Guided Discussion: Non-Euclidean Geometry

Slide Components

Problems

Walter Johnson Math Team

Guided Discussion: Non-Euclidean Geometry

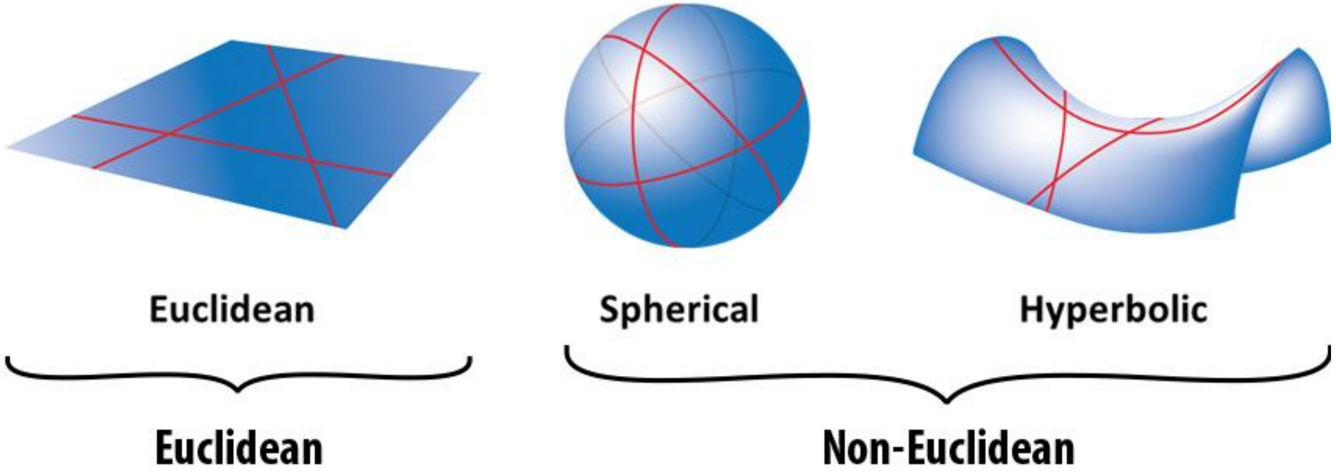
*In **Euclidean Geometry**, a triangle has 180 degrees.*

*In **Hyperbolic Geometry**, a triangle has less than 180 degrees.*

*In **Elliptic Geometry**, a triangle has more than 180 degrees.*

Why is it called Non-Euclidean Geometry?

Because it rejects Euclid's 5th postulate (The parallel postulate)



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*In **Euclidean Geometry**, a triangle has 180 degrees.*

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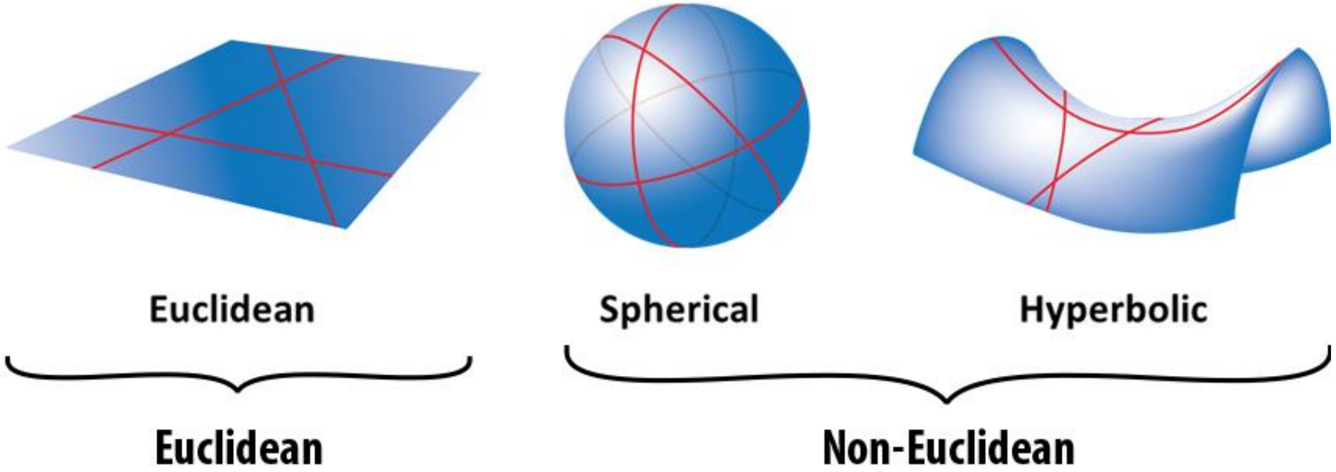
There are three spaces of geometry

- *Euclidean*
  - *No curvature, just strait lines*
- *Hyperbolic*
  - *Negative curvature*
- *Elliptic*
  - *Positive curvature*



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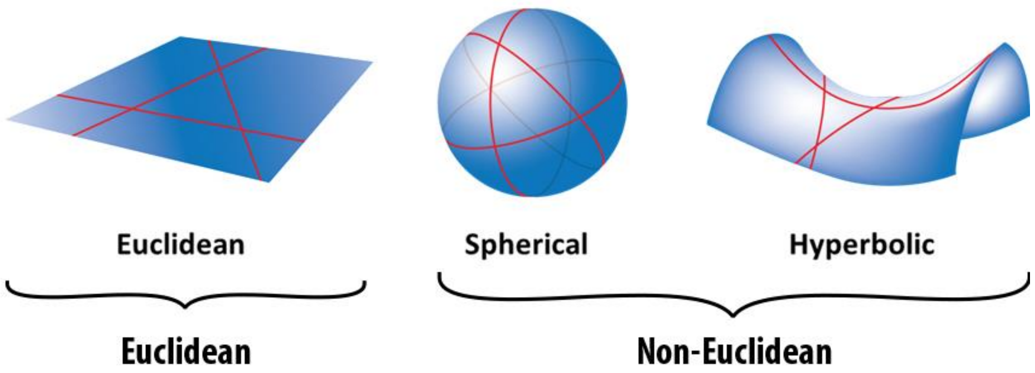

There are three spaces of geometry

- *Euclidean*
  - *For every point not on a given line, there exists exactly one line crossing that point parallel to the given line.*
- *Hyperbolic*
  - *For every point not on a given line, there exists at least 2 lines parallel to the given line*
- *Elliptic*
  - *For every point not on a given line, there is no parallel line crossing it.*

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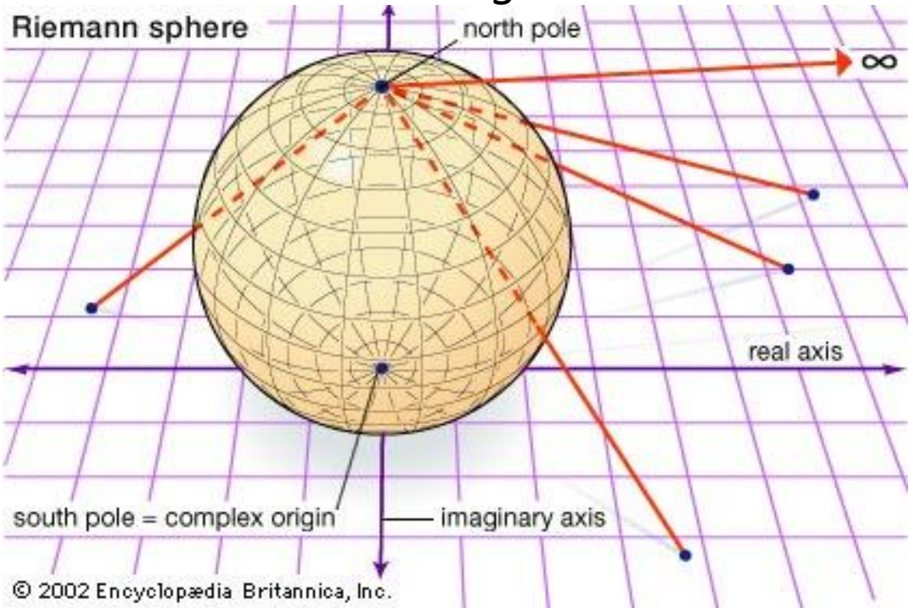
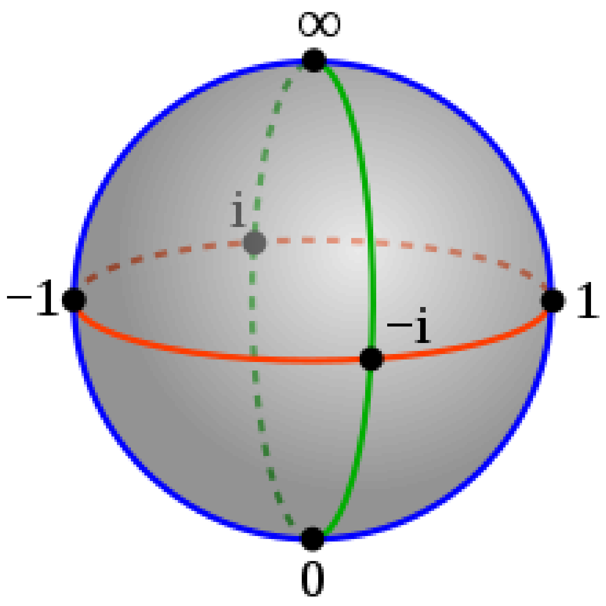


In Elliptic Geometry, we analyze the Riemann Sphere, which maps (almost) every point on a sphere to the complex plane with rays extending through the sphere. The sphere defines the "Extended Complex Numbers":

$$\mathbb{C} \text{ and } \infty = \mathbb{C}_\infty$$



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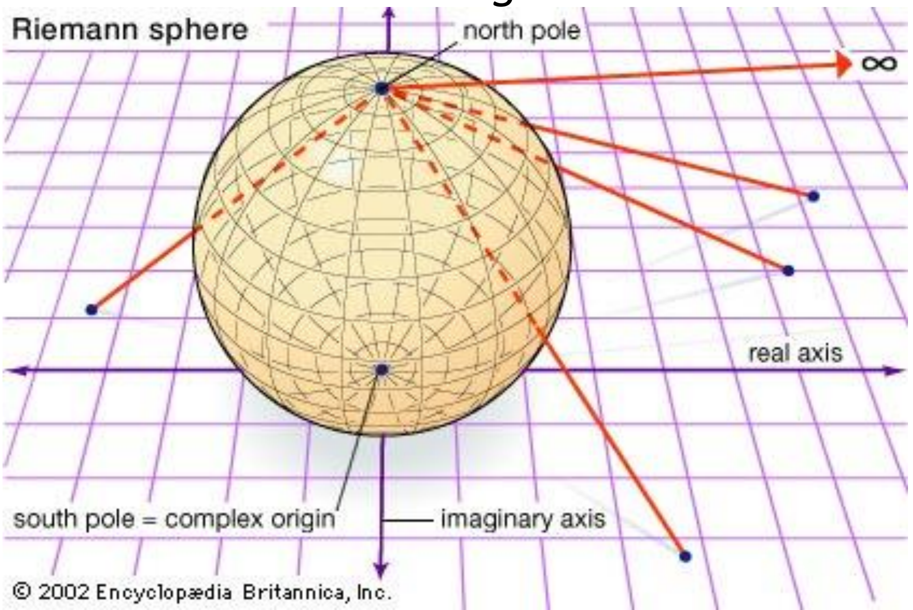
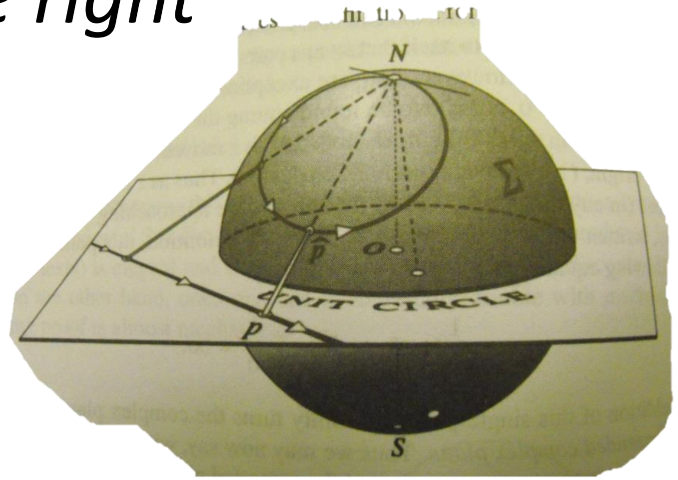
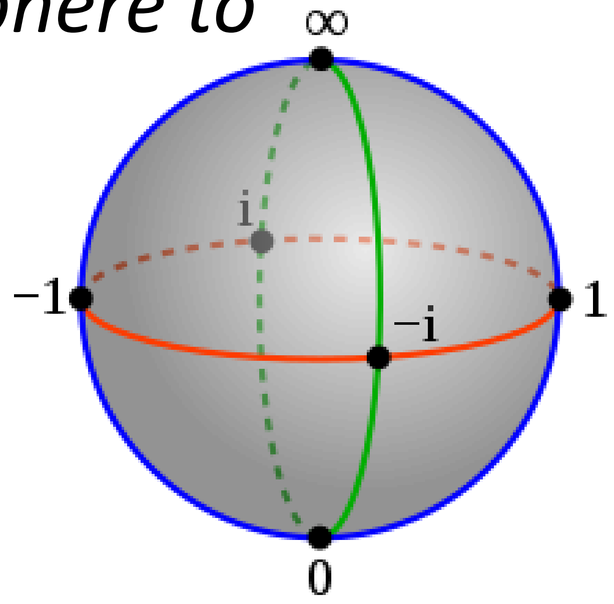


We see that where we place the Reimann sphere changes the radius of the circle traced out by the projection through the center of the sphere.

Placing it directly halfway through the plane gives us the sphere to the right





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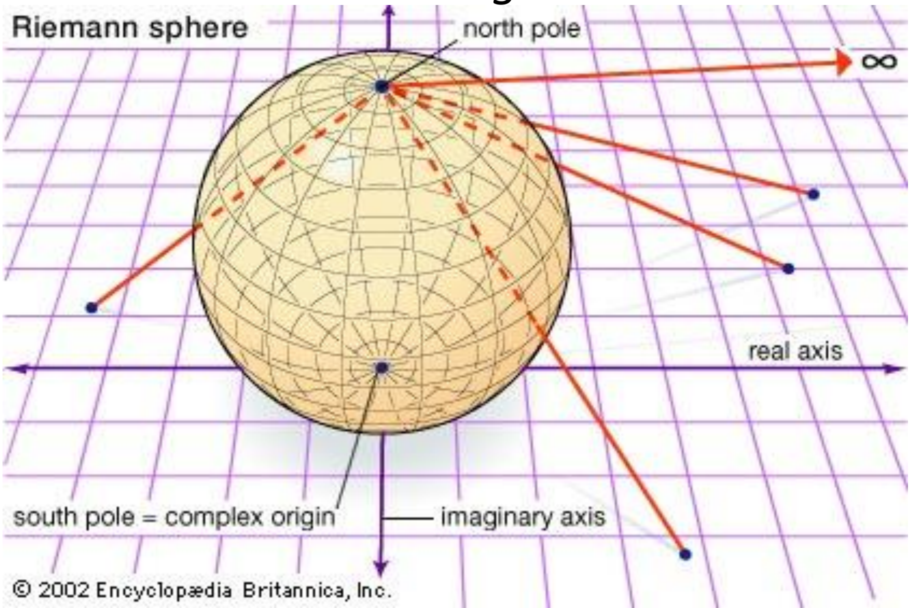
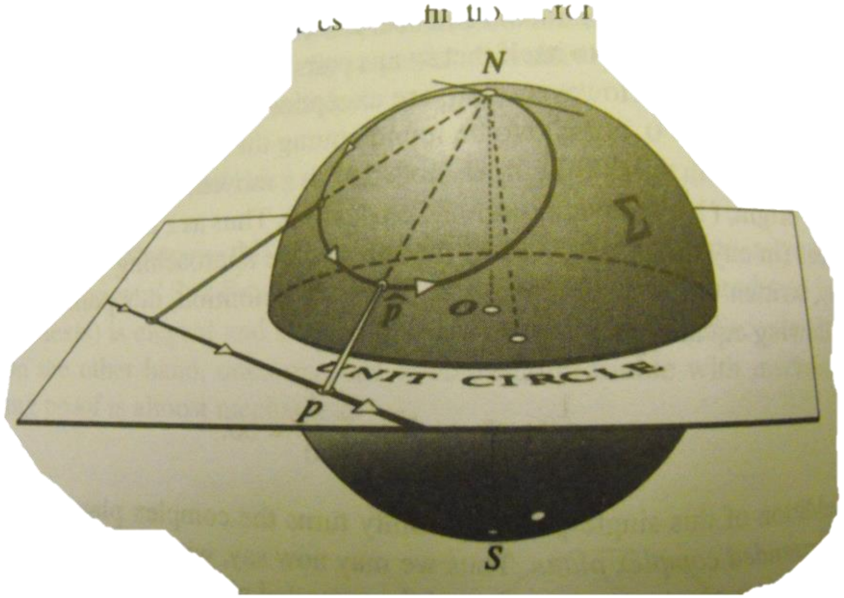
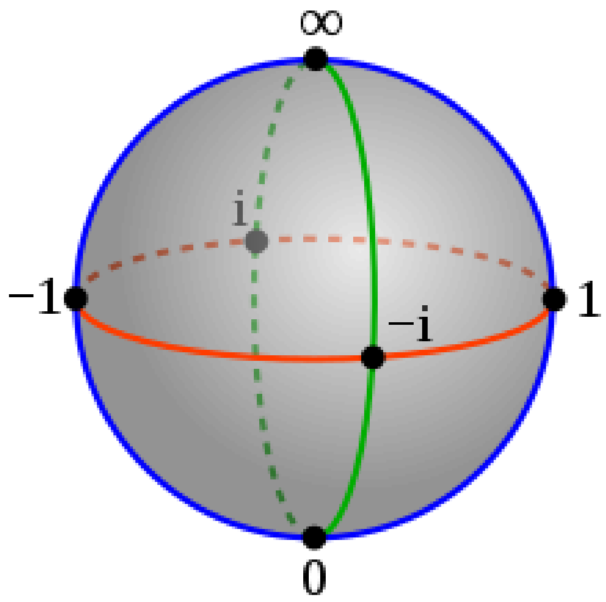
We also see that tracing out a circle on the sphere corresponds to a line on the plane.







Guided Discussion: Intro to Topology/Knot Theory

Slide Components

Problems

Walter Johnson Math Team

Guided Discussion: *Topology*

Topology *is the branch of mathematics pertaining to objects which can be continuously deformed into one another. "Squishy Geometry"*

Mathematical Knots are more rigorously defined, as I'm sure you can imagine, but we don't have to get into that

Unfortunately, deep analysis with Topology requires intense set theory, analysis and an undergraduate textbook, but there's still cool stuff to learn in 40 minutes!

Guided Discussion: Topology

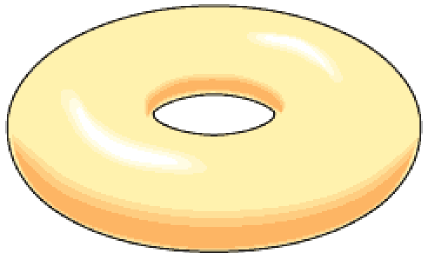
Topology *is the branch of mathematics pertaining to objects which can be continuously deformed into one another. "Squishy Geometry"*

*****Geometry cares about measurement. Topology does not!***

What are continuous deformations?

Well, they are deformations which take an object and stretch, bend, twist, or shrink it to become another.

In topology we do not consider measurements which are considered in Geometry.



doughnut

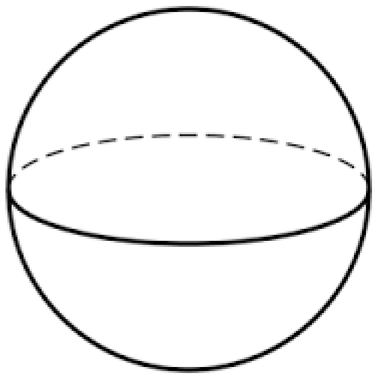
Guided Discussion: *Topology*

***Topology** is the branch of mathematics pertaining to objects which can be continuously deformed into one another. “Squishy Geometry”*

*****Geometry cares about measurement. Topology does not!***

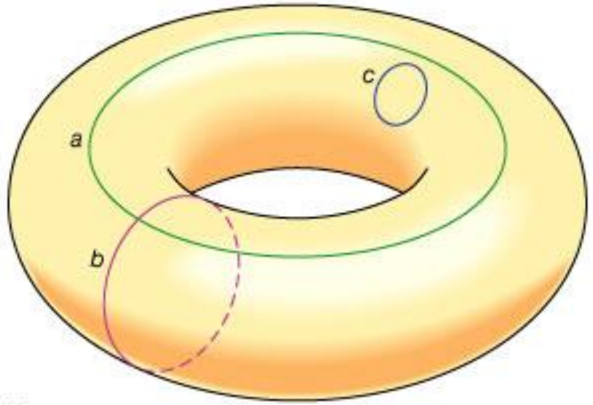
*****Look at this torus for a second. Can we move each of these escribed circles to match each other?***

If we escribed closed loops on the sphere, could we move them along the surface of the sphere to match each other?



How about the torus?

How many different distinct loops are there in each of these figures?



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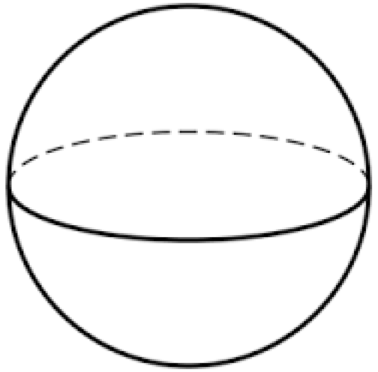
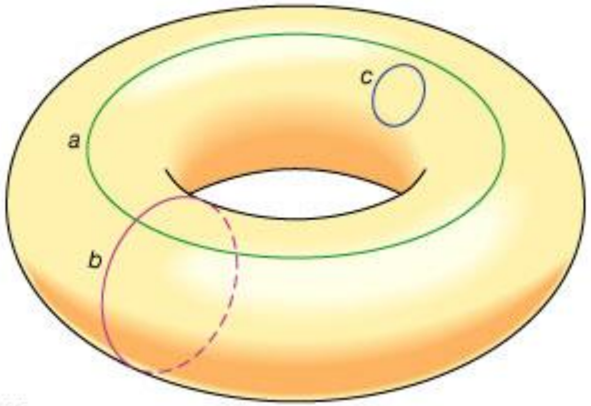
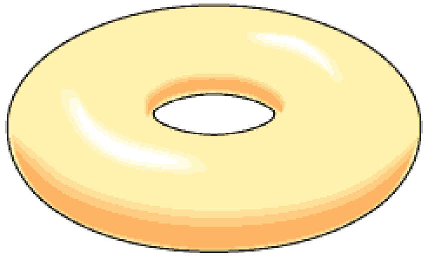
*****Look at this torus for a second. Can we move each of these escribed circles to match each other?***

On the sphere, we can see that there is only one such circle, as any circle escribed can map to another.

How about for the torus?







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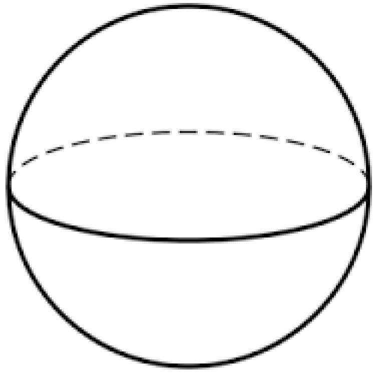
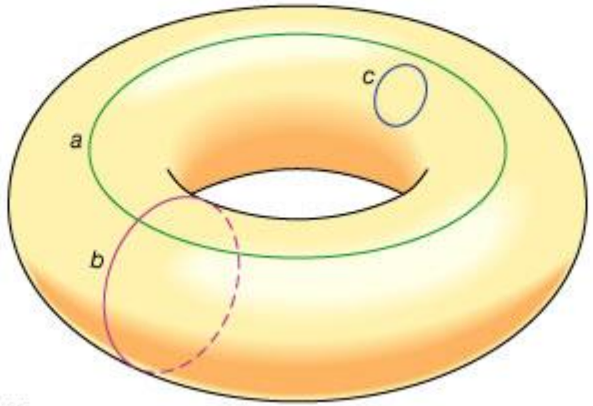
*****Look at this torus for a second. Can we move each of these escribed circles to match each other?***

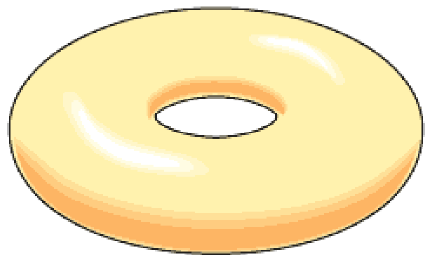
For the torus, we see that there are 3 distinct types of closed loops on it that cannot map to each other.

Can these loops be shrunk on the surface of the torus to come to a single point?

Do these loops contain points?







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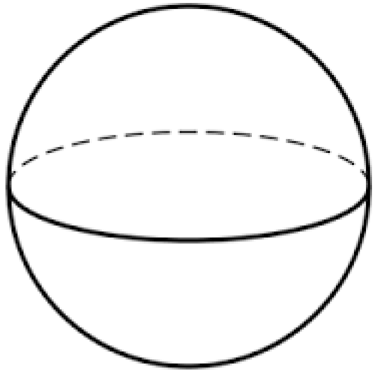
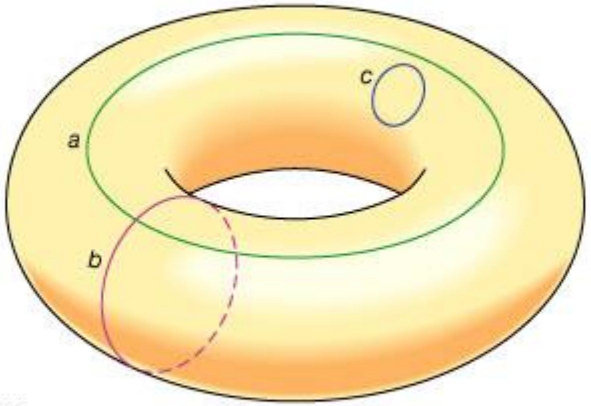
*****Look at this torus for a second. Can we move each of these escribed circles to match each other?***

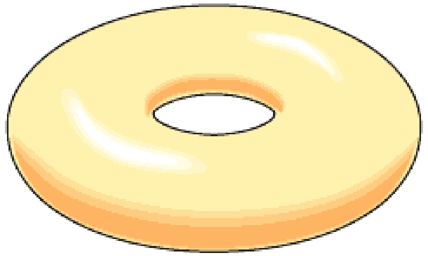
Can these loops be shrunk on the surface of the torus to come to a single point?

Only one of them can (the blue one)

But on the sphere, all of them can (the only one)







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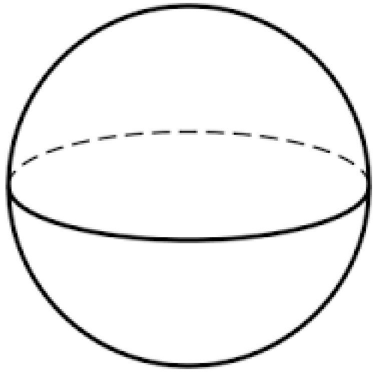
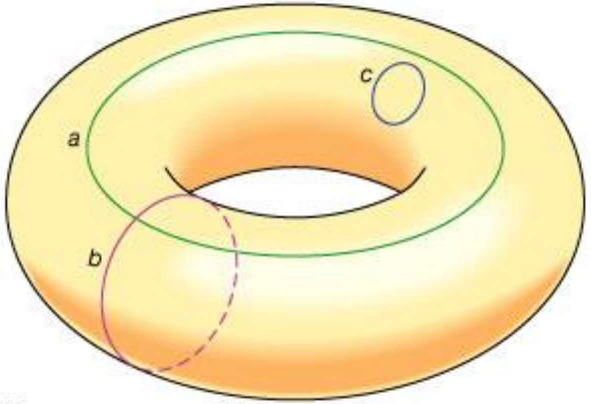
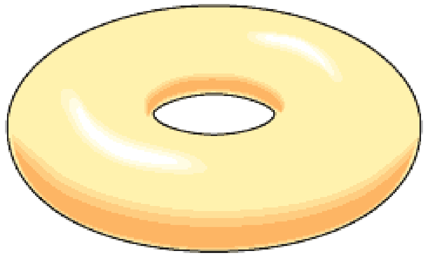
*****Look at this torus for a second. Can we move each of these escribed circles to match each other?***

Wondering why there’s a torus turning into a coffee cup?

Because using this analysis on topological objects shows that a torus and a coffee cup are topologically equivalent.







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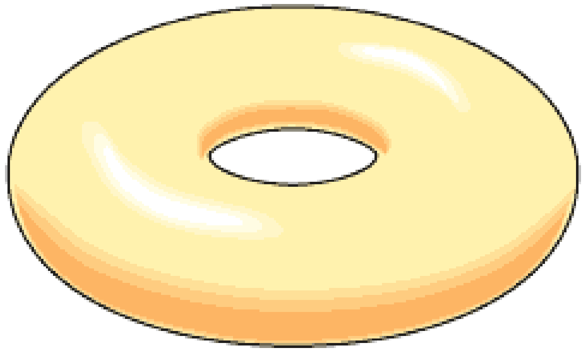
*****Look at this torus for a second. Can we move each of these enscribed circles to match each other?***

Because using this analysis on topological objects shows that a torus and a coffee cup are topologically equivalent.

How many closed loops can we have on a torus? On a coffee cup?

What are their properties?



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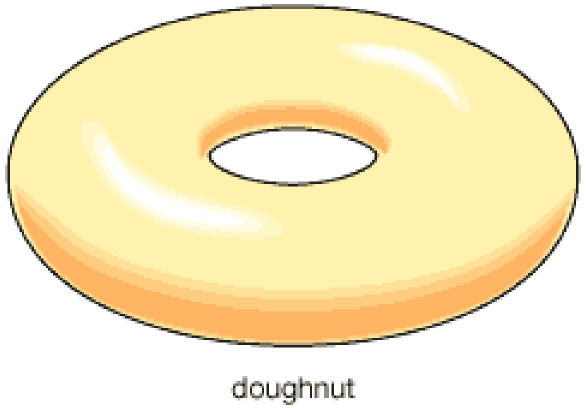
What are their properties?

Well, each has 3 different kinds of loops, 2 occurring with no ability to close to a point, and another occurring that can close to a point.



Guided Discussion: Topology

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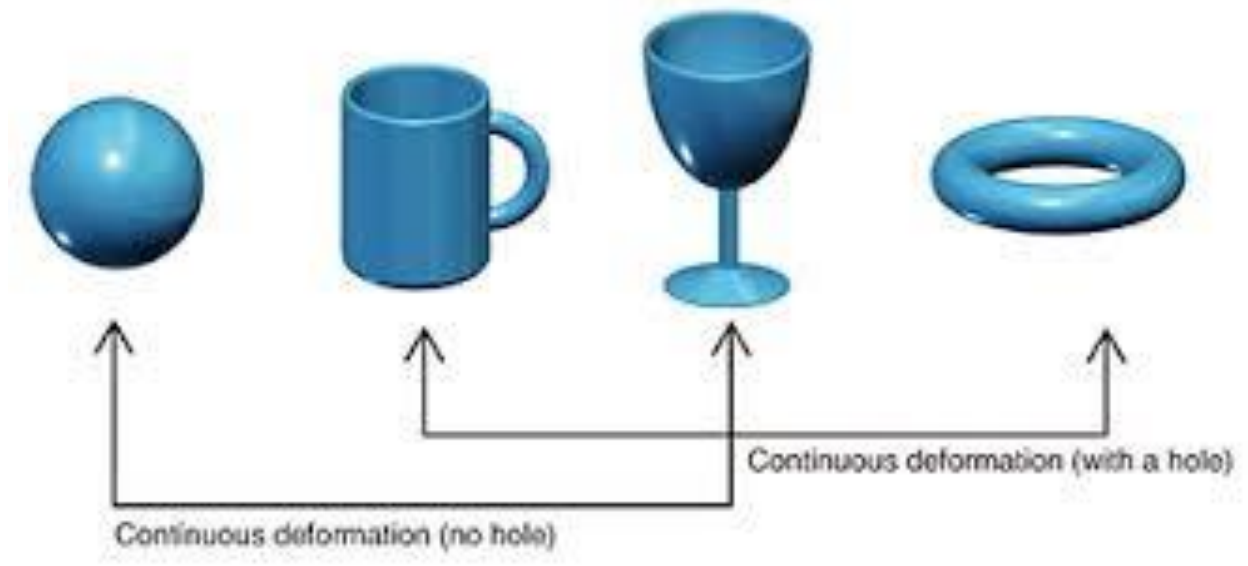
We see that in basic topology, we can really just analyze the amount of holes in things, and that the torus and the coffee cup only have one hole each, making them ***homeomorphic****.*



Guided Discussion: Topology

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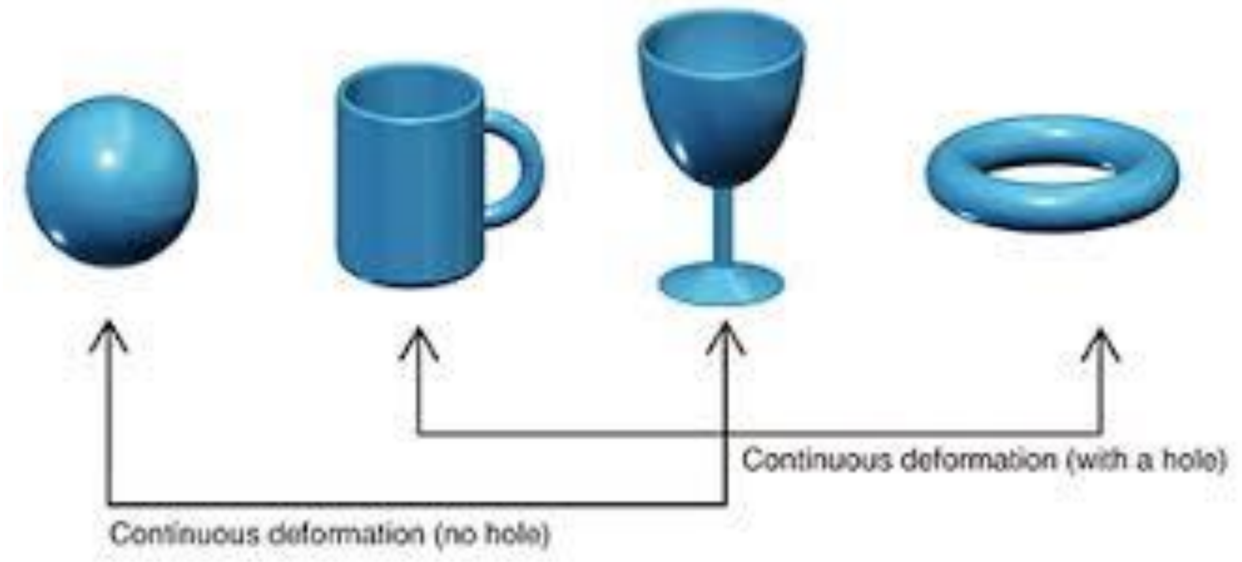
*We see that in basic topology, we can really just analyze the amount of holes in things, and that the torus and the coffee cup only have one hole each, making them **homeomorphic**. Along with other fields of mathematics, **Invariants** are a key tool to distinguishing topological objects.*



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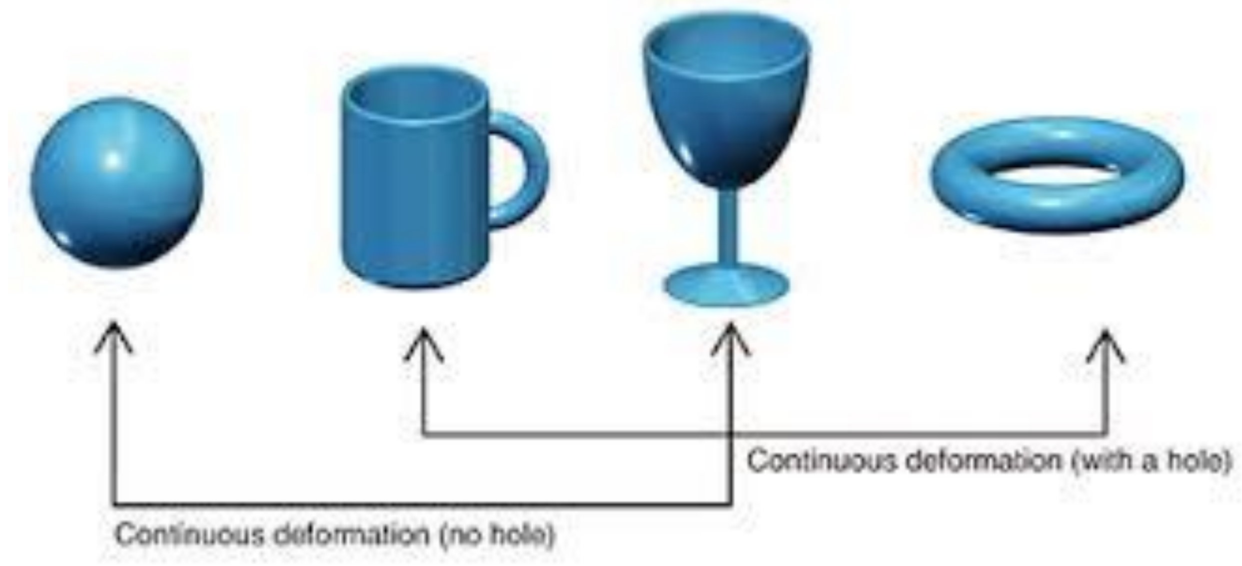
In actual topology, we would give these surfaces their own distinct polynomials, called "Alexander Polynomials," which would allow us to analyze them and higher dimensional objects more thoroughly.



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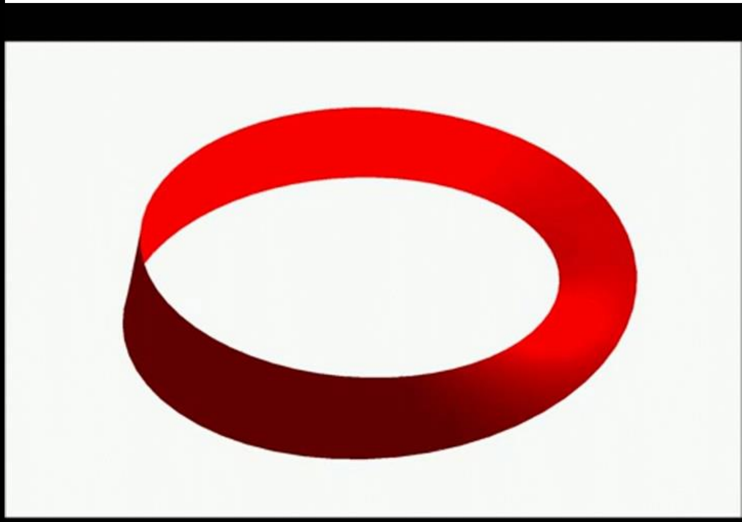
You've probably seen the mobius strip before, but in basic topology, how cutting through the center of such and object changes the object itself is studied. What happens when you cut a Mobius Strip down the middle?



Guided Discussion:
Knot Theory

Topology *is the branch of mathematics pertaining to objects which can be continuously deformed into one another. "Squishy Geometry"*

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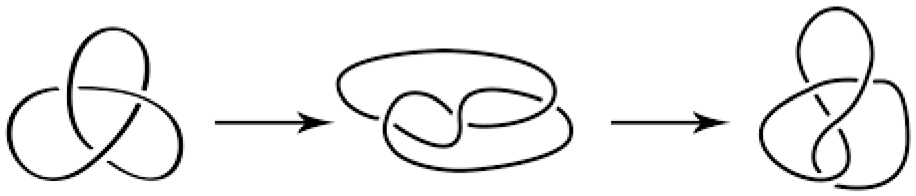
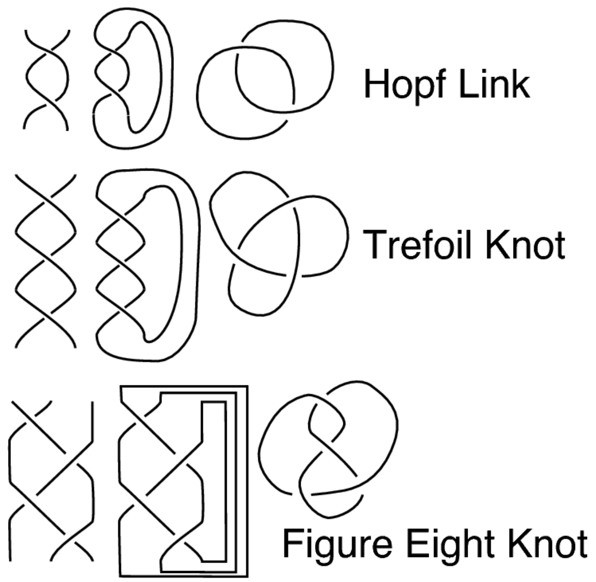
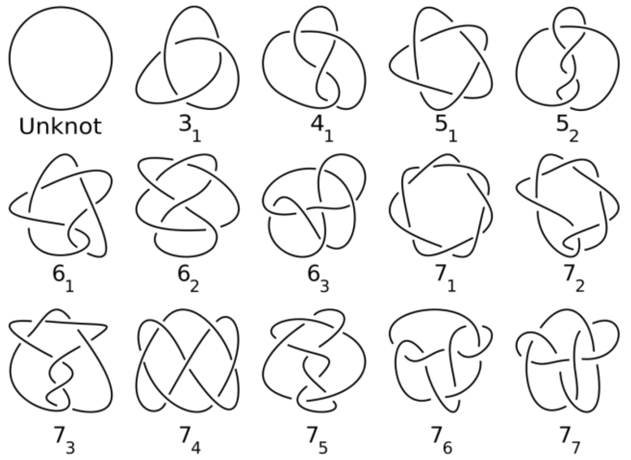
Trefoil Knot*, as pictured below, is a basic knot, but as we see we can reshape it to be the knot to the right.*



In Knot Theory, they study Knots. This might sound boring, and it may be boring, but some interesting stuff does come out of these fields.

Introductory stuff looks a lot like these diagrams





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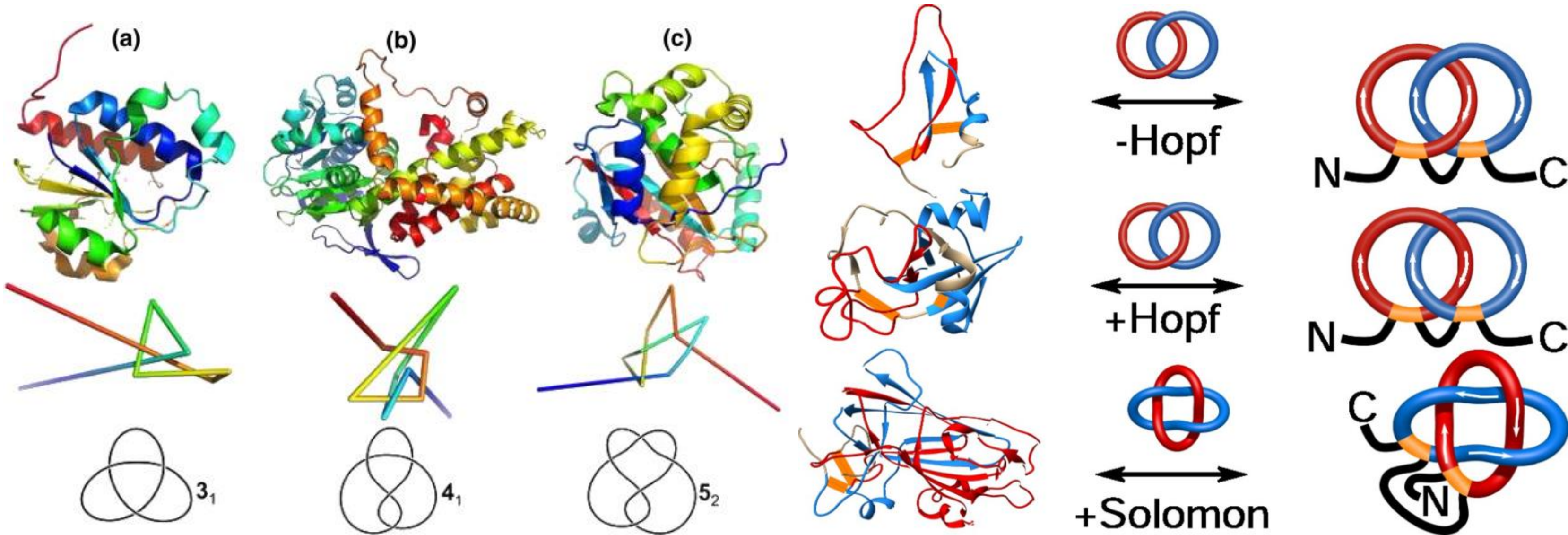

Guided Discussion: *Knot Theory*

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Trefoil Knot*, pictured to the right.*

In Knot Theory, they study Knots. This might sound boring, and it may be boring, but some interesting stuff does come out of these fields.

Below is a representation of the Seifert algorithm on a trefoil knot. Do you see how using this Seifert Surface, we can generalize this knot to be of a certain escribed figure on a torus?

Videos:

Topology Application:

https://www.youtube.com/watch?v=AmgkSdhK4K8&t=12s

Topology In Concept:

https://www.youtube.com/watch?v=yuVqxCSsE7c&t=971s