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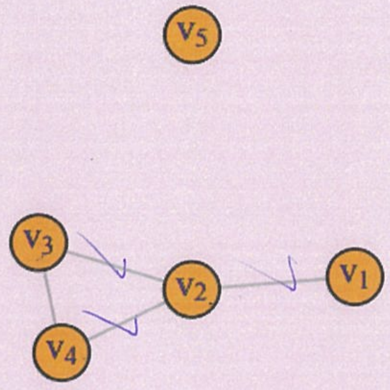
Write down the adjacency matrix for the following graph:




$$
\begin{array}{c c}
 & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

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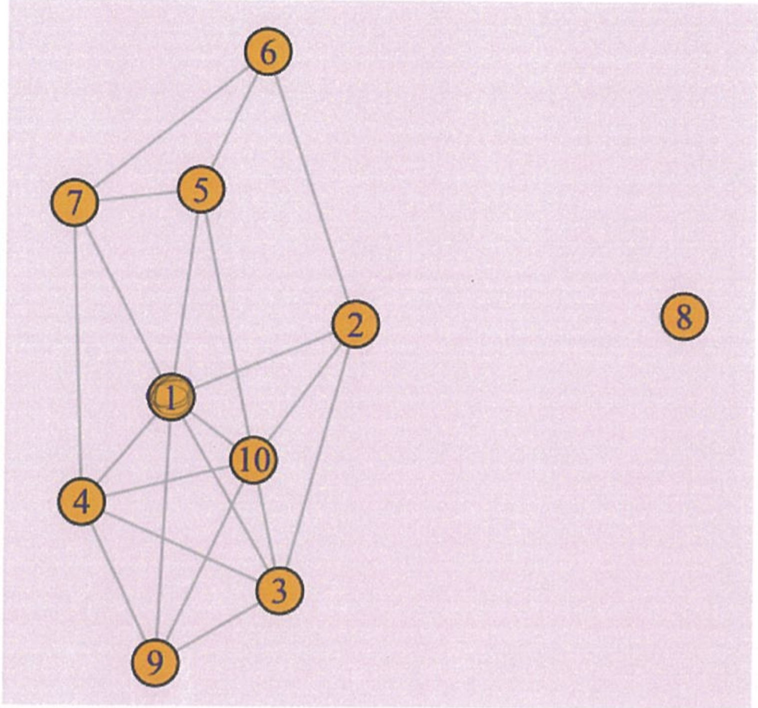
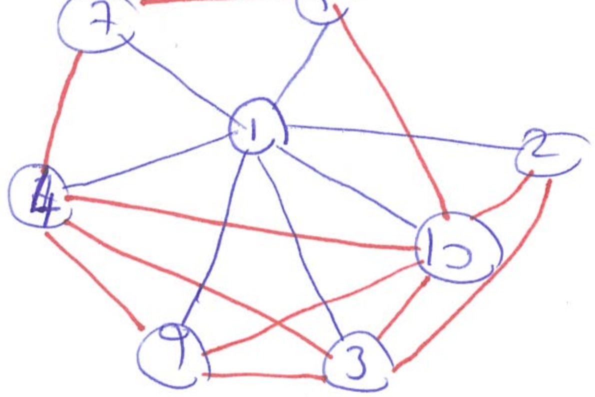



Degree 1 of $v_1$

1.5



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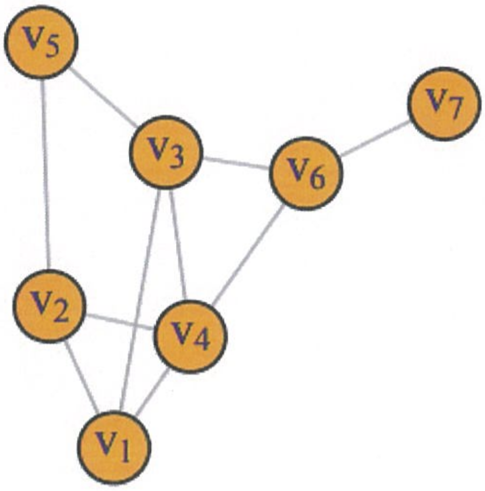
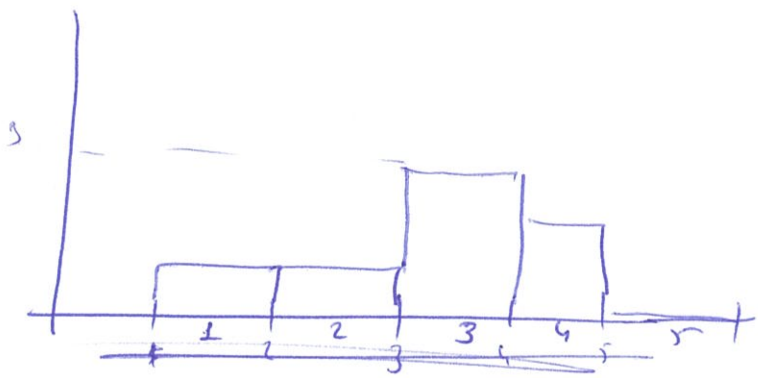



$V_1 : 3$
$V_2 : 3$
$V_3 : 4$
$V_4 : 4$
$V_5 : 2$
$V_6 : 3$
$V_7 : 1$

Compute the degree of each vertex to obtain the distribution.

| Degree | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 0 | 1 | 1 | 3 | 2 | 0 |



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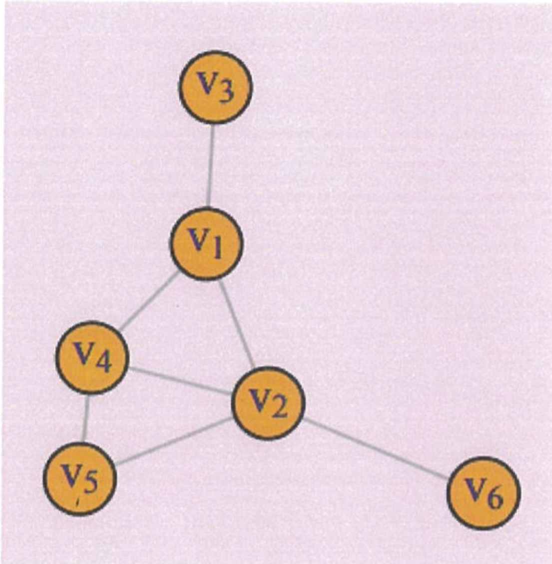


Density

$$D = \frac{2|E|}{|V|(|V|-1)} = \frac{\cancel{2}\cdot 7}{\underset{3}{\cancel{6}}\cdot 5} = \frac{7}{15}$$

$|E| = 7$

$|V| = 6$

Diameter : Longest shortest path

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ |
|---|---|---|---|---|---|---|
| $v_1$ : | | 1 | 1 | 1 | 2 | 2 |
| $v_2$ : | | | 2 | 1 | 1 | 1 |
| $v_3$ | | | | 2 | 3 | 3 |
| $v_4$ | | | | | 1 | 2 |
| $v_5$ | | | | | | 2 |
| $v_6$ | | | | | | |

$\longrightarrow$ Diameter = 3

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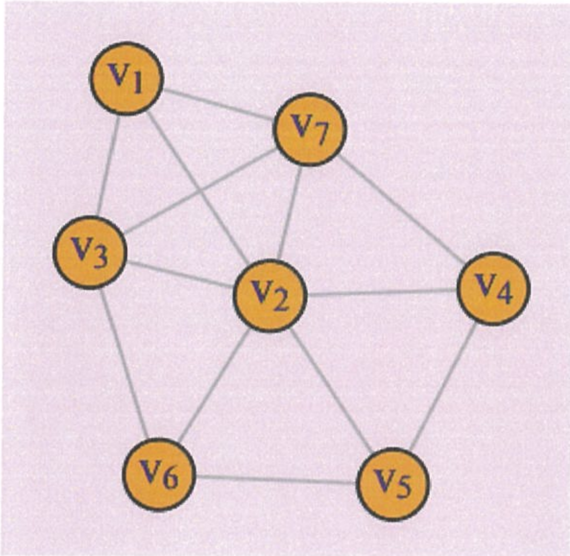

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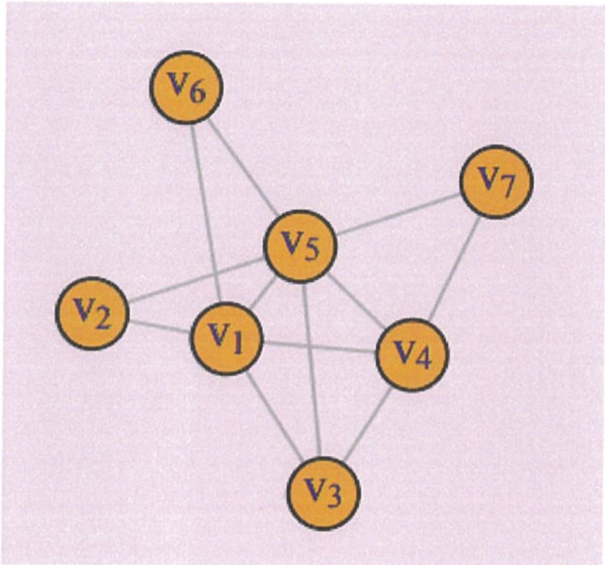


$$
\begin{array}{lccccccccl}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & & \\
c(v_1) = & & 1 & +1 & +1 & +1 & +1 & +2 & = 7 & \\
c(v_2) = & 1 & & +2 & +2 & +1 & +2 & +2 & = 10 & \\
c(v_3) = & 1 & +2 & & +1 & +1 & +2 & +2 & = 9 & \\
c(v_4) = & 1 & +2 & +1 & & +1 & +2 & +1 & = 8 & \\
c(v_5) = & 1 & +1 & +1 & +1 & & +1 & +1 & = 6 & \to 1 \\
c(v_6) = & 1 & +2 & +2 & +2 & +1 & & +2 & = 10 & \\
c(v_7) = & & & & & & & & = 10 &
\end{array}
$$

$v_5$ is the center according to closeness centrality.

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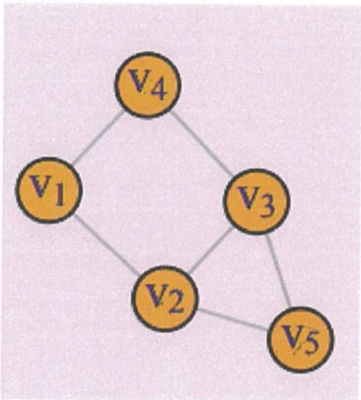


$v_1 : v_2$

$v_1 : v_2 : v_3$ , $v_1 : v_4 : v_3$

$v_1 : v_4$

$v_1 : v_2 : v_5$

$v_2 : v_3$

$v_2 : v_3 : v_4$ , $v_2 : v_1 : v_4$

$v_2 : v_5$

$v_3 : v_4$

$v_3 : v_5$

$v_4 : v_3 : v_5$

$c_b(v_1) = \frac{1}{2}$

$c_b(v_2) = \frac{1}{2} + \frac{1}{1} = \frac{3}{2}$

$c_b(v_3) = \frac{1}{2} + \frac{1}{1} = \frac{3}{2}$

$c_b(v_4) = \frac{1}{2}$

$c_b(v_5) = 0$

$v_2$ and $v_3$ are equally essential in the network.