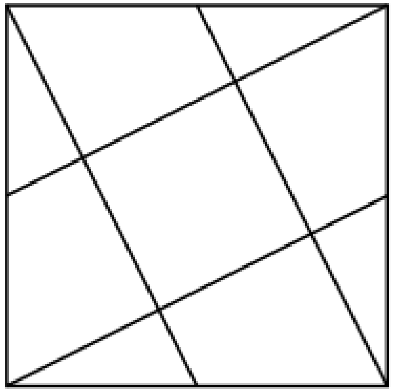
Warm Up!

The area of this larger square is 1. We join each vertex with the midpoint of an adjacent side. What is the area of the smaller square?



Given

$$f(x) = 1 - x + x^2 \cdots + x^{10}$$

We substitute $x = y + 1$ *to obtain the polynomial*

$$g(x) = a_0 + a_1 y + a_2 y^2 \cdots + a_{10} y^{10}$$

What is $a_0 + a_1 + \cdots + a_{10}$*?*

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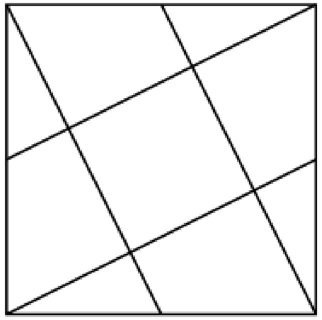


For the smaller square of side length x, we see that this forms right triangles with hypotenuse 1 and side lengths x and $2x$. Using Pythagorean theorem, we find

$$x = \frac{1}{\sqrt{5}}$$

And thus the area is

$$x^2 = \frac{1}{5}$$

Warm Up!

Given

$$f(x) = 1 - x + x^2 \cdots + x^{10}$$

We substitute $x = y + 1$ *to*

obtain the polynomial

$$g(x)$$
$$= a_0 + a_1 y + a_2 y^2 \cdots + a_{10} y^{10}$$

What is $a_0 + a_1 + \cdots + a_{10}$*?*

We can see that this sum equals g(1) which equals $f(2)$, and so we find

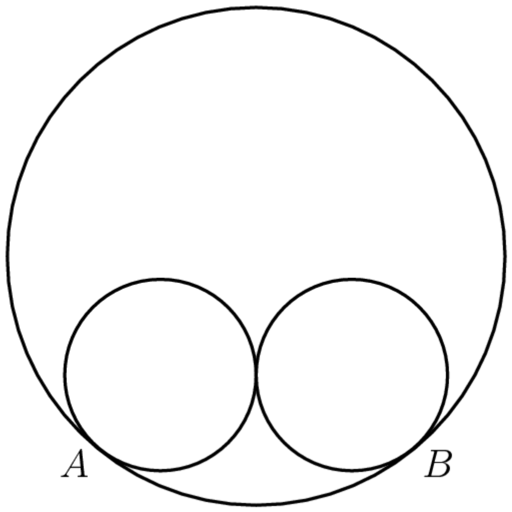
$$f(2) = 1 - 2 + 4 - 8 \cdots + 1028$$

$$f(2) = 683$$

Warm Up!

Set $S = \{1,2,3,4,5,6,7,8,9\}$. *For each subset* A *of* S, *John writes on the whiteboard the sum of the elements in* A. *Marsha then writes the sum of those sums. What number does Marsha write?*

Two circles of radius 5 *are externally tangent and internally tangent to a circle of radius 13 at points* A, B. *The distance* $AB = {}^{m}/_{n}$. *Find* $m + n$



Warm Up!

Set $S = \{1,2,3,4,5,6,7,8,9\}$. For each subset A of S, Kamala writes on the whiteboard the sum of the elements in A. Marsha then writes the sum of those sums. What number does Marsha write?

Each element is included in 1 subset with 9 elements, each element is included in 1 subset with only 1 element, and is included in 8 subsets with 2 elements, and is included in 8 subsets with 8 elements. Each element is included in $\binom{8}{n}$ sets with n elements.

Warm Up!

Set $S = \{1,2,3,4,5,6,7,8,9\}$*. For each subset* A *of* S*, Kamala writes on the whiteboard the sum of the elements in* A*. Marsha then writes the sum of those sums. What number does Marsha write?*

Each element is included in $\binom{8}{n}$ sets with n elements.

By our Pascal's triangle, we know that this sum is

$$2^8 = \sum_{k=0}^{n} \binom{8}{k}$$

And as this is for each element, our final sum is

$$(1 + 2 + \cdots + 9)(2^8)$$

Warm Up!

Set $S = \{1,2,3,4,5,6,7,8,9\}$*. For each subset* A *of* S*, Kamala writes on the whiteboard the sum of the elements in* A*. Marsha then writes the sum of those sums. What number does Marsha write?*

$$(1 + 2 + \cdots + 9)(2^8)$$

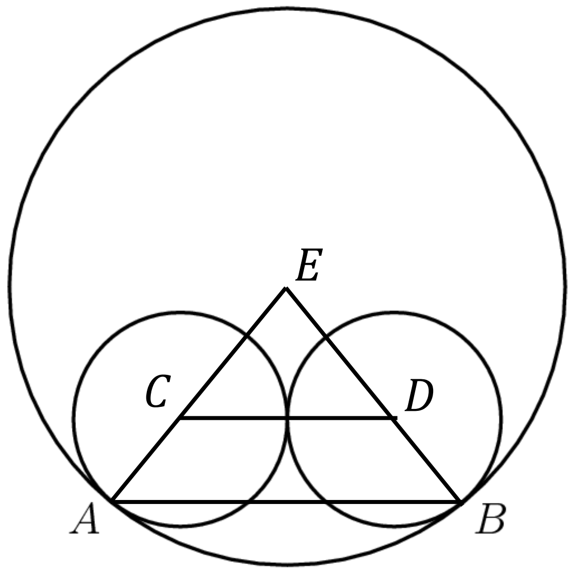
$$45 * 2^8 = 11520$$

Warm Up!

Two circles of radius 5 *are externally tangent and internally tangent to a circle of radius 13 at points* A, B*. The distance* $AB = {}^{m}/_{n}$*. Find* $m+n$

We draw two similar triangles, seeing radii $AE = BE = 13$, $EC = ED = 8$ and $CD = 10$, so we make the similar relation

$$\frac{AB}{AE} = \frac{AB}{13} = \frac{8}{10} = \frac{CD}{CE}$$



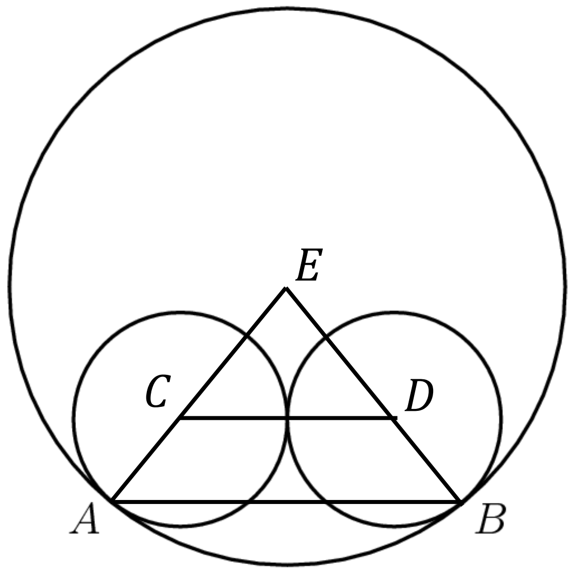
Warm Up!

Two circles of radius 5 *are externally tangent and internally tangent to a circle of radius 13 at points* A, B. *The distance* $AB = {}^{m}/_{n}$. *Find* $m + n$

$\frac{AB}{AE} = \frac{AB}{13} = \frac{8}{10} = \frac{CD}{CE}$, So we find $AB = 13 * 8/10 =$

$52/5 = m/n$

$52 + 5 = 57$



Warm Up!, AMC 12B #14

Let $ABCDE$ be a pentagon inscribed in a circle such that $AB = CD = 3$ and $BC = DE = 10$, and $AE = 14$. The sum of the lengths of the diagonals of $ABCDE$ is equal to ${}^{m}/_{n}$ where m and n are relatively prime positive integers. What is $m + n$?

Triangle ABC is a right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$ respectively, so that $\overline{AI} \geq \overline{AE}$ and $AIME$ is a cyclic quadrilateral. The area of EMI is 2, the length $\overline{CI} = \frac{a \pm \sqrt{b}}{c}$. Find $a + b + c$