Radiometry and Photometry

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GLOSSARY

Illuminance, E_v The area density of luminous flux, the luminous flux per unit area at a specified point in a specified surface that is incident on, passing through, or emerging from that point in the surface (unit: $\mathrm{lm} \cdot \mathrm{m}^{-2} = \mathrm{lux}$).

Irradianc, E_e The area density of radiant flux, the radiant flux per unit area at a specified point in a specified surface that is incident on, passing through, or emerging from that point in the surface (unit: $\mathrm{watt} \cdot \mathrm{m}^{-2}$).

Luminance, L_v The area and solid angle density of luminous flux, the luminous flux per unit projected area and per unit solid angle incident on, passing through, or emerging from a specified point in a specified surface, and in a specified direction in space (units: $\mathrm{lumen} \cdot \mathrm{m}^{-2} \cdot \mathrm{sr}^{-1} = \mathrm{cd} \cdot \mathrm{m}^{-2}$).

Luminous efficacy, K_r The ratio of luminous flux in lumens to radiant flux (total radiation) in watts in a beam of radiation (units: lumen/watt).

Luminous flux, Φ_v The $V(\lambda)$-weighted integral of the spectral flux Φ_λ over the visible spectrum (unit: lumen).

Luminous intensity, I_v The solid angle density of luminous flux, the luminous flux per unit solid angle incident on, passing through, or emerging from a point in space and propagating in a specified direction (units: $\mathrm{lm} \cdot \mathrm{sr}^{-1} = \mathrm{cd}$).

Photopic spectral luminous efficiency function, $V(\lambda)$ The standardized relative spectral response of a human observer under photopic (cone vision) conditions over the wavelength range of visible radiation.

Projected area, A_o Unidirectional projection of the area bounded by a closed curve in a plane onto another plane making some angle θ to the first plane.

Radiance, L_e The area and solid angle density of radiant flux, the radiant flux per unit projected area and per unit solid angle incident on, passing through, or emerging from a specified point in a specified surface, and in a specified direction in space (units: watt $\cdot$ m$^{-2}\cdot$ sr^{-1}).

Radiant flux, Φ_e The time rate of flow of radiant energy (unit: watt).

Radiant intensity, I_e The solid angle density of radiant flux, the radiant flux per unit solid angle incident on, passing through, or emerging from a point in space and propagating in a specified direction (units: watt $\cdot$ sr^{-1}).

Solid angle, Ω The area A on a sphere of the radial projection of a closed curve in space onto that sphere, divided by the square r^2 of the radius of that sphere.

Spectral radiometric quantities The spectral "concentration" of quantity Q, denoted Q_λ, is the derivative $dQ/d\lambda$ of the quantity with respect to wavelength λ, where "Q" is any one of: radiant flux, irradiance, radiant intensity, or radiance.

RADIOMETRY is a system of language, mathematical formulations, and instrumental methodologies used to describe and measure the propagation of radiation through space and materials. The radiation so studied normally is confined to the ultraviolet (UV), visible (VIS), and infrared (IR) parts of the spectrum, but the principles are applicable to radiant energy of any form that propagates in space and interacts with matter in known ways, similar to those of electromagnetic radiation. This includes other parts of the electromagnetic spectrum and to radiation composed of the flow of particles where the trajectories of these particles follow known laws of ray optics, through space and through materials. Radiometric principles are applied to beams of radiation at a single wavelength or those composed of a broad range of wavelengths. They can also be applied to radiation diffusely scattered from a surface or volume of material. Application of these principles to radiation propagating through absorbing and scattering media generally leads to mathematically sophisticated and complex treatments when high precision is required. That important topic called *radiative transfer*, is not treated in this article.

Photometry is a subset of radiometry, and deals only with radiation in the visible portion of the spectrum. Photometric quantities are defined in such a way that they incorporate the variations in spectral sensitivity of the human eye over the visible spectrum, as a spectral weighting function built into their definition.

In determining spectrally broadband radiometric quantities, no spectral weighting function is used (or one may consider that a weighting "function" of unity (1.0) is applied at all wavelengths).

The scope of this treatment is limited to definitions of the primary quantities in radiometry and photometry, the derivations of several useful relationships between them, the rudiments of setting up problems in radiation transfer, short discussions of material properties in a radiometric context, and a very brief discussion of electronic detectors of electromagnetic radiation. The basic design of radiometers and photometers and the principles of their calibration are described as well.

Until the latter third of the 20th century, the fields of radiometry and photometry developed somewhat independently. Photometry was beset with a large variety of different quantities, names of those quantities, and units of measurement. In the 1960s and 1970s several authors contributed articles aimed at bringing order to the apparent confusion. Also, the International Lighting Commission (CIE, *Commission International de l'Eclairage*) and the International Electrotechnical Commission (CEI, *Commision Electrotechnic International*) worked to standardize a consistent set of symbols, units, and nomenclature, culminating in the *International Lighting Vocabulary*, jointly published by the CIE and the CEI. The recommendations of that publication are followed here. The CIE has become the primary international authority on terminology and basic concepts in radiometry and photometry.

I. BACKGROUND

A. Units and Nomenclature

Radiant flux is defined as the time rate of flow of energy through space. It is given the Greek symbol Φ and the metric unit watt (a joule of energy per second). An important characteristic of radiant flux is its distribution over the electromagnetic spectrum, called a *spectral distribution* or *spectrum*. The Greek symbol λ is used to symbolize the wavelength of monochromatic radiation, radiation having only one frequency and wavelength. The unit of wavelength is the meter, or a submultiple of the meter, according to the rules of *System International*, the international system of units (the metric system). The unit of frequency is the hertz (abbreviated Hz), defined to be a cycle (or period) per second. The symbol for frequency is the Greek ν. The relationship between frequency ν and wavelength λ is shown in the equation

$$\lambda\nu = c, \tag{1}$$

where c is the speed of propagation in the medium (called the "speed of light" more familarly). The spectral concentration of radiant flux at (or around) a given wavelength λ is given the symbol Φ_λ, the name *spectral radiant flux*, and the units watts per unit wavelength. An example of this is the watt per nanometer (abbreviated W/nm). The names, definitions, and units of additional radiometric quantities are provided in Section II.

The electromagnetic spectrum is diagramed in Fig. 1. The solar and visible spectral regions are expanded to the right of the scale. Though sound waves are not electromagnetic waves, the range of human-audible sound is shown in Fig. 1 for comparison.

The term "light" can only be applied in principle to electromagnetic radiation over the range of *visible* wavelengths. Radiation outside this range is invisible to the human eye and therefore cannot be called light. Infrared and ultraviolet radiation cannot be termed "light."

Names and spectral ranges have been standardized for the ultraviolet, visible, and infrared portions of the spectrum. These are shown in Table I.

TABLE I CIE Vocabulary for Spectral Regions

Name	Wavelength range
UV-C	100 to 280 nm
UV-B	280 to 315 nm
UV-A	315 to 400
VIS	Approx. 360–400 to 760–800 nm
IR-A[a]	780 to 1400 nm
IR-B	1.4 to 3.0 μm
IR-C[b]	3 μm to 1 mm

[a] Also called "near IR" or NIR.
[b] Also called "far IR" or FIR.

B. Symbols and Naming Conventions

When the wavelength symbol λ is used as a subscript on a radiometric quantity, the result denotes the concentration of the quantity at a specific wavelength, as if one were dealing with a monochromatic beam of radiation at this wavelength only. This means that the range $\Delta\lambda$ of wavelengths in the beam, around the wavelength λ of definition, is infinitesemally small, and can therefore be defined in terms of the mathematical derivative as follows. Let Q be a radiometric quantity, such as flux, and ΔQ be the amount of this quantity over a wavelength interval $\Delta\lambda$ centered at wavelength λ. The *spectral* version of quantity Q, at wavelength λ, is the derivative of Q with respect to wavelength, defined to be the limit as $\Delta\lambda$ goes to zero of the ratio $\Delta Q/\Delta\lambda$.

$$Q_\lambda = \frac{dQ}{d\lambda}. \tag{2}$$

This notation refers to the "concentration" of the radiometric quantity Q, at wavelength λ, rather than to its functional dependence on wavelength. The latter would be notated as $Q_\lambda(\lambda)$. Though seemingly redundant, this notation is correct within the naming convention established for the field of radiometry.

When dealing with the optical properties of *materials* rather than with concentrations of flux at a given wavelength, the subscripting convention is not used. Instead, the functional dependence on wavelength is notated directly, as with the spectral transmittance: $T(\lambda)$. *Spectral optical properties* such as this one are spectral weighting functions, not flux distributions, and their functional dependence on wavelength is shown in the conventional manner.

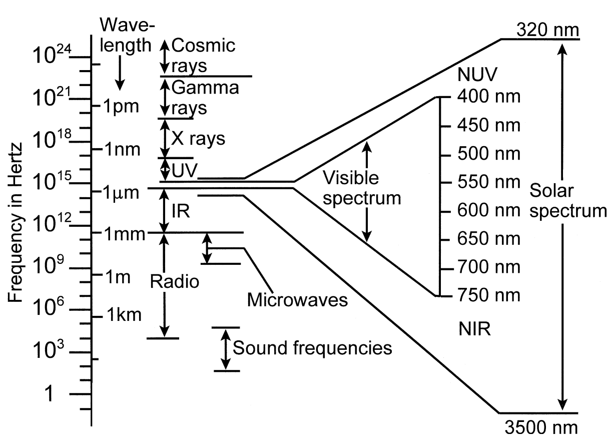

FIGURE 1 Wavelength and frequency ranges over the electromagnetic spectrum.

C. Geometric Concepts

In radiometry and photometry one is concerned with several geometrical constructs helpful in defining the spatial characteristics of radiation. The most useful are areas, plane angles, and solid angles.

The areas of interest are planar ones (including small differential elements of area used in definitions and derivations), nonplanar ones (areas on curved surfaces), and what are called *projected areas*. The latter are areas resulting when an original area is projected at some angle θ, as viewed from an infinite distance away. Projected areas are unidirectional projections of the area bounded by a closed curve in a plane onto another plane, one making angle θ to the first, as illustrated in Fig. 2.

A plane angle is defined by two straight lines intersecting at a point. The space between these lines in the plane defined by them is the plane angle. It is measured in radians (2π radians in a circle) or degrees (360 degrees to a circle). In preparation for defining solid angle it is pointed out that the plane angle can also be defined in terms of the radial projection of a line segment in a plane onto a point, as illustrated in Fig. 3.

> A *plane angle* is the quotient of the arc length s and the radius r of a radial projection of segment C of a curve in a plane onto a circle of radius r lying in that plane and centered at the vertex point P about which the angle is being defined.

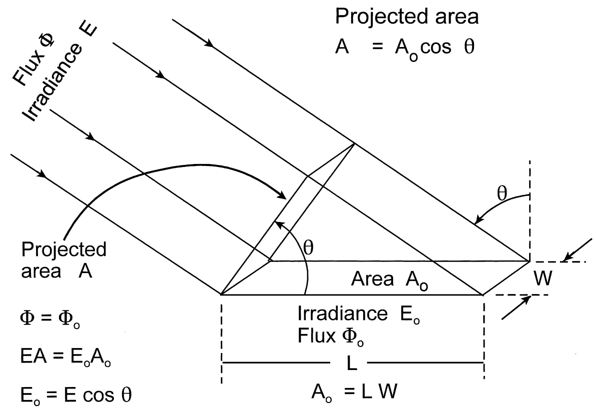

FIGURE 2 Illustration of the definition of projected areas.

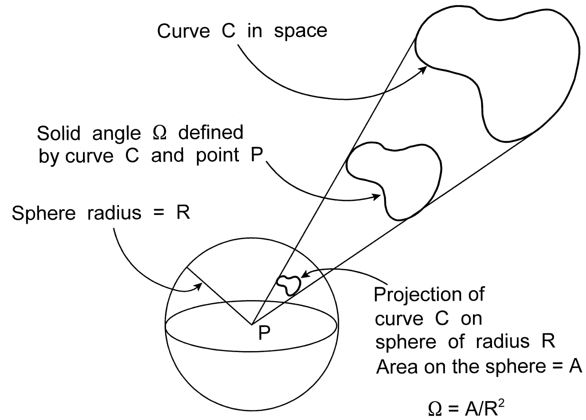

FIGURE 4 Definition of the solid angle.

If θ is the angle and s is the arc length of the projection onto a circle of radius r, then the defining equation is

$$\theta = \frac{s}{r}. \tag{3}$$

According to Eq. (3), the plane angle is a dimensionless quantity. However, to aid in communication, it has been given the unit *radian*, abbreviated *rad*. The radian measure of a plane angle can be converted to degree measure with the multiplication of a conversion constant, $180/\pi$.

A similar approach can be used to define *solid angle.* A solid angle is defined by a closed curve in space and a point, as illustrated in Fig. 4.

> A *solid angle* is the quotient of the area A and square of the radius r of a radial projection of a closed curve C in space onto a sphere of radius r centered at the vertex point P relative to which the angle is being defined.

If Ω is the solid angle being defined, A is the area on the sphere enclosed by the projection of the curve onto that sphere, and r is the sphere's radius, then the defining equation is

$$\Omega = \frac{A}{r^2} \tag{4}$$

According to Eq. (4), the solid angle is dimensionless. However, to aid in communication, it has been given the unit *steradian*, abbreviated *sr*. Since the area of a sphere is 4π times the square of its radius, for a unit radius sphere the area is 4π and the solid angle subtended by it is 4π sr. The solid angle subtended by a hemisphere is $2\pi\, sr$. It is important to note that the area A in Eq. (4) is the area *on the sphere* of the projection of the curve C. It is not the area of a plane cut through the sphere and containing the projection of curve C. Indeed, the projections of some curves in space onto a sphere do not lie in a plane.

One which *does* is of particular interest—the projection of a circle in a plane perpendicular to a radius of the sphere, as illustrated in Fig. 5, which also shows a hemispherical solid angle. Let α be the plane angle subtended by the radius of the circle at the center of the sphere, called the "half-angle" of the cone. It can be shown that the solid angle Ω subtended by the circle is given by

$$\Omega = 2\pi(1 - \cos\alpha). \tag{5}$$

If $\alpha = 0$ then $\Omega = 0$ and if $\alpha = 90^\circ$ then $\Omega = 2\pi\, sr$, as required. A derivation of Eq. (5) is provided on pp. 28–30 of McCluney (1994).

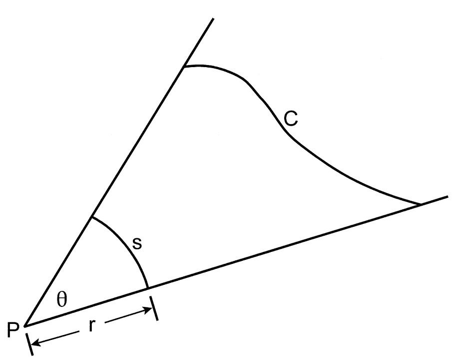

FIGURE 3 Definition of the plane angle.

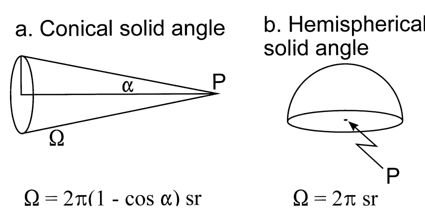

FIGURE 5 (a) Geometry for determining the solid angle of a right circular cone. α is the "half-angle" of the cone. (b) Geometry of a hemispherical solid angle.

D. The Metric System

To clarify the symbols, units, and nomenclature of radiometry and photometry the international system of units and related standards known as *the metric system* was embraced. There have been several versions of the metric system over the last couple of centuries. The current modernized one is named *Le System International d'Unites* (SI). It was established in 1960 by international agreement. The *Bureau International des Poids et Mesures* (BIPM) regularly publishes a document containing revisions and new recommendations on terminology and units. The International Standards Organization (ISO) publishes standards on the practical uses of the SI system in a variety of fields. Many national standards organizations around the world publish their own standards governing the use of this system, or translations of the BIPM documents, into the languages of their countries. In the United States the units metre and litre are spelled meter and liter, respectively.

The SI system calls for adherence to standard prefixes for standard orders of magnitude, listed in Table II. There are some simple rules governing the use of these prefixes. The prefix symbols are to be printed in roman type without spacing between the prefix symbol and the unit symbol. The grouped unit symbol plus its prefix is inseparable but may be raised to a positive or negative power and combined with other unit symbols. Examples: cm^2, nm, μm, klx, 1 $cm^2 = 10^{-4}$ m^2. No more than one prefix can be used at a time. A prefix should never be used alone, except in descriptions of systems of units.

There are now two classes of units in the SI system:

- Base units and symbols: meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd). Note that the abbreviations of units named for a person are capitalized, but the full unit name is not. (For example, the watt was named for James Watt and is abbreviated "W.")

TABLE II SI Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	decka, deca	da	10^{-24}	yocto	y

- Derived units: joule ($= \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2} = \mathrm{N} \cdot \mathrm{m}$), watt ($= \mathrm{J} \cdot \mathrm{s}^{-1}$), lumen ($= \mathrm{cd} \cdot \mathrm{sr}$), and lux ($= \mathrm{lm} \cdot \mathrm{m}^{-2}$). These are formed by combining base units according to algebraic relations linking the corresponding physical quantities. The laws of chemistry and physics are used to determine the algebraic combinations resulting in the derived units. Also included are the units of angle (radian, rad), and solid angle (steradian, sr).

A previously separate third class called supplementary units, combinations of the above units and units for plane and solid angle, was eliminated by the General Conference on Weights and Measures (CGPM, *Conference Generale des Poids et Mesures*) during its 9–12 October 1995 meeting. The radian and steradian were moved into the SI class of derived units.

Some derived units are given their own names, to avoid having to express every unit in terms of its base units. The symbol "·" is used to denote multiplication and "/" denotes division. Both are used to separate units in combinations. It is permissible to replace "·" with a space, but some standards require it to be included. In 1969 the following additional non-SI units were accepted by the International Committee for Weights and Measures for use with SI units: day, hour, and minute of time, degree, minute and second of angle, the litre (10^{-3} m^3), and the tonne (10^3 kg). In the United States the latter two are spelled "liter" and "metric ton," respectively.

The worldwide web of the internet contains many sites describing and explaining the SI system. A search on "The Metric System" with any search engine should yield several. The United States government site at http://physics.nist.gov/cuu/Units/ is comprehensive and provides links to other web pages of importance.

E. The I-P System

The most prominent alternative to the metric system is the inch-pound or the so-called "English" system of units. In this system the foot and pound are units for length and mass. The British thermal unit (Btu) is the unit of energy. This system is used little for radiometry and photometry around the world today, with the possible exception of the United States, where many illumination engineers still work with a mixed metric/IP unit, the foot-candle ($\mathrm{lumen} \cdot \mathrm{ft}^{-2}$) as their unit of illuminance. There are about 10.76 square feet in a square meter. So one foot-candle equals about 10.76 lux. The I-P system is being deprecated. However, in order to read older texts in radiometry and photometry using the I-P system, some familiarity with its units is advised. Tables 10.3 and 10.4 of McCluney (1994) provide conversion factors for many non-SI units.

II. RADIOMETRY

A. Definitions of Fundamental Quantities

There are five fundamental quantities of radiometry: radiant energy, radiant flux, radiant intensity, irradiance, and radiance. Each has a photometric counterpart, described in the next section.

Radiant energy, Q, is the quantity of energy propagating into, through, or emerging from a specified surface area in a specified period of time (unit: joule). Radiant energy is of interest in applications involving pulses of radiation, or exposure of a receiving surface to temporally continuous radiant energy over a specific period of time. An equivalent unit is the watt · sec.

Radiant flux (power), Φ, is the time rate of flow of radiant energy (unit: watt). One watt is 1 J sec^{-1}. The defining equation is the derivative of the radiant energy Q with respect to time t.

$$\Phi = \frac{dQ}{dt}. \tag{6}$$

Radiant flux is the quantity of energy passing through a surface or region of space per unit time. When specifying a radiant flux value, the spatial extent of the radiation field included in the specification should be described.

Irradiance, E, is the area density of radiant flux, the radiant flux per unit area at a specified point in a specified surface that is incident on, passing through, or emerging from that point in the surface (unit: watt · m^{-2}). All directions in the hemispherical solid angle producing the radiation at that point are to be included. The defining equation is

$$E = \frac{d\Phi}{ds_o}, \tag{7}$$

where $d\Phi$ is an infinitesimal element of radiant flux and ds_o is an element of area in the surface. (The subscript "o" is used to indicate that this area is in an actual surface and is not a projected area.) The flux incident on a point in a surface can come from any direction in the hemispherical solid angle of incidence, or all of them, with any directional distribution. The flux can also be that leaving the surface in any direction in the hemispherical solid angle of emergence from the surface.

The irradiance leaving a surface can be called the *exitance* and can be given the symbol M, to distinguish it from the irradiance incident on the surface, but it has the same units and defining equation as irradiance. (The term *emittance*, related to the *emissivity*, is reserved for use in describing a dimensionless optical property of a material's surface and cannot be used for emitted irradiance.)

Since there is no mathematical or physical distinction between flux incident upon, passing through, or leaving a surface, the term irradiance is used throughout this article to describe the flux per unit area in all three cases.

Irradiance is a function of position in the surface specified for its definition.

When speaking of irradiance, one should be careful both to describe the surface and to indicate at which point on the surface the irradiance is being evaluated, unless this is very clear in the context of the discussion, or if the irradiance is known or assumed to be constant over the whole surface.

Radiant intensity, I, is the solid angle density of radiant flux, the radiant flux per unit solid angle incident on, passing through, or emerging from a point in space and propagating in a specified direction (units: watt · sr^{-1}). The defining equation is

$$I = \frac{d\Phi}{d\omega}, \tag{8}$$

where $d\Phi$ is an element of flux incident on or emerging from a point within element $d\omega$ of solid angle in the specified direction. The representation of $d\omega$ in spherical coordinates is illustrated in Fig. 6.

Radiant intensity is a function of direction from its point of specification, and may be written as $I(\theta, \phi)$ to indicate its dependence upon the spherical coordinates (θ, ϕ) specifying a direction in space. Its definition is illustrated in Fig. 7.

Intensity is a useful concept for describing the directional distribution of radiation from a point source (or a source very small compared with the distance from it to the observer or detector of that radiation). The concept can be applied to extended sources having the same intensity at all points, in which case it refers to that subset of the radiation emanating from the entire source of finite and known area which flows into the same infinitesimal solid angle direction for each point in that area. (The next quantity

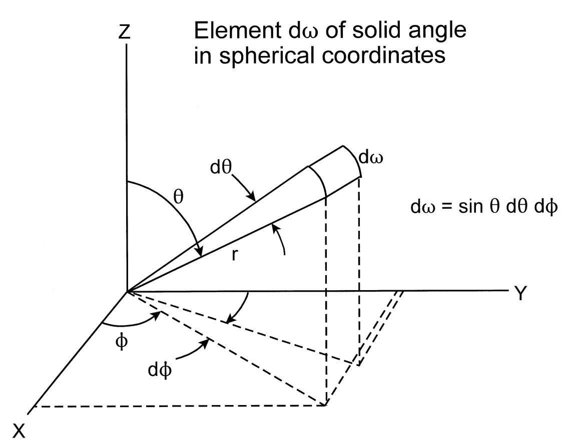

FIGURE 6 Representation of the element of solid angle $d\omega$ in Spherical coordinates.

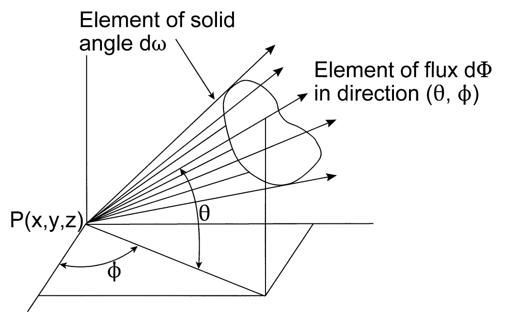

FIGURE 7 Geometry for the definition of Intensity.

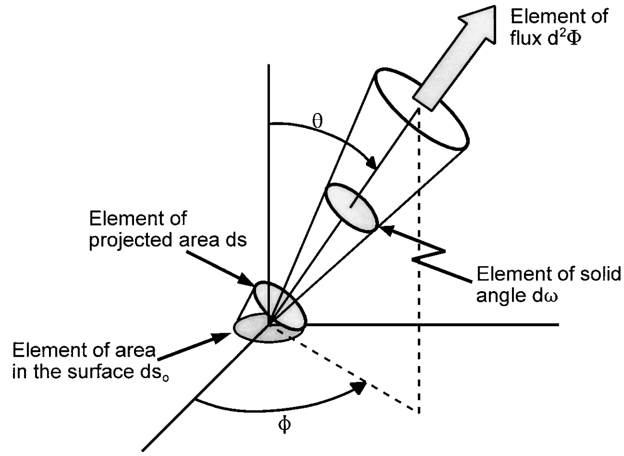

FIGURE 8 Geometry for the definition of radiance.

to be described, *radiance*, is generally a more appropriate quantity for describing the directional distribution of radiation from nonpoint sources.)

When speaking of intensity, one should be careful to describe the point of definition and the direction of radiation from that point for clarity of discourse, unless this is obvious in the context of the discussion, or if it is known that the intensity is constant for all directions.

The word "intensity" is frequently used in optical physics. Most often the radiometric quantity being described is not intensity but irradiance.

Radiance, L, is the area and solid angle density of radiant flux, the radiant flux per unit projected area and per unit solid angle incident on, passing through, or emerging from a specified point in a specified surface, and in a specified direction (units: watt · m^{-2} · sr^{-1}). The defining equation is

$$L = \frac{d^2\Phi}{d\omega\, ds}$$

or (9)

$$L = \frac{d^2\Phi}{d\omega\, ds_o \cos\theta},$$

where $ds = ds_o \cos\theta$ is the projected area, the area of the projection of elemental area ds_o along the direction of propagation to a plane perpendicular to this direction, $d\omega$ is an element of solid angle in the specified direction and θ is the angle this direction makes with the normal (perpendicular) to the surface at the point of definition, as illustrated in Fig. 8.

Radiance is a function of both position and direction. For many real sources, it is a strongly varying function of direction. It is the most general quantity for describing the propagation of radiation through space and transparent or semitransparent materials. The radiant flux, radiant intensity, and irradiance can be derived from the radiance by the mathematical process of integration over a finite surface area and/or over a finite solid angle, as demonstrated in Section IV.B.

Since radiance is a function of position in a defined surface as well as direction from it, it is important when speaking of radiance to specify the surface, the point in it, and the direction from it. All three pieces of information are important for the proper specification of radiance. For example, we may wish to speak of the radiance emanating from a point on the ground and traveling upward toward the lens of a camera in an airplane or satellite traveling overhead. We specify the location of the point, the surface from which the flux emanates, and the direction of its travel toward the center of the lens. Since the words "radiance" and "irradiance" can sound very similar in rapidly spoken or slurred English, one can avoid confusion by speaking of the point and the surface that is common to both concepts, and then to clearly specify the direction when talking about radiance.

B. Definitions of Spectral Quantities

The spectral or wavelength composition of the five fundamental quantities of radiometry is often of interest. We speak of the "spectral distribution" of the quantities and by this is meant the possibly varying magnitudes of them at different wavelengths or frequencies over whatever spectral range is of interest. As before, if we let Q represent any one of the five radiometric quantities, we define the spectral "concentration" of that quantity, denoted Q_λ, to be the derivative of the quantity with respect to wavelength λ. (The derivative with respect to frequency ν or wavenumber $(1/\nu)$ is also possible but less used.)

$$Q_\lambda = \frac{dQ}{d\lambda}. \tag{10}$$

This defines the radiometric "quantity" per unit wavelength interval and can also be called the spectral power density. It has the same units as those of the quantity Q

TABLE III Symbols and Units of the Five Spectral Radiometric Quantities

Quantity	Spectral radiant energy	Spectral radiant flux	Spectral irradiance	Spectral intensity	Spectral radiance
Symbol	Q_λ	Φ_λ	E_λ	I_λ	L_λ
Units	$\mathrm{J \cdot nm^{-1}}$	$\mathrm{W \cdot nm^{-1}}$	$\mathrm{W \cdot m^{-2} \cdot nm^{-1}}$	$\mathrm{W \cdot sr^{-1} \cdot nm^{-1}}$	$\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}$

divided by wavelength. The spectral radiometric quantity Q_λ is in one respect the more fundamental of the two, since it contains more information, the spectral distribution of Q, rather than just its total magnitude. The two are related by the integral

$$Q = \int_0^\infty Q_\lambda \, d\lambda. \tag{11}$$

If Q_λ is zero outside some wavelength range, (λ_1, λ_2) then the integral of Eq. (11) can be replaced by

$$Q = \int_{\lambda_1}^{\lambda_2} Q_\lambda \, d\lambda. \tag{12}$$

The symbols and units of the spectral radiant quantities are listed in Table III.

III. PHOTOMETRY

A. Introduction

Photometry is a system of language, mathematical formulations, and instrumental methodologies used to describe and measure the propagation of *light* through space and materials. In consequence, the radiation so studied is confined to the visible (VIS) portion of the spectrum. Only light is visible radiation.

In photometry, all the radiant quantities defined in Section II are adapted or specialized to indicate the human eye's response to them. This response is built into the definitions. Familiarity with the five basic radiometric quantities introduced in that section makes much easier the study of the corresponding quantities in photometry, a subset of radiometry.

The human eye responds only to light having wavelengths between about 360 and 800 nm. Radiometry deals with electromagnetic radiation at all wavelengths and frequencies, while photometry deals only with visible light—that portion of the electromagnetic spectrum which stimulates vision in the human eye.

Radiation having wavelengths below 360 nm, down to about 100 nm, is called *ultraviolet*, or UV, meaning "beyond the violet." Radiation having wavelengths greater than 830 nm, up to about 1 mm, is called *infrared*, or IR, meaning "below the red." "Below" in this case refers to the *frequency* of the radiation, not to its wavelength. (Solving (1) for frequency yields the equation $\nu = c/\lambda$, showing the inverse relationship between frequency and wavelength.) The infrared portion of the spectrum lies *beyond* the red, having frequencies below and wavelengths above those of red light. Since the eye is very insensitive to light at wavelengths between 360 and about 410 nm and between about 720 and 830 nm, at the edges of the visible spectrum, many people cannot see radiation in portions of these ranges. Thus, the visible edges of the UV and IR spectra are as uncertain as the edges of the VIS spectrum.

The term "light" should only be applied to electromagnetic radiation in the visible portion of the spectrum, lying between 380 and 770 nm. With this terminology, there is no such thing as "ultraviolet light," nor does the term "infrared light" make any sense either. Radiation outside these wavelength limits is radiation—not light—and should not be referred to as light.

B. The Sensation of Vision

After passing through the cornea, the aqueous humor, the iris and lens, and the vitreous humor, light entering the eye is received by the retina, which contains two general classes of receptors: rods and cones. Photopigments in the outer segments of the rods and cones absorb radiation and the absorbed energy is converted within the receptors, into neural electrochemical signals which are then transmitted to subsequent neurons, the optic nerve, and the brain.

The cones are primarily responsible for day vision and the seeing of color. Cone vision is called *photopic* vision. The rods come into play mostly for night vision, when illumination levels entering the eye are very low. Rod vision is called *scotopic* vision. An individual's relative sensitivity to various wavelengths is strongly influenced by the absorption spectra of the photoreceptors, combined with the spectral transmittance of the preretinal optics of the eye. The relative spectral sensitivity depends on light level and this sensitivity shifts toward the blue (shorter wavelength) portion of the spectrum as the light level drops, due to the shift in spectral sensitivity when going from cones to rods.

The spectral response of a human observer under photopic (cone vision) conditions was standardized by the International Lighting Commission the *International de l'Eclairage* (CIE), in 1924. Although the actual spectral response of humans varies somewhat from person to person, an agreed standard response curve has been adopted, as shown graphically in Fig. 9 and listed numerically in Table IV.

The values in Table IV are taken from the *Lighting Handbook* of the Illuminating Engineering Society of North America (IESNA). Since the symbol $V(\lambda)$ is normally used to represent this spectral response, the curve in Fig. 9 is often called the "V-lambda curve."

The 1924 CIE spectral luminous efficiency function for photopic vision defines what is called "the CIE 1924 Standard Photopic Photometric Observer." The official values were originally given for the wavelength range from 380 to 780 nm at 10-nm intervals but were then "completed by interpolation, extrapolation, and smoothing from earlier values adopted by the CIE in 1924 and 1931" to the wavelength range from 360 to 830 nm on 1-nm intervals and these were then recommended by the International Committee of Weights and Measures (CIPM) in 1976. The values below 380 and above 769 are so small to be of little value for most photometric calculations and are therefore not included in Table IV.

Any individual's eye may depart somewhat from the response shown in Fig. 9, and when light levels are moderately low, the other set of retinal receptors (rods) comes into use. This regime is called "scotopic vision" and is characterized by a different relative spectral response.

The relative spectral response curve for scotopic vision is similar in shape to the one shown in Fig. 9, but the peak is shifted from 555 to about 510 nm. The lower wavelength cutoff in sensitivity remains at about 380 nm, however, while the upper limit drops to about 640 nm. More information about scotopic vision can be found in various books on vision as well as in the *IESNA Lighting Handbook*. The latter contains both plotted and tabulated values for the scotopic spectral luminous efficiency function.

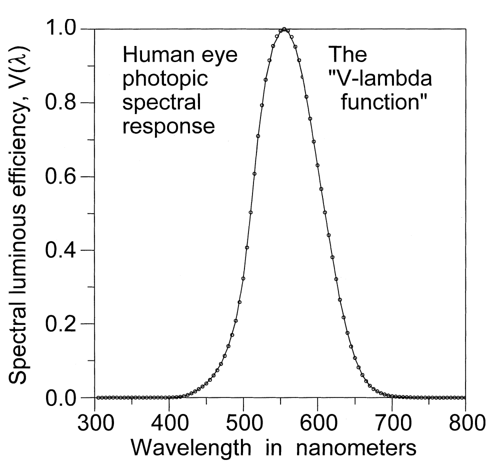

FIGURE 9 Human photopic spectral luminous efficiency.

C. Definitions of Fundamental Quantities

Five fundamental quantities in radiometry were defined in Section II.A. The photometric ones corresponding to the last four are easily defined in terms of their radiometric counterparts as follows. Let $Q_\lambda(\lambda)$ be one of the following: spectral radiant flux Φ_λ, spectral irradiance E_λ, spectral intensity I_λ, or spectral radiance L_λ. The corresponding photometric quantity, Q_v is defined as follows:

$$Q_\mathrm{v} = 683 \int_{380}^{770} Q_\lambda(\lambda) V(\lambda)\, d\lambda \tag{13}$$

with wavelength λ having the units of nanometers.

The subscript v (standing for "visible" or "visual") is placed on photometric quantities to distinguish them from radiometric quantities, which are given the subscript *e* (standing for "energy"). These subscripts may be dropped, as they were in previous sections, when the meaning is clear and no ambiguity results. Four fundamental radiometric quantities, and the corresponding photometric ones, are listed in Table V, along with the units for each.

To illustrate the use of (13), the conversion from spectral irradiance to illuminance is given by

$$E_\mathrm{v} = 683 \int_{380}^{770} E_\lambda(\lambda)\ V(\lambda)\ d\lambda. \tag{14}$$

The basic unit of luminous flux, the lumen, is like a "light-watt." It is the luminous equivalent of the radiant flux or power. Similarly, luminous intensity is the photometric equivalent of radiant intensity. It gives the luminous flux in lumens emanating from a point, per unit solid angle in a specified direction, and therefore has the units of lumens per steradian or lm/sr, given the name *candela*. This unit is one of the seven base units of the metric system. More information about the metric system as it relates to radiometry and photometry can be found in Chapter 10 of McCluney (1994).

Luminous intensity is a function of direction from its point of specification, and may be written as $I_v(\theta, \phi)$ to indicate its dependence upon the spherical coordinates (θ, ϕ) specifying a direction in space, illustrated in Fig. 6.

Illuminance is the photometric equivalent of irradiance and is like a "light-watt per unit area." Illuminance is a function of position (x, y) in the surface on which it is defined and may therefore be written as $E_v(x, y)$. Most light meters measure illuminance and are calibrated to read in lux. The lux is an equivalent term for the lumen per square meter and is abbreviated *lx*.

In the inch-pound (I-P) system of units, the unit for illuminance is the lumen per square foot, or lumen $\cdot$ ft^{-2}, which also has the odd name "foot-candle," abbreviated "fc," even though connection with candles and the candela

TABLE IV Photopic Spectral Luminous Efficiency $V(\lambda)$

Wavelength λ, nm	Standard values	Values interpolated at intervals of 1 nm								
		1	2	3	4	5	6	7	8	9
380	.00004	.000045	.000049	.000054	.000058	.000064	.000071	.000080	.000090	.000104
390	.00012	.000138	.000155	.000173	.000193	.000215	.000241	.000272	.000308	.000350
400	.0004	.00045	.00049	.00054	.00059	.00064	.00071	.00080	.00090	.00104
410	.0012	.00138	.00156	.00174	.00195	.00218	.00244	.00274	.00310	.00352
420	.0040	.00455	.00515	.00581	.00651	.00726	.00806	000889	.00976	.01066
430	.0116	.01257	.01358	.01463	.01571	.01684	.01800	.01920	.02043	.02170
440	.023	.0243	.0257	.0270	.0284	.0298	.0313	.0329	.0345	.0362
450	.038	.0399	.0418	.0438	.0459	.0480	.0502	.0525	.0549	.0574
460	.060	.0627	.0654	.0681	.0709	.0739	.0769	.0802	.0836	.0872
470	.091	.0950	.0992	.1035	.1080	.1126	.1175	.1225	.1278	.1333
480	.139	.1448	.1507	.1567	.1629	.1693	.1761	.1833	.1909	.1991
490	.208	.2173	.2270	.2371	.2476	.2586	.2701	.2823	.2951	.3087
500	.323	.3382	.3544	.3714	.3890	.4073	.4259	.4450	.4642	.4836
510	.503	.5229	.5436	.5648	.5865	.6082	.6299	.6511	.6717	.6914
520	.710	.7277	.7449	.7615	.7776	.7932	.8082	.8225	.8363	.8495
530	.862	.8739	.8851	.8956	.9056	.9149	.9238	.9320	.9398	.9471
540	.954	.9604	.9961	.9713	.9760	.9083	.9480	.9873	.9902	.9928
550	.995	.9969	.9983	.9994	1.0000	1.0002	1.0001	.9995	.9984	.9969
560	.995	.9926	.9898	.9865	.9828	.9786	.9741	.9691	.9638	.9581
570	.952	.9455	.9386	.9312	.9235	.9154	.9069	.8981	.8890	.8796
580	.870	.8600	.8496	.8388	.8277	.8163	.8046	.7928	.7809	.7690
590	.757	.7449	.7327	.7202	.7076	.6949	.6822	.6694	.6565	.6437
600	.631	.6182	.6054	.5926	.5797	.5668	.5539	.5410	.5282	.5156
610	.503	.4905	.4781	.4568	.4535	.4412	.4291	.4170	.4049	.3929
620	.381	.3690	.3575	.3449	.3329	.3210	.3092	.2977	.2864	.2755
630	.265	.2548	.2450	.2354	.2261	.2170	.2082	.1996	.1912	.1830
640	.175	.1672	.1596	.1523	.1452	.1382	.1316	.1251	.1188	.1128
650	.107	.1014	.0961	.0910	.0862	.0816	.0771	.0729	.0688	.0648
660	.061	.0574	.0539	.0506	.0475	.0446	.0418	.0391	.0366	.0343
670	.032	.0299	.0280	.0263	.0247	.0232	.0219	.0206	.0194	.0182
680	.017	.01585	.01477	.01376	.01281	.011,92	.01108	.01030	.00956	.00886
690	.0082	.00759	.00705	.00656	.00612	.00572	.00536	.00503	.00471	.00440
700	.0041	.00381	.00355	.00332	.00310	.00291	.00273	.00256	.00241	.00225
710	.0021	.001954	.001821	.001699	.001587	.001483	.001387	.001297	.001212	.001130
720	.00105	.000975	.000907	.000845	.000788	.000736	.000668	.000644	.000601	.000560
730	.00052	.000482	.000447	.000415	.000387	.000360	.000335	.000313	.000291	.000270
740	.00025	.000231	.000214	.000198	.000185	.000172	.000160	.000149	.000139	.000130
750	.00012	.000111	.000103	.000096	.000090	.000084	.000078	.000074	.000069	.000064
760	.00006	.000056	.000052	.000048	.000045	.000042	.000039	.000037	.000035	.000032

is mainly historical and indirect. The I-P system is being discontinued in photometry, to be replaced by the metric system, used exclusively in this treatment. For more information on the connections between modern metric photometry and the antiquated and deprecated units, the reader is directed to Chapter 10 of McCluney (1994). As with radiant exitance, illuminance leaving a surface can be called luminous exitance.

Luminance can be thought of as "photometric brightness," meaning that it comes relatively close to describing physically the subjective perception of "brightness." Luminance is the quantity of light flux passing through a point in a specified surface in a specified direction, per unit projected area at the point in the surface and per unit solid angle in the given direction. The units for luminance are therefore $\mathrm{lm} \cdot \mathrm{m}^{-2} \cdot \mathrm{sr}^{-1}$. A more common unit for

TABLE V Basic Quantities of Radiometry and Photometry

Radiometric quantity	Symbol	Units	Photometric quantity	Symbol	Units
Radiant flux	Φ_e	watt (W)	Luminous flux	Φ_v	lumen (lm)
Radiant intensity	I_e	W/sr	Luminous intensity	I_v	lumen/sr = candela (cd)
Irradiance	E_e	W/m^2	Illuminance	E_v	$lumen/m^2$ = lux (lx)
Radiance	L_e	$W \cdot m^{-2} \cdot sr^{-1}$	Luminance	L_v	$lm \cdot m^{-2} \cdot sr^{-1} = cd/m^2$

luminance is the $cd \cdot m^{-2}$, which is the same as the lumen per steradian and per square meter.

D. Luminous Efficacy of Radiation

Radiation luminous efficiacy, K_r, is the ratio of luminous flux (light) in lumens to radiant flux (total radiation) in watts in a beam of radiation. It is an important concept for converting between radiometric and photometric quantities. Its units are the lumen per watt, lm/W.

Luminous efficacy is not an efficiency since it is not a dimensionless ratio of energy input to energy output—it is a measure of the effectiveness of a beam of radiation in stimulating the perception of light in the human eye. If Q_v is any of the four photometric quantities (Φ_v, E_v, I_v, or L_v) defined previously and Q_e is the corresponding radiometric quantity, then the luminous efficacy associated with these quantities has the following defining equation:

$$K_r = \frac{Q_v}{Q_e} \, [lm \cdot W^{-1}] \tag{15}$$

Q_e is an integral over all wavelengths for which Q_λ is nonzero, while Q_v depends on an integral (13) over only the visible portion of the spectrum, where $V(\lambda)$ is nonzero. The luminous efficacy of a beam of infrared-only radiation is zero since none of the flux in the beam is in the visible portion of the spectrum. The same can be said of ultraviolet-only radiation.

The International Committee for Weights and Measures (CPIM), meeting at the International Bureau of Weights and Measures near Paris, France, in 1977 set the value 683 lm/W for the spectral luminous efficacy (K_r) of monochromatic radiation having a wavelength of 555 nm in standard air. In 1979 the candela was redefined to be the luminous intensity in a given direction, of a source emitting monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of $1/683$ W/sr. The candela is one of the seven fundamental units of the metric system. As a result of the redefinition of the candela, the value 683 shown in Eq. (13) is not a recommended good value for K_r but instead follows from the definition of the candela in SI units. (Prior to 1979, the candela was realized by a platinum approximation to a blackbody. After the 1979 redefinition of the candela, it can be realized from the absolute radiometric scale using any of a variety of absolute detection methods discussed in Section X.)

IV. COMMONLY USED GEOMETRIC RELATIONSHIPS

There are several important spatial integrals which can be developed from the definitions of the principal radiometric and photometric quantities. This discussion of some of them will use radiometric terminology, with the understanding that the same derivations and relationships apply to the corresponding photometric quantities.

A. Lambertian Sources and the Cosine Law

To simplify some derivations, an important property, approximately exhibited by some sources and surfaces, is useful. Any surface, real or imaginary, whose radiance is independent of direction is said to be a *Lambertian radiator*. The surface can be self-luminous, as in the case of a source, or it can be a reflecting or transmitting one. If the radiance emanating from it is independent of direction, this radiation is considered to be Lambertian. A Lambertian radiator can be thought of as a window onto an isotropic radiant flux field.

Finite Lambertian radiators obey Lambert's cosine law, which is that the flux in a given direction leaving an element of area in the surface varies as the cosine of the angle θ between that direction and the perpendicular to the surface element: $d\Phi(\theta) = d\Phi(0) \cos\theta$. This is because the projected area in the direction θ decreases with the cosine of that angle. In the limit, when $\theta = 90$ degrees, the flux drops to zero because the projected area is zero.

There is another version of the cosine law. It has to do not with the radiance *leaving* a surface but with how radiation from a uniform and collimated beam (a beam with all rays parallel to each other and equal in strength) incident on a plane surface is distributed over that surface as the angle of incidence changes.

This is illustrated as follows: A horizontal rectangle of length L and width W receives flux from a homogeneous beam of collimated radiation of irradiance E, making an angle θ with the normal (perpendicular) to the plane of the rectangle, as shown in Fig. 2. If Φ is the flux over the projected area A, given by E times A, this same flux Φ_o will be falling on the larger horizontal area $A_o = L \cdot W$, producing horizontal irradiance $E_o = \Phi_o/A$. The flux is the same on the two areas ($\Phi = \Phi_o$). Equating them gives

$$EA = E_o A_o. \tag{16}$$

But $A = A_o \cos\ \theta$ so that

$$E_o = E \cos\theta. \tag{17}$$

This is another way of looking at the cosine law. Although it deals with the irradiance falling on a surface, if the surface is perfectly transparent, or even imaginary, it will also describe the irradiances (or exitances) emerging from the other side of the surface.

B. Flux Relationships

Radiance and irradiance are quite different quantities. Radiance describes the angular distribution of radiation while irradiance adds up all this angular distribution over a specified solid angle Ω and lumps it together. The fundamental relationship between them is embodied in the equation

$$E = \int_\Omega L(\theta, \phi) \cos\theta\, d\omega \tag{18}$$

for a point in the surface on which they are defined. In this and subsequent equations, the lower case $d\omega$ is used to identify an *element* of solid angle $d\omega$ and the upper case Ω to identify a finite solid angle.

If $\Omega = 0$ in Eq. (18), there is no solid angle and there can be no irradiance! When we speak of a collimated beam of some given irradiance, say E_o, we are talking about the irradiance contained in a beam of nearly parallel rays, but which necessarily have some small angular spread to them, filling a small but finite solid angle Ω, so that (18) can be nonzero. A perfectly collimated beam contains no irradiance, because there is no directional spread to its radiation—the solid angle is zero. Perfect collimation is a useful concept for theoretical discussions, however, and it is encountered frequently in optics. When speaking of collimation in experimental situations, what is usually meant is "quasi-collimation," nearly perfect collimation.

If the radiance $L(\theta, \phi)$ in Eq. (18) is constant over the range of integration (over the hemispherical solid angle), then it can be removed from the integral and the result is

$$E = \pi L. \tag{19}$$

This result is obtained from Eq. (18) by replacing $d\omega$ with its equivalence in spherical coordinates, $\sin\theta\, d\theta\, d\phi$, and integrating the result over the angular ranges of 0 to 2π for ϕ and 0 to $\pi/2$ for θ.

A constant radiance surface is called a Lambertian surface so that (19) applies only to such surfaces.

It is instructive to show how Eq. (18) can be derived from the definition of radiance. Eq. (9) is solved for $d^2\Phi$ and the result divided by ds_o. Since $d^2\Phi/ds_o = dE$ by Eq. (7), we have

$$dE = L \cos\theta\, d\omega. \tag{20}$$

Integrating (20) yields (18).

Similarly, one can replace the quotient $d^2\Phi/d\omega$ with the differential dI [from Eq. (8)] in Eq. (9) and solve for dI. Integrating the result over the source area S_o yields

$$I = \int_{S_o} L \cos\theta\, ds_o. \tag{21}$$

Intensity is normally applied only to point sources, or to sources whose area S_o is small compared with the distance to them. However, Eq. (21) is valid, even for large sources, though it is not often used this way.

Solving (8) for $d\Phi = I d\omega$, writing $d\omega$ as da_o/R^2, and dividing both sides by da_o yields the expression

$$E = \frac{I\ d\omega}{da_o} = \frac{I\ da_o}{da_o R^2} = \frac{I}{R^2} \tag{22}$$

for the irradiance E a distance R from a point source of intensity I, on a surface perpendicular to the line between the point source and the surface where E is measured. This is an explicit form for what is known as "the inverse square law" for the decrease in irradiance with distance from a point source. The inverse square law is a consequence of the definition of solid angle and the "filling" of that solid angle with flux emanating from a point source.

Next comes the conversion from radiance L to flux Φ. Let the dependence of the radiance on position in a surface on which it is defined be indicated by generalized coordinates (u, v) in the surface of interest. Let the directional dependence be denoted by (θ, ϕ), so that L may be written as a function $L(\text{u, v}, \theta, \phi)$ of position and direction. Solve (9), the definition of radiance, for $d^2\Phi$. The result is

$$d^2\,\Phi = L \cos\theta\, ds_o\, d\omega. \tag{23}$$

Integrating (23) over both the area S_o of the surface and the solid angle Ω of interest yields

$$\Phi = \int_{S_o}\int_\Omega L(u, v, \theta, \phi) \cos\theta\ d\omega\ ds_o. \tag{24}$$

In spherical coordinates, $d\omega$ is given by $\sin\theta\ d\theta\ d\phi$. Letting the solid angle Ω over which (23) is integrated extend to the full 2π sr of the hemisphere, we have the total flux emitted by the surface in all directions.

$$\Phi = \int_{S_o} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L(u, v, \theta, \phi)\ \cos\theta\ \sin\theta\ d\theta\ d\phi\ ds_o. \tag{25}$$

V. PRINCIPLES OF FLUX TRANSFER

Only the geometrical aspects of flux transfer through a lossless and nonscattering medium are of interest in this section. The effects of absorption and scattering of radiation as it propagates through a transparent or semitransparent medium from a source to a receiver are outside the scope of this article. The effects of changes in the refractive index of the medium, however, are dealt with in Section V.E.

All uses of flux quantities in this section refer to both their radiant (subscript e) and luminous (subscript v) versions. The subscripts are left off for simplicity. When the terms radiance and irradiance are mentioned in this section, the discussion applies equally to luminance and illuminance, respectively.

A. Source/Receiver Geometry

The discussion begins with the drawing of Fig. 10 and the definition of radiance L in (9):

$$L = \frac{d^2\Phi}{d\omega\, ds_o \cos\theta}, \tag{26}$$

where θ is the angle made by the direction of emerging flux with respect to the normal to the surface of the source, ds_o is an infinitesimally small element of area at the point of definition in the source, and $d\omega$ is an element of solid angle from the point of definition in the direction of interest.

In Fig. 10 are shown an infinitesimally small element ds_o of area at a point in a source, an infinitesimal element da_o of area at point P on a receiving surface, the distance R between these points, and the angles θ and ψ between the line of length R between the points and the normals to the surfaces at the points of intersection, respectively.

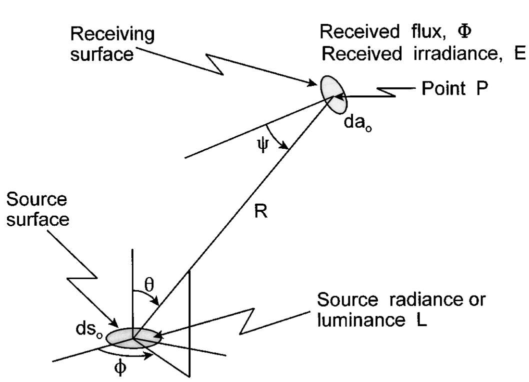

FIGURE 10 Source/receiver geometry.

B. Fundamental Equations of Flux Transfer

The element $d\omega$ of solid angle subtended by element of projected receiver area $da = da_o \cos\psi$ at distance R from the source is

$$d\omega = \frac{da}{R^2} = \frac{da_o \cos\psi}{R^2} \tag{27}$$

so that, solving (26) for $d^2\Phi$ and using (27), the element of flux received at point P from the element ds_o of area of the source is given by

$$d^2\Phi = L\frac{ds_o \cos\theta\, da_o \cos\psi}{R^2} \tag{28}$$

with the total flux Φ received by area A_o from source area S_o being given by

$$\Phi = \int_{S_o}\int_{A_o} L\frac{ds_o \cos\theta\, da_o \cos\psi}{R^2}. \tag{29}$$

This is the fundamental (and very general within the assumptions of this section) equation describing the transfer of radiation from a source surface of finite area to a receiving surface of finite area. Most problems of flux transfer involve this integration (or a related version shown later, giving the irradiance E instead of the flux). For complex or difficult geometries the problem can be quite complex analytically because in such cases L, θ, ψ, and R will be possibly complicated functions of position in both the source and the receiver surfaces. The general dependency of L on direction is also embodied in this equation, since the direction from a point in the source to a point in the receiver generally changes as the point in the receiver moves over the receiving surface.

The evaluation of (29) involves setting up diagrams of the geometry and using them to determine trigonometric and other analytic relationships between the geometric variables in (29). If the problem is expressed in cartesian coordinates, for example, then the dependences of L, θ, ψ, R, ds_o, and da_o upon those coordinates must be determined so that the integrals in (29) can be evaluated.

Two important simplifications allow us to address a large class of problems in radiometry and photometry with ease, by simplifying the mathematical analysis.

The first results when the source of radiance is known to be Lambertian and to have the same value for all points in the source surface. This makes L constant over all ranges of integration, both the integration over the source area and the one over the solid angle of emerging directions from each point on the surface. In such a case, the radiance can be removed from all integrals over these variables. The remaining integrals are seen to be purely geometric in character. The second simplification arises when one doesn't want the total flux over the whole receiving surface—only the flux per unit area at a point on that surface, the irradiance E at point P in Fig. 10. In this case, we can divide both sides of (28) by the element of area in the receiving surface, da_o, to get

$$dE = L\frac{\cos\theta\cos\psi}{R^2}ds_o. \tag{30}$$

This equation is the counterpart of (28) when it is the irradiance E of the receiving surface that is desired. For the total irradiance at point P, one must integrate this equation over the portion S_o of the source surface contributing to the flux at P.

$$E = \int_{S_o} L\frac{\cos\theta\cos\psi}{R^2}ds_o. \tag{31}$$

When L is constant over direction, it can be removed from this integral and one is left with a simpler integration to perform. Equation (31) is the counterpart to (29) when it is the irradiance E at the receiving point that is of interest rather than the total flux over area A_o.

C. Simplified Source/Receiver Geometries

If the source area S_o is small with respect to the distance R to the point P of interest (i.e., if the maximum dimension of the source is small compared with R), then R^2, $\cos\psi$, and $\cos\theta$ do not vary much over the range of integration shown in (31) and they can be removed from the integral. If L does not vary over S_o, then it also can be removed from the integral, even if L is direction dependent, because the range of integration over direction is so small; that is, only the one direction from the source to point P in the receiver is of interest. We are left with an approximate version of (30) for small homogeneous sources some distance from the point of reception:

$$E \approx L\frac{S_o\cos\theta\cos\psi}{R^2}. \tag{32}$$

This equation contains within it both the cosine law and the inverse square law.

If the source and receiving surfaces face each other directly, so that θ and ψ are zero, both of the cosines in this equation have values of unity and the equation is still simpler in form.

D. Configuration Factor

In analyzing complicated radiation transfer problems, it is frequently helpful to introduce what is called the *configuration factor*. Alternate names for this factor are the *view*, *angle*, *shape*, *interchange*, or *exchange factor*. It is defined to be the fraction of total flux from the source surface that is received by the receiving surface. It is given the symbol F_{s-r} or F_{1-2}, indicating flux transfer from source to receiver or from Surface 1 to Surface 2. In essence, it indicates the details of how flux is transferred from a source area of some known form to a reception area. Its value is most evident when the source radiance is of such a nature that it can be taken from the integrals, leaving integrals over only geometric variables.

The geometry can still be quite complex, making analytical expressions for F_{1-2} difficult to determine and calculate. Many important geometries have already been analyzed, however, and the resulting configuration factors published.

In many problems, one is most concerned with the magnitude and spectral distribution of the source radiance and the corresponding spectral irradiance in a receiving surface, rather than with the geometrical aspects of the problem expressed by the shape factor. It is very convenient in such cases to separate the spectral variations from the geometrical ones. Once the configuration factor has been determined for a situation with nonchanging geometry, it remains constant and attention can be focused on the variable portion of the solution.

A general expression for the configuration factor results from dividing (29) for the flux Φ_r on the receiver by (24) for the total flux Φ_s, emitted by the source.

$$F_{s-r} = \frac{\Phi_r}{\Phi_s} = \frac{\int_{S_o}\int_{A_o}\frac{L\cos\theta\cos\psi\,ds_o\,da_o}{R^2}}{\int_{S_o}\int_{2\pi}L\cos\theta\,d\omega\,ds_o}. \tag{33}$$

This is the most general expression for the configuration factor. If the source is Lambertian and homogeneous, or if S_o and A_o are small in relation to R^2 then L can be removed from the integrals, resulting in

$$F_{s-r} = \frac{\int_{S_o}\int_{A_o}\frac{\cos\theta\cos\psi\,ds_o\,da_o}{R^2}}{\pi S_o} \tag{34}$$

a more conventional form for the configuration factor. As desired, it is purely geometric and has no radiation components. For homogeneous Lambertian sources of radiance L, the flux to a receiver, Φ_{s-r} is given by

$$\Phi_{s-r} = \pi\, S_o L F_{s-r} \tag{35}$$

E. Effect of Refractive Index Changes

For a ray propagating through an otherwise homogeneous medium without losses, it can be shown that the quantity L/n^2 is invariant along the ray. L is the radiance and n is the refractive index of the medium. If the refractive index is constant, the radiance L is constant along the ray. This is known as the *invariance of radiance*.

Suppose this ray passes through a (specular) interface between two isotropic and homogeneous media of different refractive indices, n_1 and n_2, and suppose there is neither absorption nor reflection at the interface. In this case it can be shown that

$$\frac{L_1}{n_1^2} = \frac{L_2}{n_2^2}. \tag{36}$$

Equation 36 shows how radiance invariance is modified for rays passing through interfaces between two media with different refractive indices.

A consequence of (36) is that a ray entering a medium of different refractive index will have its radiance altered, but upon emerging back into the original medium the original radiance will be restored, neglecting absorption, scattering, and reflection losses.

This is what happens to rays passing through the lens of an imaging system. The radiance associated with every ray contributing to a point in an image is the same as when that ray left the object on the other side of the lens (ignoring reflection and transmission losses in the lens). Since this is true of all rays making up an image point, the radiance of an image formed by a perfect, lossless lens equals the radiance of the object (the source).

This may seem paradoxical. Consider the case of a focusing lens, one producing a greater irradiance in the image than in the object. How can a much brighter image have the same radiance as that of the object? The answer is that the increased flux *per unit area* in the image is balanced by an equal reduction in the flux *per unit solid angle* incident on the image. This trading of flux per unit area for flux per unit solid angle is what allows the radiance to remain essentially unchanged.

VI. SOURCES

A. Introduction

The starting point in solving most problems of radiation transfer is determining the magnitude and the angular and spectral distributions of emission from the source. The optical properties of any materials on which that radiation is incident are also important, especially their spectral and directional properties. This section provides comparative information about a variety of sources commonly found in radiometric and photometric problems within the UV, VIS, and IR parts of the spectrum. Definitions used in radiometry and photometry for the reflection, transmission, and absorption properties of materials are provided in Section VII.

Spectral distributions are probably the most important characteristics of sources that must be considered in the design of radiometric systems intended to measure all or portions of those distributions. The matching of a proper detector/filter combination to a given radiation source is one of the most important tasks facing the designer. Section VIII deals with detectors.

B. Blackbody Radiation

All material objects above a temperature of absolute zero emit radiation. The hotter they are, the more they emit. The constant agitation of the atoms and molecules making up all objects involves accelerated motion of electrical charges (the electrons and protons of the constituent atoms). The fundamental laws of electricity and magnetism, as embodied in Maxwell's equations, predict that any accelerated motion of charges will produce radiation. The constant jostling of atoms and molecules in material substances above a temperature of absolute zero produces electromagnetic radiation over a broad range of wavelengths and frequencies.

1. Stefan–Boltzmann Law

The total radiant flux emitted from the surface of an object at temperature T is expressed by the Stefan–Boltzmann law, in the form

$$M_{\mathrm{bb}} = \sigma T^4, \tag{37}$$

where M_{bb} is the exitance of (irradiance leaving) the surface in a vacuum, σ is the Stefan–Boltzmann constant ($5.67031 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$), and T is the temperature in degrees kelvin. The units for M_{bb} in (37) are $\mathrm{W \cdot m^{-2}}$. Using (37), a blackbody at 27°C, $(27 + 273 = 300\ \mathrm{K})$, emits at the rate of 460 W/m^2. At 100°C this rate increases to 1097 W/m^2.

Equation (37) applies to what is called a *perfect* or *full* emitter, one emitting the maximum quantity of radiation possible for a surface at temperature T. Such an emitter is called a *blackbody*, and its emitted radiation is called *blackbody radiation*.

A blackbody is defined as an ideal body that allows all incident radiation to pass into it (zero reflectance) and that absorbs internally all the incident radiation (zero transmittance). This must be true for all wavelengths and all angles of incidence. According to this definition, a blackbody is a perfect absorber, having absorptance 1.0 at all wavelengths and directions. Due to the law of the conservation of energy, the sum of the reflectance R and absorptance A of an opaque surface must be unity, $A + R = 1.0$. Thus, if a blackbody has an absorptance of 1.0, its reflectance must be zero. Accordingly, a perfect blackbody at room temperature would appear totally black to the eye, hence the origin of the name. Only a few surfaces, such as carbon black, carborundum, and gold black, approach a blackbody in these optical properties.

The radiation emitted by a surface is in general distributed over a range of angles filling the hemisphere and over a range of wavelengths. The angular distribution of radiance from a blackbody is constant; that is, the radiance is independent of direction; it is Lambertian. Specifically, this means that $L_\lambda(\theta, \phi) = L_\lambda(0, 0) = L_\lambda$. Thus, the relationship between the spectral radiance $L_{bb\lambda}$ and spectral exitance $M_{bb\lambda}$ of a blackbody is given by (19), repeated here as

$$M_{bb\lambda} = \pi L_{bb\lambda}. \tag{38}$$

If $L_{bb\lambda}$ is in $W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}$ then the units of $M_{bb\lambda}$ will be $W \cdot m^{-2} \cdot nm^{-1}$.

2. Greybodies

Imperfect emitters, which emit less than a blackbody at any given temperature, can be called *greybodies* if their spectral shape matches that of a blackbody. If that shape differs from that of a blackbody the emitter is called a *nonblackbody*.

The Stefan–Boltzmann law still applies to greybodies, but an optical property factor must be included in (37) and (38) for them to be correct for greybodies. That is the *emissivity* of the surface, defined and discussed in Section VII.C.3.

3. Planck's Law

As the temperature changes, the spectral distribution of the radiation emitted by a blackbody shifts. In 1901, Max Planck made a radical new assumption—that radiant energy is quantized—and used it to derive an equation for the spectral radiant energy density in a cavity at thermal equilibrium (a good theoretical approximation of a blackbody). By assuming a small opening in the side of the cavity and examining the spectral distribution of the emerging radiation, he derived an equation for the spectrum emitted by a blackbody. The equation, now called Planck's blackbody spectral radiation law, accurately predicts the spectral radiance of blackbodies in a vacuum at any temperature. Using the notation of this text the equation is

$$L_{bb\lambda} = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)}, \tag{39}$$

where $h = 6.626176 \times 10^{-34}$ J · s is Planck's constant, $c = 2.9979246 \times 10^8$ m/s is the speed of light in a vacuum, and $k = 1.380662 \times 10^{-23}$ J · K^{-1} is Boltzmann's constant. Using these values, the units of $L_{bb\lambda}$ will be $W \cdot m^{-2} \cdot \mu m^{-1} \cdot sr^{-1}$. Plots of the spectral distribution of a blackbody for different temperatures are illustrated in Fig. 11. Each curve is labeled with its temperature in degrees Kelvin. Insignificant quantities of blackbody radiation lie in the visible portion of the spectrum for temperatures below about 1000 K. With increasing temperatures, blackbody radiation first appears red, then white, and at very high temperatures it has a bluish appearance.

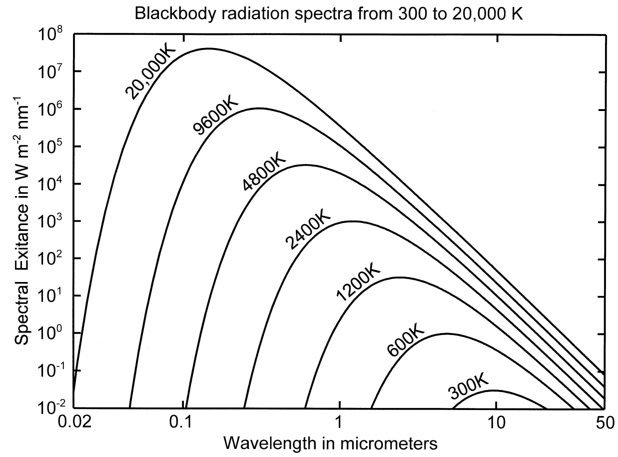

FIGURE 11 Exitance spectra for blackbodies at various temperatures from 300 to 20,000 K, calculated using Eq. (47).

From (38), the spectral exitance $M_{bb\lambda}$ of a blackbody at temperature T is just the spectral radiance $L_{bb\lambda}$ given in (39) multiplied by π.

$$M_{bb\lambda} = \frac{2\pi hc^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)}. \tag{40}$$

4. Luminous Efficacy of Blackbody Radiation

Substituting (40) for the hemispherical spectral exitance of a blackbody into (11) for E_e and (14) for E_v, for each of several different temperatures T, one can calculate the radiation luminous efficacy K_{bb} of blackbody radiation as a function of temperature. Some numerical results are given in Table VI, where it can be seen that, as expected, the luminous efficacy increases as the body heats up to white hot temperatures. At very high temperatures K_{bb} declines, since the radiation is then strongest in the ultraviolet, outside of the visible portion of the spectrum.

5. Experimental Approximation of a Blackbody

The angular and spectral characteristics of a blackbody can be approximated with an arrangement similar to the one shown in Fig. 12. A metal cylinder is hollowed out to form a cavity with a small opening in one end. At the opposite end is placed a conically shaped "light trap," whose purpose is to multiply reflect incoming rays, with maximum absorption at each reflection, in such a manner that a very large number of reflections must take place before any incident ray can emerge back out the opening. With the absorption high on each reflection, a vanishingly small

TABLE VI Blackbody Luminous Efficacy Values

Temperature in degrees K	Luminous efficacy in lm/W
500	7.6×10^{-13}
1,000	2.0×10^{-4}
1,500	0.103
2,000	1.83
2,500	8.71
3,000	21.97
4,000	56.125
5,000	81.75
6,000	92.9
7,000	92.8
8,000	87.3
9,000	79.2
10,000	70.6
15,000	37.1
20,000	20.4
30,000	7.8
40,000	3.7
50,000	2.0

fraction of incident flux, after being multiply reflected and scattered, emerges from the opening. In consequence, only a very tiny portion of the radiation passing into the cavity through the opening is reflected back out of the cavity.

The temperature of the entire cavity is controlled by heating elements and thick outside insulation so that all surfaces of the interior are at precisely the same (known) temperature and any radiation escaping from the cavity will be that emitted from the surfaces within the cavity. The emerging radiation will be rendered very nearly isotropic by the multiple reflections taking place inside (at least over the useful solid angle indicated in Fig. 12, for which the apparatus is designed).

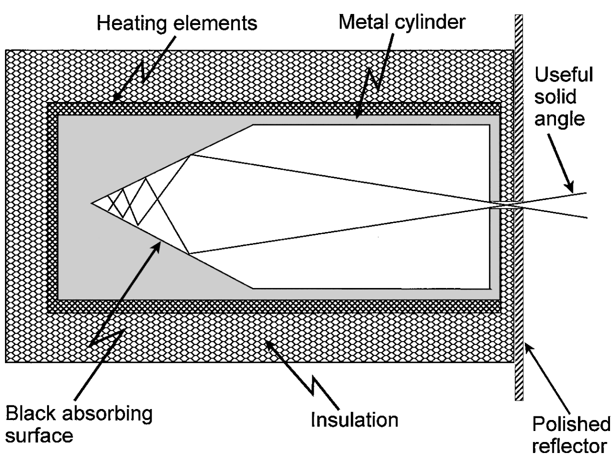

FIGURE 12 Schematic diagram of an approximation to a blackbody.

C. Electrically Powered Sources

Modern tungsten halogen lamps in quartz envelopes produce output spectra that are somewhat similar in shape to those of blackbody distributions. A representative spectral distribution is shown in Fig. 13. This lamp covers a wide spectral range, including the near UV, the visible, and much of the infrared portion of the spectrum. Only the region from about 240 to 2500 nm is shown in Fig. 13.

Although quartz halogen lamps produce usable outputs in the ultraviolet region, at least down to 200 nm, the output at these short wavelengths is quite low and declines rapidly with decreasing wavelength. Deuterium arc lamps overcome the limitations of quartz halogen lamps in this spectral region, and they do so with little output above a wavelength of 500 nm except for a strong but narrow emission line at about 660 nm. The spectral irradiance from a deuterium lamp is plotted in Fig. 14.

Xenon arc lamps have a more balanced output over the visible but exhibit strong spectral "spikes" that pose problems in some applications. Short arc lamps, such as those using xenon gas, are the brightest manufactured sources, with the exception of lasers. Because of the nature of the arc discharges, these lamps emit a continuum of output over wavelengths covering the ultraviolet and visible portions of the spectrum.

The spectral irradiance outputs of the three sources just mentioned cover the near UV, the visible, and the near IR. They are plotted along with the spectrum of a 50-W mercury-vapor arc lamp in Fig. 14. Mercury lamps emit strong UV and visible radiation, with strong spectral

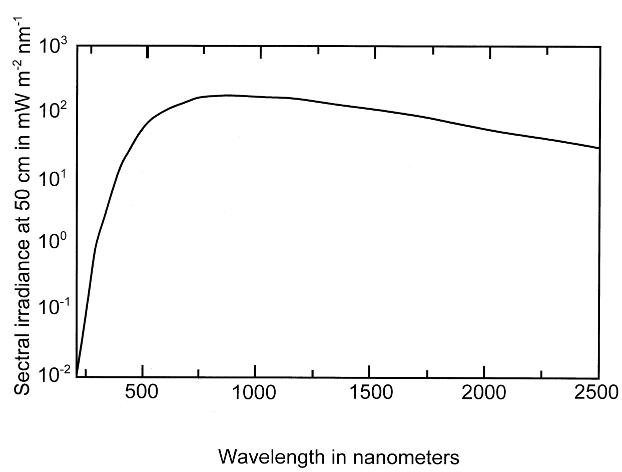

FIGURE 13 Spectral irradiance from a quartz halogen lamp 50 cm from the filament.

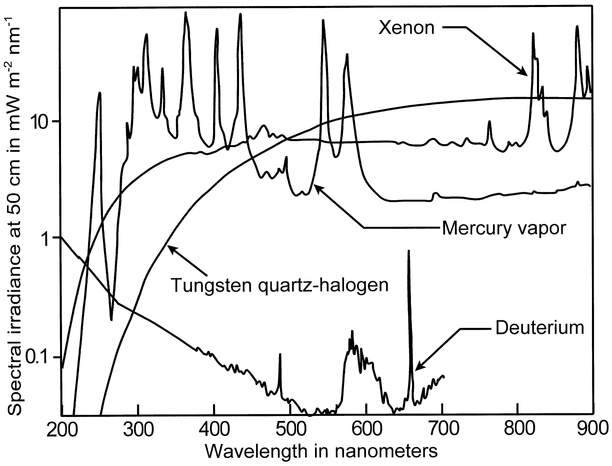

FIGURE 14 Spectral irradiance distributions for four sources of radiation.

lines in the ultraviolet superimposed over continuous spectra.

Tungsten halogen lamps have substantial output in the near infrared. For sources with better coverage of IR-A and IR-B, different sources are more commonly used. Typical outputs from several infrared laboratory sources are shown in Fig. 15. The sources are basically electrical resistance heaters, ceramic and other substances that carry electrical current, which become hot due to ohmic heating, and which emit broadband infrared radiation with moderately high radiance.

In addition to these relatively broadband sources, there are numerous others that emit over more restricted spectral ranges. The light-emitting diode (LED) is one example. It is made of a semiconductor diode with a P-N junction designed so that electrical current through the junction in the forward bias direction produces the emission of optical radiation. The spectral range of emission is limited, but not so much to be considered truly monochromatic. A sample LED spectrum is shown in Fig. 16. LEDs are efficient converters of electrical energy into radiant flux.

Lasers deserve special mention. An important characteristic of lasers is their extremely narrow spectral output distribution, effectively monochromatic. A consequence of this is high optical *coherence*, whereby the phases of the oscillations in electric and magnetic field strength vectors are preserved to some degree over time and space. Another characteristic is the high spectral irradiance they can produce. For more information the reader is referred to modern textbooks on lasers and optics. Most gas discharge lasers exhibit a high degree of collimation, an attribute with many useful optical applications.

A problem with highly coherent sources in radiometry and photometry is that not all of the relationships developed so far in this article governing flux levels are strictly correct. The reason is the possibility for constructive and destructive *interference* when two coherent beams of the same wavelength overlap. The superposition of two or more coherent monochromatic beams will produce a combined irradiance at a point that is not always a simple sum of the irradiances of the two beams at the point of superposition. A combined irradiance level can be more and can be less than the sum of the individual beam irradiances, since it depends strongly on the phase difference between the two beams at the point of interest. The predictions of radiometry and photometry can be preserved whenever they are averaged over many variations in the phase difference between the two overlapping beams.

D. Solar Radiation and Daylight

Following its passage through the atmosphere, direct beam solar radiation exhibits the spectral distribution shown in Fig. 17. The fluctuations at wavelengths over 700 nm are the result of absorption by various gaseous constituents of the atmosphere, the most noticeable of which are water vapor and CO_2. The V-lambda curve is also shown in Fig. 17 for comparison.

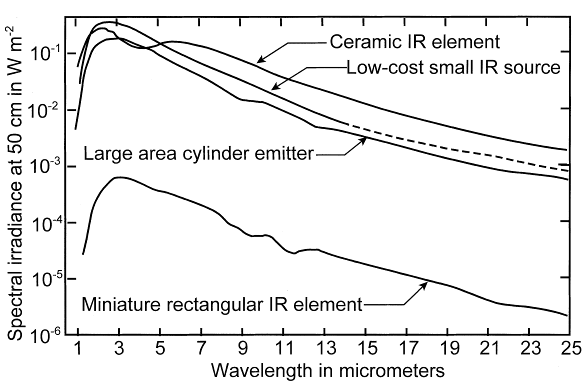

FIGURE 15 Spectral irradiance distributions from four sources of infrared radiation.

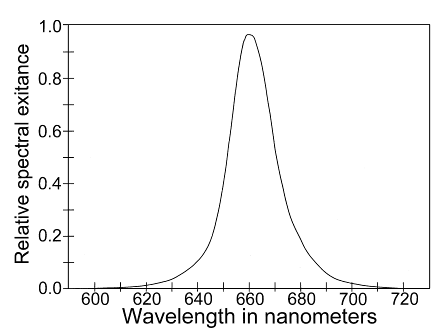

FIGURE 16 Relative spectral exitance of a red light-emitting diode.

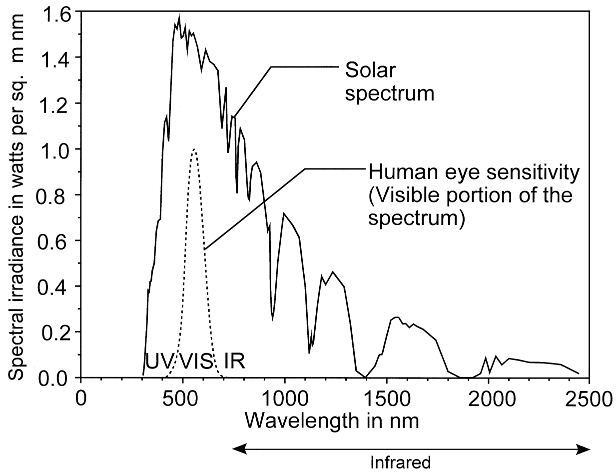

FIGURE 17 Spectral irradiance of terrestrial clear sky direct beam solar radiation.

The spectral distribution of blue sky radiation is similar to that shown in Fig. 17, but the shape is skewed by what is called Rayleigh scattering, the scattering of radiation by molecular-sized particles in the atmosphere. Rayleigh scattering is proportional to the inverse fourth power of the wavelength. Thus, blue light is scattered more prominently than red light. This is responsible for the blue appearance of sky light. (The accompanying removal of light at short wavelengths shifts the apparent color of beam sunlight toward the red end of the spectrum, responsible for the orange-red appearance of the sun at sunrise and sunset.) The spectral distribution of daylight is important in the field of colorimetery and for many other applications, including the daylight illumination of building interiors.

VII. OPTICAL PROPERTIES OF MATERIALS

A. Introduction

Central to radiometry and photometry is the interaction of radiation with matter. This section provides a discussion of the properties of real materials and their abilities to emit, reflect, refract, absorb, transmit, and scatter radiation. Only the rudiments can be addressed here, dealing mostly with terminology and basic concepts. For more information on the optical properties of matter, the reader is directed to available texts on optics and optical engineering, as well as other literature on material properties.

B. Terminology

The improved uniformity in symbols, units, and nomenclature in radiometry and photometry has been extended to the optical properties of materials. Proper terminology can now be identified for the processes of reflection, transmission, and emission of radiant flux by or through material media. Although symbols have been standardized for most of these properties, there are a few exceptions.

To begin, the CIE definitions for *reflectance*, *transmittance*, and *absorptance* are provided:

1. *Reflectance* (for incident radiation of a given spectral composition, polarization and geometrical distribution) (ρ): Ratio of the reflected radiant or luminous flux to the incident flux in the given conditions (unit: 1)
2. *Transmittance* (for incident radiation of given spectral composition, polarization and geometrical distribution) (τ): Ratio of the transmitted radiant or luminous flux to the incident flux in the given conditions (unit: 1)
3. *Absorptance*: Ratio of the absorbed radiant or luminous flux to the incident flux under specified conditions (unit: 1)

These definitions make explicit the point that radiation incident upon a surface can have nonconstant distributions over the directions of incidence, over polarization state, and over wavelength (or frequency). Thus, when one wishes to measure these optical properties, it must be specified how the incident radiation is distributed in wavelength and direction and how the emergent detected radiation is so distributed if the measurement is to have meaning. Polarization effects are not dealt with here. The wavelength dependence of radiometric properties of materials is indicated with a functional lambda λ thus: $\tau(\lambda)$, $\rho(\lambda)$, and $\alpha(\lambda)$.

The directional dependencies are indicated by specifying the spherical angular coordinates (θ, ϕ) of the incident and emergent beams.

In other fields it is common to assign the ending *-ivity* to *intensive*, inherent, or bulk properties of materials. The ending *-ance* is reserved for the *extensive* properties of a fixed quantity of substance, for example a portion of the substance having a certain length or thickness. (Sometimes the term *intrinsic* is used instead of intensive and *extrinsic* is used instead of extensive.) Figure 18 illustrates the difference between intrinsic and extrinsic reflection properties and introduces the concept of interface reflectivity. An example from electronics is the 30 ohm electrical resistance of a 3 cm length of a conductor having a *resistivity* of 10 ohms/cm.

According to this usage in radiometry, *reflectance* is reserved for the fraction of incident flux reflected (under defined conditions of irradiation and reception) from a finite and specified portion of material, such as a 1-cm-thick plate of fused silica glass having parallel, roughened

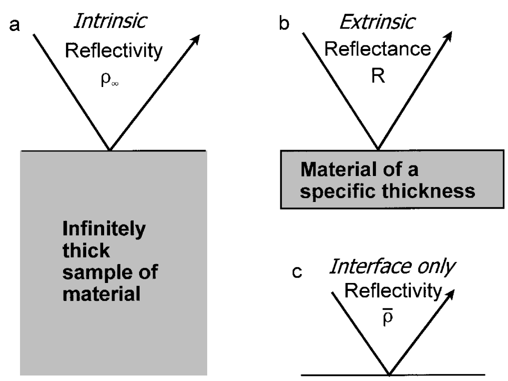

FIGURE 18 (a) Intrinsic versus (b) extrinsic reflection properties of a material. (c) Interface reflectivity.

surfaces in air. The *reflectivity* of a material, such as BK7 glass, would refer to the ratio of reflected to incident flux for the perfectly smooth (polished) interface between an optically infinite thickness of the material and some other material, such as air or vacuum. The infinite thickness is specified to ensure that reflected flux from no other interface can contribute to that reflected by the interface of interest, and to ensure the collection of subsurface flux scattered by molecules of the material.

CIE definitions for the intrinsic optical properties of matter read as follows:

1. *Reflectivity* (of a material) (ρ_∞): Reflectance of a layer of the material of such a thickness that there is no change of reflectance with increase in thickness (unit: 1)
2. *Spectral transmissivity* (of an absorbing material) ($\tau_{\mathrm{i,o}}(\lambda)$): Spectral internal transmittance of a layer of the material such that the path of the radiation is of unit length, and under conditions in which the boundary of the material has no influence (unit: 1)
3. *Spectral absorptivity* (of an absorbing material) ($\alpha_{\mathrm{i,o}}(\lambda)$): Spectral internal absorptance of a layer of the material such that the path of the radiation is of unit length, and under conditions in which the boundary of the material has no influence (unit: 1)

One can further split the reflectivity ρ_∞ into interface-only and bulk property components. We use the symbol $\bar{\rho}$, *rho* with a bar over it, to indicate the interface contribution to the reflectivity. The interface transmissivity $\bar{\tau}$ is included in this notational custom. Since there is presumed to be no absorption when radiation passes through or reflects from an interface, $\bar{\tau} + \bar{\rho} = 1.0$.

To denote the optical properties of whole objects, such as parallel sided plates of a material of specific thickness, we use upper case Roman font characters, as with the symbol R for reflectance, and the "-ance" suffix.

This terminology is summarized as follows:

$\bar{\rho}$ Reflectivity of an interface
ρ Reflectivity of a pure substance, including both bulk and interface processes
R Reflectance of an object
$\bar{\tau}$ Transmissivity of an interface
τ (Internal) linear transmissivity of (a unit length of) a transparent or partially transparent substance, away from interfaces; unit: m^{-1}
T Transmittance of an object
α (Internal) linear absorptivity of (a unit length of) a transparent or partially transparent substance, away from interfaces; unit: m^{-1}
A Absorptance of an object

C. Surface and Interface Optical Properties

1. Conductor Optical Properties

A perfect conductor, characterized by infinitely great conductivity, has an infinite refractive index and penetration of electromagnetic radiation to any depth is prohibited. This produces perfect reflectivity. Real conductors such as aluminum and silver do not have perfect conductivities nor do they have perfect reflectivities. Their reflectivities are quite high, however, over broad spectral ranges. They are therefore useful in radiometric and photometric applications. Unprotected mirrors made of these materials, unfortunately, tend to degrade with exposure to air over time and they are seldom used without protective overcoatings. The normal incidence spectral reflectances of optical quality glass mirrors coated with aluminum, with aluminum having a magnesium fluoride protective overcoat, with aluminum having a silicon monoxide overcoat, with silver having a protective dielectric coating, and with gold are shown in Fig. 19. The reflectance of these surfaces, already quite high at visible and infrared wavelengths, increases with incidence angle, approaching unity at 90°.

2. Nonconductor Optical Properties

Consider the extremely thin surface region of a perfectly smooth homogeneous and isotropic dielectric material, its interface with another medium such as air, water, or a vacuum, an interface normally too thin to absorb significant quantities of the radiation incident on it. Absorption is not considered in this discussion since it is considered to be a bulk or volume characteristic of the material. Radiation incident upon an interface between two different materials is split into two parts. Some is reflected, and the rest is transmitted. The fraction of incident flux that is reflected is called the interface reflectivity $\bar{\rho}$ and the fraction

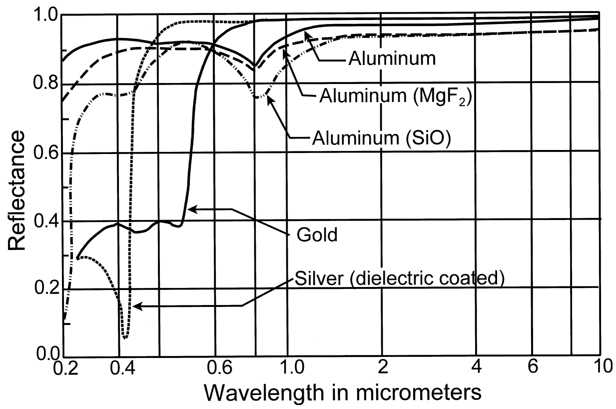

FIGURE 19 Spectral reflectances of commercially available metallic mirror materials.

transmitted is the interface transmissivity $\bar{\tau}$. The variations of $\bar{\rho}$ and $\bar{\tau}$ with angle of incidence are given by Fresnel's formulas, which can be found in most optical textbooks.

When the bulk medium optical properties are considered, the situation is more complicated, since the transmitted flux can be absorbed and "re-reflected" and/or scattered by the medium below the interface, by direction- and wavelength-dependent processes.

When both interface and interior optical processes are considered together, the spectral and directional variations in transmissivity and reflectivity become still more important, and the absorptivity of the medium also comes into play. The wavelength dependence of the optical properties of materials is indicated with a functional λ notation thus: $\tau(\lambda)$, $\alpha(\lambda)$, and $\rho(\lambda)$. The direction of an element of solid angle $d\omega$ is indicated using the spherical angular coordinates (θ, ϕ), illustrated in Fig. 6. Using these coordinates, the directional dependence of optical properties is indicated with the functional notation: $\tau(\theta, \phi)$ and $\rho(\theta, \phi)$, and the combined spectral and directional properties thus: $\tau(\lambda, \theta, \phi)$ and $\rho(\lambda, \theta, \phi)$.

3. Surface Emission Properties

The emissive properties of greybody and nonblackbody surfaces are characterized by their *emissivity* ϵ. Emissivity is the ratio of the actual emission of thermal radiant flux from a surface to the flux that would be emitted by a perfect blackbody emitter at the same temperature. According to the terminology guidelines given earlier, the term *emissivity* should be reserved for the surface of an infinitely thick slab of pure material with a polished surface, while emittance would apply to a finite thickness of an actual object. For substances opaque at the wavelengths of emission, however, the intrinsic and extrinsic versions of ϵ are the same, leading to two acceptable names for the same quantity.

As was the case for reflectance and transmittance, *emittance* is in general a directional quantity and can be specified as $\epsilon(\theta, \phi)$. The directional emittance at normal incidence ($\theta = 0$) is called the *normal emittance*. The average of the directional emittance over the whole hemispherical solid angle is called the *hemispherical emittance*. The emittances shown in Table VII are for hemispherical emittance into a vacuum.

The spectral exitance $M_\lambda(\lambda)$ of a nonblackbody can be specified using the spectral emittance $\epsilon(\lambda)$:

$$M_\lambda(\lambda) = \epsilon(\lambda) M_{bb\lambda}(\lambda) \tag{41}$$

with $M_{bb\lambda}$ being given by (40).

4. Directional Optical Properties

Radiation incident at a point in a surface can come to that point from many directions. The concept of a pencil of rays, rays filling a right circular conical solid angle, like the shape of the tip of a well-sharpened wooden pencil, is useful in describing the directional dependences of transmittance and reflectance, for both theoretical treatments and in practical measurements.

In making transmittance or reflectance measurements, a sample to be tested is illuminated with radiation filling some solid angular range of directions. The reflected or transmitted flux is then collected over another range of directions within some second solid angle.

In order for the transmittance or reflectance value to have meaning, either theoretically or experimentally, it is essential that the directions and solid angles of incidence and emergence be specified. These tell the ranges of angles involved in the measurement.

In discussing reflectance and transmittance, there are three categories of solid angles of interest, and several different definitions of reflectance and transmittance using combinations of these. The three solid angle categories are *directional*, *conical*, and *hemispherical*. They are illustrated in Fig. 20. There are nine possible combinations

TABLE VII Hemispherical Emittance Values for Typical Materials

Material	Emittance from 4 to 16 μm
White paint	0.90
Black asphalt and roofing tar	0.93
Light concrete	0.88
Pine wood	0.60
Stainless steel	0.18 to 0.28
Galvanized sheet metal	0.13 to 0.28
Aluminum sheet metal	0.09
Polished aluminum	0.05 to 0.08

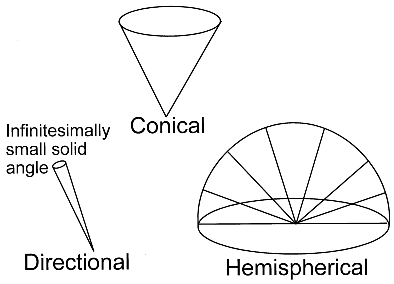

FIGURE 20 Geometry for directional, conical, and hemispherical solid angles.

of these three kinds of solid angle, resulting in the nine names for them given below. The most commonly used ones are indicated in bold face type.

- **Bidirectional**
- Directional–conical
- Directional–hemispherical
- Conical–directional
- Hemispherical–directional
- **Biconical**
- **Conical–hemispherical**
- Hemispherical–conical
- **Bihemispherical**

The first five of these are mainly found in theoretical discussions. The last four are used in reflectance and transmittance measurements. Solar optical property standards published by various organizations refer to conical–hemispherical measurements. The reason is that for most practical problems, it is only the total transmitted or reflected irradiance due to the directly incident beam alone that is of interest. For other applications and more general or more complex situations, the biconical definition is the most important (see Fig. 21). Theoretical treatments

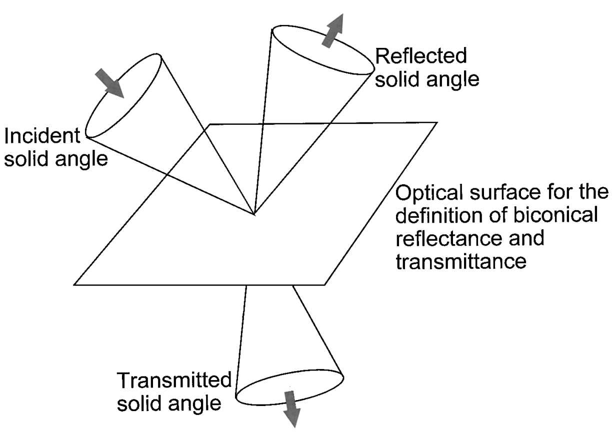

FIGURE 21 Geometry for the definition of biconical transmittance and reflectance.

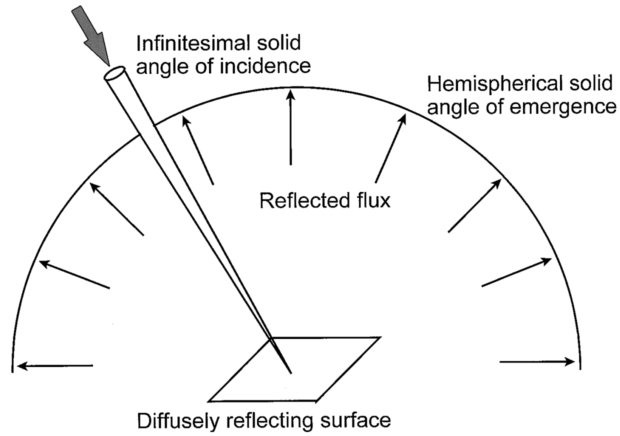

FIGURE 22 Geometry for the definition of directional–hemispherical reflectance.

of radiative transfer deal almost exclusively with the "directional" versions of the definitions. Sometimes the terminology "bidirectional" is used to refer to biconical measurements. This is appropriate when the solid angles involved are small. Example geometries are shown in Figs. 22 and 23.

VIII. THE DETECTION OF RADIATION

There is considerable variety in the kinds of devices (called *detectors* or *sensors*) available for the detection and measurement of optical radiation. Some respond to the heat produced when radiant energy is absorbed by a surface. Some convert this heat into mechanical movement, and some convert it into electricity. Photographic emulsions convert incident radiation into chemical changes

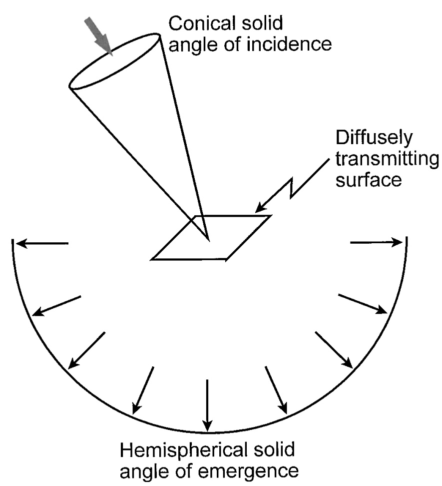

FIGURE 23 Geometry for the definition of conical–hemispherical transmittance.

made visible by the development process. Other detectors convert electromagnetic radiation directly into electrical energy.

Many electrical effects have been devised to amplify the typically small electrical signals produced by detectors to levels easier to measure. There are unavoidable small fluctuations found in the output signals of all detectors which mask or obscure the signal resulting from incident radiation. This is called *noise*, and various means have been devised to reduce its effect on measurement results.

Most detectors with high sensitivity (strong response to weak flux levels) have nonuniform spectral responses. Often the inherent spectral response of the detector is not the one desired for the application. In most such cases it is possible to add a spectrally selective filter, producing a combined filter/detector response closer to what is desired. Matching filters with detectors for this purpose can be difficult, but it is one of the most important problems in radiometry and photometry.

The output voltage or current of most detectors depends on more than just the strength of the incident flux. Temperature T can have an effect, as can the direction θ of incident radiation. If we combine all these dependencies into one single spectral response function, $R(\lambda, \Phi_\lambda, T, \theta, x, y, z, \ldots)$, we can write an equation for the output signal $S(\lambda)$ at wavelength λ as a function of the incident spectral radiant flux Φ_λ.

$$S(\lambda) = R(\lambda, \Phi_\lambda, T, \theta, x, y, z, \ldots)\,\Phi_\lambda + S_o, \qquad (42)$$

where x, y, and z are other physical parameters on which the detector's output might depend and S_o is the "dark signal," the signal output of the detector (be it current or voltage) when the flux on it is zero.

Considering only the spectral dependency in the above equation, the total output signal S, in terms of the incident spectral irradiance $E_\lambda(\lambda)$, a spectral altering filter transmittance $T(\lambda)$, the detector responsivity $R(\lambda)$, and the detector area A will be given by

$$S = A\int_0^\infty E_\lambda(\lambda)\,T(\lambda)\,R(\lambda)\,d\lambda + S_o. \qquad (43)$$

If the detector spectral response $R(\lambda)$ is constant, at the value R_o, over some wavelength range of interest, a spectrum altering filter will not be needed and the only integral remaining in (43) is over the spectral irradiance. The result is the simpler equation

$$S = AE_eR_o + S_o, \qquad (44)$$

which may be solved for the incident irradiance.

$$E_e = k(S - S_o), \qquad (45)$$

where $k = 1/AR_o$ is the calibration constant for a detector used as a normal incidence irradiance meter and S_o is the *dark signal*.

Values are published by manufacturers for the responsivity and other characteristics of their detectors. These performance figures are approximate and are used mainly for the selection of a detector with the proper characteristics for the given application—not for calibration purposes, with a notable exception, described in Section X.C.

It is important to note that the smaller the detector, the lower the noise level produced. There is therefore usually a noise penalty for using a detector having a sensitive surface significantly larger than the incident beam. The unused area contributes to both the dark current and to the noise but not to the signal. The signal-to-noise ratio (SNR) of a detector can therefore be improved by using a detector only as large as needed to match the beam of flux placed on the detector by the conditioning optics.

Often the flux incident on a detector is "chopped," is made to switch on and off at some frequency f. Much of the noise in such detectors can be suppressed from the output signal if the alternating output signal from the detector is amplified only at the chopping frequency f. The larger the frequency bandwidth Δf of this amplification circuit, the greater the noise in the amplified signal. This leads to the concept of *noise equivalent power*, or NEP, of the detector. This is the flux incident on the detector, in units of watts, which produces an amplified signal just equal to the *root mean square* (*rms*) of the noise. It is generally desirable to have a low value of the NEP, which is quoted in units of $\mathrm{W \cdot Hz^{-1/2}}$. The lower the value of the NEP the lower the flux the detector can measure with a good SNR. *Detectivity D* is the reciprocal of NEP. *Normalized detectivity, D**, is the detectivity normalized for detector area and frequency bandwidth. It has units of $\mathrm{Hz^{\frac{1}{2}} \cdot (cm^2)^{1/2} \cdot W^{-1}}$.

The spectral detectivities of a variety of detectors are shown in Fig. 24. One thing is clear from those plots. Broad spectral coverage generally comes at the expense of detectivity.

Once an appropriate detector has been selected, it will generally be placed in an optical–mechanical system having the effect of conditioning the flux prior to its receipt by the detector. This conditioning can consist of chopping, focusing incident flux into a narrow conical solid angular range of angles, and/or spectral filtering.

IX. RADIOMETERS AND PHOTOMETERS, SPECTRORADIOMETERS, AND SPECTROPHOTOMETERS

A. Introduction

Radiometer is the term given to an instrument designed to measure radiant flux. Some radiometers measure the radiant flux contained in a beam having a known solid

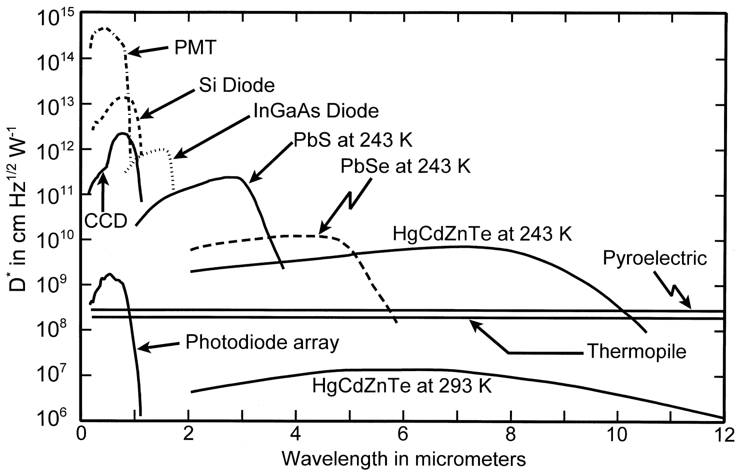

FIGURE 24 Representative spectral normalized detectivities of a variety of detectors.

angle and cross-sectional area. Others measure the flux received from a large range of solid angles. Some radiometers measure over a large wavelength range. These are termed *broadband*. Others perform measurements only over a narrow spectral interval. When the shape of the spectral response of a broadband radiometer is made to match the human spectral photopic efficiency function, the V-lambda curve, it is called a *photometer.*

Some narrow spectral interval radiometers are made which scan the position of their narrow spectral interval across the spectrum. These are called *spectroradiometers.* They are used to measure the spectral flux, irradiance, or radiance received by them.

Spectrophotometers are misnamed. This term is generally applied to neither radiometers nor photometers but to transmissometers or to reflectometers—instruments measuring an optical property—which scan over a range of monochromatic wavelengths. In spite of the inclusion of "photo" in the name, the human photopic spectral response function (the V-lambda curve) is generally not employed in the use of spectrophotometers. They might therefore more properly be called *spectral transmissometers* (or *reflectometers*).

Radiometers are divided into radiance and irradiance subclasses. Instruments with intentionally broad spectral coverage are called *broadband radiometers.* Photometers are similarly divided into luminance and illuminance versions.

B. Spectral Response Considerations

In the practical use of radiance (and irradiance) meters, it is especially important to be cognizant of the spectral limitations of the meter and to include these limits when reporting measurement results. Flux entering the meter having wavelengths outside its range of sensitivity will not be measured. In such cases, the measurements will only sample a portion of the incident flux and should be so reported. Well-built photometers do not share this characteristic. If the spectral response of a photometer strictly matches the shape of the V-lambda curve, then flux outside the visible wavelength range *should* not be measured, will not be measured, will not be recorded, and cannot be reported. Furthermore, in this case of perfect spectral correction, *any* spectral distribution of radiation incident on the photometer will be measured correctly without spectral response errors. On the other hand, if a photometer's spectral response does *not* quite match the shape of the V-lambda curve, the resulting errors can be small or large depending upon the spectral distribution of flux from the source over the spectral region of the departure from $V(\lambda)$ response. Consider, for example, the case of a measurement of the illuminance from a Helium Neon laser beam at wavelength 632.8 nm. This wavelength is at the red edge of the visible spectrum and a relatively small error in the V-lambda correction of a photometer at this wavelength can yield a large error in the measurement of illuminance from this source.

C. Cosine Correction

A consequence of the cosine law is that the output of a perfect irradiance meter illuminated uniformly with collimated radiation fully filling its sensing area will decrease with the cosine of the angle of incidence as that angle increases from zero to 90°. Such behavior is called "good cosine correction." Most detectors do not have this desirable characteristic by themselves. To restore good cosine response, some correction method is needed if a detector is to work properly as an irradiance or illuminance meter. Furthermore, the housings of many detectors shade their sensitive surfaces at some angles of incidence, again calling for some means of angular response correction.

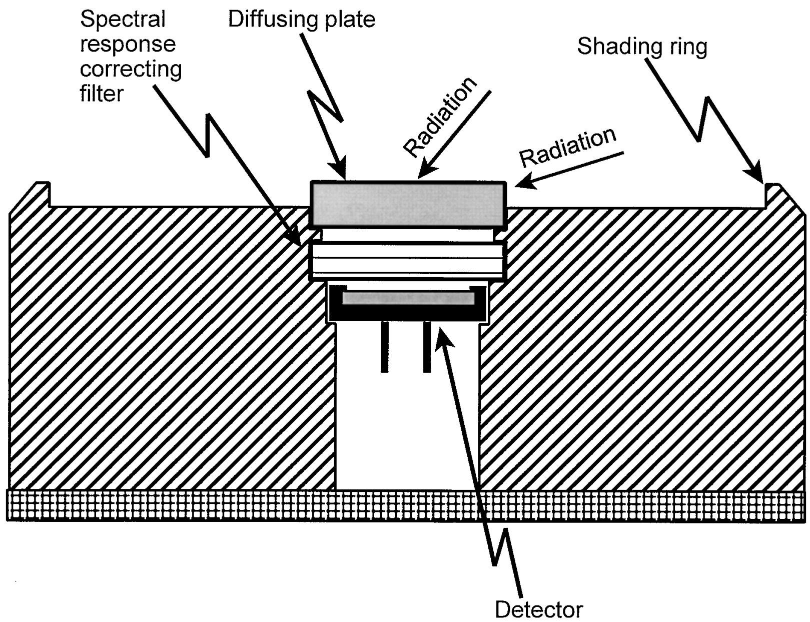

FIGURE 25 Schematic illustration of the features of an irradiance/illuminance meter.

A common method, shown in Fig. 25, for providing the needed correction is to cover the detector with a sheet of milk-white, highly diffusing, semitransparent material having good (and ideally constant) hemispherical–conical spectral transmittance over the spectral range of good detector sensitivity. The idea is that no matter how the incident radiation falls on this material, a fixed and constant fraction of it will be delivered to the detector over a range of angles. In practice, no diffusing sheet has been found that satisfies this ideal perfectly.

What is done is to experiment with a variety of diffusing materials, surface roughnesses, and geometrical configurations until a combination is found that provides reasonably good cosine correction.

A solution to this problem is to limit the size of the diffusing sheet and allow it to extend above the detector housing so that at large angles of incidence some of the incident flux will be received by the *edge* of the sheet, this edge being perpendicular to the front surface. Thus, as more and more flux is reflected from the front of the sheet, more and more will be transmitted through its edge, since in this case the incidence angle is decreasing and the exposed area incrases. At an 80° angle of incidence, for example, little flux will enter through the front face of the diffuser, but much more will enter through the edge.

A problem remains, however. True cosine response drops to zero at 90°, whereas the exposed edge of the diffuser receives considerable quantities of flux at this angle. The cosine corrector must be designed so that no flux can reach the detector for angles of incidence at and greater than 90°. The usual solution to this requirement is illustrated generically in Fig. 25.

As the angle of incidence increases, more and more flux will reach the edge of the detector until the angle of incidence approaches 90°, at which point the shading ring begins to shade the edge. Finally, at 90°, the diffuser is shaded completely and no flux can reach it. The design of the specific dimensions of this cosine correction scheme depends strongly on the biconical optical properties of the diffusing sheet, the geometrical placement of the detector below it, and the angular response characteristics of the detector itself. Finding the right geometry is often a hit-or-miss proposition. Even if a good design is found, the quality of the corrected cosine response can suffer if the properties of the diffusing material change in time, from batch to batch of manufacture, or with the wavelength of incident radiation. Making a good cosine corrector is one of the most difficult problems in the manufacture of good quality, accurate irradiance and illuminance meters.

A way of providing better cosine correction than the one diagramed in Fig. 25 is through the use of an integrating sphere. An arrangement for utilizing the desirable properties of the integrating sphere is illustrated in Fig. 26. The approach is based on the following idealized principle. Flux entering a small port in a hollow sphere whose interior surface is coated with a material of extremely high diffuse reflectance will be multiply reflected (scattered) a large number of times, in all directions, with little loss on each reflection. If a small hole is placed in the side of the sphere and shielded from flux that has not been reflected at least once by the sphere, the flux emerging from this hole

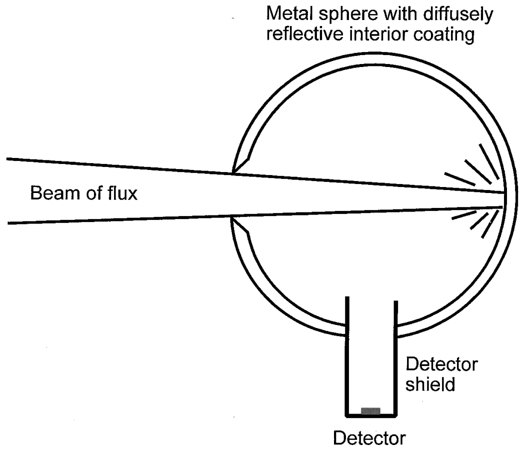

FIGURE 26 Integrating sphere cosine correction.

will be a fixed and constant fraction of the flux entering the other hole, regardless how the flux entering the input port is distributed in angle.

Real integrating spheres, with reflectances less than 1.0 and with entrance and exit ports of finite areas cannot achieve this idealized performance. They can be made to approach it closely, but they are not efficient at delivering flux to the detector for measurement, so irradiance and illuminance meters employing integrating spheres generally suffer lower sensitivities.

X. CALIBRATION OF RADIOMETERS AND PHOTOMETERS

A. Introduction

Radiometers and photometers involve a number of components, all contributing to the overall sensitivity of the instrument to incident radiation. Although one could in principle determine the contribution of each individual component to the overall calibration of the instrument, in practice this procedure is seldom used. Instead, the complete instrument is calibrated all at once.

Calibration is usually a two-step process. First one determines the mathematical transformation needed to convert an output electrical signal into an estimate of the input flux in the units desired for the quantity being measured. Second, one ensures the accuracy of this transformation over time as the characteristics of the components making up the radiometer or photometer change or drift.

There are two approaches to calibrating or recalibrating a radiometer/photometer. In the first case, one uses the radiometer/photometer to measure flux from a standard source whose emitted flux is known accurately in the desired units and then applies a suitable transformation to convert the output signal to the proper magnitude and units of the standard input. For this to work, it is critical that the overall response of the radiometer/photometer be constant over the period of time between calibrations. The output conversion transformation can be either in hardware, where the sensitivity of the radiometer is adjusted so that it reads correctly, or in "software," where a calibration constant is multiplied by the output signal to convert it to the proper value and units every time a measurement is made.

In the second approach to calibration, one measures the flux from an uncalibrated source, first with the device to be calibrated, and then with an already-calibrated standard radiometer/photometer having identical field of view and spectral response. The output of the device is then calibrated to be identical to the measured result and units obtained with the standard radiometer/photometer. Once the calibration is performed, or the calibration transformation is known, it can be applied to subsequent measurements and the device is thereby said to be calibrated.

B. Standard Sources

For radiometers and photometers whose calibration drifts slowly over time, one can calibrate the device when it is first fabricated and then recalibrate it periodically over some acceptable time period. For most accurate results, it is advisable to recalibrate frequently at first, and to then increase the time interval between recalibrations only after a history of drift has been established. For precise radiometry and photometry, a *working standard* or *transfer standard* is used to make frequent calibration checks between (or even during) measurements to account for the effects of small residual drifts in the calibration of a radiometer or photometer.

Historically, the focus of calibration was on the preparation of standard sources, most notably standard lamps, which produce a known and constant quantity of flux giving a known irradiance at a fixed distance from the emitting element. A typical measurement configuration is illustrated schematically in Fig. 27(a). There has been a shift to the use of calibrated detection standards; that is, detectors whose responsivity is sufficiently constant and reproducible over time to make it possible to calibrate other detectors or radiometers based on these standard detectors. Standard lamps are still available as calibrated sources, however. These generally produce a fixed output flux distribution with wavelength. Because of the possibility of nonlinear response effects, radiometers and photometers should be calibrated only over their ranges of linearity, within which a standard lamp can be found.

Many nations maintain primary standards for radiometry (and photometry) in national laboratories dedicated to this purpose. In the United States, such standards are maintained by the National Institute of Standards and

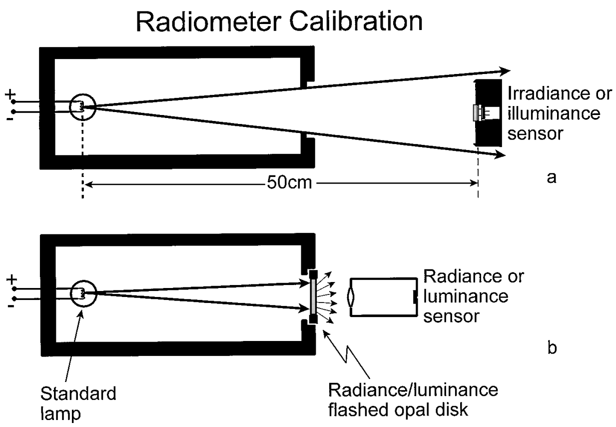

FIGURE 27 (a) Calibration arrangement for irradiance/illuminance. (b) Calibration arrangement for radiance/luminance.

Technology in Gaithersburg, MD. From these are derived secondary standards (also called transfer standards) that can be maintained at private laboratories or by other organizations for the purpose of calibrating and recalibrating commercially and custom produced radiometers and photometers. Working standards are standards derived from secondary standards but which are designed and intended for easy and repeated use to check the calibration of a radiometric or photometric system periodically during or between measurements.

1. Calibration of Radiance and Irradiance Meters

Calibrations using a standard lamp frequently utilize specially designed tungsten filament lamps whose emitting characteristics are known to be quite constant over a period of time if the lamp is not frequently used. Such lamps must be operated with precisely the same electrical current through the filament as when their calibrations were initially set. Specially designed power supplies are made for use with such lamps. These power supplies ensure the constancy of this filament current and also keep track of how many hours the filament has been operated since initial calibration.

One can obtain irradiance standard lamps commercially and use them for the calibration of broadband irradiance sensors. They must be operated according to manufacturer specifications and care must be taken to avoid stray light from the source reflecting from adjacent objects and into the radiometer being calibrated.

Over the years, researchers at the National Institute of Standards and Technology have worked to develop improved standards of spectral radiance and irradiance for the ultraviolet, visible, and near infrared portions of the spectrum. The publications of that U.S. government agency should be consulted for the details.

2. Calibration of Luminance and Illuminance Meters

Standard sources of radiance and irradiance that emit usable quantities of radiation over the visible portion of the spectrum can be used as standards for the calibration of photometers if the photometric outputs of these sources is known. Commercial radiometric and photometric standards laboratories generally can supply photometric calibrations for their radiometric sources for modest additional cost. The most common source is the incandescent filament lamp, with its characteristic spectral output distribution. If the primary use of the photometer being calibrated is to measure light levels derived from sources with similar spectral distributions, and if the V-lambda correction of the photometer is good, then use of tungsten filament standard lamps is an acceptable means of calibration.

If the photometer is intended for measurement of radiation with substantially different spectral distribution and the V-lambda correction is not good, then significant measurement errors can result from calibration using tungsten sources. Fortunately, other standard spectral distributions have been defined. They are based on phases of daylight (primarily for colorimetric applications). Sources exhibiting approximations of these distributions have been developed. For cases of imperfect V-lambda correction, it is recommended that calibration sources be used that more closely match the distributions to be measured with the photometer.

C. Calibrated Detectors

Calibrated silicon photodetectors are now available as transfer or working standards based on the NIST *absolute spectral responsivity scale.* Current information about NIST calibration services can be found at web site http://www.physics.nist.gov.

D. National Standards Laboratories

Anyone concerned with calibration of radiometers and photometers can benefit greatly from the work of the National Institute of Standards and Technology and its counterparts in other countries.

NSSN offers a web-based comprehensive data network on national, foreign, regional, and international standards and regulatory documents. A cooperative partnership between the American National Standards Institute (ANSI), U.S. private-sector standards organizations, government agencies, and international standards organizations, NSSN can help in the identification and location of national standards laboratories offering services in radiometry and photometry outside the United States.

World Wide Web address: http://www.nssn.org. The web address for the International Standards Organization (ISO) is http://www.iso.ch. A list of national metrology laboratories can be found at http://www.vnist.gov/oiaa/national.htm.

ACKNOWLEDGMENT

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