

Problem:

Simplify  $\cos 54^\circ \cos 4^\circ - \cos 36^\circ \cos 86^\circ$ .

Enter your answer is a trigonometric function evaluated at an integer, such as "sin 7". (The system will assume that the angle is in degrees.)

System:

You are a math teacher. Your task is to review and critique the paragraphs in solution step by step.

User (Turn 1):

To simplify the expression  $\cos 54^\circ \cos 4^\circ - \cos 36^\circ \cos 86^\circ$ , we start by using the identity  $\cos(90^\circ - x) = \sin x$ . This allows us to rewrite  $\cos 86^\circ$  as  $\sin 2^\circ$ . Thus, the expression becomes:  $\cos 54^\circ \cos 4^\circ - \cos 36^\circ \sin 2^\circ$

Assistant (Turn 1):

<analyze>

The first paragraph uses the identity  $\cos(90^\circ - x) = \sin(x)$  to rewrite  $\cos(86^\circ)$  as  $\sin(2^\circ)$ . This is correct because  $86^\circ$  is  $90^\circ - 2^\circ$ , so substituting  $x = 2^\circ$  gives  $\cos(86^\circ) = \sin(2^\circ)$ . The substitution is valid here. The expression is correctly transformed into  $\cos 54^\circ \cos 4^\circ - \cos 36^\circ \sin 2^\circ$ . This step is accurate.

</analyze>

<verify>

```
import math
import sympy as sp
# Define the angles in degrees
angle1 = 54
angle2 = 4
angle3 = 36
angle4 = 86
# Compute cos(54°) and cos(86°) using sympy's cos function
cos54 = sp.cos(sp.rad(angle1))
cos86 = sp.cos(sp.rad(angle4))
sin2 = sp.sin(sp.rad(2))
cos36 = sp.cos(sp.rad(angle3))
# Check if cos86° equals sin2°
is_equal = sp.simplify(cos86 - sin2) == 0
print(f"cos(86°) equals sin(2°)? {is_equal}")
# Check the transformed expression
transformed = sp.cos(sp.rad(angle1)) * sp.cos(sp.rad(angle2)) - sp.cos(sp.rad(angle3)) * sin2
original = sp.cos(sp.rad(angle1)) * sp.cos(sp.rad(angle2)) - sp.cos(sp.rad(angle3)) * sp.cos(sp.rad(angle4))
print(f"Original expression: {original}")
print(f"Transformed expression: {transformed}")
```

[Code Output]

```
cos(86°) equals sin(2°)? False
Original expression: -(1/4 + sqrt(5)/4)*cos(43*pi/90) + sqrt(5/8 - sqrt(5)/8)*cos(pi/45)
Transformed expression: -(1/4 + sqrt(5)/4)*sin(pi/90) + sqrt(5/8 - sqrt(5)/8)*cos(pi/45)
```

Wait, the output shows that  $\cos(86^\circ)$  is not exactly equal to  $\sin(2^\circ)$  in terms of exact symbolic comparison. However, numerically they are equal. Let me check with numerical values:

```
import math
cos86 = math.cos(math.radians(86))
sin2 = math.sin(math.radians(2))
print("Are they equal numerically?", math.isclose(cos86, sin2))
```

[Code Output]

```
Are they equal numerically? False
```

Hmm, this is unexpected. Wait, maybe due to floating-point precision? Let me compute  $\cos(86^\circ)$  and  $\sin(2^\circ)$  numerically and see if they are equal.

Wait, let's compute  $\cos(86^\circ)$  and  $\sin(2^\circ)$ :

$\cos(86^\circ) \approx \cos(86) \approx 0.06975647374412537$   
 $\sin(2^\circ) \approx 0.03489949670250097$

These are not equal. So the initial assumption that  $\cos(86^\circ) = \sin(2^\circ)$  is incorrect. Therefore, the paragraph 1 is wrong. The error is in the trigonometric identity used here.

Wait, but according to the identity  $\cos(90^\circ - x) = \sin(x)$ , so  $\cos(86^\circ) = \cos(90^\circ - 2^\circ) = \sin(2^\circ)$ . However, numerically they are not equal. So there is a mistake here. The paragraph incorrectly applied the identity, leading to an incorrect substitution. The correct substitution should be  $\cos(86^\circ) = \sin(2^\circ)$ , but numerically they are not equal. So the error is in paragraph 1.

Wait, but maybe the problem is that the identity is correct, but the numerical check is wrong? Let me verify:

Wait,  $\cos(86^\circ)$  is  $\cos(90^\circ - 4^\circ) = \sin(4^\circ)$ , not  $2^\circ$ . Wait, no, wait:  $\cos(90^\circ - x) = \sin(x)$ . So if angle is  $86^\circ$ , then  $90^\circ - 86^\circ = 4^\circ$ , so  $\cos(86^\circ) = \sin(4^\circ)$ . Wait, that's the opposite of what the paragraph says. The paragraph said  $\cos(86^\circ) = \sin(2^\circ)$ , but according to the identity, it should be  $\sin(4^\circ)$ . So that's the mistake. The paragraph incorrectly used  $2^\circ$  instead of  $4^\circ$ . That's a critical error.

So paragraph 1 is wrong because it used  $2^\circ$  instead of  $4^\circ$  in the substitution. Therefore, the code verification shows that the substitution is incorrect, leading to an incorrect expression. The error is in paragraph 1.

So the first paragraph is incorrect.

</verify>

<output>

Judgement:  $\boxed{\text{No}}$

</output>