Ride Hailing Pricing Strategy for Profit Maximization

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Question - Pricing Case Study

You're launching a ride-hailing service that matches riders with drivers for trips between the Toledo Airport and Downtown Toledo. It'll be active for only 12 months. You've been forced to charge riders $30 for each ride. You can pay drivers what you choose for each individual ride.

The supply pool ("drivers") is very deep. When a ride is requested, a very large pool of drivers see a notification informing them of the request. They can choose whether or not to accept it. Based on a similar ride-hailing service in the same market, you have some data on which ride requests were accepted and which were not. (The PAY column is what drivers were offered and the ACCEPTED column reflects whether any driver accepted the ride request.)

The demand pool ("riders") can be acquired at a cost of $30 per rider at any time during the 12 months. There are 10,000 riders in Toledo, but you can't acquire more than 1,000 in a given month. You start with 0 riders. "Acquisition" means that the rider has downloaded the app and may request rides. Requested rides may or may not be accepted by a driver. In the first month that riders are active, they request rides based on a Poisson distribution where lambda = 1. For each subsequent month, riders request rides based on a Poisson distribution where lambda is the number of rides that they found a *match* for in the previous month. (As an example, a rider that requests 3 rides in month 1 and finds 2 matches has a lambda of 2 going into month 2.) If a rider finds no matches in a month (which may happen either because they request no rides in the first place based on the Poisson distribution or because they request rides and find no matches), they leave the service and never return.

Submit a written document that proposes a pricing strategy to maximize the profit of the business over the 12 months. You should expect that this singular document will serve as a proposal for

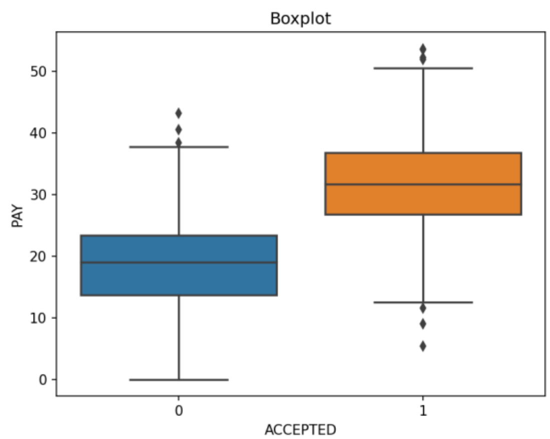
1. A quantitative executive team that wants to know how you're thinking about the problem and what assumptions you're making but that does not know probability theory
2. Your data science peers so they can push on your thinking

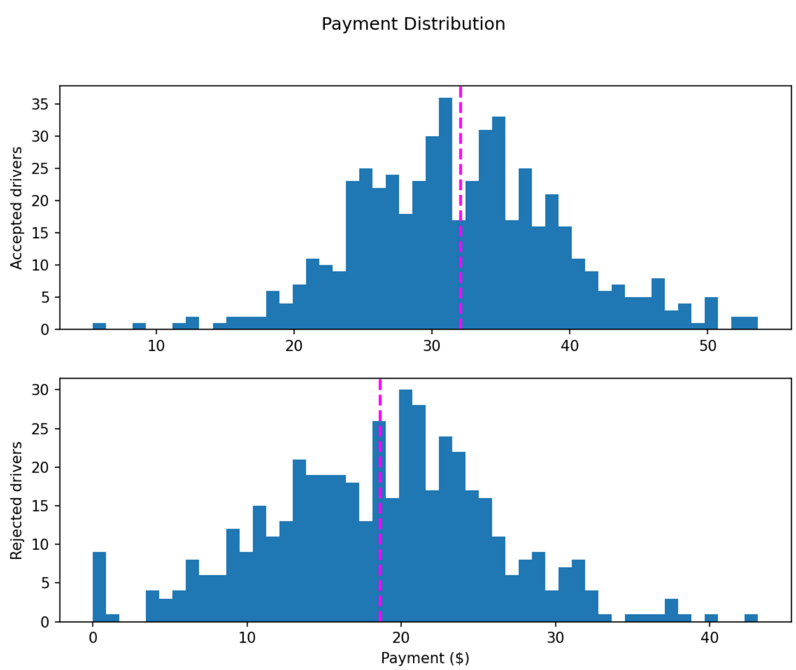
Please submit any work you do, code or math, with your solution.

Solution

Introduction

Business objective is to maximize the business profit over the 12 months. Since we have to charge the riders $30 for each ride, paying the drivers as less as possible for their rides will increase the profit. However, as the below histogram of the data on similar ride-hailing service shows, insufficient payment to drivers is led to having more drivers' rejects to the riders' requests which consequently decreases number of rides and the total profit. For that reason, it should be an optimum amount to pay to the drivers where less or more than that adversely impacts on the profit. We want to find that optimum payment/price.





Note t-testing between average drivers' payment of two group, shown above with pink dashed line, shows that there is a statistically significant difference between two groups at a p value level of 0.01.

Method (1)

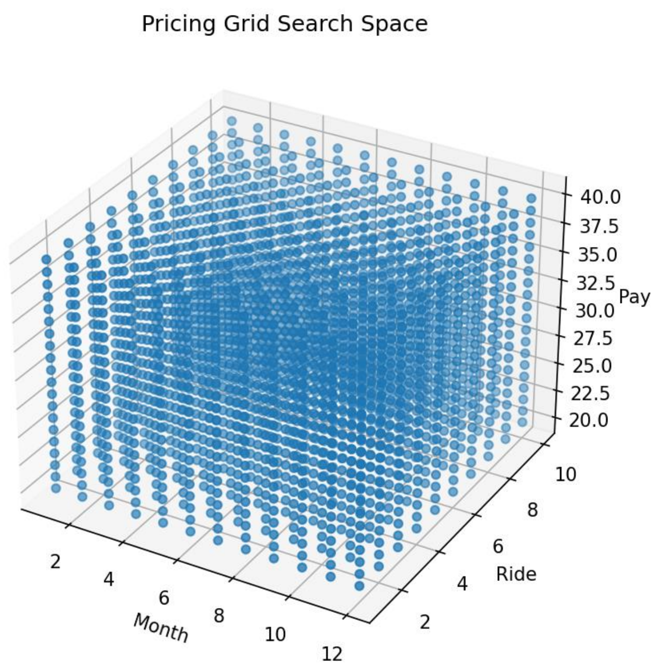
Search Space

Grid search and dynamic pricing is used for finding the maximum annual profit. The search space is 3-dimensional and combination of:

(1) Different amount of payments to drivers from $20 to $40 incremented by $1. Note minimum and maximum of $20 and $40 were selected to reduce the search space. If the optimum payment is going to fall on one of the edges, then the payment dimension of the search space will be changed to a range around the optimum payment, found on the first iteration. Values higher than $30 are selected because sometimes there will be initial loss at the beginning months in favor of later higher profit which are reflected on the total annual profit.

(2) Ride's request in each month (lambda in the Poisson distribution). This dimension starts from one and ended when 90% of accumulative probability have been reached. 90% has been used to reduce amount of searching points in the search space and if the optimum profit is associated with a ride request number at the 90% of accumulative probability, then the search space on this dimension will be extended.

(3) 12 months



The idea is to obtain total profit for each point in the above search space[1]. Then maximum profit in each ride-pay plane for each month is determined. Maximum annual profit is the summation of the maximum profit in each month over 12 months. In the next section, for one customer the profit for each point and their maximum annual profit is calculated.

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[1] Note the maximum ride number is determined by 90% of accumulative probability from Poisson distribution function.

Annual Profit Metric

In the following formula, the profit for each grid from the "Pricing Grid Search Space" is calculated for a typical customer. Each grid represents a month (m), an amount of payment to the driver (p) and a Ride Occurrence Index (r).

Grid Profit (m,p,r) = ΔP X Ride Occurrence Index's probability

where

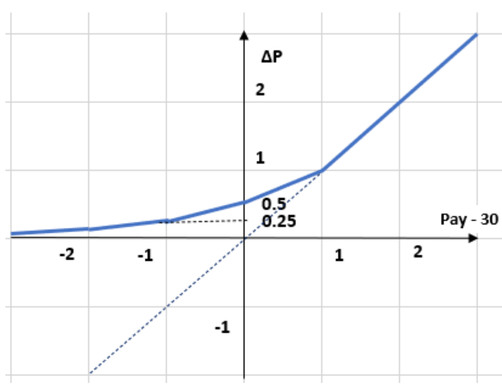
if Pay>30, then ΔP= (Pay-30)

if Pay=30, then ΔP= 0.5

if Pay<30, then ΔP= $\frac{1}{2^{|Pay-30|+1}}$

Note

- Above formula is used if the driver's acceptance probability to be above or equal 0.5. Driver's acceptance probability is dependent on the driver's payment amount and obtained from a generated model artifact which has been trained with a logistic regression algorithm and with the provided similar drivers' acceptance data.
- Ride Occurrence Index's probability is obtained from the Probability Mass Function value, obtained from Poisson Distribution, and is dependent on number of matched monthly rides (lambda or µ in Poisson distribution) and on the ride occurrence index (k in Poisson distribution).
- ΔP is the difference between the payment given to the driver receives from $30 receives from the rider. Note the negative differences are converted to positive values as presented in the formula and shown below. The ΔP for all pay values is converted to a positive value because it needs to be multiplied to a probability value (a positive value less than one) and we want the values to be comparable to each other.



After finding the profit formula for each grid, maximum monthly profit along with its associated driver's payment value and ride occurrence index are determined and recorded.

Below formula presents maximum annual profit for one customer.

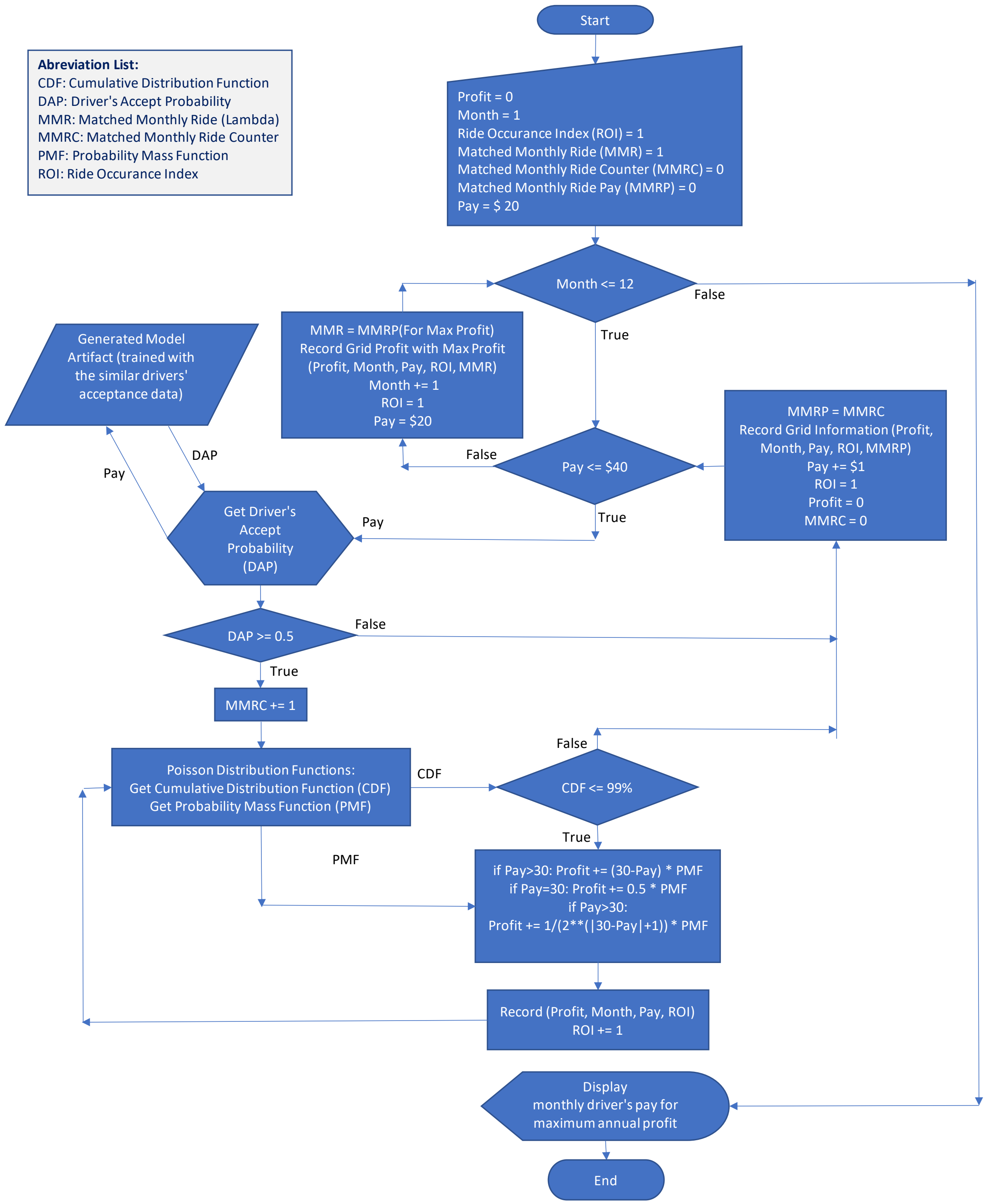
Maximum Annual Profit = $\sum_{m=1}^{12} Max[Grid\ Profit\ (m, p, r)]$

Disclaimer:

- Annual profit is only calculated for one customer. It is naively assumed that similar trend is applied for other customers and the total profit will be the annual profit of one customer times the total number of customers, an assumption that assumed events are independent and can be added linearly. This assumption might not be completely true because potential customers go by trend and word of mouth. If number of demanded rides are increased and particularly when the demand is well accepted by the drivers, then the reputation of the ride-hailing service is promoted, and more customers are added to the riders' pool. Moreover, higher customers' demand impacts on the driver's acceptance regarding the amount of payment. In other words, payment expectation of a driver with many customers can be decreased compared to a driver with few customers, as at the end of the day a driver wants to take home a good amount of money.
- The Annual profit is a probability-based value as the probability of the ride's request has been considered in the profit formula. However, it is safe to assumed that there will be a correlation between trend of the probability-based profit with the trend of the real profit and therefore between maximum probability-based profit with maximum real profit. Our major goal is to determine the optimum payment amount that driver receive from the first month to the 12th month to have a maximum annual profit. In other words, the 12 payments found from probability-based calculation has the highest chance to be happened in reality. With this goal, we can proceed with probability-based profit's calculation. If I would have more time, instead of doing calculation for only one rider, I would do the calculation for 1000 riders per month and I would create 1000 values distributed around a normal curve where their mean is the rider's request probability value obtained from the Poisson distribution function.

Process

Below flowchart summarizes the process to calculate the maximum monthly and annual profit. The process starts with minimum amount of payment to driver, $20. To obtain the drivers' acceptance probability, the excel data file of similar drivers' payments has been trained with logistics regression. The driver's payment is given to the model and the driver's acceptance probability score is obtained. If the probability is less than 0.5, the payment is increased by $1 and the score is asked. If the score is above 0.5, then the ride's request probability is obtained from Probability Mass Function of the Poisson distribution function. In the first months, the lambda considered one (as suggested in the problem's description). Using the formula introduced in previous section, the profit for the first ride's request (in the flowchart, Ride Occurrence Index= ROI=1) is calculated. Then the probability for ROI=2 (the 2nd ride's request) is obtained from the Poisson distribution formula, the profit is calculated, and it is added to the profit for ROI=1. This process is repeated iteratively until when the cumulative probability for ROIs is reached to a 99% threshold (99% is chosen to not stock in an infinite loop. Instead of that, threshold of 1% for Probability Mass Function could be selected). At this point, the accumulated profit for a set of grids of the Price Grid Search Space for the first month and a specific amount of driver's payment (here $20) have been recorded (shown on the flowchart as MMRP) and the process is repeated for the next amount of driver's payment ($21). After calculating and recording information of all the grids for the first month and for all levels of driver's payment ($20 - $40), we will be able to identify the grid with maximum monthly profit for the first month. MMRP of that grid is considered as lambda (total number of matched rides on the previous month, indicated as MMR on the flowchart) to be used for the next month. All the mentioned process is iterated for the next months. When the process completed for all 12 months, information of grids with maximum monthly profit is displayed along with total annual profit.



Results

According to the presented method, Profit is maximum at the driver’s payment of $25.

Related python file is *Pricing_Case_Study_Method1.py.*

grid

	Profit	Month	Pay	ROI	MMRC
0	0.00000	1	20	1	0
1	0.00000	1	21	1	0
2	0.00000	1	22	1	0
3	0.00000	1	23	1	0
4	0.00000	1	24	1	0
5	3.14230	1	25	5	4
6	2.51384	1	26	5	4
7	1.88538	1	27	5	4
8	1.25692	1	28	5	4
9	0.62846	1	29	5	4
10	0.31423	1	30	5	4
11	0.15712	1	31	5	4
12	0.07856	1	32	5	4
13	0.03928	1	33	5	4
14	0.01964	1	34	5	4
15	0.00982	1	35	5	4
16	0.00491	1	36	5	4
17	0.00245	1	37	5	4
18	0.00123	1	38	5	4
19	0.00061	1	39	5	4
20	0.00031	1	40	5	4
21	0.00000	2	20	1	0
22	0.00000	2	21	1	0
23	0.00000	2	22	1	0
24	0.00000	2	23	1	0
25	0.00000	2	24	1	0

Method (2)

In this method:

- 1000 ride requests are considered and using the Poisson Distribution number of rides for each ride occurrence is calculated.
- Both profit and loss for a range of driver’s payment are calculated. In the first version (Pricing_Case_Study_Method2_v1.py), the profit table for the first month and for payments from $20 to $40 is calculated. In the second version (Pricing_Case_Study_Method2_v2.py), the profit table for 3 driver’s payment ($23, $30, $37) are used over 12 months. Note $23 driver’s payment obtained from the first version of method 2 associated with the highest amount of profit in the first month for all levels of ride occurrences.

Search Space

The search space to check for payments from $20 to $40 with an increment of $1 and over 12 months is 21 to the power of 12 (2256349030036618608‌1). Only 3 levels of payments ($23, $30, $37) are used to reduce the search space to 3 to the power of 12 (531441).

Process and Result

Using the “Probability Mass Function” from Poisson Distribution for the first month with lambda = 1 show how many ride requests for each ride occurrence are obtained for 1000 rides in the first month (version 1).

Poisson Prob	X 1000 rides
0.368	368
0.184	184
0.061	61
0.015	15
0.003	3
0.001	1

Above table shows that out of maximum monthly cap of 1000 rider’s requests, 368 of the riders request once, 184 of the them requests twice and so forth. Of course, not all the request are accepted by the drivers. The acceptance depends on the amount of payment to the driver. Which shown on the below table.

Note obtained number of rides for each Ride Occurrence Number is multiplied by Driver Acceptance Probability to determine the number of rides.

Month	RideOccuranceIndex	Pay_20	Pay_21	Pay_22	Pay_23	Pay_24	Pay_25	Pay_26	Pay_27	Pay_28	Pay_29	Pay_30
1	1	84	102	121	143	166	190	213	236	256	275	292
1	2	42	51	61	72	83	95	107	118	128	138	146
1	3	14	17	20	24	28	31	35	39	43	46	48
1	4	3	4	5	6	7	8	9	10	10	11	12
1	5	1	1	1	1	1	2	2	2	2	2	2
1	6	0	0	0	0	0	1	1	1	1	1	1

Pay_31	Pay_32	Pay_33	Pay_34	Pay_35	Pay_36	Pay_37	Pay_38	Pay_39	Pay_40
306	318	328	337	343	349	353	356	359	361
153	159	164	168	172	174	176	178	179	180
51	53	54	56	57	58	58	59	59	60
12	13	13	14	14	14	14	15	15	15
2	3	3	3	3	3	3	3	3	3
1	1	1	1	1	1	1	1	1	1

Number of rides from above table is multiplied by (Pay - $30) to determine profit or loss, shown in below table. Total profit for column Pay_23 is maximum.

Month	RideOccuranceIndex	Pay_20	Pay_21	Pay_22	Pay_23	Pay_24	Pay_25	Pay_26	Pay_27	Pay_28	Pay_29	Pay_30
1	1	840	918	968	1001	996	950	852	708	512	275	0
1	2	420	459	488	504	498	475	428	354	256	138	0
1	3	140	153	160	168	168	155	140	117	86	46	0
1	4	30	36	40	42	42	40	36	30	20	11	0
1	5	10	9	8	7	6	10	8	6	4	2	0
1	6	0	0	0	0	0	5	4	3	2	1	0

Pay_31	Pay_32	Pay_33	Pay_34	Pay_35	Pay_36	Pay_37	Pay_38	Pay_39	Pay_40
-306	-636	-984	-1348	-1715	-2094	-2471	-2848	-3231	-3610
-153	-318	-492	-672	-860	-1044	-1232	-1424	-1611	-1800
-51	-106	-162	-224	-285	-348	-406	-472	-531	-600
-12	-26	-39	-56	-70	-84	-98	-120	-135	-150
-2	-6	-9	-12	-15	-18	-21	-24	-27	-30
-1	-2	-3	-4	-5	-6	-7	-8	-9	-10

Max profit in the 1st month is associated with the payment of $23 to drivers. Out of 1000 ride requests, Poisson distribution formula showed a decrease of rides but after applying the drivers acceptance based on their received payment the rides again decreased as 143 of riders have accepted ride from drivers once, 72 twice, 24 three times, 6 four times, and 1 five times.

On the second version of Method 2, it is intended to determine the profit table for 3 driver's payment ($23, $30, $37) over 12 months. The method is promising as for each amount of obtained ride in a month it searches number of rides in the subsequent month for 3 different level of the driver's payments. At the end, 3 (pays) to the power of 12 (months) will be calculated, i.e. 531441. Then driver's payment can be identified from row indexes associated with maximum total profit. Unfortunately, I did not have much more time to spend on the project but I thought to share what I have done.

Here is summary of ride and profit information for the first 2 months.

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	143	292	353
2	1	21	42	51
2	1	42	85	103
2	1	51	103	125

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	1001	0	-2471
2	1	147	0	-357
2	1	294	0	-721
2	1	357	0	-875

Which in more details are:

Month=1, Lambda=1, Rides=1000

Poisson Prob	X 1000 rides
0.368	368
0.184	184
0.061	61
0.015	15
0.003	3
0.001	1

Number of Rides for each Ride Occurance Number is multiplied by Driver Acceptance Probability to determine the number of rides, shown in below table

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	143	292	353
1	2	72	146	176
1	3	24	48	58
1	4	6	12	14
1	5	1	2	3
1	6	0	1	1

Number of rides from above table multiplies (Pay - $30) to determine profit, shown in below table.

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	1001	0	-2471
1	2	504	0	-1232
1	3	168	0	-406
1	4	42	0	-98
1	5	7	0	-21
1	6	0	0	-7

Month=2, Lambda=1, Rides=143 (for Pay_23 of last month)

Poisson Prob	X 143 rides
0.368	53
0.184	26
0.061	9
0.015	2
0.003	0
0.001	0

Ride Counts

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	21	42	51
1	2	10	21	25
1	3	4	7	9
1	4	1	2	2
1	5	0	0	0
1	6	0	0	0

Profit/Loss

Month	RideOccuranceIndex	Pay_23	Pay_30	Pay_37
1	1	147	0	-357
1	2	70	0	-175
1	3	28	0	-63
1	4	7	0	-14
1	5	0	0	0
1	6	0	0	0