Introduction to ANOVA and Experimental Design

Which of the following are true statements about the difference between an experimental and observational study?

Answer: Researchers have control over the treatment in an experimental study.

A difference across sample means (for example, through visual assessment) implies a difference across population means.

Answer:

False

Researchers are conducting an experiment with one treatment of five levels. However, they also believe that an additional continuous variable will impact the response. Given this information, which method is most appropriate?

Answer:

Analysis of covariance (ANCOVA)

What role does randomization play in statistical inference?

Answer:

Randomization helps mitigate the risk of applying the treatment to experimental units in some systematic way that would affect the causal conclusions of an experiment.

The One-Way ANOVA and ANCOVA Models

What is a factor?

Answer:

A factor is a discrete, categorical variable used to predict or explain the response variable.

An ANCOVA model includes additional variables over and above ANOVA. These additional variables are sometimes referred to as:

Answer:

- Predictors
- Explanatory variables
- Features
- Covariates

The means model for one-way ANOVA is given as:

$Y_{i,j} = \mu_j + \epsilon_{i,j}$, where $\epsilon_{i,j} \sim iidN(0, \sigma^2)$

In this model, μ_j can be interpreted as the mean of the response over all units in the sample.

Answer:

False

The effects model for one-way ANOVA is given as:

$Y_{i,j} = \mu + \tau_j + \epsilon_{i,j}$, where $\epsilon_{i,j} \sim iidN(0, \sigma^2)$.

In this model, μ can be interpreted as the mean response limited to the units in the j^{th} level of the factor, τ_j.

Answer:

False

In a 2014 paper titled "Involving Children in Meal Preparation," published in the journal *Appetite*, researchers hoped to determine the effect of child participation in meal preparation (factor with two levels) on caloric intake (response). In one group, children participated in the preparation of a meal. In a second group, children did not participate.

Which of the following is a correct interpretation of μ in the one-way ANOVA effects model?

Answer:

μ is the population mean of caloric intake, across both meal preparation groups.

In a 2014 paper titled "Involving Children in Meal Preparation," published in the journal *Appetite*, researchers hoped to determine the effect of child participation in meal preparation (factor with two levels) on caloric intake (response). In group one, children participated in the preparation of a meal. In group two, children did not participate.

Which of the following is a correct interpretation of μ_2 in the one-way ANOVA means model?

Answer:

μ_2 is the mean of the caloric intake in the meal preparation group where children did not participate.

ANOVA Variance Decomposition

If $\bar{Y}_j \neq \bar{Y}_k$ for some $j \neq k$, then there are differences, with respect to the population mean of a continuous variable, across groups of experimental units.

Answer:

False (further investigation needs to be conducted to see if the differences reflect real differences in the population, rather than differences due to sampling variability)

In the context of one-way ANOVA,

$\sum_{j=1}^{J} \sum_{i=1}^{n_j} (Y_{i,j} - \bar{Y}_{..})^2$

is the treatment sum of squares.

Answer:

False

In the context of one-way ANOVA,

$\sum_{j=1}^{J}(\bar{Y}_j - \bar{Y}_{..})^2$

is a measure of:

Answer:

Between group variability in the sample

In the context of one-way ANOVA,

$\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{i,j} - \bar{Y}_{..})^2$

is a measure of:

Answer:

Total variability in the sample

In the context of one-way ANOVA,

$\sum_{i=1}^{n_j}(Y_{i,j} - \bar{Y}_j)^2$

is a measure of:

Answer:

Within group variability in the sample

In the context of one-way ANOVA, the between groups variability is equal to the total variability plus the within group variability.

Answer:

False

ANOVA Sums of Squares and the F-Test

A very small p-value suggests that the differences with respect to the mean of a continuous variable across groups of experimental units is very small.

Answer:

False

In the space below, type R code to run a one-way ANOVA, using the `aov()` function. Assume that the response is called `response`, there is one predictor called `predictor`, and the data frame is called `data`.

Answer:

```
aov(response ~ predictor, data = data)
```

In the context of one-way ANOVA, the null hypothesis for the (full) F-test is:

H_0: there are no differences with respect to the mean of a continuous variable across groups of experimental units.

Answer:

True

Using the code output below, calculate the residual standard error, $\hat{\sigma}$.

Call: aov(formula = y ~ f1, data = df1)

Terms	f1	Residuals
Sum of Squares	820.2125	117.1587
Deg. of Freedom	2	247

Answer:

0.6887138

Using the code output below, conduct the (full) F-test. Then, use the result of that test to choose the most accurate statement below.

Call: aov(formula = y ~ f1, data = df1)

Terms	f1	Residuals
Sum of Squares	820.2125	117.1587
Deg. of Freedom	2	247

Answer:

Yes, there is some evidence that there are differences with respect to the mean of the response variable y across the three levels of factor $f1$.

ANOVA and ANCOVA as Regression Models

A one-way ANOVA model with a J-level factor is a multiple linear regression model with $J-1$ predictor/explanatory variables, and a continuous response.

Answer:

True

Consider an experiment conducted to study the effectiveness of different hand washing techniques (factor) on the prevalence of bacteria (response). The experiment tested four different methods—washing with water only, washing with regular soap, washing with antibacterial soap (ABS), and spraying hands with antibacterial spray (AS) (containing 65% ethanol as an active ingredient). Ten different hand washings were included within each level (i.e., ten people washed with water only, with regular soap, etc.).

In the regression context, which of the following correctly gives the dimensions of the design matrix, X?

Answer:

40 rows and 4 columns

40 rows and 4 columns

--

Consider an experiment conducted to study the effectiveness of different hand washing techniques (factor) on the prevalence of bacteria (response). The experiment tested four different methods—washing with water only, washing with regular soap, washing with antibacterial soap (ABS), and spraying hands with antibacterial spray (AS) (containing 65% ethanol as an active ingredient). Ten different hand washings were included within each level (i.e., ten people washed with water only, with regular soap, etc.).

Whitney mistakenly sets up a one-way ANOVA regression model with four indicator variables—one for each level. What consequences does this mistake have?

Answer:

- The regression model is non-identifiable.
- $\hat{\beta} = (X^T X)^{-1} X^T Y$ cannot be accurately computed from the data.
- The matrix $X^T X$ (where X is the design matrix) is not invertible.

Which of the following are benefits of casting one-way ANOVA as a linear regression model?

Answer:

- Casting one-way ANOVA as a linear regression model allows us to rely on least squares (or maximum likelihood) to estimate our parameters.
- Casting one-way ANOVA as a linear regression model allows us to rely on the interpretation of regression parameters to answer our research questions of interest.
- Casting one-way ANOVA as a linear regression model provides a set of inference techniques (e.g., t-tests, F-tests) that might help us answer research questions of interest.

One-Way ANOVA Interpretation in the Regression Context

In the one-way ANOVA regression model, the intercept term β_0 is the expected response in the baseline/reference/control group.

Answer:

True

Consider a one-way ANOVA regression model with a factor of five levels: a control group and four treatments.

$Y_i = \beta_0 + \sum_{j=1}^{4} \beta_j X_{i,j} + \epsilon_i.$

The mean of the response variable in the third treatment group is $\beta_0 + \beta_3$.

Answer:

True

Consider a one-way ANOVA regression model with a factor of five levels: a control group and four treatments.

$Y_i = \beta_0 + \sum_{j=1}^{4} \beta_j X_{i,j} + \epsilon_i.$

The mean of the response variable in the third treatment group is β_3.

Answer:

False

The ANCOVA Model

Analysis of Covariance (ANCOVA) can help answer the question: are there differences, with respect to the population mean of a response variable, across groups, adjusting for several continuous variables thought to be correlated with the response?

Answer:

True

Analysis of Covariance (ANCOVA) can help answer the question: how does the relationship between a continuous response and continuous predictor differ, on average, across groups?

Answer:

True

Social science researchers are sometimes interested in what social or cultural factors influence happiness. One such research question might be: are married people happier than non-married people? Select the most promising modeling approach for answering this question.

Answer:

- Since other variables like income level influence happiness, an ANCOVA model with a continuous happiness measure as the response, a two-level factor that records marital status, and other variables like income level as predictors/explanatory variables.
- Since other variables like income level influence happiness, a multiple linear regression model with a continuous happiness measure as the response, a two-level factor that records marital status, and other variables like income level as predictors/explanatory variables.

Beyond the Full F-test

The full F-test can tell researchers which groups differ with respect to the mean of a continuous response.

Answer:

False

The hypotheses specified in post hoc comparisons are specified before looking at the data.

Answer:

False

Post hoc comparisons performed without adjusting for type I error rates is a form of data dredging.

Answer:

True

Markus is conducting a study on the effect of eating dark chocolate on health. In the study, Markus recruits $n = 24$ individuals, and splits them into three groups:

- A control group that eats no dark chocolate.
- A group that eats one ounce of dark chocolate per day for six weeks.
- A group that eats one ounce of dark chocolate per day for six weeks and performs at least 30 minutes of exercise four times per week.

Markus and his team measured 10 different health markers, including blood pressure, blood sugar, and body fat percentage, before and after the six-week period. The data analysis showed that blood sugar levels were lower in the dark chocolate (no exercise) group.

Which of the following would help avoid data dredging:

Answer:

- Specify any hypotheses about the relationships between dark chocolate and health markers before conducting the study.
- If hypotheses about the relationships between dark chocolate and health markers cannot be specified before conducting the study, then, to achieve an overall false positive rate of 5%, the team should set the familywise type I error rate to 5%, and adjust individual hypothesis test type I error rates accordingly.

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Markus and his team measured 10 different health markers, including blood pressure, blood sugar, and body fat percentage, before and after the six-week period.

Suppose that 10 tests relating each of the health markers to dark chocolate consumption (group 2 above) are independent of one another, and each have a significance level of $\alpha = 0.1$. What is the probability of at least one of these tests to yield a false positive?

Answer:

0.6513216

Planned Comparisons: Defining Contrasts

Planned hypothesis tests should not be conducted if the p-value for the full F-test is greater than the pre-specified significance level for the full F-test.

Answer:

True

Which of the following is not a necessary condition for successfully conducting planned comparisons?

Answer:

- Planned comparisons must have a significance level greater than or equal to the familywise error rate.
- Planned comparisons must be conducted regardless of the outcome of the full F-test.

Suppose that a one-way ANOVA is conducted, with a factor of 5 levels. The contrast using $c = (1, -1, -1, 0, -1)$ and $\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)$ is equivalent to the null hypothesis $H_0 : \mu_1 - \mu_2 - \mu_3 = \mu_5$.

Answer:

True

Let $\gamma = (\gamma_1, \ldots, \gamma_4)$ be a set of parameters. Then $\gamma_1 - \gamma_2 + 2\gamma_3 - 4\gamma_4$ is a contrast.

Answer:

False

Markus is conducting a study on the effect of eating dark chocolate on health. In the study, Markus recruits $n = 24$ individuals, and splits them into three groups:

- A control group that eats no dark chocolate.
- A group that eats one ounce of dark chocolate per day for six weeks.
- A group that eats one ounce of dark chocolate per day for six weeks and performs at least 30 minutes of exercise four times per week.

Markus and his team measured 10 different health markers, including blood pressure, blood sugar, and body fat percentage, before and after the six-week period. Before conducting the study, the team knew that they would want to study relationships between each of the 10 health markers and dark chocolate consumption. The subsequent data analysis showed that blood sugar levels were lower in the dark chocolate (no exercise) group.

Which of the following criteria for planned comparisons were violated, based on the description above?

Answer:

The number of planned hypothesis tests is no more than the corresponding degrees of freedom (number of groups minus one).

Planned and Unplanned Comparisons

Consider a one-way ANOVA with a J level factor. How many pairwise comparisons are possible (i.e., hypotheses of the form $H_0 : \mu_i = \mu_j$ for $i \neq j$)?

Answer:

$\binom{J}{2} = \frac{J(J-1)}{2}$

Pairwise comparisons can still be conducted when the one-way ANOVA model assumptions are violated.

Answer:

False

The test statistic for the Tukey Method:

The test statistic for the Tukey Method:

$$q_{j,k} = \frac{\bar{Y}_j - \bar{Y}_k}{\sqrt{\frac{\hat{\sigma}^2}{r}}},$$

is always positive.

Answer:

True

Tukey's method always compares the mean of each group to the mean of every other group (i.e., makes all pairwise comparisons).

Answer:

True

The Bonferroni method always compares the mean of each group to the mean of every other group (i.e., makes all pairwise comparisons).

Answer:

False

Consider performing $m > 1$ post hoc comparisons. Using Bonferroni's method, the type I error rate for each individual test will always be smaller than the familywise type I error rate.

Answer:

True

Type II Error and Power in the ANOVA Context

The larger the sample size in a given one-way ANOVA analysis, the larger the power of associated tests (e.g., pairwise comparisons).

Answer:

True

The power of tests associated with a one-way ANOVA analysis are impacted by:

- Sample size
- The significance level
- The true size of the mean differences across groups
- The within group variability

Prospective power analyses are widely thought to be epistemically unjustified.

Answer:

False

Prospective power analyses are often used to:

Answer:

- Choose a sample size to achieve a particular power level
- Estimate the power of a given study design

Motivating the Two-way ANOVA Model

Suppose that a group of potential pet food customers were surveyed on the type of pet they owned (factor A). Individuals had either cats or dogs. Then, individuals were asked about their purchasing preferences for pet food (factor B). Specifically, cat owners were only asked about cat foods produced by Brands X, Y, and Z. Dog owners were only asked about dog foods produced by brands X, Y, and Z.

Factors A and B are...

Answer:

Nested

Researchers designed an experiment to study factors affecting the particle size in the production of polyvinyl chloride (PVC) plastic. In the experiment, three operators (Factor A, levels X, Y, and Z) used eight different devices called resin railcars (Factor B, levels 1-8) to produce PVC. There were 24 total particle size measurements in the study.

Answer:

There is not enough information to conclude that the experiment is balanced or unbalanced.

Researchers designed an experiment to study factors affecting the particle size in the production of polyvinyl chloride (PVC) plastic. In the experiment, three operators (Factor A, levels X, Y, and Z) used eight different devices called resin railcars (Factor B, levels 1-8) to produce PVC. There were 24 total particle size measurements in the study.

Suppose that, for resin railcar 1, operator X produces a larger particle size than operator Y, but a lower particle size for resin railcar 2. This is an example of...

Answer:

An interaction between Factor A and Factor B.

Researchers designed an experiment to study factors affecting the particle size in the production of polyvinyl chloride (PVC) plastic. In the experiment, three operators (Factor A, levels X, Y, and Z) used eight different devices called resin railcars (Factor B, levels 1-8) to produce PVC. There were 24 total particle size measurements in the study. However, there was no measurement of particle size for operator Y and resin railcar 3.

Answer:

- This experiment is unbalanced.
- This experiment contains replication.

Suppose that a group of potential pet food customers were surveyed on the type of pet they owned (factor A). Individuals had either cats or dogs. Then, individuals were asked about their purchasing preferences for pet food (factor B). Specifically, cat owners were only asked about dogs. Then, individuals were asked about their purchasing preferences for pet food (factor B). Specifically, cat owners were only asked about cat foods produced by Brands X, Y, and Z. Dog owners were only asked about dog foods produced by those same brands.

This study can be analyzed using a two-way ANOVA model because...

Answer:

There are two factors.

The Two-way ANOVA Model

The means model and the effects model for two-way ANOVA are equivalent.

Answer:

True

Consider the means model for two-way ANOVA as described in our video:

$$Y_{ijk} = \mu_{jk} + \epsilon_{ijk}$$

where Y_{ijk} is the i^{th} measurement of the response in the j^{th} level of the τ factor and the k^{th} level of the α factor; and $i = 1, \ldots, n_{jk}$. n_{jk} is the total number of observations in the study/experiment.

Answer:

False

In the two-way ANOVA effects model, τ_j can be interpreted as:

Answer:

- The true difference between the grand mean and the mean of the response in the j^{th} level of the τ factor, holding the α factor constant.
- The true difference between the grand mean and the mean of the response in the j^{th} level of the τ factor, holding the α factor at its mean value.
- The true difference between the grand mean and the mean of the response in the j^{th} level of the τ factor, adjusting for the α factor.

The error term in the two-way ANOVA means model is assumed to be distributed as:

Answer:

$$\epsilon_{ijk} \sim iid\, N(0, \sigma^2).$$

Consider the two-way ANOVA effects model:

$$Y_{ijk} = \mu + \tau_j + \alpha_k + \epsilon_{ijk}$$

and suppose that, for all levels of α, the mean response for units in τ_1 is greater than the mean response for units in τ_2. Then, necessarily, the move from τ_1 to τ_2 causes the increase in the mean response.

Answer:

False

Factor A	Factor B	Response
1	1	10.9
2	1	9.2
3	1	8.0
1	2	8.6
2	2	11.1
3	2	7.3
1	3	9.9
3	3	8.5

Suppose that the data frame above is an entire two-factor study. Does this data frame constitute a full factorial design?

Answer:

No

The Two-way ANOVA Model as a Regression Model

A two-way ANOVA model in regression form will always have as many indicator variables as factor level combinations.

Answer:

False

Suppose that we have one factor, τ, with 3 levels and another factor, α, at 3 levels. Assuming no interaction, the regression form of this model is:

$$Y_i = \beta_0 + \beta_1\tau_{i,1} + \beta_2\tau_{i,2} + \beta_3\alpha_{i,1} + \beta_4\alpha_{i,2} + \epsilon_i$$

where $\epsilon_i \sim iid\, N(0, \sigma^2)$.

Answer:

True

Suppose that the following regression model corresponding to a two-way ANOVA is correct (factors τ with levels 1-2 and α with levels 1-2):

$$Y_i = \beta_0 + \beta_1\tau_{i,1} + \beta_2\alpha_{i,1} + \epsilon_i$$

where:

- $\tau_{i,1} = 1$ if the i^{th} unit is in the first level of τ, and $\tau_{i,1} = 0$ if the i^{th} unit is in the second level of τ.
- $\alpha_{i,1} = 1$ if the i^{th} unit is in the first level of α, and $\alpha_{i,1} = 0$ if the i^{th} unit is in the second level of α.
- $\epsilon_i \sim iid\, N(0, \sigma^2)$.

The mean of the response for all units in the second level of τ and the second level of α is:

Answer:

$\mu_{2,2} = \beta_0$

Suppose that the following regression model corresponding to a two-way ANOVA is correct (factors τ with levels 1-4, and α with levels 1-2):

$$Y_i = \beta_0 + \beta_1\tau_{i,1} + \beta_2\tau_{i,3} + \beta_3\tau_{i,4} + \beta_4\alpha_{i,1} + \epsilon_i$$

where:

- $\tau_{i,1} = 1$ if the i^{th} unit is in the first level of τ and $\tau_{i,1} = 0$ if the i^{th} unit is in any other level of τ.
- $\tau_{i,3} = 1$ if the i^{th} unit is in the third level of τ and $\tau_{i,3} = 0$ if the i^{th} unit is in any other level of τ.
- $\tau_{i,4} = 1$ if the i^{th} unit is in the fourth level of τ and $\tau_{i,4} = 0$ if the i^{th} unit is in any other level of τ.
- $\alpha_{i,1} = 1$ if the i^{th} unit is in the first level of α and $\alpha_{i,1} = 0$ if the i^{th} unit is in the second level of α.
- $\epsilon_i \sim iid\, N(0, \sigma^2)$.

The mean of the response for all units in the second level of τ and the second level of α is:

Answer:

$\mu_{2,2} = \beta_0$

Interaction Terms in the Two-way ANOVA Model: Definitions and Visualizations

Interaction plots can tell researchers whether there is a statistically significant interaction between factors (with respect to the mean of a continuous response variable).

Answer:

False

Interaction plots...

Answer:

- Are helpful visualizations for gaining insight into the nature of interactions in a two-way ANOVA.
- Show precisely how the sample means of the response change as a function of the factors.

Interactions in the Two-way ANOVA Model: Formal Tests

In the two-way ANOVA model with interactions, the interaction term describes how the relationship between a factor and the response differs as a function of the level of the other factor.

Answer:

True

Consider a two-way ANOVA model with two factors, τ and α, each of two levels, with the following regression form:

$$Y_i = \beta_0 + \beta_1\tau_{i,2} + \beta_2\alpha_{i,2} + \beta_3\tau_{i,2}\alpha_{i,2} + \epsilon_i,$$

where the typical definitions of the indicator variables hold (e.g., $\tau_{i,2} = 1$ when the i^{th} unit is in the second level of τ and $\tau_{i,2} = 0$ otherwise).

Whether there is an interaction in the data can be successfully tested with an F-test.

Answer:

True

Consider a two-way ANOVA model with two factors, τ, of three levels, and α of two levels, with the following regression form:

$$Y_i = \beta_0 + \beta_1\tau_{i,2} + \beta_2\tau_{i,3} + \beta_3\alpha_{i,2} + \beta_4\tau_{i,2}\alpha_{i,2} + \beta_5\tau_{i,3}\alpha_{i,2} + \epsilon_i,$$

where the typical definitions of the indicator variables hold (e.g., $\tau_{i,2} = 1$ when the i^{th} unit is in the second level of τ and $\tau_{i,2} = 0$ otherwise).

Whether there is an interaction in the data can be successfully tested with a t-test.

Answer:

False

Consider a two-way ANOVA model with two factors, τ, of three levels, and α of two levels, with the following regression form:

$$Y_i = \beta_0 + \beta_1\tau_{i,2} + \beta_2\tau_{i,3} + \beta_3\alpha_{i,2} + \beta_4\tau_{i,2}\alpha_{i,2} + \beta_5\tau_{i,3}\alpha_{i,2} + \epsilon_i,$$

where the typical definitions of the indicator variables hold (e.g., $\tau_{i,2} = 1$ when the i^{th} unit is in the second level of τ and $\tau_{i,2} = 0$ otherwise).

Suppose that there is, in fact, no interaction between factors at any level. Then the mean of the response for units in the second level of the τ factor and the second level of the α factor is:

Answer:

$\mu_{2,2} = \beta_0 + \beta_1 + \beta_3$

Consider a two-way ANOVA model with two factors, τ, of three levels, and α of two levels, with the following regression form:

$$Y_i = \beta_0 + \beta_1\tau_{i,2} + \beta_2\tau_{i,3} + \beta_3\alpha_{i,2} + \beta_4\tau_{i,2}\alpha_{i,2} + \beta_5\tau_{i,3}\alpha_{i,2} + \epsilon_i,$$

where the typical definitions of the indicator variables hold (e.g., $\tau_{i,2} = 1$ when the i^{th} unit is in the second level of τ and $\tau_{i,2} = 0$ otherwise).

Suppose that there is, in fact, an interaction between factors. Then the mean of the response for units in the first level of the τ factor and the first level of the α factor is:

Answer:

$\mu_{1,1} = \beta_0$

Researchers designed an experiment to study factors affecting the particle size in the production of polyvinyl chloride (PVC) plastic. In the experiment, three operators (Factor A, levels X, Y, and Z) used eight different devices called resin railcars (Factor B, levels 1-8) to produce PVC. Suppose that there were several replications in the study.

If the interaction terms in this study are statistically significant, then we cannot easily interpret the marginal effect of resin railcar in particle size.

Answer:

True

In the two-way ANOVA context, when using the F-test to test whether an interaction exists for a particular dataset, the model with the

In the two-way ANOVA context, when using the F-test to test whether an interaction exists for a particular dataset, the model with the interaction term is the reduced model.

Answer:

False

Two-way ANOVA Hypothesis Testing (no interaction)

When conducting planned comparisons in a two-way ANOVA, it is important to:

Answer:

- Specify all tests before observing the data.
- Make sure that the full F-test is statistically significant.

Researchers designed an experiment to study factors impacting the foam index for espresso. In the experiment, three espresso brewing machines (Factor A, machines X, Y, and Z) were used. Researchers also tested whether foam index was impacted by filtered or unfiltered water (Factor B, levels 1-2).

Researchers can conduct up to $2 - 1 = 1$ planned test without correcting for the familywise type I error.

Answer:

False

Testing the "marginal effect" of factor τ in a two-way ANOVA model with factors τ and α means that we test the effect of α while holding τ at its average value.

Answer:

False

The testing of combined effects in the context of two-way ANOVA implies that there are not interactions present.

Answer:

False

In the context of two-way ANOVA, before testing for marginal effects, researchers should be reasonably sure that there are significant interactions between factors.

Answer:

False

When conducting several hypothesis tests after the data have been observed, one must adjust the p-values of those tests to correct for the individual type I error rate.

Answer:

False

Researchers designed an experiment to study factors affecting the particle size in the production of polyvinyl chloride (PVC) plastic. In the experiment, three operators (Factor A, levels X, Y, and Z) used eight different devices called resin railcars (Factor B, levels 1-8) to produce PVC. There were enough observations to reasonably rule out a statistically significant interaction. Let $\alpha = 0.05$.

Mean Comparisons	Difference Between Means	Adjusted p-value
Operator 2 - Operator 1	-0.263	0.794
Operator 3 - Operator 1	-1.506	0.002
Operator 3 - Operator 2	-1.244	0.011

Answer:

- Averaging over all resin railcars, there is a statistically significant difference between the particle size of PVC between operator 3 and operator 1.
- Averaging over all resin railcars, there is a statistically significant difference between the particle size of PVC between operator 3 and operator 2.

The Conceptual Framework of Experimental Design

Gina is a soccer player looking to improve her soccer skills. One measure of a player's skills is their "plus/minus" score. The plus/minus score subtracts a point for every goal surrendered while the player is on the field, and adds a point for every goal scored while the player is on the field.

Suppose, through detailed analysis, Gina notices that, if she performs an aerobic conditioning workout in a given week, she is more likely to have a higher plus/minus score that week. From this, she concludes that the aerobic conditioning workout causes the increase in her plus/minus score.

In reasoning in this way, Gina is most likely employing which theory of causality?

Answer:

The probabilistic theory of causality

Gina is a soccer player looking to improve her soccer skills. One measure of a player's skills is their "plus/minus" score. The plus/minus score subtracts a point for every goal surrendered while the player is on the field, and adds a point for every goal scored while the player is on the field.

One week, Gina performs an aerobic conditioning workout, and notices that her plus/minus score is higher that week. From this, she reasons: if I did not perform this aerobic conditioning workout, I would not have increased my plus/minus score.

In reasoning in this way, Gina is most likely employing which theory of causality?

Answer:

The counterfactual theory of causality

Beth and Jessie's Ice Cream Company is attempting to perfect their recipe for vegan chocolate ice cream. One change in the recipe is related to the milk substitute. In some recipes, they use oat milk, and in others, almond milk.

Through extensive focus group analysis, they notice an interesting trend: the use of oat milk increases the odds of a high mean consumer rating at a given focus group by a factor of two. They conclude that the oat milk is the cause of the odds increase.

In reasoning in this way, Beth and Jessie are most likely employing which theory of causality?

Answer:

The probabilistic theory of causality

Beth and Jessie's Ice Cream Company is attempting to perfect their recipe for vegan chocolate ice cream. One change in the recipe is related to the milk substitute. In some recipes, they use oat milk, and in others, almond milk.

Beth and Jessie hire several food scientists who explain to them that oat milk is sweeter, and there is a stable (stochastic) positive relationship between sweetness and consumer ratings. Based on this information, Beth and Jessie conclude that oat milk is the cause of the higher consumer rating.

Answer:

The structural model theory of causality

Consider an experiment exploring factors related to the amount of pressure needed to have mountain climbing ropes fail. The experiment studied three factors, each with two levels:

- Abrasion: whether the rope had an abrasion on it or not
- Dirt: whether the rope was dirty or clean
- Soaked: whether the rope was soaked in water or not

Two replicates were recorded for each combination of factor levels.

Suppose that each of these factors is negatively associated with the response — for example, an abrasion decreases the amount of pressure needed to have the rope fail. However, rope failure is actually largely due to a fourth variable, namely, whether the rope has been "fatigued" (that is, whether a climber has fallen on it before). Which condition for causal reasoning from experimental data is not met?

Answer:

Nonspuriousness

Consider a study of the effectiveness of an "active learning" teaching method on student learning. In the study, 100 different senior-level high school math classes were randomly chosen from all such classes in the state of Colorado. Each class had $n = 25$ students. Among the 100 classes, 50 were randomly chosen, and teachers were asked to teach a lesson using a new active learning teaching method; in the other classes, teachers used the standard "lecture" teaching method. The response in this experiment was an exam, which was administered to each student and measured the extent to which each student learned the content of the lesson.

Identify the experimental units in this study.

Answer:

The experimental units are the 100 senior-level high school classes.

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Identify the sampling units in this study.

Answer:

The sampling units are the 2500 students in the senior-level high school classes.

The Completely Randomized Design

In a randomized experiment, controls are assigned at random to different levels of a treatment factor.

Answer:

False

In a randomized experiment, a control is a baseline to which treatments are compared.

Answer:

True

In a completely randomized design, every experimental unit has the same chance of being assigned to the control or treatment groups.

Answer:

True

The use of one-way ANOVA in a completely randomized design provides the justification for the "correct temporal relationship" condition of causality.

Answer:

False

In a completely randomized design, randomization helps with:

Answer:

Nonspuriousness

A completely randomized design is inappropriate if we have reason to believe that:

Answer:

- Experimental units are not homogeneous.
- The one-way ANOVA assumptions are violated in the analysis of the experiment.

The Randomized Complete Block Design (RCBD)

Blocking is a technique used to include other factors in an experiment, in order to reduce undesirable variability in the response.

Answer:

True

In a randomized complete block design, units are randomly assigned to blocks.

Answer:

False

If the nuisance variable in question is known and controllable, then we should measure it and use it as a blocking factor.

Answer:

True

Randomization reduces the effect of unknown nuisance variables that might be correlated with the response.

Answer:

True

One-way ANOVA can be used to analyze the results of a randomized complete block design.

Answer:

False

The Factorial Design

One factor at a time (OFAT) designs...

Answer:

- Produce estimates of treatment effects that are less precise than those produced by an appropriately designed experiment.
- Often require more resources, such as time, energy, and material.
- Cannot detect or estimate interactions.

Factorial designs are experimental designs that consist of two or more treatment factors.

Answer:

True

Any factorial design always includes experimental units in all possible factor level combinations.

Answer:

False

A $2^4 \times 3^2 \times 5^2$ factorial design is a design with four factors of two levels, two factors with three levels, and two factors of five levels.

Answer:

True

A 2^2 factorial design (with factors τ and α) can help researchers answer which of the following questions:

Answer:

- Are the effects of factor τ significant?
- Are the effects of α significant?
- Do τ and α interact?

Further Issues in Experimental Design

Two factors are crossed if there is at least one observation in every factor level combination.

Answer:

True

Factor A is nested within factor B if each level of A occurs within only one level of B.

Answer:

True

Nested factor designs and crossed factor designs share the same hypotheses and thus can be analyzed in the same way.

Answer:

False

If experimental treatments (factor levels) in a particular experiment are selected at random from a larger class of possible treatments, then a fixed effects model is likely most appropriate for the analysis of the experiment.

Answer:

False