Distributed Nash Equilibrium Seeking On a Consensus based Gaming

Ryan J. Kung

ryankung@ieee.org

April 29, 2018

Introduction

- Game
- Nash Equilibrium
- Distributed Game

Game

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, ..., N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, ..., x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i, then player i has no direct access to player j's action.

Nash Equilibrium

Nash equilibrium is an action profile on which no player can gain more payoff by unilaterally changing its own action.

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$$

Cournot's duopoly Model

- the players are the firms
- the actions of each firm are the set of possible outputs Q
- the payoff of each firm is its profit.

where:

$$
\begin{aligned}
\text{p}{:=} &\quad \text{Price, Inverse demand function} \\
u_i{:=} &\quad \text{Profit of player i} \\
c_i{:=} &\quad \text{Total Cost Function}
\end{aligned}
$$

Cournot's duopoly Model

p:=	Price, Inverse demand function
u_i:=	Profit of player i
c_i:=	Total Cost Function

We have:

$$u_i(q_1, q_2) = q_i p(q) - c_i(q_i)$$

Thus the response function $\dot{r}_i : Q_1 \rightarrow Q_2$:

$$\dot{r}_1 = \dot{r}_2 = \frac{1-c}{3}$$

Distributed

A system is distributed if the message transmission delay is not negligible compared to the time between events in a single process. [1]

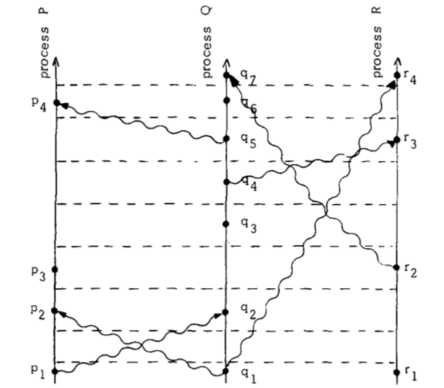


Figure: Lamport Timestamp

Distributed Game

We defined Distributed Gaming as a series of Game Behaviors and Strategis which is Distributed. A Gaming is Distributed if the message transaction delay is not negligible compared to the time between event in classic gaming behavior.

Formalize

Consider a game with N players. The set of players is denoted by $\mathbb{N} = 1, 2, ..., N$. The payoff function of player i is $f_i(x)$, where $x = [x_1, x_2, ..., x_N]^T \in R^N$ is the vector of players' actions and $x_i \in R$ is the action of player i, then player i has no direct access to player j's action.

Suppose that if player j is not a neighbor of player i, then player i has no direct access to player $j's$ action

Distributed Nash Equilibrium

- Sensitive Game
- Distributed Network

Example A

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

With Repeated Advantage Solution:

	L
U	(1,3)

But when we discuss Mixed Strategy, if more than 1% $players_2$ chose R, D is better than U:

Example A :: Mixed Strategy

Mixed Strategy σ is a probability distribution over pure strategy.

A Mixed Strategy for player i can be present as a vector:

$$(\sigma_i(U), \sigma_i(M), \sigma_i(D))$$

Payoff Function u_i over σ:

$$\sum_{i \in S} \left(\prod_{j=i}^{I} \sigma_i(s_u) \right) u_j(s)$$

Example A :: Mixed Strategy

	L	R
U	(1,3)	(-3,0)
M	(-2,0)	(1,3)
D	(0,1)	(0,1)

Let:

$$\sigma_2 = (0.99, 0.01)$$

Then:

$$u_1 = -5.94 \iff \sigma_1 = (1, 0, 0)$$
$$u_1 = 2 \iff \sigma_1 = (0, 0, 1)$$

Example B

Prisoner's Dilemma

	M	S
M	(1,1)	(-1, 2)
S	(2,-1)	(0,0)

If players are sensitive on the uncertainty of s_{-1}, they may not choose the rational strategy.

Distributed Nash Equilibrium

The action of player i is updated according to

$$\dot{x} = k_i \frac{\partial f_i}{\partial x_i}(y_i), i \in \mathbb{N} \tag{0}$$

where

$$\dot{y}_i = [y_{i1}, y_{i2}, ..., y_{iN}]^T \in R$$
$$k_i = \delta \bar{k}$$

δ as a small positive parameter, $\bar{k}_i$ is a fixed positive parameter. $y_{ij}, \forall i, j \in \mathbb{N}$ is player i's estimate on player j's action, which are generated by:

$$\dot{y}_{ij} = -\left(\sum_{k=1}^{N} a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right)$$

Distributed Nash Equilibrium

Let $\tau = \delta t$, at the τ-time scale:

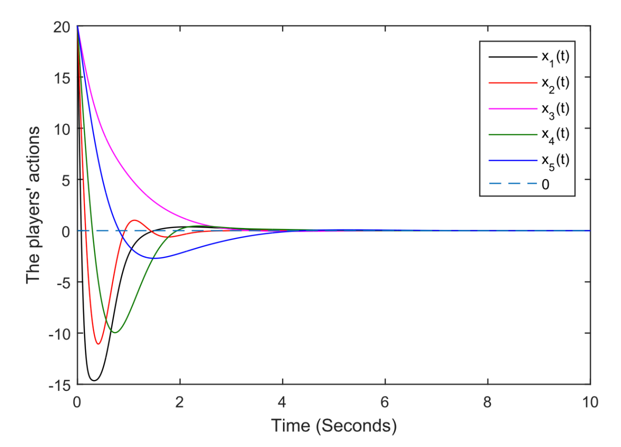
$$\frac{dx_i}{d_\tau} = \bar{k}_i \frac{\partial f_i}{\partial x_i}(y_i)$$

$$\delta \frac{dy_{ij}}{d\tau} = -\left(\sum_{k=1}^{N} a_{ik}(y_{ik} - y_{kj}) + a_{ij}(y_{ij} - x_y) \right) \text{[?]} \tag{1}$$

Distributed Nash Equilibrium

Easy to know, by setting δ to zero, The reduced system is:

$$\frac{dx_i}{d\tau} = \bar{k}_i \frac{\partial f_i}{\partial x_i}(x) \quad \forall i \in \mathbb{N} \tag{2}$$



Under Switching Topologies

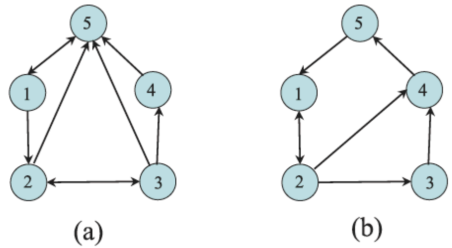
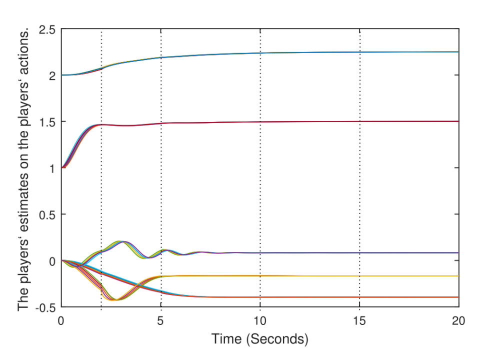


Fig. 1. Communication graph in (a) is $\mathcal{G}^1$ and the communication graph in (b) is $\mathcal{G}^2$.

Player i can be regarded as a virtual leader, who provides its action as a reference signal to be followed by $-i$, For each X^*, there is a constant δ that for every $\delta \in (\delta_{max}, \delta_{min})$, Nash Equilibrium X^* is asymptotically stable. [2]

On loss of Communication



Player i can be regarded as a virtual leader, who provides its action as a reference signal to be followed by $-i$, For each X^*, there is a constant δ that for every $\delta \in (\delta_{max}, \delta_{min})$, Nash Equilibrium X^* is asymptotically stable. [2]

Conclusions

- Exists Distributed Nash Equilibrium
- Distributed Nash Equilibrium can be sensitive
- Distributed Nash Equilibrium is stable under different network Topologies

Reference

- Leslie Lamport.
  Time, clocks and the ordering of events in a distributed system.
  pages 558–565, July 1978.

- M. Ye and G. Hu.
  Distributed nash equilibrium seeking by a consensus based approach.
  IEEE Transactions on Automatic Control, 62(9):4811–4818, Sept 2017.

- M. Ye and G. Hu.
  Distributed nash equilibrium seeking in multiagent games under switching communication topologies.
  IEEE Transactions on Cybernetics, PP(99):1–10, 2017.

Thanks

Repository of this Slides:

- https://github.com/RyanKung/dneq4eos