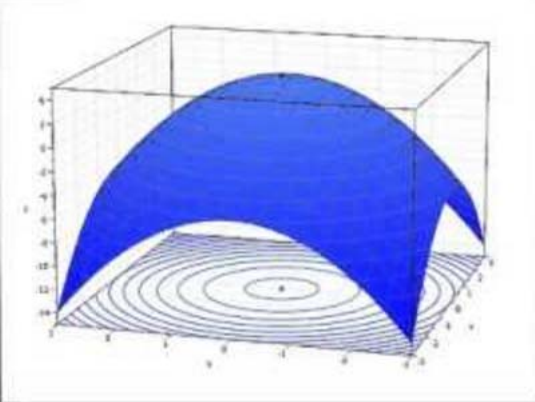


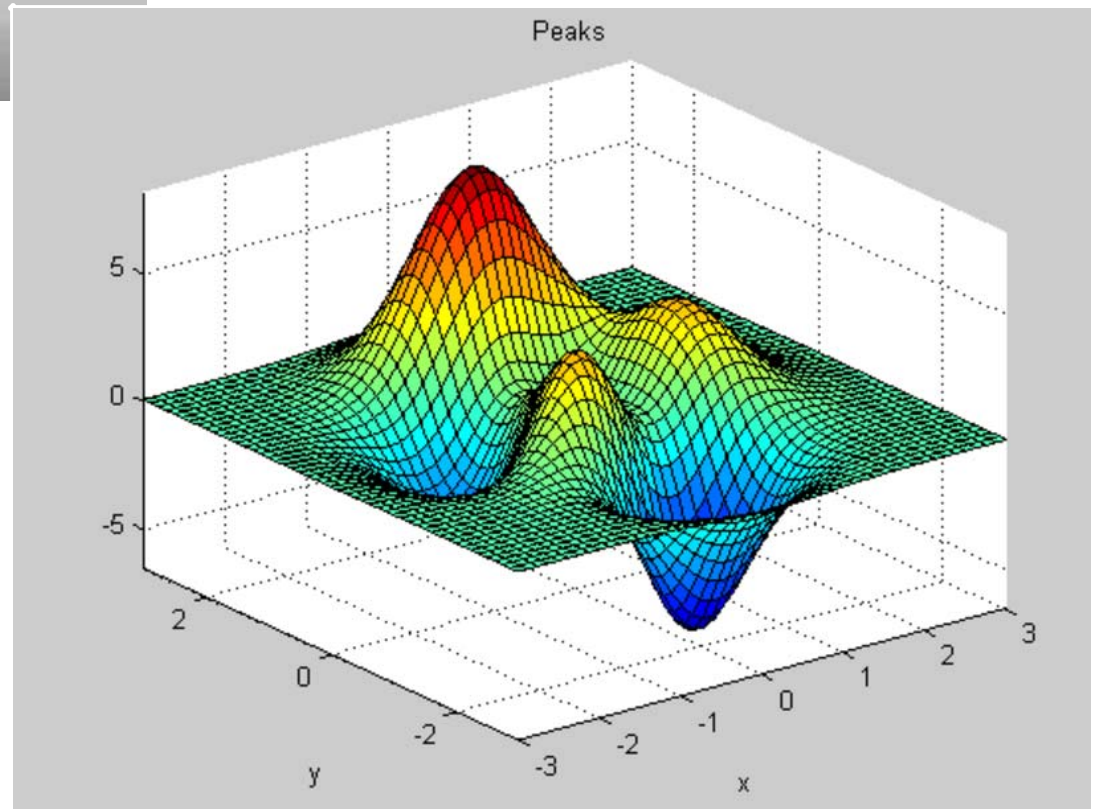
# Optimization

## Nonlinear programming



Find the **Location** of the Maximum or Minimum Value

<https://ocw.mit.edu/courses/sloan-school-of-management/15-084j-nonlinear-programming-spring-2004/15-084jf04.jpg>



[http://www.math.uwaterloo.ca/~hwolkowi/henry/reports/talks.d/t09talks.d/09waterloomatlab.d/optimTipsWebinar/html/optimTipsTricksWalkthrough\\_06.png](http://www.math.uwaterloo.ca/~hwolkowi/henry/reports/talks.d/t09talks.d/09waterloomatlab.d/optimTipsWebinar/html/optimTipsTricksWalkthrough_06.png)

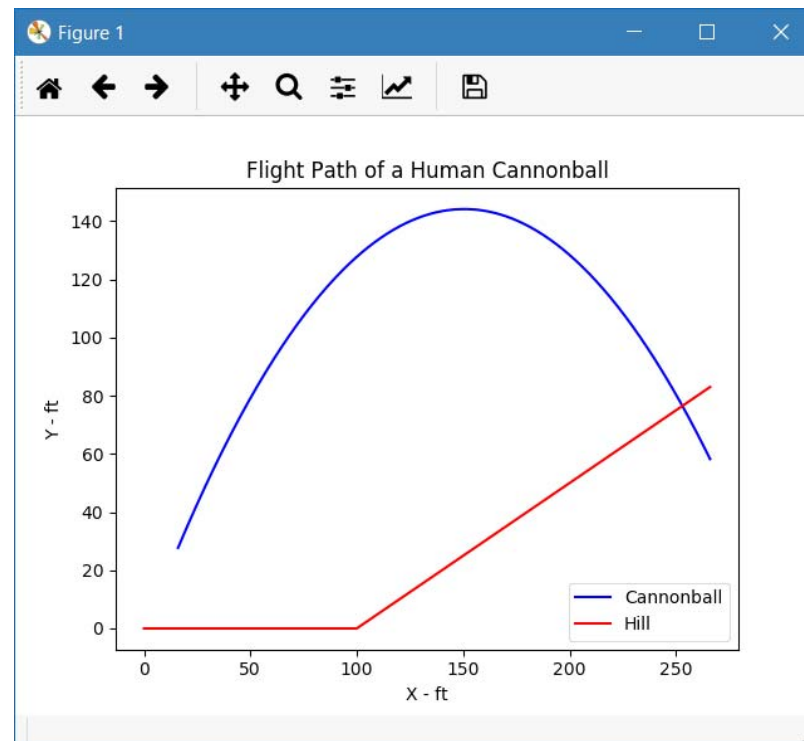
# Engineering Optimization

Choose the values of Design Parameters or Operating Parameters to achieve maximum GOOD or minimum BAD, subject to Constraints

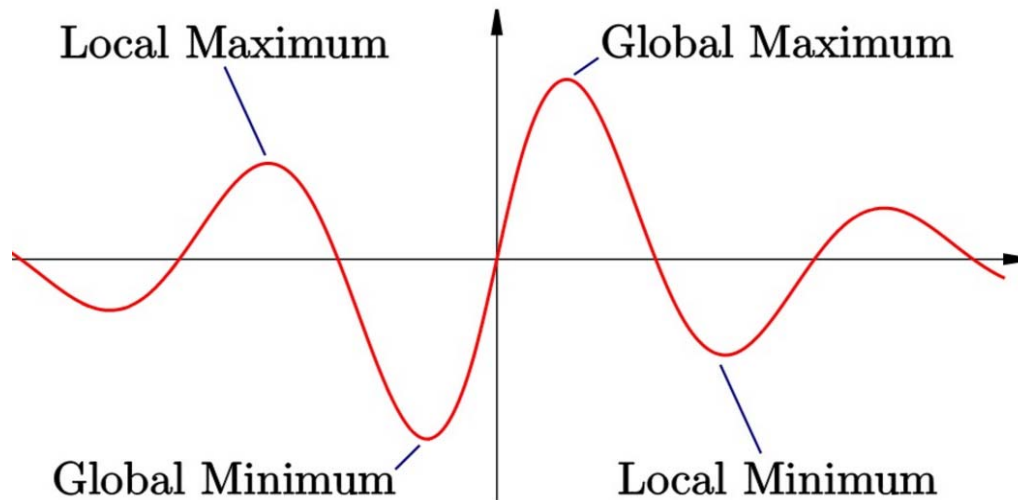
Variable and Functional Constraints

Good and Bad for:

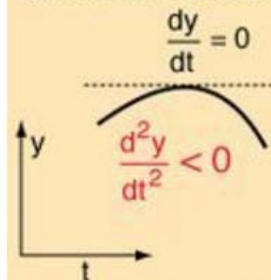
Airplane  
Race Car  
Commuter Car  
Umbrella  
Camera  
Human Canon



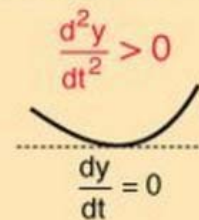
# One - Degree of Freedom (dof) Problem Unconstrained



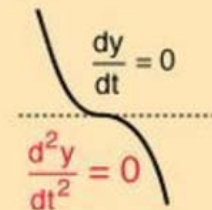
The second derivative demonstrates whether a point with zero first derivative is a maximum, a minimum, or an inflexion point.



For a **maximum**, the second derivative is negative. The slope of the curve ( first derivative) is at first positive, then goes through zero to become negative.

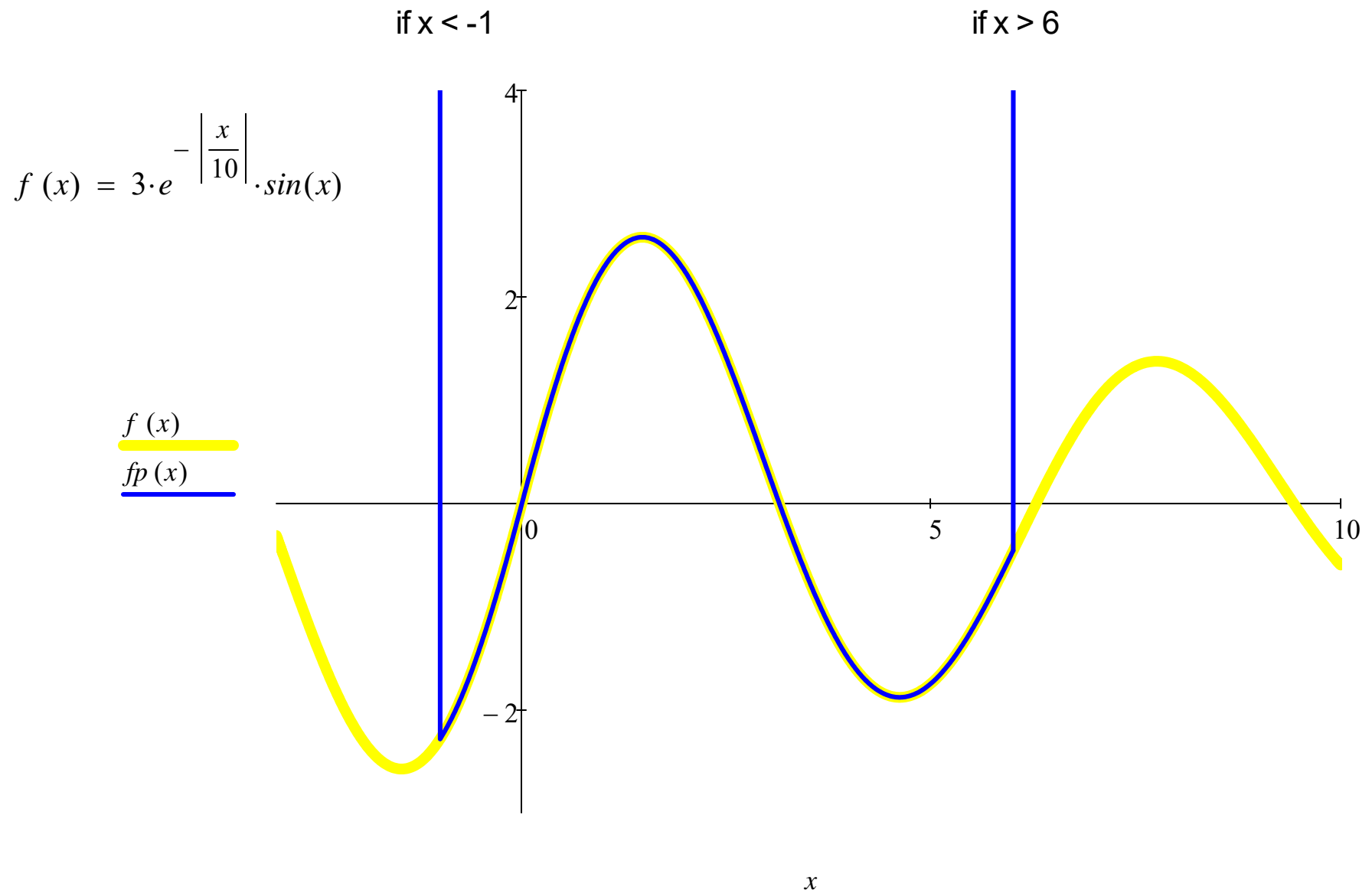


For a **minimum**, the second derivative is positive. The slope of the curve = first derivative is at first negative, then goes through zero to become positive.



For an **inflexion point**, the second derivative is zero at the same time the first derivative is zero. It represents a point where the curvature is changing its sense. Inflexion points are relatively rare in nature.

## Constraints - and Local minima and maxima



# Constraints and Penalty Functions - for Unconstrained MINIMIZERS

if  $x < -1$

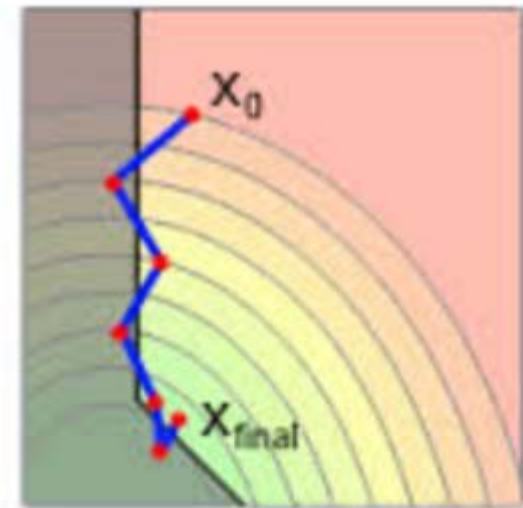
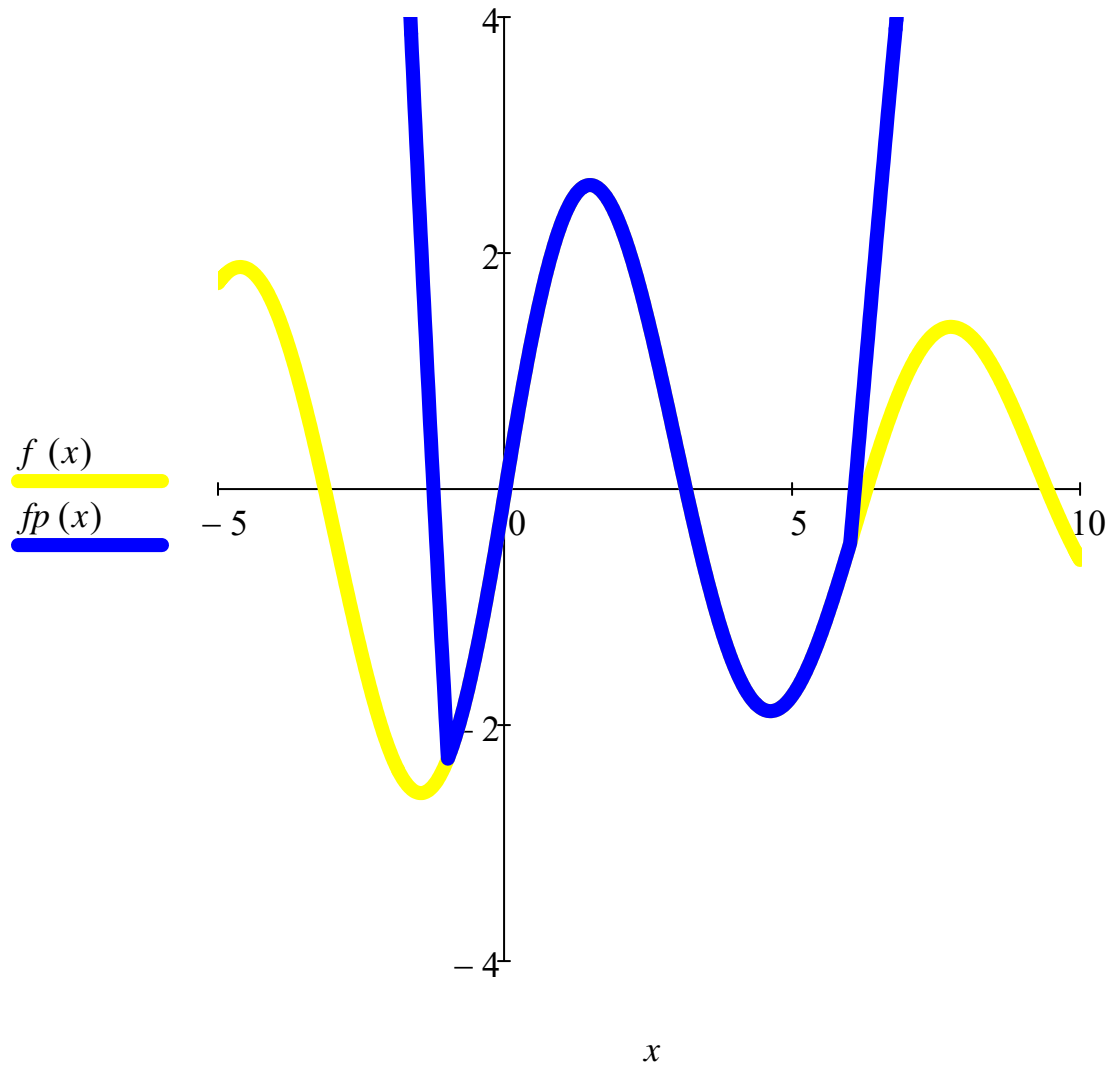
$$p = 10 \cdot (-1 - x)$$

if  $x > 6$

$$p = 4 \cdot (x - 6)$$

$$f(x) = 3 \cdot e^{-\left|\frac{x}{10}\right|} \cdot \sin(x)$$

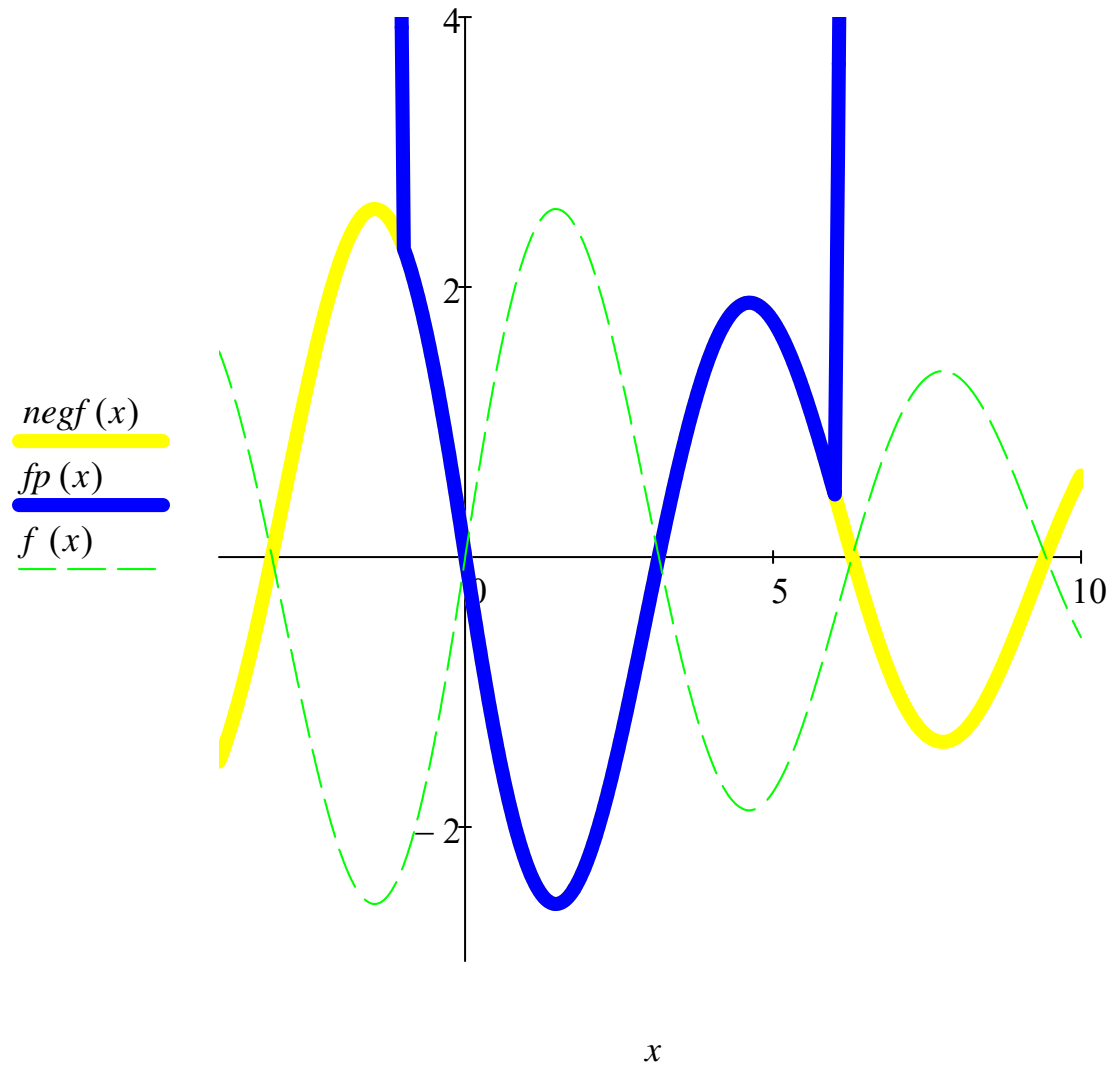
Usually slopes around  $10^6$



Penalty method

To Maximize a function, using a MINIMIZER, use the NEGATIVE of the function, and use POSITIVE penalties

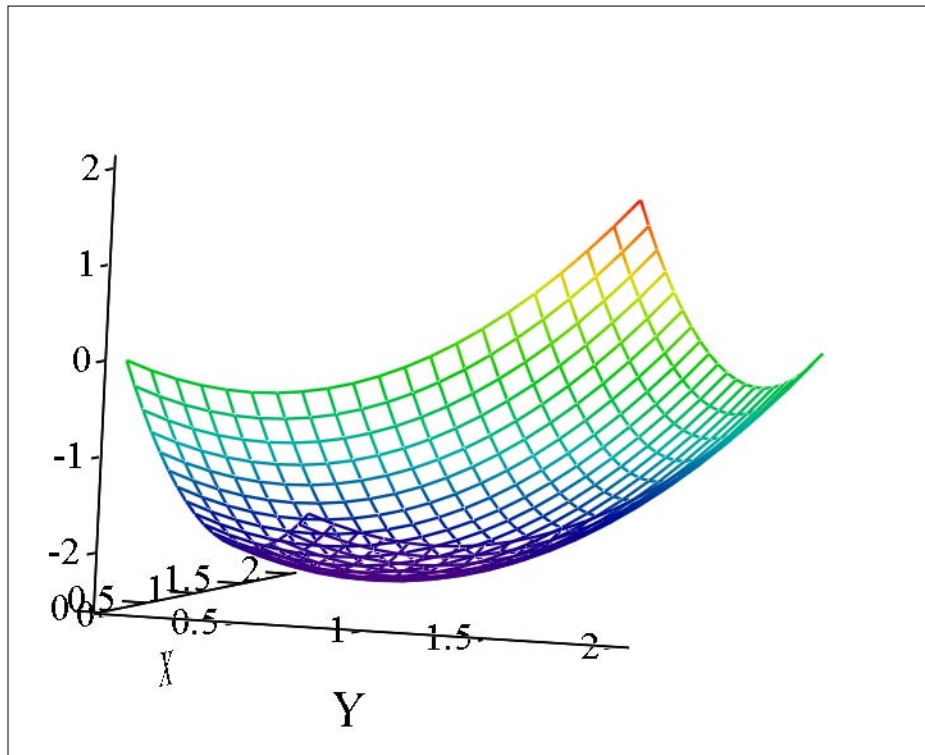
$$negf(x) = (-1) \cdot 3 \cdot e^{-\left|\frac{x}{10}\right|} \cdot \sin(x)$$



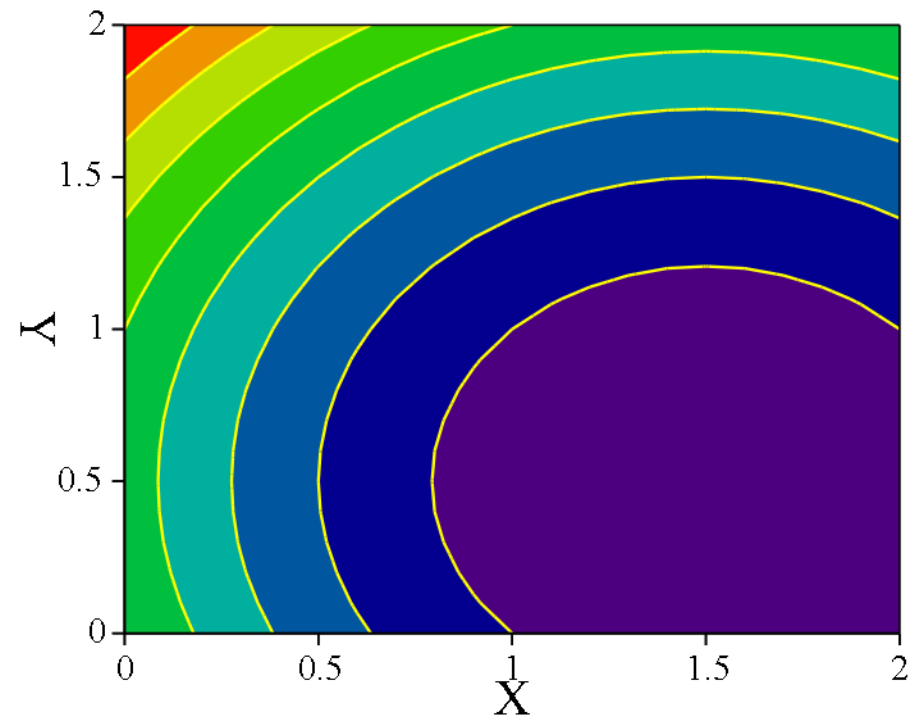
## A 2D Example

$$z(x,y) = y^2 - y + x^2 - 3x$$

We will play with these in Python

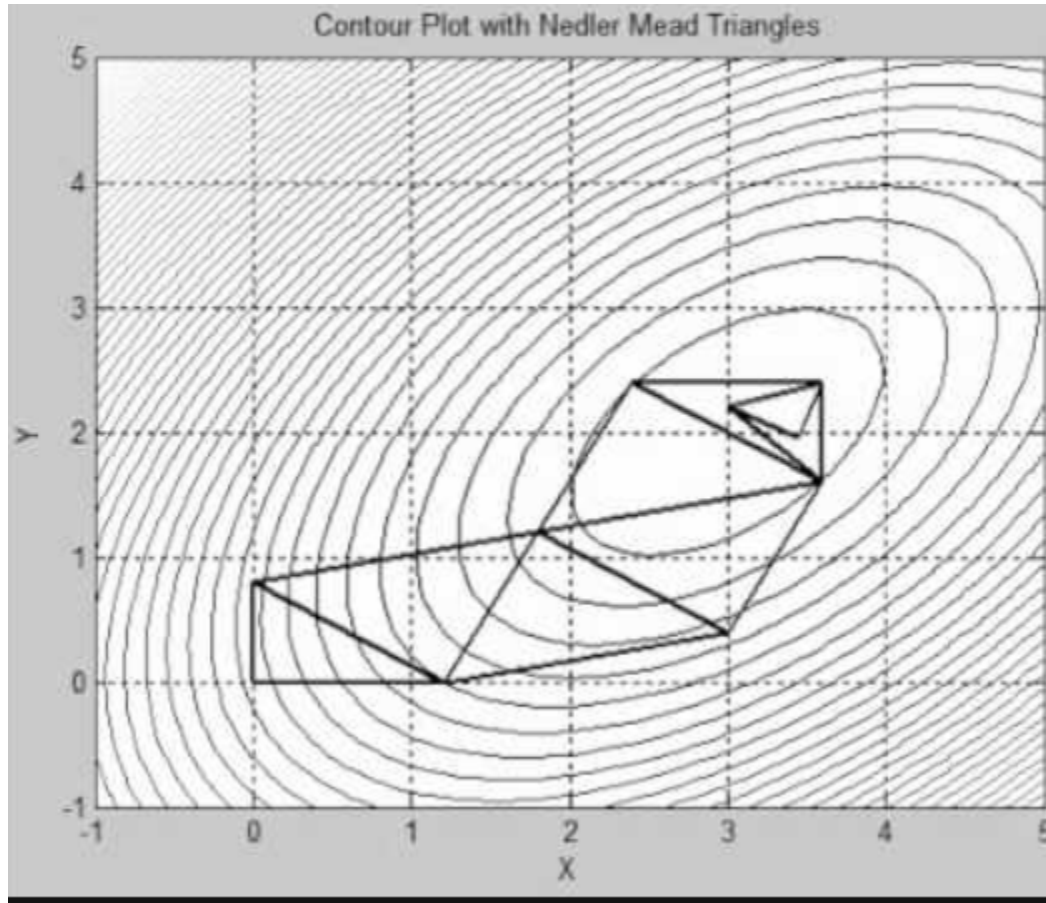


z



z

# Nelder - Mead Simplex Method - an Unconstrained Minimizer



<http://www.jakubkonka.com/images/nelder-mead/no-minimum.png>



## SciPy Documentation - A really poor example for Beginners

[SciPy.org](#)[Docs](#)[SciPy v0.19.0 Reference Guide](#)[SciPy Tutorial](#)

### Nelder-Mead Simplex algorithm (`method='Nelder-Mead'`)

In the example below, the `minimize` routine is used with the *Nelder-Mead* simplex algorithm

```
>>> import numpy as np
>>> from scipy.optimize import minimize
```

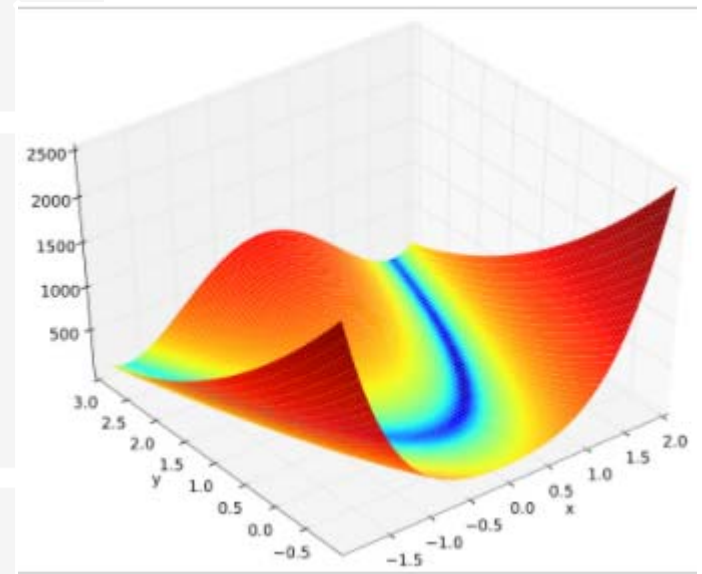
```
>>> def rosen(x):
...     """The Rosenbrock function"""
...     return sum(100.0*(x[1:]-x[:-1]**2.0)**2.0 + (1-x[:-1])**2.0)
```

```
>>> x0 = np.array([1.3, 0.7, 0.8, 1.9, 1.2])
>>> res = minimize(rosen, x0, method='nelder-mead',
...               options={'xtol': 1e-8, 'disp': True})
Optimization terminated successfully.
    Current function value: 0.000000
    Iterations: 339
    Function evaluations: 571
```

```
>>> print(res.x)
[ 1.  1.  1.  1.  1.]
```

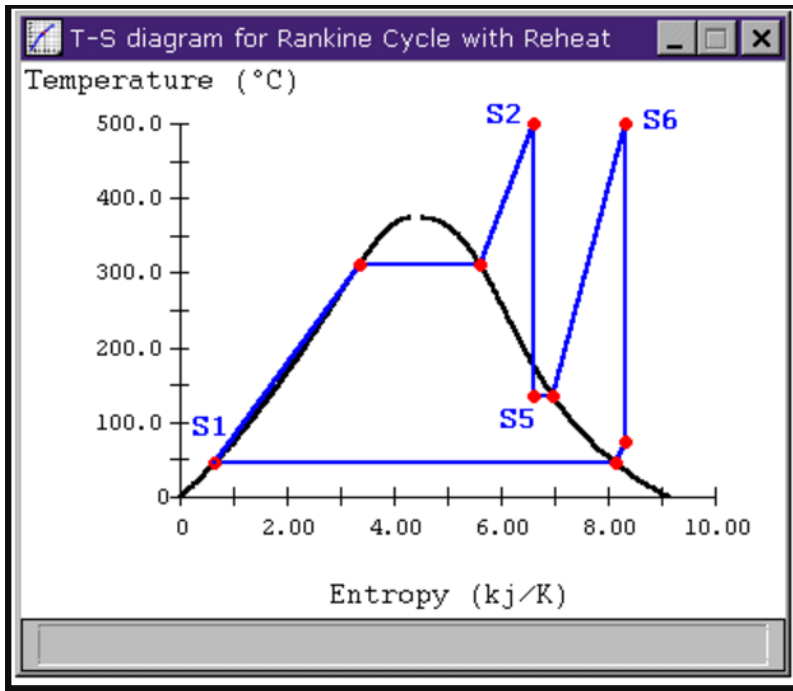
$$f(\mathbf{x}) = \sum_{i=1}^{N-1} 100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2$$

$$f(x, y) = (a - x)^2 + b(y - x^2)^2$$



<https://docs.scipy.org/doc/scipy/reference/tutorial/optimize.html>

## Rankine Cycle Optimization - Reheat Pressure and Temperature



<http://www.qrg.northwestern.edu/thermo/design-library/reheat/rhtTs.gif>

```

5  def optimize_Rankine(p_high,p_low,t_high,t_mid_max,
6                        turbine_efficiency=0.90, pump_efficiency=0.85):
7      def eff(vals):...
21     vals=minimize(eff,(p_low*100,t_high),method='Nelder-Mead',
22                   options={'fatol':0.01, 'xatol':1.0})
23     p_mid=vals.x[0]
24     t_mid=vals.x[1]
25     eff=vals.fun
26     count=vals.nfev
27
28     return p_mid, t_mid, eff, count
29

```