

Data Analysis 3

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Introduction

In this paper we will be looking at data related to calves. The data comes from an experiment designed to study the impact dietary treatments given to pregnant heifers had on the development of the calves. The study was conducted over a three year period and involved three different dietary treatments given to select groups of heifers in the final trimester. In total the data has 22 variables for 120 entries, though some data points are missing.

For more information on the experiment, the data, or any other files used in this paper see our [Github page](https://github.com/RyanLalicker/Data-Analysis-2-STAT-325-825) which can be found at <https://github.com/RyanLalicker/Data-Analysis-2-STAT-325-825>. The coding languages used in the paper are R and SAS. The corresponding code can be found in *Appendix A - R Code* and *Appendix B - SAS Code* respectively.

Exploring the Data

Variables

As mentioned above the experiment used three different dietary treatments. These were DDG, CON, and MET. For the first two trimesters the heifers were given one of seven developmental treatments, found in `Development.Treatment`, and then in the final trimester the each was given one of the three treatments mentioned above. This is recorded in the `Calan.Treatment` column of the data set.

The heifers were placed into one of four pens by weight, which can be seen in the column `Pen #`. They were then artificially inseminated from an assigned sire, which we will assume was done randomly since the client says weight was not a factor. The sire is represented by the column of the same name and has six unique entries.

Upon the birth of the calves, several measurements were taken. These include the sex of the calf, weights taken at both birth and slaughter, and scores of both the calf's vigor and the ease of birth. The variable names line up with these descriptions.

Other variables, such as the id of the calf, length of gestation for the heifer, and postmortem scoring such as hot carcass weight (HCW) are included as well. (Saner (2024)). Note two birthdays are included in the data, `Birth.date` and `Birth.date.1`. These variables will not be used in the models below so no further investigation was done on our part to determine the differences.

The client's main focus is the effect the third trimester treatment and the sex of a calf have on the calf's vigor score, ease of birth score, and final body weight. Therefore, these are the variables we will place more of an emphasis on, while exploring the effect some of the other variables may have.

Missing Values

UPDATE THIS AFTER SEEING WHAT VARIABLES ARE NEEDED FOR THE MODEL

The data contains some missing values. In total 53 rows in the data set are missing at least one variable. Figure 1 shows which columns have the most missing data. As we can see the values for the variable `DMI`, which according to the USDA represents the dry matter intake for a cow, is missing for two-thirds of the entries. (USDA). Given the number of missing values is this large, it is probably best to not use this variable in our models. Some other variables, including the final body weight of the calf represented by `Final.Calf.BW`, are missing in 19 entries. Of the other four variables the client was most interested in, none have more than ten missing values.

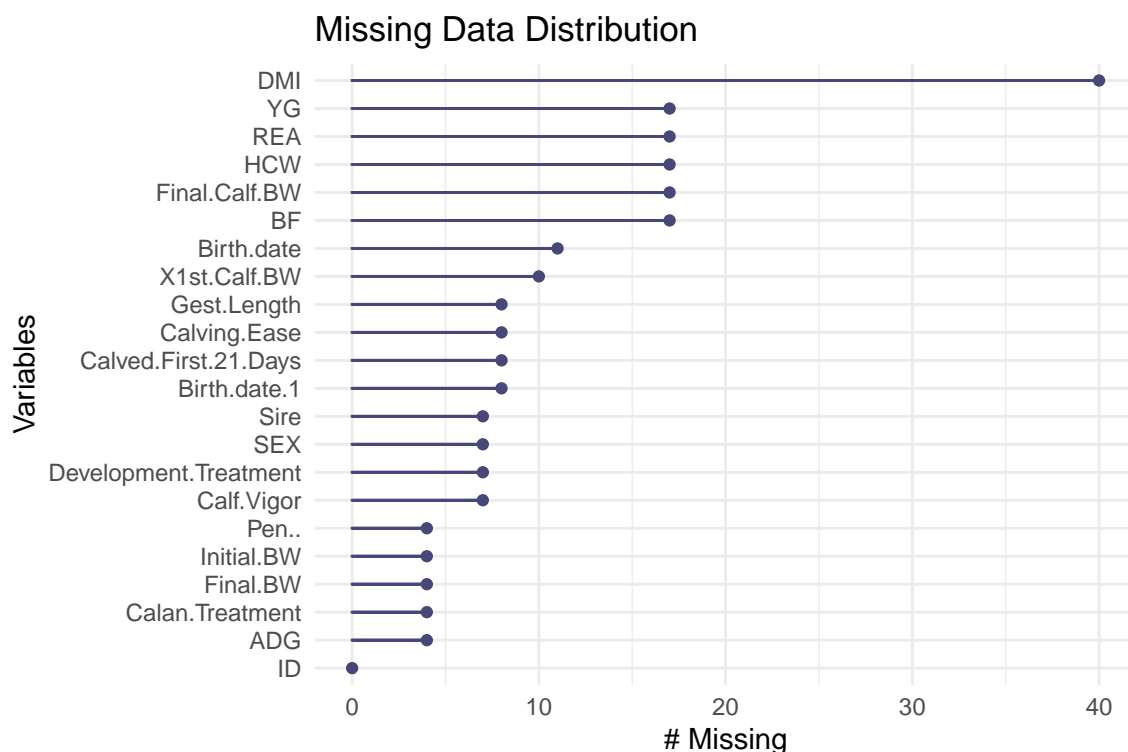


Figure 1: Chart counting the number of missing values for each variable within the data.

Cleaning the Dataset

To clean up the data set let's first focus on some of the quantitative variables. For these we can impute the missing values with the median or mean. We will use the median due to its greater resistance to potential outliers. This reduces the number of rows with at least one missing variable to 15. Much of the improvement comes from DMI being a quantitative variable, but to reiterate what was discussed earlier, this variable will not be used in any models.

The categorical variables are not as simple. Let's consider the third trimester treatment. The MET treatment was used in 40 cases, while the other two treatments were only used 38 times, meaning there are four missing values. If we used the mode as Memon, Wamala, and Kabano (2023) suggests, this would make 42 instances of the MET treatment. However, it seems very possible that the missing entries were split between the CON and DDG treatments to make an even 40 uses each. For this reason, rows with missing values for the categorical variables used in the model will be removed. This results in the models using 113 entries of the original 120, which is something we are comfortable with.

RETURN AND VERIFY NUMBERS BEFORE SUBMITTING.

Summary Statistics

Let's take a closer look at what the three dependent variables the client is interested in. Figure 2 shows several summary statistics for each. The calving ease and calf vigor are each scores given. Looking at the minimum and maximum values of each it would seem they are scored in a three-point and five-point system respectively, both only using integers. We can also see from the median and 75th percentiles that both seem very skewed towards the low end of the scale. While the imputing done previously may be exaggerating the curve, Figure 1 shows less than ten values were imputed. This indicates to us the skew was already present before cleaning the data.

Variable	Mean	Median	SD	25th Percentile	75th Percentile	Min	Max
Calving.Ease	1.132743	1	0.4331039	1	1	1	3
Calf.Vigor	1.362832	1	0.8244256	1	1	1	5
Final.Calf.BW	1290.106195	1283	122.0959344	1227	1360	932	1690

Figure 2: Summary Statistics for Dependent Variables

The final weight of calf is the third variable in Figure 2. The mean and median are relatively similar given the large standard deviation. While the previous two variables discussed had some concerns on top of being counting variables on a scale, the final weight does not present the same issues. Further investigation into the approximate distribution of the final weight is needed though.

Let's look at a histogram and a Q-Q plot for the final weight of the calves in Figure 3. The bin width for the histogram comes from the Freedman-Diaconis rule. (William (2023)). The histogram appears to follow an approximately normal distribution. The Q-Q plot mostly follows this as most points follow the linear trend represented by the red line.

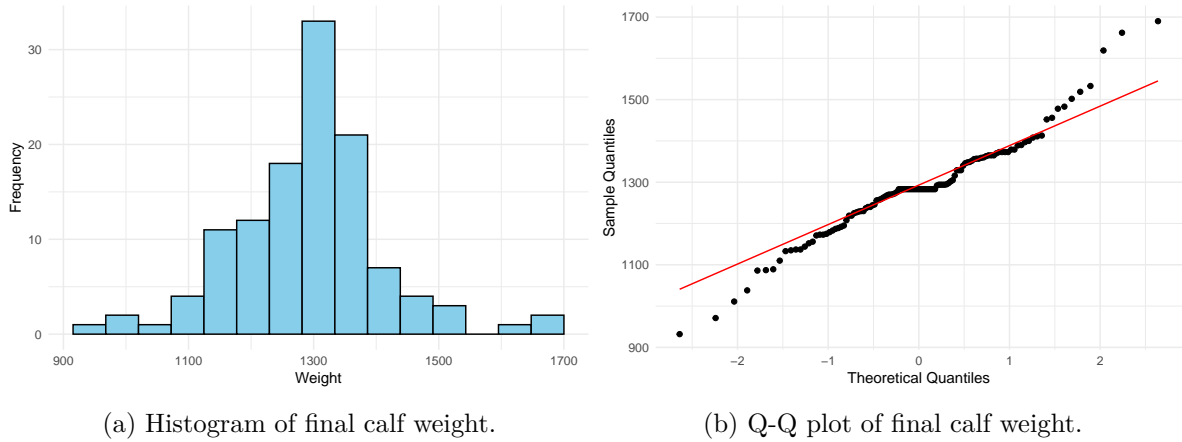


Figure 3: Plots used to check if the distribution of the final calf weight is normal.

Before moving on we want to look at plots of the scoring variables as well. While we suspect a heavy skew for each, the histograms in Figure 4 verify this. It is important to remember that these two variables are not continuous like the weight variable, so the types of models used will vary.

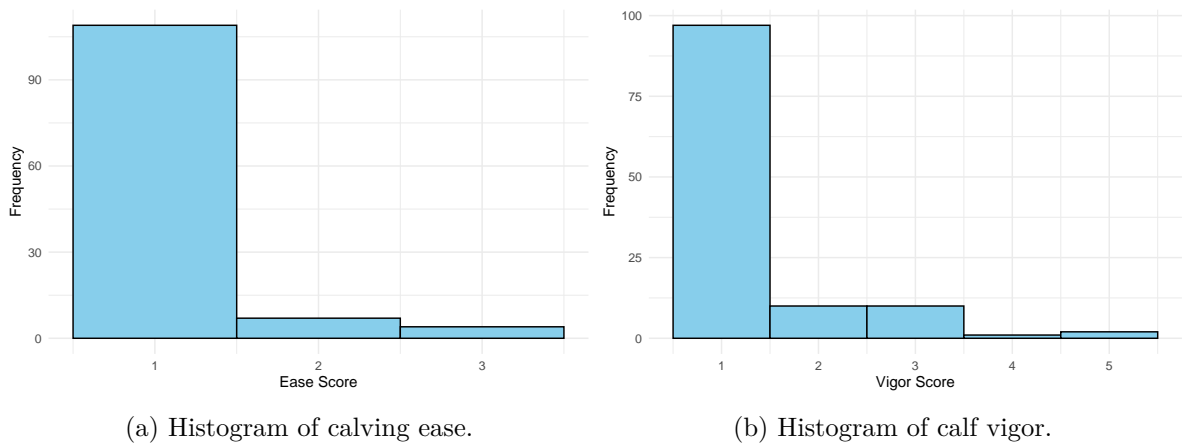


Figure 4: Histograms of scoring variables.

Exploring the Data

Before looking at potential models, let's explore how some of the variables interact with each other. While we will be able to include other explanatory variables, the client specifically mentions using the third trimester treatment and the sex of a calf as explanatory variables of interest. The table in Figure 5 shows the breakdown of treatment by sex. Note HFR stands

for heifer and STR stands for steer. Although not every group has an equivalent number of subjects, this is nothing we are concerned about. Please note that the total occurrences per treatment are different than discussed above since rows containing missing values were removed.

	HFR	STR
CON	20	17
DDG	15	23
MET	19	19

Figure 5: Table showing the breakdown of treatment by sex.

One of the key assumptions for some of the models we will be discussing later is that the explanatory variables of the model are not highly correlated with each other. If this assumption is violated, multicollinearity is present. Since both the treatment and the sex of the calf are categorical, we can use the Pearson’s chi-squared on the table in Figure 5 to determine if multicollinearity is a problem for these variables. (Bhalla (2017)). The results of the test, shown in Figure 6, indicate multicollinearity is not a problem since the p-value is well above any commonly used significance level such as 0.05.

Metric	Value
Statistic	1.71
Degrees of Freedom	2
P-Value	0.42538

Figure 6: Chi-squared test for treatment and sex.

Now let’s consider how these variables affect the final body weight. The boxplot shown in Figure 7 indicates the highest median weights come from the MET treatment, followed closely by DDG. Additionally, the steers are heavier on average than the heifers. This indicates both likely have a use in predicting the final weight of a calf. The variance differs by both treatment and sex, but the only outlier is a heifer calf with the MET treatment.

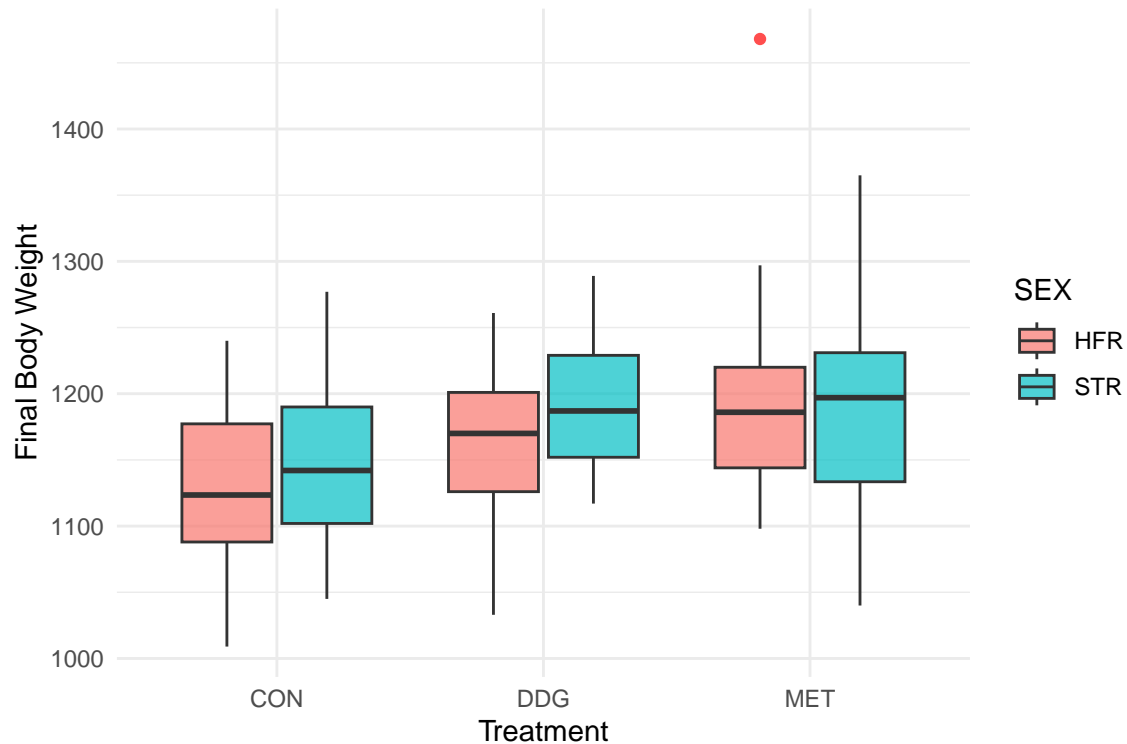


Figure 7: Final body weight by treatment and sex.

Models for Calving Ease

Since the client is interested in three different dependent variables we will need to handle one variable at a time. For the next two models we will be focused on calving ease.

Ordinal Logistic Regression Model

Since calving ease is a score from one to three it can be considered an ordinal variable. This means instead of treating it as a quantitative variable as we did previously, it could be considered an ordered categorical variable. One method of modeling ordinal variables is with ordinal logistic regression, or OLS. This uses one or more independent variables to predict the ordinal value of the dependent variable. A key assumption of OLS, outside of the dependent variable

being ordinal, is no multicollinearity between independent variables. (“Ordinal Regression” (n.d.)).

Let’s attempt to apply an OLS to the calving ease variable. In this simple case we will use the third trimester treatment and the calf’s sex as the independent variables. As we saw in Figure 6, multicollinearity is not a problem with these variables, so we may procede.

START BACK HERE

Call:

```
polr(formula = Calving.Ease ~ Calan.Treatment * SEX, data = data2,
      Hess = TRUE)
```

Coefficients:

	Value	Std. Error	t value
Calan.TreatmentDDG	-16.7932	0.6341	-26.4836
Calan.TreatmentMET	-1.1127	1.2003	-0.9270
SEXSTR	-0.2646	0.9739	-0.2717
Calan.TreatmentDDG:SEXSTR	15.7936	0.6341	24.9073
Calan.TreatmentMET:SEXSTR	1.9542	1.5240	1.2822

Intercepts:

	Value	Std. Error	t value
1 2	1.7963	0.6222	2.8870
2 3	2.9177	0.7396	3.9449

Residual Deviance: 79.01625

AIC: 93.01625

	Value	Std. Error	t value	p-value
Calan.TreatmentDDG	-16.7931566	0.6340969	-26.4835793	1.498375e-154
Calan.TreatmentMET	-1.1127063	1.2002826	-0.9270369	3.539074e-01
SEXSTR	-0.2645934	0.9738552	-0.2716968	7.858551e-01
Calan.TreatmentDDG:SEXSTR	15.7936403	0.6340968	24.9073017	6.201312e-137
Calan.TreatmentMET:SEXSTR	1.9541929	1.5240486	1.2822379	1.997592e-01
1 2	1.7962564	0.6221810	2.8870320	3.888946e-03
2 3	2.9177029	0.7396133	3.9449031	7.983226e-05

Multinomial Logistic Regression

weights: 21 (12 variable)

initial value 124.143189


```

iter 10 value 36.581074
iter 20 value 34.686003
iter 30 value 34.676842
iter 40 value 34.671003
iter 50 value 34.670618
final value 34.670617
converged

```

Call:

```
multinom(formula = Calving.Ease ~ Calan.Treatment * SEX, data = data2)
```

Coefficients:

```

(Intercept) Calan.TreatmentDDG Calan.TreatmentMET SEXSTR
2 -1.734529 -20.7937399 -1.15589609 -0.2803915
3 -19.374020 0.7406987 -0.05867808 5.9970139
Calan.TreatmentDDG:SEXSTR Calan.TreatmentMET:SEXSTR
2 -4.122079 0.4628294
3 9.545318 11.8261828

```

Std. Errors:

```

(Intercept) Calan.TreatmentDDG Calan.TreatmentMET SEXSTR
2 0.6262084 4.879631e-09 1.203222 0.9791894
3 103.8662689 1.036460e+02 103.645620 103.4249381
Calan.TreatmentDDG:SEXSTR Calan.TreatmentMET:SEXSTR
2 1.697924e-09 1.755282
3 1.036477e+02 103.646552

```

Residual Deviance: 69.34123

AIC: 93.34123

Pairwise Comparisons

SEX = HFR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	5.14e-17	NaN	12	NaN	NaN
CON - MET	-1.17e-17	NaN	12	NaN	NaN
DDG - MET	-6.31e-17	2.44e-10	12	0.000	1.0000

SEX = STR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	2.57e-18	NaN	12	NaN	NaN
CON - MET	-3.30e-17	NaN	12	NaN	NaN

DDG - MET -3.56e-17 4.85e-10 12 0.000 1.0000

Results are averaged over the levels of: Calving.Ease
P value adjustment: tukey method for varying family sizes

Models for Calf Vigor

Ordinal Logistic Regression Model

Call:

```
polr(formula = Calf.Vigor ~ Calan.Treatment * SEX, data = data2,  
      Hess = TRUE)
```

Coefficients:

	Value	Std. Error	t value
Calan.TreatmentDDG	-0.3742	0.8178	-0.4576
Calan.TreatmentMET	-0.1616	0.7599	-0.2127
SEXSTR	-1.6731	1.1513	-1.4532
Calan.TreatmentDDG:SEXSTR	1.8571	1.4086	1.3184
Calan.TreatmentMET:SEXSTR	1.7870	1.3787	1.2961

Intercepts:

	Value	Std. Error	t value
1 2	1.0761	0.5133	2.0964
2 3	1.7628	0.5425	3.2493
3 4	3.3313	0.7403	4.5000
4 5	3.7477	0.8448	4.4360

Residual Deviance: 160.1861

AIC: 178.1861

	Value	Std. Error	t value	p-value
Calan.TreatmentDDG	-0.3742104	0.8177783	-0.4575940	6.472442e-01
Calan.TreatmentMET	-0.1616211	0.7599288	-0.2126792	8.315772e-01
SEXSTR	-1.6730691	1.1513128	-1.4531838	1.461727e-01
Calan.TreatmentDDG:SEXSTR	1.8570878	1.4086188	1.3183750	1.873782e-01
Calan.TreatmentMET:SEXSTR	1.7870376	1.3787308	1.2961468	1.949249e-01
1 2	1.0760995	0.5133124	2.0963834	3.604819e-02
2 3	1.7628214	0.5425236	3.2492990	1.156898e-03
3 4	3.3312996	0.7402816	4.5000439	6.793943e-06
4 5	3.7477448	0.8448396	4.4360433	9.162743e-06

Pairwise comparisons

SEX = HFR:

contrast	estimate	SE	df	z.ratio	p.value
CON - DDG	0.374	0.818	Inf	0.458	0.8910
CON - MET	0.162	0.760	Inf	0.213	0.9754
DDG - MET	-0.213	0.850	Inf	-0.250	0.9661

SEX = STR:

contrast	estimate	SE	df	z.ratio	p.value
CON - DDG	-1.483	1.147	Inf	-1.293	0.3990
CON - MET	-1.625	1.150	Inf	-1.413	0.3342
DDG - MET	-0.143	0.716	Inf	-0.199	0.9784

P value adjustment: tukey method for comparing a family of 3 estimates

Multinomial Logistic Regression

```
# weights: 35 (24 variable)
initial value 181.866484
iter 10 value 72.424368
iter 20 value 70.354245
iter 30 value 70.292462
iter 40 value 70.288076
final value 70.287948
converged
```

Call:

```
multinom(formula = Calf.Vigor ~ Calan.Treatment * SEX, data = data2)
```

Coefficients:

	(Intercept)	Calan.TreatmentDDG	Calan.TreatmentMET	SEXSTR
2	-2.014904	0.2231357	-38.7656488	-47.2209029
3	-2.014776	-0.4702127	0.6930633	-0.7578969
4	-37.507025	-7.3618177	4.6924230	13.9963447
5	-2.708032	-14.1380100	-42.7800640	-29.0708500
	Calan.TreatmentDDG:SEXSTR	Calan.TreatmentMET:SEXSTR		
2	46.815435	86.7486656		
3	1.045754	-42.0187508		
4	27.982098	0.2371585		
5	6.461777	71.9199389		

Std. Errors:

	(Intercept)	Calan.TreatmentDDG	Calan.TreatmentMET	SEXSTR
2	0.7527781	1.072385	0.4331898	0.466635
3	0.7527356	1.284531	0.9398243	1.276397
4	483.2990336	483.298955	963.9381822	483.298693
5	1.0327936	1313.649758	0.4874010	0.487401

	Calan.TreatmentDDG:SEXSTR	Calan.TreatmentMET:SEXSTR
2	6.674788e-01	0.4331898
3	1.807792e+00	NaN
4	4.832990e+02	963.9378422
5	2.989303e-07	0.4874010

Residual Deviance: 140.5759

AIC: 188.5759

Models for Final Calf Weight

Linear Model

Call:

```
lm(formula = Final.Calf.BW ~ Calan.Treatment * SEX, data = data2)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-311.20	-61.13	7.89	54.59	344.59

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1261.000	25.531	49.390	<2e-16 ***
Calan.TreatmentDDG	-17.800	39.000	-0.456	0.6490
Calan.TreatmentMET	-43.895	36.579	-1.200	0.2328
SEXSTR	84.412	37.666	2.241	0.0271 *
Calan.TreatmentDDG:SEXSTR	3.519	53.429	0.066	0.9476
Calan.TreatmentMET:SEXSTR	30.115	52.830	0.570	0.5699

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 114.2 on 107 degrees of freedom

Multiple R-squared: 0.1645, Adjusted R-squared: 0.1255

F-statistic: 4.214 on 5 and 107 DF, p-value: 0.001553

Tukey comparison

Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

Fit: `lm(formula = Final.Calf.BW ~ Calan.Treatment * SEX, data = data2)`

Linear Hypotheses:

	Estimate	Std. Error	t value	Pr(> t)
DDG - CON == 0	-17.80	39.00	-0.456	0.892
MET - CON == 0	-43.89	36.58	-1.200	0.455
MET - DDG == 0	-26.09	39.44	-0.662	0.786

(Adjusted p values reported -- single-step method)

Linear Mixed Model

This will need to be redone in SAS if we stick with it.

Linear mixed model fit by REML [`'lmerMod'`]

Formula: `Final.Calf.BW ~ Calan.Treatment * SEX + (1 | Pen..) + (1 | Sire)`

Data: `data2`

REML criterion at convergence: 1329.9

Scaled residuals:

Min	1Q	Median	3Q	Max
-2.91784	-0.55007	-0.02894	0.53915	2.74964

Random effects:

Groups	Name	Variance	Std.Dev.
Sire	(Intercept)	1800	42.43
Pen..	(Intercept)	0	0.00
Residual		11711	108.22

Number of obs: 113, groups: Sire, 6; Pen.., 4

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	1266.813	30.104	42.081

Calan.TreatmentDDG	-23.290	37.427	-0.622
Calan.TreatmentMET	-36.938	34.934	-1.057
SEXSTR	90.685	35.958	2.522
Calan.TreatmentDDG:SEXSTR	8.894	50.781	0.175
Calan.TreatmentMET:SEXSTR	20.064	50.729	0.396

Correlation of Fixed Effects:

```

(Intr) Cl.TDDG Cl.TMET SEXSTR C.TDDG:
Cln.TrtmDDG -0.535
Cln.TrtmMET -0.564  0.462
SEXSTR      -0.548  0.445  0.479
C.TDDG:SEXS 0.393 -0.729 -0.335 -0.703
C.TMET:SEXS 0.397 -0.324 -0.695 -0.715 0.503
optimizer (nloptwrap) convergence code: 0 (OK)
boundary (singular) fit: see help('isSingular')

```

Tukey comparison

Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

```
Fit: lmer(formula = Final.Calf.BW ~ Calan.Treatment * SEX + (1 | Pen..) +
(1 | Sire), data = data2)
```

Linear Hypotheses:

	Estimate	Std. Error	z value	Pr(> z)
DDG - CON == 0	-23.29	37.43	-0.622	0.808
MET - CON == 0	-36.94	34.93	-1.057	0.540
MET - DDG == 0	-13.65	37.60	-0.363	0.930

(Adjusted p values reported -- single-step method)

Pariwise comparison

SEX = HFR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	23.29	38.1	104	0.612	0.8141
CON - MET	36.94	35.4	103	1.043	0.5514
DDG - MET	13.65	38.8	102	0.352	0.9340

SEX = STR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	14.40	35.1	102	0.410	0.9114
CON - MET	16.87	36.8	102	0.459	0.8907
DDG - MET	2.48	34.7	104	0.072	0.9972

Degrees-of-freedom method: kenward-roger

P value adjustment: tukey method for comparing a family of 3 estimates

ANCOVA Model

This includes initial weight.

Call:

```
lm(formula = Final.Calf.BW ~ Calan.Treatment * SEX + Initial.BW,  
    data = data2)
```

Residuals:

Min	1Q	Median	3Q	Max
-291.16	-74.56	-0.57	63.79	327.87

Coefficients:

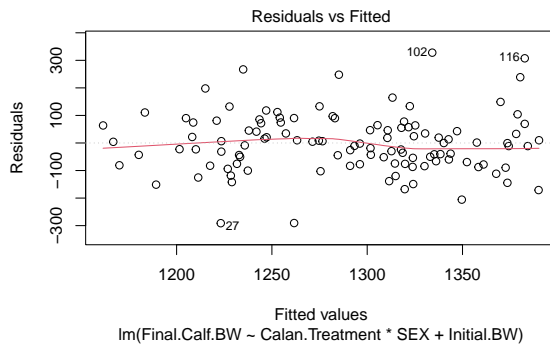
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	717.7214	180.1893	3.983	0.000125 ***
Calan.TreatmentDDG	-22.9430	37.6139	-0.610	0.543193
Calan.TreatmentMET	-49.3793	35.2893	-1.399	0.164652
SEXSTR	79.3296	36.3293	2.184	0.031194 *
Initial.BW	0.5348	0.1757	3.044	0.002949 **
Calan.TreatmentDDG:SEXSTR	15.5936	51.6312	0.302	0.763230
Calan.TreatmentMET:SEXSTR	36.3226	50.9424	0.713	0.477404

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

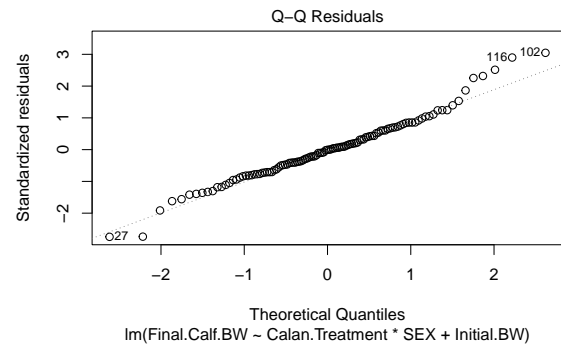
Residual standard error: 110 on 106 degrees of freedom

Multiple R-squared: 0.2317, Adjusted R-squared: 0.1882

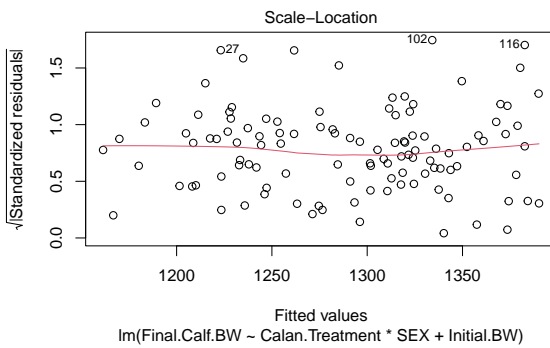
F-statistic: 5.326 on 6 and 106 DF, p-value: 7.747e-05



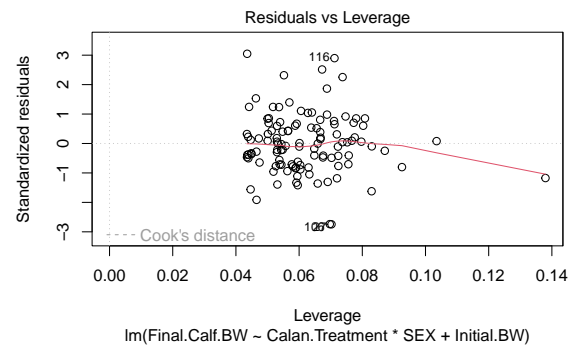
(a)



(b)



(c)



(d)

Post Hoc Test

SEX = HFR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	22.94	37.6	106	0.610	0.8150
CON - MET	49.38	35.3	106	1.399	0.3449
DDG - MET	26.44	38.0	106	0.696	0.7665

SEX = STR:

contrast	estimate	SE	df	t.ratio	p.value
CON - DDG	7.35	35.3	106	0.208	0.9763
CON - MET	13.06	36.7	106	0.355	0.9328
DDG - MET	5.71	34.2	106	0.167	0.9847

P value adjustment: tukey method for comparing a family of 3 estimates

Model 3 - Binary Logistic Regression with GLM

Call:

```
glm(formula = Calf.Vigor.Binary ~ Calan.Treatment * SEX + Initial.BW,  
     family = binomial(link = "logit"), data = data)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-2.5460511	5.1116605	-0.498	0.618
Calan.TreatmentDDG	-0.9116950	1.2106253	-0.753	0.451
Calan.TreatmentMET	0.4047963	0.8435442	0.480	0.631
SEXSTR	-1.0458581	1.2072829	-0.866	0.386
Initial.BW	0.0007978	0.0049818	0.160	0.873
Calan.TreatmentDDG:SEXSTR	1.7983853	1.7099003	1.052	0.293
Calan.TreatmentMET:SEXSTR	-0.5212389	1.6825267	-0.310	0.757

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 80.667 on 112 degrees of freedom
Residual deviance: 77.041 on 106 degrees of freedom
(7 observations deleted due to missingness)
AIC: 91.041

Number of Fisher Scoring iterations: 5

$$y_{ijklmn} = ENTER - MODEL - HERE$$

where y_{ijklm} represents the *dependent variable*, ...

![Picture of SAS Output](filename.png){width="3in"}

Conclusion

Recomendation

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Appendix A - R Code

Appendix B - SAS Code

Appendix C - Additional SAS Output

