

Approximated Scatter Models for Stray Light Analysis

Richard N. Pfisterer



John Newhart

In the absence of measured data, stray light estimates can be daunting for engineers who do not regularly make scatter calculations. Here's how to make reasonable approximations.

Optical designers and engineers frequently need to model optical phenomena before they can accomplish component characterization. For example, for a proposal effort, they might have to estimate scatter from optical elements in order to establish a budget for stray light. Illumination engineers may do a “back-of-the-envelope” calculation to determine how energy is scattered by a diffuser plate. This article describes how to make such approximations.

Optical surface scatter

The *ABg* model is available in most optical design and engineering software to describe scatter from smooth optical surfaces. The model is typically written:

$$\text{BSDF}(\beta - \beta_0) = \frac{A}{B + (\beta - \beta_0)^g} \quad [\text{sr}^{-1}], \quad (1)$$

where $\beta = \sin(\theta_{\text{scatter}})$, $\beta_0 = \sin(\theta_{\text{specular}})$, and A , B and g are the specific model parameters. Unfortunately, the parameters are not all directly related to the phenomenology they describe: The ratio of the A and B parameters is equal to the peak bidirectional scattering distribution function (BSDF) in the specular direction. The angle at which the BSDF transitions from a constant value to a power-law falloff is given by $B^{1/g}$. The slope g is identical to the power-law

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falloff of the 2-D power-spectral density (PSD) of the surface roughness.

In order for *ABg* to be a valid model for optical surface scatter:

- ▶ The surface roughness must be isotropic. Diamond-turned (or otherwise ruled) surfaces and diffraction gratings cannot use the *ABg* model without modification.
- ▶ The rms surface roughness must be much less than the incident wavelength. Typically, this criterion is less than 1 percent; a common surface with 20-Å-rms roughness illuminated by visible light at 5,876 Å easily meets this condition, so this is not overly restrictive. Conversely, engineers improperly use this scatter model to describe surfaces with hundreds of micrometers of surface roughness; the hardware results then have little correlation to the computer model. (This

restriction is imposed by Rayleigh-Rice scatter theory and is a common assumption made in the derivation of most scatter models.)

- ▶ The spatial frequencies of the surface roughness must be bandwidth-limited; that is, only “real” scatter is considered, and evanescent scatter components are ignored. At normal incidence, the spatial frequency bandwidth limits are $1/D$ and $1/\lambda$, where D is the surface diameter and λ is the incident wavelength.

Interestingly enough, these parameters are not arbitrary. For most optical surfaces, the slope parameter g is bounded between 1 and 2.5, with a “most commonly observed” value of 1.5. Occasionally, surfaces are measured with a slope value of 2.75 or higher; these may be referred to as “superpolished” surfaces. Typically, they are fabricated with specific attention to the removal of as much surface damage as possible during the grinding process; this is usually accomplished by hydrofluoric acid etching or the equivalent. The reduction of surface damage dramatically changes the PSD power-law falloff.

Likewise, the transition angle $B^{1/g}$, when measured, is typically 0.01 radians from specular. Because of the near-specular measurement limitations in modern scatterometers, we do not often observe

BSDF data rolling over to a constant value. However, it must transition to a constant value in order to obey conservation of energy. Theorists postulate that the transition angle can be between 0.0001 and 0.01 radians from specular, if it is not seen in the BSDF measurement data.

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One obvious problem with ABg is that, with the exception of the slope g , the parameters themselves do not relate to anything tangible to the engineer. In order to make that connection, we invoke two other relationships. First, according to Rayleigh-Rice theory and subject to the restrictions described above, the total integrated scatter (TIS) of a surface is

$$\text{TIS} = \left(\frac{2\pi\Delta n\sigma}{\lambda} \right)^2, \quad (2)$$

where Δn is the refractive index difference across the surface ($\Delta n = 2$ for a mirror), σ is the rms surface roughness, and λ is the incident wavelength.

Next, we have to relate the TIS to the ABg model; for any scatter function this relationship is

$$\text{TIS} = \iint \text{BSDF}(\theta, \phi) \sin\theta \cos\theta d\theta d\phi. \quad (3)$$

This integral can be evaluated in closed form. By equating the two expressions for the TIS, the tangible rms surface roughness is related to the “intangible” ABg parameters.

Mirror and lens surface scatter

Assuming that $B^{1/g} = 0.01$ radians from specular, we derive a simple expression for the A parameter for a mirror surface, shown in the table to the right.

Take an example material such as conventionally polished BK7 glass: Using a power-law falloff of 1.5, a wavelength of 5876 Å, and a 20 Å rms roughness, the appropriate ABg parameters would be $A = 0.000161$, $B = 0.001$ and $g = 1.5$.

By making the same reasonable assumptions, one can calculate the A

[Lambertian scatter models for selected surfaces]	
Surface description	BSDF (1/sr)
Ideal white diffuser	$1/\pi$
Typical matte white paper at normal incidence	$0.85/\pi$
Typical diffuse black paint at normal incidence	$0.05/\pi$
Ideal absorber	$0/\pi$

parameter for a transmissive lens surface. In this case, the index difference across the surface boundary is explicit.

Non-optical surface scatter

Scatter models for non-optical surfaces are more complex because the underlying phenomenology can be complicated. Vendors may add glass spheres to black paints to improve the scatter properties, for example, with the resulting light interaction requiring an involved calculation. In these cases, it is best to work from actual scatter measurements.

When this is not practical or possible, one must make judicious use of reasonable approximations. A Lambertian scatter model is common for some applications:

$$\text{BSDF} = \frac{\rho}{\pi} [sr^{-1}], \quad (4)$$

where ρ is the hemispheric reflectivity. Note that the Lambertian scatter model is degenerate in the sense that it is not dependent on the incident and scattered angles; this makes it convenient for back-of-the-envelope calculations. The

hemispheric reflectivity is a bounded quantity: It must be between 0 and 1, so it is not arbitrary. This provides a sanity check of the model’s validity.

Except for thermal (blackbody) emission, nothing observed in nature is truly Lambertian. Spectralon is widely considered one of the most Lambertian substances known, but it exhibits an increased specularity in reflected P -polarized light for large angles of incidence.

A caution

Experienced stray light analysts may take exception to simplistic forms for stray light calculations, citing, for example, that aluminum mirrors do not always follow the wavelength-dependence described here. Others may point out that coating a surface changes its scatter properties or that scatter from a “real” black paint becomes decidedly less Lambertian at higher angles of incidence. Such perspectives are true and should be considered.

At the same time, keep in mind that stray light software from the 1970s and early 1980s had only two scatter models available: Lambertian and a variant of the ABg model. A great many systems were designed, built and successfully deployed based on stray light calculations using these simplistic forms. In his classic paper on estimating stray light levels, Greynolds notes that only these two scatter models are necessary to develop a good understanding of the scatter properties for well-baffled systems (Proc. SPIE, **257**, 39, 1980). So it’s important to recognize those instances in which a quick estimate may be more valuable than a detailed answer for which the boss must wait several weeks. ▲

[ABg parameters for surfaces]		
Mirrors		
g	B	A
2.5	0.00001	$1.40 (\sigma/\lambda)^2$
2	0.0001	$5.46 (\sigma/\lambda)^2$
1.5	0.001	$13.92 (\sigma/\lambda)^2$
1	0.01	$25.51 (\sigma/\lambda)^2$
Lens surfaces		
g	B	A
2.5	0.00001	$0.35 (\sigma \Delta n/\lambda)^2$
2	0.0001	$1.37 (\sigma \Delta n/\lambda)^2$
1.5	0.001	$3.50 (\sigma \Delta n/\lambda)^2$
1	0.01	$6.35 (\sigma \Delta n/\lambda)^2$

Richard N. Pfisterer (richp@photonengr.com) is president of Photon Engineering, LLC.