# The Mathematics of Feedforward Neural Networks

This reading provides you with the opportunity to check your understanding of the mathematics of feedforward networks. We will provide you with network parameters and an input and suggest you try to compute the output by hand first, before looking at the correct solution on the next page. Have fun!

Let's consider a tiny (to make things easier!) feedforward neural network with: (1) an input layer with 3 neurons, (2) a hidden layer with 2 neurons, and (3) an output layer with 1 neuron. We'll use the ReLU activation function in the hidden layer and a sigmoid activation function in the output layer.

## Given Parameters and Input

Input: $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Weights: $\mathbf{W_1} = \begin{bmatrix} 0.1 & 0.3 & 0.5 \\ 0.2 & 0.4 & 0.6 \end{bmatrix}, \quad \mathbf{W_2} = \begin{bmatrix} 0.7 & 0.8 \end{bmatrix}$

Biases: $\mathbf{b_1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad \mathbf{b_2} = \begin{bmatrix} 0.2 \end{bmatrix}$

Compute the output corresponding to the given input! (Reminder: We suggest you try this on your own before checking the solution on the next page.)

## Solution

### Step 1: Hidden Layer Calculation

Relevant input and parameters:

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.1 & 0.3 & 0.5 \\ 0.2 & 0.4 & 0.6 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

Calculate $z_1$:

$$\begin{aligned} z_1 &= W_1 \times x + b_1 \\ &= \begin{bmatrix} 0.1 & 0.3 & 0.5 \\ 0.2 & 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \\ &= \begin{bmatrix} 0.1 \times 1 + 0.3 \times 2 + 0.5 \times 3 \\ 0.2 \times 1 + 0.4 \times 2 + 0.6 \times 3 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \\ &= \begin{bmatrix} 2.3 \\ 2.9 \end{bmatrix} \end{aligned} \tag{1}$$

Apply the ReLU activation function:

$$h = \mathrm{ReLU}(z_1) = \begin{bmatrix} \max(0, 2.3) \\ \max(0, 2.9) \end{bmatrix} = \begin{bmatrix} 2.3 \\ 2.9 \end{bmatrix}$$

### Step 2: Output Layer Calculation

Relevant input and parameters:

$$h = \begin{bmatrix} 2.3 \\ 2.9 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.7 & 0.8 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0.2 \end{bmatrix}$$

Calculate $z_2$:

$$\begin{aligned} z_2 &= W_2 \times h + b_2 \\ &= \begin{bmatrix} 0.7 & 0.8 \end{bmatrix} \times \begin{bmatrix} 2.3 \\ 2.9 \end{bmatrix} + \begin{bmatrix} 0.2 \end{bmatrix} \\ &= \begin{bmatrix} 0.7 \times 2.3 + 0.8 \times 2.9 \end{bmatrix} + \begin{bmatrix} 0.2 \end{bmatrix} \\ &= \begin{bmatrix} 4.13 \end{bmatrix} \end{aligned}$$

Apply the sigmoid activation function:

$$y = \text{sigmoid}(z_2) = \begin{bmatrix} \frac{1}{1+e^{-4.13}} \end{bmatrix} \approx \begin{bmatrix} 0.984 \end{bmatrix}$$

**Result:** The output of the feedforward neural network for the input $x$ is approximately $\begin{bmatrix} 0.984 \end{bmatrix}$. Assuming binary classification, this would usually be interpreted as prediction of the positive class.