

# Distributed Nash Equilibrium Seeking On a Consensus based Gaming



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# Introduction

- Game
- Nash Equilibrium
- Distributed Game

# Game

Consider a game with  $N$  players. The set of players is denoted by  $\mathbb{N} = 1, 2, \dots, N$ . The payoff function of player  $i$  is  $f_i(x)$ , where  $x = [x_1, x_2, \dots, x_N]^T \in R^N$  is the vector of players' actions and  $x_i \in R$  is the action of player  $i$ , then player  $i$  has no direct access to player  $j$ 's action.

# Nash Equilibrium

**Nash equilibrium is an action profile on which no player can gain more payoff by unilaterally changing its own action.**

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$$

# Cournot's duopoly Model

- the players are the firms
- the actions of each firm are the set of possible outputs (any nonnegative amount)
- the payoff of each firm is its profit.

where:

$P :=$  Price, Inverse demand function

$f_i :=$  Profit of player  $i$

$TC_i :=$  Total Cost Function

# Cournot's duopoly Model

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We have:

# Example A

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

With Repeated Advantage Solution:

	L
U	(1,3)

# Distributed Game

We defined Distributed Gaming as a series of Game Behaviors and Strategies which is Distributed. With Lamport's definition on 1978, A Gaming is Distributed if the message transaction delay is not negligible compared to the time between event in classic gaming behavior.



# Distributed

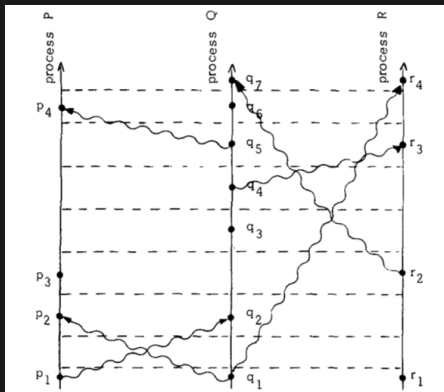


Figure: Lamport Timestamp

# Formalize

Consider a game with  $N$  players. The set of players is denoted by  $\mathbb{N} = 1, 2, \dots, N$ . The payoff function of player  $i$  is  $f_i(x)$ , where  $x = [x_1, x_2, \dots, x_N]^T \in R^N$  is the vector of players' actions and  $x_i \in R$  is the action of player  $i$ , then player  $i$  has no direct access to player  $j$ 's action.

**Suppose that if player  $j$  is not a neighbor of player  $i$ , then player  $i$  has no direct access to player  $j$ 's action**

# Example B

## Prisoner's Dilemma

	W	A
W	(1,1)	(-1, 2)
A	(2,-1)	(0,0)

If players are sensitive on the uncertainty of  $s_{-1}$ , they may not choose the rational strategy.

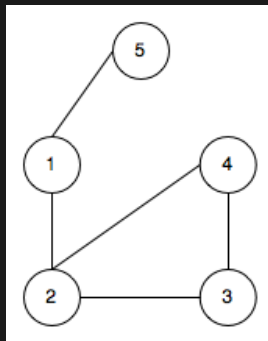


Figure: Communication graph for the players in the numerical example

# Distributed Nash Equilibrium

The action of player  $i$  is updated according to

$$\dot{x} = k_i \frac{\partial f_i}{\partial x_i}(y_i), i \in \mathbb{N} \quad (1)$$

where

$$\begin{aligned} \dot{y}_i &= [y_{i1}, y_{i2}, \dots, y_{iN}]^T \in R \\ k_i &= \delta \bar{k} \end{aligned}$$

$\delta$  as a small positive parameter,  $\bar{k}_i$  is a fixed positive parameter.  
 $y_{ij}, \forall i, j \in \mathbb{N}$  is player  $i$ 's estimate on player  $j$ 's action, which are generated by:

$$\dot{y}_{ij} = \left( \sum_{k=1}^N a_{ik}(y_{ik} - y_{kj} + a_{ij}(y_{ij} - x_y)) \right)$$

# Distributed Nash Equilibrium

Let  $\tau = \delta t$

# Thanks

Repository of this Slides:

- <https://github.com/RyanKung/dneq4eos>

