

Assignment 2: Context-Free Languages

Computer Science 311
Spring 2016

Due April 28th, 2016

Problem 1 Let A be the complement of the language $\{a^n b^n \mid n \geq 0\}$.

- a) Show that A is not regular.
- b) Create a context-free grammar that describes A .

Problem 2 Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar:

$$\begin{aligned}\langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF} - \text{THEN} \rangle \mid \langle \text{IF} - \text{THEN} - \text{ELSE} \rangle \\ \langle \text{IF} - \text{THEN} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF} - \text{THEN} - \text{ELSE} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\ \langle \text{ASSIGN} \rangle &\rightarrow \text{a} := 1\end{aligned}$$

$$\begin{aligned}\Sigma &= \{\text{if, condition, then, else, a} := 1\} \\ V &= \{\langle \text{STMT} \rangle, \langle \text{IF} - \text{THEN} \rangle, \langle \text{IF} - \text{THEN} - \text{ELSE} \rangle, \langle \text{ASSIGN} \rangle\}\end{aligned}$$

Show that G is ambiguous.

Problem 3 The languages $\{0^s 1^s 2^t \mid s, t \geq 0\}$ and $\{0^s 1^t 2^t \mid s, t \geq 0\}$ are context-free. Use those languages to answer the following:

- a) Write down CFGs for each of the languages. [5 Points]
- b) Use the fact that both languages are context-free to show that the class of context-free languages is *not* closed under intersection. (Use the context-free pumping lemma to show that the resulting language is not context-free.) [5 Points]
- c) Use the result from part *b* and DeMorgan's Laws to show that the class of context-free languages are *not* closed under complement. [5 Points]

Problem 4 Prove that any derivation of a string s with a length greater than zero using a grammar in Chomsky Normal Form contains $2|s| - 1$ productions.