Lesson: Linear regression diagnostic methods

Module: Regression Diagnostics

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The Linear Regression Model

<u>Definition/Assumptions</u> of the linear regression model:

1. *Linearity*
2. *Independence*
3. *Homoskedasticity (constant variance)*
4. *Normality*

Motivation

So far, we've mainly taken for granted that the MLR assumptions have been met. But what if they aren't?

Diagnostic methods:

1. Graphical techniques

2. Numerical techniques

Motivation

A note on the validity assumption

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Lesson: Violations of the linearity assumption

Module: Regression Diagnostics

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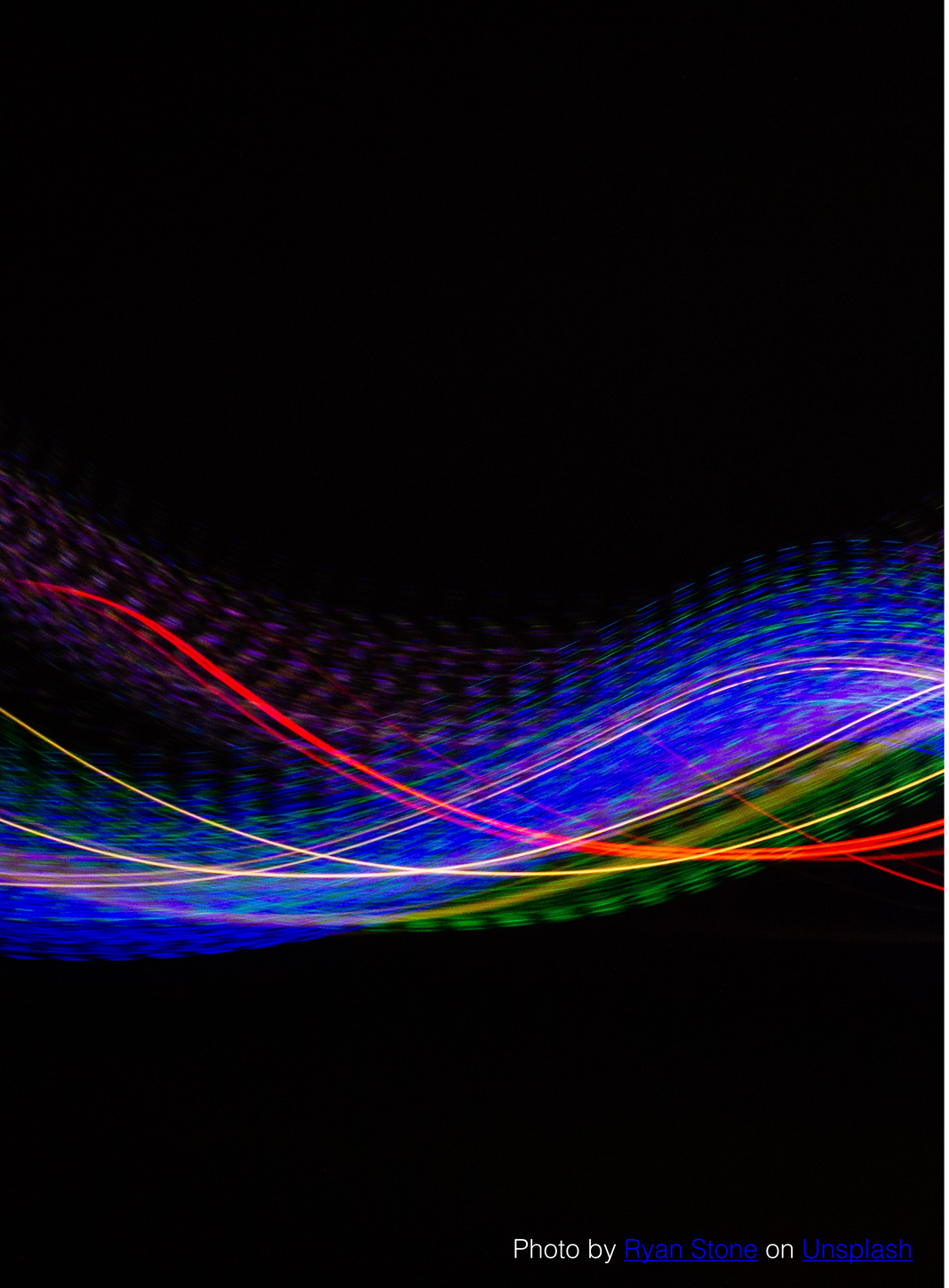

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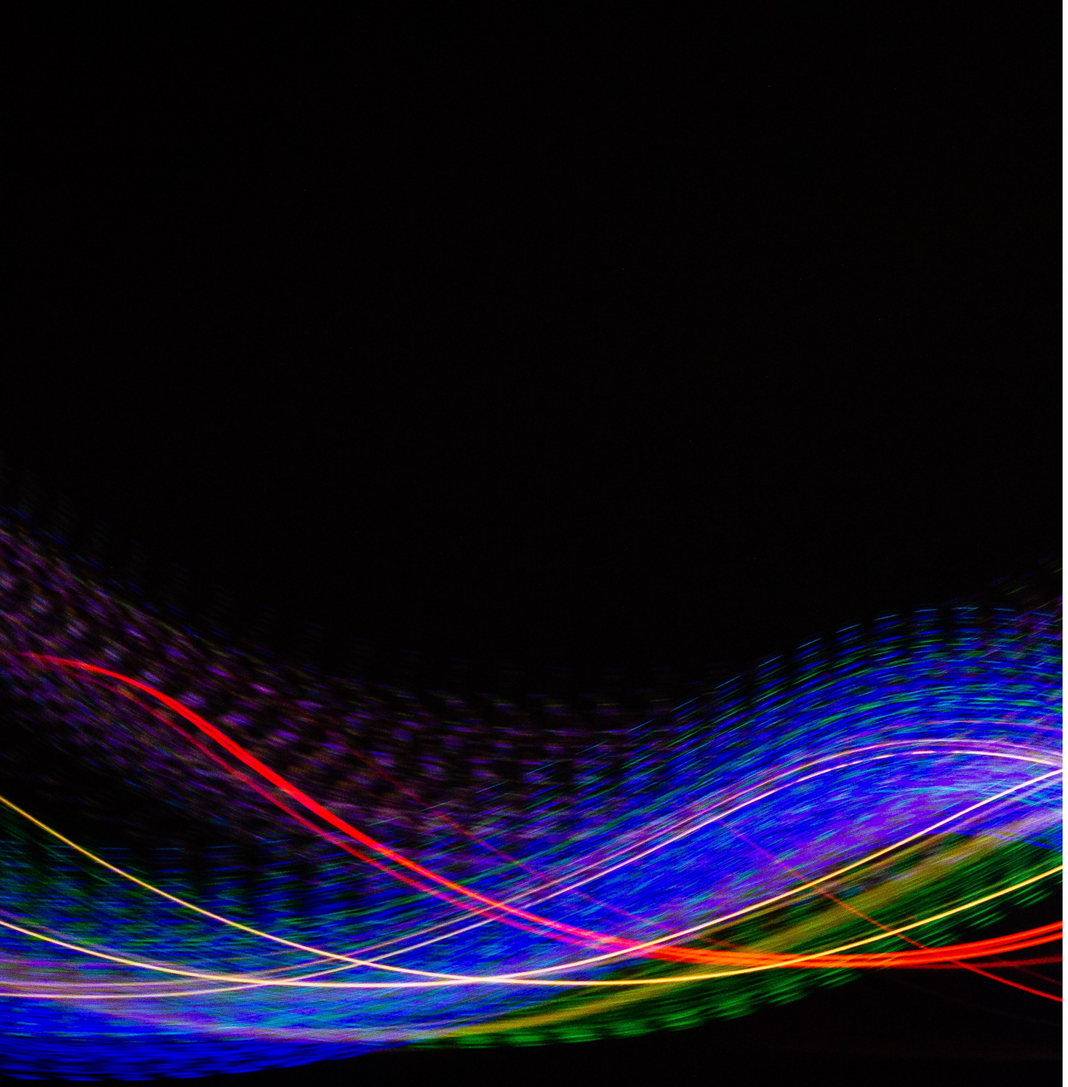
$\mathbf{Y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where $E(\boldsymbol{\varepsilon}) = 0$.

Or equivalently, that for $i = 1, \ldots, n$,

$Y_i = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_p x_{i,p} + \varepsilon_i$, where $E(\varepsilon_i) = 0$.



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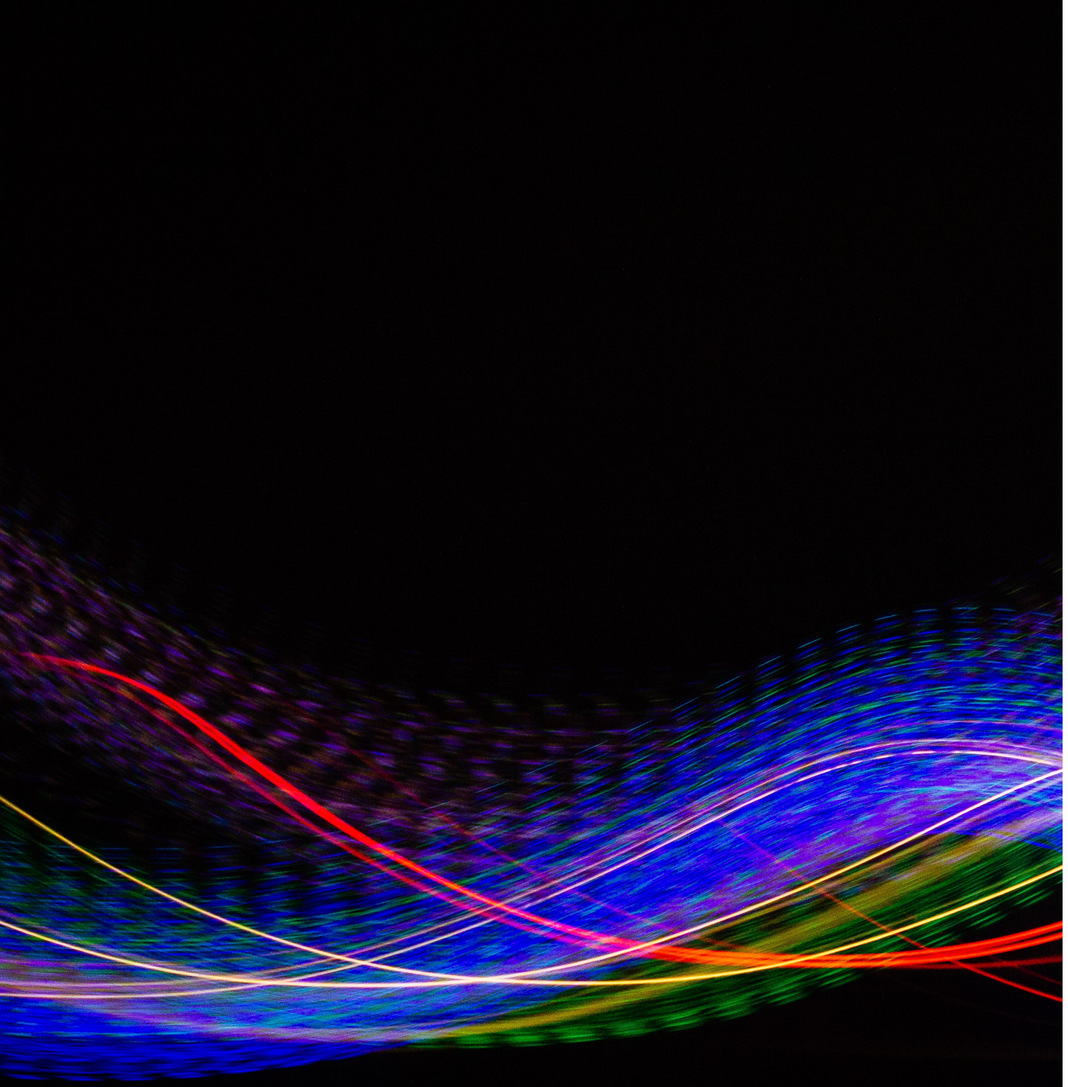
This specification means that, for each $i = 1, \ldots, n$ and $j = 1, \ldots, p$:

1. $E(Y_i)$ is a linear function of $x_{i,j}$, holding all other $x_{i,k}$, $k \neq j$ fixed.

2. The slope of the line that relates $E(Y_i)$ to $x_{i,j}$ does not depend on the values of any other $x_{i,k}$, $k \neq j$.

3. The effects of each $x_{i,j}$ on the



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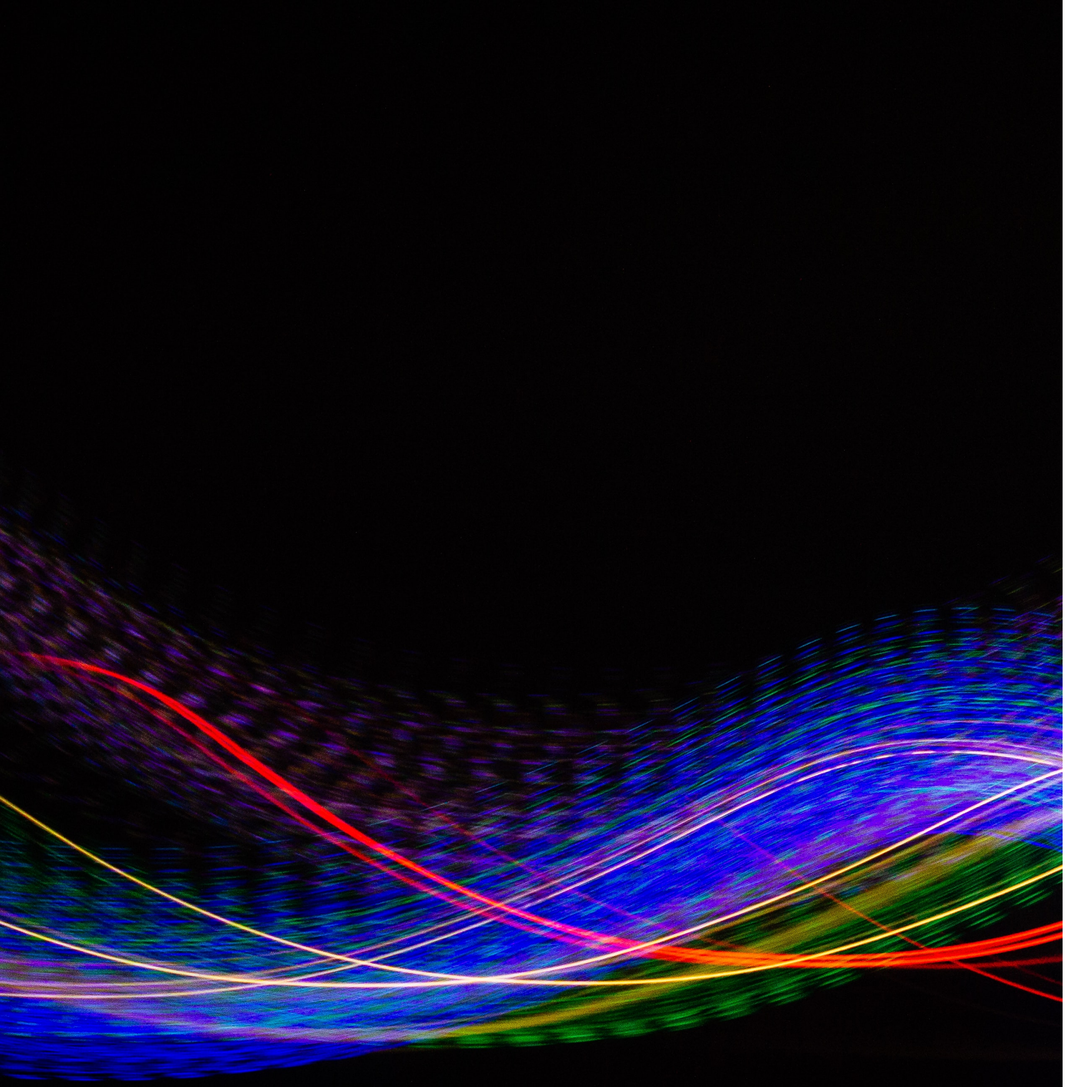
If this assumption is not met:

1. The LS estimator is not unbiased.
2. The model will have low explanatory power.
3. The inferences (e.g., t-tests, F-test) based on the misspecified regression model will be biased and misleading.
4. The predictions from the model are less likely to be accurate.



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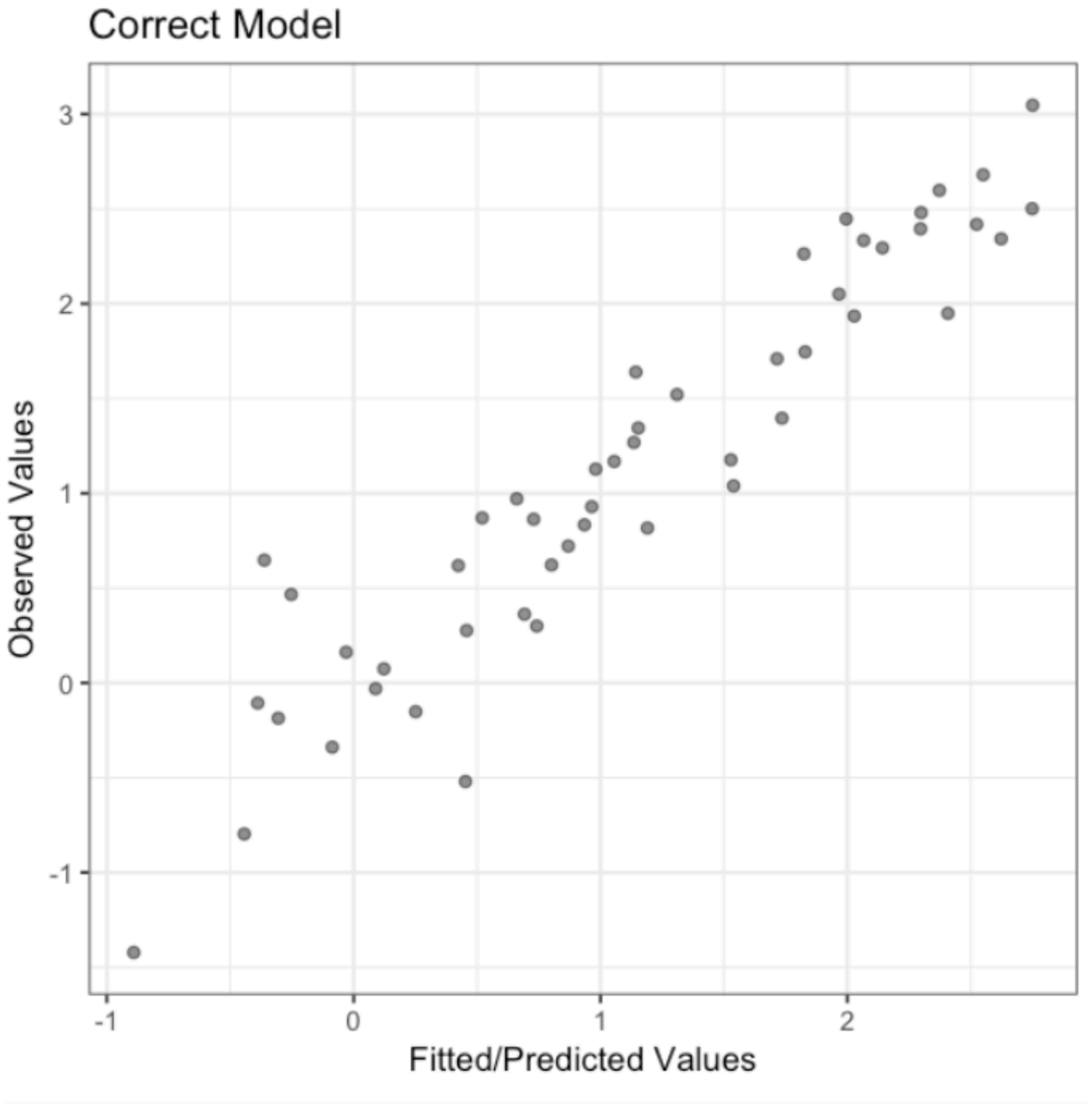
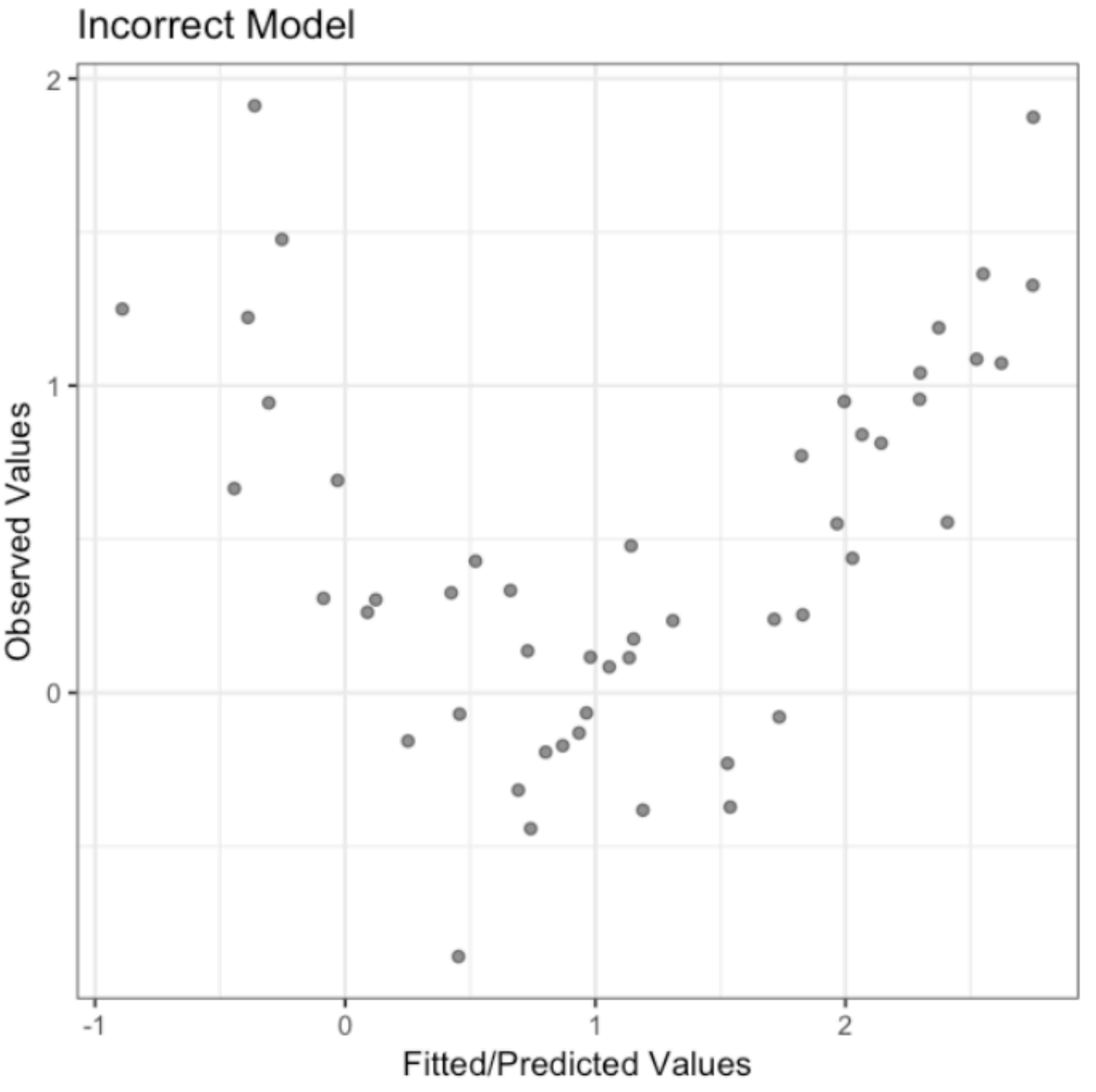
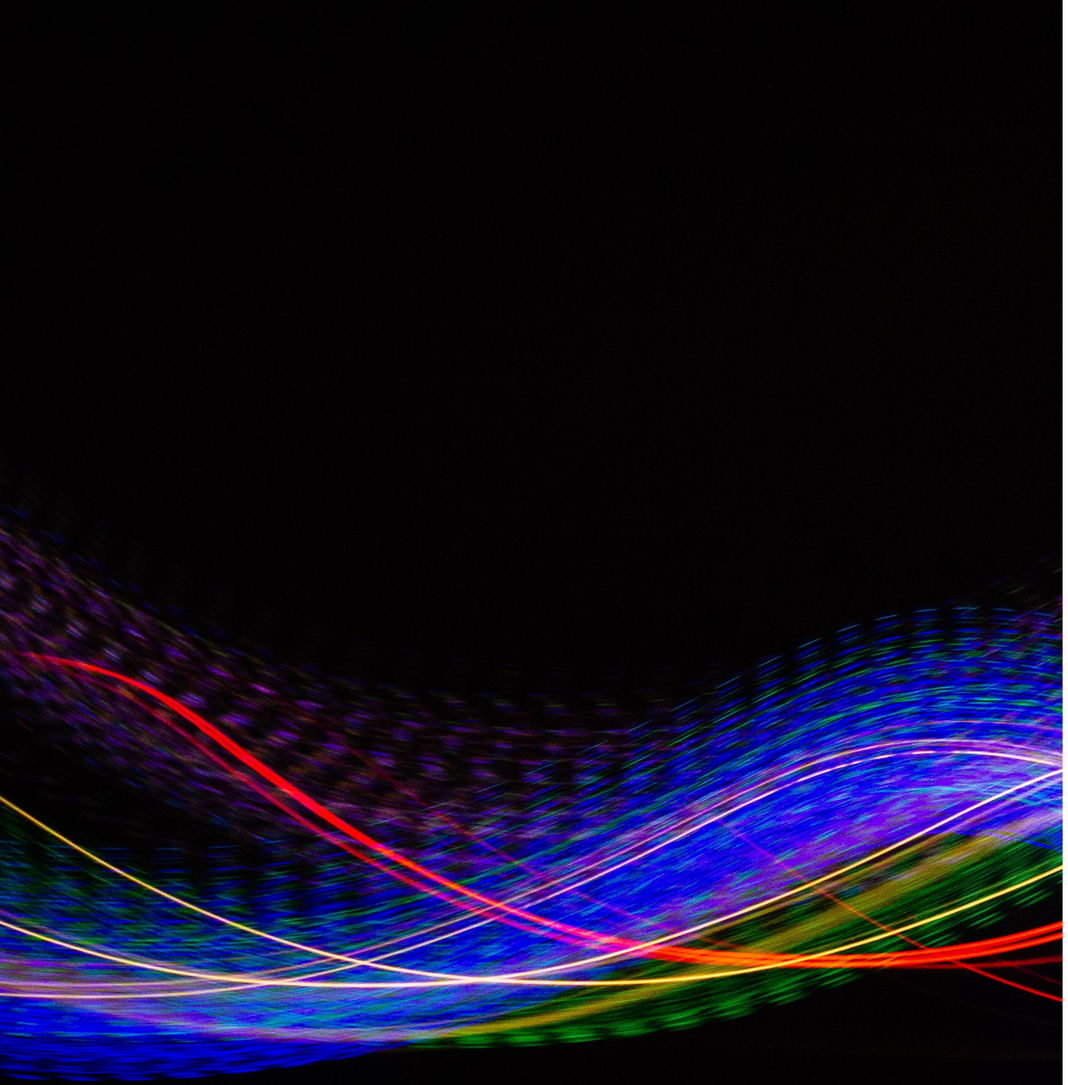
Fitted/Predicted vs. observed value plots


Correct Model
Observed Values
Fitted/Predicted Values
3
2
1
0
-1
-1
0
1
2


Incorrect Model
Observed Values
Fitted/Predicted Values
2
1
0
-1
0
1
2



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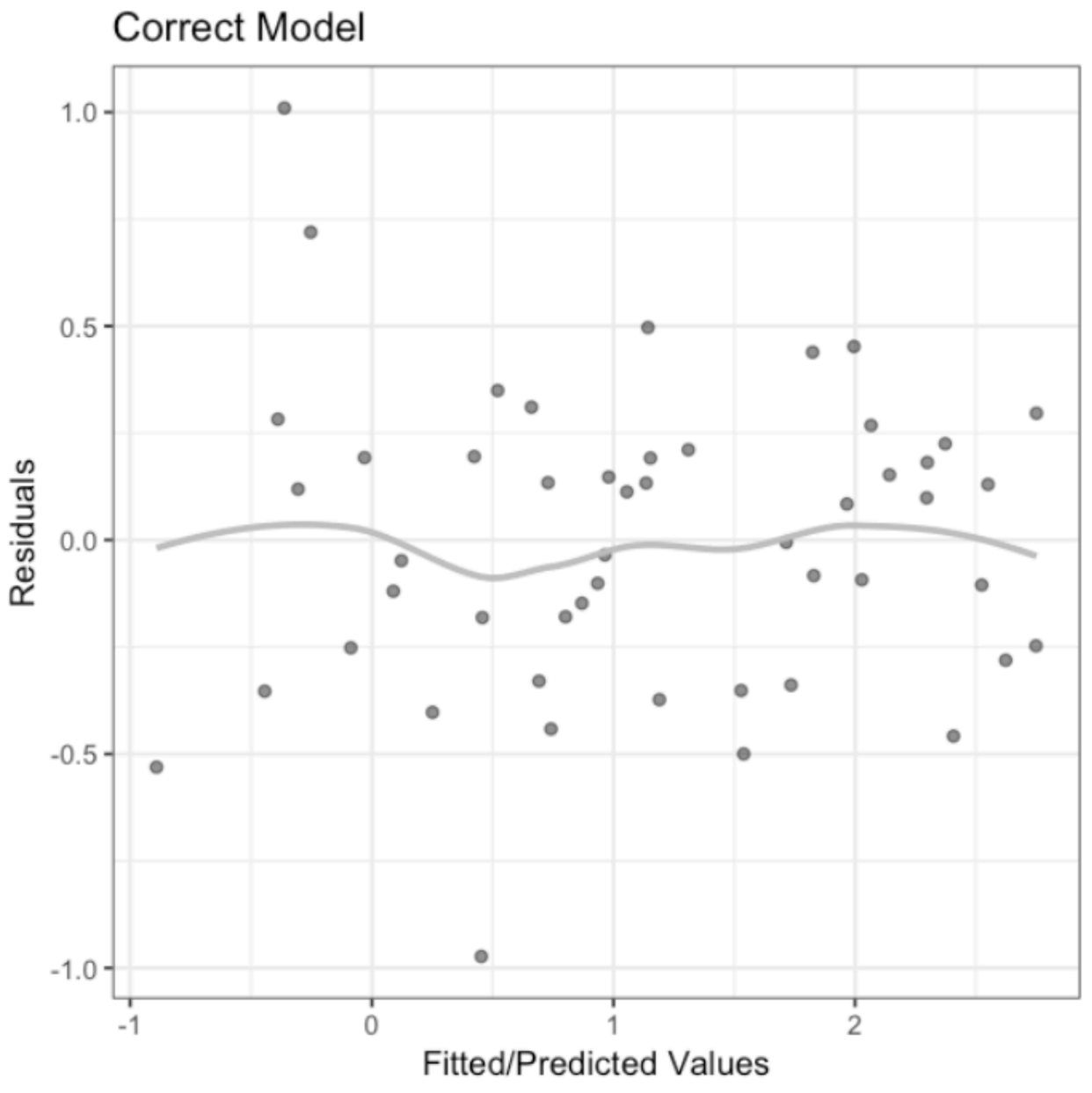
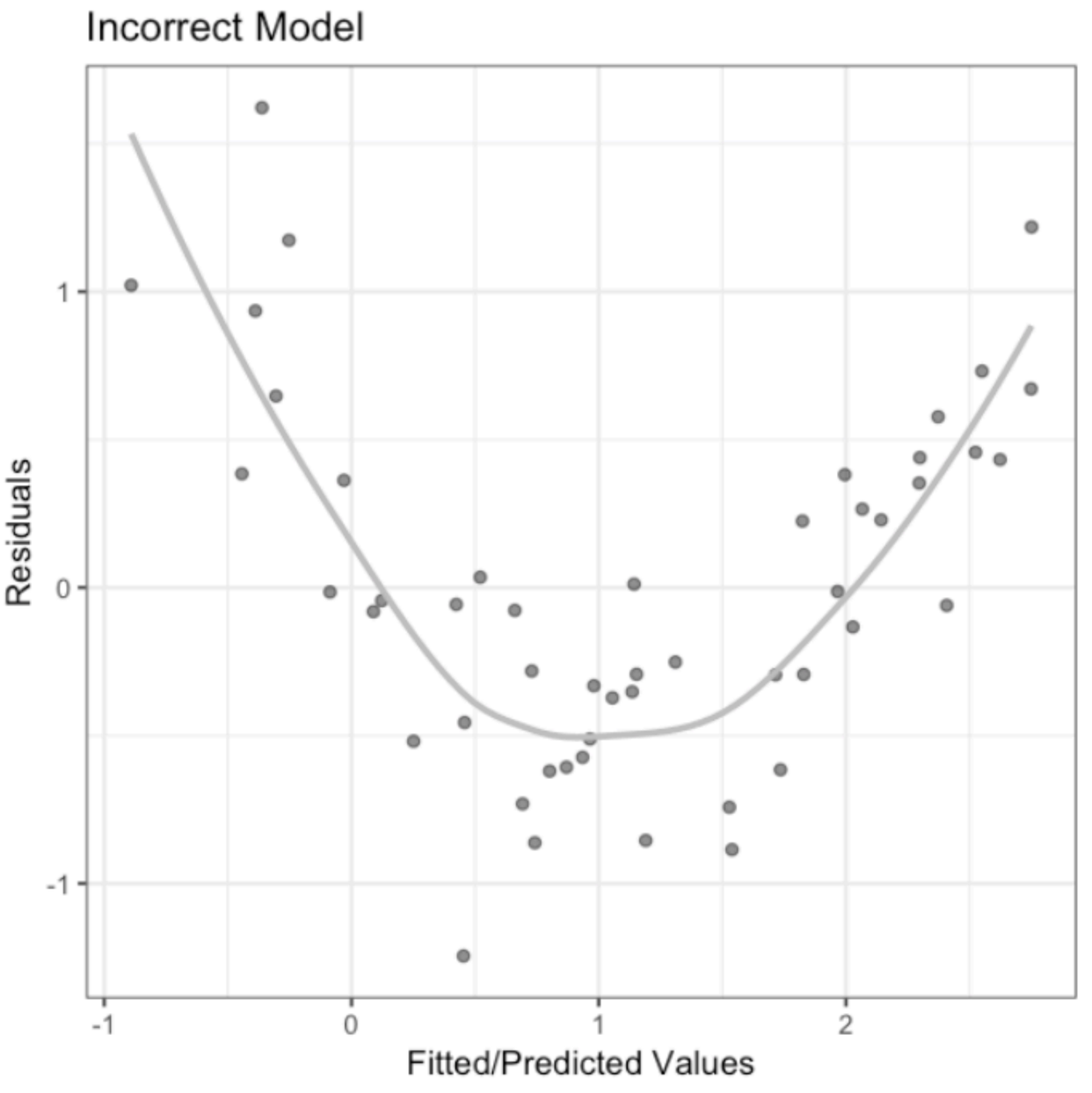
Residual vs. fitted/predicted value plots


Correct Model
Residuals
1.0
0.5
0.0
-0.5
-1.0
-1
0
1
2
Fitted/Predicted Values


Incorrect Model
Residuals
1
0
-1
-1
0
1
2
Fitted/Predicted Values

Lesson: Violations of the independence assumption

Module: Regression Diagnostics

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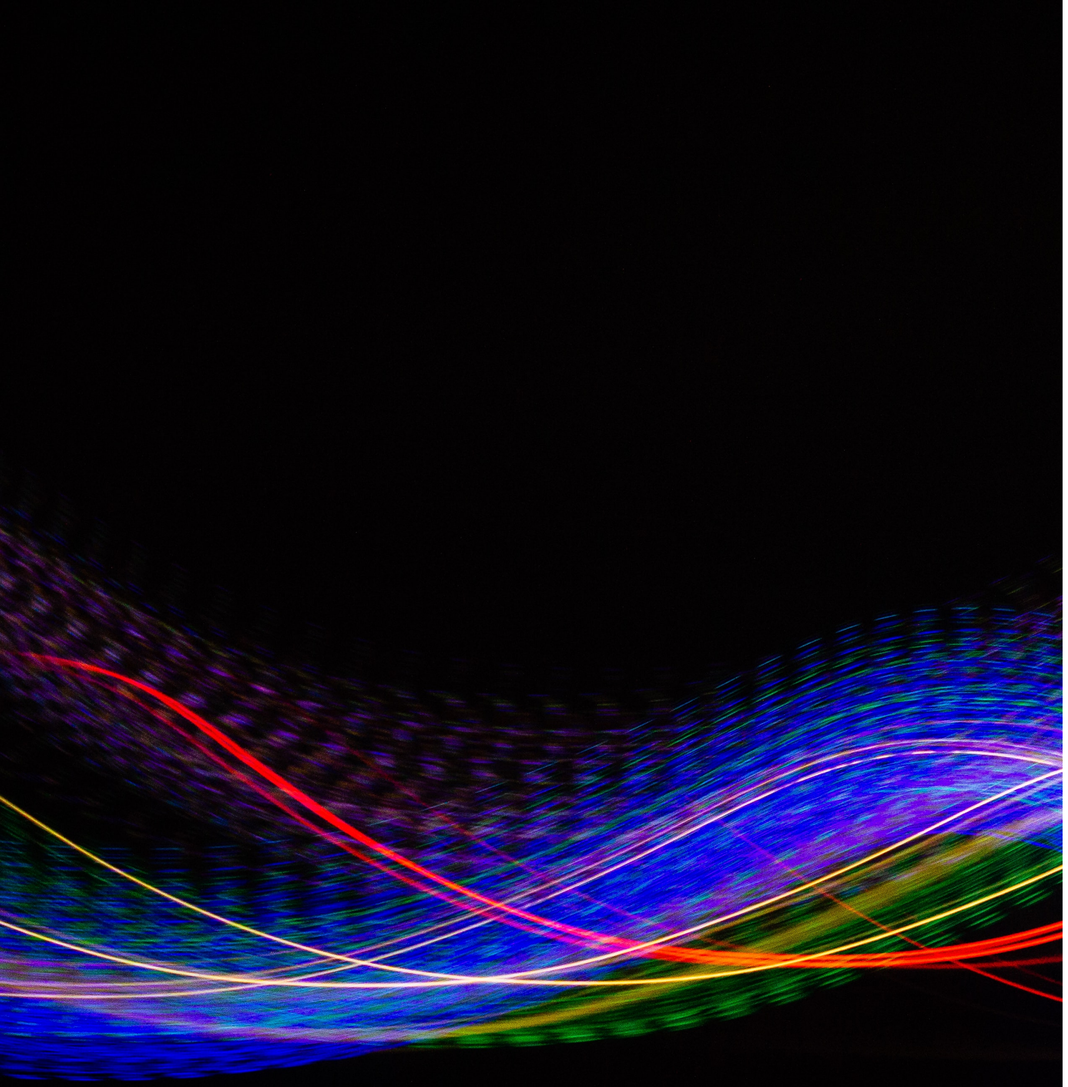

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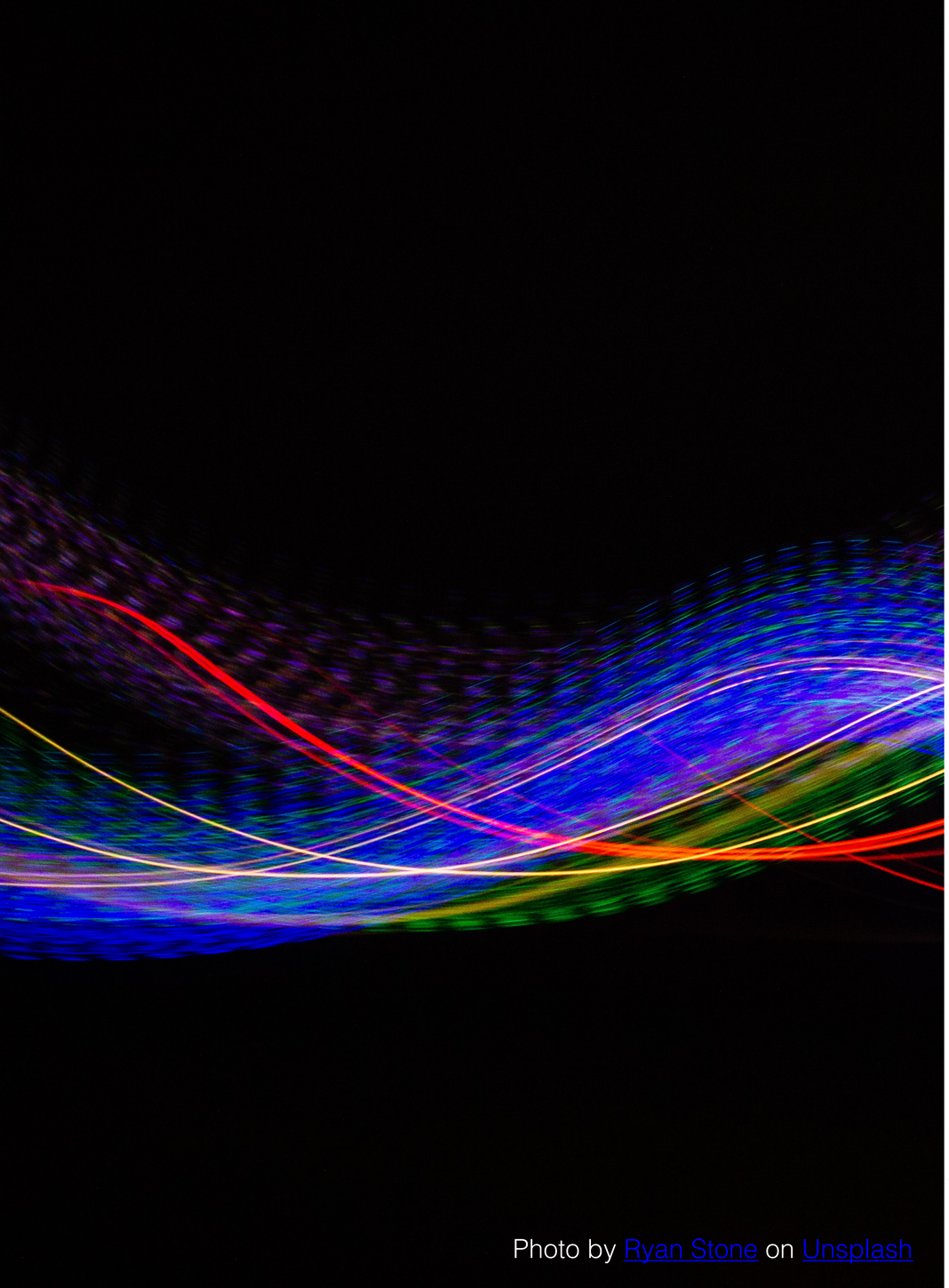
Uncorrelated/Independent Errors. This violation might occur, and be serious, when measuring ***spatial*** or ***temporal*** data.

1. Residuals vs time/index.

2. Successive residual plot.

3. Durbin-Watson test.



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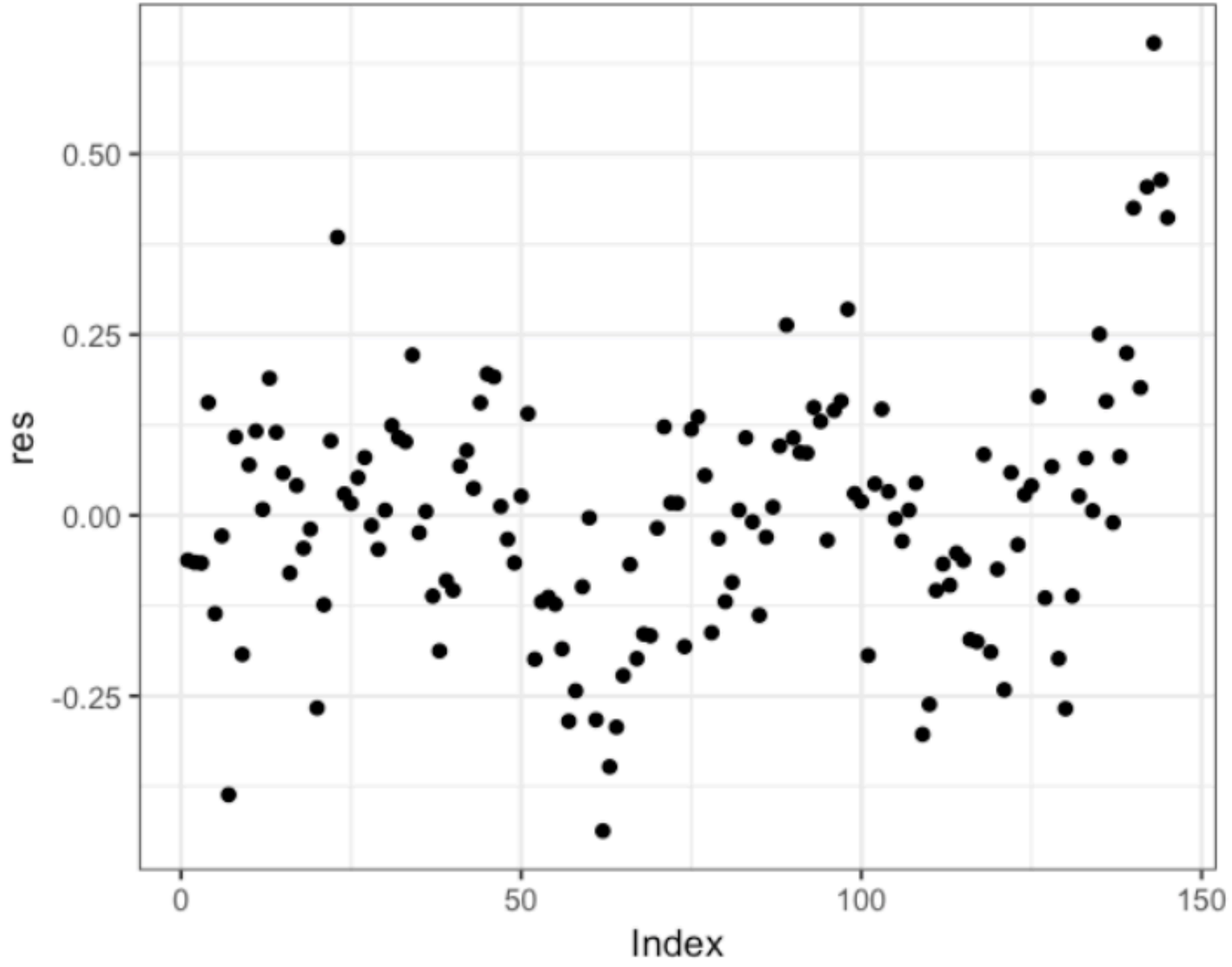
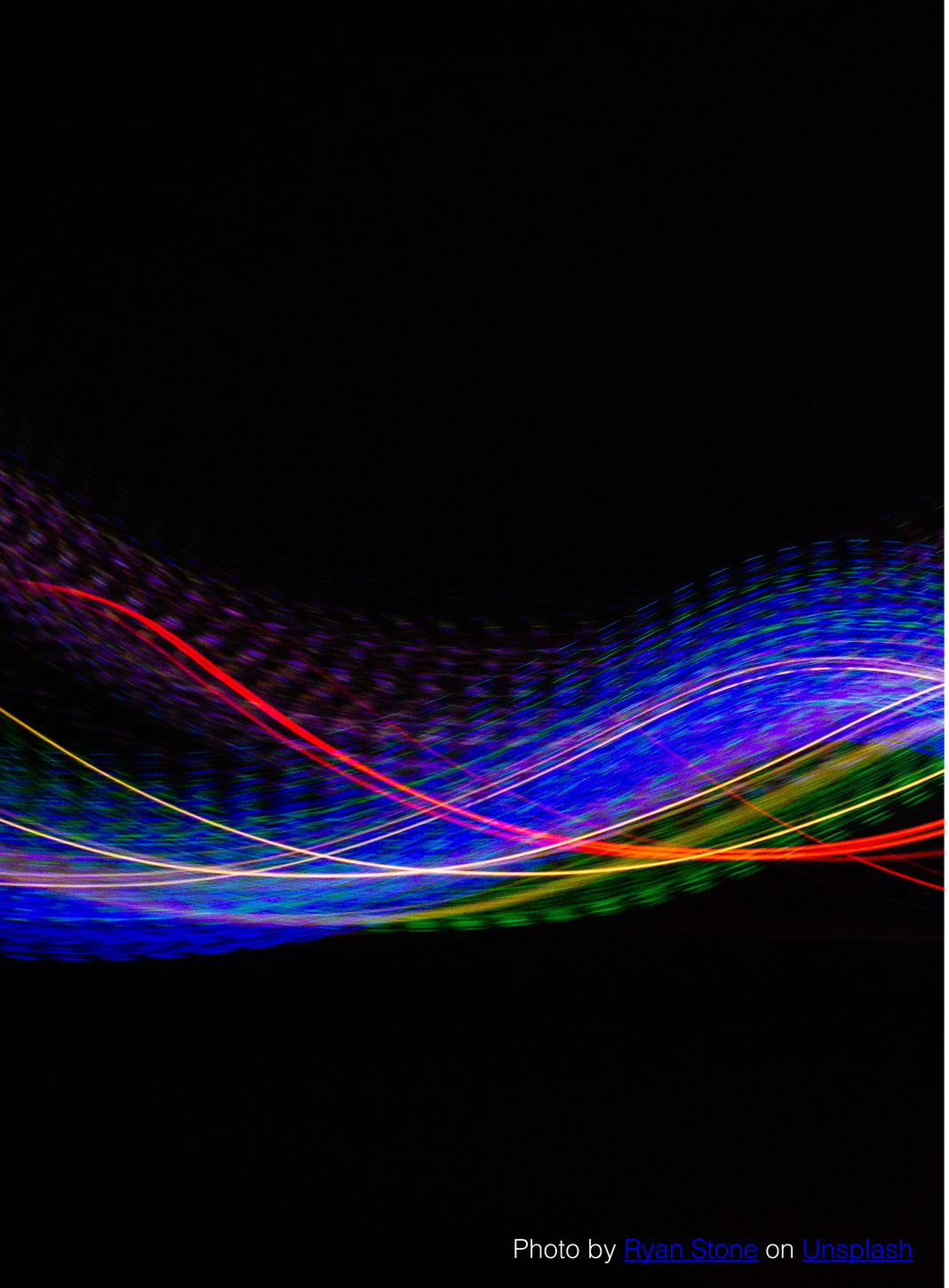
Residual vs index plot





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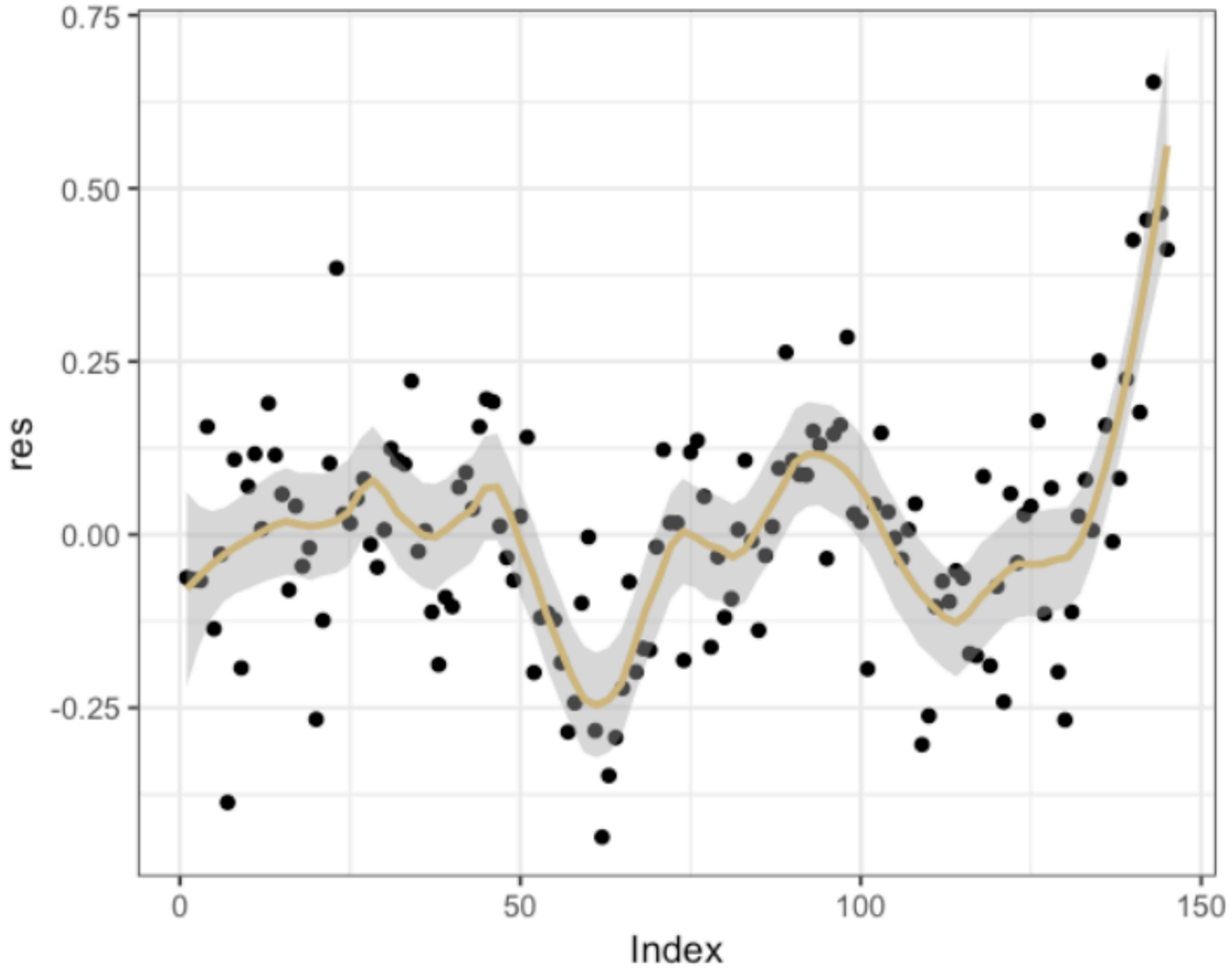
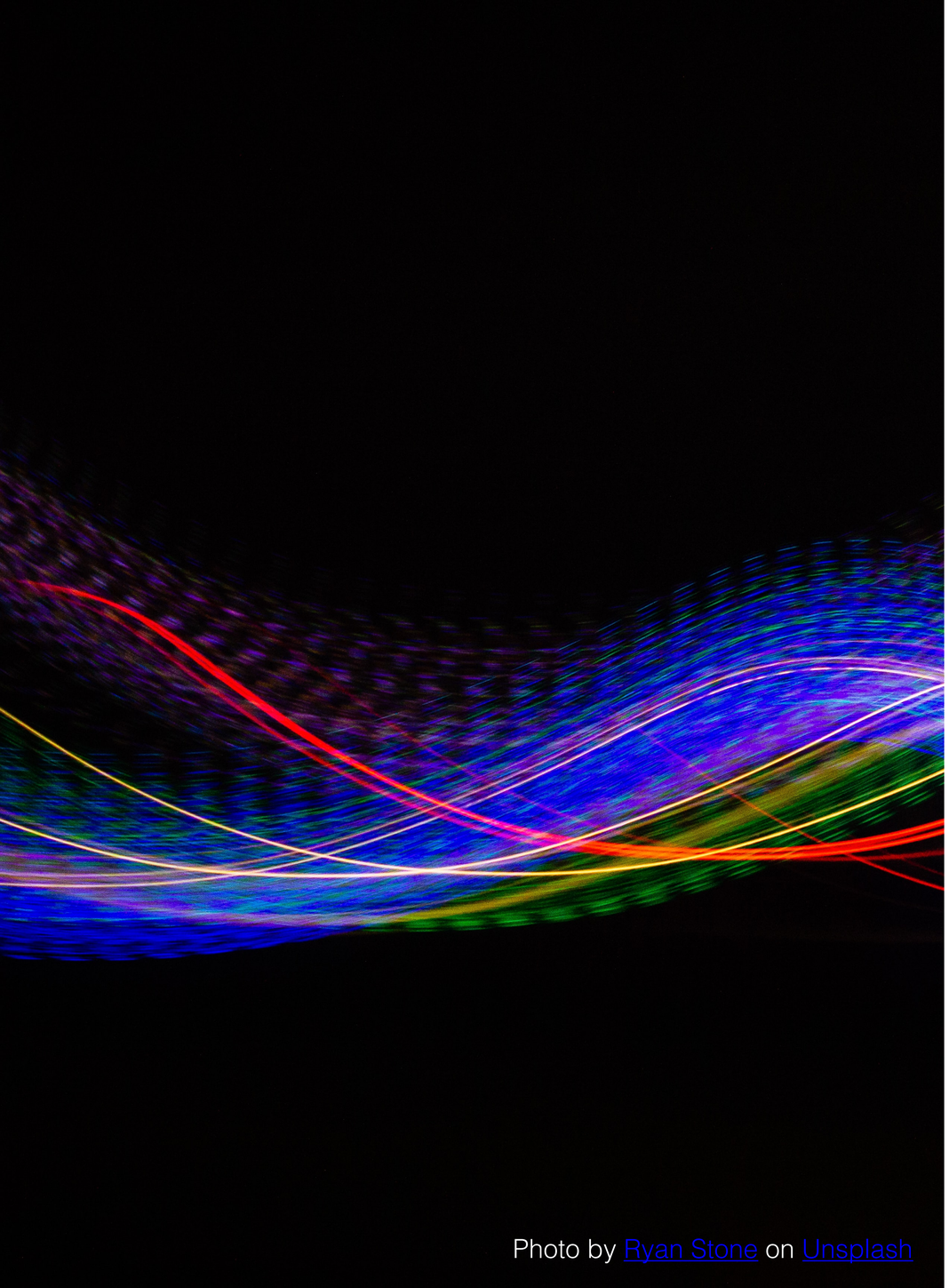
Residual vs index plot





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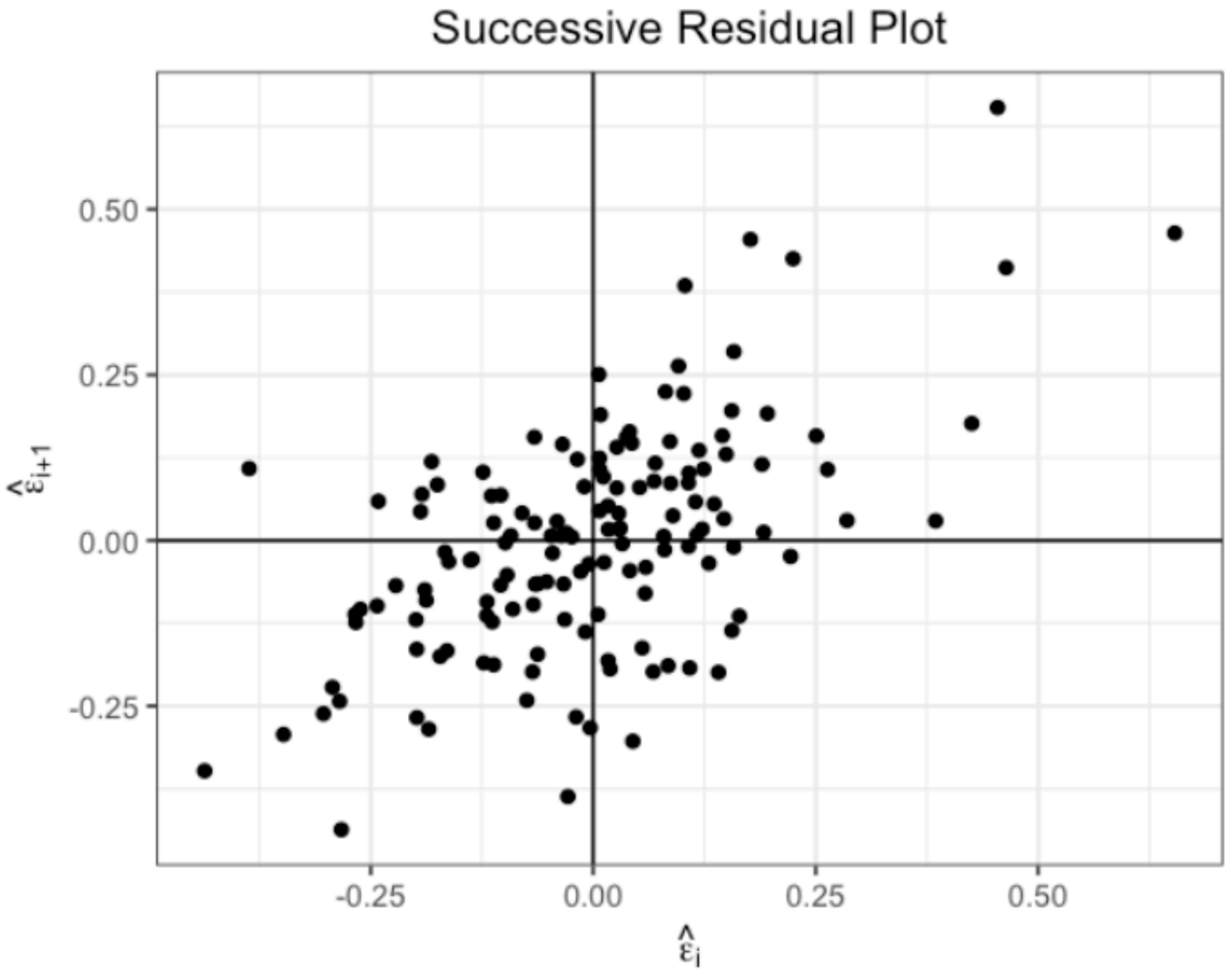

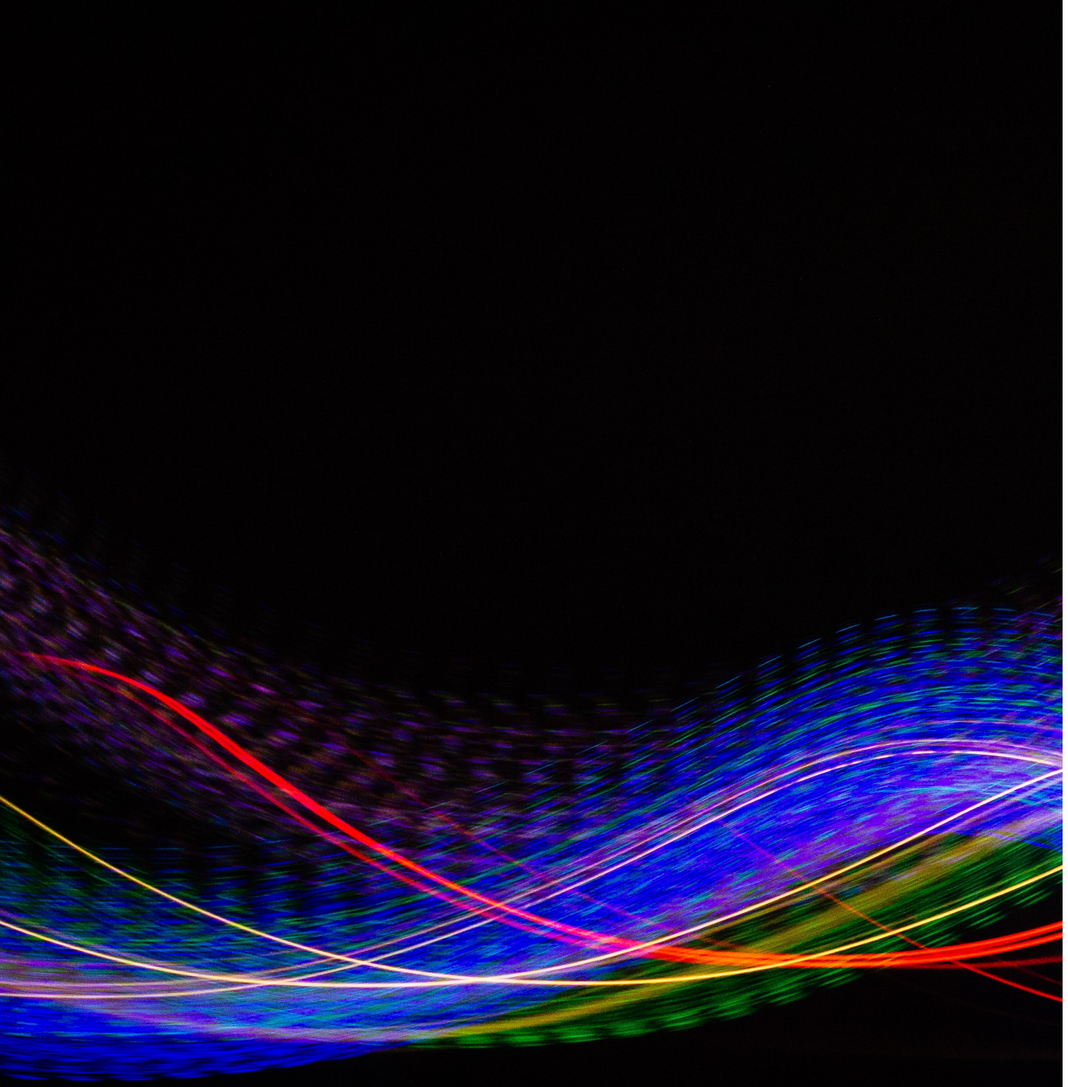

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What solutions are available for violations of this assumption?

1. Generalized least squares

2. Time Series/Spatial Statistics

Lesson: Violations of the constant variance assumption

Module: Regression Diagnostics

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Constant Variance Assumption. Recall that one of our assumptions is:

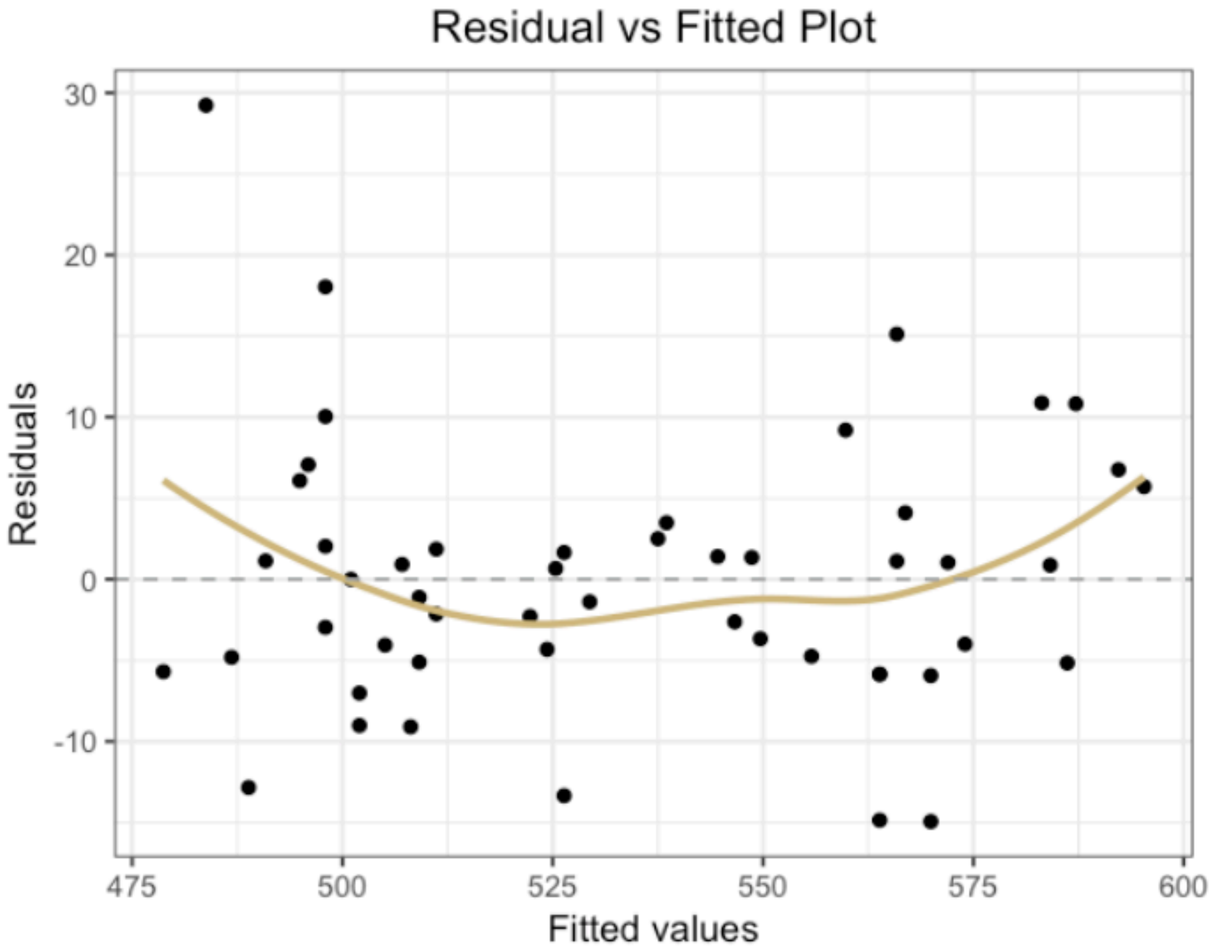
Even under this assumption, the residuals—our estimator of the error—are not constant variance:

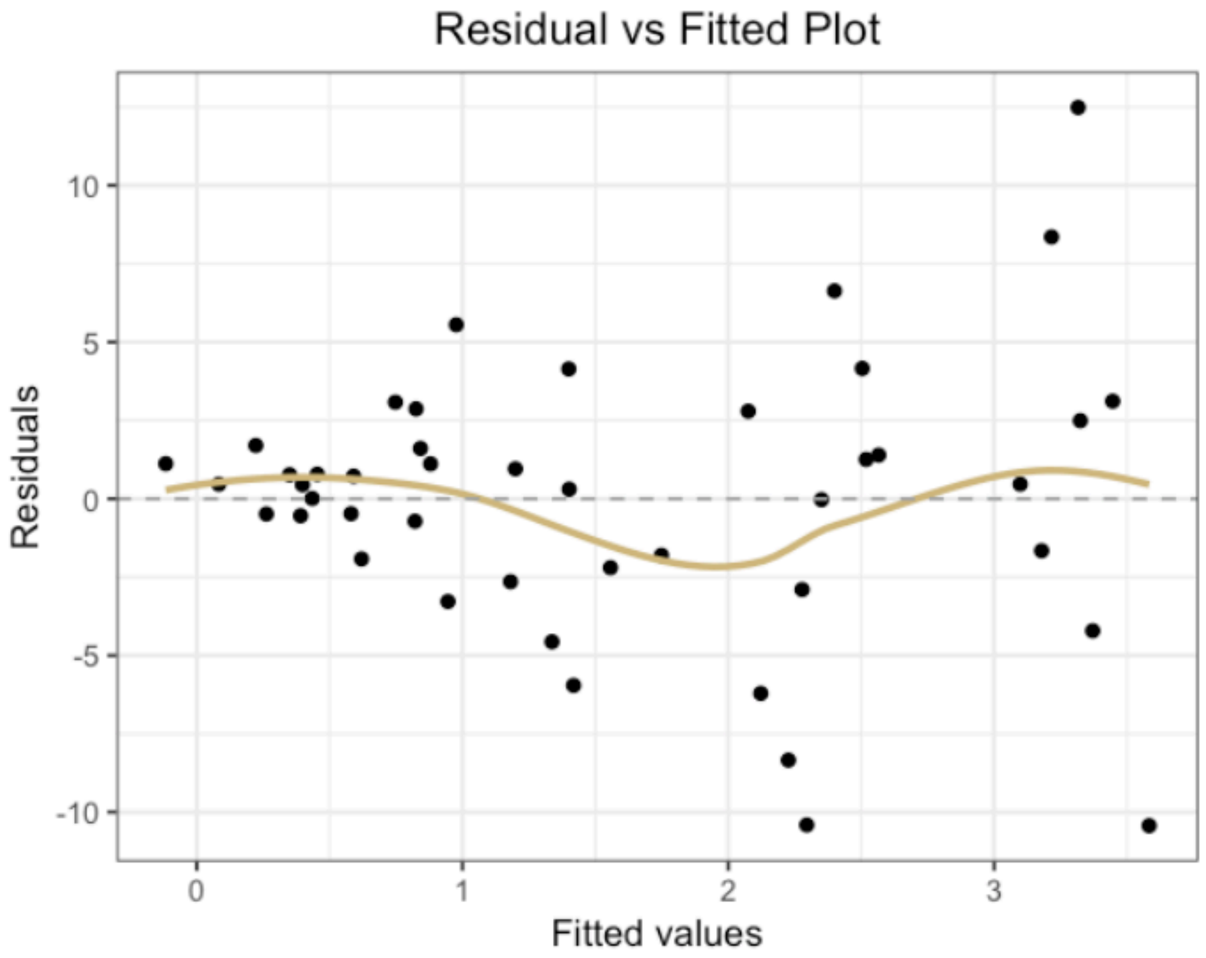
Constant Variance Diagnostics:

1. Residual vs. Fitted plot.

2. Residual vs. potential predictor plot.

Checking Error Assumptions





Solutions to non constant variance:

1. Transformation of the response.

2. Weighted/Generalized Least Squares.

Lesson: Violations of the normality assumption

Module: Regression Diagnostics

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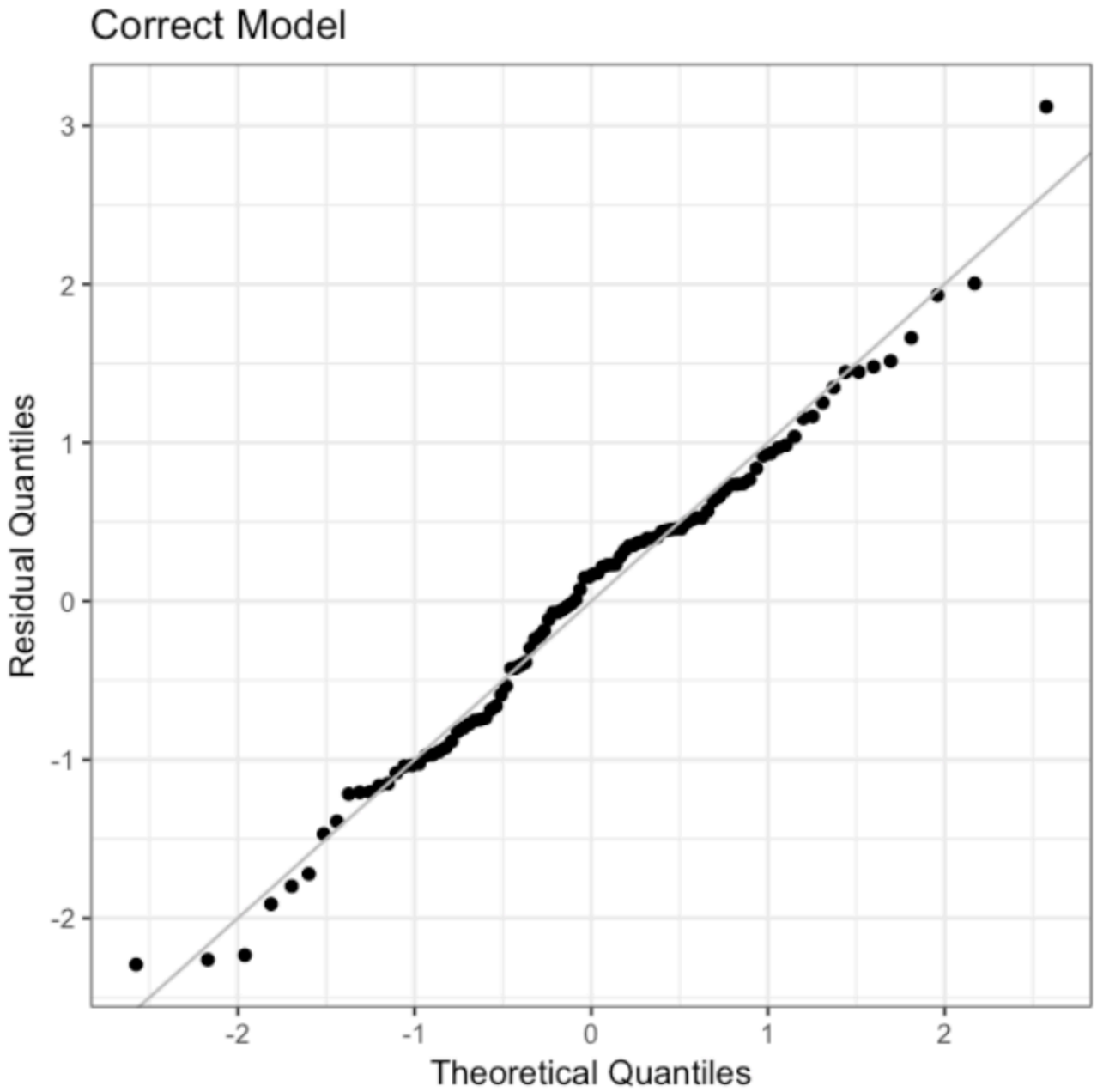
(Height of Hallgrímskirkja in Reykjavik, Iceland)

Normality Assumption.

To diagnose deviations from normality, we can look at:

1. QQ plots
2. Residual vs fitted plots
3. Shapiro-Wilk test for normality

(Height of Hallgrímskirkja in Reykjavik, Iceland)



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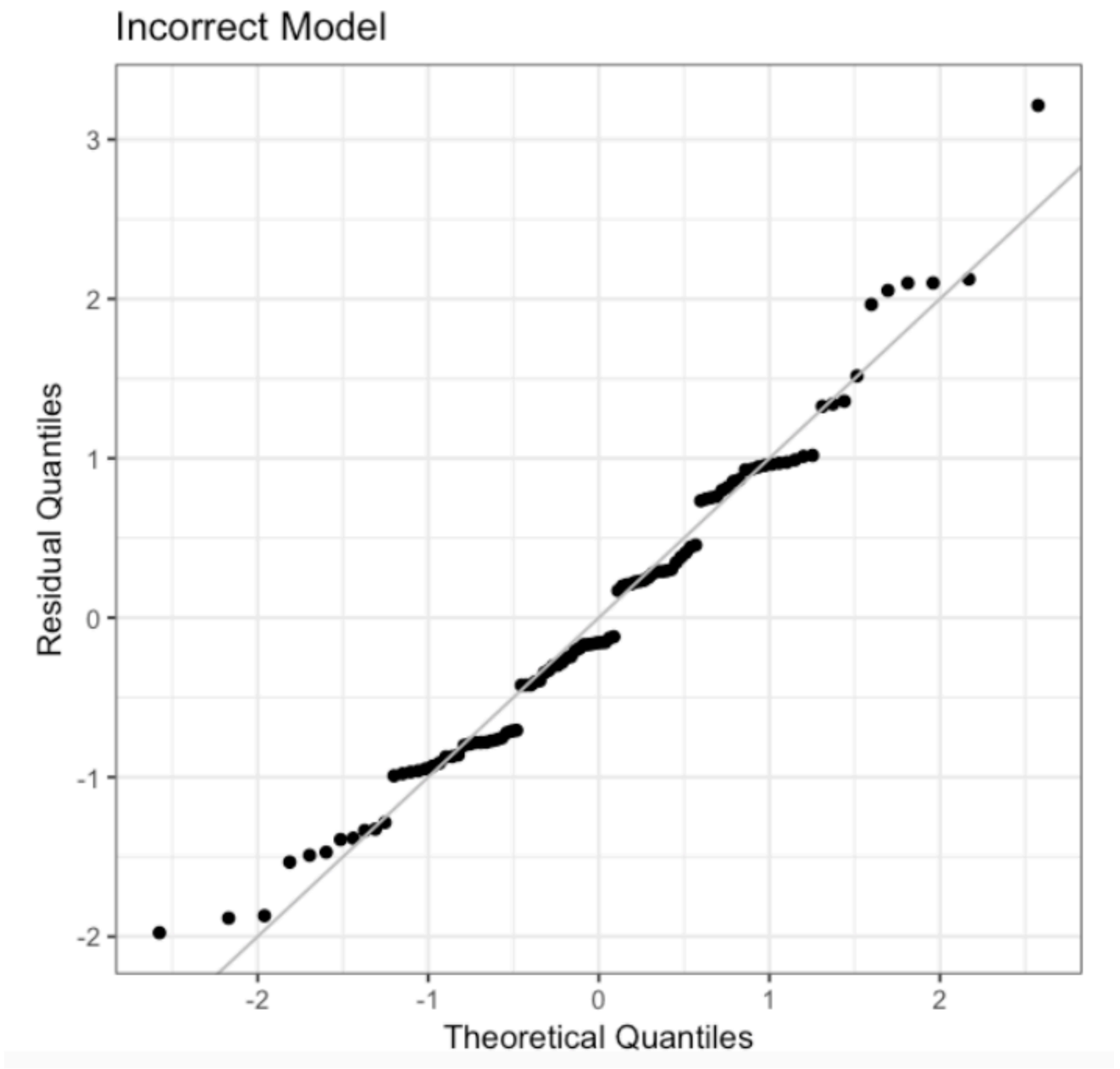
(Height of Hallgrímskirkja in Reykjavik, Iceland)



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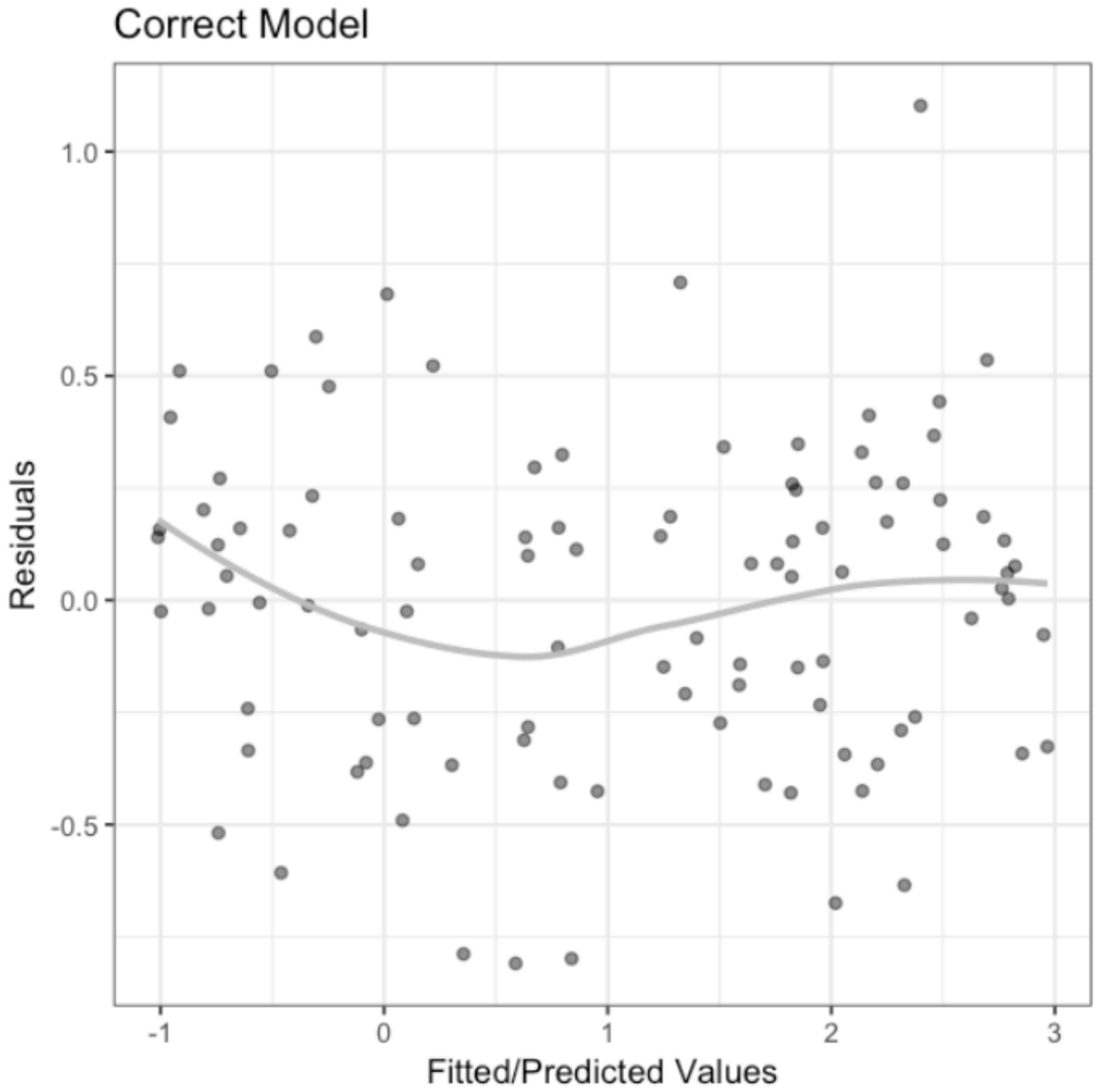
(Height of Hallgrímskirkja in Reykjavik, Iceland)



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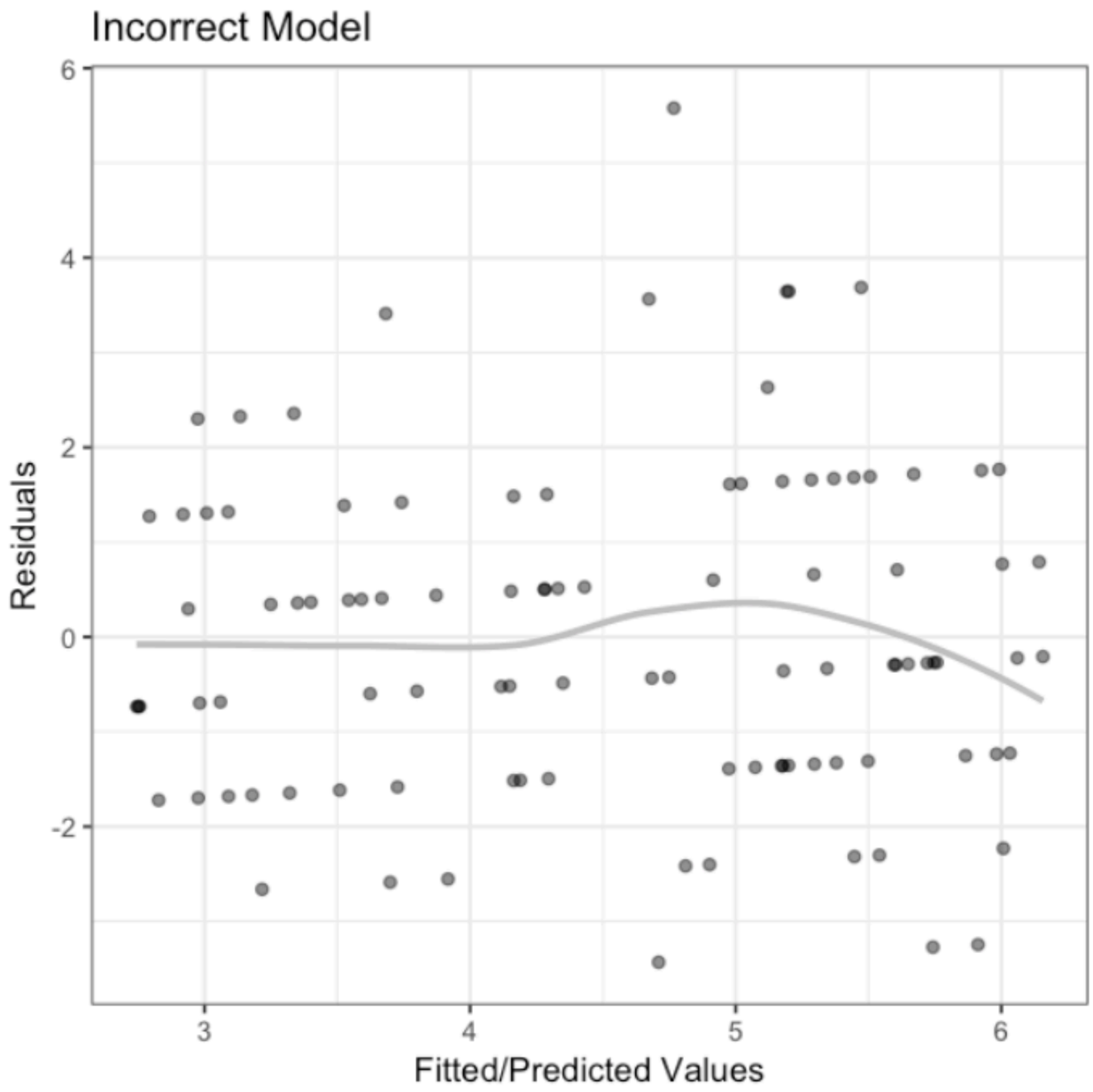
(Height of Hallgrímskirkja in Reykjavik, Iceland)



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H_0: The sample (residuals) came from a normal distribution

H_1: The sample (residuals) came from a non-normal distribution

```
shapiro.test(df.diagnostics$r_correct)
```

```
        Shapiro-Wilk normality test

data:  df.diagnostics$r_correct
W = 0.98711, p-value = 0.4449
```

Photo by Cassidy Dickens on Unsplash

(Height of Hallgrímskirkja in Reykjavik, Iceland)

Solutions:

1. Transformations.

2. *Generalized linear models (GLMs)*

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