Distributed Nash Equilibrium Seeking On a Consensus based Gaming

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Formulation

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, ..., N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, ..., x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i, then player i has no direct access to player j's action.

Formulation

Nash equilibrium is an action profile on which no player can gain more payoff by unilaterally changing its own action.

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$$

Cournot's duopoly Model

- the players are the firms
- the actions of each firm are the set of possible outputs (any nonnegative amount)
- the payoff of each firm is its profit.

where:

$$
\begin{aligned}
\text{P}:= &\quad \text{Price, Inverse demand function} \\
f_i:= &\quad \text{Profit of player i} \\
TC_i:= &\quad \text{Total Cost Function}
\end{aligned}
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We have:

Example A

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

With Repeated Advantage Solution:

	L
U	(1,3)

Example B

Prisoner's Dilemma

	W	A
W	(1,1)	(-1, 2)
A	(2,-1)	(0,0)

If players are sensitive on the uncertainty of s_{-1}, they may not choose the rational strategy.

Lamport TimeStamp

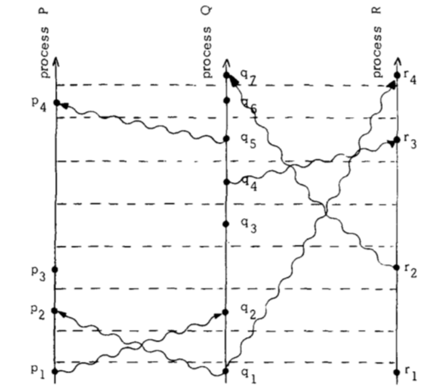

Figure: Lamport Timestamp

Definition

We defined Distributed Gaming as a series of Game Behaviors and Strategis which is Distributed. With Lamport's defination on 1978, A Gaming is Distributed if the message transaction delay is not negligible compared to the time between event in classic gaming behavior.

Distributed Nash Equilibrium

$$\dot{x} = k_i \frac{\partial f_i}{\partial x_i}(y_i), i \in \mathbb{N} \tag{1}$$