Modeling Crossflow-induced Propeller Autorotation  
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***Abstract*—The design of VTOL aircraft, particularly eVTOL aircraft, often relies on the use of multiple electrically powered thrust-vectoring propellers to meet the high thrust requirements of vertical flight. Many configurations employ aft-mounted propellers fixed in a vertical orientation, dedicated exclusively to providing extra lift during vertical take-off and landing. These propellers, left unpowered at cruise, experience crossflow—air moving parallel to the plane of rotation—which in turn induces autorotation. Modeling the aerodynamic behavior resulting from this crossflow-induced autorotation requires a modified mathematical framework—an adaptation of blade element theory—formulated specifically for crossflow conditions. This report presents the formulation of such a model and introduces software developed to solve it numerically, which, given propeller geometry and initial crossflow conditions, computes both transient and steady-state solutions for the propeller’s kinematic and aerodynamic states. These solutions inform the design and optimization of propeller systems for VTOL and eVTOL aircraft by providing insight into design parameters that drive both desired and undesired aerodynamic behavior. Notably, they reveal a strong positive correlation between blade pitch and cruise drag, suggesting that variable-pitch propellers, by leveling the blades at cruise, may offer advantages in improving aerodynamic efficiency.**

***Nomenclature***cBlade chord length (m)  
CdSection drag coefficient  
CℓSection lift coefficient  
DTotal propeller drag (N)  
DhPropeller hub drag (N)  
DnDrag from the nth propeller blade (N)  
Fy,nSide-force from the nth propeller blade (N)  
FyTotal propeller side-force (N)  
hPropeller hub height (m)  
IzMoment of inertia about the z-axis (kgm2)  
KvMotor velocity constant ((rads/s)/V)  
KtMotor torque constant (Nm/A)  
LTotal propeller lift (N)  
LnLift from the nth propeller blade (N)  
NNumber of blades  
RMotor resistance (Ω)  
rRadius (m)  
Hub Reynolds number  
RpPropeller radius (m)  
RhPropeller hub radius (m)   
tTime (s)  
Freestream kinematic viscosity(m2/s)  
Relative velocity vector (m/s)  
VℓRelative longitudinal velocity (m/s)  
VtRelative transverse velocity (m/s)  
Freestream velocity vector (m/s)  
Angular acceleration vector (rads/s2)  
βSection blade pitch (rads)  
θPropeller angular position (rads)  
Freestream air density (kg/m3)  
Net torque (Nm)  
Total Aerodynamic Torque (Nm)  
Aerodynamic Torque on the nth blade (Nm)  
Motor torque (Nm)  
Angular velocity vector (rads/s)

***Abbreviations***EMFElectromotive force  
GUI Graphical user interface  
RK4 Fourth-order Runge-Kutta  
SIMDSingle instruction, multiple data  
VTOLVertical take-off and landing  
eVTOLElectric vertical take-off and landing

1. INTRODUCTION

An unpowered propeller, oriented so that its plane of rotation is parallel to a freestream, will autorotate as energy from the flow is imparted to the propeller’s blades. Understanding the aerodynamics of this crossflow-induced autorotation is critical for the design of modern eVTOL aircraft, which often combine the use of forward wing-mounted thrust-vectoring propellers and aft wing-mounted vertically fixed propellers, both of which may experience crossflow-induced autorotation. Modeling this phenomenon requires a novel mathematical approach, as classical blade element theory assumes airflow collinear with the rotation axis, not perpendicular, as in the case of crossflow-induced autorotation [1]. Adapting classical blade element theory to account for propeller crossflow produces a system of nonlinear ordinary differential equations governing the propeller’s motion and aerodynamics. Specialized software developed to numerically solve this system enables detailed analysis of the geometric factors driving transient aerodynamic behavior. This analysis streamlines the optimization of propeller designs for the cruise crossflow conditions.

1. FORMULATION OF A MATHEMATICAL MODEL

To formulate a mathematical model for crossflow-induced autorotation, several simplifying assumptions are introduced. The flow is assumed incompressible, the upstream velocity profile is taken to be linear, and the induced velocity from the propeller’s rotation is considered negligible. A coordinate system is established to define the aerodynamic forces acting on the propeller—a foundation for deriving the equations of motion. The following sections detail the governing assumptions, coordinate system, aerodynamic force modeling, and equations of motion for the autorotation.

1. *Governing Assumptions*

As typically cruising speeds of current eVTOL aircraft fall bellow Mach 0.3, incompressible flow may be assumed if the propeller’s maximum tip speed also meets this threshold. Conveniently, angular velocities during autorotation are significantly less than powered operation, producing tip speeds under the Mach 0.3 threshold for reasonably sized propellers. This assumption permits the use of subsonic aerodynamic equations and airfoil experimental data to develop a model for aerodynamic forces.

In addition, the velocity profile is assumed to be linear—an assumption that holds well for propellers mounted ahead of flow-disrupting elements. Whereas when mounted, for example, behind a wing and subject to propeller-shed vortices, the more relevant case, this assumption becomes a likely source of error. Nevertheless, this assumption serves as a clear starting point for developing a simplified mathematical model.

Lastly, we assume that velocity induced by the propeller’s rotation is negligible. As in the assumption of incompressible flow, the propeller’s significantly lower rotation rate during autorotation justifies the simplification. It follows that the formulation avoids the mathematical complications of adapting blade element momentum theory as there is no longer a need to model the propeller’s suction produced by a high rotation rate. As such, mathematical formulation reduces to an adaptation of only bade element theory [2].

1. *Coordinate System*The coordinate system for the model’s mathematical

formulation is shown in Fig. 1.

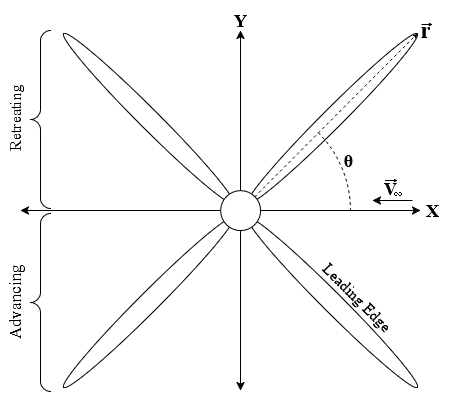


Fig. 1: The right-handed coordinate system for the propeller autorotation model.

The propeller rotates about the z-axis, while the freestream (crossflow) velocity vector, , is directed along the negative x-axis. As the propeller rotates, the blades cyclically pass through regions where they either advance or retreat into the airflow, providing an increase or decrease in the relative velocity, *V*, as defined in (1).

(1)

Equation (1) defines the relative velocity as the vector sum of the circular velocity—the cross product of the angular velocity, , and the radial position vector, —and the freestream velocity vector, . This relative velocity is then resolved into two components: one parallel to the blade chord line and one perpendicular to it. The parallel component, referred to as the relative transverse velocity, *Vt*, is used to compute the aerodynamic forces acting on the blades through integration from the blade root to the tip and is given by (2).

(2)  
  
The perpendicular component, the relative longitudinal velocity, is obtained by (3), although it is not used in the subsequent mathematical formulation.

(3)  
  
However, incorporating the longitudinal velocity into the aerodynamic force modeling could improve the model’s accuracy by capturing drag due to spanwise flow along the blade and should be considered in future work.

1. *Modeling Aerodynamic Forces*

As in blade element theory, the total aerodynamic forces acting on each blade can be estimated by integrating from the blade root at *r = Rh* to the blade tip at *r = Rp*. Each discretization along the blade’s span is taken as an airfoil section with width *dr*, chord length *c*, lift coefficient *Cℓ*, drag coefficient *Cd*, and pitch *β*—all of which are functions of the radial position *r*. The lift produced by the nth blade is obtained using (4).

(4)  
  
Similarly, the contribution of aerodynamic torque from the nth blade is found using (5). The sign function is applied to the transverse velocity because the dot product in (2), permits the possibility of reversed airflow (i.e., from the blade’s trailing edge to the blade’s leading edge) when a retreating blade moves faster than the opposing freestream. In such cases, aerodynamic torque is clockwise.

(5)  
  
To compute the propeller drag contribution from each blade, the sine of the angular position, *θ*, is introduced in the integration to extract the x-axis component of the airfoil section drag, as shown in (6).

(6)  
  
By instead using the cosine function, the side-force, *Fy*, acting along the y-axis, may be obtained from (7).

(7)

The total lift, drag, torque, and side-force for the  
propeller is found by summing the contributions from each blade; however, the hub drag and the motor’s resistive torque are also included in the total drag and torque, respectively. The hub drag is found using the hub Reynolds number given by (8).

(8)

Using the hub Reynolds number, the hub drag is estimated from the piecewise function, (9), which is derived from experimental data reported by Brennen [3].

(9)

Secondly, the electric motor’s resistive torque, , is obtained using (10), which models the motor’s internal resistance to rotation using the motor velocity constant, *Kv*, torque constant, *Kt*, and motor resistance, *R*.

(10)

Using (9) and (10), the summations for total lift, drag, net torque, and side-force are given in (11), (12), (13), and (14), respectively.

(11)

(12)

(13)

(14)

1. *Equations of Motion*
2. SOLVING THE MODEL NUMERICALLY
3. *Applying a 4th-order Runge-Kutta Method*
4. *Developing Specialized Software*
5. VALIDATION
6. RESULTS
7. CONCLUSION

REFERENCES

[1] H. Glauert, “The Airscrew: Blade Element Theory,” in *The Elements of Aerofoil and Airscrew Theory*, London: Cambridge University Press, 1926, pp. 208–222

[2] H. Glauert, “The Airscrew: Momentum Theory,” in *The Elements of Aerofoil and Airscrew Theory*, London: Cambridge University Press, 1926, pp. 199–208

[3] C. E. Brennen, “An Internet Book on Fluid Dynamics: Drag on a Sphere and Cylinder”. Pasadena, CA: California Institute of Technology, 2006. [Online]. Available: http://brennen.caltech.edu/ fluidbook/externalflows/drag/dragonasphere.pdf

[4] R. King, “Propeller Crossflow Autorotation Solver (v1.0.0),” Apr. 2025. [Source code]. Available:https://github.com/RyanKingSoftware/ propeller-crossflow-autorotation-solver