

Distributed Nash Equilibrium Seeking On a Consensus based Gaming



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Introduction

- Game
- Nash Equilibrium
- Distributed Game

Game

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, \dots, N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, \dots, x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i , then player i has no direct access to player j 's action.

Nash Equilibrium

Nash equilibrium is an action profile on which no player can gain more payoff by unilaterally changing its own action.

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$$

Cournot's duopoly Model

- the players are the firms
- the actions of each firm are the set of possible outputs (any nonnegative amount)
- the payoff of each firm is its profit.

where:

$P :=$ Price, Inverse demand function

$f_i :=$ Profit of player i

$TC_i :=$ Total Cost Function

Cournot's duopoly Model

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We have:

Example A

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

With Repeated Advantage Solution:

	L
U	(1,3)

Distributed Game

We defined Distributed Gaming as a series of Game Behaviors and Strategies which is Distributed. With Lamport's definition on 1978, A Gaming is Distributed if the message transaction delay is not negligible compared to the time between event in classic gaming behavior.

Distributed

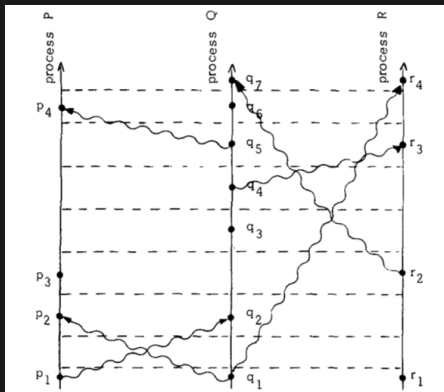


Figure: Lamport Timestamp

Formalize

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, \dots, N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, \dots, x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i , then player i has no direct access to player j 's action.

Suppose that if player j is not a neighbor of player i , then player i has no direct access to player j 's action

Example B

Prisoner's Dilemma

	W	A
W	(1,1)	(-1, 2)
A	(2,-1)	(0,0)

If players are sensitive on the uncertainty of s_{-1} , they may not choose the rational strategy.

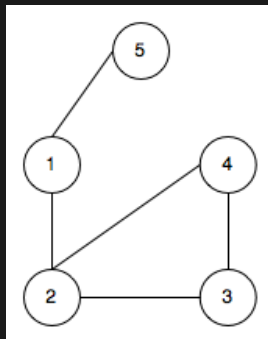


Figure: Communication graph for the players in the numerical example

Distributed Nash Equilibrium

The action of player i is updated according to

$$\dot{x} = k_i \frac{\partial f_i}{\partial x_i}(y_i), i \in \mathbb{N} \quad (1)$$

where

$$\begin{aligned} \dot{y}_i &= [y_{i1}, y_{i2}, \dots, y_{iN}]^T \in R \\ k_i &= \delta \bar{k} \end{aligned}$$

δ as a small positive parameter, \bar{k}_i is a fixed positive parameter.
 $y_{ij}, \forall i, j \in \mathbb{N}$ is player i 's estimate on player j 's action, which are generated by:

$$\dot{y}_{ij} = - \left(\sum_{k=1}^N a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right)$$

Distributed Nash Equilibrium

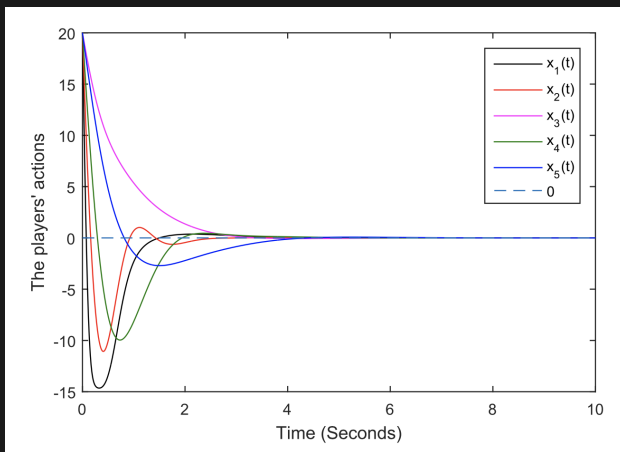
Let $\tau = \delta t$, at the τ -time scale:

$$\begin{aligned}\frac{dx_i}{d\tau} &= \bar{k}_i \frac{\partial f_i}{\partial x_i}(y_i) \\ \delta \frac{dy_{ij}}{d\tau} &= - \left(\sum_{k=1}^N a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right)\end{aligned}\tag{2}$$

Distributed Nash Equilibrium

Easy to know, by setting δ to zero, The reduced system is:

$$\frac{dx_i}{d\tau} = \bar{k}_i \frac{\partial f_i}{\partial x_i}(x) \quad \forall i \in \mathbb{N} \quad (3)$$



Thanks

Repository of this Slides:

- <https://github.com/RyanKung/dneq4eos>

