

# Math 442 Homework 5

- Due Thursday February 7 at start of class.
  - If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper.
  - **Collaboration Policy:** You are welcome (and encouraged) to work on the homework in groups. However, each student must write up the homework on their own, and must use their own wording (i.e. don't just copy the solutions from your friend). If you do collaborate with others, please list the name of your collaborators at the top of the homework.
  - You are encouraged (though not required) to type up your solutions. If you choose to do this, I strongly recommend that you use the typesetting software LaTeX. LaTeX is used by the entire mathematics community, and if you intend to go into math, you'll need to learn it sooner or later. "The Not So Short Introduction to LaTeX" is a good place to start. This guide can be found at <http://tug.ctan.org/info/lshort/english/lshort.pdf>. You can also download the .tex source file for this homework and take a look at that.
  - Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.
1. Prove that every tree with at least two vertices has a vertex of degree one. You may not use the fact that  $v = e + 1$ .
  2. Let  $G$  be a (simple) planar graph that contains no cycle of length  $\leq k$ . Prove that
$$e \leq \frac{k+1}{k-1}v.$$
  3. Prove that if a graph has no closed paths of odd length then it is bipartite.
  4. If a simple connected planar graph consists of 5 vertices of degree 4, and 4 vertices of degree 3, then how many faces does it have? Give an example of such a graph.
  5. For exactly which values of  $k$  is  $Q_k$  planar? Prove your answer.
  6. Prove that every simple connected planar graph with fewer than 12 vertices contains a vertex of degree at most 4.
  7. Let  $G$  be a simple graph with at least 11 vertices. Prove that  $G$  and  $\overline{G}$  are not both planar.