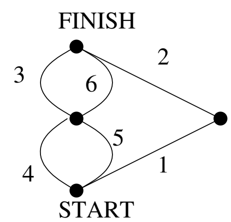
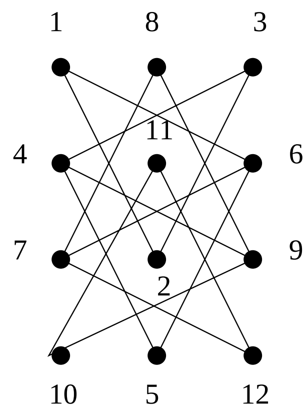
Math 442 Homework 2 Solutions

1. Yes, by the demolition of the bridge and route given below.



2. Here is an example of such a tour.



Observe that on an odd board there will always be one more square of one colour (say black) than the other, and a knight will always move from a square to a square of the other colour. Hence one can never find a knight's tour on an odd board since to get to all the squares once you would need to move

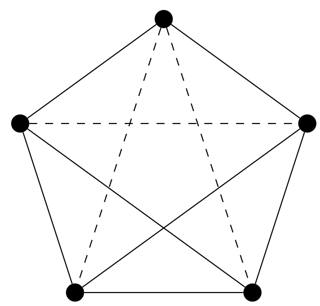
$$BWBWBW\dots WB$$

but there is no way to return from a black square to a black square without visiting an already visited white square.

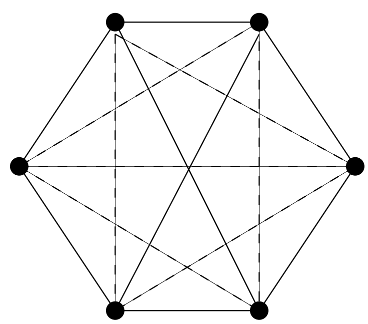
3.

$$((4t-5w)(x+y))((((2x+1)+y)+(3+5(w^2)))+(y+(w+z))).$$

4. Here is one such coloring.



5. No. Here is a counter-example.



6. First note that if G is a simple graph on n vertices then the maximum degree of a vertex is $n-1$, and this vertex must be connected to every other vertex.

Now we do a proof by contradiction. Assume that the n vertices of G each have different degree, then by our above observation the degrees of the vertices must be $0, 1, 2, \ldots, n-1$. However the vertex of degree $n-1$ must be connected to every other vertex, and hence no vertex of degree 0 can exist, which is a contradiction.