MATH 442

Homework 2

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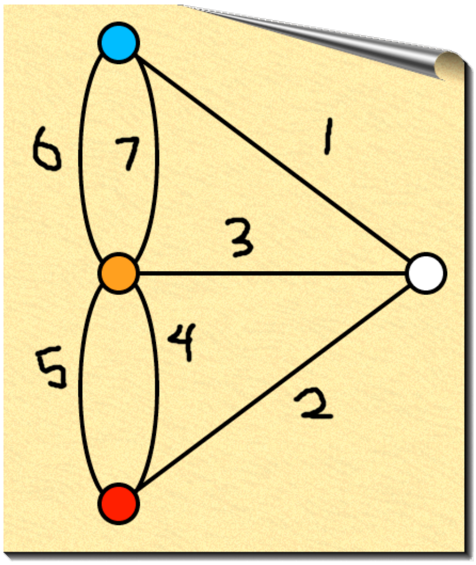
1. The local council in Königsberg eventually decide to demolish one bridge. Does there exist a bridge they can demolish so the citizens can find a route through the town crossing each bridge only once and *not* finish up where they started? Explain your answer.

Answer:



Labelled figure of the initial Königsberg Bridge problem, taken from Wikipedia.

Yes. If bridges 1 or 2 are demolished, then there is now a path so that the citizens can traverse the town, crossing every bridge exactly once and not end up where they started.

If bridge 1 is demolished, the path starting from Red is:

$5 \to 6 \to 7 \to 4 \to 2 \to 3$ ending at Orange.

If bridge 2 is demolished, the path starting from Blue is:

$6 \to 5 \to 4 \to 7 \to 1 \to 3$ ending at Orange.

Another possibility is to demolish bridge 3.

2. Show that a knight can tour each square on a 3×4 chessboard – though without finishing at the starting square.

Answer:
Given a 3x4 chessboard, we are able to divide the board into exactly 6 black and 6 white spaces. Using the same approach as we did in class, we are able to construct a bipartite

graph. Using this graph, we can move the knight from a black space to a white space and vice versa, and as there are exactly 6 black and white spaces, the knight can tour every square.

1	2	3	4
5	6	7	8
9	10	11	12

For this arrangement, the traversal is:

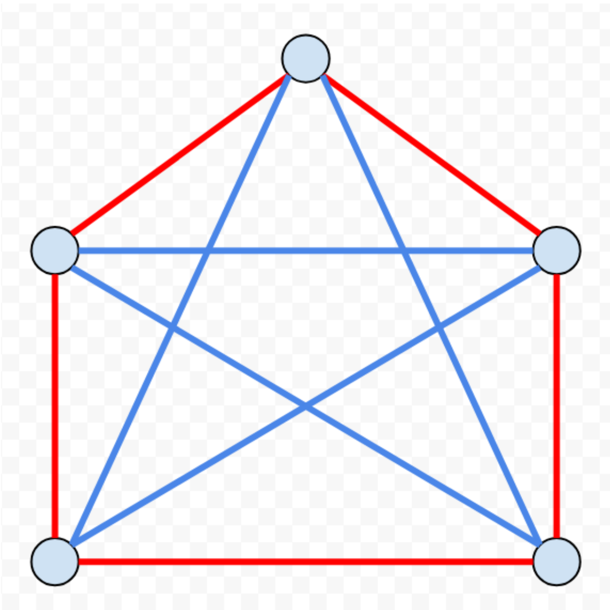
$1 \to 7 \to 9 \to 2 \to 8 \to 10 \to 3 \to 12 \to 6 \to 4 \to 11 \to 5$

3. Write down the expression given by the parse tree.

Answer:
(((4 * t) - (5 * w)) * (x + y)) * ((y + (w + z)) + (((2 * x) + 1 + y) + (5 * (w^2) + 3)))

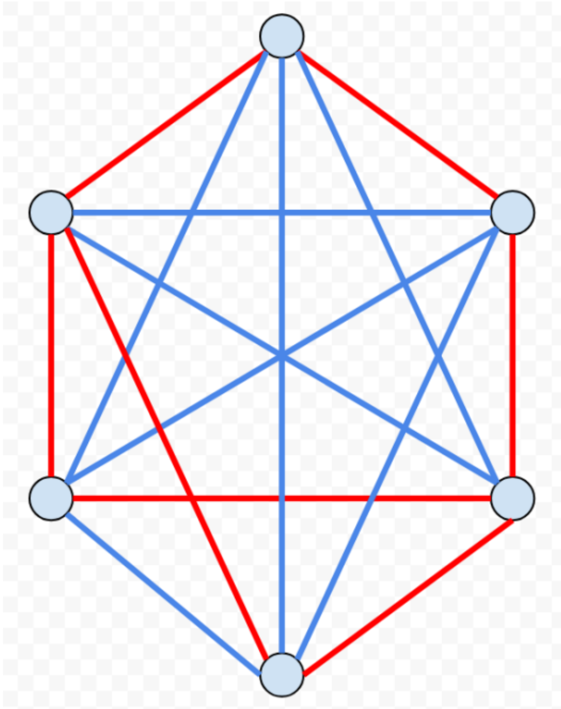
4. Consider a graph with five vertices and all $\binom{5}{2} = 10$ edges between the vertices. Color the edges red and blue so that there does not exist a monochromatic triangle.

Answer:



5. In a party of 6 people is it true that either there exists 4 people who all do know each other or there exists 4 people who all do not know each other? Justify your answer.

Answer:



Another way to think of this is to look at Q4, in which we showed that for a graph with 5 vertices, there is a colouring such that there does not exist a monochromatic triangle. Thus, for a graph with 6 vertices, if we colour the edges correspondingly, there is a colouring such that there does not exist a monochromatic square. The figure above is simply one example of such a colouring.

6. Prove that every (simple) graph with at least two vertices contains 2 vertices with the same degree.

Answer:
Let G be a simple graph with n vertices where $n \geq 2$.

Suppose that G is a connected graph, then each vertex must have a degree from the set $\{1, n-1\}$. As the graph has n vertices, by the pigeon-hole principle, there must be two vertices that have the same degree.

Suppose that G is not a connected graph, then each vertex cannot have a degree of $n-1$ as that would be a connected graph. Thus, each vertex must have a degree from the set $\{0, n-2\}$. As the graph has n vertices, by the pigeon-hole principle, there must be two vertices that have the same degree.