Warm Up!

The set of real numbers x for which

$$\frac{1}{x-2009}+\frac{1}{x-2010}+\frac{1}{x-2011}\geq 1$$

Is the union of intervals of the form

$a<x<b$. What is the sum of the lengths of these intervals?

For every integer $n \geq 2$, let $pow(n)$ be the largest power of the largest prime which divides n. For example,

$$pow(144)=pow\left(2^4 * 3^2\right)=3^2$$

What is the largest integer m such that

2010^m divides $\prod_{n=2}^{5300} pow(n)$?

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$a < x < b$. What is the sum of the lengths of these intervals?

The range is the same as the range of

$$\frac{1}{x-1}+\frac{1}{x}+\frac{1}{x+1} \geq 1$$

We see this gives us the distances between the beginnings of vertical asymptotes and where they meet the line $y = 1$

We find these intersections with

$$\frac{1}{x-1}+\frac{1}{x}+\frac{1}{x+1} = 1$$

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What is the largest integer m such that 2010^m divides $\prod_{n=2}^{5300} pow(n)$?

Largest prime factor of 2010 is 67

We see $67 * 80 > 5300$, so for each number where $67 * n$ where $1 < n \leq 79$, each of these have 67 as a factor once except for where $n = 67$, and so we count this twice. We also exclude the numbers $71, 73$ and 79 as these are primes greater than 67. So we have

$$79 - 3 + 1 = 77$$

Warm Up!

The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is equal to the reverse of these digits. What is $|x - y|$?

A) 24*, B)* 48*, C)* 54*, D)* 66*, E)* 70

For a positive integer n and non-zero digits $a, b,$ and c let A_n be the n-digit integer each of whose digits is a, and the same for B_n, but let C_n be the $2n$-digit integer each of whose digits is c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that

$$C_n - B_n = A_n^2?$$

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$$\frac{x+y}{2} = 10a + b$$

$$\sqrt{xy} = 10b + a$$

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A) 24, *B)* 48, *C)* 54, *D)* 66, *E)* 70

$$\frac{x^2 + 2xy + y^2}{4} = 100a^2 + 20ab + b^2$$

$$xy = 100b^2 + 20ab + a^2$$

$$\frac{x^2 + 2xy + y^2}{4} - xy = \frac{x^2 - 2xy + y^2}{4} = \left(\frac{x - y}{2}\right)^2$$

Warm Up!

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A) 24, *B)* 48, *C)* 54, *D)* 66, *E)* 70

$$\frac{x^2+2xy+y^2}{4}-xy=\frac{x^2-2xy+y^2}{4}=\left(\frac{x-y}{2}\right)^2$$

$$\left(\frac{x-y}{2}\right)^2=99a^2-99b^2$$

$$x-y=6\sqrt{11(a^2-b^2)}$$

Warm Up!

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$$C_n - B_n = A_n^2?$$

Break this down.

$$A_n = aaa \cdots aaa.0$$

$$A_n = (10^n - 1) * a$$

We find

$$\left(10^{2n} - 1\right) * c - (10^n - 1) * b = (10^n - 1)^2 * a^2$$

What do we know about coefficient matching?

Warm Up!

Let $m > 1$ and $n > 1$ be integers.

Suppose the product of the solutions for

x of the equation

$$8(\log_n x)(\log_m x) - 7\log_n x$$
$$- 6\log_m x - 2020 = 0$$

Is the smallest possible integer.

What is $m + n$?

An unfair coin lands on heads with a probability of ${}^{1}/_{4}$. When tossed $n > 1$ times, the probability of exactly two heads is the same as the probability of exactly 3 heads. What is the value of n?

Guided Discussion: AMC Season

What to expect, Approaches to Problems

Walter Johnson Math Team

Guided Discussion: Subtopics

There are a lot of types of problems. We're going to review some basic techniques to approaching all these problems to remind ourselves and get all around ready for the AMC!

Probability	Common, Hard,
Complex Numbers	Uncommon, Medium, 15-25
Functions	Common, M-Hard, 8-23
Geometry	Common, M-Hard, 10-25
Triangles	Uncommon, Hard, 10-21
Trig Equations	Rare, Hard, 15-25
Combinatorics	Rare, Hard, 13-23
Series	Uncommon, Medium, 10-20
Logarithms	Common, M-Hard,
Number Theory	Uncommon, Easy,
Word Problems	Common, Medium, 10-20

Guided Discussion: Probability

For a particular peculiar pair of dice, the probability of rolling a 1, 2, 3, 4, 5 and 6, on each die are in the ratio of 1 : 2 : 3 : 4 : 5 : 6. What is the probability of rolling a total of 7 on the two die?

How can we model this?

- *Infinite Series*
- *Modeling with 3-D Figures*
- *Systems of Equations*

Guided Discussion: Logarithms

For the harder problems, you'll get familiar over time with how problems are structured and what they are really asking for.

**Don't be afraid if you find a problem splits up into multiple directions for different solutions!*

- *Identities*
- *Modeling with Ranges and Domains*
- *Don't be afraid of the Sigma!*
- *Reciprocal Rule*

Guided Discussion: Functions

Never back away from a polynomials problem without modeling the equation and writing everything you have on paper.

- *Try to just go at problems, don't be intimidated, and go for problems.*
- *Vieta's Formulas for Monic Polynomials*
- n*-roots for* $n-1$ *degree polynomials*
- *Nested radicals (not common)*
- *Multinomial Theorem*

Guided Discussion: Functions

Four positive integers, $a, b, c,$ and d have a product of $8!$ And satisfy

$$ab + a + b = 524$$

Type equation here.

- *Consider* $P(0)$ *= constant term = product of all roots*
- *Consider* $P(1) = a_1 + a_2 \cdots$
- *Newton Sums,* $P_k = x_1^k + x_2^k + \cdots$
- $a_n P_1 + a_{n-1} = 0$
- $a_n P_2 + a_{n-1} P_1 + 2a_{n-2} = 0$
- *Systems of Equations*
- *Cauchy-Schwarz*

Guided Discussion: Counting

Easier problems will ask you how many numbers below 2020 satisfy a quantity. Don’t think in terms of splitting this up

- *Sum of n numbers*

$$\frac{n(n+1)}{2}$$

- *Combinatorics (Identities), Name some!*

Guided Discussion: Complex Numbers

- *Euler's Formula,* $e^{i\pi} = -1$
- $\operatorname{cis}\theta = \cos\theta + i\sin\theta$
- *This inscribes regular polygons in the unit circle on the complex plane*
- *Conjugation!*
- *If a polynomial* $f(x)$ *has real coefficients but complex solutions, it's complex solutions come in conjugates.*

Guided Discussion: Geometry

Always start just by getting a feel for a problem. Notice things if they give you a diagram, if they don't, notice that they don't.

**Typypically* —

*Typically problems will be broken up in an order for Geometry problems, making them a little easier if you do know where you're going.

- *Power of a Point*
- *Ptolemy's Theorem (Cyclic Quadrilaterals)*
- *Brahmagupta's Formula*

$$A_{CQ} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- *Don't be afraid to try out anything*
- *Identities*