MAE 3013 Mechanical and Aerospace Engineering Analysis
Chapter 19 Numerics in General

School of Mechanical and Aerospace Engineering
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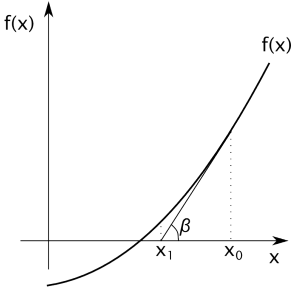
Newton-Raphson Method

- Consider the function shown below. We want to find the location where the function crosses the x-axis.



- Starting from point x_0, a tangent line is drawn to the curve at the point $f(x_0)$. The location where this line intersects the x-axis will be our next approximation of the root.

Newton-Raphson Method, cont.

- The slope of the line can be given by:

$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

solving for x_1 yields:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Newton-Raphson Method, cont.

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- This can be generalize to:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example

Use the same example as before: $x - 2\sin x = 0$.

Secant Method

What if the derivate of the function is unknown? In the secant method the difference formula is used to approximate the derivative:

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

This approximation is substituted into the Newton-Raphson formula to obtain:

$$x_{n+1} = x_n - f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

Note we need two starting values x_n and x_{n-1}.

Example

Using the same equation as before: $x - 2\sin x = 0$

Newton's Method for a Set of Equations

What if we have an N-equation, N-unknown set of non-linear equations? The Newton-Raphson Method can be extended from:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

to: $$\bar{x}_{new} = \bar{x}_{old} - [J(\bar{x}_{old})]^{-1} * \overline{f(\bar{x}_{old})}$$

The Jacobian $$J(\bar{x}_{old}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & . \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & . \\ . & . & . \end{bmatrix}$$ is a matrix of partial deris.

Notice that we need to take the inverse of a matrix .... or do we?

Newton's Method for a Set of Equations

$$\bar{x}_{new} = \bar{x}_{old} - [J(\bar{x}_{old})]^{-1} * \overline{f(\bar{x}_{old})}$$

Define: $\Delta x = [J(\bar{x}_{old})]^{-1} * \overline{f(\bar{x}_{old})}$

Which can be rearranged as: $[J(\bar{x}_{old})]\Delta x = \overline{f(\bar{x}_{old})}$

Which resembles A x = b, and can be solve $[J(\bar{x}_{old})]\Delta x = \overline{f(\bar{x}_{old})}$

Procedure: given a vector of functions, $\overline{f(\bar{x})}$ and an initial guess: $\bar{x}$

Form the Jacobian: $J(\bar{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & . \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & . \\ . & . & . \end{bmatrix}$

and use GE to solve: $[J(\bar{x}_{old})]\Delta x = \overline{f(\bar{x}_{old})}$ for: Δx

and finally: $\bar{x}_{new} = \bar{x}_{old} - \Delta x$

Example

Example:

$$\overline{f(x,y)} = \begin{bmatrix} \sin(x) - 3y \\ e^y + \cos(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$