Lecture 1: Introduction

ECE 481 - Digital Control Systems

Yash Vardhan Pant

Based on course notes by Professor Chris Nielsen.

Outline

[X] Course outline

[ ] Some examples of Digital Control

[ ] Digital Feedback Control

[ ] Sampled-data systems and themes in this course

[ ] Continuous-time Control Systems

[ ] Discrete-time Control Systems

[ ] Sampled-data Control Systems

[ ] Discretizing Continuous-time Controllers
    <> Ideal sampling and zero-order hold
    <> Preserving linearity with discretizations

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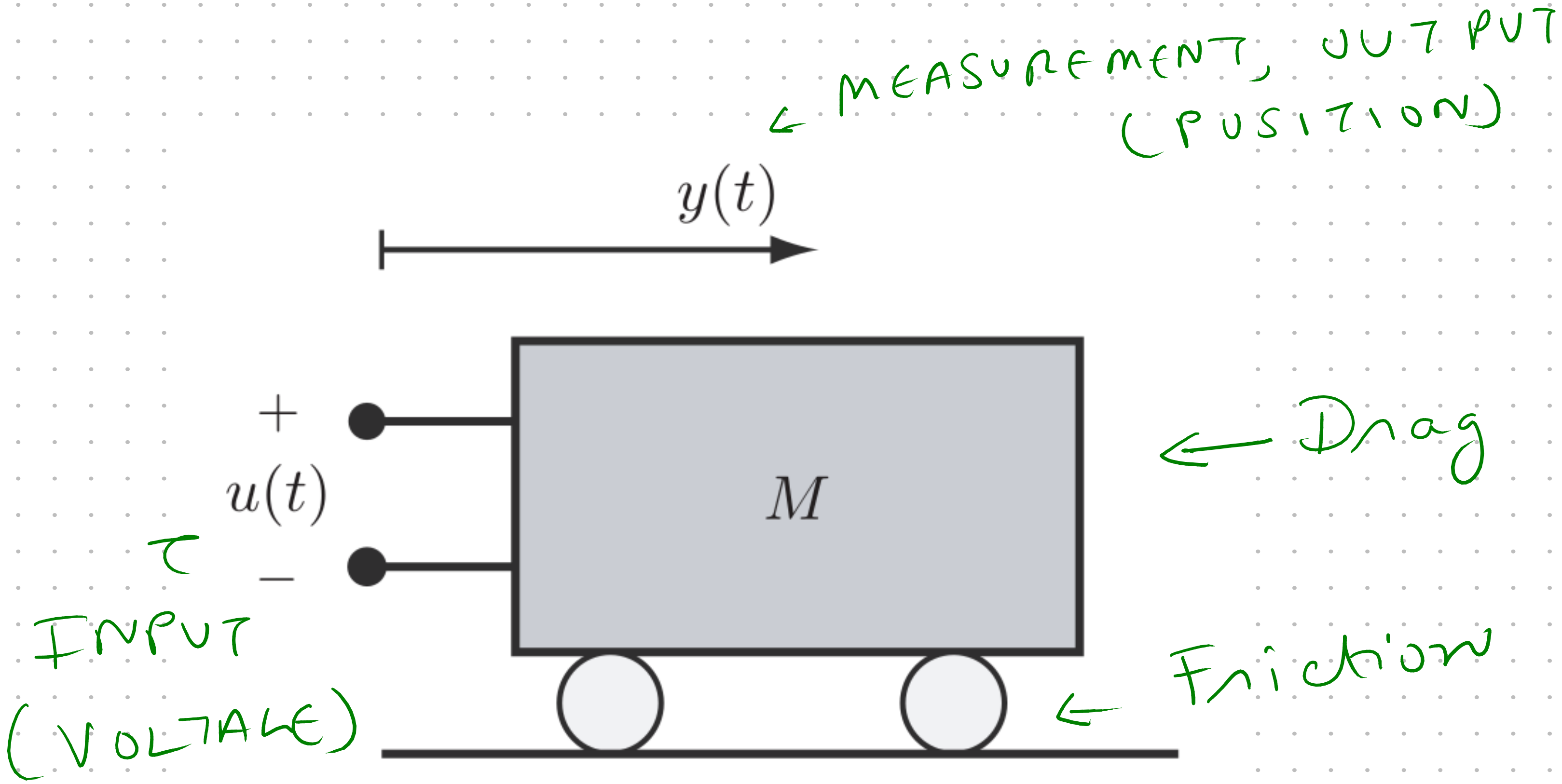
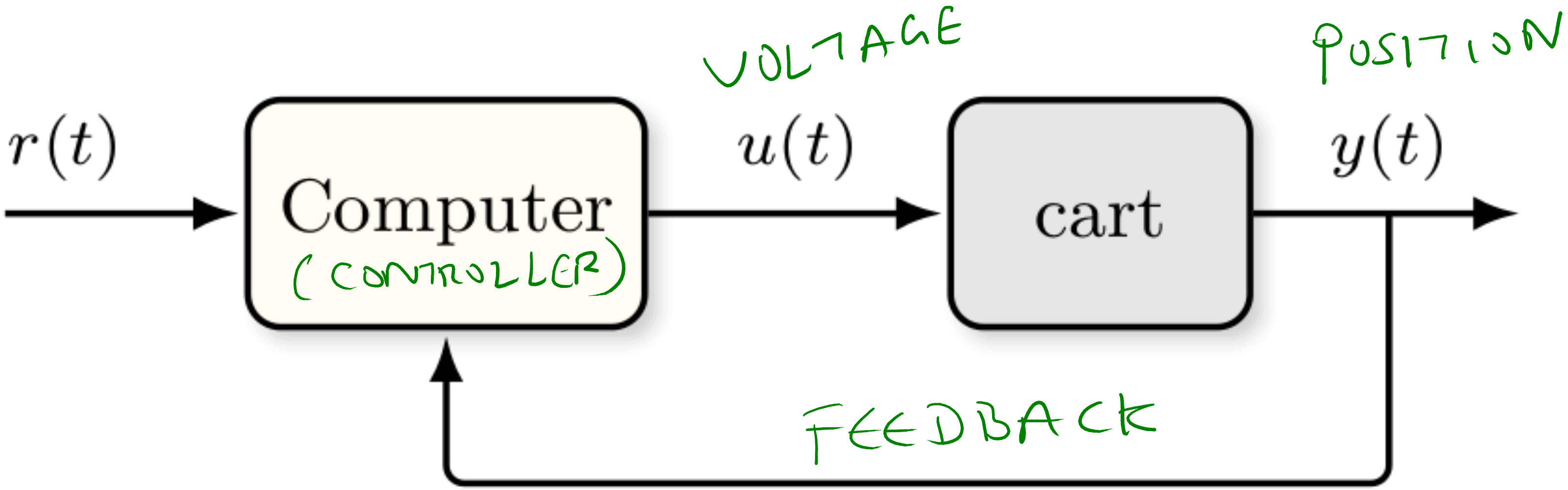
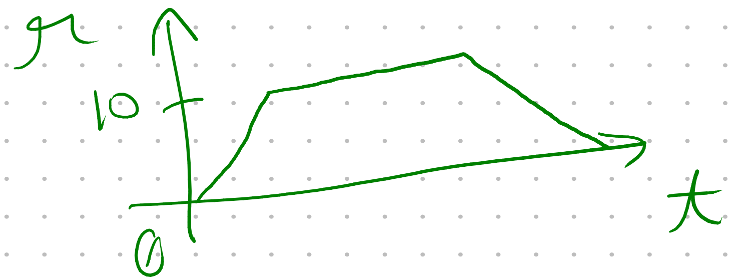
Example: A cart with a motor drive



Feedback control with a computer in the loop



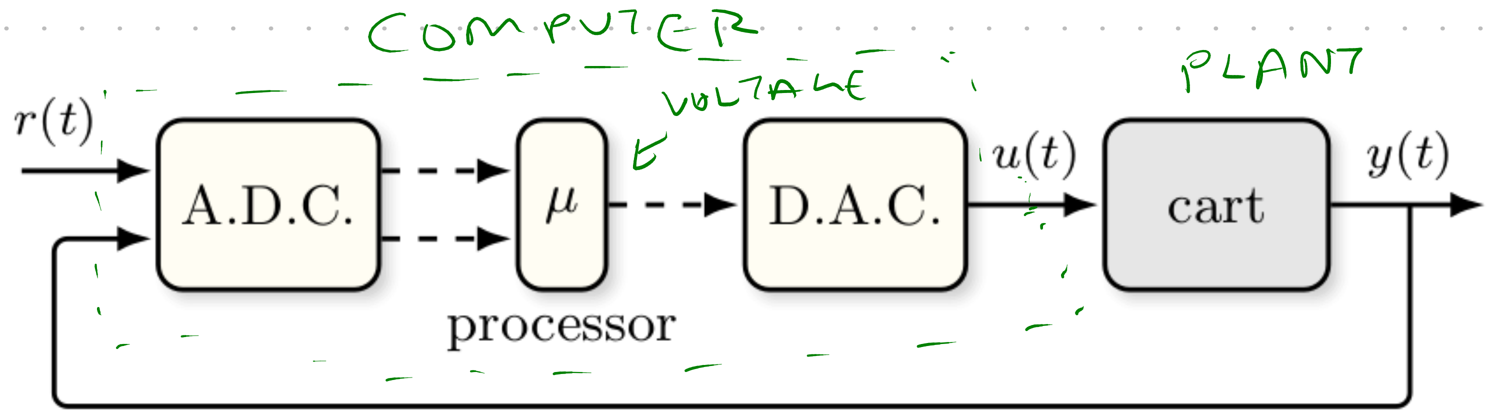
r(t): Reference position signal



Computer: Gets a measurement and computes a control input.

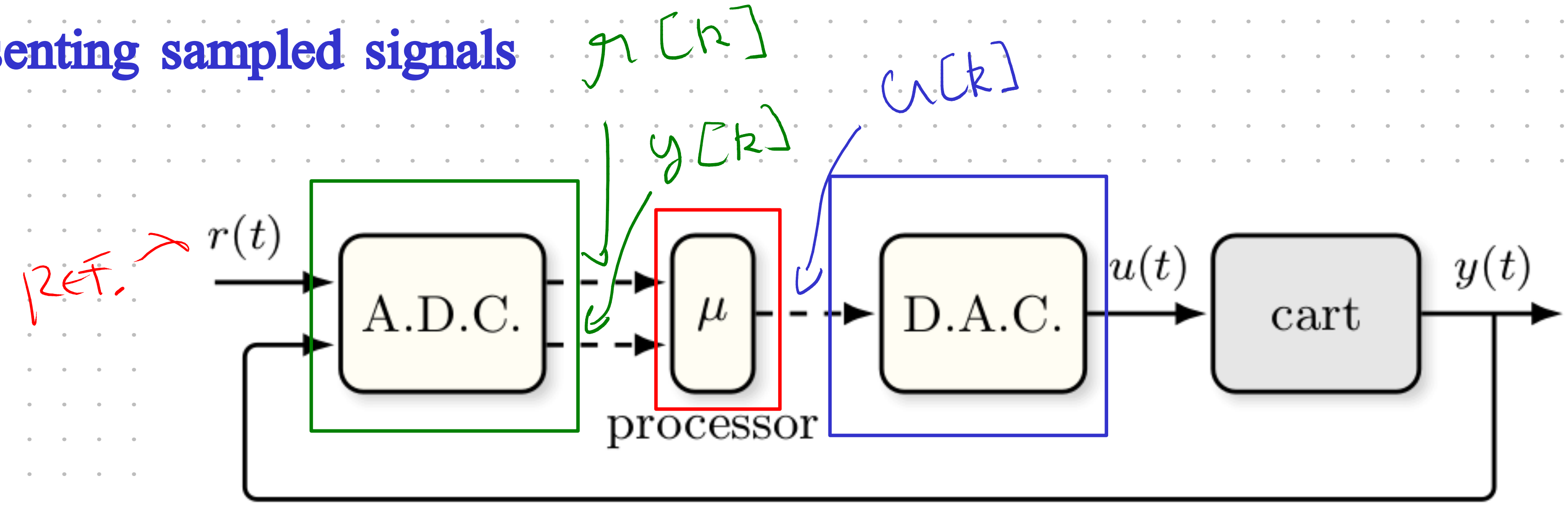
A more detailed model

— CONTINUOUS TIME (CT)
- - - DISCRETE TIME (DT)



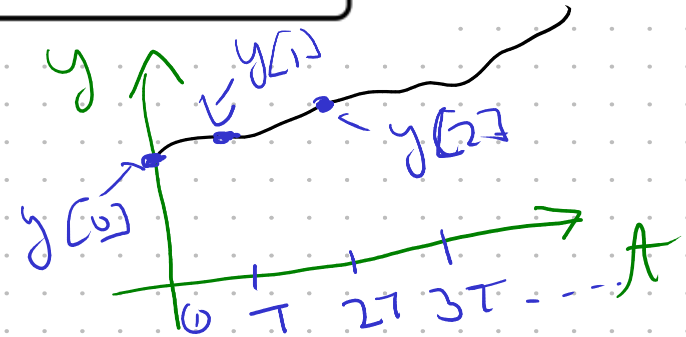
A.D.C: Analog-to-Digital converter
D.A.C: Digital-to-Analog converter

Representing sampled signals



Let T be the sampling period, and k an integer (positive)

y(t) is sampled to get $y(kT)$ / $y[k]$



REFERENCE

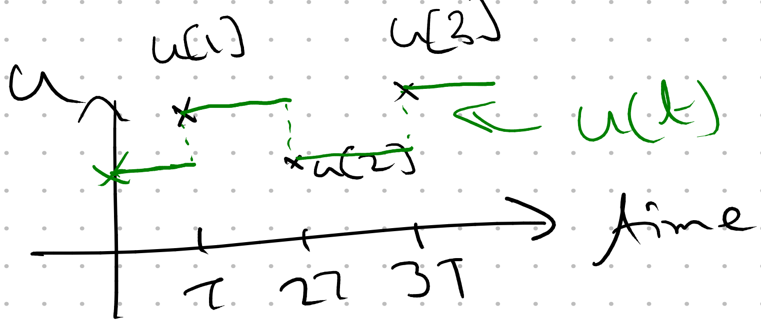
A Proportional controller is implemented on the computer

$$u(kT) = K_p(r(kT) - y(kT))$$

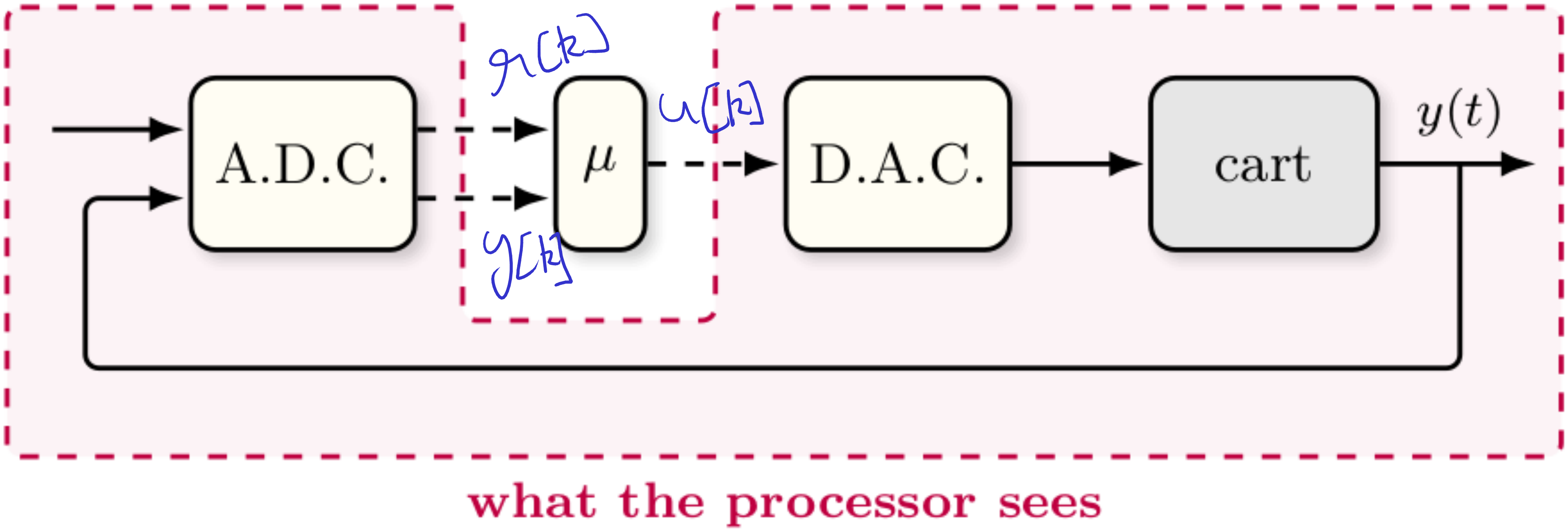
$$u[k] = K_p(\underbrace{r[k] - y[k]}_{\text{error}})$$

The DT control signal is interpolated to a CT control signal that is applied to the motor

$$\underbrace{u(t) = u(kT) \quad \forall\, t \,.\, kT \leq t < (k+1)T}_{\text{ZOH (zero order hold)}}$$

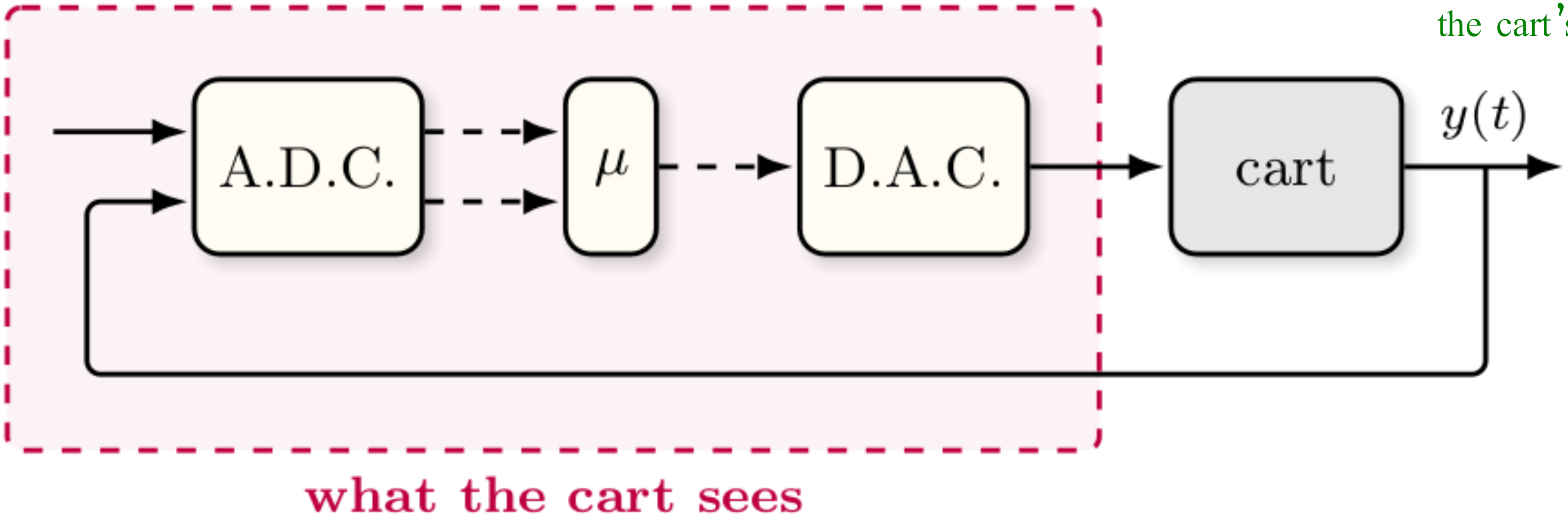


What does the processor see (CT or DT)?



What does the cart see (CT or DT)?

What is an analog sensor that measure the cart's position?



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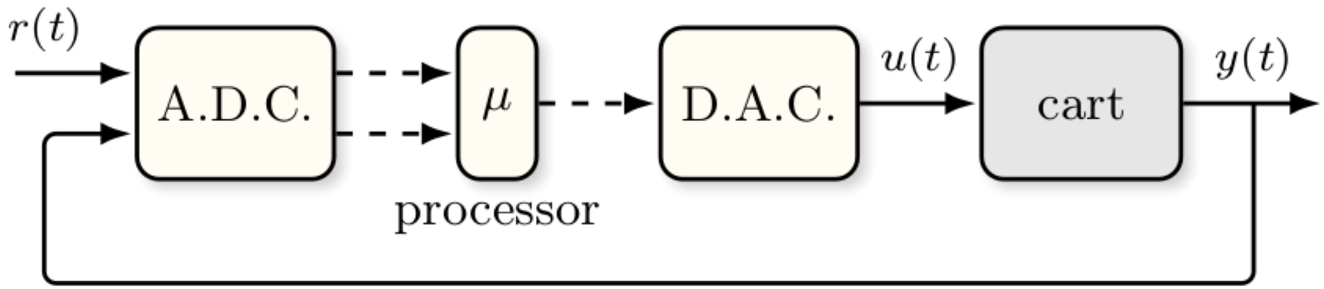
Sampled-data Systems:



Definition: Feedback systems that have a mixture of continuous-time and discrete-time elements are called sampled-data systems.

Some themes in the course:

1. DT signals and systems theory is similar to CT theory, and has parallels such as:
   - DT has difference equations instead of differential equations.
   - z-transforms instead of Laplace transforms.

2. Sampled-data systems are fundamentally not time-invariant
   - They are, in fact, periodically time-varying.
   - The C2D and D2C conversions (A.D.C, D.A.C) cause this.

3. Digital control implementations are subject to hardware limitations, but, can achieve specifications that are hard to achieve via analog control alone. e.g.,
   - Obstacle avoidance in autonomous navigation: https://www.youtube.com/watch?v=tltYkv8bjAw
   - Multi-UAV timed-tasks: https://www.youtube.com/watch?v=xBQnEweVwZs

4. Physical systems (such as the cart) are represented as a sampled-data system. However there are also applications that are purely discrete-time control systems, e.g., inventory/supply-chain control.

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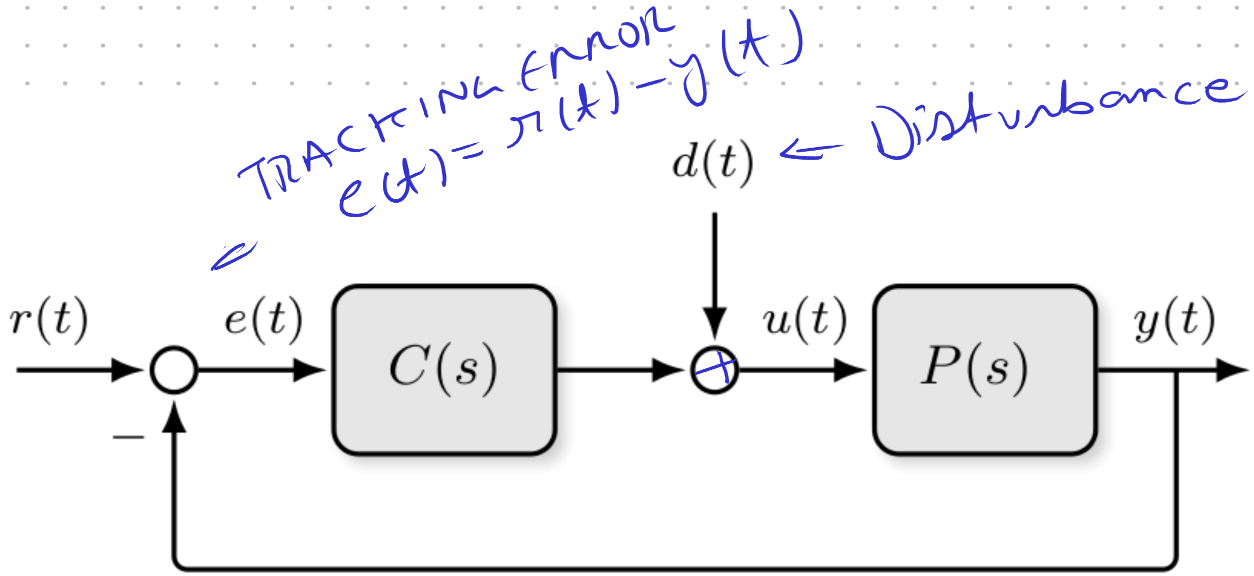
Continuous Time Control:



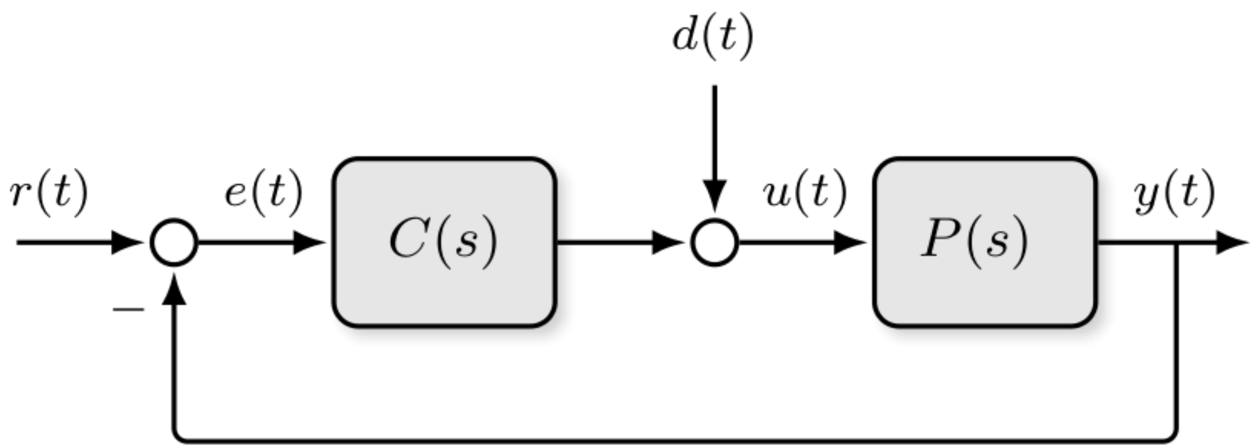
Typical control objectives:

- Closed-loop stability
- Transient performance (e.g., maximum overshoot, settling time etc.)
- Robustness to model uncertainty
- Disturbance rejection
- Tracking: $y(t)$ tracks $r(t)$

Continuous time Control Design Problem.

Given a set of control objectives and a plant model $P(s)$, design a control law $C(s)$ such that the closed-loop system satisfies the objectives

Continuous Time Control:



Continuous control implementations were implemented using analog devices and circuits (mostly in the past). This leads to disadvantages such as:

- Inflexible: Changes in control require changes in the circuit, making them difficult to maintain too.
- Difficult to adapt controller to changes in the plant (adaptive control).
- Difficult to implement sophisticated controllers, e.g., Obstacle-avoidance in racing: https://www.youtube.com/watch?v=tltYkv8bjAw

With computers becoming cheaper, most modern control designs and implementations have moved to the discrete-time domain.

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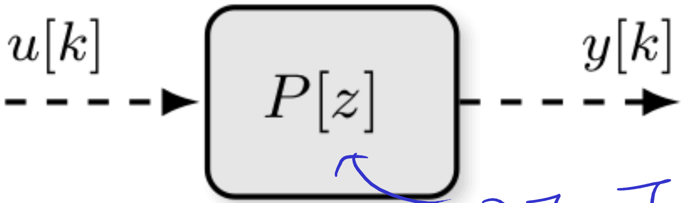
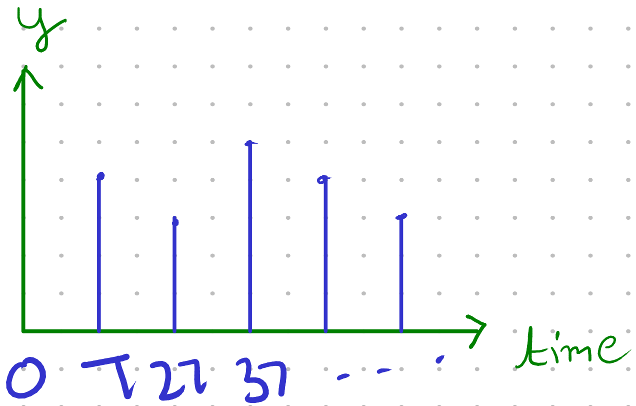
Discrete-time Control Systems

Consider a discrete-time (DT) system that takes in DT inputs and outputs DT signals (recall: z-transforms)



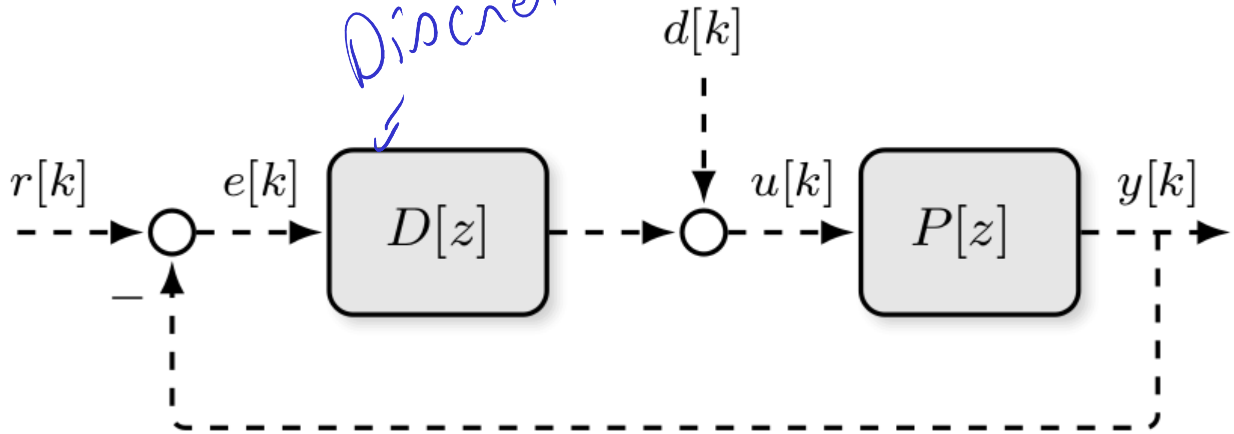
DT Transfer function (TF)

Discrete-time signals:



Discrete-time Control Systems

Discrete time controller



Discrete-time feedback control system (from the embedded processor's perspective).

Systems: $D[z]$, $P[z]$

Signals:

INDEPENDENT (EXOGENOUS) : $r[k]$, $d[k]$

DEPENDENT : $e[k]$, $u[k]$, $y[k]$

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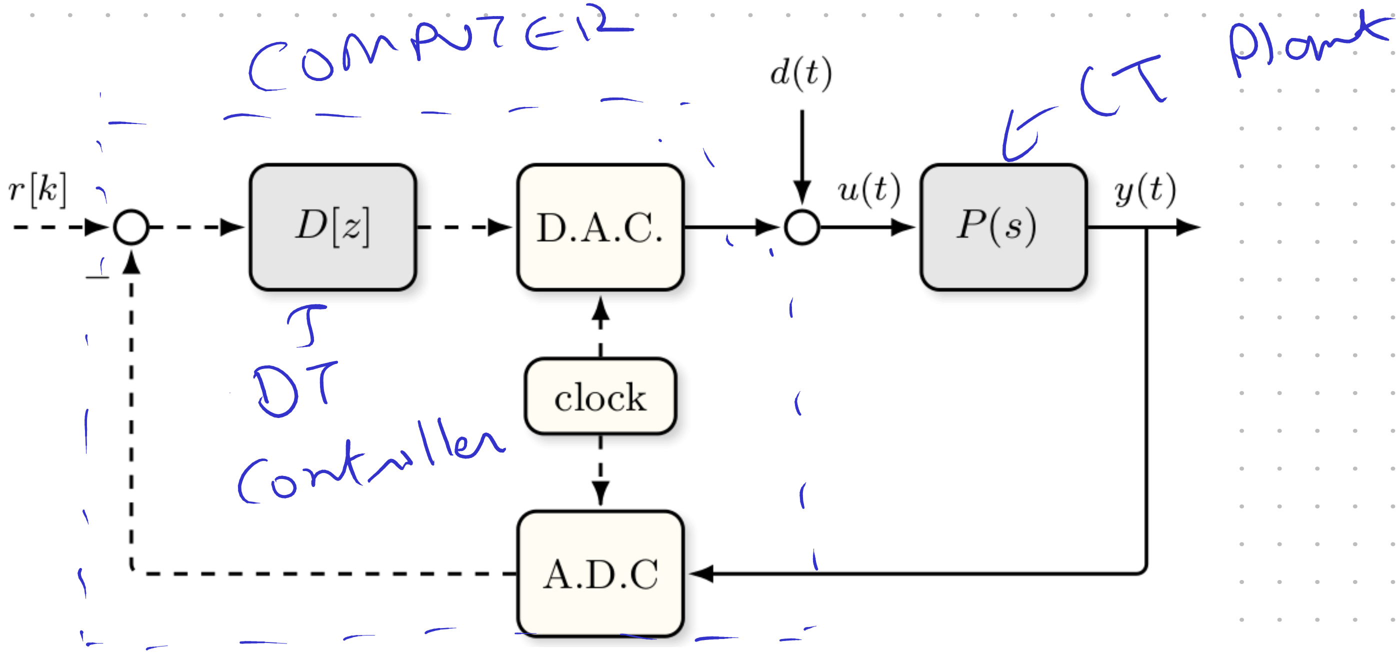
Sampled-data Control Systems

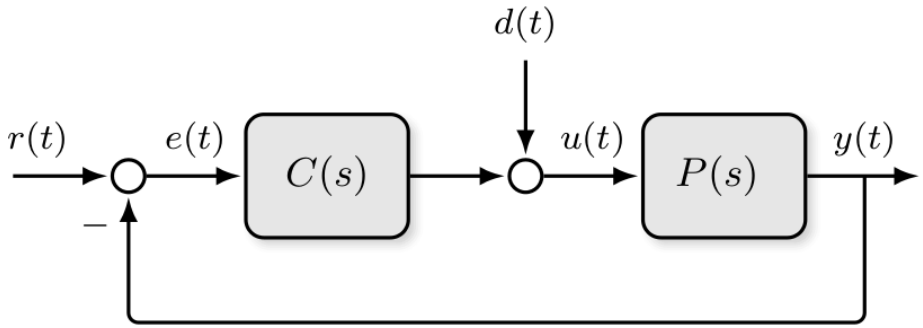


Mixture of CT and DT elements

Sampled-data Control Design Problem

Given: a) continuous-time performance specifications, b) Plant model P(s), design a digital control law D[z] such that the closed-loop system satisifes the specifications.

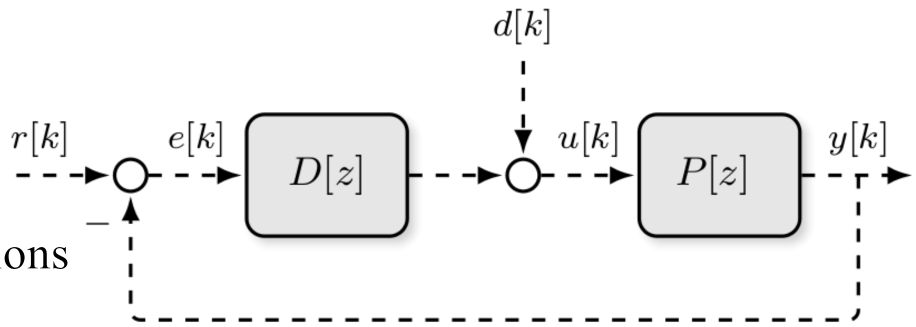
Two common approaches to solving this design problem:



1. Emulation:

- Ignore the ADC and DAC and design a CT controller
- Discretize the controller to get D[z] (C2D)
- Advantage: Straightforward, lets you use what you learn in ECE380/equivalent.
- Disadvantage: Needs fast sampling, otherwise inaccurate

2. Direct (DT) design:



- Discretize P(s) to get P[z]
- Treat the sampled-data system as a DT system
- Design D[z] to make P[z] satisfy discrete-time specifications
- Advantage: Accounts for sampling periods explicitly
- Disadvantage: Analysis can be intricate

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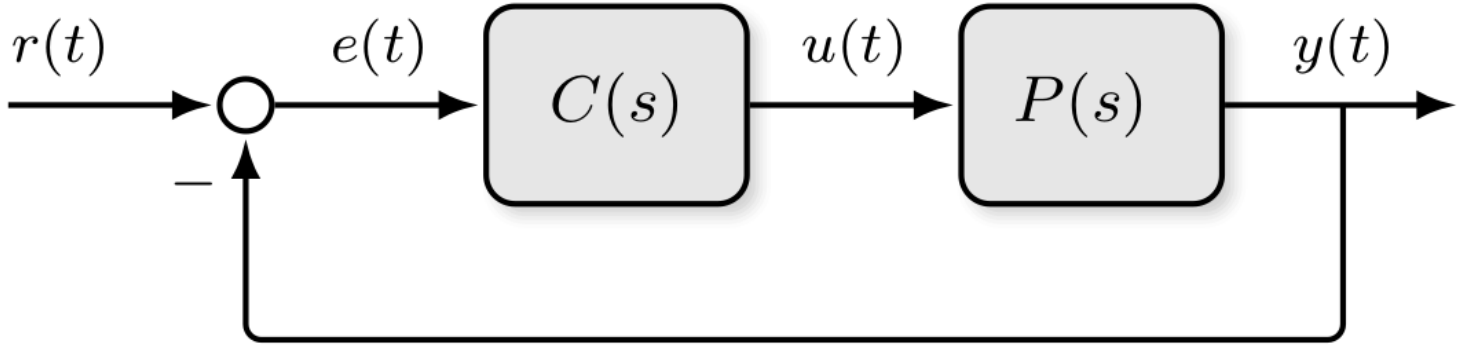
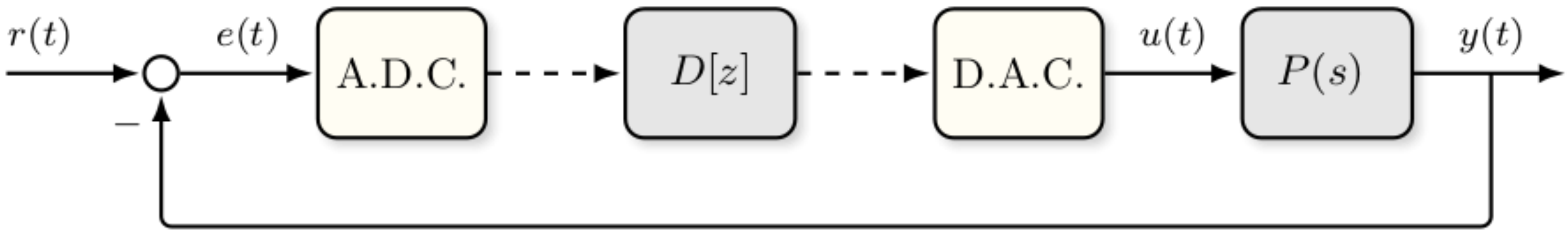
A first look at discretization of continuous-time controllers

Let us start with a continuous controller C(s) that works well for the plant P(s)



We want to design a discrete-time controller D(z) so that the system above is well-approximated.



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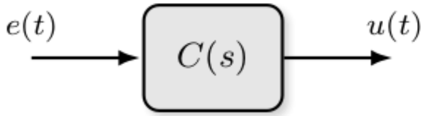
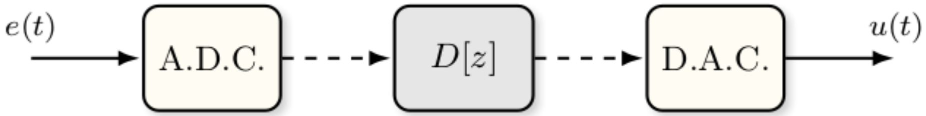
Ideal sample and zero-order hold

We want to approximate the continuous-time control law





Let T be the sampling time (period)

$$y(t) \to y(kT), \quad k \in \mathbb{Z}_{\geq 0}$$

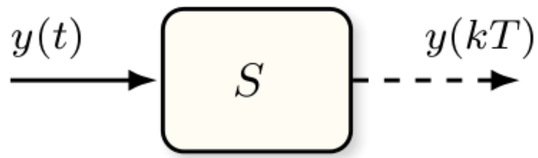
$$y[k]$$

Approximation method:

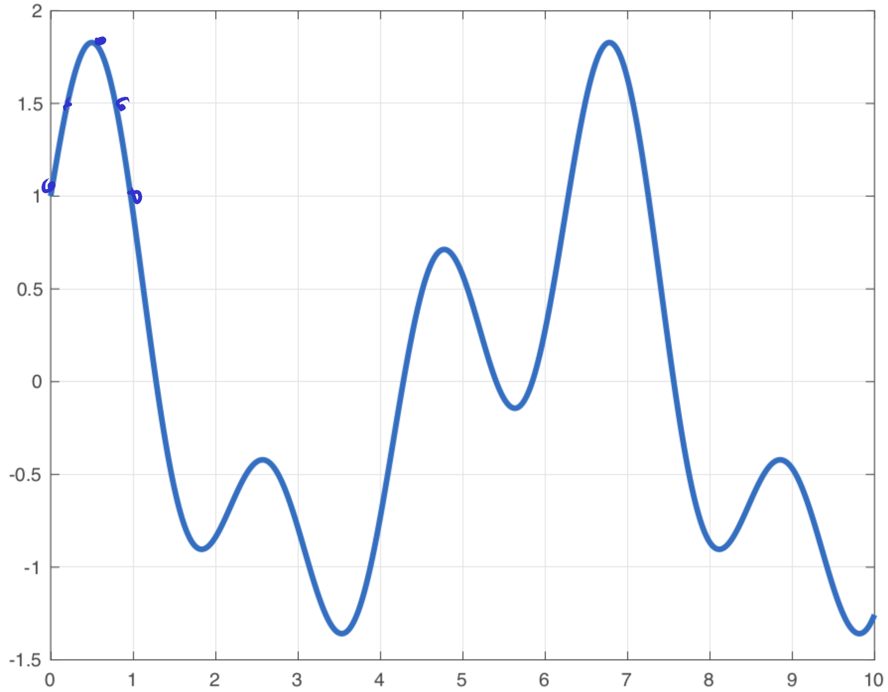
- Model the A.D.C as an ideal sampler
- Model the D.A.C using zero-order hold

Ideal sampler

Get a discrete-time representation of a continuous time signal

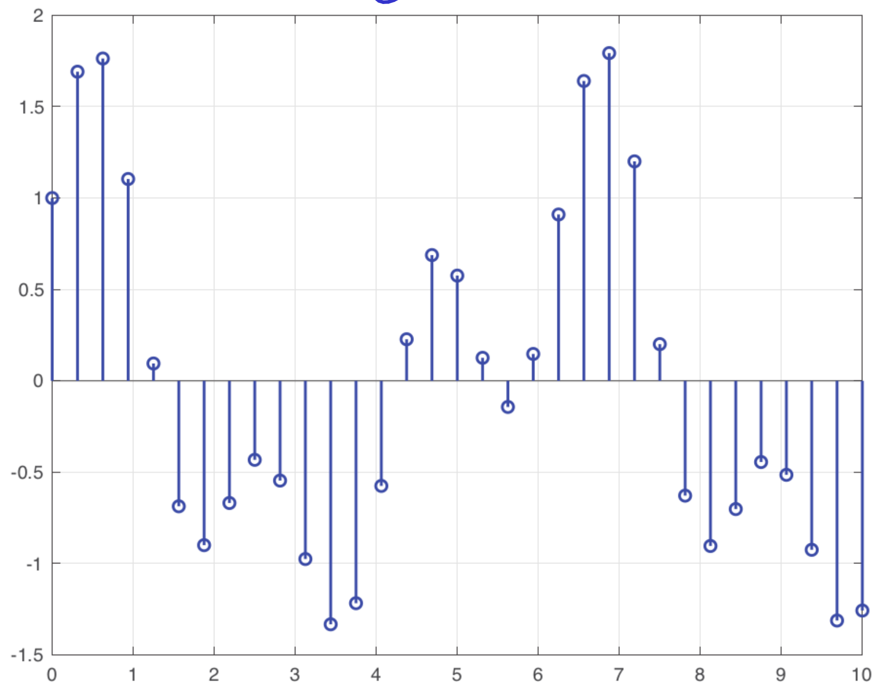


CT



y(t): Input to S

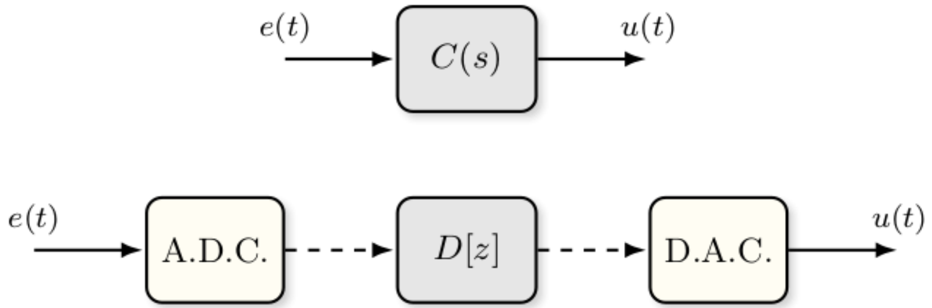
DT



y[k]: Output of S

Ideal sample and zero-order hold

We want to approximate the continuous-time control law



Let T be the sampling period

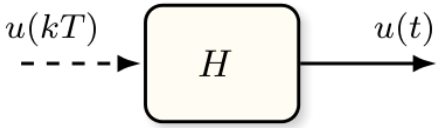
$y(t)$ is sampled to produce $y(kT)$, $k \in \mathbb{Z}_{\geq 0}$

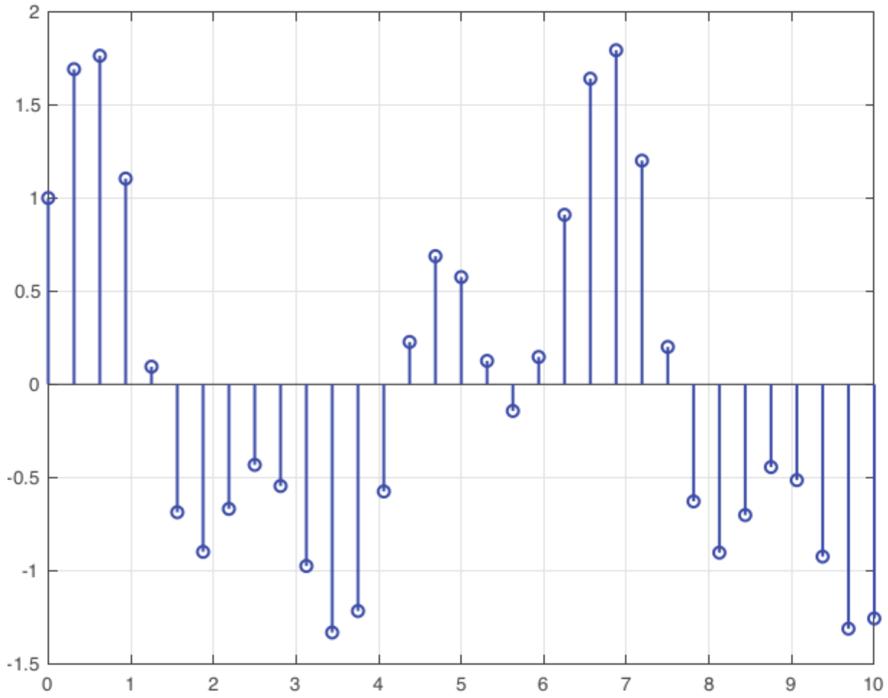
Shorthand $y[k]$

Approximation method:
- Model the A.D.C as an ideal sampler
- Model the D.A.C using zero-order hold

Zero-order hold

Get a continuous-time signal from a discrete-time signal

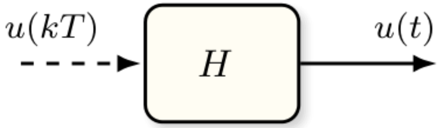


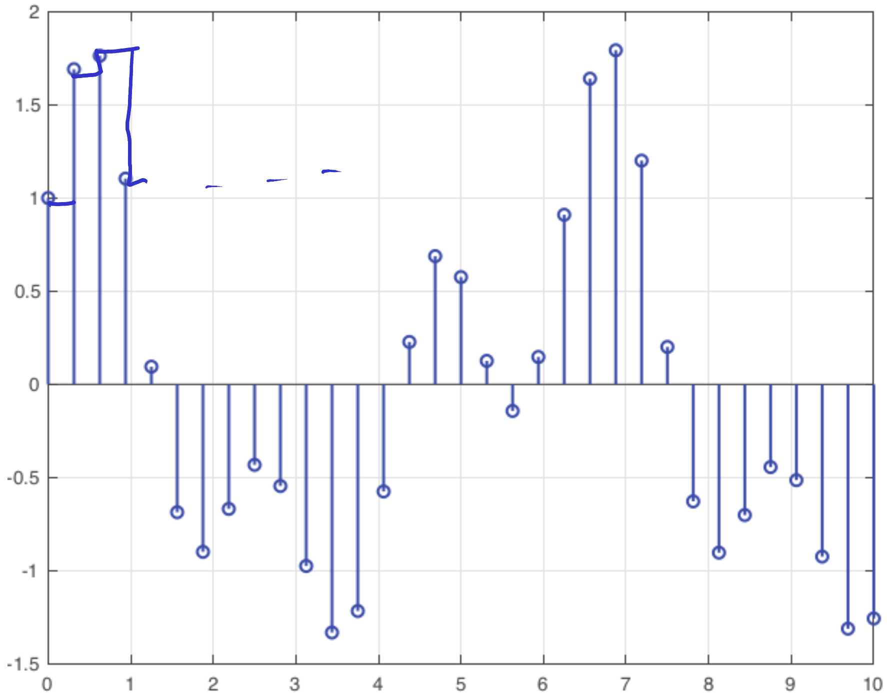


u[k]: Input to the ZOH

Zero-order hold

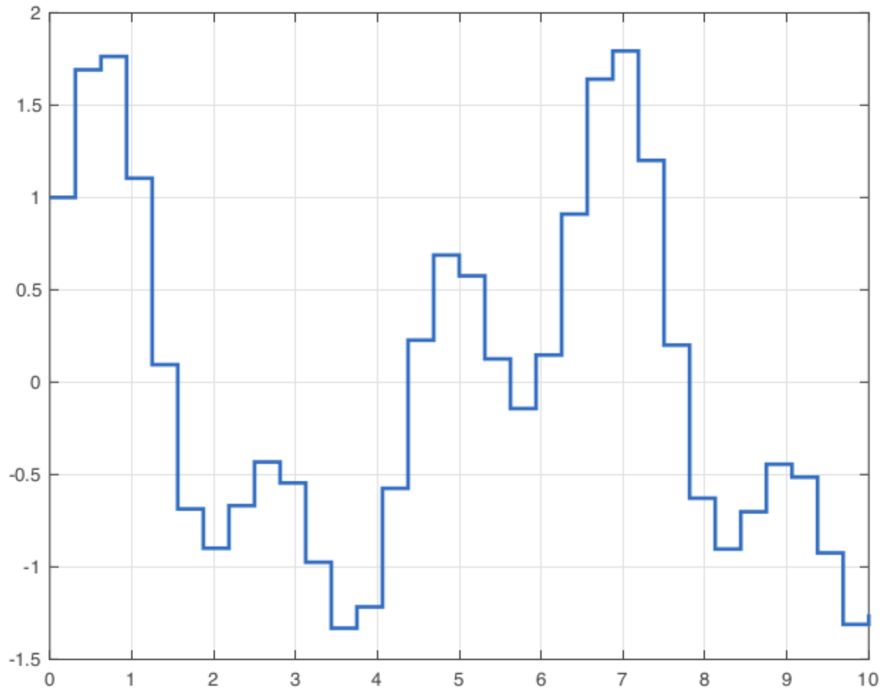
Get a continuous-time signal from a discrete-time signal





u[k]: Input to the ZOH

ZOH



u(t): Output of the ZOH

$$u(t) = u[k], \quad \forall t \,.\; kT \leq t < (k+1)T$$

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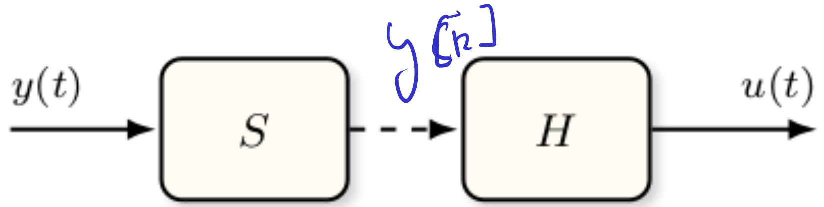
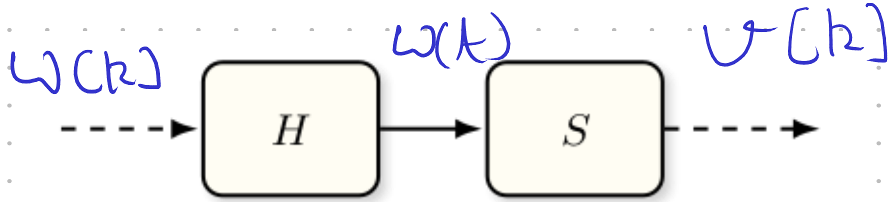
Preserving linearity with ideal sampling and ZOH

Excercise: Prove that the ideal sampler S, and zero-order hold H are both linear systems.

As a consequence, the compositions sample-and-hold and hold-and-sample are both linear systems (ignoring quantization).



Sample-then-hold: H o S



Hold-then-sample: S o H

$$u(t) = \underbrace{H(S}_{H \circ S}(y(t)))$$

{composition}

$$v[k] = \underbrace{S(H}_{S \circ H}(w[k]))$$

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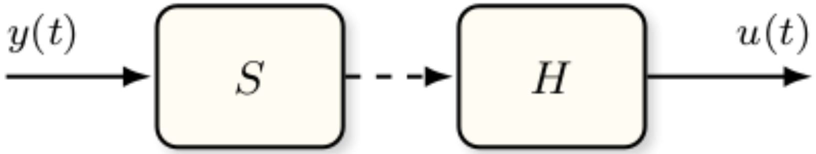
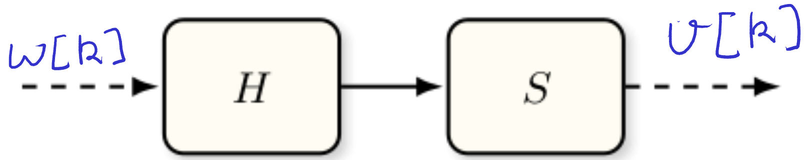


Sample-then-hold: H o S

$$u(t) = \underbrace{H(S(y(t)))}_{H \circ S}$$

composition



Hold-then-sample: S o H

$$v[k] = \underbrace{S(H(w[k]))}_{S \circ H}$$

H o S: Not a Linear Time-Invariant System!
(No transfer function representation, see Section 4.2 in notes)

S o H: Simply the identity system

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