# *Warm Up!* UMD HS MC, #23, #9

*Find the number of integers between* $1$ *and* $100$*, inclusive, that can be written as the sum of non-negative integers such that all digits* $0-9$ *are used exactly once*

*(e.g.* $90 = 0 + 1 + 52 + 3 + 4 + 6 + 7 + 8 + 9$*)*

*Compute*

$$\frac{\log_{10} 8 * \log_{10} 16}{\log_{10} 2 * \log_{10} 4}$$

# *Warm Up!* UMD HS Math Competition, #23

*Find the number of integers between $1$ and $100$, inclusive, that can be written as the sum of non-negative integers such that all digits $0-9$ are used exactly once*

We see the smallest number in this form is $0+1+2+3+4+5+6+7+8+9=45$, and thus all other numbers representable this way are greater than $45$.

Now we also see, the only way to increase this value is to take certain digits into the 10s place.

# *Warm Up!* *UMD HS Math Competition, #23*

*Find the number of integers between $1$ and $100$, inclusive, that can be written as the sum of non-negative integers such that all digits $0 - 9$ are used exactly once*

Now we can see that if we take any digit, $n$, to the 10s place, it replaces $n$ with $n0$.

We see the difference between the two is $10n - n = 9n$. Because the smallest value is $45$, we know the rest of the integers are going to be multiples of $9$. This gives us the possible numbers

$$45, 54, 63, 72, 81, 90, 99$$

Thus, there are 7 such integers

# *Warm Up!* UMD HS Math Competition, #9

*Compute*

$$\frac{\log_{10} 8 * \log_{10} 16}{\log_{10} 2 * \log_{10} 4}$$

We see all of these can be expressed as $\log_{10}(2^n)$, and via the rules of logarithms

$$\log_{10}(2^n) = n \log_{10} 2$$

And thus our expression becomes

$$\frac{3\log_{10}(2) * 4\log_{10}(2)}{\log_{10}(2) * 2\log_{10}(2)} = \frac{12}{2} = 6$$

# *Warm Up!* UMD HS MC, #20, #14

*Suppose $f(x)$ is a function that satisfies*

$$f(x) + 5f\left({}^{1}/{}_{x}\right) = 3 + x$$

*For all non-zero real numbers $x$.*

*What is $f(4)$?*

*Let $p$ be the smallest prime number with $2019$ digits.*

*What is the remainder when $p^2$ is divided by $12$?*

# *Warm Up!* UMD HS Math Competition, #20

*Suppose $f(x)$ is a function that satisfies*

$$f(x) + 5f\left({}^{1}/_{x}\right) = 3 + x$$

*For all non-zero real numbers $x$.*

*What is $f(4)$?*

The solution is almost completely algebraic. We see

$$f(4) + 5f\left({}^{1}/_{4}\right) = 3 + 4 = 7$$

And

$$f\left({}^{1}/_{4}\right) + 5f(4) = 3 + {}^{1}/_{4}$$

And so we can solve for $f({}^{1}/_{4})$ and plug this into our previous equation

# *Warm Up!* *UMD HS Math Competition, #20*

*Suppose $f(x)$ is a function that satisfies*

$$f(x) + 5f\left({}^{1}/_{x}\right) = 3 + x$$

*For all non-zero real numbers $x$.*

*What is $f(4)$?*

And so we can solve for $f({}^{1}/_{4})$ and plug this into our previous equation

$$f\left({}^{1}/_{4}\right) = 3 + {}^{1}/_{4} - 5f(4)$$

And

$$f(4) + 5\left(3 + {}^{1}/_{4} - 5f(4)\right) = 7$$

$$-24f(4) + 15 + {}^{5}/_{4} = 7$$

# *Warm Up!* UMD HS Math Competition, #20

*Suppose $f(x)$ is a function that satisfies*

$$f(x) + 5f\left({}^{1}/_{x}\right) = 3 + x$$

*For all non-zero real numbers $x$.*

*What is $f(4)$?*

$$-24f(4) + 15 + {}^{5}/_{4} = 7$$

$$-24f(4) = 7 - 15 - {}^{5}/_{4}$$

$$f(4) = \frac{37}{96}$$

# *Warm Up!* *UMD HS Math Competition, #14*

*Let $p$ be the smallest prime number with $2019$ digits.*

*What is the remainder when $p^2$ is divided by $12$?*

We know the remainder of $p/12$ must be either 1, 5, 7 or 11, because if it wasn't any of those, it wouldn't be a prime number as it would be a multiple of either 2 or 3.

Thus we can examine this problem in $mod\ 12$ then.

# *Warm Up!* UMD HS Math Competition, #14

*Let $p$ be the smallest prime number with $2019$ digits.*

*What is the remainder when $p^2$ is divided by $12$?*

We see $1^2 \equiv 1 \, mod \, 12$, $5^2 \equiv 25 \equiv 1 \, mod \, 12$,

$7^2 \equiv 49 \equiv 1 \, mod \, 12$, $11^2 \equiv 121 \equiv 1 \, mod \, 12$

And so the remainder when $p^2$ is divided by 12, the remainder will be

$$1$$

# *Warm Up!* UMD HS MC, #16, #12

*Let $S$ be the set of integers $n$ with*

$1 \leq n \leq 2019$ *such that* $1$ *does not*

*occur in the decimal expansion of $n$.*

*How many integers are in $S$?*

*How many real numbers satisfy*

$$\log_{10}(x) = \sin(x)$$

*Where $x$ is in radians?*

# *Warm Up!* *UMD HS Math Competition #16*

*Let $S$ be the set of integers $n$*

*with*

*$1 \leq n \leq 2019$ such that $1$ does*

*not occur in the decimal*

*expansion of $n$.*

*How many integers are in $S$?*

At first glance, we see all numbers $1000 \leq n \leq 1999$ will not satisfy, and thus we focus on which numbers are counted between $1 \leq n \leq 999$. We see for these numbers, we can permutate on the digits, where the digits just can't be 1, and thus there are 9 possibly digits for the 9 decimal places, excluding 0, and thus we get $9^3 - 1 = 729 - 1 = 729$ for all numbers in that range.

# *Warm Up!* *UMD HS Math Competition #16*

*Let $S$ be the set of integers $n$ with $1 \leq n \leq 2019$ such that 1 does not occur in the decimal expansion of $n$.*

*How many integers are in $S$?*

Now, we count the numbers greater than $2000$ which satisfy. We see $2000, 2002, 2003, \cdots 2019$ satisfy and the rest don't, and this is 9 numbers. Our final sum is

$$728 + 9 = 737$$

# *Warm Up!* *UMD HS Math Competition #12*

*How many real numbers satisfy*

$$\log_{10}(x) = \sin(x)$$

*Where $x$ is in radians?*

The maximum value of the $\sin x$ function is 1, and thus the only points of intersection will be between $0$ and when $\log_{10} x = 1$, which is satisfied when $x = 10$.

Thus we find it intersects 3 times