$$

$$= \left[U\left(t\right)\right]_a^b + i\cdot\left[v\left(t\right)\right]_a^b$$

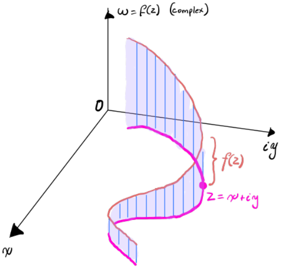
$$= [U(b) + i \cdot V(b)] - [U(a) + i \cdot V(a)]$$

$$= [W(b) - W(a)]$$

$$= [W(t)]_a^b$$

$$\lim_{n \to \infty} \left(\sum_{i=1}^{n} \left[f\left(z_i^*\right) \cdot \Delta z \right] \right)$$

Δz



ω = f(z) (complex)
0
iy
f(z)
z = x + iy
x

$$z = (x, y)$$

C

$$x = x(t), \quad y = y(t) \qquad (a \leq t \leq b)$$

$$x\left(t\right)$$

$y\left(t\right)$

t

Z

$$= z(t)$$

$$= x(t) + i \cdot y(t)$$

$$\int_C f(z)\,\mathrm{d}\,z$$

$$\int_C f(z)\,\mathrm{d}\,z = \int_{z_1}^{z_2} f(z)\,\mathrm{d}\,z$$

$$\int_C \left(f\left(z\right)\right) \mathrm{d}\, z = \int_a^b \left(f\left[z\left(t\right)\right] \cdot z'\left(t\right)\right) \mathrm{d}\, t$$

$$-C$$

$$\int_{-C} (f(z))\, \mathrm{d}\, z$$

$$= \int_{-b}^{-a} \left(f\left[z\left(-t\right)\right] \cdot \frac{\mathrm{d}}{\mathrm{d}\,t}\left(z\left(-t\right)\right) \right) \mathrm{d}\,t$$

$$= -\int_a^b \left(f\left[z\left(-t\right)\right] z'\left(-t\right)\right) \mathrm{d}\, t$$

$$= -\int_C (f(z))\, \mathrm{d}\, z$$

$$\oint f(z)\,\mathrm{d}z$$

$$|\alpha + \beta| \leq |\alpha| + |\beta|$$

$$\implies \left| \int (w(t)) \, \mathrm{d}\, t \right|$$

$$\leq \int |w(t|) \, \mathrm{d}\, t$$

L

$$|f(z)| \leq M$$

$$\left|\int (f(z))\,\mathrm{d}z\right| \leq M \cdot L$$

$f(z)$

$$\int_C \left(\frac{1}{z}\right) \mathrm{d}\, z$$

$$C : |z| = 1$$

$$= \int_{\pi}^{\pi} \left(\frac{1}{e^{i\theta}} \cdot \frac{\mathrm{d}}{\mathrm{d}\,\theta} \left(e^{i\theta} \right) \right) \mathrm{d}\,\theta$$

$$= \int_{\pi}^{\pi} \left(\frac{1}{e^{i\theta}} \cdot i \cdot e^{i\theta} \right) \mathrm{d}\,\theta$$

$$= \int_{\pi}^{\pi} (i)\, \mathrm{d}\,\theta$$

$$= [i \cdot \theta]_{\pi}^{\pi}$$

$$= 0$$

$$F(z)$$

$$F'(z) = f(z)$$

$(+C)$

$$\int_C f(z)\,\mathrm{d}\,z = \int_{z_1}^{z_2} f(z)\,\mathrm{d}\,z = \left[F(z)\right]_{z_1}^{z_2}$$

$$F'(z) = f(z)$$

$$= \int_a^b f\left(z\left(t\right)\right) \mathrm{d}\, t$$

$$= \int_a^b U\left(z\left(t\right)\right) \mathrm{d}\, t + i \cdot \int_a^b b\left(z\left(t\right)\right) \mathrm{d}\, t$$

$$= F\left(z\left(b\right)\right) - F\left(z\left(a\right)\right)$$

$$= F\left(z_2\right) - F\left(z_1\right)$$

$$= [F(z)]_{z_1}^{z_2}$$

$$= [U(t)]_a^b + i \cdot [V(t)]_a^b$$

$$= U(b) - U(a) + i \cdot [V(b) - V(a)]$$

$$= [U(b) + V(b)] - i \cdot [U(a) - V(b)]$$

$$= F(b) - F(a)$$

$$= \left[F\left(z\right)\right]_{a}^{b}$$

f

$$\oint_C f(z)\,\mathrm{d}\,z = 0$$

$$F(b) - F(a)$$

$c_1, c_2, c_3 \ldots$

$$\int_C f(z)\,\mathrm{d}\,z + \sum_{n=1}^{k} \left[\int_{c_n} f(z)\,\mathrm{d}\,z \right]$$

$$\int_{C_1} f(z)\,\mathrm{d}\,z + \int_{c_2} f(z)\,\mathrm{d}\,z$$

$$\int_{C_1} f(z)\,\mathrm{d}\,z - \int_{c_2} f(z)\,\mathrm{d}\,z$$

$$\int_{C_1} f(z)\,\mathrm{d}\,z$$

$$= \int_{c_2} f(z)\,\mathrm{d}\,z$$

c_2

C_1

$$\oint_{|z-z_0|=r} (z-z_0)^n \,\mathrm{d}\, z = \begin{cases} 0 & if n \neq -1 \\ 2\pi i & \quad if n = -12 \end{cases}$$

$$\int_{|z|=1} \frac{1}{z^2+2z+2}\,\mathrm{d}\,z$$

$$= \int_{|z|=1} \frac{1}{(z + (1+i)) \cdot (z + (1-i))} \, \mathrm{d}\, z$$

$$z = -1 \pm i$$

z_0

$$\int_C \frac{f(z)}{z - z_0} \, \mathrm{d}\, z = 2\pi i \cdot f(z_0)$$

$$\int_{|z|=1} \frac{\cos z}{z^3 + 9z} \, \mathrm{d}\, z$$

$$\int_{|z|=1} \frac{\cos z}{z^3 + 9z} \,\mathrm{d}\, z = \int_{|z|=1} \frac{1}{z} \cdot \frac{\cos z}{z^2 + 9z} \,\mathrm{d}\, z$$

$$\frac{\cos z}{z^3+9z}$$

$$z = 0$$

$$z = 3i$$

$$z = -3i$$

$$z_0 = 0$$

$$f(z) = \frac{\cos z}{z^2+9}$$

$$\int_{|z|=1} \frac{\frac{\cos z}{z^2+9}}{(z-0)}\,\mathrm{d}\,z$$

$$= 2\pi i \cdot \frac{\cos(0)}{0^2 + 9}$$

$$= i \cdot \frac{2\pi}{9}$$

$$\int_C \frac{f(z)}{(z-z_0)^{n+1}}\,\mathrm{d}\,z = f^n(z_0)\cdot\frac{2\pi i}{n!}$$

The derivative is defined as:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta x}{\Delta y}\right) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right) = \lim_{x \to a}\left(\frac{f(x)-f(a)}{x-a}\right)$$

a function will be differentiable at a if $\left.\frac{dy}{dx}\right|_{x=a}$ exists

if a function is differentiable at a it must hence be continuous

if the derivative is a continuous function, the derivative will exist for all x.

A continuous function will be any function:

$$\lim_{x \to a}\left(f(x)\right) = f(a) \quad \forall x \in \mathbb{R}$$

This can be restated as 3 statements that are, in conjunction, equivalent

all of these must be satisfied

1. $f(a)$ exists
2. $\lim_{x \to a}\left(f(x)\right)$ exists ⟶ this implies: $\lim_{x \to a^-}\left(f(x)\right)$ exists, $\lim_{x \to a^+}\left(f(x)\right)$ exists, $\lim_{x \to a}\left(f(x)\right) = f(a)$
3. $\lim_{x \to a}\left(f(x)\right) = f(a)$

The derivative value of some function at some x-value is the limit of the slope of the secant line from x to $(x+\Delta x)$ as Δx approaches zero.