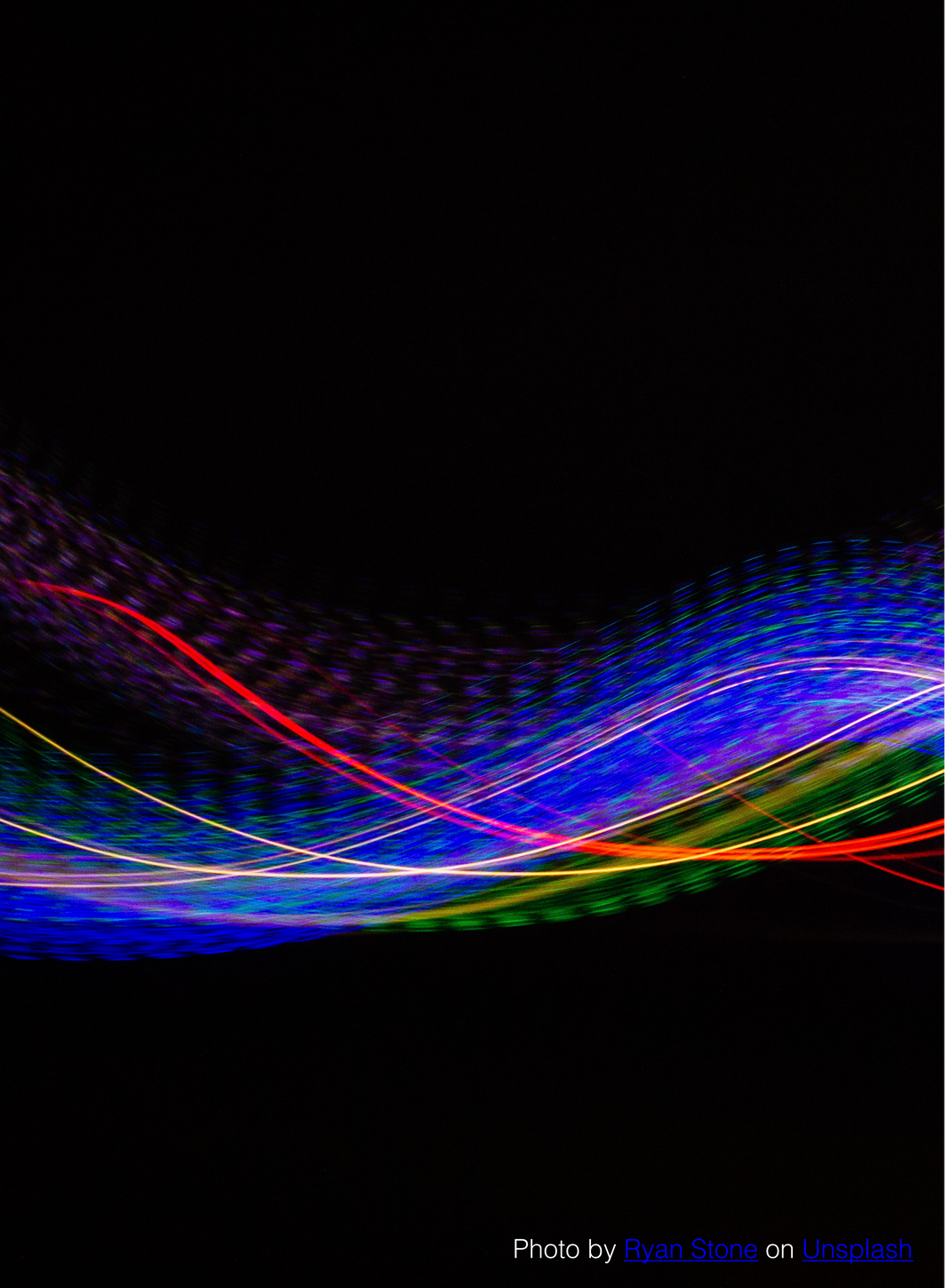
Lesson: Differentiating prediction and explanation

Module: Prediction and Explanation in Linear Regression Analysis



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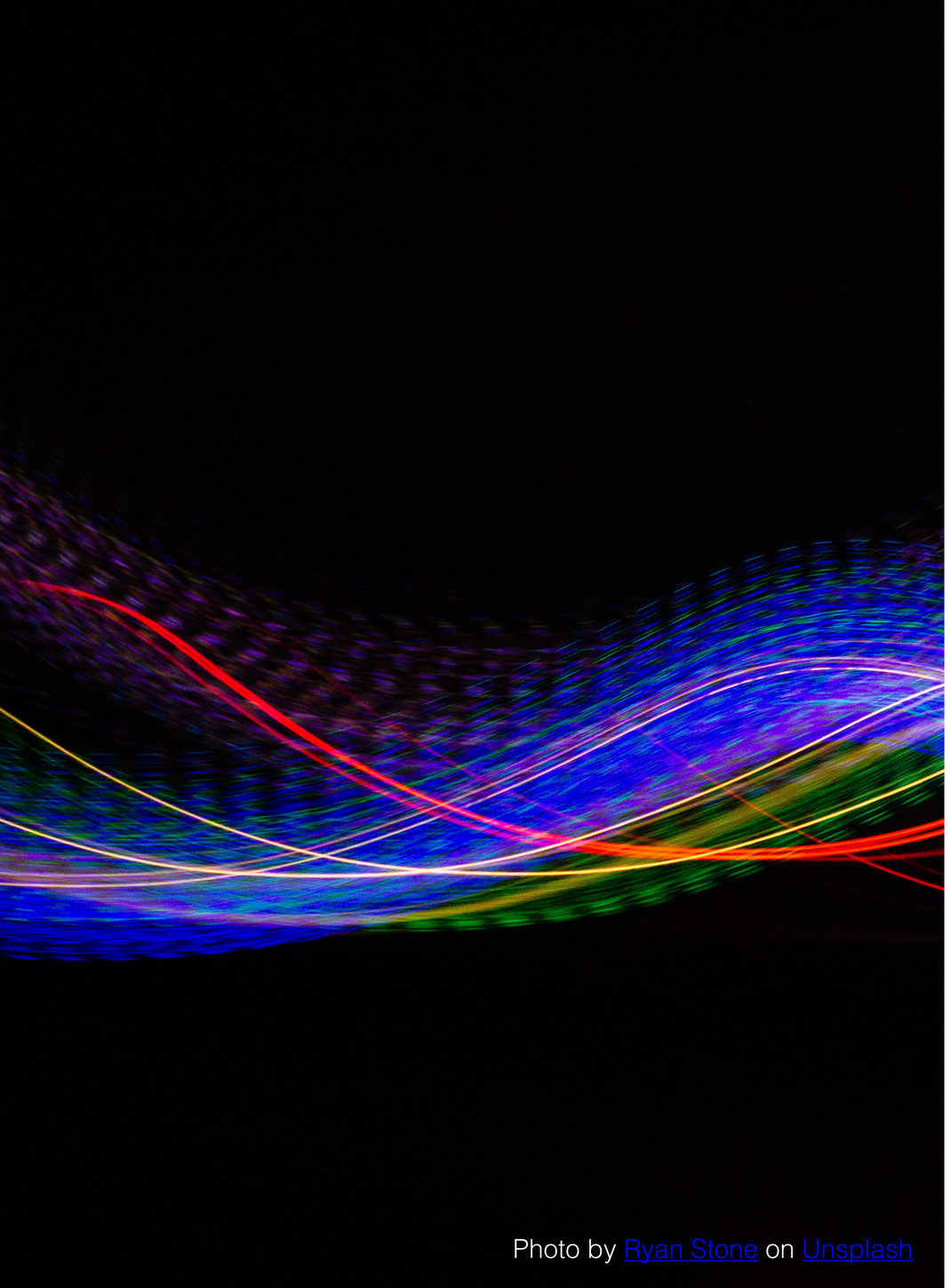
An *explanandum* is a set of propositions that refer to the thing being explained.

An *explanans* is the set of propositions that do the explaining.

Explanatory modeling is an explanans that makes use of statistical models for testing and providing causal explanations.



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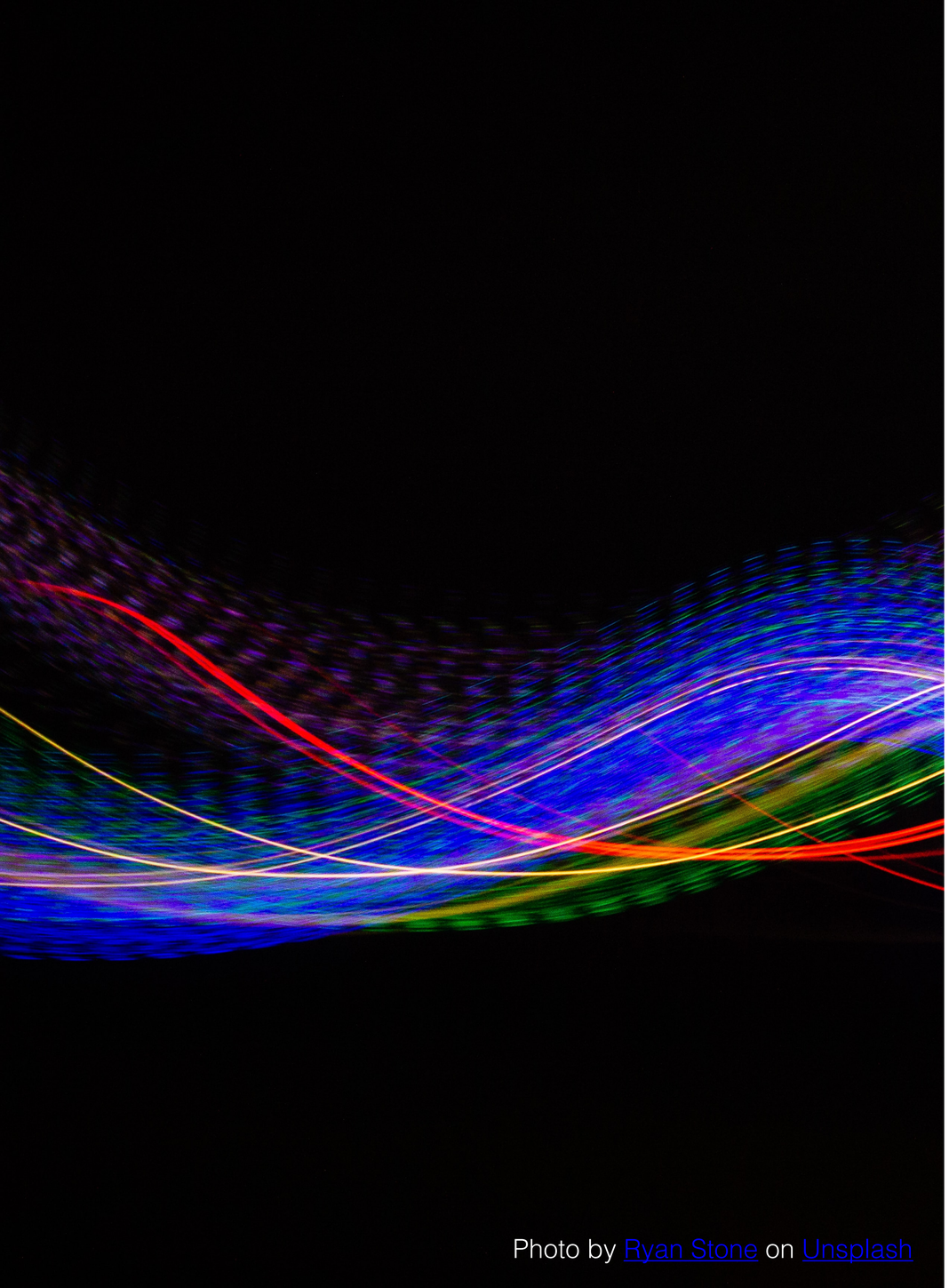
Explanatory models often:

1. Have parameters with a **physical meaning** (e.g., Hooke's Law).

2. Arise in the context of an **experiment design**.

3. Require additional assumptions if the data come from an **observational study**.



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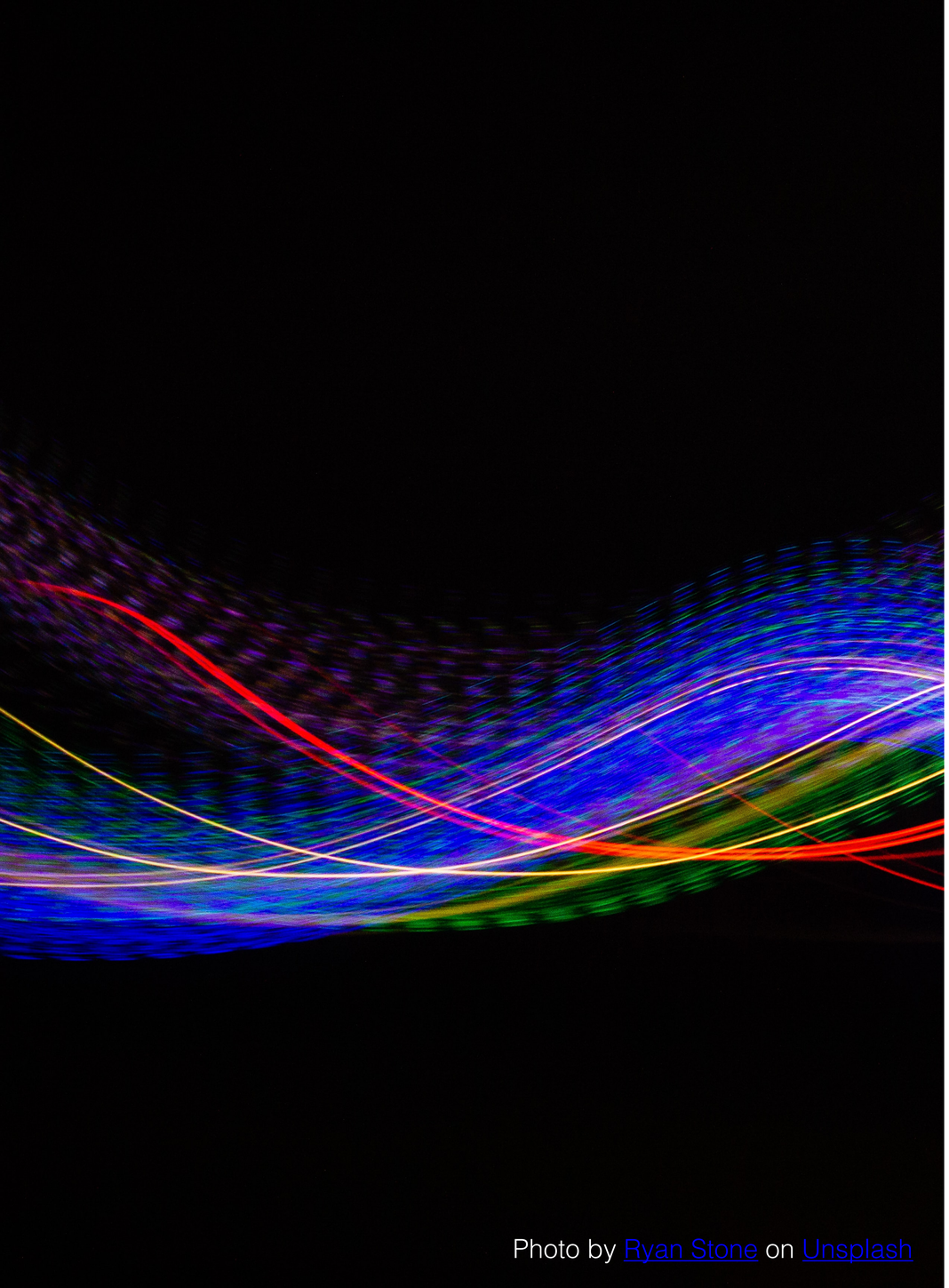
A *prediction* is an empirical consequence of a theory that has not yet been verified at the time the theory was constructed.

A *predictive model* is a statistical model used to provide a value (or range of values) of the response based on values of the predictors that were not used to train the model.



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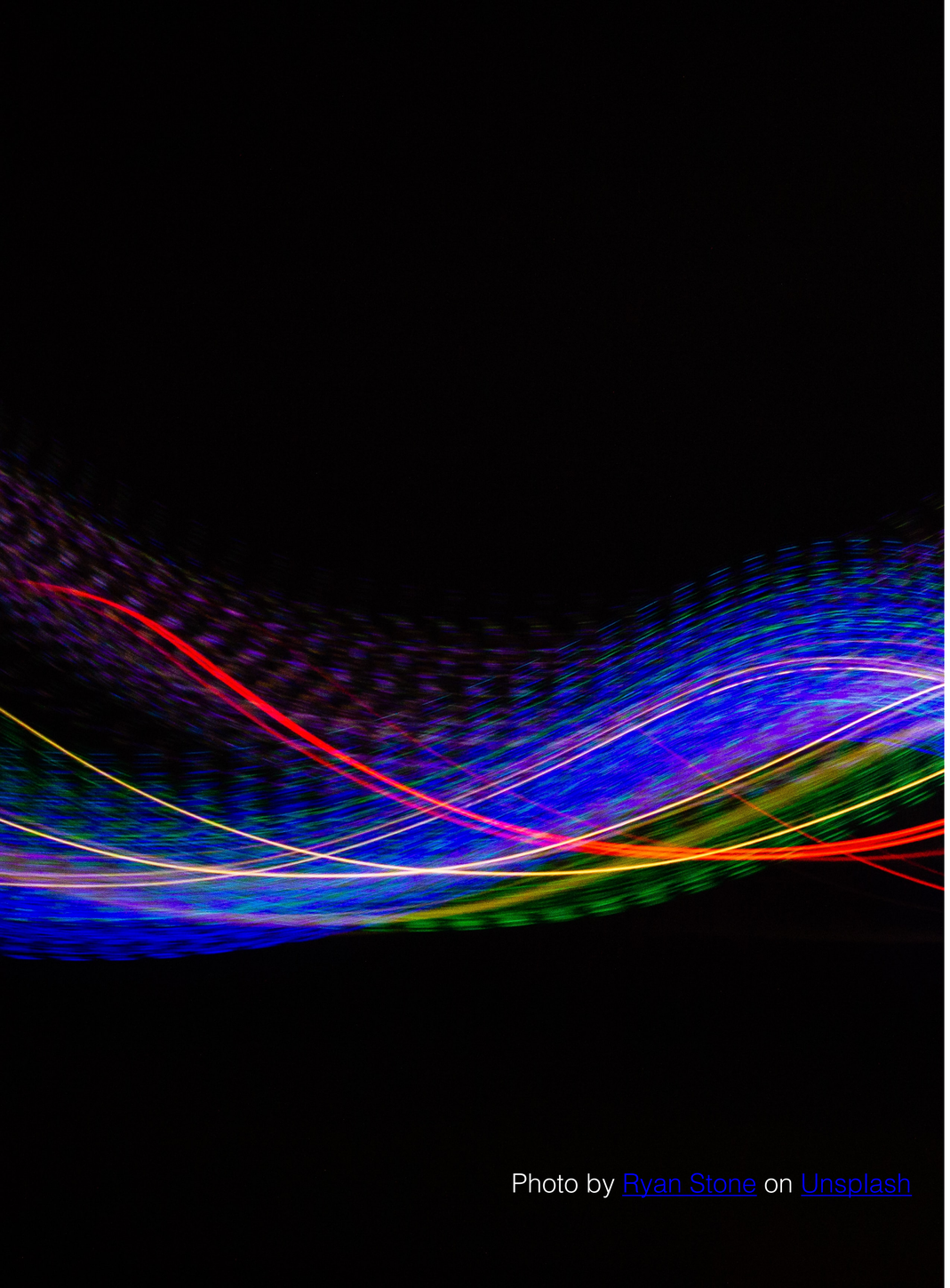
Difference between prediction and explanation?



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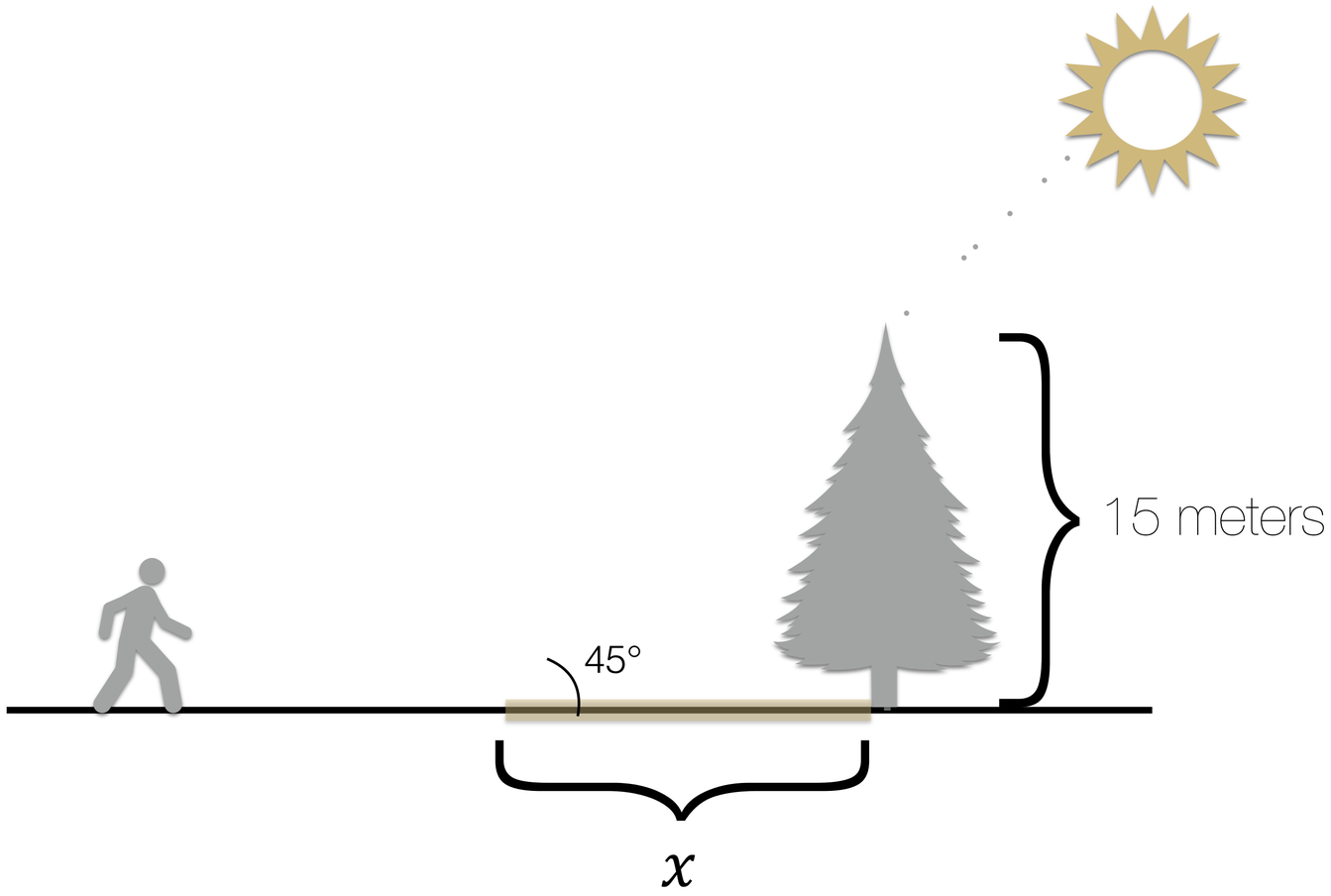

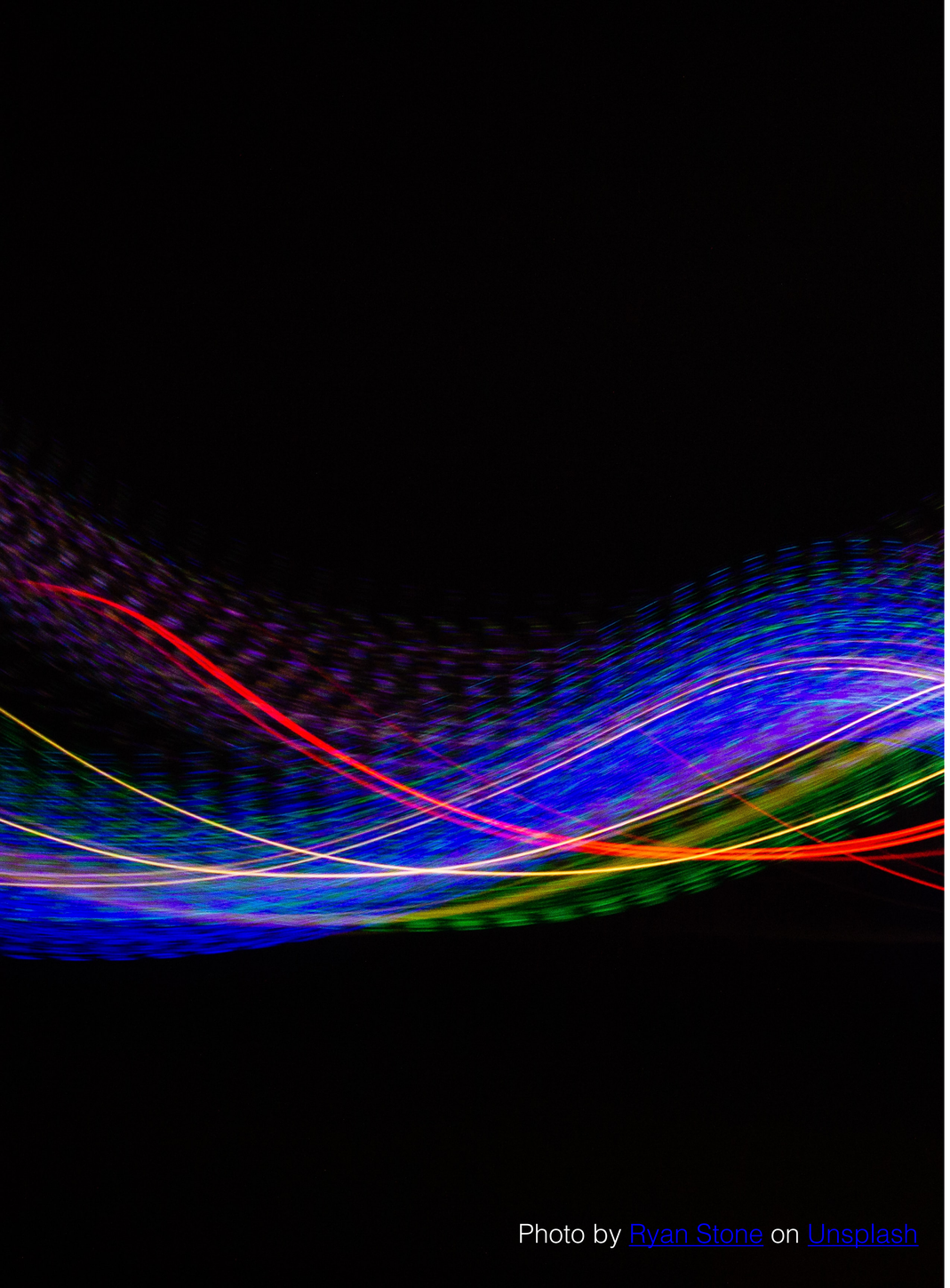

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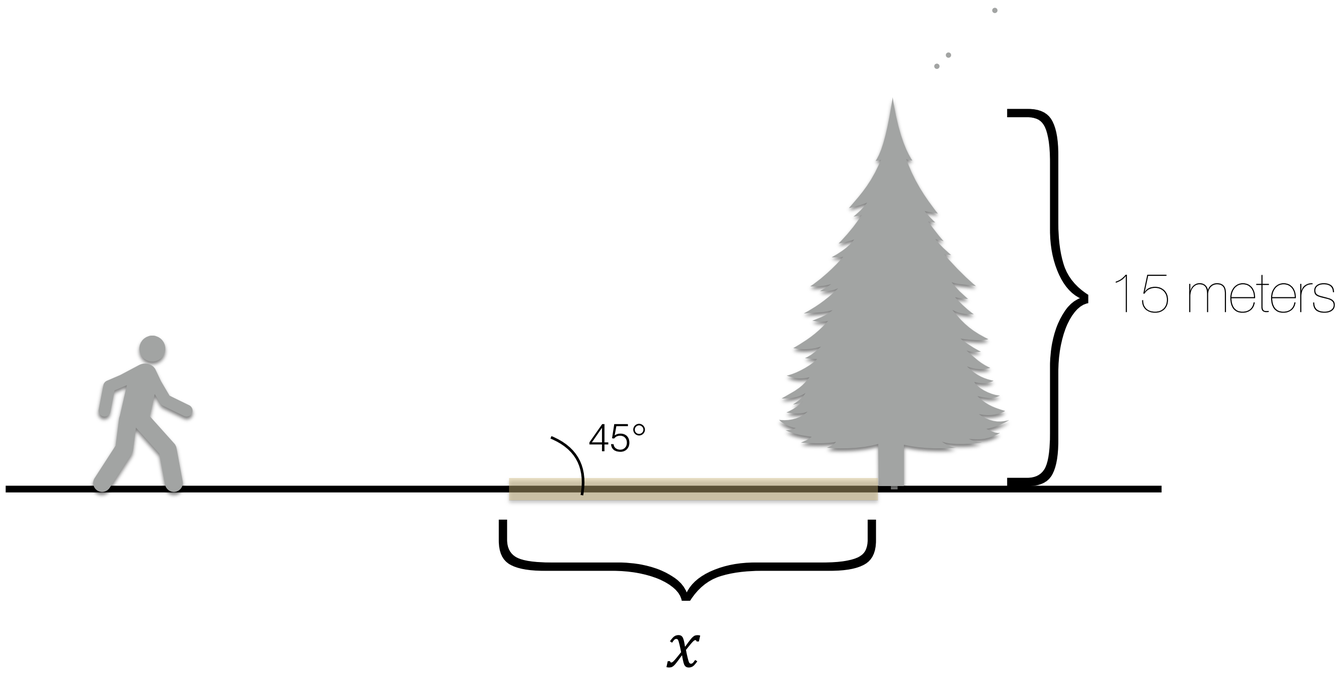
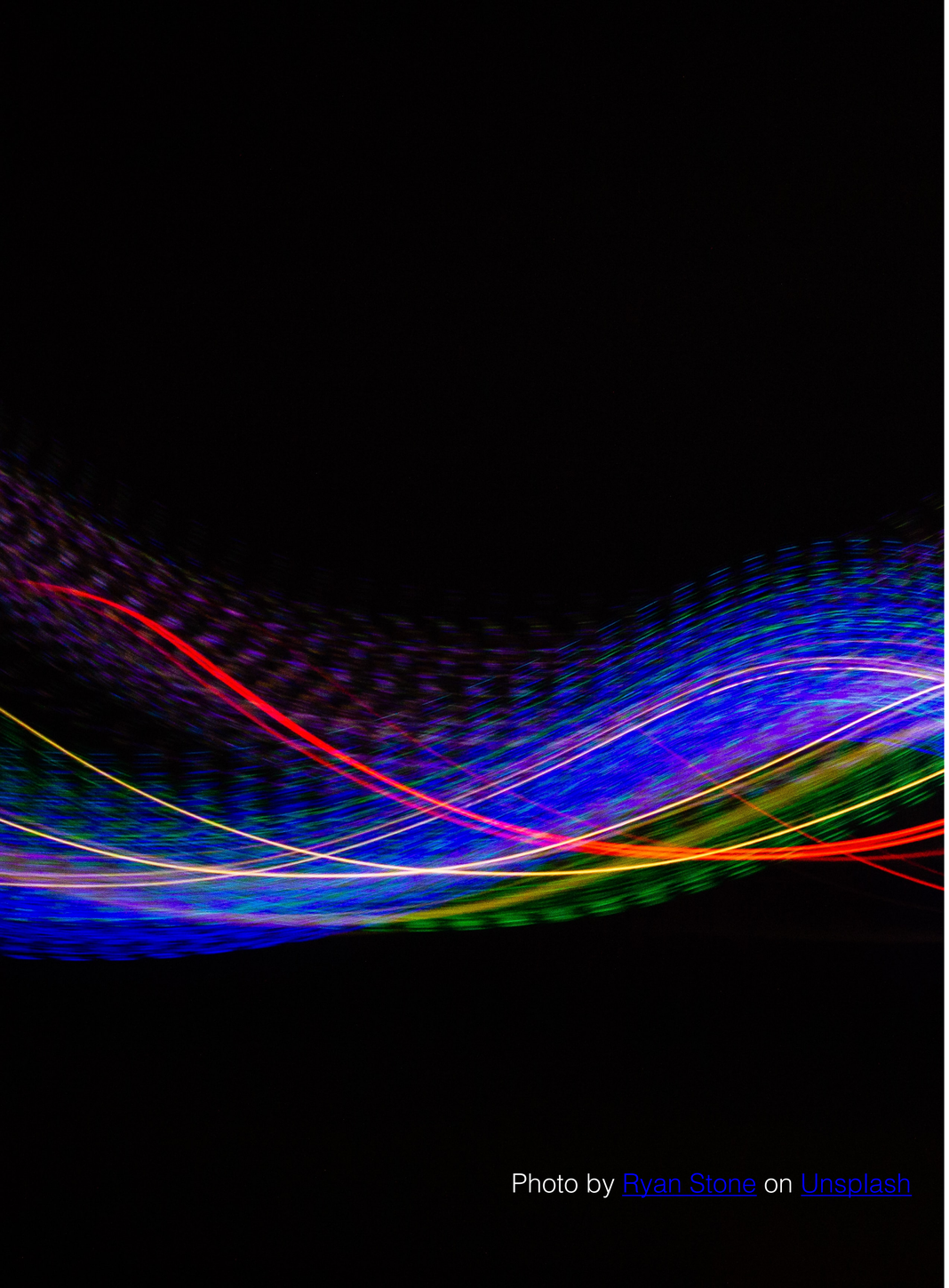
$$tan(45°) = 1 = x/15$$

$$\Longrightarrow x = 15$$



The tree height, angle of incidence, and laws of mathematics explain the shadow.



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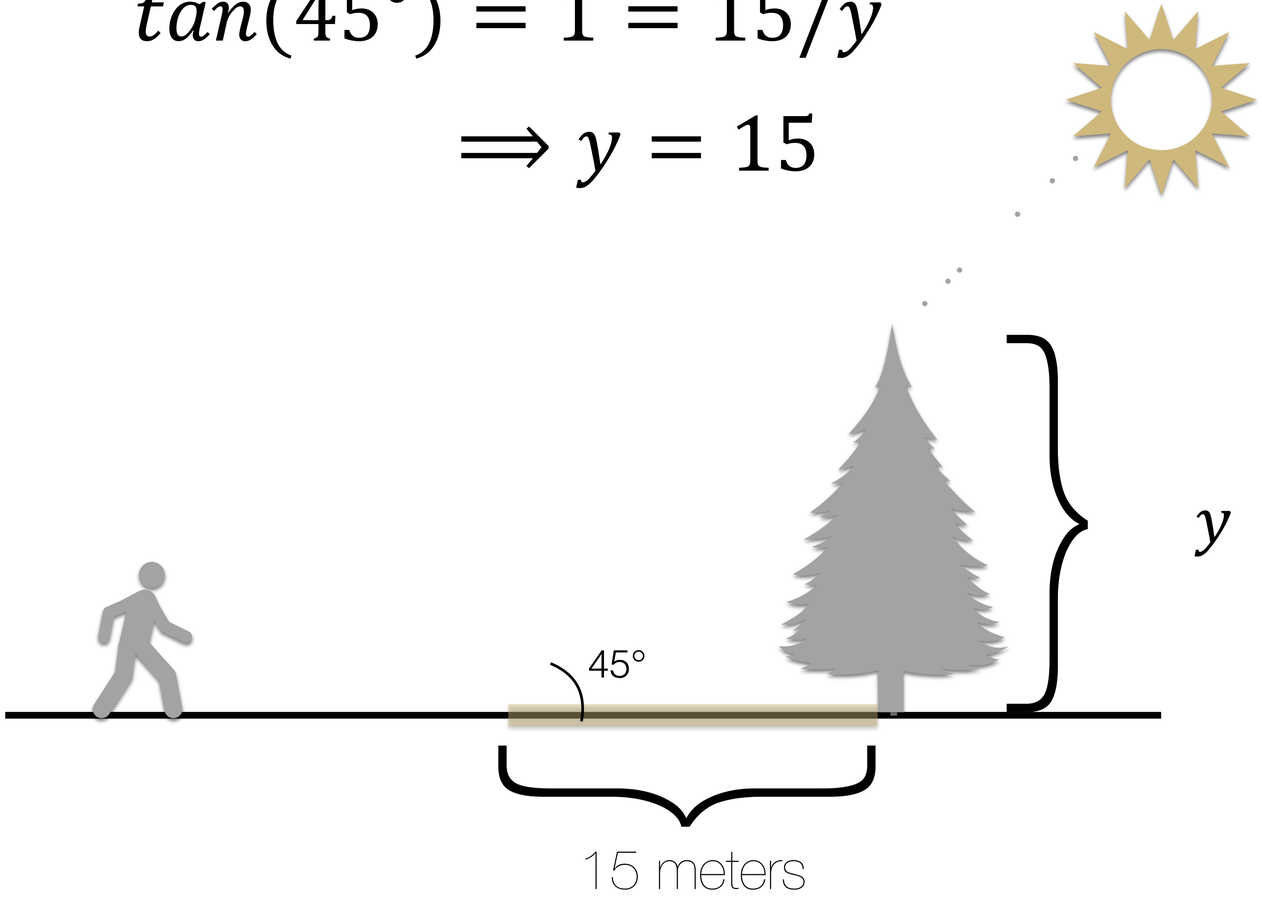
$$tan(45°) = 1 = 15/y$$

$$\Longrightarrow y = 15$$



The length of the shadow, angle of incidence, and laws of mathematics *do not explain* the tree height.

$$F = kx$$

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Lesson: Point estimates for prediction

Module: Prediction and Explanation in Linear Regression Analysis

$$\hat{\mathbf{y}} = X\hat{\boldsymbol{\beta}}$$

$$\hat{\mathbf{y}}^{\star} = X^{\star}\hat{\boldsymbol{\beta}}$$

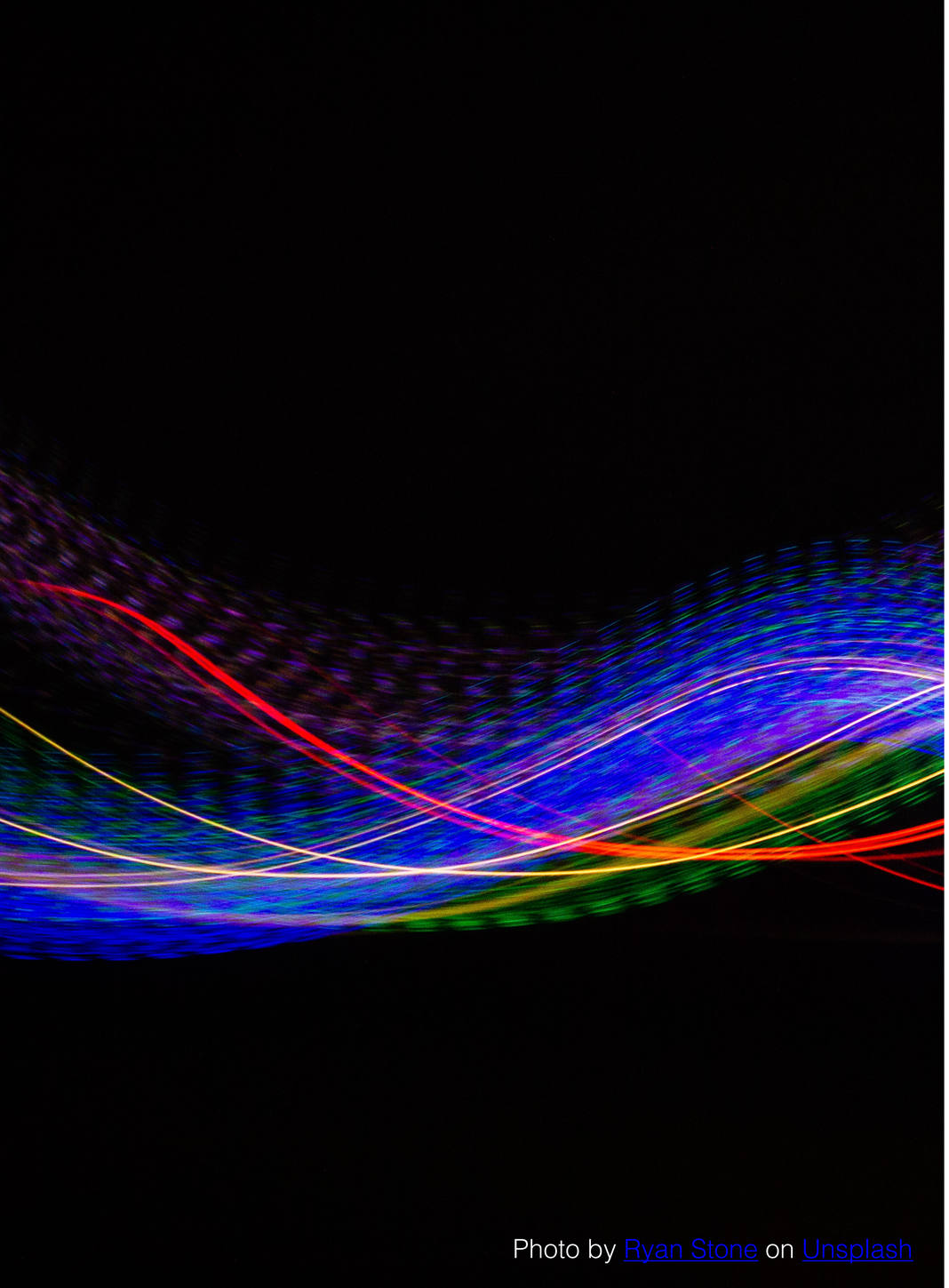
$$\hat{y}_i^\star = \mathbf{x}_i^\star \hat{\boldsymbol{\beta}}$$

$$MSPE = \frac{1}{k}\sum_{i=1}^{k}\left(y_i^\star - \hat{y}_i^\star\right)^2$$

$$= \frac{1}{k}\sum_{i=1}^{k}\left(y_i^\star - \mathbf{x}_i^\star\hat{\boldsymbol{\beta}}\right)^2$$

Lesson: Interval estimates for prediction

Module: Prediction and Explanation in Linear Regression Analysis



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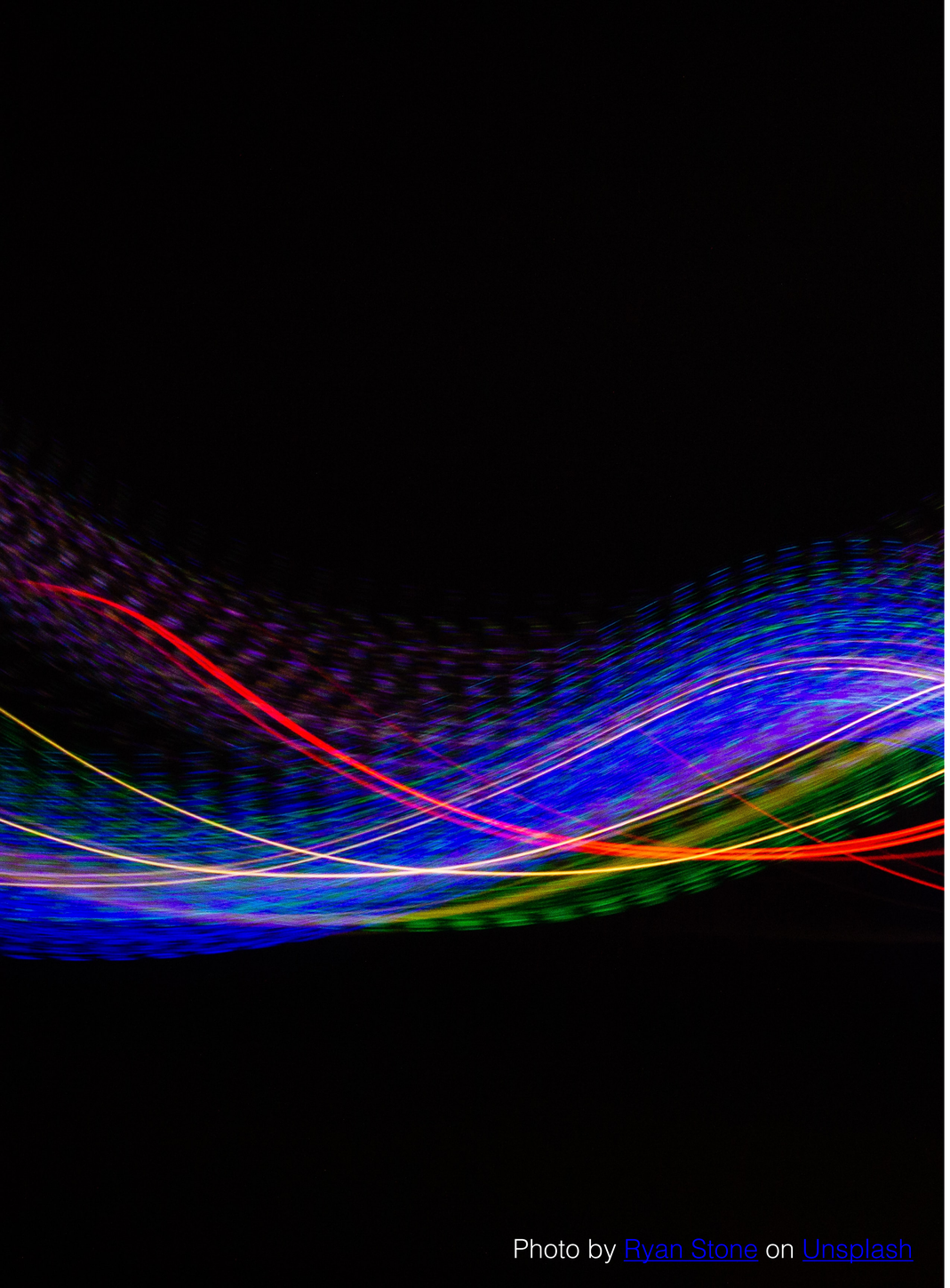
Prediction vs explanation



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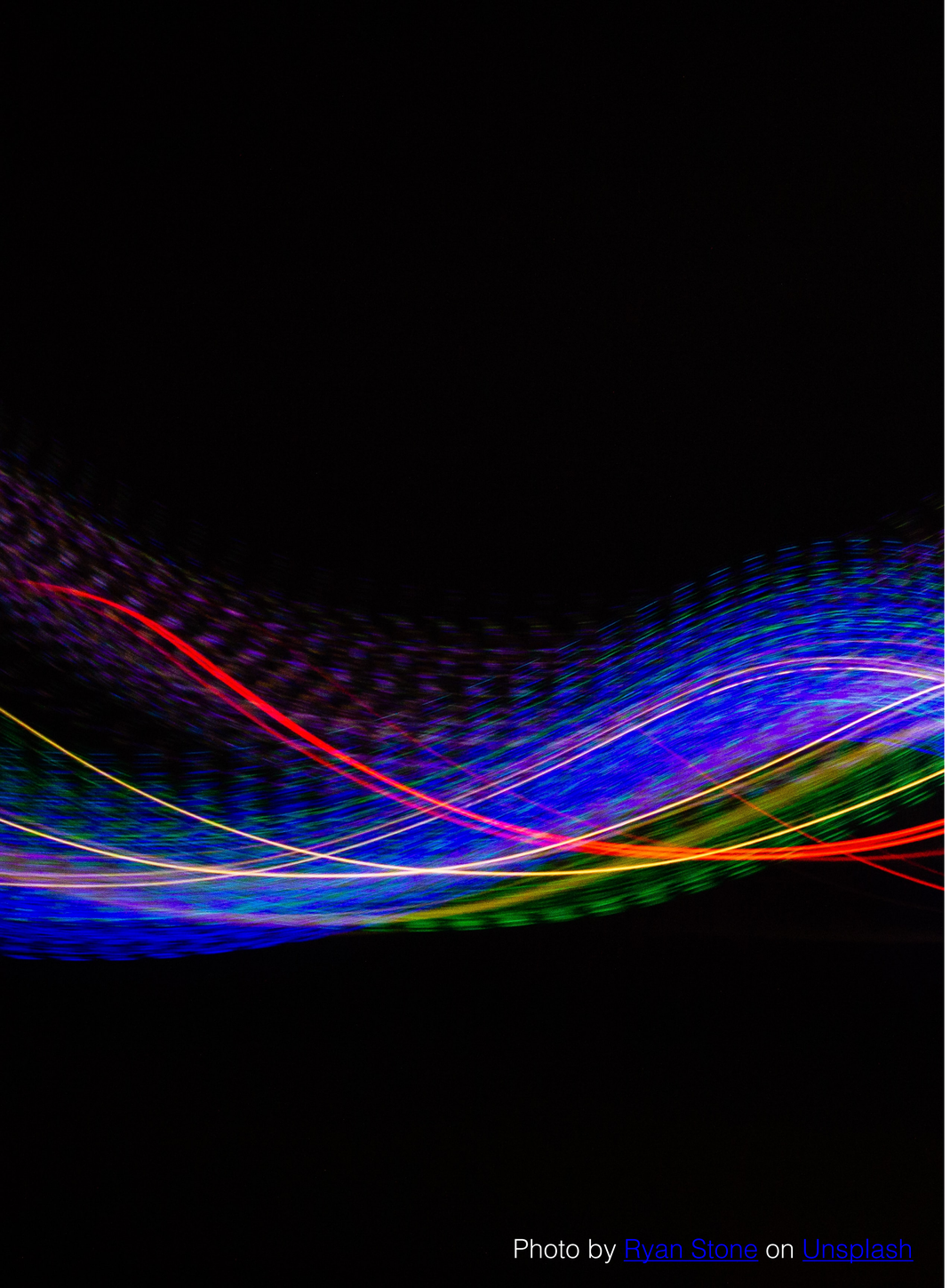
$$\hat{y}_i \pm t_{\alpha/2}(n-(p+1))\sqrt{\hat{\sigma}^2 \mathbf{x}_i^{\star}(X^T X))^{-1}\mathbf{x}_i^{\star T}}$$



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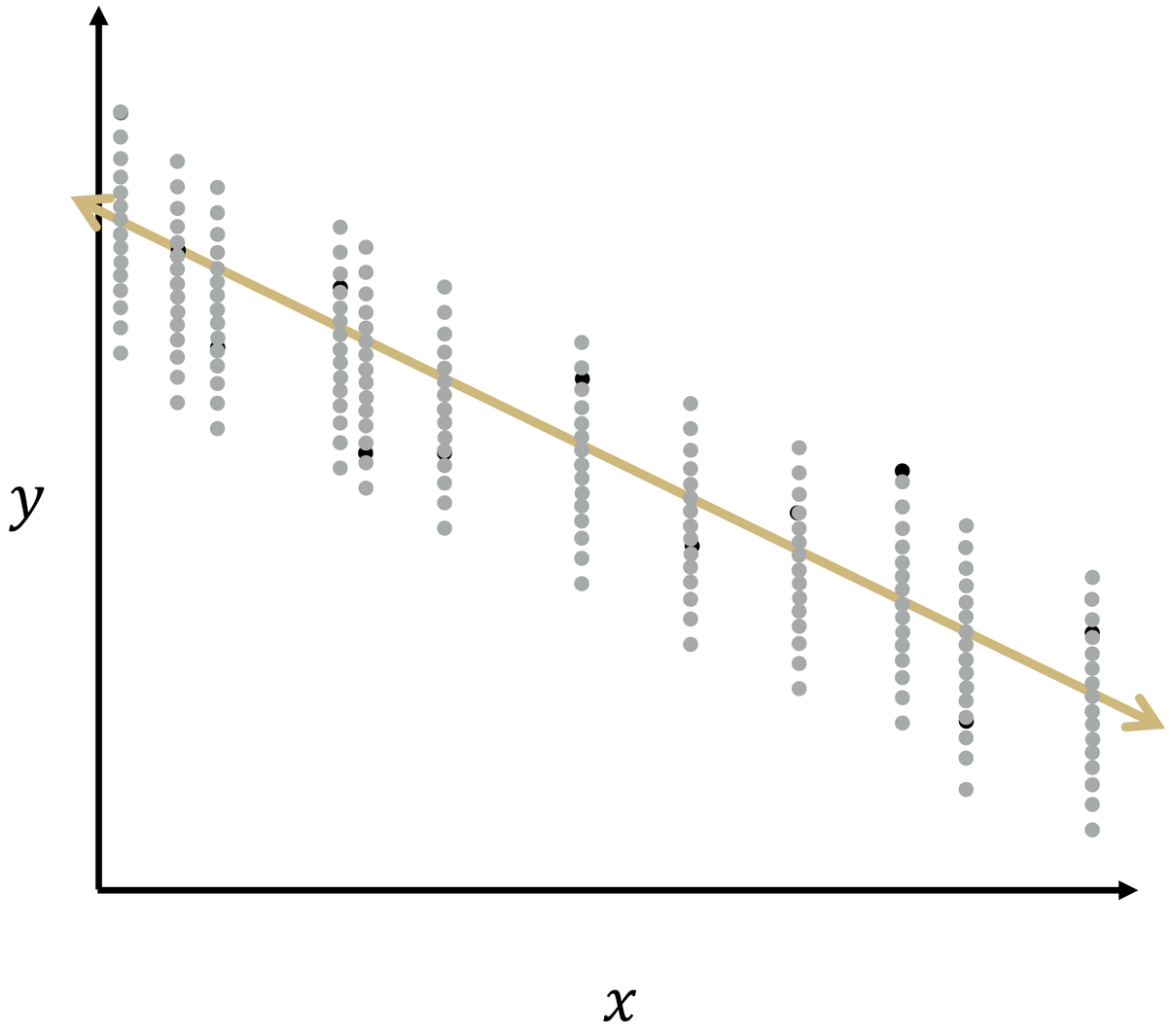

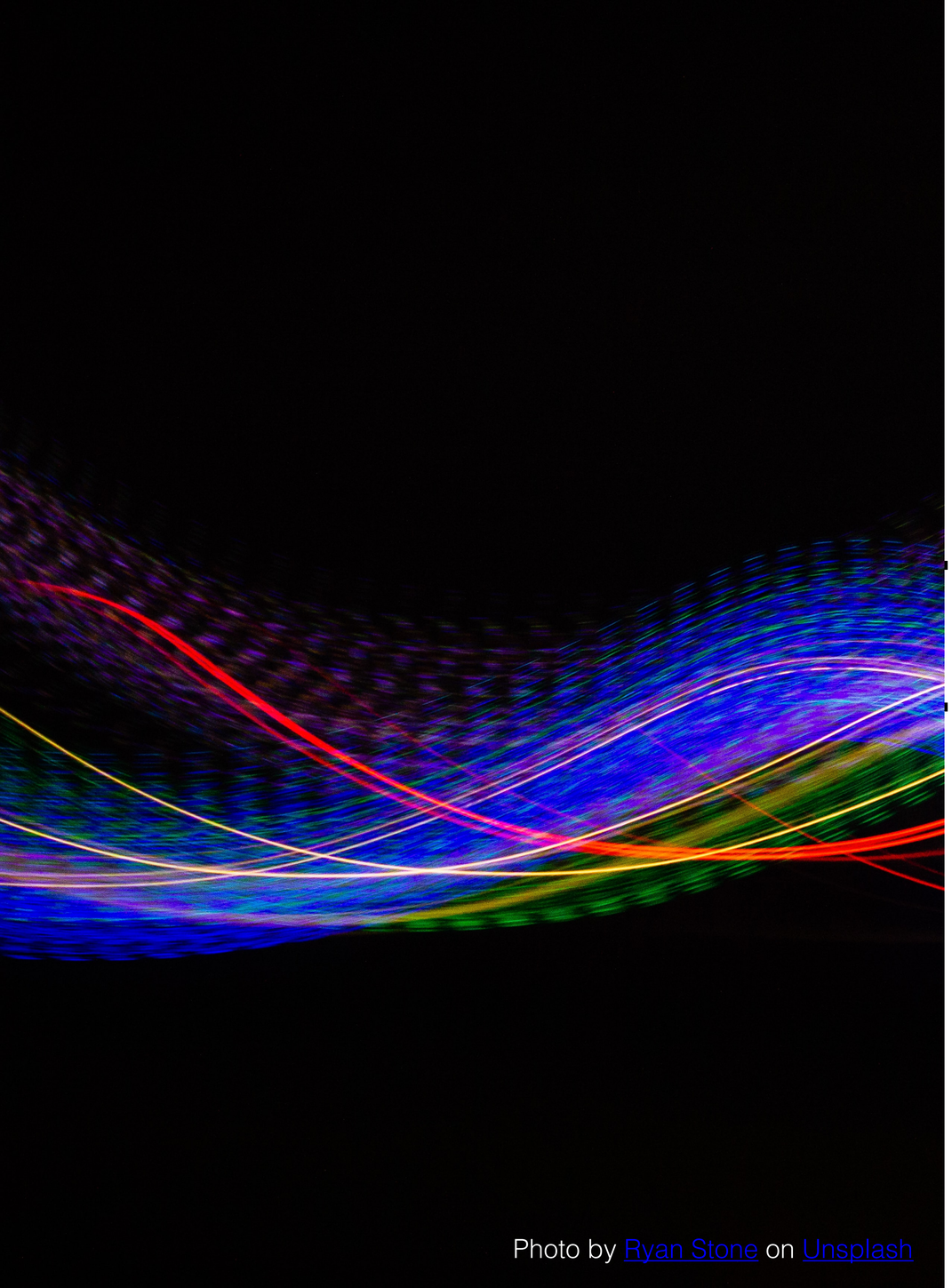

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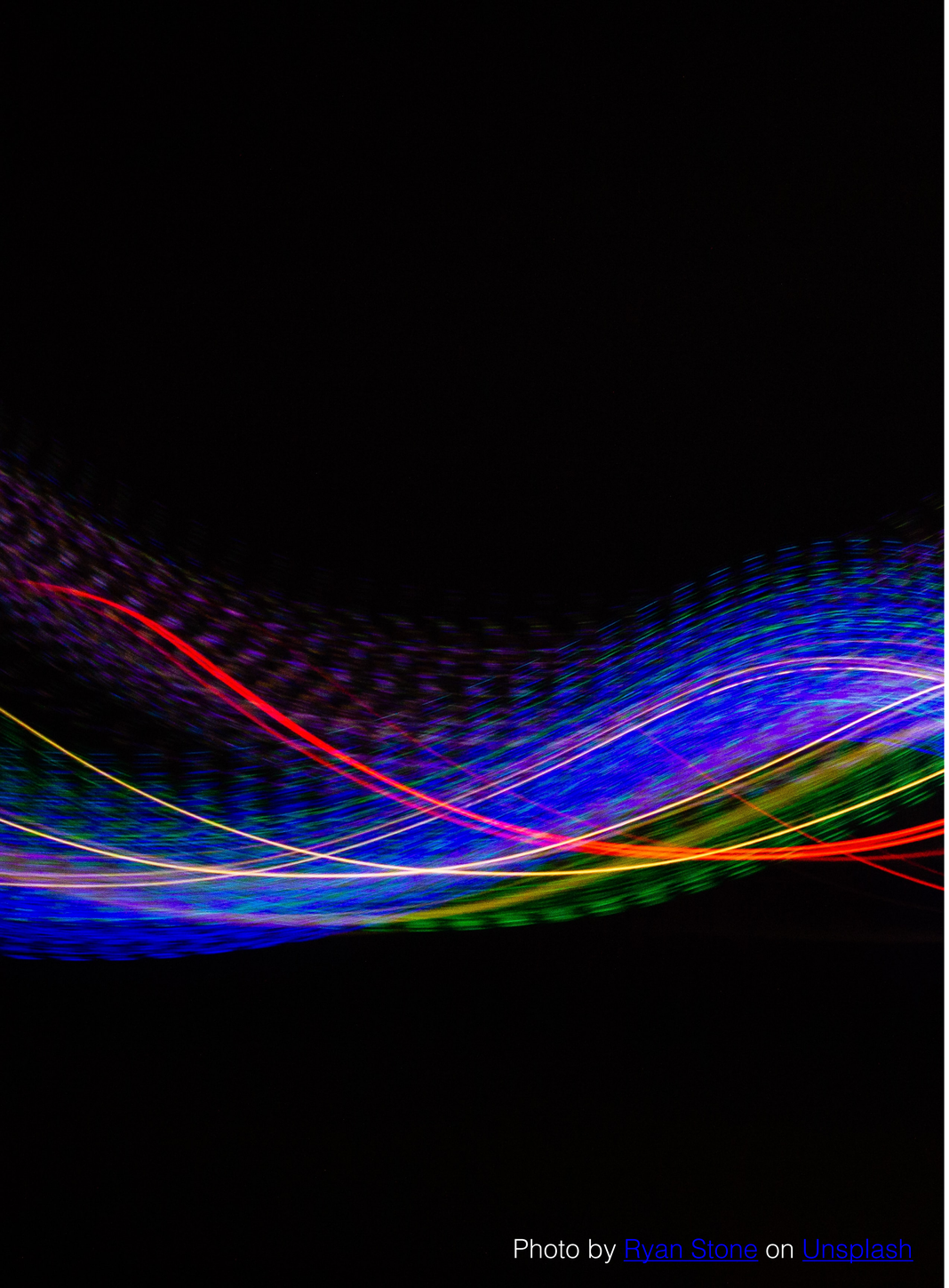
- **Confidence interval** for the *mean response*.
- **Prediction interval** for a *future observation* or *observation not used to train the model*.



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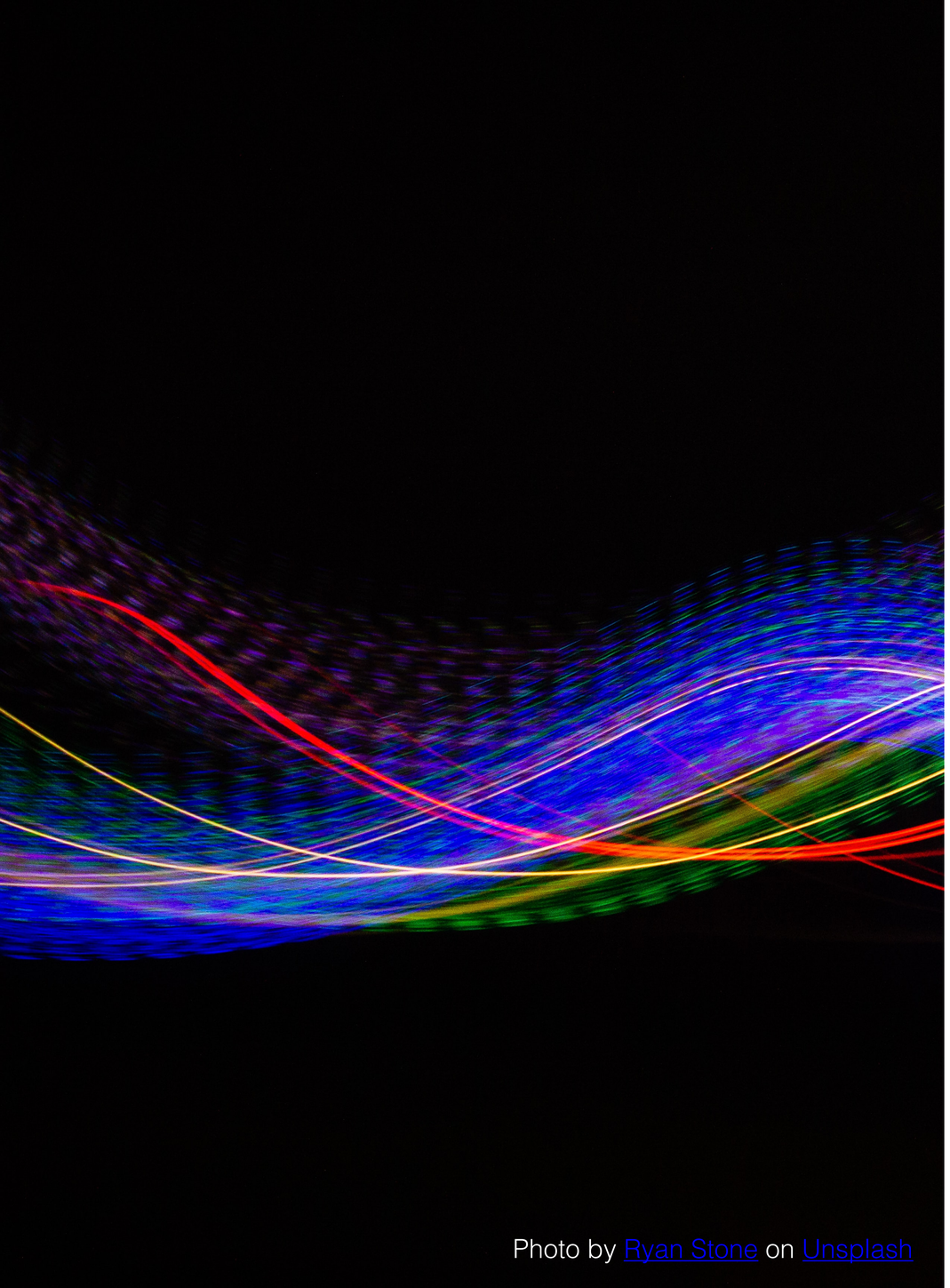
For our prediction interval, we'll need:

$$Var(\mathbf{x}_i^\star \hat{\beta} + \varepsilon_i) =$$

Thus, a prediction interval for a ***future observation*** or ***observation not used to train the model,*** given predictor $\mathbf{x}_i^\star$ is:



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Prediction interval for a future observation in R:

predict(lm_data, new = $\mathbf{x}_i^\star$, interval = “prediction”)

Lesson: When prediction goes wrong

Module: Prediction and Explanation in Linear Regression Analysis

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What can go wrong with predictions:

1. Poorly fitting model (overfitting, not enough data to fit properly).

2. Quantitative extrapolation.

3. Qualitative extrapolation.

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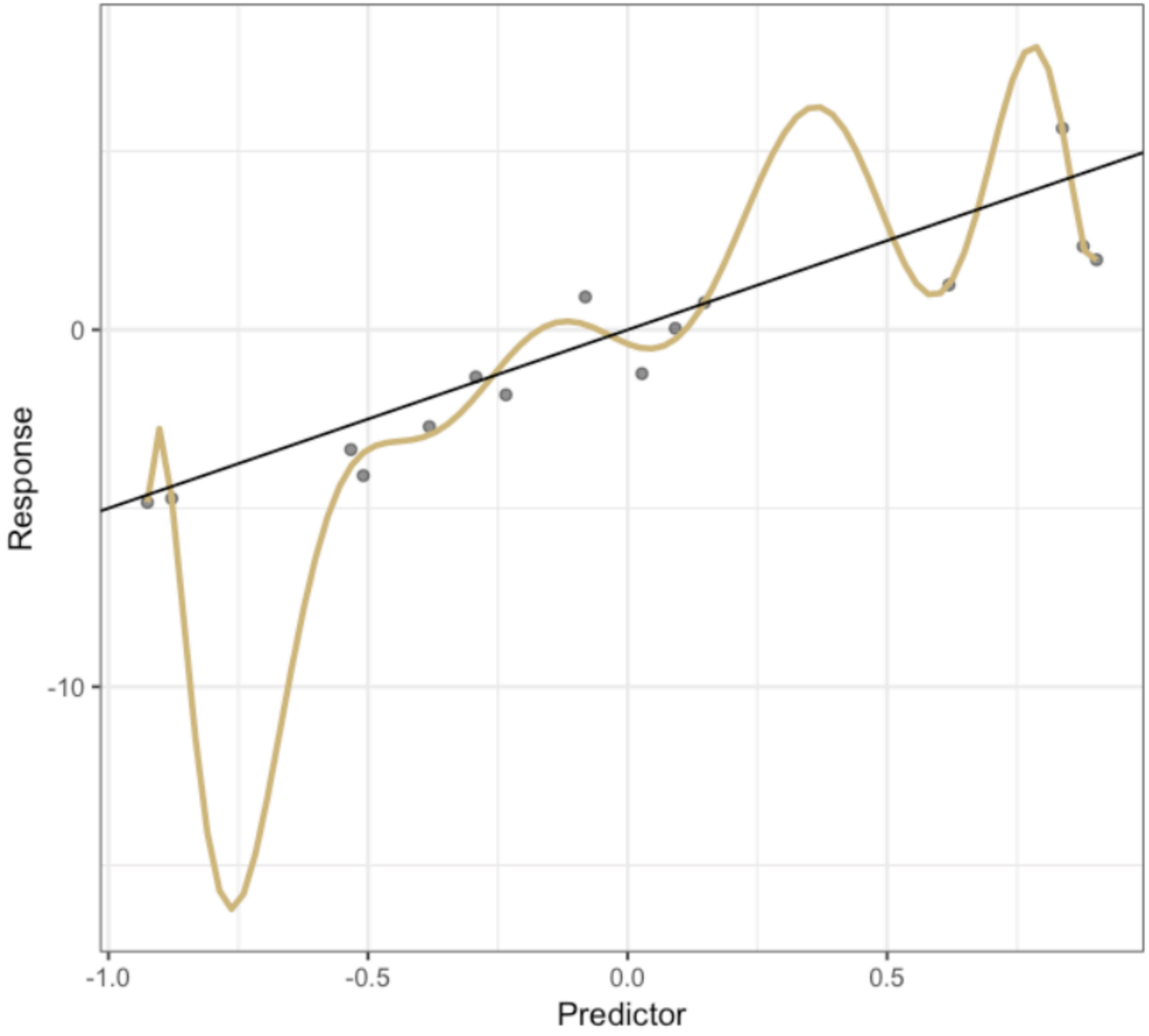

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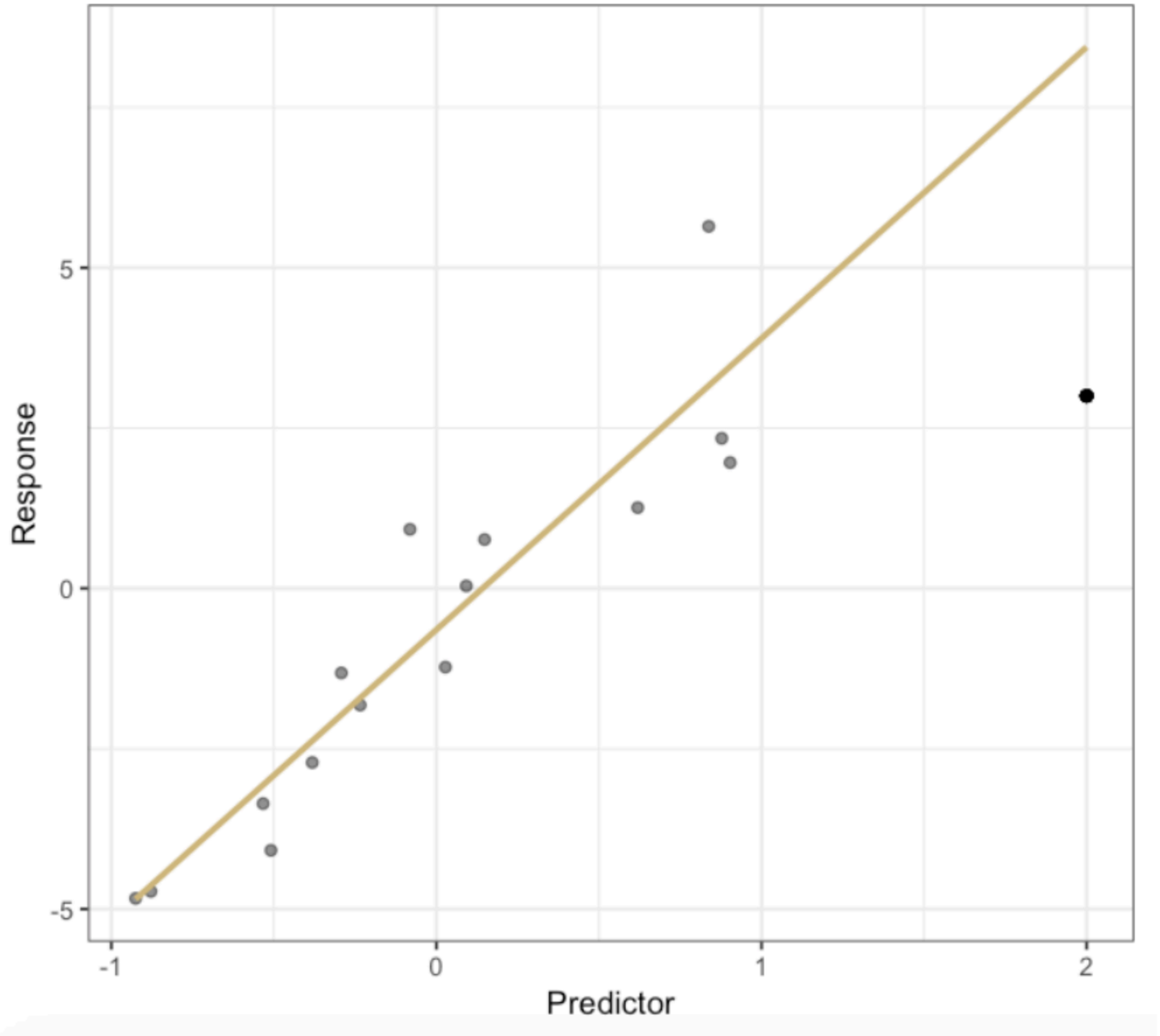

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Lesson: Defining causality

Module: Prediction and Explanation in Linear Regression Analysis

Recall: explanation typically means *causal explanation.* So:

1. What is a cause?
   (metaphysical/ontological question)

2. How can we learn about causes?
   (epistemological question)

Definition: The *relata* are the terms that show up in the causal relation.

Ontological questions:

1. What are the *relata* in a causal relationship?

2. How many *relata* are there?

Is a causal relationship ***deterministic*** or ***probabilistic***?

- A theory about the nature of causality is ***deterministic*** if the effect necessarily follows from the cause.

- A theory about the nature of causality is ***probabilistic*** if the existence of a cause impacts the probability of an effect.

Counterfactual theory of causality posits that C causes E if, in the absence of C, E would not have occurred (or would have been less likely to occur).

The fundamental problem of causal inference: at most, only one of many *potential outcomes* can be observed for a given experimental or observational study, and thus, causal effects cannot be measured directly.

This is a problem for a scientific of statistical philosophy that posits that *observations* drive the generation of knowledge!

Ways of circumventing the fundamental problem of causal inference:

1. Find close substitutes

2. Conduct randomized experiments

3. Make additional assumptions about observational data

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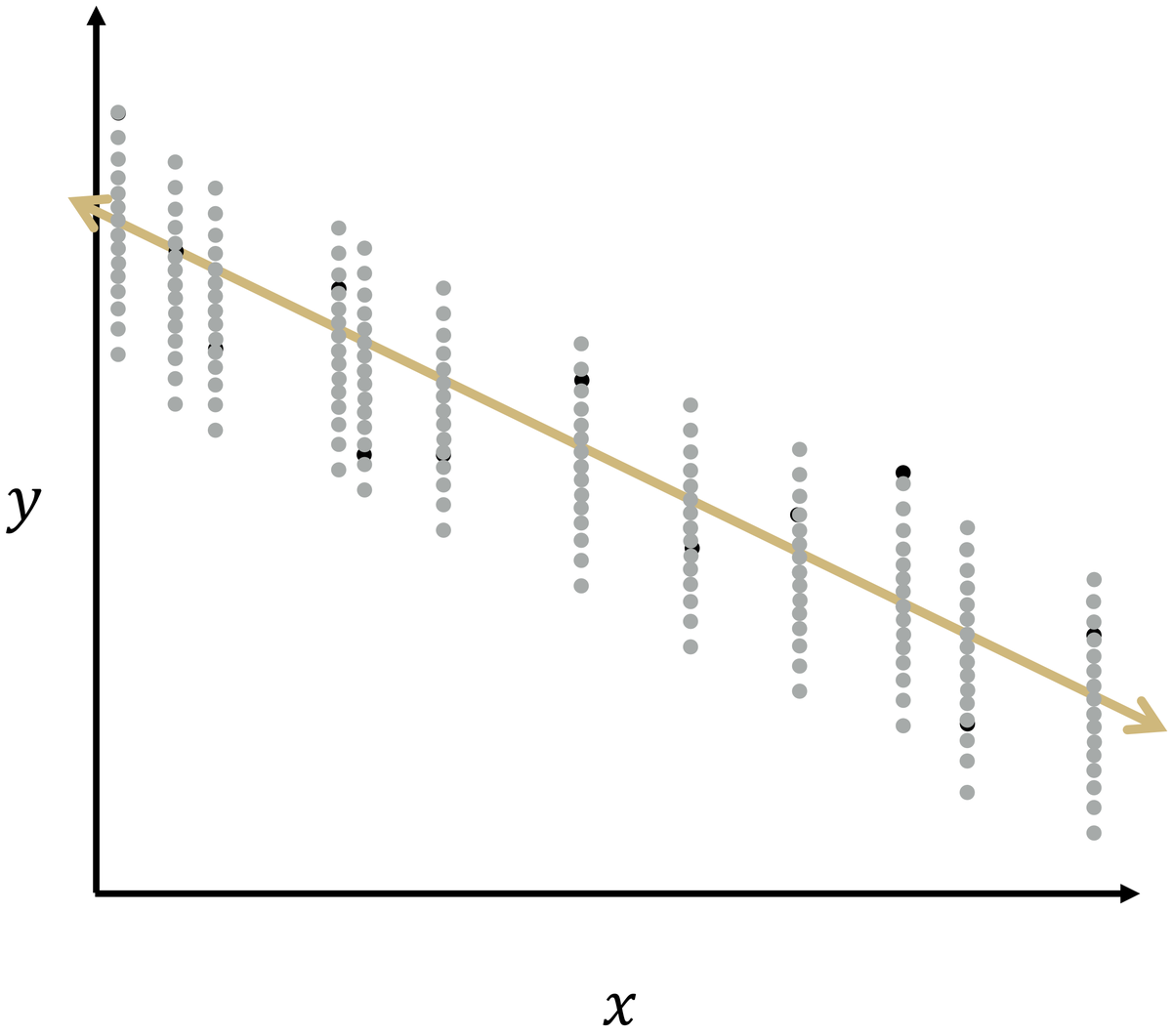
y
x

Recall that prediction is one of the main purposes of using regression models. Suppose for now that we have a model $\mathbf{Y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ that fits the data well (we'll say more later about checking wither the model fits the data).

Given a new set of predictors, $\mathbf{x}^\star = (1, x_1^\star, \ldots, x_p^\star)^T$, the estimated average value of the response is:

Suppose we'd like to know the *average sales in a market with advertising budgets* $x_1^\star, x_2^\star, and x_3^\star$. The point estimate is:

But what about an interval estimate?