

# MATH 442

## Homework 2

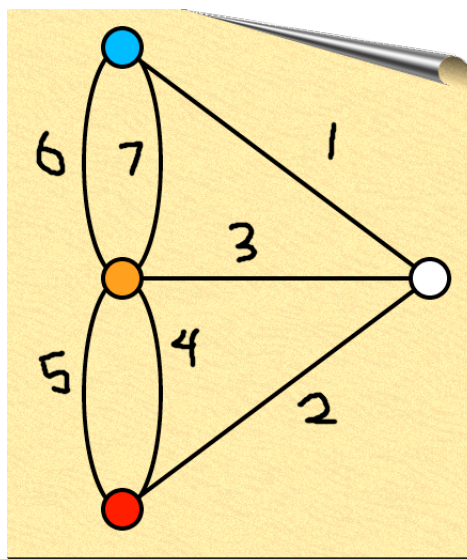
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1. The local council in Königsberg eventually decide to demolish one bridge. Does there exist a bridge they can demolish so the citizens can find a route through the town crossing each bridge only once and *not* finish up where they started? Explain your answer.

Answer:



Labelled figure of the initial Königsberg Bridge problem, taken from Wikipedia.

Yes. If bridges 1 or 2 are demolished, then there is now a path so that the citizens can traverse the town, crossing every bridge exactly once and not end up where they started.

If bridge 1 is demolished, the path starting from Red is:

$5 \rightarrow 6 \rightarrow 7 \rightarrow 4 \rightarrow 2 \rightarrow 3$  ending at Orange.

If bridge 2 is demolished, the path starting from Blue is:

$6 \rightarrow 5 \rightarrow 4 \rightarrow 7 \rightarrow 1 \rightarrow 3$  ending at Orange.

2. Show that a knight can tour each square on a  $3 \times 4$  chessboard – though without finishing at the starting square.

Answer:

Given a  $3 \times 4$  chessboard, we are able to divide the board into exactly 6 black and 6 white spaces. Using the same approach as we did in class, we are able to construct a bipartite graph. Using this graph, we can move the knight from a black space to a white space and

vice versa, and as there are exactly 6 black and white spaces, the knight can tour every square.

1	2	3	4
5	6	7	8
9	10	11	12

For this arrangement, the traversal is:

$1 \rightarrow 7 \rightarrow 9 \rightarrow 2 \rightarrow 8 \rightarrow 10 \rightarrow 3 \rightarrow 12 \rightarrow 6 \rightarrow 4 \rightarrow 11 \rightarrow 5$

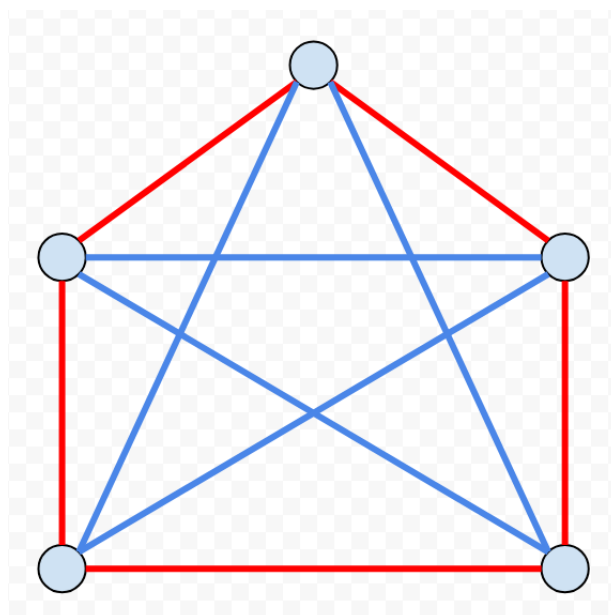
3. Write down the expression given by the parse tree.

Answer:

$((4 * t) - (5 * w)) * (x + y) * ((y + (w + z)) + (((2 * x) + 1 + y) + (5 * (w^2) + 3)))$

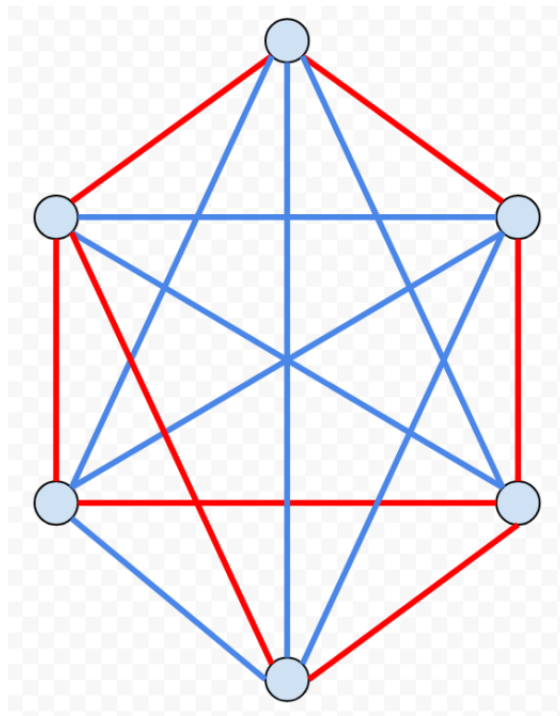
4. Consider a graph with five vertices and all  $\binom{5}{2} = 10$  edges between the vertices. Color the edges red and blue so that there does not exist a monochromatic triangle.

Answer:



5. In a party of 6 people is it true that either there exists 4 people who all do know each other or there exists 4 people who all do not know each other? Justify your answer.

Answer:



Another way to think of this is to look at Q4, in which we showed that for a graph with 5 vertices, there is a colouring such that there does not exist a monochromatic triangle. Thus, for a graph with 6 vertices, if we colour the edges correspondingly, there is a colouring such that there does not exist a monochromatic square. The figure above is simply one example of such a colouring.

**6.** Prove that every (simple) graph with at least two vertices contains 2 vertices with the same degree.

Answer:

Let  $G$  be a simple graph with  $n$  vertices where  $n \geq 2$ .

Suppose that  $G$  is a connected graph, then each vertex must have a degree from the set  $\{1, n-1\}$ . As the graph has  $n$  vertices, by the pigeon-hole principle, there must be two vertices that have the same degree.

Suppose that  $G$  is not a connected graph, then each vertex cannot have a degree of  $n-1$  as that would be a connected graph. Thus, each vertex must have a degree from the set  $\{0, n-2\}$ . As the graph has  $n$  vertices, by the pigeon-hole principle, there must be two vertices that have the same degree.