Warm Up!

Triangle ABC *is a right triangle with* $AB = AC = 3$. *Let* M *be the midpoint of hypotenuse* $\overline{BC}$. *Points* I *and* E *lie on sides* $\overline{AC}$ *and* $\overline{AB}$ *respectively, so that* $\overline{AI} \geq \overline{AE}$ *and* $AIME$ *is a cyclic quadrilateral. The area of* EMI *is 2, the length* $\overline{CI} = \frac{a \pm \sqrt{b}}{c}$. *Find* $a + b + c$

Let S *be the set of positive integers* n *for which* ${}^{1}/_{n}$ *has the repeating decimal expansion* $0.\overline{ab} = 0.ababab\ldots$ *with* a *and* b *distinct. What is the sum of elements of* S*?*

Warm Up!

Triangle ABC is a right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$ respectively, so that $\overline{AI} \geq \overline{AE}$ and $AIME$ is a cyclic quadrilateral. The area of EMI is 2, the length $\overline{CI} =$ $\frac{a \pm \sqrt{b}}{c}$. Find $a + b + c$

The answer will be drawn

Warm Up!

Let S be the set of positive integers n for which $^1/_n$ has the repeating decimal expansion $0.\overline{ab} = 0.ababab\ldots$ with a and b distinct. What is the sum of elements of S?

We see that $\frac{100}{n} = ab.\overline{ab}$ and if we subtract our original expression from this we get $\frac{99}{n} = ab$.

The factors of 99 are $1, 3, 9, 11, 33, 99$, but only $11, 33$ and 99 give us distinct values for a and b

$$11 + 33 + 99 = 143$$

Warm Up!

Let $ABCDE$ be a pentagon inscribed in a circle such that $AB = CD = 3$ and $BC = DE = 10$, and $AE = 14$. The sum of the lengths of the diagonals of $ABCDE$ is equal to ${}^{m}/_{n}$ where m and n are relatively prime positive integers.

What is $m + n$?

Given x and y are distinct nonzero real numbers such that

$$x + \frac{2}{x} = y + \frac{2}{y}$$

What is xy?

Warm Up!

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The answer will be drawn

Warm Up!

Given x and y are distinct nonzero real numbers such that

$$x + \frac{2}{x} = y + \frac{2}{y}$$

What is xy?

Multiplying both sides by xy we find

$$x^2y + 2y = xy^2 + 2x$$

$$(x - y)(xy - 2) = 0$$

$$xy = 2$$

Or we swap:

$$x - y = \frac{2}{y} - \frac{2}{x}$$

$$x - y = \frac{2x - 2y}{xy}$$

$$xy = \frac{2(x - y)}{x - y}$$

County Competition: Review Week 1

Identities

Problems

Walter Johnson Math Team

Warm Up!

Find the sum of the two smallest 3-digit base-10 numbers that are not only palindromic but stay so when converted to base-7

Trying out the smallest palindromes we find

$$121_{10} = 232_7$$

And

$$171_{10} = 333_7$$

$$121 + 171 = 292$$

Warm Up!

Find the exact value of $\sin(x)$

given

$$\cos x = \frac{1}{\cot x}$$

Just manipulating our given we find

$$\cos x = \tan x = \frac{\sin x}{\cos x}$$

$$\sin x = \cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x + \sin x - 1 = 0$$

Warm Up!

Find the exact value of $\sin(x)$

given

$$\cos x = \frac{1}{\cot x}$$

$$\sin^2 x + \sin x - 1 = 0$$

Solving for this ϕ like quadratic we see

$$\sin x = \frac{-1 \pm \sqrt{5}}{2}$$

However $-1 - \sqrt{5}/2$ does not suffice the domain of

$\sin x$

Warm Up!

Find the exact value of $\sin(x)$

given

$$\cos x = \frac{1}{\cot x}$$

$$\sin x = \frac{-1 + \sqrt{5}}{2}$$

Warm Up!

Consider circle C with equation

$$(x-4)^2+(y+5)^2=36$$

Circles A, B, C each have equal radius and there are three points of tangency among the circles, each intersecting once. What is the slope of the line connecting the centers of circles A and C if the tangent where B and C intersect is located at $(4, 1)$?

Recognize that circles B and C are directly on top of each other, making the triangle formed by their centers an equilateral triangle with one side verticle.

With the $x, 2x, x\sqrt{3}$ triangles we know the absolute value of the slope is

$$\frac{1}{\sqrt{3}}$$

Warm Up!

What is the remainder when

$$3^1 + 3^2 + \cdots + 3^{100}$$

Is divided by 13?

Analyze with modular arithmetic.

$$3^1 \equiv 3 \ mod\ 13$$

$$3^2 \equiv 9 \ mod\ 13$$

$$3^3 \equiv 1 \ mod\ 13$$

As we see $1 + 3 + 9 \equiv 0 \ mod\ 13$, what matters is

what $100 \equiv 1 \ mod\ 13$

Warm Up!

What is the remainder when

$$3^1 + 3^2 + \cdots + 3^{100}$$

Is divided by 13?

As we see $1 + 3 + 9 \equiv 0 \; mod \; 13$, what matters is what $100 \equiv 1 \; mod \; 13$

$$3^1 + 3^2 + \cdots + 3^{100} \equiv 3^1 \equiv 3 \; mod \; 13$$

Our answer is

$$3$$