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GenPRM: Scaling Test-Time Compute of Process Reward Models via Generative Reasoning

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Recent advancements in Large Language Models (LLMs) have shown that it is promising to utilize Process Reward Models (PRMs) as verifiers to enhance the performance of LLMs. However, current PRMs face three key challenges: (1) limited process supervision and generalization capabilities, (2) dependence on scalar value prediction without leveraging the generative abilities of LLMs, and (3) inability to scale the test-time compute of PRMs. In this work, we introduce GenPRM, a generative process reward model that performs explicit Chain-of-Thought (CoT) reasoning with code verification before providing judgment for each reasoning step. To obtain high-quality process supervision labels and rationale data, we propose Relative Progress Estimation (RPE) and a rationale synthesis framework that incorporates code verification. Experimental results on ProcessBench and several mathematical reasoning tasks show that GenPRM significantly outperforms prior PRMs with only **23K** training data from **MATH** dataset. Through test-time scaling, a **1.5B** GenPRM outperforms **GPT-4o**, and a **7B** GenPRM surpasses **Qwen2.5-Math-PRM-72B** on ProcessBench. Additionally, GenPRM demonstrates strong abilities to serve as a critic model for policy model refinement. This work establishes a new paradigm for process supervision that bridges the gap between PRMs and critic models in LLMs. Our code, model, and data are available in `https://ryanliu112.github.io/GenPRM`.

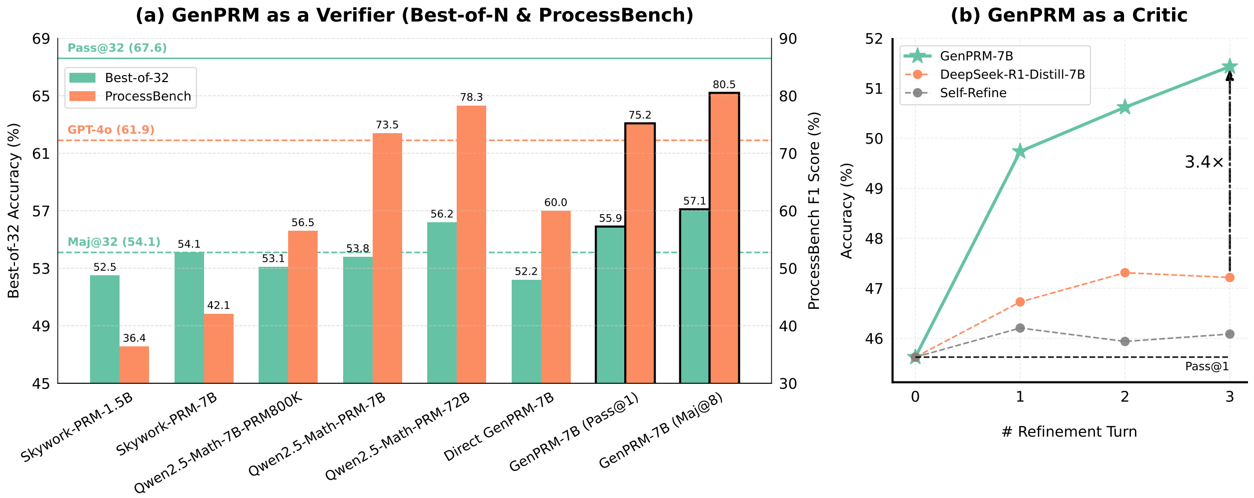

Figure 1: GenPRM achieves state-of-the-art performance across multiple benchmarks in two key roles: (a) **As a verifier**: GenPRM-7B outperforms all classification-based PRMs of comparable size and even surpasses **Qwen2.5-Math-PRM-72B** via test-time scaling. (b) **As a critic**: GenPRM-7B demonstrates superior critique capabilities, achieving **3.4×** greater performance gains than DeepSeek-R1-Distill-Qwen-7B after 3 refinement iterations.

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[*] Equal contribution

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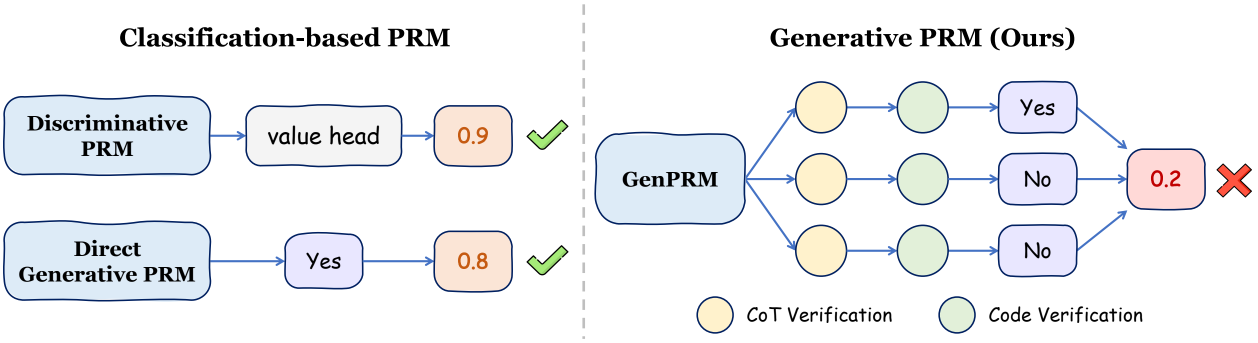

Figure 2: Comparison between GenPRM (right) and previous classification-based PRMs (left).

1. Introduction

Large Language Models (LLMs) have shown significant advances in recent years (OpenAI, 2023; Anthropic, 2023; OpenAI, 2024a,b; DeepSeek-AI et al., 2025). As OpenAI o1 demonstrates the great effectiveness of scaling test-time compute (OpenAI, 2024a), an increasing number of researches focus on Test-Time Scaling (TTS) methods to improve the reasoning performance of LLMs (Snell et al., 2025; Liu et al., 2025).

Effective TTS requires high-quality verifiers, such as Process Reward Models (PRMs) (Liu et al., 2025). However, existing PRMs face several limitations. They exhibit limited process supervision capabilities and struggle to generalize across different models and tasks (Zheng et al., 2024; Zhang et al., 2025c; Liu et al., 2025). Furthermore, most current approaches train PRMs as classifiers that output scalar values, neglecting the natural language generation abilities of LLMs, which are pre-trained on extensive corpora. This classifier-based modeling inherently prevents PRMs from leveraging test-time scaling methods to enhance process supervision capabilities. These limitations lead us to the following research question: ***How can generative modeling enhance the process supervision capabilities of PRMs while enabling test-time scaling?***

In this work, we address these challenges through a generative process reward model, named GenPRM. Specifically, GenPRM differs from classification-based PRMs in that GenPRM redefines process supervision as a generative task rather than a discriminative scoring task by integrating Chain-of-Thought (CoT) (Wei et al., 2022) reasoning and code verification processes before providing final judgment. To improve conventional hard label estimation, we propose Relative Progress Estimation (RPE), which leverages a relative criterion for label estimation. Additionally, we introduce a rationale synthesis framework with code verification to obtain high-quality process supervision reasoning data. A comparison of our method with previous classification-based methods is presented in Figure 2.

Our contributions can be summarized as follows:

1. We propose a generative process reward model that performs explicit CoT reasoning with code verification and utilizes Relative Progress Estimation to obtain accurate PRM labels.
2. Empirical results on ProcessBench and common mathematical reasoning tasks demonstrate that GenPRM outperforms prior classification-based PRMs. Additionally, smaller GenPRM models can surpass larger PRMs via TTS.
3. We provide a new perspective on PRMs in this work, fully leveraging their TTS capabilities, reshaping their applications, and opening new directions for future research in process supervision.

2. Preliminaries

2.1. Markov Decision Process

Following Liu et al. (2025), we formulate the test-time scaling process with PRMs as a Markov Decision Process (MDP) defined by $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$, where $\mathcal{S}$ is the state space, $\mathcal{A}$ is the action space, P represents transition dynamics, $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is the reward function, and $\gamma \in [0, 1]$ is the discount factor. Starting with a prompt set $\mathcal{X}$ and an initial state $s_1 = x \sim \mathcal{X}$, the policy model π_θ generates an action $a_1 \sim \pi_\theta(\cdot \mid s_1)$.[1] Unlike traditional RL methods with stochastic transitions (Liu et al., 2022, 2024), transitions in LLMs are deterministic, i.e., $s_{t+1} = P(\cdot \mid s_t, a_t) = [s_t, a_t]$, where $[\cdot, \cdot]$ denotes string concatenation. This process continues until the episode terminates (i.e., generating the `[EOS]` token), obtaining a trajectory of T steps: $\tau = \{a_1, a_2, \cdots, a_T\}$. The goal is to optimize either the reward of each step (as in search-based methods) or the reward over the full response (as in Best-of-N sampling).

2.2. Supervised Fine-Tuning

Supervised Fine-Tuning (SFT) trains a model to predict the next token based on prior context. For a dataset $\mathcal{D}_{\text{SFT}} = \{(x^{(i)}, y^{(i)})\}_{i=1}^{N}$, the SFT loss is:

$$\mathcal{L}_{\text{SFT}}(\theta) = -\mathbb{E}_{(x,y)\sim\mathcal{D}_{\text{SFT}}} \left[\sum_{t=1}^{|y|} \log \pi_\theta(y_t \mid x, y_{1:t-1}) \right], \tag{1}$$

where π_θ represents a model with parameters θ.

2.3. Test-Time Scaling

In this work, we consider two test-time scaling methods, including majority voting and Best-of-N.

Majority Voting. Majority voting (Wang et al., 2023) selects the answer that appears the most frequently among all solutions.

Best-of-N. Best-of-N (BoN) (Brown et al., 2024; Snell et al., 2025) selects the best answer from N candidate solutions.

3. Method

In this section, we first describe how to develop GenPRM and integrate the reasoning process with code verification. We then introduce how to scale test-time compute of policy models using GenPRM and apply TTS for GenPRM. Last, we present the improved label estimation method and data generation and filtering framework of GenPRM.

3.1. GenPRM and Test-Time Scaling

3.1.1. From Discriminative PRM to Generative PRM

Discriminative PRM. Assume we have a PRM dataset $\mathcal{D}_{\text{Disc}} = \{(s_t, a_t), r_t\}$, where $r_t \in \{0, 1\}$ for PRM labels with hard estimation. The discriminative PRM r_ψ is trained via cross-entropy loss (Skywork

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[1] Following Snell et al. (2025); Liu et al. (2025), we refer to models that generate solutions as policy models.

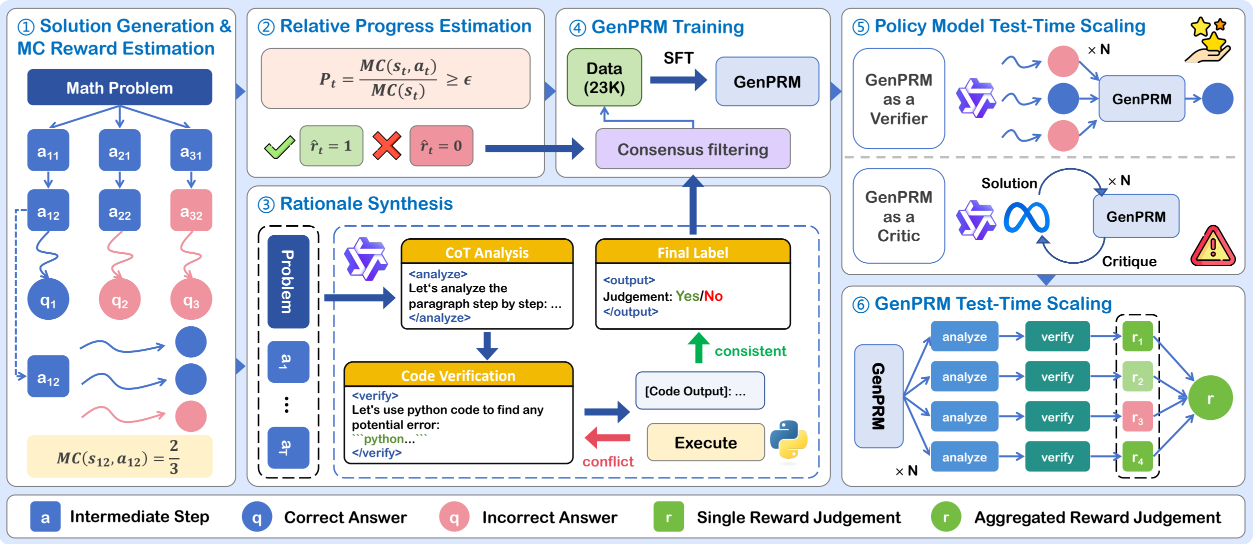

Figure 3: Overall framework of GenPRM. Our framework consists of six key parts: ① The policy model generates solution steps, with MC scores estimated from rollout trajectories. ② Our proposed RPE derives accurate PRM labels. ③ High-quality process supervision data is synthesized through CoT reasoning augmented with code verification. ④ We apply consensus filtering followed by SFT to train GenPRM. ⑤ The trained GenPRM functions as a verifier or critic, enabling enhanced test-time scaling for policy models. ⑥ The performance of GenPRM further improves through test-time scaling.

o1 Team, 2024; Zhang et al., 2025c):

$$\mathcal{L}_{\text{CE}}(\psi) = -\mathbb{E}_{(s_t,a_t,r_t)\sim\mathcal{D}_{\text{Disc}}}\left[r_t \log r_\psi(s_t,a_t) + (1-r_t)\log(1-r_\psi(s_t,a_t))\right]. \tag{2}$$

Direct Generative PRM. With a dataset $\mathcal{D}_{\text{Direct-Gen}} = \{(s_t, a_t), r_t\}$, where r_t is `Yes` for a correct step and `No` otherwise, the direct generative PRM (Xiong et al., 2024) is trained through SFT to predict `Yes` or `No` for each step. For step t, we use the probability of the `Yes` token as the predicted process reward $\hat{r}_t$:

$$\hat{r}_t = r_\psi(\texttt{Yes} \mid s_t, a_t). \tag{3}$$

Generative PRM. By equipping the direct generative PRM with an explicit reasoning process like CoT (Wei et al., 2022), we obtain a generative PRM. Let $v_{1:t-1}$ denote the rationale from step 1 to $t-1$ and v_t denote the rationale for step t. Assume we have a dataset $\mathcal{D}_{\text{Gen}} = \{(s_t, a_t, v_{1:t-1}), (v_t, r_t)\}$. GenPRM learns to reason and verify each step via SFT on this dataset. The generative process reward $\hat{r}_t$ can be obtained via the following equation:

$$\hat{r}_t = r_\psi(\texttt{Yes} \mid s_t, a_t, v_{1:t-1}, v_t), \quad \text{where } v_t \sim r_\psi(\cdot \mid s_t, a_t, v_{1:t-1}) \tag{4}$$

Generative PRM with Code Verification. If we only verify the reasoning step with CoT based on natural language, the process may lack robustness in certain complex scenarios (Zhu et al., 2024; Gou et al., 2024). The difference between the generative PRM and the generative PRM with code verification is that the latter generates code to verify the reasoning step by executing it and provides the judgment based on the execution results. At step t, after generating the rationale v_t containing CoT and code, we execute the code and obtain feedback f_t. Given the current state s_t, action a_t, previous rationales $v_{1:t-1}$, and previous corresponding execution feedback $f_{1:t-1}$, the PRM first generates the rationale v_t. After execution and obtaining the feedback f_t, we compute the final generative process reward as follows:

$$\hat{r}_t = r_\psi(\texttt{Yes} \mid s_t, a_t, v_{1:t-1}, f_{1:t-1}, v_t, f_t), \quad \text{where } v_t \sim r_\psi(\cdot \mid s_t, a_t, v_{1:t-1}, f_{1:t-1}) \tag{5}$$

In the following sections, we refer to GenPRM as this generative PRM type with code verification. The effectiveness of CoT and code verification can be found in Section 4.4.

3.1.2. *Test-Time Scaling*

Policy Model TTS: GenPRM as a Verifier. To scale the test-time compute of policy models, we can sampling multiple responses from policy models and then use GenPRM as a verifier to select the final answer (Snell et al., 2025) in the way of parallel TTS.

Policy Model TTS: GenPRM as a Critic. By equipping the PRM with generative process supervision abilities, GenPRM can be naturally used as a critic model to refine the outputs of policy models and we can scale the refinement process with multiple turns in a sequential TTS manner.

GenPRM TTS. When evaluating each solution step, we first sample N reasoning verification paths and then use majority voting to obtain the final prediction by averaging the rewards. For GenPRM without code verification, the rewards are computed as follows:

$$\hat{r}_t = \frac{1}{N}\sum_{i=1}^{N} r_\psi(\texttt{Yes} \mid s_t, a_t, v^i_{1:t-1}, v^i_t). \tag{6}$$

And we can further incorporate code verification and execution feedback into this reasoning process:

$$\hat{r}_t = \frac{1}{N}\sum_{i=1}^{N} r_\psi(\texttt{Yes} \mid s_t, a_t, v^i_{1:t-1}, f^i_{1:t-1}, v^i_t, f^i_t). \tag{7}$$

Then the rewards can be used for ranking the responses of policy models or be converted into binary labels through a threshold 0.5 for judging the correctness of the step. The discussion of using code verification can be found at Table 5.

3.2. Synthesizing Data of GenPRM

In this section, we introduce our pipeline for synthesizing training data of GenPRM. The pipeline consists of three stages: (1) generating reasoning paths and obtaining PRM labels via Monte Carlo (MC) estimation; (2) evaluating the progress of each step via Relative Progress Estimation; and (3) synthesizing rationales with CoT and code verification, and inferring LLM-as-a-judge labels with consensus filtering.

3.2.1. *Solution Generation and Monte Carlo Estimation*

Solution Generation with Step Forcing. We utilize the 7.5K problems from the training set of the MATH dataset (Hendrycks et al., 2021) as the problem set. For each problem, we use Qwen2.5-7B-Instruct (Yang et al., 2024a) as the generation model to collect multiple solutions. Since using “`\n\n`” for step division does not consider the semantics of each step and may result in overly fine-grained division, we apply a step forcing approach to generate solutions. Specifically, we add “`Step 1:`” as the prefix for the generation model to complete the response. For a response with T reasoning steps, the format is as follows:

The response format with step forcing

Step 1: {step content}
...
Step T: {step content}

The proportion of correct paths versus incorrect paths varies significantly depending on the difficulty of the problems. To ensure a sufficient number of correct and incorrect paths, we sample up to 2048 paths for both hard and easy problems. If no correct or incorrect paths are found after sampling 2048 responses, we discard the corresponding problems.

Balancing the Precision and Efficiency of MC Estimation. Following Math-Shepherd (Wang et al., 2024b), we estimate the probability of correctness for each step using completion-based sampling. For each reasoning step s_t, we generate K completion trajectories using a completion model, specifically Qwen2.5-Math-7B-Instruct (Yang et al., 2024b), and use MC estimation to calculate the probability that the current step a_t is correct (Wang et al., 2024b; Zhang et al., 2025c):

$$MC(s_t, a_t) = MC(s_{t+1}) = \frac{1}{K}\sum_{j=1}^{K} \mathbb{1}(q_j = q^*), \tag{8}$$

where q_j is the answer of the j-th response, q^* is the ground-truth answer, and $\mathbb{1}$ is the indicator function. However, it is difficult for the completion model to reach the correct answer for hard problems even when the original step is correct, leading to incorrect results for MC estimation. To address this and balance the computation cost, we use a dynamic K based on the estimated Pass@1 $MC(s_1)$:

$$K = \begin{cases} 128 & \text{if } 0 \leq MC(s_1) < 0.1, \\ 64 & \text{if } 0.1 \leq MC(s_1) < 0.9, \\ 32 & \text{if } 0.9 \leq MC(s_1) < 1. \end{cases} \tag{9}$$

3.2.2. Relative Progress Estimation

Previous work has shown that hard label estimation is better than soft label estimation for PRMs (Zhang et al., 2025c). However, after MC estimation, we observe that although the MC score of many steps is greater than 0, the steps are incorrect, as also noted by Zhang et al. (2025c). On the other hand, we assume that a positive step should be both correct and beneficial. A reasoning step is considered as a beneficial one if it is easier to reach the correct answer by adding this step as the generation prefix. To address these issues, we propose Relative Progress Estimation (RPE), which shares a similar idea with relative advantage estimation in GRPO (Shao et al., 2024; DeepSeek-AI et al., 2025), to improve conventional hard label estimation.

Specifically, the MC score is an empirical estimation of the current state s_t. To evaluate the quality of the current action a_t, it is natural to compare the MC score of the next state s_{t+1} with that of the current state s_t, since $s_{t+1} = [s_t, a_t]$. For each response, if the first erroneous step is step t' (i.e., $MC(s_{t'}) = 0$), we set the MC score of the following steps to 0. Our RPE P_t for step t is defined as follows:

$$P_t = \frac{MC(s_t, a_t)}{MC(s_t)}, \tag{10}$$

where $MC(s_1)$ is the estimated Pass@1 computed in the solution generation phase. However, we empirically find that using a strict criterion where progress is always greater than 1 leads to unsatisfactory performance, as shown in Table 3. To address this, we estimate the final reward label $\hat{r}_t$ by introducing a threshold ϵ:

$$\hat{r}_t = \begin{cases} 1 & \text{if } P_t \geq \epsilon, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

We also discuss another form of relative progress $P_t = MC(s_t, a_t) - MC(s_t)$ in Table 3 in Section 4.4.

3.2.3. *Rationale Generation, Verification and Filtering*

To obtain high-quality rationale data, we use QwQ-32B (Qwen Team, 2025) as the rationale generation model and introduce a three-step pipeline that automatically generates and verifies the rationale of each reasoning step. Given a problem x with a ground-truth answer q^* and candidate steps $\{a_1, \cdots, a_T\}$, the generation and verification proceed as follows:

Step 1: Code-Based Rationale Generation. To evaluate the correctness of a_t, we synthesize step-by-step CoT analysis. It has been shown that program-based reasoning improves verification outcomes (Zhu et al., 2024). Based on CoT analysis, we continue to synthesize code-based rationales to verify a_t based on the problem and historical steps $\{a_1, \cdots, a_{t-1}\}$. We prompt the rationale generation model to surround the CoT with `<analyze>` and `</analyze>`, and the code with `<verify>` and `</verify>`. The prompt for rationale generation is shown in Table A.2.

Step 2: Code Execution and Verification. With the generated code, we execute it and obtain the feedback f_t for step t. The execution feedback is formatted as `[Code output: {execution result}]` and is concatenated to the generated CoT and code as the prefix for the subsequent generation. If the execution result is inconsistent with the generated CoT verification, we observe that QwQ-32B performs self-reflection behaviors until reaching a consensus.

Step 3: Label Judgment and Consensus Filtering. After generating and verifying the rationale data of all candidate steps, the rationale generation model finally outputs an number. If all steps are inferred to be correct, the number will be -1, otherwise will be the index of the first erroneous step. For each solution, if there is at least one process label with RPE is not consistent with the labels generated by LLM-as-a-judge (Zheng et al., 2023), we discard the entire solution and only retain the one with all labels consistent. After consensus filtering, we discard approximately 51% of the data and finally obtain a dataset containing 23K problems with reasoning steps and rationale data.

4. Experiments

In this section, we aim to answer the following questions:

- **Q1:** How does GenPRM perform compared with previous PRMs? (§4.2, §4.3)
- **Q2:** How does the performance of GenPRM scale with more test-time compute? (§4.2, §4.3)
- **Q3:** How does GenPRM benefit policy model test-time scaling? (§4.3)
- **Q4:** How do the components and hyperparameters influence GenPRM? (§4.4)

4.1. Setup

Benchmarks. We evaluate GenPRM and baseline methods on ProcessBench (Zheng et al., 2024), a benchmark designed to assess process supervision capabilities in mathematical reasoning tasks.[2]

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[2] Our evaluation code is adapted from `https://github.com/QwenLM/ProcessBench`.

Additionally, we conduct BoN and critic refinement experiments using MATH (Hendrycks et al., 2021), AMC23 (AI-MO, 2024b), AIME24 (AI-MO, 2024a), and Minerva Math (Lewkowycz et al., 2022). For BoN response generation, we employ Qwen2.5-Math-7B-Instruct (Yang et al., 2024b) and Gemma-3-12b-it (Gemma Team and Google DeepMind, 2025) as policy models. For policy model TTS with GenPRM as the critic, we use Gemma-3-12b-it (Gemma Team and Google DeepMind, 2025) and Qwen2.5-7B Instruct (Yang et al., 2024a) as generators.

Baselines. For ProcessBench and BoN experiments, we compare GenPRM with the following methods:

- **Math-Shepherd-PRM-7B** (Wang et al., 2024b): This method trains a PRM using hard labels computed based on MC estimation.
- **RLHFlow series** (Xiong et al., 2024): Includes RLHFlow-PRM-Mistral-8B and RLHFlow-PRM-Deepseek-8B.
- **Skywork-PRM series** (Skywork o1 Team, 2024): Comprises Skywork-PRM-1.5B and Skywork-PRM-7B.
- **EurusPRM** (Cui et al., 2025): EurusPRM-Stage1 and EurusPRM-Stage2 are trained as implicit PRMs (Yuan et al., 2024).
- **Qwen2.5-Math series** (Zheng et al., 2024; Zhang et al., 2025c): Qwen2.5-Math-7B-Math-Shepherd and Qwen2.5-Math-7B-PRM800K are trained with Math-Shepherd (Wang et al., 2024b) and PRM800K (Lightman et al., 2024), respectively. For Qwen2.5-Math-PRM-7B and Qwen2.5-Math-PRM-72B, the training data is applied consensus filtering using LLM-as-a-judge (Zheng et al., 2023).
- **RetrievalPRM-7B** (Zhu et al., 2025): The method enhances PRM with retrieved questions and corresponding steps.
- **Universal-PRM-7B** (Tan et al., 2025): The method proposes an automated framework using ensemble prompting and reverse verification.
- **Dyve-14B** (Zhong et al., 2025): This method dynamically applies fast or slow verification for each reasoning step.
- **Direct Generative PRM-7B**: The method trains a direct generative PRM with the original language head via SFT using the same data as GenPRM, but without CoT and code verification.

For critic experiments, we use the following methods for comparison:

- **Self-Refine** (Madaan et al., 2023): This method uses the generator to self-critique and refine the solution.
- **DeepSeek-R1-Distill-Qwen-7B** (DeepSeek-AI et al., 2025): This model is fine-tuned based on Qwen2.5-Math-7B (Yang et al., 2024a) using high-quality reasoning data generated by DeepSeek-R1 (DeepSeek-AI et al., 2025).

Implementation Details. For RPE, we set $\epsilon = 0.8$ across all experiments, with ablation studies presented in Section 4.4. Rationale data is generated using QwQ-32B (Qwen Team, 2025) and the prompt template is shown in Table A.2. Our base models are from the DeepSeek-R1-Distill series (DeepSeek-AI et al., 2025), specifically the 1.5B, 7B, and 32B parameter variants. The training configuration for our method uses a batch size of 64 and a learning rate of 2.0×10^{-6}. During evaluation, we employ a temperature of 0.6. For critique refinement experiments, we extract content within the `<analyze></analyze>` tags, focusing exclusively on steps predicted as negative by the policy model. The baseline methods utilize standardized prompt templates (detailed in Table A.2) to ensure consistent critique generation formats.

Table 1: ProcessBench results reported with F1 scores. The results of GenPRM are shaded. For 1.5B PRMs, **bold** indicates the best Pass@1 or scores superior to GPT-4o. For 7-8B and 14-72B PRMs, **bold** denotes the best Pass@1 or scores superior to Qwen2.5-Math-PRM-72B.

Model	**# Samples**	**GSM8K**	**MATH**	**Olympiad Bench**	**Omni-MATH**	**Avg.**
Proprietary LLMs (Critic)						
GPT-4o-0806	unk	79.2	63.6	51.4	53.5	61.9
o1-mini	unk	93.2	88.9	87.2	82.4	87.9
PRMs (1.5B)						
Skywork-PRM-1.5B	unk	**59.0**	48.0	19.3	19.2	36.4
GenPRM-1.5B (Pass@1)	23K	52.8	**66.6**	**55.1**	**54.5**	**57.3**
GenPRM-1.5B (Maj@8)	23K	51.3	**74.4**	**65.3**	**62.5**	**63.4**
PRMs (7-8B)						
Math-Shepherd-PRM-7B	445K	47.9	29.5	24.8	23.8	31.5
RLHFlow-PRM-Mistral-8B	273K	50.4	33.4	13.8	15.8	28.4
RLHFlow-PRM-Deepseek-8B	253K	38.8	33.8	16.9	16.9	26.6
Skywork-PRM-7B	unk	70.8	53.6	22.9	21.0	42.1
EurusPRM-Stage1	463K	44.3	35.6	21.7	23.1	31.2
EurusPRM-Stage2	30K	47.3	35.7	21.2	20.9	31.3
Qwen2.5-Math-7B-Math-Shepherd	445K	62.5	31.6	13.7	7.7	28.9
Qwen2.5-Math-7B-PRM800K	264K	68.2	62.6	50.7	44.3	56.5
Qwen2.5-Math-PRM-7B	$\sim$344K	82.4	77.6	67.5	66.3	73.5
RetrievalPRM-7B	404K	74.6	71.1	60.2	57.3	65.8
Universal-PRM-7B	unk	**85.8**	77.7	67.6	66.4	74.3
Direct Generative PRM-7B	23K	63.9	65.8	54.5	55.9	60.0
GenPRM-7B (Pass@1)	23K	78.7	**80.3**	**72.2**	**69.8**	**75.2**
GenPRM-7B (Maj@8)	23K	81.0	**85.7**	**78.4**	**76.8**	**80.5**
PRMs (14-72B)						
Dyve-14B	117K	68.5	58.3	49.0	47.2	55.8
Qwen2.5-Math-PRM-72B	$\sim$344K	**87.3**	80.6	74.3	71.1	78.3
GenPRM-32B (Pass@1)	23K	83.1	**81.7**	72.8	**72.8**	77.6
GenPRM-32B (Maj@8)	23K	85.1	**86.3**	**78.9**	**80.1**	**82.6**

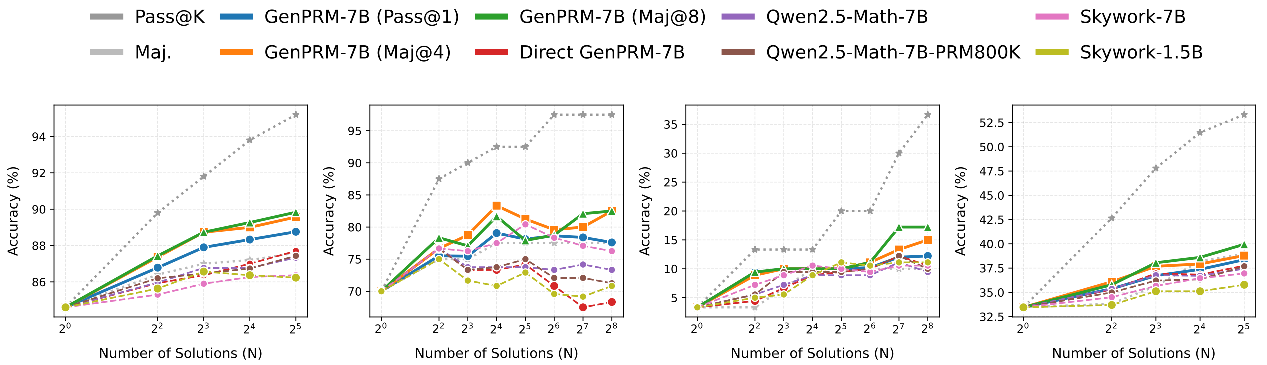
4.2. ProcessBench Results

GenPRM outperforms classification-based PRMs on ProcessBench. As shown in Table 1, GenPRM-7B significantly outperforms direct generative PRM and surpasses **all** previous PRMs with parameters less than 72B on ProcessBench. Also, GenPRM-1.5B outperforms Skywork-PRM-1.5B by a large margin. It is noteworthy that GenPRM is trained with merely **23K** data from **MATH** (Hendrycks et al., 2021) only. By comparing the detailed results in Table 6, we can find that the performance gain of GenPRM mainly comes from the stronger abilities of finding **erroneous** steps and we provide concrete cases in Appendix C. These results demonstrating the superiority of generative modeling of PRM.

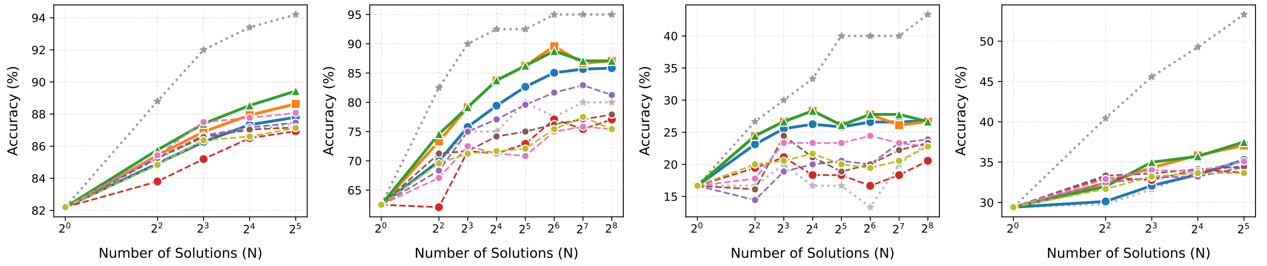
GenPRM enables smaller PRMs surpass $10\times$ larger PRMs and GPT-4o via TTS. We also compare the TTS results of GenPRM in Table 1 and find that GenPRM-1.5B surpasses GPT-4 and GenPRM-7B exceeds Qwen2.5-Math-PRM-72B on ProcessBench via simply majority voting, showing that scaling test-time compute is highly effective for GenPRM. We also find that the performance improvement of scaling the test-time compute on **harder** problems is larger than that of easier questions.

4.3. Policy Model Test-Time Scaling Results

GenPRM as a Verifier. The results in Figure 4 (a)-(d) show that GenPRM outperforms the baselines on MATH, AMC23, AIME24, and Minerva Math with Qwen2.5-Math-7B-Instruct (Yang et al., 2024b) as the generation model. The advantage of GenPRM becomes larger by scaling the test-time compute of GenPRM and the generation model. Figure 4 (e)-(h) demonstrates that GenPRM generalizes well to responses with Gemma-3-12b-it (Gemma Team and Google DeepMind, 2025) as the generation model.



(a) MATH (Qwen) (b) AMC23 (Qwen) (c) AIME24 (Qwen) (d) Minerva Math (Qwen)



(e) MATH (Gemma) (f) AMC23 (Gemma) (g) AIME24 (Gemma) (h) Minerva Math (Gemma)

Figure 4: BoN results with different generation models on multiple mathematical benchmarks.

GenPRM as a Critic. We also conduct experiments by using GenPRM as a critic to refine the outputs of the policy model. The results in Table 2 and Figure 1 (right) show that GenPRM exhibits strong critique abilities than the baselines, significantly improving the performance of the policy model and the performance continues to increase with more refinement based on the critic feedback.

4.4. Analysis

Label Estimation Method and Criterion. To explore how different label estimation influences GenPRM, we conduct experiments with the following methods: (1) hard label (Wang et al., 2024b; Zhang et al., 2025c); (2) RPE in (10); and (3) a RPE variant ($P_t = MC(s_t, a_t) - MC(s_t)$). For the RPE and its variant, we use different thresholds ϵ for evaluation and set the labels as correct by checking whether $P_t \geq \epsilon$. The results in Table 3 show that RPE and its variant outperforms hard label estimation and RPE with $\epsilon = 0.8$ achieves the best result. By scaling test-time compute with majority voting, the results in Table 4 demonstrate that RPE with $\epsilon = 0.8$ still reaches the best.

Table 2: Results of critique refinement experiments. The results of GenPRM are shaded. For each refinement turn, the highest values are **bolded**.

Critic Model	***Gemma-3-12b-it as Generator***					***Qwen2.5-7B-Instruct as Generator***					**Avg.**
	AMC23	AIME24	MATH	Minerva Math	**Avg.**	AMC23	AIME24	MATH	Minerva Math	**Avg.**	
Zero-shot	64.1	15.8	83.8	31.9	48.9	51.6	7.1	76.2	34.5	42.4	45.7
Turn 1											
Generator	66.6	15.8	84.7	33.3	50.1	50.6	8.0	76.8	34.0	42.4	**46.3**
DeepSeek-R1-Distill-7B	69.1	17.9	84.6	33.0	51.2	50.6	6.3	77.7	34.7	42.3	**46.8**
GenPRM-7B	**74.1**	**19.6**	**86.0**	**35.3**	**53.8**	**57.5**	**8.3**	**80.6**	**36.5**	**45.7**	**49.8**
Turn 2											
Generator	66.6	18.0	84.8	31.6	50.3	49.8	8.0	76.9	31.8	41.6	**46.0**
DeepSeek-R1-Distill-7B	70.9	18.3	85.0	33.5	51.9	51.9	7.9	78.1	32.8	42.7	**47.3**
GenPRM-7B	**75.0**	**21.3**	**86.9**	**35.6**	**54.7**	**59.4**	**9.6**	**82.2**	**35.0**	**46.6**	**50.7**
Turn 3											
Generator	67.8	18.1	85.0	32.1	50.8	49.7	8.1	77.1	30.8	41.4	**46.1**
DeepSeek-R1-Distill-7B	69.6	18.8	85.0	33.4	51.7	51.9	8.3	78.2	32.7	42.7	**47.2**
GenPRM-7B	**76.2**	**22.8**	**86.7**	**36.0**	**55.4**	**62.7**	**9.3**	**82.9**	**34.9**	**47.5**	**51.5**

Table 3: Results of GenPRM with different label estimation method and threshold on ProcessBench, reported with Pass@1. The best results are shown in **bold**.

Estimation Method	**Positive Label Criterion**	**GSM8K**	**MATH**	**Olympiad Bench**	**Omni-MATH**	**Avg.**
$P_t = MC(s_t, a_t)$ (hard label)	$P_t > 0$	72.9	78.9	**73.2**	68.0	73.2
$P_t = MC(s_t, a_t) - MC(s_t)$	$P_t \geq -0.1$	77.3	79.9	70.8	68.5	74.1
	$P_t \geq -0.3$	76.8	79.6	71.1	69.0	74.1
	$P_t \geq -0.5$	75.8	80.2	72.8	68.6	74.3
$P_t = \frac{MC(s_t, a_t)}{MC(s_t)}$	$P_t \geq 0.1$	74.8	78.7	71.6	68.7	73.5
	$P_t \geq 0.5$	75.7	79.2	70.4	68.5	73.5
	$P_t \geq 0.8$	**78.7**	**80.3**	72.2	**69.8**	**75.2**
	$P_t \geq 1.0$	76.4	77.4	68.1	67.2	72.3

Table 4: Results of GenPRM with different label estimation method and threshold on ProcessBench, reported with Maj@8. The best results are shown in **bold**.

Estimation Method	**Positive Label Criterion**	**GSM8K**	**MATH**	**Olympiad Bench**	**Omni-MATH**	**Avg.**
$P_t = MC(s_t, a_t)$ (hard label)	$P_t > 0$	75.1	83.8	**80.6**	74.4	78.5
$P_t = MC(s_t, a_t) - MC(s_t)$	$P_t \geq -0.1$	79.8	85.1	78.0	74.5	79.4
	$P_t \geq -0.3$	80.9	**86.5**	78.1	75.0	80.2
	$P_t \geq -0.5$	78.1	85.6	79.1	73.4	79.1
$P_t = \frac{MC(s_t, a_t)}{MC(s_t)}$	$P_t \geq 0.1$	77.0	84.6	78.1	75.3	78.7
	$P_t \geq 0.5$	78.0	85.2	78.2	74.3	78.9
	$P_t \geq 0.8$	81.0	85.7	78.4	**76.8**	**80.5**
	$P_t \geq 1.0$	**81.1**	84.1	76.0	74.7	79.0

Reasoning Components. To understand how each reasoning component influence GenPRM, we conduct experiments by training GenPRM with: (1) CoT data only, (2) code verification data only, and (3) full data. During inference phase, we also compare several variants. For example, GenPRM trained with full data can be used to only verify each step with CoT only by stopping generation at `</analyze>` token. The results in Table 5 show that: (1) the improvement of GenPRM mainly comes from CoT reasoning; (2) generating code and reasoning with code execution result improves the process verification performance as well.

Table 5: Results on ProcessBench of GenPRM with different reasoning components, reported with Maj@8. The best results are shown in **bold**.

Training		**Inference**			**GSM8K**	**MATH**	**Olympiad Bench**	**Omni-MATH**	**Avg.**
CoT	Code	CoT	Code	Code Exec.					
✗	✗	✗	✗	✗	63.9	65.8	54.5	55.9	60.0
✗	✓	✗	✓	✗	67.0	70.8	61.6	57.4	64.2
✗	✓	✗	✓	✓	70.6	76.6	67.3	63.9	69.6
✓	✗	✓	✗	✗	76.4	83.0	**80.5**	75.4	78.8
✓	✓	✗	✓	✗	60.1	66.7	59.9	59.2	61.5
		✗	✓	✓	61.3	74.7	68.1	62.0	66.5
		✓	✗	✗	78.8	85.1	78.7	74.9	79.3
		✓	✓	✗	**81.0**	85.1	78.1	75.5	79.9
		✓	✓	✓	**81.0**	**85.7**	78.4	**76.8**	**80.5**

5. Related Work

Process Reward Models. Process reward models have been proved to be effective for providing step-wise scores and are superior to outcome reward models in mathematical reasoning tasks (Uesato et al., 2022; Lightman et al., 2024). However, annotating a process supervision dataset such as PRM800K (Lightman et al., 2024) requires significant human costs. To mitigate this cost, prior works utilize Monte Carlo estimation (Wang et al., 2024b) and binary search (Luo et al., 2024) for automated label generation. Subsequent research improves PRMs through methods such as advantage modeling (Setlur et al., 2025), Q-value rankings (Li and Li, 2025), implicit entropy regularization (Zhang et al., 2024a), retrieval-augmented generation (Zhu et al., 2025), and fast-slow verification (Zhong et al., 2025). Furthermore, the community has developed high-quality open-source PRMs, including the RLHFlow series (Xiong et al., 2024), Math-psa (Wang et al., 2024a), Skywork series (Skywork o1 Team, 2024), and Qwen2.5-Math series (Zheng et al., 2024; Zhang et al., 2025c). Recently, a line of works focus on extending PRMs to other tasks, including coding (Zhang et al., 2024b), medical tasks (Jiang et al., 2025), agentic tasks (Choudhury, 2025), general domain tasks (Zhang et al., 2025a; Zeng et al., 2025), and multimodal tasks (Wang et al., 2025). Current studies also focus on benchmarking PRMs (Zheng et al., 2024; Song et al., 2025) to systematically evaluate their performance.

Large Language Model Test-Time Scaling. Scaling test-time computation is an effective method for improving performance during the inference phase (OpenAI, 2024a,b; DeepSeek-AI et al., 2025). TTS is commonly implemented with external verifiers (e.g., ORMs and PRMs) or strategies (e.g., beam search and MCTS) (Wu et al., 2025; Snell et al., 2025; Beeching et al., 2024; Liu et al., 2025). In this work, we scale the test-time computation of a generative PRM with an explicit reasoning process and GenPRM can also serve as a verifier or a critic model in external TTS.

Enhancing the Generative Abilities of Reward Models. Previous research has investigated methods to enhance the generative capabilities of reward models using CoT reasoning (Ankner et al., 2024; Zhang et al., 2025b; Mahan et al., 2024). For instance, CLoud reward models (Ankner et al., 2024) are trained to generate critiques for responses and predict rewards using an additional reward head. GenRM-CoT (Zhang et al., 2025b) and GenRM (Mahan et al., 2024) train generative reward models that perform CoT reasoning before making final predictions via SFT and preference learning, respectively. CTRL (Xie et al., 2025) demonstrates that critic models exhibit strong discriminative abilities when utilized as generative reward models. Prior to these works, GRM (Yang et al., 2024c) regularizes the hidden states of reward models with a text generation loss.

6. Conclusion

In this work, we propose GenPRM, a generative process reward model that performs explicit reasoning and code verification for process supervision and enables scaling the test-time compute of PRMs. Experimental results on ProcessBench and several mathematical datasets show GenPRM outperforms prior PRMs. We also demonstrate that the performance of GenPRM increases via test-time scaling and GenPRM is effective as a critic model. We believe that this work provides perspectives on PRMs by demonstrating the strong TTS abilities of PRMs and extending the applications of PRMs.

Limitations. First, GenPRM provides process supervision by generative reasoning, which introduces additional computation during inference phase. Future work will investigate how to prune the reasoning process dynamically (Zhong et al., 2025). Although GenPRM focuses mainly on mathematical reasoning tasks, it is worth to explore how to apply generative reasoning on coding and general reasoning tasks in the future (Zhang et al., 2025a). Additionally, it would be interesting to leverage RL to incentivize the generative reasoning abilities of GenPRM.

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A. Experimental Details

A.1. Scoring and Voting Methods

PRM-Last. PRM-Last considers the process reward of the last step of the entire LLM response as the final score, i.e., $\text{score} = r_T$.

PRM-Avg. PRM-Avg computes the mean process reward across all steps as the final score, i.e., $\text{score} = \frac{1}{T}\sum_{t=1}^{T} r_t$.

PRM-Min. PRM-Min uses the minimum process reward across all steps as the final score, i.e., $\text{score} = \min_r \{r_t\}_{t=1}^{T}$.

A.2. Implementation Details

Prompt for CoT and code rationale generation is shown in Table A.2.

Prompt for CoT and code rationale generation

[System]:
You are a math teacher. Your task is to review and critique the paragraphs in solution step by step with python code.

[User]:
The following is the math problem and a solution (split into paragraphs, enclosed with tags and indexed from 1):

[Math Problem]

{problem}

[Solution]

{solution_section}

Your task is to verify the correctness of paragraph in the solution. Split your verification by ‘`### Paragraph {{ID}}`‘.

Your verification for each paragraph should be constructed by 2 parts, wrapped by ‘`<analyze></analyze>`‘ and ‘`<verify></verify>`‘ separately.

1. In ‘`<analyze></analyze>`‘ part, you need to analyze the reasoning process and explain why the paragraph is correct or incorrect in detail.
2. In ‘`<verify></verify>`‘ part, you must write **Python code** in the form of ‘‘‘`python\n{{CODE}}\n`‘‘‘ to verify every details that can be verified by code. You can import PyPI (i.e., ‘sympy‘, ‘scipy‘ and so on) to implement complicated calculation. Make sure to print the critic results in the code. Every code will be executed automatically by system. You need to analyze the ‘[Code Output]‘ after code executing.

>Pay attention that you must follow the format of ```python\n{{CODE}}\n``` when you write the code, otherwise the code will not be executed.

After all verifications, if you identify an error in a paragraph, return the **index of the paragraph where the earliest error occurs**. Otherwise, return the **index of -1 (which typically denotes "not found")**. Please put your final answer (i.e., the index) within box in the form of `$\\boxed{{INDEX}}$`.

Following Zheng et al. (2024); Zhang et al. (2025c), we use the prompt in Table A.2 to evaluate LLM-as-a-judge methods on ProcessBench (Zheng et al., 2024).

Evaluation prompt for LLM-as-a-judge methods on ProcessBench

I will provide a math problem along with a solution. They will be formatted as follows:

[Math Problem]

<math_problem>
...(math problem)...
</math_problem>

[Solution]

<paragraph_1>
...(paragraph 1 of solution)...
</paragraph_1>

...

<paragraph_n>
...(paragraph n of solution)...
</paragraph_n>

Your task is to review each paragraph of the solution in sequence, analyzing, verifying, and critiquing the reasoning in detail. You need to provide the analyses and the conclusion in the following format:

<analysis_1>
...(analysis of paragraph 1)...
</analysis_1>

...

<analysis_n>
...(analysis of paragraph n)...
</analysis_n>

<conclusion>

Correct/Incorrect
</conclusion>

* When you analyze each paragraph, you should use proper verification, recalculation, or reflection to indicate whether it is logically and mathematically valid. Please elaborate on the analysis process carefully.

* If an error is detected in any paragraph, you should describe the nature and cause of the error in detail, and suggest how to correct the error or the correct approach. Once a paragraph is found to contain any error, stop further analysis of subsequent paragraphs (as they may depend on the identified error) and directly provide the conclusion of "Incorrect."

For instance, given a solution of five paragraphs, if an error is found in the third paragraph, you should reply in the following format:

<analysis_1>
...(analysis of paragraph 1)...
</analysis_1>

<analysis_2>
...(analysis of paragraph 2)...
</analysis_2>

<analysis_3>
...(analysis of paragraph 3; since an error is found here, also provide detailed critique and correction guideline)...
</analysis_3>

<conclusion>
Incorrect
</conclusion>

Note that the analyses of paragraphs 4 and 5 should be skipped as the paragraph 3 has been found to contain an error.

* Respond with your analyses and conclusion directly.

The following is the math problem and the solution for you task:

[Math Problem]

{tagged_problem}

[Solution]

{tagged_response}

Prompt for critique generation

[User]:
The following is a math problem and my solution. Your task is to review and critique the paragraphs in solution step by step. Pay attention that you should not solve the problem and give the final answer. All of your task is to critique. Output your judgement of whether the paragraph is correct in the form of ‘`\\boxed{{Yes|No}}`‘ at the end of each paragraph verification:

[Math Problem]

{problem}

[Solution]

<paragraph_{idx}>
{paragraph_content}
</paragraph_{idx}>

B. Additional Results

We provide full results of ProcessBench in Table 6.

Model Size. We investigate the impact of model size on GenPRM by evaluating variants with 1.5B, 7B, and 32B parameters. As shown in Table 7, scaling the model from 1.5B to 7B parameters yields substantial performance gains ($57.3 \rightarrow 75.2$ and $63.4 \rightarrow 80.5$). However, further increasing the model size to 32B provides only marginal improvements, suggesting that the 7B variant offers the best balance between efficiency and effectiveness.

Data Size. To assess the influence of training data volume, we train GenPRM on progressively larger subsets of ProcessBench (25%, 50%, and 100% of the full dataset). Table 8 demonstrates that Pass@1 F1 scores improve rapidly with initial data increases, but the growth rate slows substantially with additional data.

Inference Tokens. We provide statistics of the reasoning tokens per step and per response in Table 9.

C. Cases

In this section, we analyze two cases to have a better understanding of GenPRM. The case in Figure 5 shows that the code execution feedback can correct the mistakes in CoT and enhance the process supervision abilities of GenPRM. The case in Figure 6 demonstrates that GenPRM provides accurate process supervision with CoT reasoning only.

Table 6: Full results of critic models and PRMs on ProcessBench.

Model	**GSM8K**			**MATH**			**OlympiadBench**			**Omni-MATH**			**Avg. F1**
	Err.	Corr.	**F1**	Err.	Corr.	**F1**	Err.	Corr.	**F1**	Err.	Corr.	**F1**	
Proprietary LLMs (Critic)													
GPT-4-0806	70.0	91.2	79.2	54.4	76.6	63.6	45.8	58.4	51.4	45.2	65.6	53.5	61.9
o1-mini	88.9	97.9	93.2	83.5	95.1	88.9	80.2	95.6	87.2	74.8	91.7	82.4	87.9
Open-Source LLMs (Critic)													
Llama-3-8B-Instruct	42.5	7.8	13.1	28.6	9.1	13.8	27.1	2.7	4.8	26.1	8.3	12.6	11.1
Llama-3-70B-Instruct	35.7	96.9	52.2	13.0	93.3	22.8	12.0	92.0	21.2	11.2	91.7	20.0	29.1
Llama-3.1-8B-Instruct	44.4	6.2	10.9	41.9	2.7	5.1	32.4	1.5	2.8	32.0	0.8	1.6	5.1
Llama-3.1-70B-Instruct	64.3	89.6	74.9	35.4	75.6	48.2	35.1	69.9	46.7	30.7	61.8	41.0	52.7
Llama-3.3-70B-Instruct	72.5	96.9	82.9	43.3	94.6	59.4	31.0	94.1	46.7	28.2	90.5	43.0	58.0
Qwen2.5-Math-7B-Instruct	15.5	100.0	26.8	14.8	96.8	25.7	7.7	91.7	14.2	6.9	88.0	12.7	19.9
Qwen2.5-Math-72B-Instruct	49.8	96.9	65.8	36.0	94.3	52.1	19.5	97.3	32.5	19.0	96.3	31.7	45.5
Qwen2.5-Coder-7B-Instruct	7.7	100.0	14.3	3.4	98.3	6.5	2.1	99.1	4.1	0.9	98.3	1.8	6.7
Qwen2.5-Coder-14B-Instruct	33.8	96.4	50.1	25.4	92.4	39.9	20.7	94.1	34.0	15.9	94.2	27.3	37.8
Qwen2.5-Coder-32B-Instruct	54.1	94.8	68.9	44.9	90.6	60.1	33.4	91.2	48.9	31.5	87.6	46.3	56.1
Qwen2-7B-Instruct	40.6	4.7	8.4	30.5	13.8	19.0	22.4	10.9	14.7	20.0	8.7	12.1	13.6
Qwen2-72B-Instruct	57.0	82.9	67.6	37.7	70.9	49.2	34.0	55.2	42.1	32.3	53.1	40.2	49.8
Qwen2.5-7B-Instruct	40.6	33.2	36.5	30.8	45.1	36.6	26.5	33.9	29.7	26.2	28.6	27.4	32.6
Qwen2.5-14B-Instruct	54.6	94.8	69.3	38.4	87.4	53.3	31.5	78.8	45.0	28.3	76.3	41.3	52.2
Qwen2.5-32B-Instruct	49.3	97.9	65.6	36.7	95.8	53.1	25.3	95.9	40.0	24.1	92.5	38.3	49.3
Qwen2.5-72B-Instruct	62.8	96.9	76.2	46.3	93.1	61.8	38.7	92.6	54.6	36.6	90.9	52.2	61.2
QwQ-32B-Preview	81.6	95.3	88.0	78.1	79.3	78.7	61.4	54.6	57.8	55.7	68.0	61.3	71.5
PRMs (1.5B)													
Skywork-PRM-1.5B	50.2	71.5	59.0	37.9	65.2	48.0	15.4	26.0	19.3	13.6	32.8	19.2	36.4
GenPRM-1.5B (Pass@1)	37.0	92.7	52.8	57.1	80.1	66.6	47.0	66.5	55.1	45.2	68.7	54.5	57.3
GenPRM-1.5B (Maj@8)	34.8	97.4	51.3	64.7	87.7	74.4	57.2	76.1	65.3	51.3	80.1	62.5	63.4
PRMs (7-8B)													
Math-Shepherd-PRM-7B	32.4	91.7	47.9	18.0	82.0	29.5	15.0	71.1	24.8	14.2	73.0	23.8	31.5
RLHFlow-PRM-Mistral-8B	33.8	99.0	50.4	21.7	72.2	33.4	8.2	43.1	13.8	9.6	45.2	15.8	28.4
RLHFlow-PRM-Deepseek-8B	24.2	98.4	38.8	21.4	80.0	33.8	10.1	51.0	16.9	10.9	51.9	16.9	26.6
Skywork-PRM-7B	61.8	82.9	70.8	43.8	62.2	53.6	17.9	31.9	22.9	14.0	41.9	21.0	42.1
EurusPRM-Stage1	46.9	42.0	44.3	33.3	38.2	35.6	23.9	19.8	21.7	21.9	24.5	23.1	31.2
EurusPRM-Stage2	51.2	44.0	47.3	36.4	35.0	35.7	25.7	18.0	21.2	23.1	19.1	20.9	31.3
Qwen2.5-Math-7B-Math-Shepherd	46.4	95.9	62.5	18.9	96.6	31.6	7.4	93.8	13.7	4.0	95.0	7.7	28.9
Qwen2.5-Math-7B-PRM800K	53.1	95.3	68.2	48.0	90.1	62.6	35.7	87.3	50.7	29.8	86.1	44.3	56.5
Qwen2.5-Math-PRM-7B	72.0	96.4	82.4	68.0	90.4	77.6	55.7	85.5	67.5	55.2	83.0	66.3	73.5
RetrievalPRM-7B	64.7	88.1	74.6	67.2	75.6	71.1	56.0	65.2	60.2	52.8	62.7	57.3	65.8
Universal-PRM-7B	-	-	85.8	-	-	77.7	-	-	67.6	-	-	66.4	74.3
Direct Generative PRM-7B	52.7	81.4	63.9	55.9	80.0	65.8	44.8	69.6	54.5	45.5	72.6	55.9	60.0
GenPRM-7B (Pass@1)	67.7	94.0	78.7	74.6	87.0	80.3	68.3	76.6	72.2	63.5	77.4	69.8	75.2
GenPRM-7B (Maj@8)	69.6	96.9	81.0	80.5	91.6	85.7	74.0	83.5	78.4	70.0	85.1	76.8	80.5
PRMs (14-72B)													
Dyve-14B	-	-	68.5	-	-	58.3	-	-	49.0	-	-	47.2	55.8
Qwen2.5-Math-PRM-72B	78.7	97.9	87.3	74.2	88.2	80.6	67.9	82.0	74.3	64.8	78.8	71.1	78.3
GenPRM-32B (Pass@1)	73.1	96.4	83.1	79.4	84.1	81.7	73.4	72.2	72.8	70.3	75.5	72.8	77.6
GenPRM-32B (Maj@8)	74.9	98.5	85.1	84.0	88.7	86.3	79.0	78.8	78.9	76.3	84.2	80.1	82.6

Table 7: Evaluation results of GenPRM with different sizes on ProcessBench.

Model Size	**Metric**	**GSM8K**	**MATH**	**OlympiadBench**	**Omni-MATH**	**Avg.**
1.5B	Pass@1	52.8	66.6	55.1	54.5	57.3
	Maj@8	81.0	74.4	65.3	62.5	63.4
7B	Pass@1	78.7	80.3	72.2	69.8	75.2
	Maj@8	81.0	85.7	78.4	76.8	80.5
32B	Pass@1	83.1	81.7	72.8	72.8	77.6
	Maj@8	85.1	86.3	78.9	80.1	82.6

Table 8: Evaluation results of GenPRM with different proportions of training data on ProcessBench.

Data Size	**Metric**	**GSM8K**	**MATH**	**OlympiadBench**	**Omni-MATH**	**Avg.**
25%	Pass@1	71.8	78.0	68.9	68.2	71.7
50%	Pass@1	78.1	80.6	71.5	67.6	74.5
100%	Pass@1	78.7	80.3	72.2	69.8	75.2

Table 9: Statistics of the output tokens of GenPRM.

	MATH	**AMC23**	**AIME24**	**Minerva Math**
Step	344.7	416.2	432.5	503.3
Response	2771.4	3200.2	4112.9	4877.1

Problem:

Simplify $\cos 54^\circ \cos 4^\circ - \cos 36^\circ \cos 86^\circ.$

Enter your answer is a trigonometric function evaluated at an integer, such as "sin 7". (The system will assume that the angle is in degrees.)

System:

You are a math teacher. Your task is to review and critique the paragraphs in solution step by step.

User (Turn 1):

To simplify the expression \$\cos 54^\circ \cos 4^\circ - \cos 36^\circ \cos 86^\circ\$, we start by using the identity \$\cos(90^\circ - x) = \sin x\$. This allows us to rewrite \$\cos 86^\circ\$ as \$\sin 2^\circ\$. Thus, the expression becomes: \$ \cos 54^\circ \cos 4^\circ - \cos 36^\circ \sin 2^\circ \$

Assistant (Turn 1):

<analyze>

The first paragraph uses the identity cos(90° - x) = sin(x) to rewrite cos(86°) as sin(2°). This is correct because 86° is 90° - 2°, so substituting x = 2° gives cos(86°) = sin(2°). The substitution is valid here. The expression is correctly transformed into cos54°cos4° - cos36°sin2°. This step is accurate.

</analyze>

<verify>

```
import math
import sympy as sp
# Define the angles in degrees
angle1 = 54
angle2 = 4
angle3 = 36
angle4 = 86
# Compute cos(54°) and cos(86°) using sympy's cos function
cos54 = sp.cos(sp.rad(angle1))
cos86 = sp.cos(sp.rad(angle4))
sin2 = sp.sin(sp.rad(2))
cos36 = sp.cos(sp.rad(angle3))
# Check if cos86° equals sin2°
is_equal = sp.simplify(cos86 - sin2) == 0
print(f"cos(86°) equals sin(2°)? {is_equal}")
# Check the transformed expression
transformed = sp.cos(sp.rad(angle1)) * sp.cos(sp.rad(angle2)) - sp.cos(sp.rad(angle3)) * sin2
original = sp.cos(sp.rad(angle1)) * sp.cos(sp.rad(angle2)) - sp.cos(sp.rad(angle3)) * sp.cos(sp.rad(angle4))
print(f"Original expression: {original}")
print(f"Transformed expression: {transformed}"
```

[Code Output]

```
cos(86°) equals sin(2°)? False
Original expression: -(1/4 + sqrt(5)/4)*cos(43*pi/90) + sqrt(5/8 - sqrt(5)/8)*cos(pi/45)
Transformed expression: -(1/4 + sqrt(5)/4)*sin(pi/90) + sqrt(5/8 - sqrt(5)/8)*cos(pi/45)
```

Wait, the output shows that cos(86°) is not exactly equal to sin(2°) in terms of exact symbolic comparison. However, numerically they are equal. Let me check with numerical values:

```
import math
cos86 = math.cos(math.radians(86))
sin2 = math.sin(math.radians(2))
print("Are they equal numerically?", math.isclose(cos86, sin2))
```

[Code Output]

```
Are they equal numerically? False
```

Hmm, this is unexpected. Wait, maybe due to floating-point precision? Let me compute cos(86°) and sin(2°) numerically and see if they are equal.

Wait, let's compute cos(86°) and sin(2°):

cos(86°) ≈ cos(86) ≈ 0.06975647374412537

sin(2°) ≈ 0.03489949670250097

These are not equal. So the initial assumption that cos(86°) = sin(2°) is incorrect. Therefore, the paragraph 1 is wrong. The error is in the trigonometric identity used here.

Wait, but according to the identity cos(90° - x) = sin(x), so cos(86°) = cos(90° - 2°) = sin(2°). However, numerically they are not equal. So there is a mistake here. The paragraph incorrectly applied the identity, leading to an incorrect substitution. The correct substitution should be cos(86°) = sin(2°), but numerically they are not equal. So the error is in paragraph 1.

Wait, but maybe the problem is that the identity is correct, but the numerical check is wrong? Let me verify:

Wait, cos(86°) is cos(90° - 4°) = sin(4°), not 2°. Wait, no, wait: cos(90° - x) = sin(x). So if angle is 86°, then 90° - 86° = 4°, so cos(86°) = sin(4°). Wait, that's the opposite of what the paragraph says. The paragraph said cos(86°) = sin(2°), but according to the identity, it should be sin(4°). So that's the mistake. The paragraph incorrectly used 2° instead of 4°. That's a critical error.

So paragraph 1 is wrong because it used 2° instead of 4° in the substitution. Therefore, the code verification shows that the substitution is incorrect, leading to an incorrect expression. The error is in paragraph 1.

So the first paragraph is incorrect.

</verify>

<output>

Judgement: \$\boxed{No}\$

</output>

Figure 5: The first case of GenPRM.

Problem:

Compute the distance between the two foci of the ellipse $x^2 + 4y^2 = 9.$

System:

You are a math teacher. Your task is to review and critique the paragraphs in solution step by step.

User (Turn 1):

Since $ x^2 + 4y^2 = 9 $, we divide both sides by 9 to obtain $\frac{x^2}{9} + \frac{4y^2}{9} = 1.$

Assistant (Turn 1):

<analyze>

Let's analyze the Paragraph 1 step by step: 1. The original equation is given as $x^2 + 4y^2 = 9$. Dividing both sides by 9 is correct because the standard form of an ellipse requires the right-hand side to be 1. This results in $\frac{x^2}{9} + \frac{4y^2}{9} = 1$. 2. The semi-major axis a is correctly identified as 3 because the denominator under x^2 is 9, so $a^2 = 9$ leading to $a = 3$. 3. The semi-minor axis b is calculated as $\frac{3}{2}$. Since the coefficient under y^2 after division is $\frac{4}{9}$, so $b^2 = \frac{9}{4}$ (since the denominator under y^2 would be $ \frac{9}{4} $), but wait, actually the equation after division is $\frac{x^2}{9} + \frac{y^2}{(9/4)} = 1$, so $b^2 = 9/4$, so $b = 3/2$. That part is correct. So paragraph 1 is correct.

</analyze>

<output>

Judgement: $\boxed{Yes}$

</output>

User (Turn 2)

The distance between the two foci of an ellipse with semi-major axis $ a $ and semi-minor axis $ b $ is given by $ 2c $, where

$

c = \sqrt{a^2 - b^2} = \sqrt{3^2 - \left(\frac{3}{2}\right)^2} = \sqrt{\frac{27}{4} - \frac{9}{4}} = \sqrt{\frac{18}{4}} = \frac{3\sqrt{2}}{2}.

$

Assistant (Turn 2)

<analyze>

Let's analyze the Paragraph 2 step by step: 1. The formula for the distance between foci is $2c$, where $c = \sqrt{a^2 - b^2}$. This is correct for an ellipse with major axis along the x-axis (since the denominator under x^2 is larger). 2. Plugging in $a = 3$ and $b = 3/2$:

- $a^2 = 9$, $b^2 = (3/2)^2 = 9/4$

- $c = \sqrt{9 - 9/4} = \sqrt{27/4}$ (Wait, 9 is 36/4, so 36/4 - 9/4 = 27/4. So sqrt(27/4) is (3√3)/2, but in the solution they have sqrt(27/4 - 9/4) = sqrt(18/4). Wait, hold on, that's an error! Wait, 9 - 9/4 is 27/4, not 18/4. Wait, 9 is 36/4, so 36/4 - 9/4 = 27/4. Therefore sqrt(27/4) is (3√3)/2. But in the solution, they wrote sqrt(27/4 - 9/4) = sqrt(18/4). Wait, that's wrong. The calculation inside the sqrt should be 27/4, not 18/4. So the error is here. Wait, but let me check again. Wait, the user's solution says sqrt(27/4 - 9/4) which is sqrt(18/4). Wait, that's a mistake. So the error is in the arithmetic here. The correct calculation should be 9 - ((3/2)^2) = 9 - 2.25 = 6.75, which is 27/4. So sqrt(27/4) is 3√3 / 2 ≈ 2.598, but the solution says sqrt(18/4) which is sqrt(4.5) ≈ 2.121. Therefore, there is an arithmetic mistake here. Therefore, paragraph 2 is incorrect. Wait, but maybe I made a mistake here? Let me recheck. The user's solution says:

"c = sqrt(3² - (3/2)^2) = sqrt(27/4 - 9/4) = sqrt(18/4) = 3√2/2."

Wait, 3 squared is 9, not 27. Wait, hold on, the user wrote 3^2 - (3/2)^2 = 9 - (9/4) = (36/4 - 9/4) = 27/4. But in their calculation, they wrote sqrt(27/4 - 9/4) which would be sqrt(18/4). Wait, that's a mistake. The correct expression is sqrt(9 - 9/4) = sqrt(27/4). Therefore, the step where they wrote sqrt(27/4 - 9/4) is incorrect. The correct calculation is sqrt(9 - 9/4) = sqrt(27/4). So the error is in the arithmetic here. Therefore, paragraph 2 is incorrect.

</analyze>

<output>

Judgement: $\boxed{No}$

</output>`

Figure 6: The second case of GenPRM.