Warm Up!

For all integers n greater than 1, define

$$a_n = \frac{1}{\log_n 2002}$$

Let $b = a_2 + a_3 + a_4 + a_5$ and

$$c = a_{10} + \cdots + a_{14}$$

Then $b - c$ equals?

In trapezoid $ABCD$, AB and CD are perpendicular to AD, with $AB + CD = BC$. $AB < CD$, and $AD = 7$.

What is $AB * CD$?

Warm Up!

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We have that $\frac{1}{\log_a b} = \log_b a$

Warm Up!

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What is $AB * CD$?

Enscribe circles to our lengths AB and CD

Warm Up!

A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

Warm Up!

A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

Consider a cross-section of these figures

Warm Up!

A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

Consider the permutations and combinations of the 16 items.

Warm Up!

A point P is selected at random from the interior of the pentagon with vertices $A = (0,2), B = (4,0), C = (2\pi + 1,0),$ *and* $E = (0,4)$. *What is the probability that* $\angle APB$ *is obtuse?*

The polynomial $p(x) = x^3 + ax^2 + bx + c$ *has the property that the average of its zeros, the product of its zeros, and the sum of it's coefficients are all equal. The* y*-intercept of the graph of* $y = p(x)$ *is 2. What is* b*?*

Warm Up!

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Consider the geometric region restricting all the possible points

Warm Up!

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Consider what we know about Vieta's Formulas and the coefficients of polynomials.

Problems:

For all positive integers n, let

$$a_n = \begin{cases} 11, & n \text{ is divisible by } 13 \text{ and } 14 \\ 13, & n \text{ is divisible by } 14 \text{ and } 11 \\ 14, & n \text{ is divisible by } 11 \text{ and } 13 \\ 0, & \text{otherwise} \end{cases}$$

Find $\sum_{n=1}^{2001} a_n$

Let $\{a_k\}$ *be a sequence of integers such that* $a_1 = 1$ *and*

$a_{m+n} = a_m + a_n + mn$, *for all positive integers* m *and* n.

Find a_{12}

Guided Discussion: AMC Season

What to expect, Approaches to Problems

Walter Johnson Math Team

Guided Discussion: Subtopics

There are a lot of types of problems. We're going to review some basic techniques to approaching all these problems to remind ourselves and get all around ready for the AMC!

Probability	Common, Hard,
Complex Numbers	Uncommon, Medium, 15-25
Functions	Common, M-Hard, 8-23
Geometry	Common, M-Hard, 10-25
Triangles	Uncommon, Hard, 10-21
Trig Equations	Rare, Hard, 15-25
Combinatorics	Rare, Hard, 13-23
Series	Uncommon, Medium, 10-20
Logarithms	Common, M-Hard,
Number Theory	Uncommon, Easy,
Word Problems	Common, Medium, 10-20

Guided Discussion: Probability

For a particular peculiar pair of dice, the probability of rolling a 1, 2, 3, 4, 5 and 6, on each die are in the ratio of 1 : 2 : 3 : 4 : 5 : 6. What is the probability of rolling a total of 7 on the two die?

How can we model this?

- *Infinite Series*
- *Modeling with 3-D Figures*
- *Systems of Equations*

Guided Discussion: Logarithms

For the harder problems, you'll get familiar over time with how problems are structured and what they are really asking for.

**Don't be afraid if you find a problem splits up into multiple directions for different solutions!*

- *Identities*
- *Modeling with Ranges and Domains*
- *Don't be afraid of the Sigma!*
- *Reciprocal Rule*

Guided Discussion: Functions

Never back away from a polynomials problem without modeling the equation and writing everything you have on paper.

- *Try to just go at problems, don't be intimidated, and go for problems.*
- *Vieta's Formulas for Monic Polynomials*
- *n-roots for $n-1$ degree polynomials*
- *Nested radicals (not common)*
- *Multinomial Theorem*

Guided Discussion: Functions

Four positive integers, a, b, c, and d have a product of $8!$ And satisfy

$$ab + a + b = 524$$

Type equation here.

- *Consider $P(0)$ = constant term = product of all roots*
- *Consider $P(1) = a_1 + a_2 \cdots$*
- *Newton Sums, $P_k = x_1^k + x_2^k + \cdots$*
- $a_n P_1 + a_{n-1} = 0$
- $a_n P_2 + a_{n-1} P_1 + 2a_{n-2} = 0$
- *Systems of Equations*

Guided Discussion: Counting

Easier problems will ask you how many numbers below 2020 satisfy a quantity. Don't think in terms of splitting this up

- *Sum of n numbers*

$$\frac{n(n+1)}{2}$$

- *Combinatorics (Identities), Name some!*

Guided Discussion: Complex Numbers

- *Euler's Formula,* $e^{i\pi} = -1$
- $\operatorname{cis}\theta = \cos\theta + i\sin\theta$
- *This inscribes regular polygons in the unit circle on the complex plane*
- *Conjugation!*
- *If a polynomial* $f(x)$ *has real coefficients but complex solutions, it's complex solutions come in conjugates.*

Guided Discussion: Geometry

Always start just by getting a feel for a problem. Notice things if they give you a diagram, if they don't, notice that they don't.

**Typically problems will be broken up in an order for Geometry problems, making them a little easier if you do know where you're going.*

- *Power of a Point*
- *Ptolemy's Theorem (Cyclic Quadrilaterals)*
- *Brahmagupta's Formula*

$$A_{CQ} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- *Don't be afraid to try out anything*
- *Identities*