TODO Overheads

Gitignore Quiz

Put the Quizz in a *.gitignore file here* Clear the bad lines

TODO Install Emacs Application Framework

This is going to be necessary to deal with not just equations but links, tables and other quirks

Install it from here

The reason for this is that generating latex preview fragments is just far too slow to be useful in any meaningful fashion.

TODO Install a live preview for equations in org-mode

Here is one example but there was a better one I was using

TODO [#C] Fix MkDocs for New ‘MD‘ Structure

TODO [#C] Put all RMD HTML Files on a GitPage

Look at Using Bookdown rather than mkdocs for this?

Unit Information

- Learning Guide
  - Zoom Tutorial
  - Zoom Lecture

Deriving the Normal Distribution

Power Series Series

A function f :

$$f(j) = \sum_{i=0}^{\infty} \left[C_n (z-a)^n \right], \ \exists z \in \mathbb{C}$$

$$f(z) = \sum_{i=0}^{\infty} \left[C_n (z-a)^n \right], \ \exists z \in \mathbb{C}$$

Is a Power Series a and will either:

- Converge only for $x = a$,

- converge $\forall x$
- converge in the circle $|z-a|<R$

Example

Take some function equal to the following power series:

$f(x)=\sum_{n=0}^{\infty}[n!\cdot x^n]$

Because the terms inside the power series has a factorial the only test that will work is the limit ratio test so we use that to evaluate convergence. [1]

let $a_n=n!\cdot x^n$:

$$\begin{aligned}\frac{\lim_{n\to\infty}|a_{n+1}|}{\lim_{n\to\infty}|a_n|}&=\lim_{n\to\infty}\left|\frac{(n+1)!\cdot x^n\cdot x}{n!\cdot x^n}\right|\\&=(n+1)\cdot|x|\\&=0 \iff x=0\end{aligned}$$

$\therefore$ The power series converges if and only $x=0$.

Representing a function as a Power Series

Ordinary functions can be represented as power series, this can be useful to deal with integrals that don't have an elementary anti-derivative.

1. Geometric Series

   First take the Series:

$$\begin{aligned}S_n&=\sum_{n=0}^{n}r^k\\&=1+r+r^2+r^3\ldots+r^{n-1}+r^n\\\implies r\cdot S_n&=r+r^2+r^3+r^4\ldots r^n+r^{n+1}\\\implies S_n-r\cdot C_n&=1+r^{n+1}\\\implies S_n&=\frac{1+r^{n+1}}{1-r}\end{aligned}$$

   So now consider the geometric series:

[1] Refer to Solving Series Strategy

$$
\begin{aligned}
\sum_{k=0}^{\infty}\left[x^k\right] &= \lim_{n\to\infty}\left[\sum_{k=0}^{n} x^k\right] \\
&= \lim_{n\to\infty}\left[\frac{1+x^{n+1}}{1-x}\right] \\
&= \frac{1+\lim_{n\to\infty}\left[x^{n+1}\right]}{1-x} \\
&= \frac{1+0}{1-x} \\
&= \frac{1}{1-x}
\end{aligned}
$$

2. Using The Geometric Series to Create a Power Series

   Take for example the function:

$$g\left(x\right) = \frac{1}{1+x^2}$$

   This could be represented as a power series by observing that:

$$\frac{1}{1-\#_1} = \sum_{n=0}^{\infty}\left[\#_1^n\right]$$

   And then simply putting in the value of $\#_1 = \left(-x^2\right)$:

$$\frac{1}{1-\left(-x^2\right)} = \sum_{n=0}^{\infty}\left[\left(-x^2\right)^n\right]$$

Calculus Rules and Series

The laws of differentiation allow the following relationships:

1. Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sum_{n=1}^{\infty} c_n\left(z-a\right)^n\right) = \sum_{n=1}^{\infty}\left[\frac{\mathrm{d}}{\mathrm{d}x}\left(c_n\left(z-a\right)^n\right)\right]$$

2. Integration

$$\int\left(\sum_{n=1}^{\infty} c_n\left(z-a\right)^n\right)\mathrm{d}x = \sum_{n=1}^{\infty}\left[c_n\left(z-a\right)^n\right]$$

Taylor Series

This is the important one, the idea being that you can use this to easily represent any function as an infinite series:

Consider the pattern formed by taking derivatives of $f(z) = \sum_{n=1}^{\infty} c_n (z-a)^n$:

$$\begin{aligned}
f(z) &= c_0 + c_1 (x-a) + c_2 (x-a)^2 + c_3 (x-a)^3 + ... \\
&\implies f(a) = c_0 \\
f'(z) &= c_1 + 2c_2 (z-a) + 3c_3 (z-a)^2 + 4c_4 (z-a)^3 \\
&\implies f'(a) = c_1 \\
f''(z) &= 2c_2 + 3 \times 2 \times c_3 (z-a) + 4 \times 3c_4 (z-a)^2 + ... \\
&\implies f''(a) = 2 \cdot c_2 \\
f'''(z) &= 3 \times 2 \times 1 \cdot c_3 + 4 \times 3 \times 2c_4 (z-a) + ... \\
&\implies f'''(a) = 3! c_3
\end{aligned}$$

Following this pattern forward:

$$\begin{aligned}
f^{(n)}(a) &= n! \cdot c_n \\
\implies c_n &= \frac{f^{(n)}(a)}{n!}
\end{aligned}$$

Hence, if there exists a power series to represent the function f, then it must be:

$$f(z) = \sum_{n=0}^{\infty} \left[\frac{f^{(n)}(a)}{n!} (x-a)^n \right]$$

If the power series is centred around 0, it is then called a *Mclaurin Series*.

1. Power Series Expansion of e

$$\begin{aligned}
f(z) = e^z &= \sum_{n=0}^{\infty} \left[\frac{f^{(n)}(0)}{n!} \cdot x^n \right] \\
&= \sum_{n=0}^{\infty} \left[\frac{e^0}{n!} x^n \right] \\
&= \sum_{n=0}^{\infty} \left[\frac{x^n}{n!} \right]
\end{aligned}$$

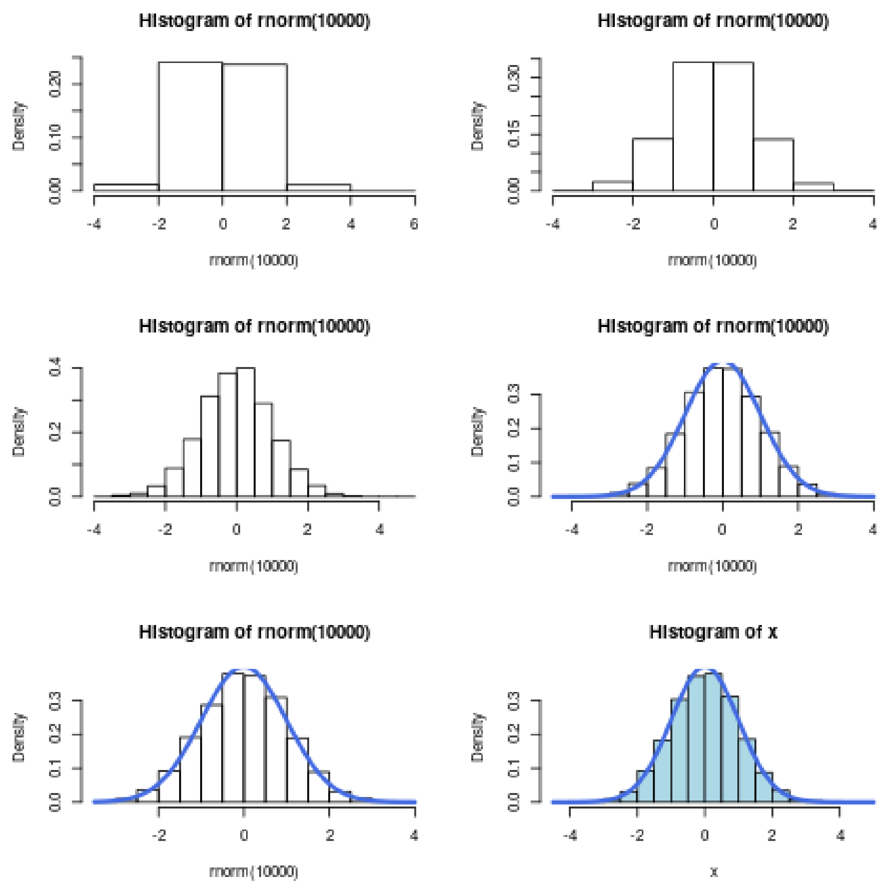
Modelling Normal Distribution

The Normal Distribution is a probability density function that is essentially modelled after observation.[fn:5]

what is the y-axis in a Density curve? ggplot2 ATTACH

Consider a histogram of some continuous normally distributed data:

```r
# layout(mat = matrix(1:6, nrow = 3))
layout(matrix(1:6, 3, 2, byrow = TRUE))


 x <- rnorm(10000, mean = 0, sd = 1)
sd(x)
hist(rnorm(10000), breaks = 5, freq = FALSE)
## curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")

hist(rnorm(10000), breaks = 10, freq = FALSE)
## curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")

hist(rnorm(10000), breaks = 15, freq = FALSE)
##  curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")

hist(rnorm(10000), breaks = 20, freq = FALSE)
curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")

hist(rnorm(10000), breaks = 25, freq = FALSE)
curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")

hist(x, breaks = 30, freq = FALSE, col = "lightblue")
curve(dnorm(x, 0, 1), add = TRUE, lwd = 3, col = "royalblue")
```



(Or in ggplot2) as described in listing *ggphist* and shown in figure *gghist*

```
library(tidyverse)
library(gridExtra)
x <- rnorm(10000)
x <- tibble::enframe(x)
head(x)
PlotList <- list()
for (i in seq(from = 5, to = 30, by = 5)) {
  PlotList[[i/5]] <- ggplot(data = x, mapping = aes(x = value)) +
    geom_histogram(aes(y = ..density..), col = "royalblue", fill = '
    stat_function(fun = dnorm, args = list(mean = 0, sd = 1))+
    theme_classic()
}

# arrangeGrob(grobs = PlotList, layout_matrix = matrix(1:6, nrow = 3
grid.arrange(grobs = PlotList, layout_matrix = matrix(1:6, nrow = 3)
```

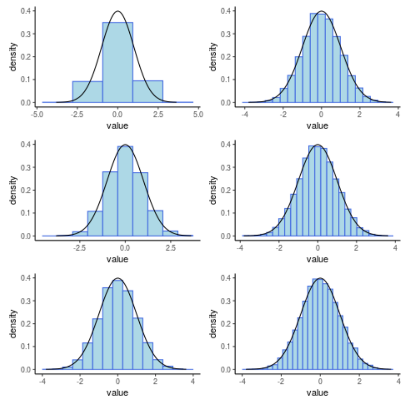


Figure 1: Histograms Generated in ggplot2

Observe that the outline of the frequencies can be made arbitrarily close to a curve given that the bin-width is made sufficiently small. This curve, known as the probability density function, represents the frequency of observation around that value, or more accurately the area beneath the curve around that point on the x-axis will be the probability of observing values within that corresponding interval.

Strictly speaking the curve is the rate of change of the probability at that point as well.

Defining the Normal Distribution

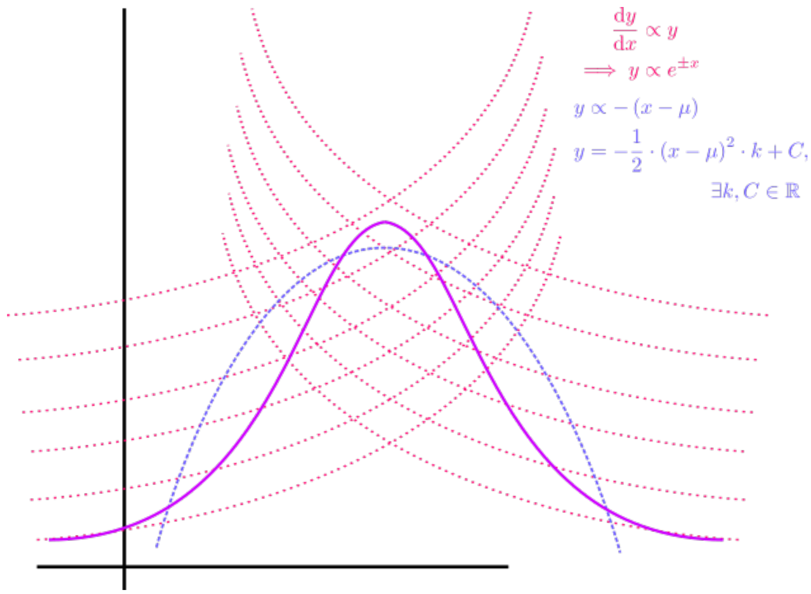
Data are said to be normally distributed if, the plot of the frequency density curve is such that:

- The rate of change is proportional to:
  - The distance of the score from the mean
    - $\frac{\mathrm{d}}{\mathrm{d}x}(f) \propto -(x-\mu)$
  - The frequencies themselves.
    - $\frac{\mathrm{d}}{\mathrm{d}x} \propto f$

If the Normal Distribution was only proportional to the distance from the mean (i.e. $(x \propto -(x-\mu))$ the model would be a parabola that dips below zero, as shown in parabolExamp, so it is necessary to provide the restriction that the rate of change is also proportional to the frequency (i.e. $y \propto y$).

let f be the frequency of observation around x, following these rules the plot would come to look something like figure *supcu*:

Bell Curve



Modelling only distance from the mean

If we presumed the frequency (which we will call f on the y-axis) was proportional only to the distance from the mean the model would be a parabola:

$$\begin{aligned}
\frac{\mathrm{d}f}{\mathrm{d}x} &\propto -(x-\mu) \\
\frac{\mathrm{d}f}{\mathrm{d}x} &= -k\,(x-\mu)\,, \quad \exists k \in \mathbb{R} \\
\int \frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x &= -\int (x-\mu)\,\mathrm{d}x
\end{aligned}$$

Using integration by substitution:

$$\begin{aligned}
&\text{let:} \quad v = x - \mu \\
&\implies \frac{\mathrm{d}v}{\mathrm{d}x} = 1 \\
&\implies \mathrm{d}v = \mathrm{d}x
\end{aligned}$$

and hence

$$
\begin{aligned}
\int \frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x &= -\int (x-\mu)\,\mathrm{d}x \\
\implies \int \mathrm{d}p &= -\int v\mathrm{d}v \\
p &= -\frac{1}{2}v^2 \cdot k + C \\
p &= -\frac{1}{2}\left(x-\mu\right)^2 \cdot k + C
\end{aligned}
$$

Clearly the problem with this model is that it allows for probabilities less than zero, hence the model needs to be refined to:

- incorporate a slower rate of change for smaller values of f (approaching 0)
- incorporate a faster rate of change for larger values of f
  - offset by the the condition that $\frac{\mathrm{d}f}{\mathrm{d}x} \propto -(x-\mu)$

Incorporating Proportional to Frequency

In order to make the curve bevel out for smaller values of f it is sufficient to implement the condition that $\frac{\mathrm{d}f}{\mathrm{d}x} \propto f$:

$$
\begin{aligned}
\frac{\mathrm{d}f}{\mathrm{d}x} &\propto f \\
\int \frac{1}{f}\cdot\frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x &= k \cdot \int \mathrm{d}x \\
\ln|f| &= k \cdot x \\
f &= C \cdot e^{\pm x} \\
f &\propto e^{\pm x}
\end{aligned}
$$

Putting both Conditions together

So in order to model the bell-curve we need:

$$
\begin{aligned}
f \propto f &\wedge f \propto -(x-\mu) \\
\implies \frac{\mathrm{d}f}{\mathrm{d}x} &\propto -f\,(x-\mu) \\
\int \frac{1}{f}\mathrm{d}f &= -k \cdot \int (x-\mu)\,\mathrm{d}x \\
\ln|f| &= -k \int (x-\mu)\,\mathrm{d}x
\end{aligned}
$$

because $f > 0$ by definition, the absolute value operators may be dispensed with:

$$
\begin{aligned}
\ln\left(f\right) &= -k\cdot\frac{1}{2}\left(x-\mu\right)^2 + C \\
f &\propto e^{\frac{(x-\mu)^2}{2}}
\end{aligned}
$$

Now that the function has been solved it is necessary to apply the IC's in order to further simplify it.

1. IC, Probability Adds to 1

   The area bound by the curve must be 1 because it represents probability, hence:

$$
\begin{aligned}
1 &= \int_{-\infty}^{\infty} f\mathrm{d}f \\
1 &= -C\int_{-\infty}^{\infty} e^{\frac{k}{2}(x-\mu)^2}\mathrm{d}f
\end{aligned}
$$

   Using integration by substitution:

$$
\begin{aligned}
\text{let:}\quad u^2 &= \frac{k}{2}\left(x-\mu\right)^2 \\
u &= \sqrt{\frac{k}{2}}\left(x-\mu\right) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= \sqrt{\frac{k}{2}}
\end{aligned}
$$

   hence:

$$
\begin{aligned}
1 &= -C\int_{-\infty}^{\infty} e^{\frac{k}{2}(x-\mu)^2} \\
1 &= \sqrt{\frac{2}{k}}\cdot C\int_{-\infty}^{\infty} e^{-u^2}\mathrm{d}u \\
1^2 &= \left(\sqrt{\frac{2}{k}}\cdot C\int_{-\infty}^{\infty} e^{-u^2}\mathrm{d}u\right)^2 \\
1^2 &= \left(\sqrt{\frac{2}{k}}\cdot C\int_{-\infty}^{\infty} e^{-u^2}\mathrm{d}u\right)\times\left(\sqrt{\frac{2}{k}}\cdot C\int_{-\infty}^{\infty} e^{-u^2}\mathrm{d}u\right)
\end{aligned}
$$

   Because this is a definite integral u is merely a dummy variable and instead we can make the substitution of x and y for clarity sake.

$$1^2 = \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x\right) \times \left(\sqrt{\frac{2}{k}} \cdot C \int_{-\infty}^{\infty} e^{-y^2} \mathrm{d}y\right)$$

Now presume that the definite integral is equal to some real constant $\beta \in \mathbb{R}$:

$$\begin{aligned}
1 &= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} e^{-y^2} \mathrm{d}y \times \beta \\
&= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \beta \cdot e^{-y^2} \mathrm{d}y \\
&= \frac{2}{k} \cdot C^2 \cdot \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x\right) e^{-y^2} \mathrm{d}y \\
&= \frac{2}{k} \cdot C^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \mathrm{d}x\mathrm{d}y
\end{aligned}$$

This integral will be easier to evaluate in polar co-ordinates, a double integral may be evaluated in polar co-ordinates using the following relationship: [fn:3]

$$\iint_D f(x, y)\, dA = \int_{\alpha}^{\beta} \int_{h_1(\phi)}^{h_2(\phi)} f(r \cdot \cos(\phi), r \cdot \sin(\phi))\, \mathrm{d}r\mathrm{d}\phi$$

hence this simplifies to:

$$\begin{aligned}
1 &= \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{(r \cdot \cos\theta)^2 + (r \cdot \sin\theta)^2} \mathrm{d}r\mathrm{d}\theta \\
1 &= \frac{2}{k} c^2 \int_0^{2\pi} \int_0^r r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta
\end{aligned}$$

Because the integrand is of the form $f'(x) \times g(f(x))$ we may use integration by substitution:

$$\begin{aligned}
\text{let:} \quad u &= -r^2 \\
\frac{\mathrm{d}u}{\mathrm{d}r} &= -2r \\
\mathrm{d}r &= -\frac{1}{2r} \mathrm{d}u
\end{aligned}$$

and hence:

$$
\begin{aligned}
1 &= \frac{2}{k}c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta \\
\implies 1 &= -\frac{2}{k}c^2 \int_0^{2\pi} \int_0^{\infty} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta
\end{aligned}
$$

$$
\begin{aligned}
1 &= \frac{2}{k}c^2 \int_0^{2\pi} \int_0^{r} r \cdot e^{r^2} \mathrm{d}r\mathrm{d}\theta \\
\implies 1 &= -\frac{2}{k}c^2 \int_0^{2\pi} \int_0^{\infty} -\frac{1}{2}e^{-u} \mathrm{d}u\mathrm{d}\theta \\
&= \frac{1}{k}c^2 \int_0^{2\pi} \int_0^{\infty} e^{-u} \mathrm{d}u\mathrm{d}\theta \\
&= \frac{1}{k}c^2 \int_0^{2\pi} \left[-e^{-u}\right]_0^{\infty} \mathrm{d}\theta \\
1 &= \frac{1}{k}c^2 2\pi \\
\implies C^2 &= \frac{k}{2\pi}
\end{aligned}
$$

So from before:

$$
\begin{aligned}
f &= -C \cdot e^{k \cdot \frac{(x-\mu)^2}{2}} \\
&= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}}
\end{aligned}
$$

so now we simply need to apply the next initial condition.

2. IC, Mean Value and Standard Deviation

   (a) Definitions

   The definition of the expected value, where $f(x)$ is a probability function is: [fn:4]

$$
\mu = E(x) = \int_a^b x \cdot f(x) \, \mathrm{d}x
$$

   That is, roughly, the sum of the expected proportion of occurence.

   The definition of the variance is:

$$
V(x) = \int_a^b (x-\mu)^2 f(x) \, \mathrm{d}x
$$

which can be roughly interpreted as the sum of the proportion of squared distance units from the mean. The standard deviation is $\sigma = \sqrt{V(x)}$.

(b) Expected Value of the Normal Distribution

The expected value of the normal distribution is μ, this can be shown rigorously:

$$\begin{aligned} \text{let:} \quad & v = x - \mu \\ \implies & \mathrm{d}v = \mathrm{d}x \end{aligned}$$

Observe that the limits of integration will also remain as $\pm\infty$ following the substitution:

$$\begin{aligned} E(v) &= \int_{-\infty}^{\infty} v \times f(v) \, \mathrm{d}v \\ &= k \cdot \int_{-\infty}^{\infty} v \cdot e^{v^2} \mathrm{d}v \\ &= \frac{1}{2} \left[e^{x^2} \right]_{\infty}^{\infty} \\ &= \frac{1}{2} \lim_{b \to \infty} \left[\left[e^{x^2} \right]_{-b}^{b} \right] \\ &= \frac{1}{2} \lim_{b \to \infty} \left[e^{b^2} - e^{(-b)^2} \right] \\ &= \lim_{b \to \infty} [0] \times \frac{1}{2} \\ &= \frac{1}{2} \times 0 \\ &= 0 \end{aligned}$$

Hence the Expected value of the standard normal distribution is $0 = x - \mu$ and so $E(x) = \mu$.

(c) Variance of the Normal Distribution

Now that the expected value has been confirmed, consider the variance of the distribution:

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \times f(x) \, \mathrm{d}x$$

Now observe that $(x - \mu)$ appears as an exponential and as a factor if this is redefined as $w = x - \mu \implies \mathrm{d}x = \mathrm{d}w$ we have:

$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2}w^2} \mathrm{d}w$$

Now the integrand is of the form $f(x) \times g(x)$ meaning that the only strategy to potentially deal with it is integration by parts:

$$\int u \mathrm{d}v = u \cdot v - \int v \mathrm{d}u$$

where:

- u is a function that simplifies with differentiation
- $\mathrm{d}v$ is something that can be integrated

$$\begin{aligned} u &= w & \mathrm{d}v &= w \cdot e^{-\frac{k}{2}w^2} \mathrm{d}w \\ \implies \mathrm{d}u &= \mathrm{d}w & \implies v &= \int w \cdot e^{\frac{k}{2}w^2} \mathrm{d}w \\ & & \implies v &= \frac{1}{k} e^{\frac{k}{2}w^2} \end{aligned}$$

Hence the value of the variance may be solved:

Now that the expected value has been confirmed, consider the variance of the distribution:

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} (x-\mu)^2 \times f(x) \mathrm{d}x \\ &= \int_{-\infty}^{\infty} (x-\mu)^2 \times \left(\sqrt{\frac{k}{2\pi}} e^{-\frac{k}{2}(x-\mu)^2} \right) \mathrm{d}x \\ &= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 \times \left(e^{-\frac{k}{2}(x-\mu)^2} \right) \mathrm{d}x \end{aligned}$$

Now observe that $(x-\mu)$ appears as an exponential and as a factor if this is redefined as $w = x - \mu \implies \mathrm{d}x = \mathrm{d}w$ we have:

$$\sigma^2 = \sqrt{\frac{k}{2}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2}w^2} \mathrm{d}w$$

Now the integrand is of the form $f(x) \times g(x)$ meaning that the only strategy to potentially deal with it is integration by parts:

$$\int u \mathrm{d}v = u \cdot v - \int v \mathrm{d}u$$

where:

- u is a function that simplifies with differentiation

- $\mathrm{d}v$ is something that can be integrated

$$\begin{aligned} & u = w & & \mathrm{d}v = w \cdot e^{-\frac{k}{2}w^2}\mathrm{d}w \\ \implies & \mathrm{d}u = \mathrm{d}w & \implies & v = \int w \cdot e^{\frac{k}{2}w^2}\mathrm{d}w \\ & & \implies & v = \frac{1}{k}e^{\frac{k}{2}w^2} \end{aligned}$$

Hence the value of the variance may be solved:

$$\begin{aligned} \sigma^2 &= \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} w^2 e^{-\frac{k}{2}w^2}\mathrm{d}w \\ &= \sqrt{\frac{k}{2\pi}} \left[u \cdot v - \int v\mathrm{d}u\right]_{\infty}^{\infty} \\ &= \sqrt{\frac{k}{2\pi}} \left(\left[\frac{-w}{k} \cdot e^{-\frac{k}{2}w^2}\right]_{\infty}^{\infty} - \frac{1}{k}\int_{-\infty}^{\infty} e^{\frac{k}{2}w^2}\mathrm{d}w\right) \\ &= \sqrt{\frac{k}{2\pi}} \left[\frac{-w}{k} \cdot e^{-\frac{k}{2}w^2}\right]_{\infty}^{\infty} - \frac{1}{k}\left(\sqrt{\frac{k}{2\pi}}\int_{-\infty}^{\infty} e^{\frac{k}{2}w^2}\mathrm{d}w\right) \end{aligned}$$

The left term evaluates to zero and the right term is the area beneath the bell curve with mean value 0 and so evaluates to 1:

$$\begin{aligned} \sigma^2 &= 0 - \frac{1}{k} \\ \implies k &= \frac{1}{\sigma^2} \end{aligned}$$

So the function for the density curve can be simplified:

$$\begin{aligned} &= -\sqrt{\frac{k}{2\pi}} \cdot e^{k \cdot \frac{(x-\mu)^2}{2}} \\ &= \sqrt{\frac{1}{2\pi\sigma^2}} \cdot e^{\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}} \end{aligned}$$

now let $z = \frac{x-\mu}{\sigma} \implies \mathrm{d}z = \frac{\mathrm{d}x}{\sigma}$, this then simplifies to:

$$f(x) = \sqrt{\frac{1}{2\pi}} \cdot e^{-\frac{1}{2}z^2}$$

Now using the power series identity from BEFORE :

$$e^{-\frac{1}{2}z^2} = \sum_{n=0}^{\infty}\left[\frac{\left(-\frac{1}{2}z^2\right)^n}{n!}\right]$$

We can solve the integral of $f\left(x\right)$ (which has no elementary integral.

$$
\begin{aligned}
f\left(x\right) &= \sqrt{\frac{1}{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2} z^2\right)^n}{n!} \right] \\
\int f\left(x\right) \mathrm{d}x &= \frac{1}{\sqrt{2\pi}} \int \sum_{n=0}^{\infty} \left[\frac{\left(-\frac{1}{2} z^2\right)^n}{n!} \right] \mathrm{d}z \\
&= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\int \frac{\left(-1\right)^{-1} z^{2n}}{2^n \cdot n!} \mathrm{d}z \right] \\
&= \frac{1}{\sqrt{2\pi}} \cdot \sum_{n=0}^{\infty} \left[\frac{\left(-1\right)^n \cdot z^{2n+1}}{2^n \left(2n+1\right) n!} \right]
\end{aligned}
$$

Although this is a power series it still gives a method to solve the area beneath the curve of the density function of the normal distribution.

Understanding the p-value

Let's say that I'm given 100 vials of medication and in reality only 10 of them are actually effective.

POS	POS	POS	POS	POS	POS	POS	POS	POS	POS
:-:	:-:	:-:	:-:	:-:	:-:	:-:	:-:	:-:	:-:
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG
NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG	NEG

We don't know which ones are effective so It is necessary for the effective medications to be detected by experiment. Let:

- the p-value be 9% for detecting a significant effect
- assume the statistical power is 70%

So this means that the corresponding errors are:

1. Of the 90 Negative Drugs, $\alpha \times 90 \approx 8$ will be identified as Positive (False Positive) a. This means 72 will be correctly identified as negative. (TN)

2. Of the 10 Good drugs $\beta \times 10 = 3$ will be labelled as negative (False Negative) b. This means 8 will be correctly identified as positive (True Positive)

These results can be summarised as:

	Really Negative	Really Positive
Predicted Negative	TNR; $(1 - \alpha)$	FNR; $\beta \times 10 = 3$
Predicted Positive	FPR; $\mathsf{FPR} = \alpha \times 90 \approx 8$	TPR $(1 - \beta)$

And a table visualising the results: