Portland State
UNIVERSITY

DEPARTMENT OF
ELECTRICAL AND COMPUTER ENGINEERING

ECE331

Lab2: Matlab

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1.1 EXPERIMENT (Characteristic of sound)

Objective:

The objective of this lab is to get familiar with Matlab. We will be using it's tools to solve electromagnetic problems. Electromagnetic concepts can be difficult to visualize from the equation. Using the graphics from Matlab will help you gain intuition about Maxwell's equations and electromagnetics general.

Set Up:

All works will be completed on Matlab. No tools or test equipment are required to be used for this lab.

Theory:
We will be looking at the matlab functions on how to test these functions:

MESHGRID: This is a convenient command to get x and y coordinates for a uniform grid for a numerical calculation. This command works as described below from the Matlab help in Fig. 1. The meshgrid command gives us two matrices X And Y that contain the x and y coordinates for each location on a uniform grid between -2 and 2 for x and y. We can then calculate a height function as given in the example. The example uses surf plot and we will use pcolor which is similar but for 2D plots.

QUIVER: This command allows you to make vector plots. It is sometimes tricky to get the length of the arrows to come out looking nice in a plot but you can play with the scaling to help plots look better. The help and example given in Matlab shows how to use this function and is given in Fig. 2.

PCOLOR: This is a color image plotting function. It is similar to imagesc but insures that the data point corresponds to the axis values. Can be particularly nice to use if The x and y data points are not on a uniform grid and this will plot them according to their actual location.

DIV: Numerical calculation of divergence

CURL: Numerical calculation of curl. NOTE: In Matlab we have to multiply the curl result by 2 to get it to agree with what we do analytically- no idea why.

DOT: The dot product of two vectors or for the magnitude of a vector can be used as (dot((R, R))^(½).

Numerical integration: We will do simple numerical integration by summing up values (also sometimes known as rectangular method). It often works well as long as the steps are reasonably small (how small depends on the problem). You will see from these exercises it is plenty good for what we are doing. The numerical integration is approximated as the below:

$$\int_a^b f(x)dx \approx \Delta x \sum_{n=0}^{N-1} f(x_n)$$

where xn = a + nΔx and Δx = (b — a)/N. You can think of Δx as the approximation to the differential element dx. When we work in cylindrical coordinates we will have to keep in mind the differential element. If a line integral is in ϕ direction then the differential element is rdϕ so the numerical $\triangle$ will need to be r$\triangle\phi$.

Publishing results in Matlab

Procedure

Pre-Lab

-Log into Matlab and run

The publishing example m file (provided to you) to do a gradient and plot the results.

Use the publish button to reproduce the results shown in Fig. 4

Review attachment for Matlab and Plot

-*Calculate the divergence and curl for following:*

$$\boldsymbol{F} = F_1\,\boldsymbol{i} + F_2\,\boldsymbol{j} + F_3\,\boldsymbol{k} = \left\langle F_1, F_2, F_3 \right\rangle$$

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\text{Div}=\nabla\cdot\boldsymbol{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\text{Curl}=\nabla\times\boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

1. Problem 3.44 (a) from book

$$\vec{\mathbf{A}} = -\hat{\mathbf{x}}\cos x \sin y + \hat{\mathbf{y}} \sin x \cos y$$

$\nabla\cdot\vec{A} = -\cos x \sin y\,\hat{x} + \sin x \cos y\,\hat{y}$

$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}$

$\sin x(\sin y) - \sin x \sin y = \boxed{0}$

Curl

$\begin{matrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\cos x \sin y & \sin x \cos y & 0 \end{matrix}$

$\left[\frac{\partial}{\partial y}[0] - \frac{\partial}{\partial z}\sin x \cos y\right] - \left[\frac{\partial}{\partial x}[0] \cdot \frac{\partial}{\partial z} - \cos x \sin y\right] + \left[\frac{\partial}{\partial x}\sin x \cos y - \frac{\partial}{\partial y} - \cos x \sin y\right]$

$0 \qquad 0 \qquad + \cos x \cos y + \cos x \cos y$

$\nabla\cdot\vec{A} = 0$

Curl $\vec{A} = \boxed{2\cos x \cos y}$

2. Problem 3.44 (c) from book

$$\vec{\mathbf{A}} = -\hat{\mathbf{x}}xy + \hat{\mathbf{y}}y^2$$

$A = -xy\,\hat{x} + y^2\,\hat{y}$

$$\nabla \cdot \vec{A} = -y + 2y = \boxed{y}$$

Curl

$$\begin{vmatrix} x & y & z \\ \frac{\partial x}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -xy & y^2 & 0 \end{vmatrix} \qquad [0-0] - [0-0] + [0-(-x)] = \boxed{x}$$

3. Problem 3.44 (i) from book

$$\vec{\mathbf{A}} = \hat{\mathbf{r}}r + \hat{\phi}r\cos\phi$$

div $\{r, r\cos(\theta), 0\} = 2 - \sin(\theta)$

(using $x=x(r,\theta)=r\cos(\theta)$ and $y=y(r,\theta)=r\sin(\theta)$)

(r: radial coordinate | θ: azimuthal angle | z: third Cartesian coordinate)

curl $\{r, r\cos(\theta), 0\} = (0, 0, 2\cos(\theta))$

in Cartesian coordinates:

$$\ldots = \frac{2}{\sqrt{\frac{x^2+y^2}{x^2}}}\,\mathbf{e}_z$$

(using $x=x(r,\theta)=r\cos(\theta)$ and $y=y(r,\theta)=r\sin(\theta)$)

(r: radial coordinate | θ: azimuthal angle | z: third Cartesian coordinate)

4. The magnetic field from a wire carrying a constant current:

$$\vec{\mathbf{B}} = \hat{\phi}\frac{\mu_0 I}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

$$\begin{array}{ccc} r & \phi & z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{\mu_0 I}{2\pi r} & 0 \end{array}$$

$$[0-0]-[0-0]+\left[\frac{\partial}{\partial r}\frac{\mu_0 I}{2\pi r}-0\right]$$

$$\text{Curl} = \boxed{-\frac{\mu_0 I}{2\pi r^2}}\hat{z}$$

Due to Maxwell's equation

$$\nabla \cdot \vec{B} = 0 \quad \therefore \quad \nabla \cdot \vec{B} = \boxed{0}$$

Experiment:

3.1 Numerical Computation of Divergence and Curl

- Plot ~A using Matlab quiver command. Make sure to label axes.
- Plot the divergence of ~A that you calculated analytically (by hand). Use the pcolor command and label axes.
- Compute the divergence of ~A using Matlab div command. Plot the results using pcolor command and be sure to label axes.
- Plot the curl of ~A that you calculated analytically (by hand). Use the pcolor command and label axes.
- Compute the curl of ~A using Matlab curl command. Plot the results using pcolor command and be sure to label axes.

For Equation from the book: **3.44 (a), (c) and (i).**

<u>Review attachment for Matlab and Plot</u>

3.2 Numerical Computation of Divergence and Curl for a Current Carrying Wire

- Plot ~A using Matlab quiver command. Make sure to label axes.
- Plot the divergence of ~A that you calculated analytically (by hand). Use the pcolor command and label axes.
- Compute the divergence of ~A using Matlab div command. Plot the results using pcolor command and be sure to label axes.
- Plot the curl of ~A that you calculated analytically (by hand). Use the pcolor command and label axes.
- Compute the curl of ~A using Matlab curl command. Plot the results using pcolor command and be sure to label axes.

<u>Review attachment for Matlab and Plot</u>

3.3 Numerical Integration and Calculating the Electric Field from a Ring of Charge

Calculate the value of the differential element for line integration.

-Get the x coordinate of the ring (Rcos (n) where n is the index of the innermost loop).
-Get the y coordinate of the ring (Rsin (n) where n is the index of the innermost loop).
-Get the R vector. This is the vector from a point on the ring to the observation point. You can and by taking R obs - R ring.
-Calculate R2 as the dot product, R R.
-Calculate the ^R unit vector as R=R.

Problem 4-10 Half Ring of Charge:

P4.10

→ refer to example 4.4 in the text.

$$dE_1 = \frac{\rho b}{4\pi\varepsilon_0} \frac{(-\hat{r}b + \hat{z}h)}{(b^2+h^2)^{3/2}} d\phi$$

$$= \frac{\rho_1 b}{4\pi\varepsilon_0} \frac{(-\hat{r}b)}{(b^2)^{3/2}} d\phi$$

$$= \frac{-\hat{r}\rho_1 b^2}{4\pi\varepsilon_0 b^3} d\phi$$

$$= \frac{-\hat{r}\rho_1}{4\pi\varepsilon_0 b} d\phi$$

Integrate the expression of ϕ from 0 to π, to obtain the electric feild E.

$$E = \int_{\phi=0}^{\pi} dE_1$$

$$= \int_{\phi=0}^{\pi} \frac{-\hat{r}\rho_1}{4\pi\varepsilon_0 b} d\phi$$

$$= \frac{-\hat{r}\rho_1}{4\pi\varepsilon_0 b} \int_{\phi=0}^{\pi} d\phi$$

$$= \frac{-\hat{r}\rho_1}{4\pi\varepsilon_0 b} (\phi)_0^{\pi}$$

Simplify the expression further.

$$E = \frac{-\hat{r}\rho_1}{4\pi\varepsilon_0 b}(\pi - 0 = \frac{-\hat{r}\rho_1}{4\varepsilon_0 b} = \frac{-\hat{r}\rho_1}{4\varepsilon_0 b}$$

<u>Review attachment for Matlab and Plot</u>

Data Collection/Plot/Image:

<u>Review attachment for Matlab and Plot</u>

Conclusion/Discussions:

At first this lab section did not look very hard. But it was more time consuming than we thought. The lab instructions were not very clear. We did not cover the Div and Curl calculation in class. On top of that we had midterms rolling over this week's lab work.

In the end, the divergence and curl calculations matched the exact calculations. Also, Maxwell's equation of the divergence of the magnetic field was confirmed.

For the last section, the directions weren't entirely clear until you look at the example code. But even then it is hard to know which plots are required and to know if the correct solution was found. In a portion of the Lab Note, The due to the programming error or interior software programming; the Plots will look similar to one another. Our Scaling for some color bar of part 3.1 on the Experiment was changed. We don't understand why notes are written on the Deliverable doc.

Question 1.

For the divergence and curl problems 3.44 (a), (c) and (i) were your numerical and exact solutions exactly the same? Explain why you think they were or were not. (5 pts)

All of the calculations for the divergence and curl all match most of the simulations. There was some error with the way the chart looks, but everything matches. The divergence calculations and curl calculations are on point. We had a lot of trouble with 3.44 (i). The Matlab function was not able to det

Question 2.

For the magnetic field from a current carrying wire: Do your divergence and curl plots agree with Maxwell's equations? Write down the relevant Maxwell's equation and explain why your plots agree or not with that expected from these equations. Comment on differences between numerical and "exact".(5 pts)

Yes, the plots agree with each other even though the scaling of one of the plots were different. The plots confirm Maxwell's equations especially so with divergence of the magnetic field. That one is the clearest to see. The Maxwell equation was: div B = 0. The plots agree because the exact plot matches the numerical plot. The only difference was the scaling in the color of the pcolor plots. However, there are slight differences due to internal or error of programming (negligible error). Otherwise, they match perfectly.

Question 3.
For the half-ring what happens to the values of Ex and Ey (at the origin) components if we had used a spacing for 0.01 which is smaller than the value used of =100? Are the results for Ex and Ey (at the origin) better or worse with the spacing for of 0.01 and why do you think you see those results? (5 pts)

I think this would result in the worst spacing of the charge representation on the math graph. The charge would be too far away for us to see the result.