Distributed Nash Equilibrium Seeking On a Consensus based Gaming

Ryan J. Kung

ryankung@ieee.org

April 28, 2018

Introduction

- Game
- Nash Equilibrium
- Distributed Game

Game

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, ..., N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, ..., x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i, then player i has no direct access to player j's action.

Nash Equilibrium

Nash equilibrium is an action profile on which no player can gain more payoff by unilaterally changing its own action.

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$$

Cournot's duopoly Model

- the players are the firms
- the actions of each firm are the set of possible outputs (any nonnegative amount)
- the payoff of each firm is its profit.

where:

$$\begin{aligned} P := & \quad \text{Price, Inverse demand function} \\ f_i := & \quad \text{Profit of player i} \\ TC_i := & \quad \text{Total Cost Function} \end{aligned}$$

Cournot's duopoly Model

P:= Price, Inverse demand function
f_i:= Profit of player i
TC_i:= Total Cost Function

We have:

$$f_i = x_i(x_i + x_{-1}) - TC_i(x_i)$$

Thus:

$$\dot{q}_1 = \dot{q}_2 = \frac{1-c}{3}$$

Distributed Game

We defined Distributed Gaming as a series of Game Behaviors and Strategis which is Distributed. With Lamport's defination on 1978, A Gaming is Distributed if the message transaction delay is not negligible compared to the time between event in classic gaming behavior.

Distributed

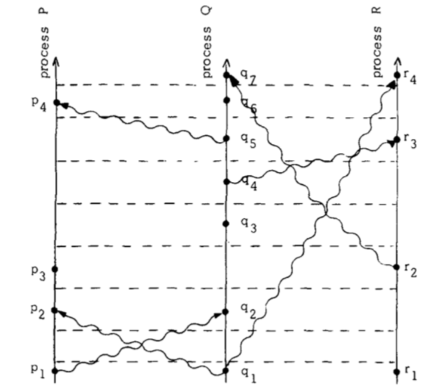


Figure: Lamport Timestamp

Formalize

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, ..., N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, ..., x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i, then player i has no direct access to player j's action.

Suppose that if player j is not a neighbor of player i, then player i has no direct access to player $j's$ action

Distributed Nash Equilibrium

- Sensitive Game
- Distributed Network

Example A

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

With Repeated Advantage Solution:

	L
U	(1,3)

But when we discuss Mixed Strategy, if more than 1% $players_2$ chose R, D is better than U:

Example A :: Mixed Strategy

Mixed Strategy σ is a probability distribution over pure strategy.

A Mixed Strategy for player i can be present as a vector:

$$(\sigma_i(U), \sigma_i(M), \sigma_i(D))$$

Payoff Function u_i over σ:

$$\sum_{i \in S} \left(\prod_{j=i}^{I} \sigma_i(s_u) \right) u_j(s)$$

Example A :: Mixed Strategy

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

Let:

$$\sigma_2 = (0.99, 0.01)$$

Then:

$$u_1 = -5.94 \iff \sigma_1 = (1, 0, 0)$$
$$u_1 = 2 \iff \sigma_1 = (0, 0, 1)$$

Example B

Prisoner's Dilemma

	W	A
W	(1,1)	(-1, 2)
A	(2,-1)	(0,0)

If players are sensitive on the uncertainty of s_{-1}, they may not choose the rational strategy.

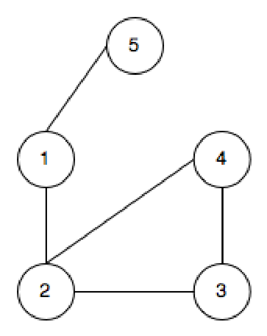


Figure: Communication graph for the players in the numerical example

Distributed Nash Equilibrium

The action of player i is updated according to

$$\dot{x} = k_i \frac{\partial f_i}{\partial x_i}(y_i), i \in \mathbb{N} \tag{-1}$$

where

$$\dot{y_i} = [y_{i1}, y_{i2}, ..., y_{iN}]^T \in R$$
$$k_i = \delta \bar{k}$$

δ as a small positive parameter, $\bar{k}_i$ is a fixed positive parameter. $y_{ij}, \forall i, j \in \mathbb{N}$ is player i's estimate on player j's action, which are generated by:

$$\dot{y_{ij}} = -\left(\sum_{k=1}^{N} a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right)$$

Distributed Nash Equilibrium

Let $\tau = \delta t$, at the τ-time scale:

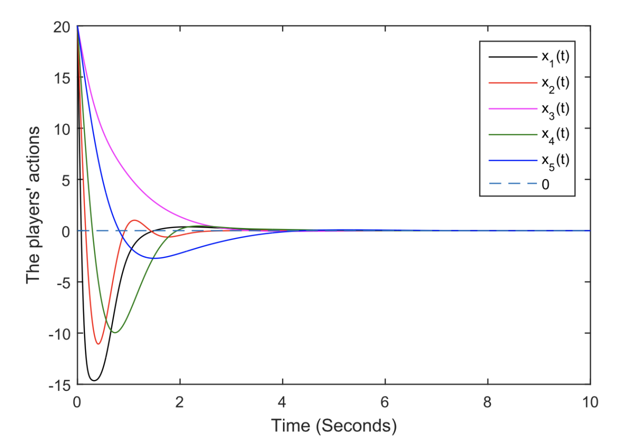
$$\frac{dx_i}{d_\tau} = \bar{k}_i \frac{\partial f_i}{\partial x_i}(y_i)$$

$$\delta \frac{dy_{ij}}{d\tau} = -\left(\sum_{k=1}^{N} a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right) \tag{0}$$

Distributed Nash Equilibrium

Easy to know, by setting δ to zero, The reduced system is:

$$\frac{dx_i}{d\tau} = \bar{k}_i \frac{\partial f_i}{\partial x_i}(x) \quad \forall i \in \mathbb{N} \tag{1}$$



Conclusions

- Pure Strategis Game and Mixed Strategis Game
- Distributed Network May Changed Nash-eq
- Distributed Nash equilibrium have single value

Thanks

Repository of this Slides:

- https://github.com/RyanKung/dneq4eos