Warm Up! UMD HS Math Competition

Let N *be the smallest integer so there is no perfect square* n^2 *such that*

$$N < n^2 < N + 100$$

What is the sum of the digits in N*?*

Let $S = \left\{\frac{1}{256}, \frac{1}{32}, \frac{1}{4}, 2, 16, 128, 1024\right\}$

How many real numbers can be written as the product of three distinct elements of S*?*

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Let N be the smallest integer so there is no perfect square n^2 such that

$$N < n^2 < N + 100$$

What is the sum of the digits in N?

Supposing x^2 is the greatest perfect square less than N. We see the next perfect square has to be at least 100 more,

$$(x+1)^2 - x^2 \leq 100$$

$$2x + 1 \leq 100$$

$$x \leq 99/2$$

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Let N be the smallest integer so there is no perfect square n^2 such that

$$N < n^2 < N + 100$$

What is the sum of the digits in N?

$$x \leq 99/2$$

We see that the first integer that works is $x = 50$

And so we see now, the first integer that satisfies N is

$$N = 50^2 = 2500$$

The sum of the digits is

$$2 + 5 + 0 + 0 = 7$$

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Let $S =$

$$\left\{\frac{1}{256}, \frac{1}{32}, \frac{1}{4}, 2,16,128,1024\right\}$$

How many real numbers can be written as the product of three distinct elements of S*?*

We see that this set is equal to

$$S = \{2^{-8}, 2^{-5}, 2^{-2}, 2^{1}, 2^{4}, 2^{7}, 2^{10}\}$$

And thus a product of three of them will be

$$x = 2^{a+b+c}$$

Where $a, b, c \in \{-8, -5, -2,1,4,7,10\}$

So now we must find all the possible distinct numbers producible as the sum of any of those numbers.

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Let $S =$

$$\left\{\frac{1}{256}, \frac{1}{32}, \frac{1}{4}, 2,16,128,1024\right\}$$

How many real numbers can be written as the product of three distinct elements of S*?*

$$a, b, c \in \{-8, -5, -2,1,4,7,10\}$$

Now we see that each element has a common difference of 3, and thus every number that can be created will be between the minimum and maximum with a difference of 3

$$-15, -15 + 3, \cdots 21 - 3,21$$

And so there are 13 such numbers.

Warm Up! UMD HS Math Competition

If

$$1000^{\log 3} + 1000^{\log 4} + 1000^{\log 5} = 1000^{\log x}$$

What is x?

For any positive integer n, let $f(n)$ be the number of 1's that appear in the base-2 representation of n. Find the following:

$$\sum_{n=1}^{1023} f(n)$$

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If

$$1000^{\log 3} + 1000^{\log 4} + 1000^{\log 5} = 1000^{\log x}$$

What is x?

We see that $1000 = 10^3$, so

$$1000^{\log 3} = 10^{3 \log 3} = 3^3$$

And so our expression becomes

$$3^3 + 4^3 + 5^3 = x^3$$

$$216 = x^3$$

$$x = 6$$

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For any positive integer n, let $f(n)$ be the number of 1's that appear in the base-2 representation of n. Find the following:

$$\sum_{n=1}^{1023} f(n)$$

There are two *distinct* ways of going about this problem. The first (the worse way), which is the way I did it, is to model a recursive function after writing some values in a table we can see clearly;

		$\sum f(n)$	$f(n)$
0	0	0	0
1	1	1	1
2	10	2	1
3	11	4	2
4	100	5	1

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For any positive integer n, let $f(n)$ be the number of 1's that appear in the base-2 representation of n. Find the following:

$$\sum_{n=1}^{1023} f(n)$$

		$\sum \boldsymbol{f(n)}$	$\boldsymbol{f(n)}$
0	000	0	0
1	001	1	1
2	010	2	1
3	011	4	2
4	100	5	1

$$\sum_{0}^{2^a-1} f(n) = 2\left(\sum_{0}^{2^{a-1}-1} f(n)\right) + 2$$

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For any positive integer n, let $f(n)$ be the number of 1's that appear in the base-2 representation of n.

Find the following:

$$\sum_{n=1}^{1023} f(n)$$

$$\sum_{0}^{2^a-1} f(n) = 2\left(\sum_{0}^{2^{a-1}-1} f(n)\right) + 2$$

$$\sum_{0}^{2^a-1} f(n) = a * 2^{a-1}$$

$$\sum_{0}^{2^{10}-1} f(n) = 10 * 2^9$$

Warm Up! *UMD HS Math Competition*

Given w is a whole number and

$$\log_3(\boldsymbol{cabin}) = w$$

Where $\boldsymbol{cabin}$ is a five digit number with distinct digits, what is $\boldsymbol{cabin}$?

What is

$$\tan(10°) * \tan(20°) * \cdots * \tan(80°)?$$

Warm Up! *UMD HS Math Competition*

Given w is a whole number and

$$\log_3(\boldsymbol{cabin}) = w$$

Where $\boldsymbol{cabin}$ is a five digit number with distinct digits, what is $\boldsymbol{cabin}$?

We can see that if w is a whole number, $\boldsymbol{cabin}$ must be in the form of 3^w. We then find the first 5 digit number in this form is $3^9 = 19683$ and since all of these digits are distinct, our number is

$$19683$$

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What is

$$\tan(10°) * \tan(20°) * \cdots * \tan(80°)$$

?

We remember that $\tan x = \frac{\sin x}{\cos x}$, and that

$$\sin(x) = \cos(90 - x)$$

And so we see that

$$\frac{\sin x}{\cos(90 - x)} = 1$$

Warm Up! *UMD HS Math Competition*

What is

$$\tan(10°) * \tan(20°) * \cdots * \tan(80°)$$

?

$$\frac{\sin x}{\cos(90 - x)} = 1$$

And so for each $\sin x$ in this list, we see there is a compliment $\cos(90 - x)$ in the denominator:

$$\frac{\sin 10° * \sin 20° * \cdots * \sin 80°}{\cos 10° * \cos 20° * \cdots \cos 80°} = 1$$