Geometry Problem Set #2

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Problems are ordered from easiest to hardest difficulty, with high probability. None of the problems require a calculator, calculus, analysis, or an abacus. If you have any questions, just ask!

1

What is the area of the region enclosed by the graph of the equation

$$x^2 + y^2 = |x| + |y|$$

2

Point O is the center of the circle circumscribed about isosceles $\triangle ABC$. If $AB = AC = 7$, and $BC = 2$, find AO.

3

A polyhedron has 12 vertices. At 6 of them, 4 edges come together; at the other 6, 3 edges come together. Compute the number of faces that the polyhedron has.

4

A point P has coordinates $P(2009, 2010)$. Let d be the distance from P to the line $\frac{1}{4}x + \frac{1}{3}y = 1$. Determine the value of $3(2009) + 4(2010) - 5d$

5

An *ARMLbar* is a 7×7 grid of unit squares with the center unit square removed. A *portion* of an *ARMLbar* is a square section of the bar, cut along the gridlines of the original bar. Compute the number of different ways there are to cut a single portion from an *ARMLbar*.

6

Vertex E of equilateral $\triangle ABE$ is in the interior of the unit square $ABCD$. Let R be the region consisting of all points inside $ABCD$ and outside $\triangle ABE$ whose distance from AD is between $\frac{1}{3}$ and $\frac{2}{3}$. What is the area of R?

7

Let C_1 and C_2 be circles defined by

$$(x-10)^2 + y^2 = 36$$

and

$$(x+15)^2 + y^2 = 81$$

respectively. What is the length of the shortest line segment $\overline{PQ}$ that is tangent to C_1 at P and to C_2 at Q?

8

Twelve tangent circles as shown all have their radius equal to 1. What is the length of the shortest path from point P to point Q that does not pass through the interior of any of the circles?

9

Let

$$S_1 = \{(x,y) | \log_{10}(1 + x^2 + y^2) \leq 1 + \log_{10}(x+y)\}$$

and

$$S_1 = \{(x,y) | \log_{10}(2 + x^2 + y^2) \leq 2 + \log_{10}(x+y)\}$$

What is the ratio of the area of S_2 to the area of S_1?

10

A circle passes through both trisection points of side $\overline{AB}$ of square $ABCD$ and intersects $\overline{BC}$ at points P and Q. Compute the greatest possible value of $\tan \angle PAQ$.

11

A rectangular box has a total surface area of 94 square inches. The sum of the lengths of all its edges is 48 inches. What is the sum of the lengths in inches of all of its interior diagonals?

12

Regular hexagon $ABCDEF$ and regular hexagon $GHIJKL$ both have side length 24. The hexagons overlap, so that G is on segment AB, B is on segment GH, K is on segment DE, and D is on segment JK. If $[GBCDKL] = \frac{1}{2}[ABCDEF]$, compute LF.

13

Let $ABCD$ be a square of side length 6, and let P, Q, R and S be points on the sides of this square as shown so that $\overline{PR} \parallel \overline{BC}$ and $\overline{QS} \parallel \overline{CD}$. If $[ARQ] = 13$, then determine $[ASP]$ (where $[ASP]$ denotes the area of triangle ASP)

14

Let ABC be a triangle. Let D be the midpoint of BC, let E be the midpoint of AD, and let F be the midpoint of BE. Let G be the point where the lines AB and CF intersect. What is the value $\frac{AG}{AB}$?

15

Let $TRIANGLE$ be an equilateral octagon with side length 10, and let α be the acute angle whose tangent is $\frac{3}{4}$. Given that the measures of the interior angles of $TRIANGLE$ alternate between $180° - \alpha$ and $90° + \alpha$, compute $[TRIANGLE]$.

16

Let $ABCD$ be a rectangle, and let E and F be points on segment AB such that $AE = EF = FB$. If CE intersects the line AD at P, and PF intersects BC at Q, determine the ratio of BQ to CQ.

17

Santa hits four houses which are along a circular path. Santa, however, to conserve time, takes a direct route from house A, to B to C to D.The distance between A and C and B and D are integers. If the distances between these homes that Santa takes are 10, 12, 11, and 13, in order, then how many possible quadrilaterals could Santa be creating?

18

For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?