

Topology

Walter Johnson Maths Honors Society

Requirements

In order to receive credit for this independent research project and be in good standing with the Walter Johnson Maths Honor Society, you must write a paper which does each of the following:

- Describe, briefly, what the field of Topology is.
- Describe, briefly, what Knot Theory is.
- Define a Seifert Surface and explain the Seifert Theorem. Construct the Seifert Surface for the Trefoil Knot.
- Define, describe and give examples of topological manifolds with the same and different intrinsic and extrinsic dimension, from 1 to 4 dimensions. State the dimensions of the Trefoil knot.
- Define and describe what the genus, orientability, and boundary of a topological manifold is, and provide examples of each.
- Describe, pictorially, what a connected sum of 2-manifolds is and what the remaining manifold is, as well as how rubber sheet squares can be used to describe them.
- Complete and describe a solution to the problem.

On average, this assignment will take about 3 hours to research and write up. You may **not** work in a group or collaborate with others.

You will be assigned to groups of 6 people, each of which has completed a different independent research project. At the end of the year, you will present your findings to your group, and listen as your peers present their findings. Your presentation must briefly discuss every subject required in your paper along with 1 of the problems you solved, of your choice.

Resources

You are provided with various resources to complete your research. You are welcome to use resources that are not given here.

Sets and Topological Spaces

A **Set** is simply a collection of objects, most times numbers, but sometimes pairs or other sets, with no other structure or order to describe them. These are some examples of sets:

$$S = \{0, 1, 4, 3, \pi, 8, 4.5\}$$

$$J = \{(0, 1), (0, 2), (1, 0), (1, 1)\}$$

A **Topological Space** is a set with some added structure, including a very vaguely defined “closeness”, similar to the idea of ordering $1 < 2$ and $2 < 3$ and thus $1 < 3$, etc.

Knot Theory

- Problems in Knot Theory by Louis H. Kauffman at University of Illinois Chicago.
- An Introduction to the Theory of Knots by Giovanni De Santi at Stanford.

Seifert Surface

- Eindhoven University of Technology article on visualizing Seifert Surfaces.
- Visual examples and explanation of Seifert Surfaces for different knots.
- Introduction to Seifert Surfaces and Their Properties by Mengtong Wang. Introduction with explanation of Seifert Algorithm.

Topological Manifolds

- Lecture notes from University of Bonn course in Topological Manifolds.
- An Introduction to Topology by E.C. Zeeman. Multiple examples of intrinsically 2 dimensional manifolds which are extrinsically 3 or 4 dimensional.
- Classification of Surfaces by Yujie Zhang at University of Chicago.

Genus, Orientability, and Boundaries

- Mathematical Encyclopedia article on the Genus of connected surfaces.
- OpenLearn article on Orientability of Surfaces.
- Northeastern article on Genus and Orientability. Start at page 9.

Problem

Determine the intrinsic and extrinsic dimension of the following objects, along with their genus and orientability. If they have a boundary, describe the boundary.

