Theory of Computation (60-354), Fall 2010
Solution to Midterm 2, 23 Nov., 2010
Total Mar ks : 40
Time: 80 minutes

Qn.1 Show that the following context-free grammar is ambiguous.

$$S \to ABA|aBaa$$
$$A \to a|aA$$
$$B \to b$$

by finding a string that has two leftmost or rightmost derivations.

[8 marks]

Ans: The string $abaa$ has the following two leftmost derivations.

1. $S \Rightarrow aBaa \Rightarrow abaa$

2. $S \Rightarrow ABA \Rightarrow aBA \Rightarrow abA \Rightarrow abaA \Rightarrow abaa$

Qn.2 Find a context-free grammar that generates the language accepted by the DPDA $P = (\{q_0, q_1\}, \{0, 1, 2\}, \{0, 1, 2, Z_0\}, \delta, q_0, Z_0\}$ whose transition function is given below:

$$\begin{gathered}\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\} \\ \delta(q_0, 0, 0) = \{(q_0, 00)\} \\ \delta(q_0, 1, Z_0) = \{(q_1, Z_0)\} \\ \delta(q_0, 1, 0) = \{(q_1, 0)\} \\ \delta(q_1, 2, 0) = \{(q_1, \epsilon)\} \\ \delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}\end{gathered}$$

It is enough to provide templates for the productions corresponding to the first two transitions.

[8 marks]

Ans:

$[q_0 Z_0 ?] \rightarrow 0[q_0 0 !][! Z_0 ?]$

$[q_0 0 ?] \rightarrow 0[q_0 0 !][! 0 ?]$

$[q_0 Z_0 ?] \rightarrow 1[q_1 Z_0 ?]$

$[q_0 0 ?] \rightarrow 1[q_1 0 ?]$

$[q_1 0 q_1] \rightarrow 2$

$[q_1 Z_0 q_1] \rightarrow \epsilon$

Qn.3 Design a context-free grammar that generates the language $L = \{a^n b^m c^{n+m} | n \geq 0, m \geq 0\}$. (**Hint**: We can rewrite the string $a^n b^m c^{n+m}$ as $a^n b^m c^m c^n$, which shows that the outer (a, c)-pairs and the inner (b, c)-pairs have to be generated by different variables). You must provide explanations for the productions of your grammar.

[8 marks]

Ans:

$S \rightarrow aSc|B$

$B \rightarrow bBc|\epsilon$

Qn.4 Use the pumping lemma (for context-free languages) to show that the language $L = \{a^i b^{2i} c^{3i} | i \geq 0\}$ over $\Sigma = \{a, b, c\}$ is not context-free. (**Hint**: You may choose the string $a^n b^{2n} c^{3n}$ in L to which to apply the PL). Clearly mention the i that you choose to show that $z' = uv^i wx^i y$ is not in L in each of the different cases that arises.

[8 marks]

Ans: Choose $z = a^n b^{2n} c^{3n}$. Let $uvwxy$ be an adversarial decomposition of z.

Case 1: vwx consists of a's (b's or c's) alone. Setting $i = 0$ we get fewer a's (b's or c's), disturbing the $1 : 2 : 3$ ratio of the a's, b's and c's.

Case 2: vwx consists of a's and b's (b's and c's). Setting $i = 0$ may maintain the ratio of a's to the b's but not their ratios with respect to the c's.

Qn.5 Use the CYK algorithm to determine whether the string $w = ababa$ is in the language generated by the following context-free grammar in CNF.

$$S \to AB$$
$$A \to BB|a$$
$$B \to AB|b$$

[8 marks]

Ans:

$S_{11} = \{A\}$, $S_{22} = \{B\}$, $S_{33} = \{A\}$, $S_{44} = \{B\}$, $S_{55} = \{A\}$

$S_{12} = \{S, B\}$, $S_{23} = \{\}$, $S_{34} = \{S, B\}$, $S_{45} = \{\}$

$S_{13} = \{\}$, $S_{24} = \{A\}$, $S_{35} = \{\}$

$S_{14} = \{A\}$, $S_{25} = \{\}$