ECE 405 Assignment 1

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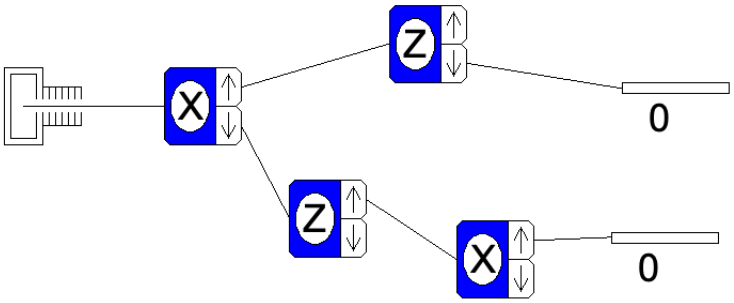
Question 1

a.



b.

$$
\begin{aligned}
P_{-z} &= |\langle -z \mid +x \rangle|^2 \\
&= \left| \langle -z | \left(\frac{1}{\sqrt{2}} |+z\rangle + \frac{1}{\sqrt{2}} |-z\rangle \right) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle -z \mid +z \rangle + \frac{1}{\sqrt{2}} \langle -z \mid -z \rangle \right|^2 \\
&= \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
P_{+x} &= |\langle +x \mid +z \rangle|^2 \ |\langle +z \mid -x \rangle|^2 \\
&= \left| \left(\frac{1}{\sqrt{2}} \langle +z| + \frac{1}{\sqrt{2}} \langle -z| \right) |+z\rangle \right|^2 \left| \langle +z| \left(\frac{1}{\sqrt{2}} |+z\rangle - \frac{1}{\sqrt{2}} |+z\rangle \right) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +z \mid +z \rangle + \frac{1}{\sqrt{2}} \langle -z \mid +z \rangle \right|^2 \left| \frac{1}{\sqrt{2}} \langle +z \mid +z \rangle - \frac{1}{\sqrt{2}} \langle +z \mid -z \rangle \right|^2 \\
&= \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \\
&= \frac{1}{4}
\end{aligned}
$$

c.

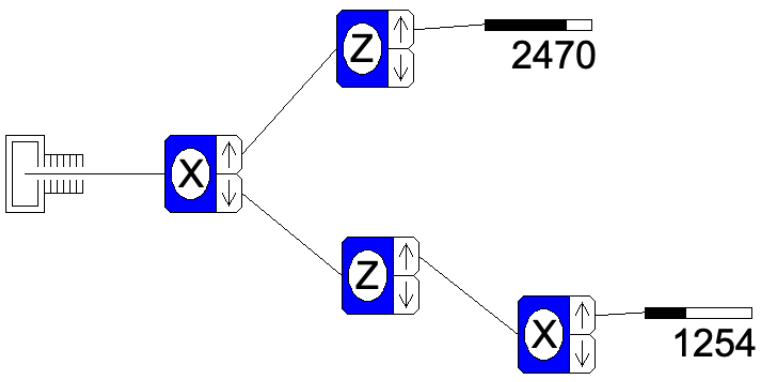
Atoms expected at $-z$ port:

$$10000 \cdot \frac{1}{2} \cdot \frac{1}{2} = 2500$$

Atoms expected at $+x$ port:

$$10000 \cdot \frac{1}{2} \cdot \frac{1}{4} = 1250$$

d.



The simulated results are consistent with the theoretical results (within a reasonable margin of error).

Using a larger number of atoms in the simulation would result in convergence to the theoretical results.

Question 2

a.

Iteration	$+x$	$-x$	$+z$	$-z$
1	2485	0	1254	1222
2	2518	0	1248	1274
3	2493	0	1260	1227
4	2431	0	1280	1206
5	2514	0	1174	1290
6	2453	0	1250	1268
7	2515	0	1261	1307
8	2516	0	1262	1276
9	2514	0	1269	1246
10	2427	0	1251	1246

b.

From the simulations, the average value for the number of atoms with spin $+x$ is given by:

$$\frac{1}{10}\sum_{i=1}^{10} +x_i = 2486.6$$

The theoretical probability is found like so:

$$\begin{aligned}
P_{+x} &= |\langle +x \mid +x \rangle|^2 \ |\langle +x \mid +z \rangle|^2 \\
&= |\langle +x \mid +z \rangle|^2 \\
&= \left| \left(\frac{1}{\sqrt{2}} \langle +z| + \frac{1}{\sqrt{2}} \langle -z| \right) |+z\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +z \mid +z \rangle + \frac{1}{\sqrt{2}} \langle -z \mid +z \rangle \right|^2 \\
&= \frac{1}{2}
\end{aligned}$$

c.

From the simulations, the average value for the number of atoms with spin $+z$ is given by:

$$\frac{1}{10} \sum_{i=1}^{10} +z_i = 1250.9$$

The theoretical probability is found like so:

$$\begin{aligned}
P_{+x} &= |\langle -z \mid -x \rangle|^2 \ |\langle -x \mid +z \rangle|^2 \\
&= \left| \langle -z| \left(\frac{1}{\sqrt{2}} |+z\rangle - \frac{1}{\sqrt{2}} |-z\rangle \right) \right|^2 \ \left| \left(\frac{1}{\sqrt{2}} \langle +z| - \frac{1}{\sqrt{2}} \langle -z| \right) |+z\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle -z \mid +z \rangle - \frac{1}{\sqrt{2}} \langle -z \mid -z \rangle \right|^2 \ \left| \frac{1}{\sqrt{2}} \langle +z \mid +z \rangle - \frac{1}{\sqrt{2}} \langle -z \mid +z \rangle \right|^2 \\
&= \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \\
&= \frac{1}{4}
\end{aligned}$$

d.

The X analyzer immediately before the second X analyzer prepares the atoms in the $+x$ state. As such, the probability of atoms being detected at the $-x$ port is 0.

Question 3

a.

The quantom state $|\psi\rangle$ must be normalized so that their probabilities $|\langle \pm z \ |\psi\rangle|$ sum to 1.

If all quantum states are not normalized, the total probability of all quantum states will not sum to 1, violating basic probability theory.

b.

Due to the law of large numbers, the experimental uncertainty (error) will converge to the theoretical value as the expirement is repeated a large number of times.
However, the theoretical uncertainty (due to the uncertainty principle) will not change as the experiment is repeated.

c.

If the SG-analyzers are not perfectly aligned, error will be introduced into the experiment (obviously). Specifically, an analyzer which is mis-aligned in the $+z$ direction (for example) will introduce bias (error) into the $+z$ and $-z$ measurements.