Laboratory #6: Closed Loop Feedback System - Design: Design of an effective feedback compensator for a closed loop dc-to-dc converter system.

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ECE 317 - Signals and Systems III

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We explored the design variations discussed in the section titled “Dominant Pole with Lead Compensation” within our textbook and decided to use the first design method. This is because upon observing the PECS simulation results of the Lead Compensated System with Zero for both 500 Hz and 150 Hz, we found that the zero at 500 Hz (determined by $\frac{f_c}{10}$), had a smaller overshoot amplitude and a shorter settling time.

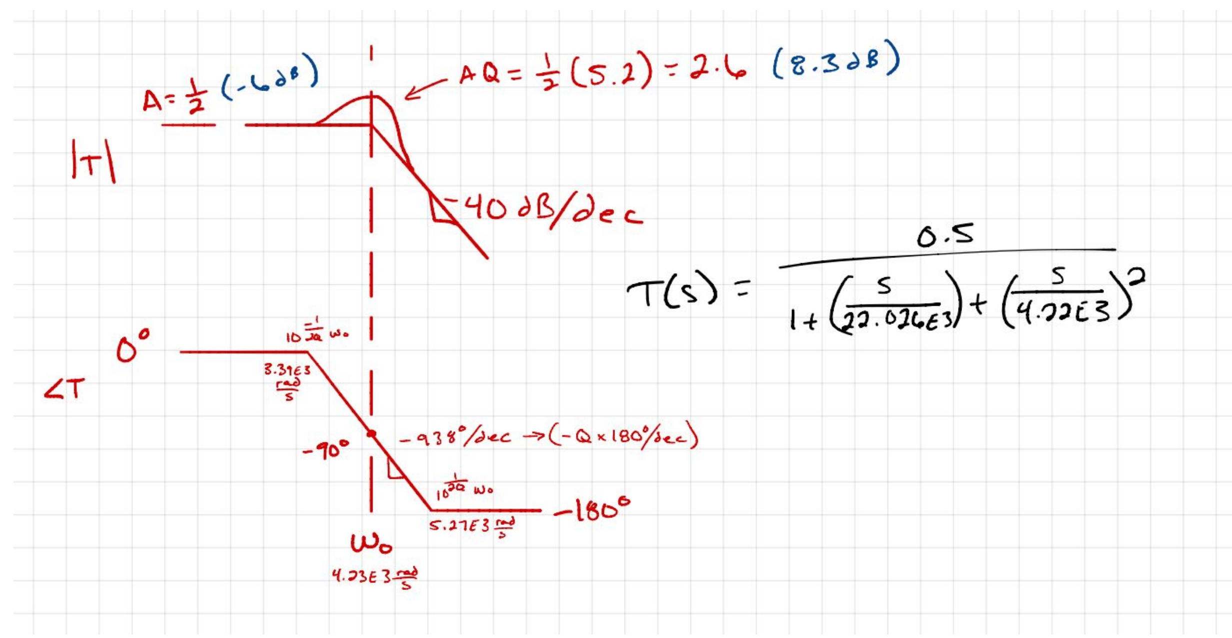

Figure #1 - Magnitude and Phase of Uncompensated Loop Gain

For a desired phase margin of 45°:

$$45° = 45° log(\frac{f_p}{f_z})$$

$$f_p = 10 f_z$$

As described in the pre-lab assignment, we should set the unity gain crossover frequency, f_c , to be ⅛ of the switching frequency. This corresponds to $f_c = 5\ kHz$.

We can now determine f_z and f_p from the following equation:

$$f_c = \sqrt{f_z f_p}$$

$$5\ kHz = \sqrt{10 f_z^{\ 2}}$$

$$f_z = \frac{5\ kHz}{\sqrt{10}}$$

$$f_z = 1.58\ kHz \text{ and } f_p = 15.8\ kHz$$

The compensator is represented by the form:

$$G_c(s) = \frac{\omega_I\,(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_z})}{s(1+\frac{s}{\omega_p})}$$

Where,

$$\omega_I = 2\pi f_I$$

$$\omega_1 = 2\pi(500\ Hz) = 3.14\ krad/s$$

$$\omega_z = 2\pi(1.58\ kHz) = 9.93\ krad/s$$

$$\omega_p = 2\pi(15.8\ kHz) = 99.27\ krad/s$$

The expression for the loop gain then becomes:

$$T(s) = T_o \frac{\omega_I(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_z})}{s(1+\frac{s}{\omega_p})[1+\frac{s}{Q\omega_o}+(\frac{s}{\omega_o})^2]}$$

Where $T_o = 0.5,\ Q = 5.2,\ and\ \omega_o = 4.22\ krad/s$ as found in lab 5.

As discussed within the design procedure in §9.7.1 Design 1: Zero at $f_1 = 500Hz$, ω_I is the remaining value that we need to determine. We can determine f_I from the following relationship:

$$T_o \frac{f_I}{f_1} = T_o G_{c_o}$$

$$f_I = f_1 G_{c_o}$$

As we recall from the lead compensation design, G_{c_o} is determined by:

$$G_{c_o} = \frac{1}{T_o}\left(\frac{f_z}{f_o}\right)^2 \frac{f_c}{f_z}$$

$$G_{c_o} = \frac{1}{0.5}\left(\frac{1.58\ kHz}{672.6\ Hz}\right)^2 \frac{5\ kHz}{1.58\ kHz}$$

$$G_{c_o} = 34.93$$

We can now determine f_I from the equation above:

$$f_I = f_1 G_{c_o}$$

$$f_I = (500\ Hz)(34.93)$$

$$f_I = 17.47\ kHz$$

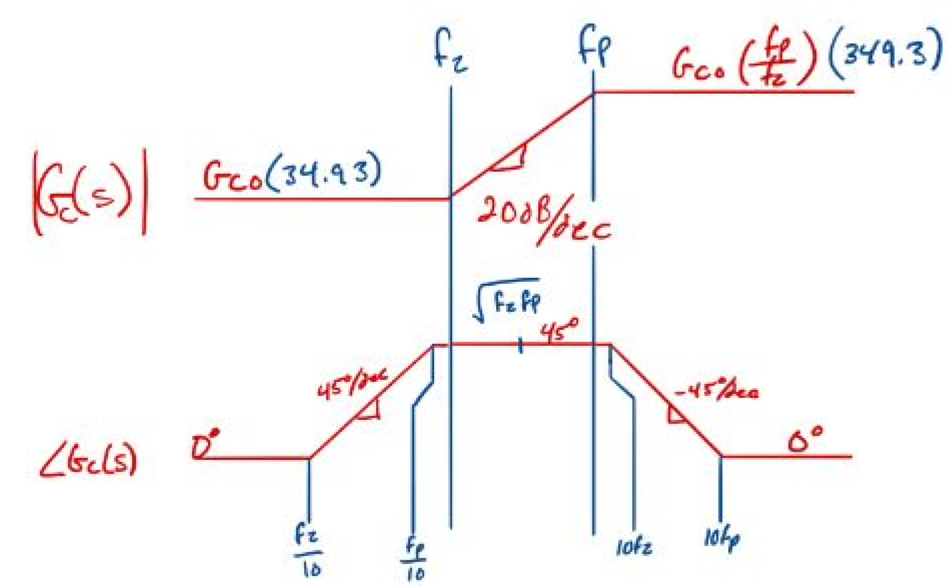

Figure #2 - Bode Plot of Lead Compensator

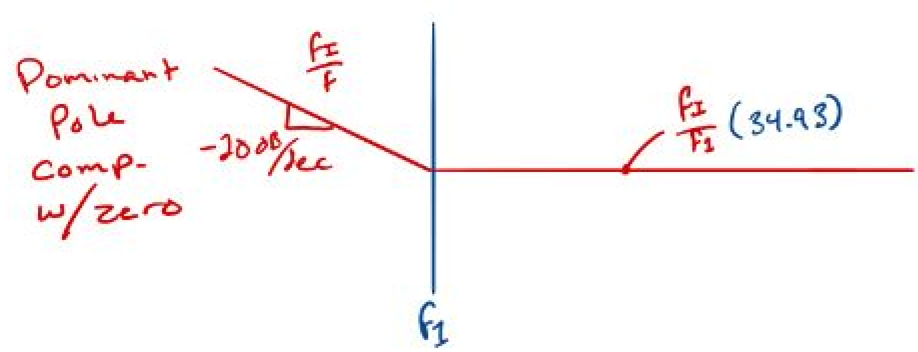

Figure #3 - Bode Plot of Dominant Pole Compensator

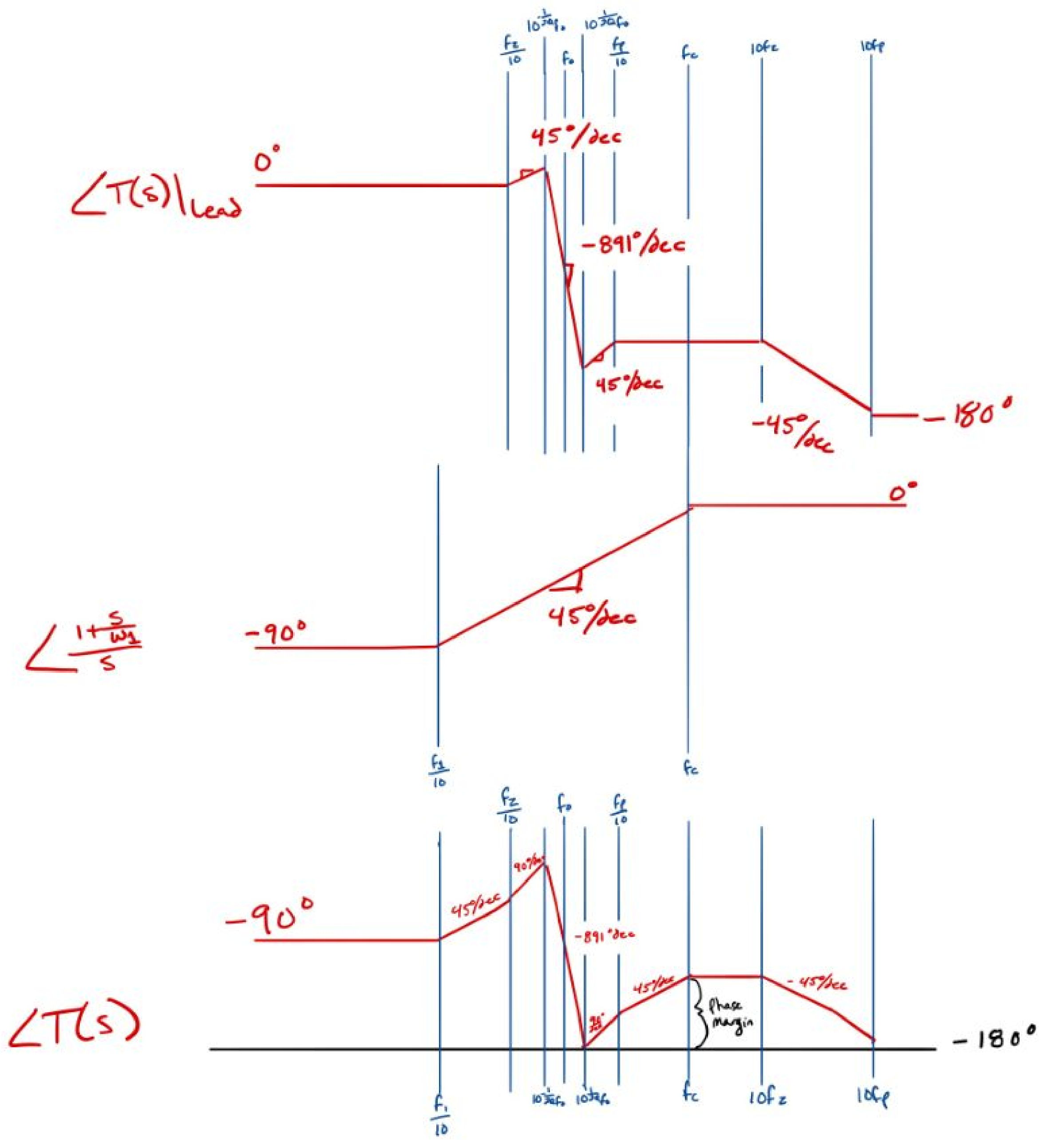

Figure #4 - Bode Plot of System with Lead plus Integral Compensator (Phase)

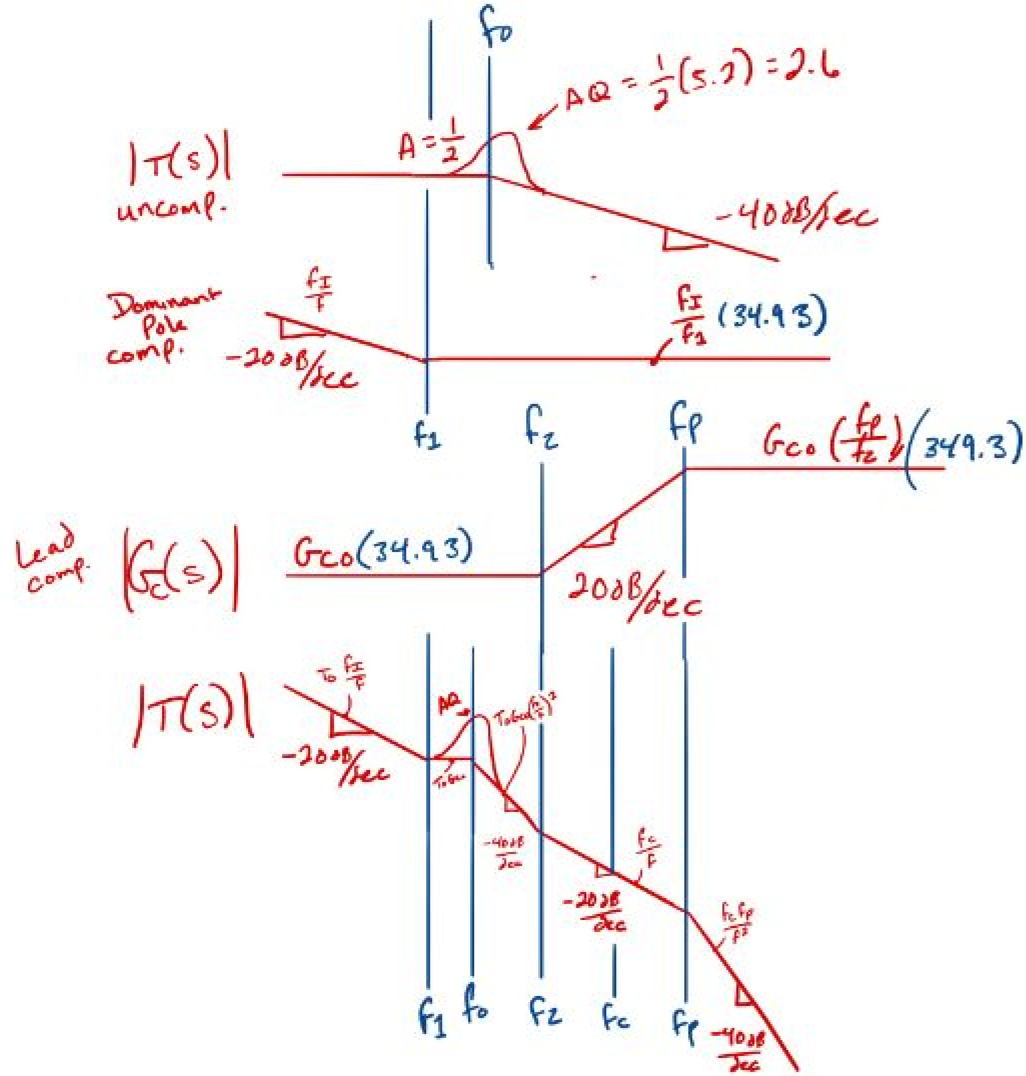

Figure #5 - Bode Plot of System with Lead plus Integral Compensator (Magnitude)

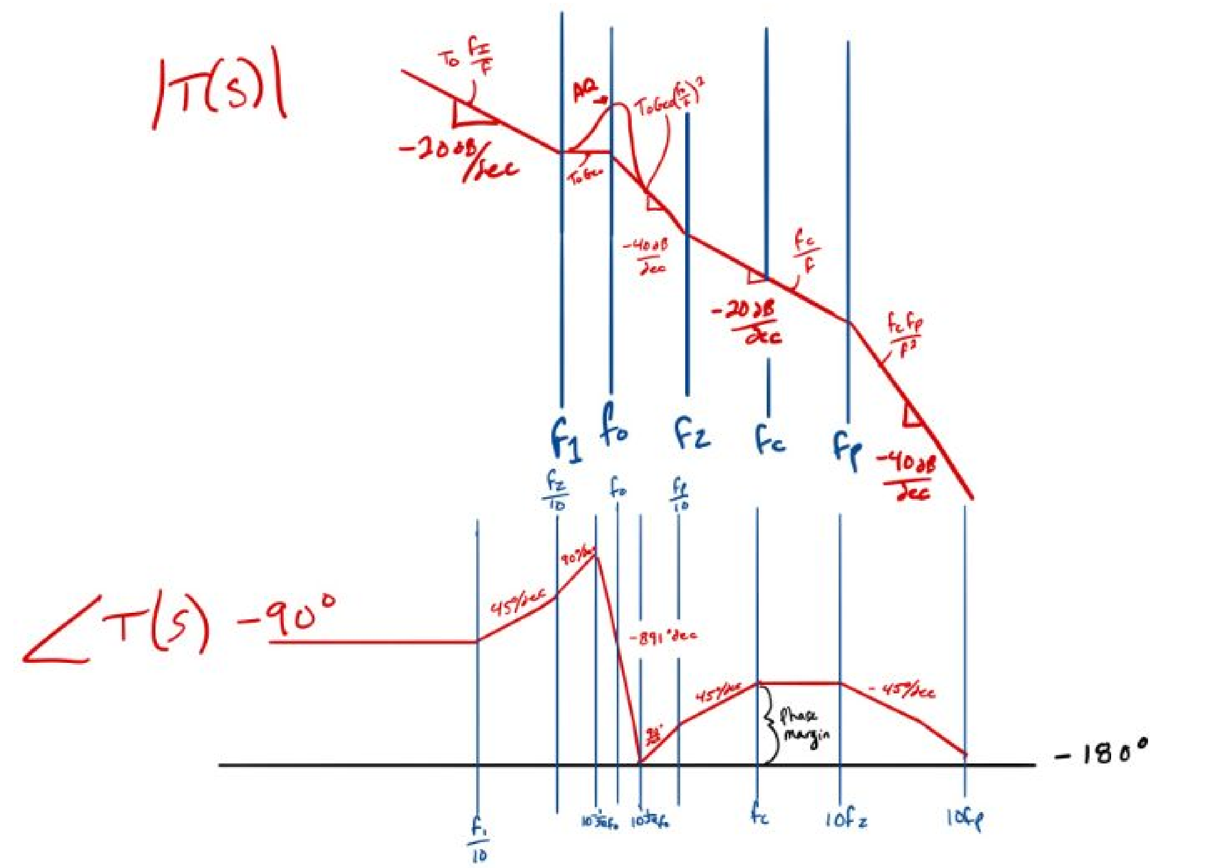

Figure #6 - Bode Plot of System with Lead plus Integral Compensator

$R1 = 100k\Omega$

$R2 = 140k\Omega$

$C1 = 1nF$

$C2 = 2.3nF$

$C3 = 2.8pF$

The peak-to-peak amplitude of the PWM modulator is 10 v.

The closed loop gain:

$$T(s) = -G_c(s)G_{pwm}G_{vd}(s)H(s)$$

Where

$$G_{pwm} = \frac{1}{V_m}$$

$$G_{vd} = \frac{V_g}{1+s(r_L C+\frac{L}{R})+s^2(LC)}$$

$$H(s) = \frac{R_b}{R_b+R_a}$$

Utilizing the values from the schematic: $R = 25\Omega$, $C = 100\mu F$, $L = 560\mu H$, $V_g = 10V$, $R_a = R_b = 1k\Omega$. The pulse width modulation output voltage was previously measured to be $V_m = 9.73V_{pp}$ and we will be modeling inductor losses more accurately with $r_L = 230m\Omega$.

The compensator is now:

$$G_c(s) = \frac{\omega_I (1+\frac{s}{\omega_1})(1+\frac{s}{\omega_z})}{s(1+\frac{s}{\omega_p})}$$

Where

$$\omega_I = 2\pi(17470),\ \omega_1 = 2\pi(500),\ \omega_z = 2\pi(1580),\ \omega_p = 2\pi(15800)$$

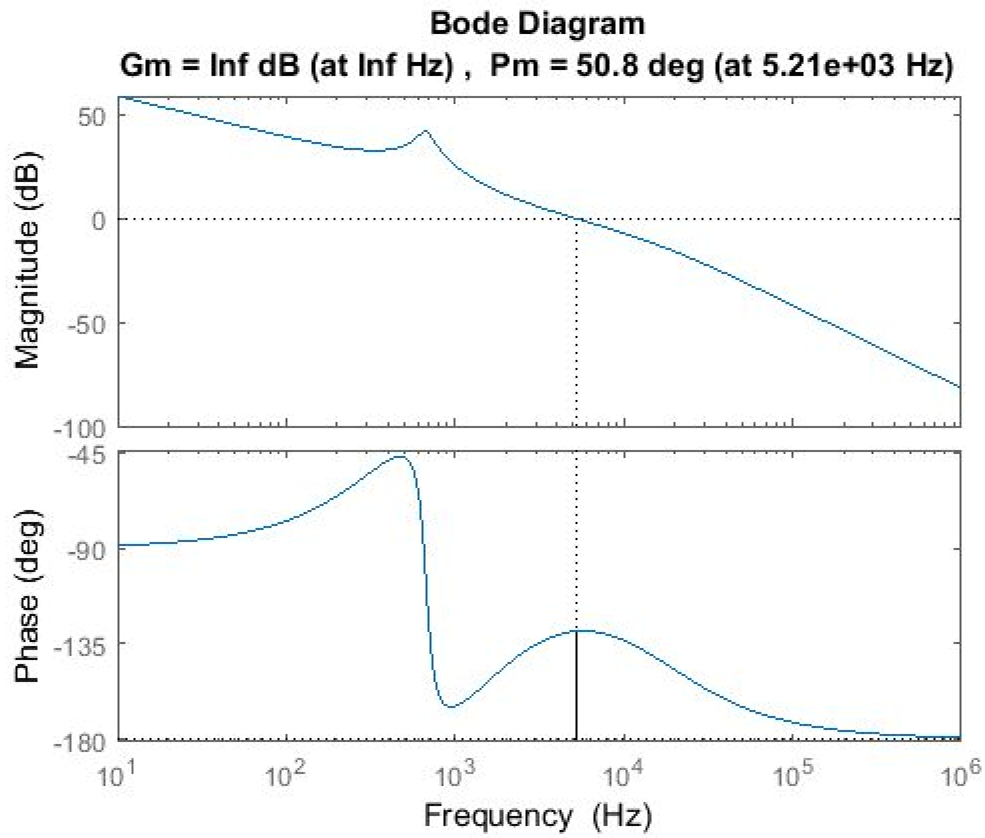

Figure #7 - Closed Loop Gain Bode Plot

The unity gain crossover frequency is 5210 Hz.

The phase margin is 50.8 degrees.

The -180 phase crossover frequency is infinite. Never crosses the -180 phase line.

Therefore, the gain margin is infinite.

The closed loop input to output voltage transfer function:

$$G_{vg-cl} = \frac{G_{vg}(s)}{1+T(s)}$$

Where

$$G_{vg}(s) = \frac{D}{1+s(r_L C+\frac{L}{R})+s^2(LC)}$$

T(s) will be the previously defined closed loop gain expression from above. Also, the duty ratio was previously established to be $D = .5V$.

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
w_I = 2*pi*17470;   % Compensator Integrator
w_l = 2*pi*500;     % Compensator zero
w_p = 2*pi*15800;   % Compensator pole
w_z = 2*pi*1580;    % Compensator zero
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
D = .5;     % Duty Ratio
% System Transfer Functions
G_vg = D/(1+s*(C*r_L+(L/R))+s^2*(L*C));     % IV to OV TF
G_c = (w_I*(1+(s/w_l))*(1+(s/w_z)))/(s*(1+(s/w_p))); % Compensator TF
G_pwm = 1/V_m;                              % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));   % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                          % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Input Voltage to Output Voltage (IV to OV) Transfer Function
G_vg_cl = G_vg/(1+T);
% Create Plot
bodemag(G_vg_cl,G_vg)               % Bode Plot
legend('G_{vg_cl}','G_{vg}')        % Create Legend
h = gcr;
h.AxesGrid.Xunits = 'Hz';           % Display Frequency in Hz
h.AxesGrid.TitleStyle.FontSize=12;  % Increase Font Size
h.AxesGrid.XLabelStyle.FontSize=12; % Increase Font Size
h.AxesGrid.YLabelStyle.FontSize=12; % Increase Font Size
xlim([1,10e3])                      % Range X-Axis 1Hz to 10kHz
```

Figure #8 - Matlab Code for Input Voltage to Output Voltage Transfer Function

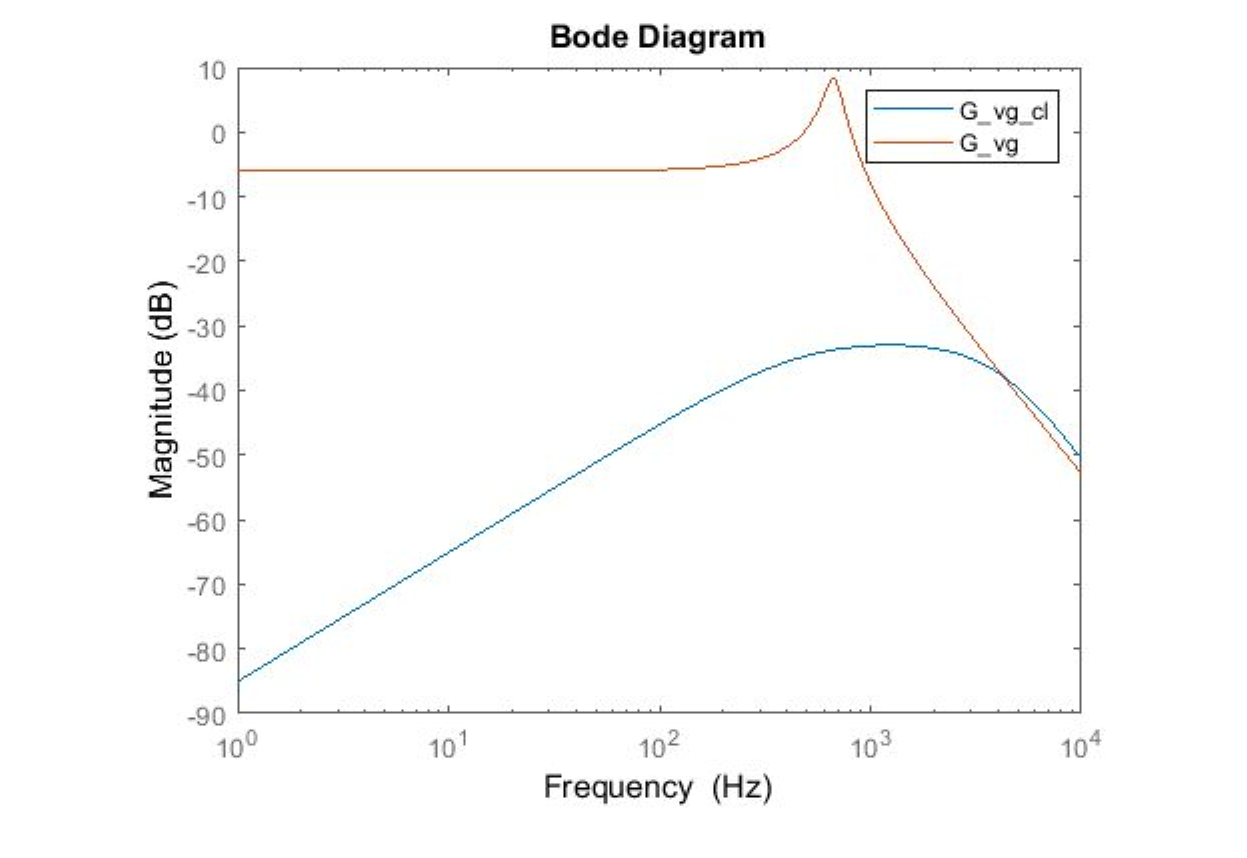

Figure #9 - Bode Plot for Input Voltage to Output Voltage Transfer Function (1Hz to 10kHz)

$$-Z_{out-cl} = \frac{Z_{out}}{1+T(s)}$$

Where

$$-Z_{out} = \quad -\frac{r_L(1+\frac{sL}{r_L})}{1+s(r_LC+\frac{L}{R})+s^2(LC)}$$

T(s) will be the previously defined closed loop gain expression from above.

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
w_I = 2*pi*17470;   % Compensator Integrator
w_1 = 2*pi*500;     % Compensator zero
w_p = 2*pi*15800;   % Compensator pole
w_z = 2*pi*1580;    % Compensator zero
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
% System Transfer Functions
Z_out = -(r_L+s*L)/(1+s*(C*r_L+(L/R))+s^2*(L*C));% OC to OV TF
G_c = (w_I*(1+(s/w_1))*(1+(s/w_z)))/(s*(1+(s/w_p))); % Compensator TF
G_pwm = 1/V_m;                                    % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));        % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                                % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Output Current to Output Voltage (OC to OV) Transfer Function
Z_out_cl = Z_out/(1+T);
% Create Plot
bodemag(Z_out_cl,Z_out)             % Bode Plot
legend('-Z_{out_cl}','-Z_{out}')    % Create Legend
h = gcr;
h.AxesGrid.Xunits = 'Hz';           % Display Frequency in Hz
h.AxesGrid.TitleStyle.FontSize=12;  % Increase Font Size
h.AxesGrid.XLabelStyle.FontSize=12; % Increase Font Size
h.AxesGrid.YLabelStyle.FontSize=12; % Increase Font Size
xlim([1,10e3])                      % Range X-Axis 1Hz to 10kHz
```

Figure #10 - Matlab Code for Output Current to Output Voltage Transfer Function

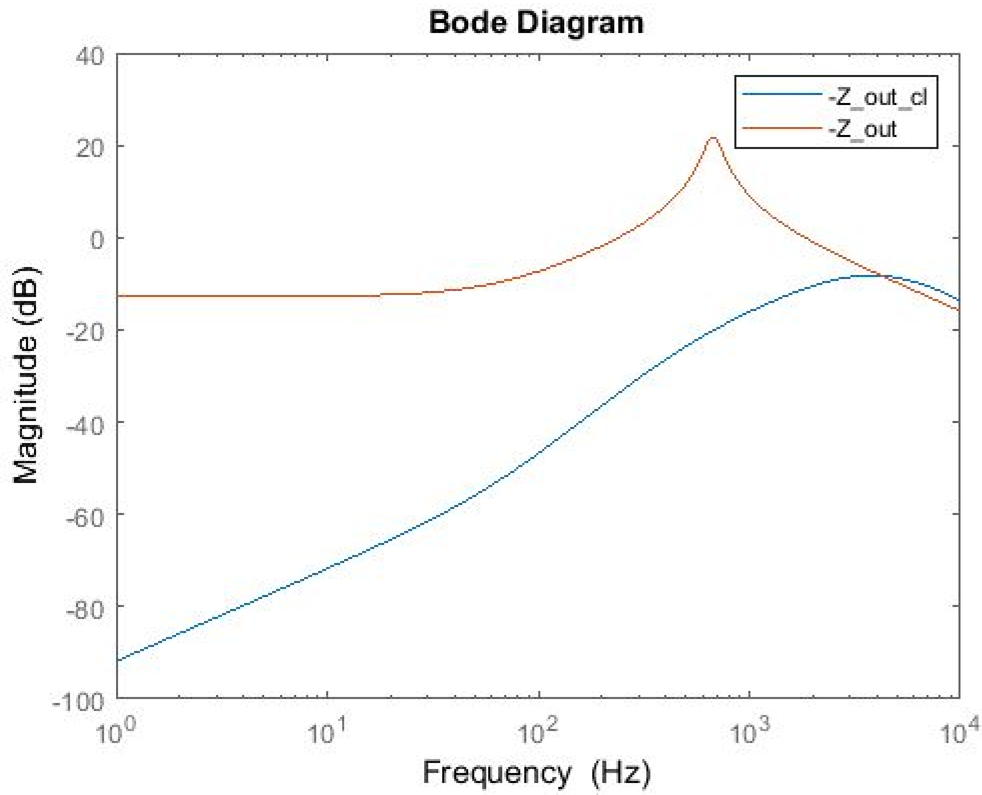

Figure #11 - Bode Plot for Output Current to Output Voltage Transfer Function (1Hz to 10kHz)

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
w_I = 2*pi*17470;   % Compensator Integrator
w_1 = 2*pi*500;     % Compensator zero
w_p = 2*pi*15800;   % Compensator pole
w_z = 2*pi*1580;    % Compensator zero
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
D = .5;     % Duty Ratio
V = D*V_g;  % Voltage from Duty Cycle (Switching)
% System Transfer Functions
G_vg = D/(1+s*(C*r_L+(L/R))+s^2*(L*C));     % IV to OV TF
G_c = (w_I*(1+(s/w_1))*(1+(s/w_z)))/(s*(1+(s/w_p))); % Compensator TF
G_pwm = 1/V_m;                              % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));   % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                          % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Input Voltage to Output Voltage (IV to OV) Transfer Function
G_vg_cl = G_vg/(1+T);
% Create Plot
t = linspace(0.02, 0.06, 1000);
u = zeros(size(t));
ind = find(t>=0.025 & t<=0.04); % Step between 0.025<t<0.04
V_g_diff = 1;
u(ind) = u(ind) + V_g_diff; % Form Input Vector Containing Step
y = lsim(G_vg_cl, u, t);    % Simulate the Step Response
plot(t, y+V);               % Add Steady State Voltage to Response and Plot
title('Closed Loop Input Voltage Step Response');
xlabel('Time(s)');
ylabel('Voltage(V)');
del_v = max(y) - min(y)     % Peak-to-Peak Output Voltage Deviation
SSE = y(ind(end))           % Steady State Error
```

Figure #12 - Matlab Code for Closed Loop IV to OV TF Step Response

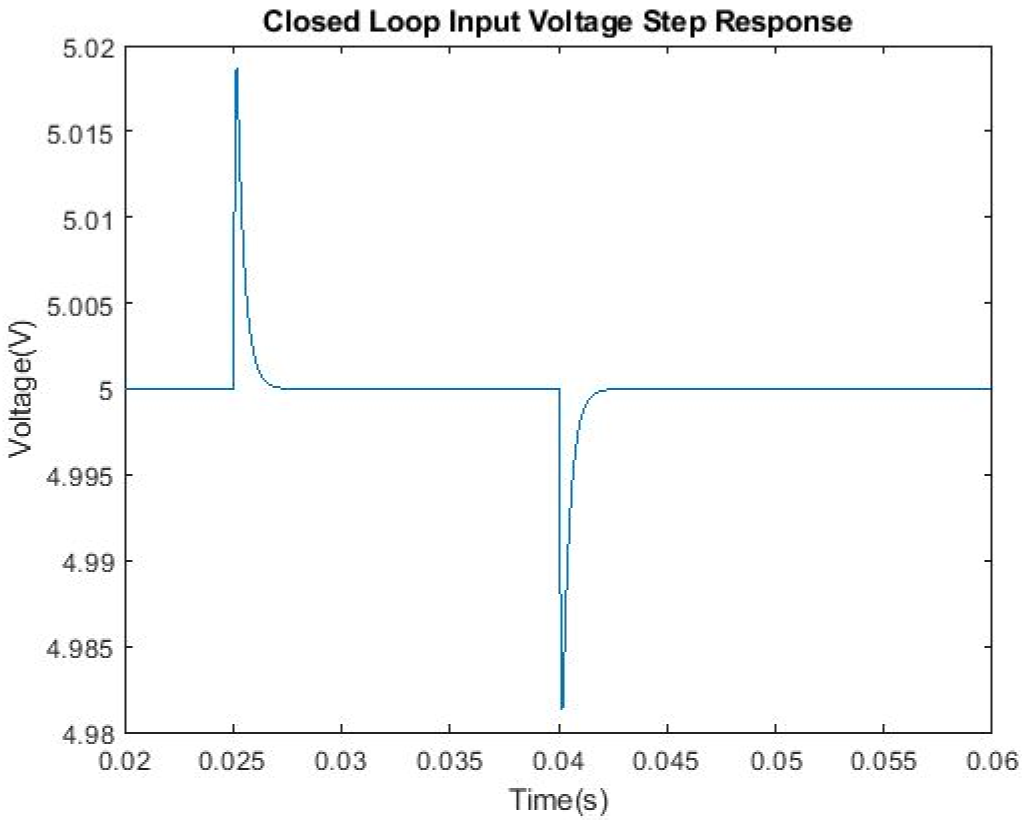

Figure #13 - Closed Loop IV to OV TF Step Response

```
del_v =

    0.0374


SSE =

  -5.4315e-18
```

Figure #14 - Peak-to-Peak Max Voltage Deviation and Steady State Error of Step Response

```matlab
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
w_I = 2*pi*17470;   % Compensator Integrator
w_1 = 2*pi*500;     % Compensator zero
w_p = 2*pi*15800;   % Compensator pole
w_z = 2*pi*1580;    % Compensator zero
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
V = .5*V_g;
% System Transfer Functions
Z_out = -(r_L+s*L)/(1+s*(C*r_L+(L/R))+s^2*(L*C));% OC to OV TF
G_c = (w_I*(1+(s/w_1))*(1+(s/w_z)))/(s*(1+(s/w_p))); % Compensator TF
G_pwm = 1/V_m;                                    % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));         % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                                % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Output Current to Output Voltage (OC to OV) Transfer Function
Z_out_cl = Z_out/(1+T);
% Create Plot
Io_1 = V/25;                % Load Current Before Step (25 ohm load)
Io_2 = V/5;                 % Load Current After Step (5 ohm load)
Io_diff = Io_2 - Io_1;      % Current Step
t = linspace(0.02, 0.06, 1000);
u = zeros(size(t));
ind = find(t>=0.025 & t<=0.04); % Step between 0.025<t<0.04
u(ind) = u(ind) + Io_diff;  % Form Input Vector Containing Step
y = lsim(Z_out_cl, u, t);  % Simulate the Step Response
plot(t,y+V)                 % Add Steady State Voltage to Response and Plot
title('Closed Loop Output Current Step Response');
xlabel('Time(s)');
ylabel('Voltage(V)');
del_v = max(y) - min(y) % Peak-to-Peak Output Voltage Deviation
SSE = y(ind(end))       % Steady State Error
```

Figure #15 - Matlab Code for Closed Loop OC to OV TF Step Response

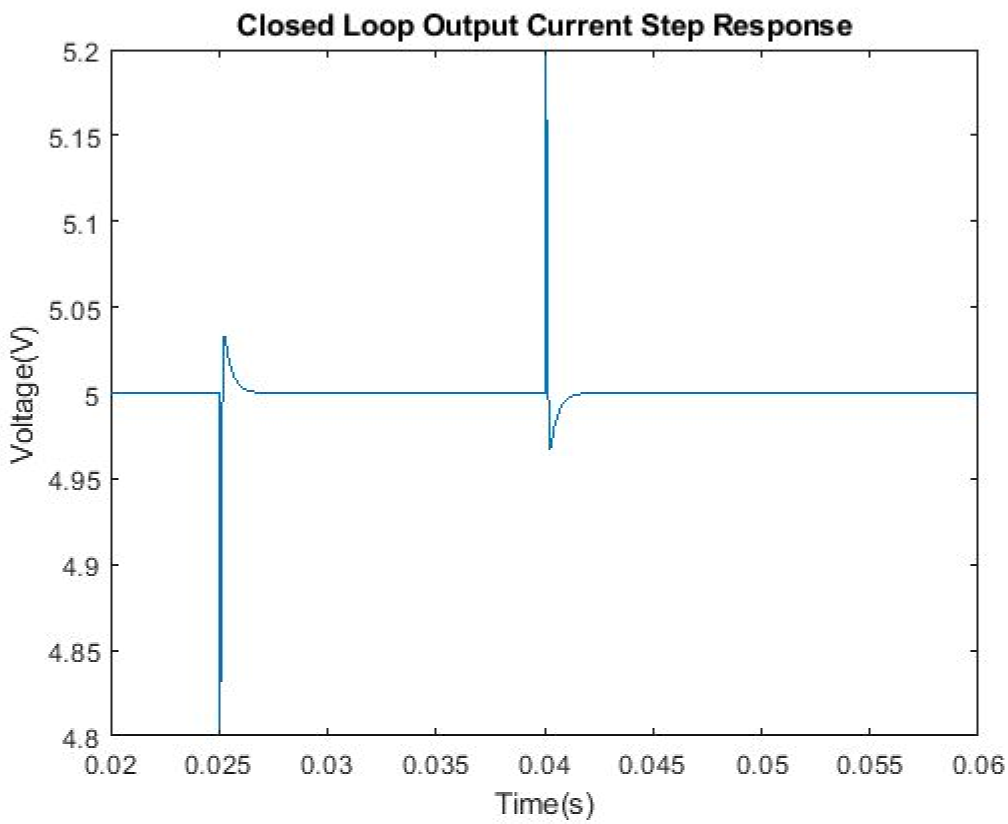

Figure #16 - Closed Loop OC to OV TF Step Response

```
del_v =

    0.3914


SSE =

   3.7404e-17
```

Figure #17 - Peak-to-Peak Max Voltage Deviation and Steady State Error of Step Response

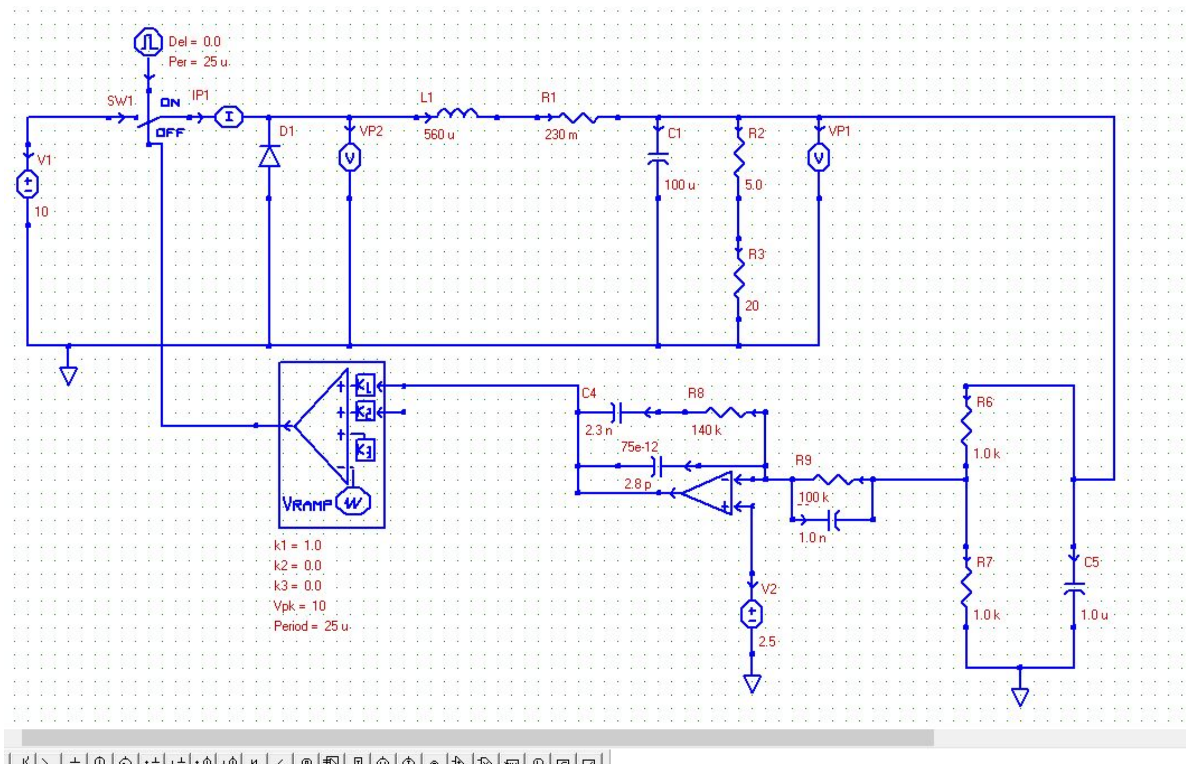

Figure #18 - PECS schematic for simulation of step response of G_{vg_CL}

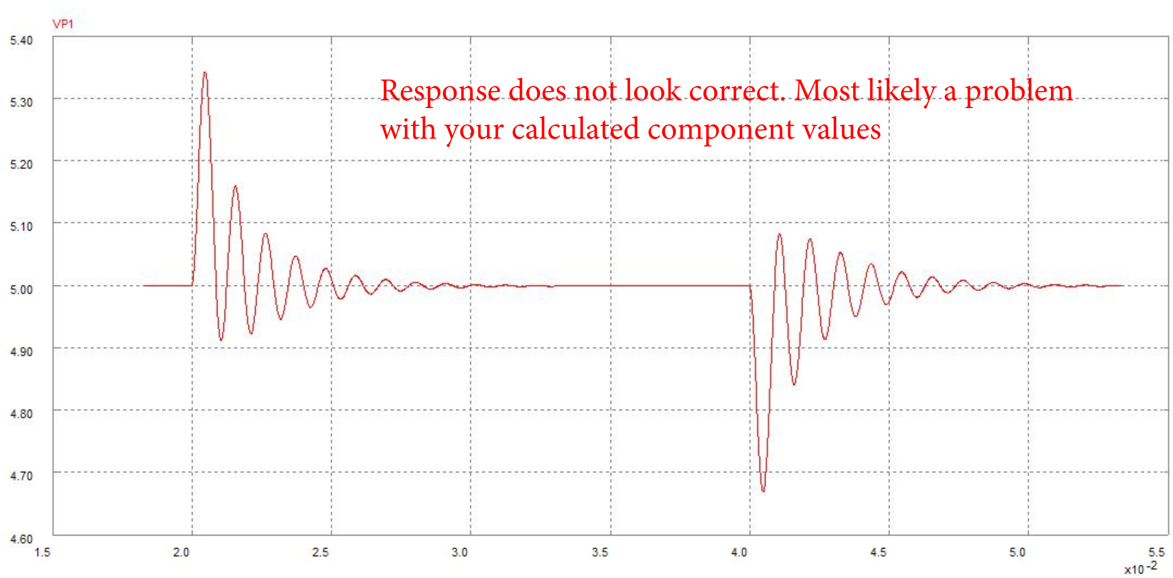

Figure #19 - Step response obtained from PECS

$\Delta V = 5.370\text{-}5V = 370mV$

$SSE = 4.99874V\text{-}4.99882V = 81\mu V$

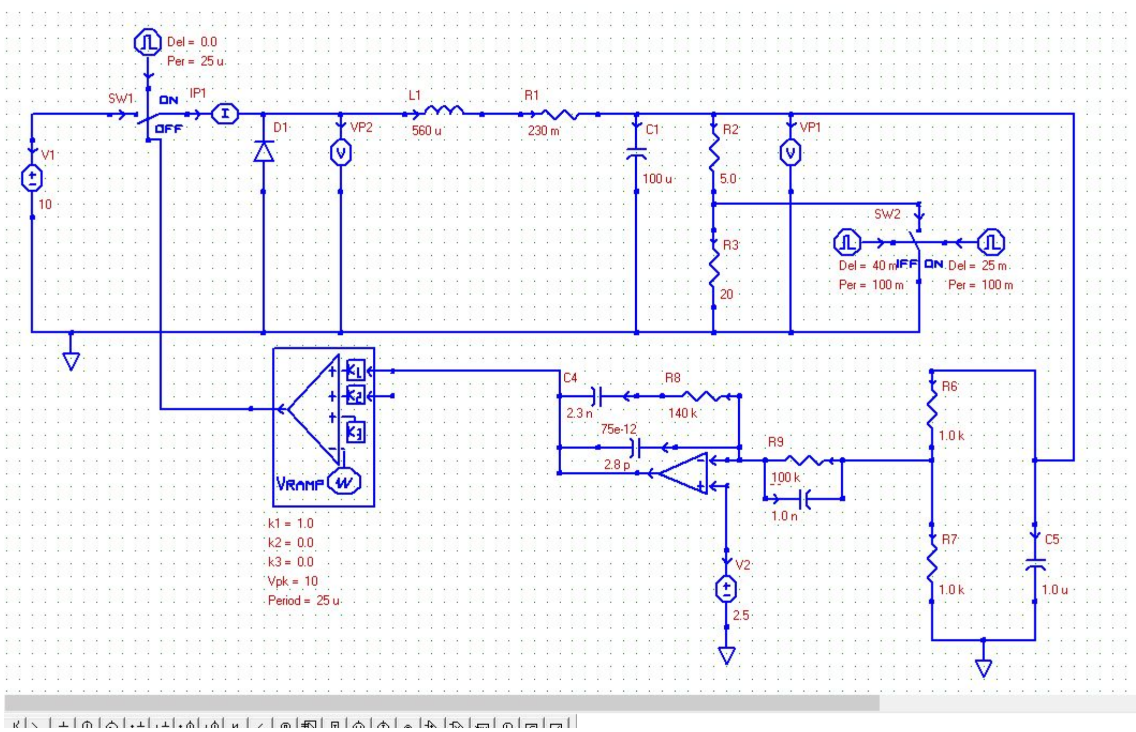

Figure #20 - PECS schematic for simulation of step response of Z_{out_CL}

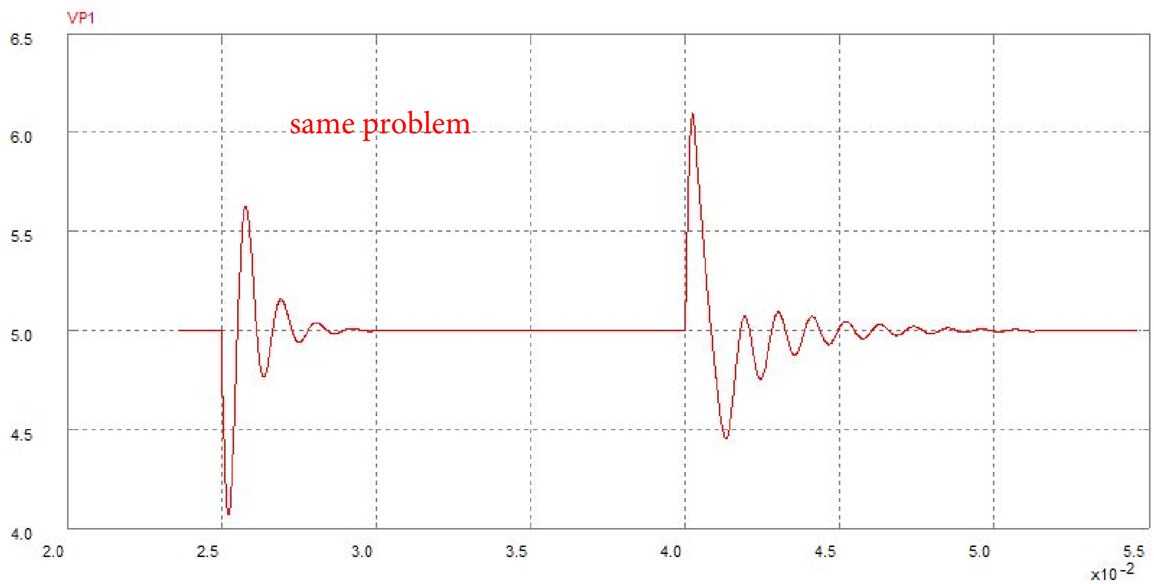

Figure #21 - Step response obtained from PECS

ΔV = 6.10-5V =1.1V

SSE = 4.99805V-4.99895V = 0.9mV

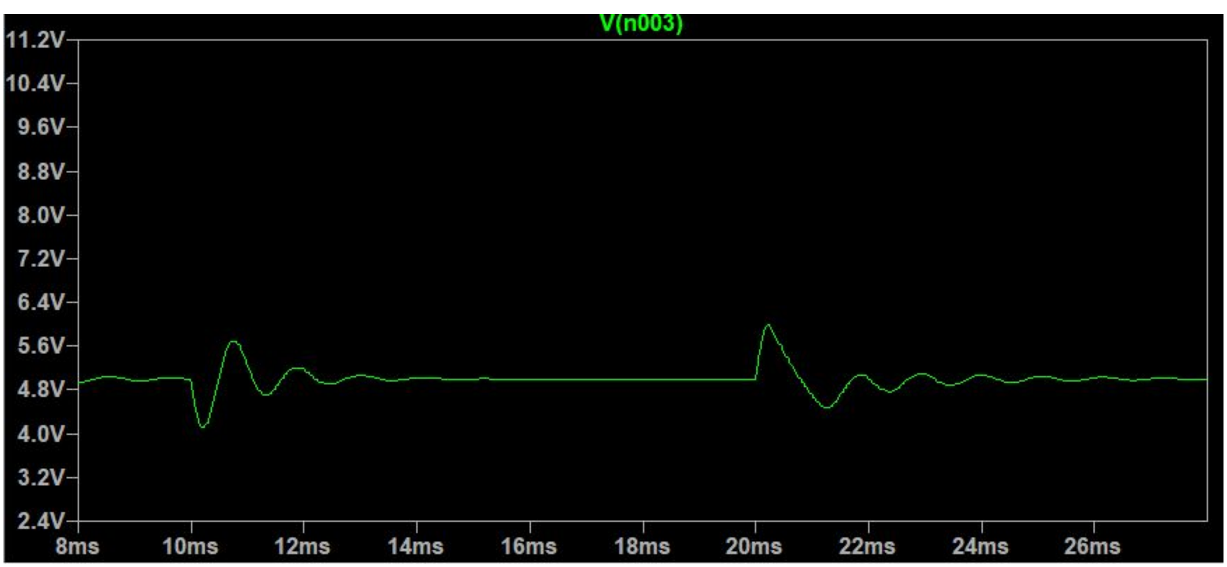

Figure #22 - Lab screenshot of step response of $-Z_{out\_CL}$

$\Delta v = 5.941 - 4.115 = 1.826V$

$SSE = 5.002 - 4.998 = 4mV$

	Phase Margin φ_{PM} (degrees)	Unity Gain Crossover, fc (kHz)	Gain Margin G_{GM} (dB)	Phase Crossover, f_{GM} (kHz)
i) Asymptotes	45	5	∞	∞
ii) Matlab	50.8	5.210	∞	∞

	Open loop* (Uncompensated) (From Lab. 4)	Integral Compensator, (from Lab. 5)	Integral + lead Compensator, (from Lab. 6, this lab)
Compensator Transfer Function $G_c(s)$	No compensator	$G_c(s) = \frac{1}{R_c C_c S}$	$G_c(s) = \frac{\omega_I(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_z})}{s(1+\frac{s}{\omega_p})}$
φ_{PM} (degrees) MATLAB	15.6	89.4	50.8
fc (kHz) MATLAB	.942	.0373 0.673	5.210
i_{out} step: Δv (mV) LAB	3310	2406	1826
i_{out} step: Δv (mV) PECS	2920	3114	1100
i_{out} step: Δv (mV) MATLAB	3410	3540	391.4
i_{out} step: SSE (mV) LAB	220	66	4
i_{out} step: SSE (mV) PECS	174	5	0.9

i_{out} step: SSE (mV) MATLAB	184	2.8	0
v_g step: Δv (mV) PECS	1150	1501	370
v_g step: Δv (mV) MATLAB	1240	1653	37.4
v_g step: SSE (mV) PECS	498	28	.081
v_g step: SSE (mV) MATLAB	498	8.1	0

The above data shows that the progression from lab 5 to 6 with the implementation of dominant pole plus lead compensation had a significant effect on the SSE in both $i_{out}step$ and $v_g step$, in which, both were very close to zero. Comparing the SSE of the open loop without compensation in lab 4 with lab 6, it is clear that by using the dominant pole plus lead compensation we were able to dramatically lower Δv and SSE. Our data for each lab across all three simulations was largely consistent. The minor discrepancies could be due to the complexity of some of the imported circuit elements used in the LTspice schematic, compared with the more simply modeled PECS schematic. Also, as previously noted in past labs, LTspice uses more defined models with parasitic impedances that affect SSE, and PECS uses a different implementation of the compensator and comparator.

Lab 6 Grading Sheet

1. In Lab demonstration of switched load feedback voltage regulator________/10
2. Task 1:
   - (i) Full documentation of your design procedure (do not copy plots from anywhere)________/20
   - (ii) Compensator parameters from simplified equations derived from your asymptotic plots________/5
   - (iii) model of the PWM modulator use a pk-pk ramp amplitude value measured in Lab.4________/1
3. Task 2a:
   - (i) Determine the loop gain transfer function________/1
   - (ii) Matlab *margin* plot in Hz________/1
   - (iii) margins and associated frequencies________/1
4. Task 2b:
   - (i) Expression for G_{vg_CL}________/1
   - (ii) Matlab code to produce *bodemag* plots of G_{vg_CL} and (open loop) G_{vg} /1
   - (iii) Matlab *bodemag* plots of G_{vg_CL} and (open loop) G_{vg} ________/1
   - (iv) (iii) in frequency range 1 Hz to 10 kHz ________/1
5. Task 2c:
   - (i) Expression for $-Z_{out\_CL}$ ________/1
   - (ii) Matlab code to produce *bodemag* plots of $-Z_{out\_CL}$ and (open loop) $-Z_{out}$ /1
   - (iii) Matlab *bodemag* plots of $-Z_{out\_CL}$ and (open loop) $-Z_{out}$ ________/1
   - (iv) (iii) in frequency range 1 Hz to 10 kHz ________/1
6. Task 3a:
   - (i) Full Matlab code for simulation of step response of G_{vg_CL}________/1
   - (ii) Step response obtained from the Matlab code________/1
   - (iii) Δv________/1
   - (iv) SSE ________/1
7. Task 3b:
   - (i) Full Matlab code for simulation of step response of $-Z_{out_CL}$________/1
   - (ii) Step response obtained from the Matlab code________/1
   - (iii) Δv________/1
   - (iv) SSE ________/1
8. Task 4a:
   - (i) PECS schematic for simulation of step response of G_{vg_CL} ________/1

(ii) Step response obtained from PECS 0 /1

(iii) Δv /1

(iv) SSE /1

9. Task 4b:

   (i) PECS schematic for simulation of step response of $-Z_{out\_CL}$ /1

   (ii) Step response obtained from PECS 0 /1

   (iii) Δv /1

   (iv) SSE /1

10. Task 5b:

    (i) Lab screen shot of step response of $-Z_{out\_CL}$ /2

    (ii) Δv /1

    (iii) SSE /1

11. Task 6a:

    (i) Loop stability margins and frequencies table /4

12. Task 6b:

    (i) Large results summary table 14 /15

13. Task 6c:

    (i) Observations on results /5

Report: /9

Total: 97 /100