Following are the super detailed solution for driving and simplifying the two functions asked in HW1 question 1.

$$\mu' = f(\mu, n, x_{n+1}) = \frac{\sum_{i=1}^{n+1} x_i}{n+1} = \frac{\sum_{i=1}^{n} x_i + x_{n+1}}{n+1} = \frac{n \cdot \mu + x_{n+1}}{n+1}$$

Since $v = \left(\sum_{i=1}^{n}(x_i - \mu)^2\right)/(n-1)$ and $v' = \left(\sum_{i=1}^{n+1}(x_i - \mu)^2\right)/n$, by multiplying $n-1$ and n to v and v' and subtracting nv' by $(n-1)v$ and then divide both side by n, we can have

$$\begin{aligned}
v' &= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + \sum_{i=1}^{n}\left((x_i-\mu')^2 - (x_i-\mu)^2\right)}{n} \\
&= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + \sum_{i=1}^{n}\left(x_i^2 + \mu'^2 - 2x_i\mu' - x_i^2 - \mu^2 + 2x_i\mu\right)}{n} \\
&= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + \sum_{i=1}^{n}\left(\mu'^2 - \mu^2 - 2x_i\cdot\mu' + 2x_i\cdot\mu\right)}{n} \\
&= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + n\mu'^2 - n\mu^2 - 2n\mu\mu' + 2n\mu^2}{n} \\
&= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + n(\mu'^2 + \mu^2 - 2\mu\cdot\mu')}{n} \\
&= \frac{(n-1)\cdot v + (x_{n+1}-\mu')^2 + n(\mu'-\mu)^2}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{x_{n+1}^2 + \mu'^2 - 2x_{n+1}\mu' + n\mu'^2 + n\mu^2 - 2n\mu\mu'}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) + \mu'(\mu' - 2x_{n+1} + n\mu' - 2n\mu)}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) + \mu'((n+1)\mu' - 2x_{n+1} - 2n\mu)}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) + \mu'(n\mu + x_{n+1} - 2x_{n+1} - 2n\mu)}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) + \mu'(-x_{n+1} - n\mu)}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) + \frac{(x_{n+1}+n\mu)(-x_{n+1}-n\mu)}{n+1}}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}^2 + n\mu^2) - \frac{(x_{n+1}+n\mu)^2}{n+1}}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{\frac{(n+1)\cdot(x_{n+1}^2+n\mu^2)}{n+1} - \frac{(x_{n+1}+n\mu)^2}{n+1}}{n} \\
&= \frac{n-1}{n}\cdot v + \frac{nx_{n+1}^2 + n^2\mu^2 + x_{n+1}^2 + n\mu^2 - x_{n+1}^2 - n^2\mu^2 - 2x_{n+1}n\mu}{n\cdot(n+1)} \\
&= \frac{n-1}{n}\cdot v + \frac{nx_{n+1}^2 + n\mu^2 - 2x_{n+1}n\mu}{n\cdot(n+1)} \\
&= \frac{n-1}{n}\cdot v + \frac{x_{n+1}^2 + \mu^2 - 2\mu x_{n+1}}{n+1} \\
&= \frac{n-1}{n}\cdot v + \frac{(x_{n+1}-\mu)^2}{n+1}
\end{aligned}$$