Modeling Crossflow-induced Propeller Autorotation  
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***Abstract*—The design of VTOL aircraft, particularly eVTOL aircraft, often relies on the use of multiple electrically powered thrust-vectoring propellers to meet the high thrust requirements of vertical flight. Many configurations employ aft-mounted propellers fixed in a vertical orientation, dedicated exclusively to providing extra lift during vertical take-off and landing. These propellers, left unpowered at cruise, experience crossflow—air moving parallel to the plane of rotation—which in turn induces autorotation. Modeling the aerodynamic behavior resulting from this crossflow-induced autorotation requires a modified mathematical framework—an adaptation of blade element theory—formulated specifically for crossflow conditions. This report presents the formulation of such a model and introduces software developed to solve it numerically, which, given propeller geometry and initial crossflow conditions, computes both transient and steady-state solutions for the propeller’s kinematic and aerodynamic states. These solutions inform the design and optimization of propeller systems for VTOL and eVTOL aircraft by providing insight into design parameters that drive both desired and undesired aerodynamic behavior. Notably, they reveal a strong positive correlation between blade pitch and cruise drag, suggesting that variable-pitch propellers, by leveling the blades at cruise, may offer advantages in improving aerodynamic efficiency.**

***Nomenclature***cBlade chord length (m)  
CdSection drag coefficient  
CℓSection lift coefficient  
DTotal propeller drag (N)  
DhPropeller hub drag (N)  
DnDrag from the nth propeller blade (N)  
hPropeller hub height (m)  
IzMoment of inertia about the z-axis (kgm2)  
KvMotor velocity constant ((rads/s)/V)  
KtMotor torque constant (Nm/A)  
LTotal propeller lift (N)  
LnLift from the nth propeller blade (N)  
NNumber of blades  
RMotor resistance (Ω)  
rRadius (m)  
RpPropeller radius (m)  
RhPropeller hub radius (m)   
tTime (s)  
Local velocity vector (m/s)  
VℓLongitudinal velocity (m/s)  
VtTransverse velocity (m/s)  
Freestream velocity vector (m/s)  
Angular acceleration vector (rads/s2)  
βSection blade pitch (rads)  
θPropeller angular position (rads)  
Freestream air density (kg/m3)  
Net torque (Nm)  
Total Aerodynamic Torque (Nm)  
Aerodynamic Torque on the nth blade (Nm)  
Motor torque (Nm)  
Angular velocity vector (rads/s)

***Abbreviations***EMFElectromotive force  
GUI Graphical user interface  
RK4 Fourth-order Runge-Kutta  
SIMDSingle instruction, multiple data  
VTOLVertical take-off and landing  
eVTOLElectric vertical take-off and landing

1. INTRODUCTION

An unpowered propeller, oriented so that its plane of rotation is parallel to a freestream, will autorotate as energy from the flow is imparted to the propeller’s blades. Understanding the aerodynamics of this crossflow-induced autorotation is critical for the design of modern eVTOL aircraft, which often combine the use of forward wing-mounted thrust-vectoring propellers and aft wing-mounted vertically fixed propellers, both of which may experience crossflow-induced autorotation. Modeling this phenomenon requires a novel mathematical approach, as classical blade element theory assumes airflow collinear with the rotation axis, not perpendicular, as in the case of crossflow-induced autorotation [1]. Adapting classical blade element theory to account for propeller crossflow produces a system of nonlinear ordinary differential equations governing the propeller’s motion and aerodynamics. Specialized software developed to numerically solve this system enables detailed analysis of the geometric factors driving transient aerodynamic behavior. This analysis streamlines the optimization of propeller designs for the cruise crossflow conditions.

1. FORMULATION OF A MATHEMATICAL MODEL

To formulate a mathematical model for crossflow-induced autorotation, several simplifying assumptions are introduced. The flow is assumed incompressible, the upstream velocity profile is taken to be linear, and the induced velocity from the propeller’s rotation is considered negligible. A coordinate system is established to define the aerodynamic forces acting on the propeller—a foundation for deriving the equations of motion. The following sections detail the governing assumptions, coordinate system, aerodynamic force modeling, and equations of motion for the autorotation.

1. *Governing Assumptions*

As the cruising speeds of current eVTOL aircraft are

typically bellow Mach 0.3, incompressible flow can be assumed if the propeller’s maximum tip speed also meets this threshold. Conveniently, angular velocities during autorotation are significantly less than powered operation, producing tip speeds under the Mach 0.3 threshold for reasonably sized propellers. This assumption permits the use of subsonic aerodynamic equations and airfoil experimental data to develop a model for aerodynamic forces.

1. *Coordinate System*
2. *Modeling Aerodynamic Forces*
3. *Equations of Motion*
4. SOLVING THE MODEL NUMERICALLY
5. *Applying a 4th-order Runge-Kutta Method*
6. *Developing Specialized Software*
7. VALIDATION
8. RESULTS
9. CONCLUSION

REFERENCES

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