Laboratory #5: Closed Loop Feedback System - Analysis: Analysis and performance of a closed loop dc-to-dc Buck converter system

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ECE 317 - Signals and Systems III

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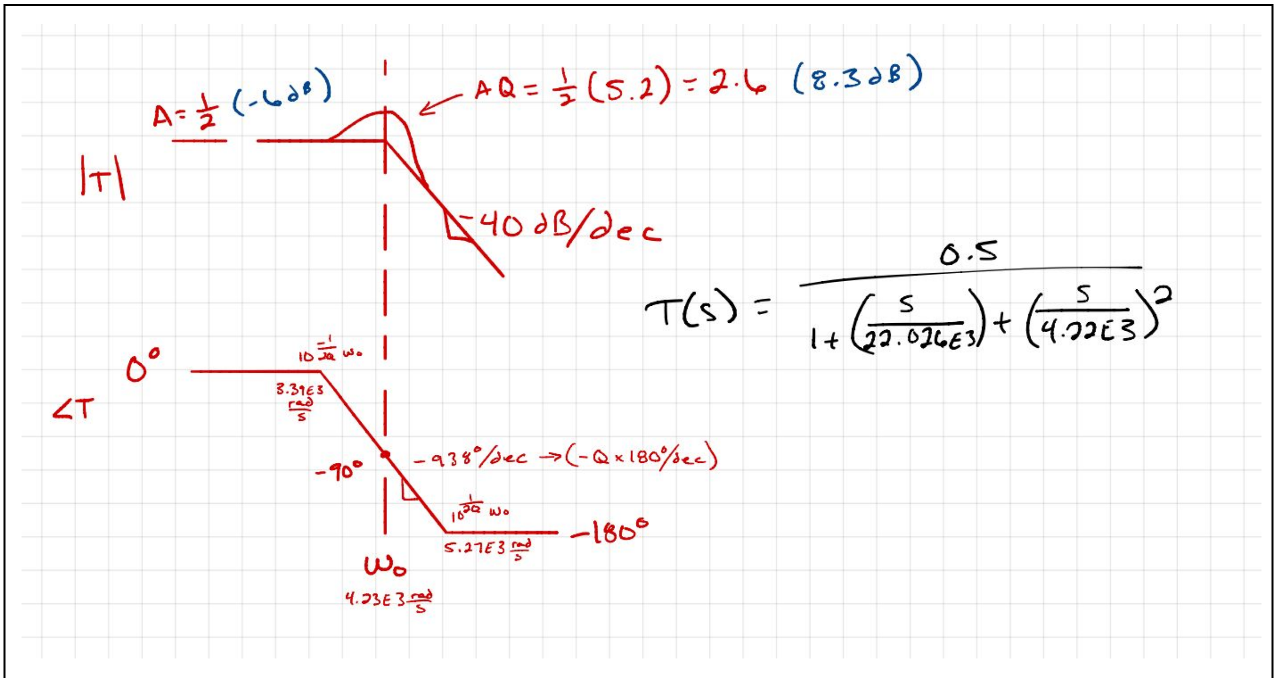
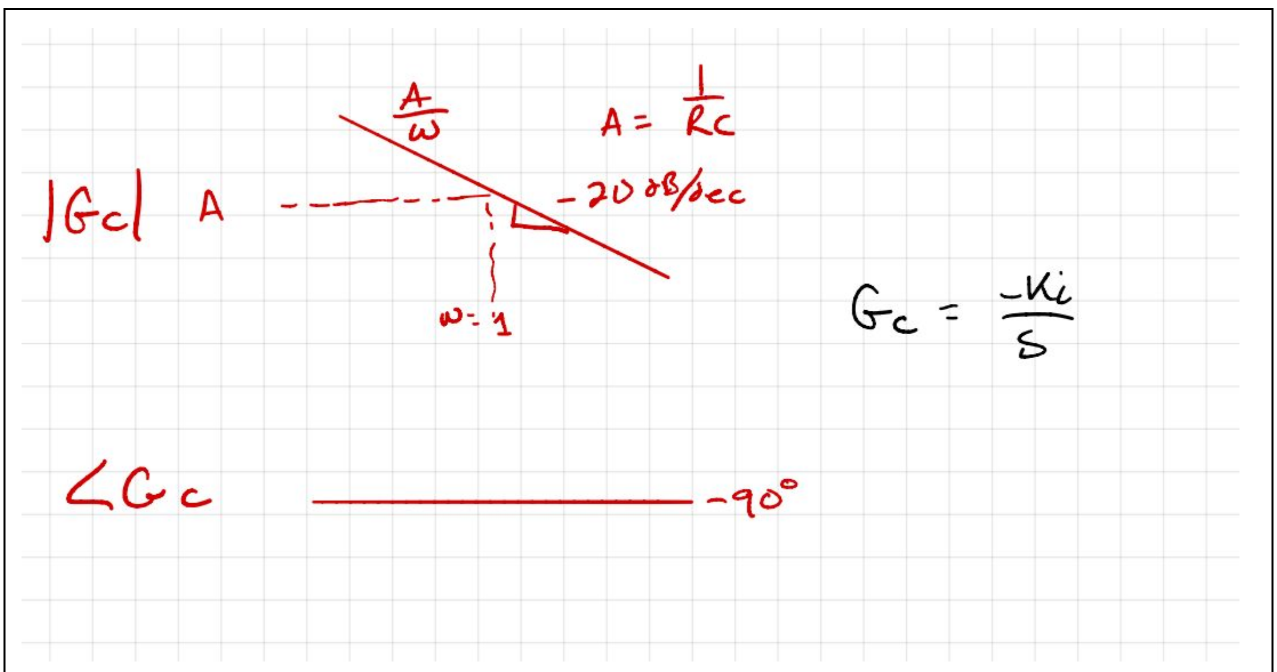
Task 1:



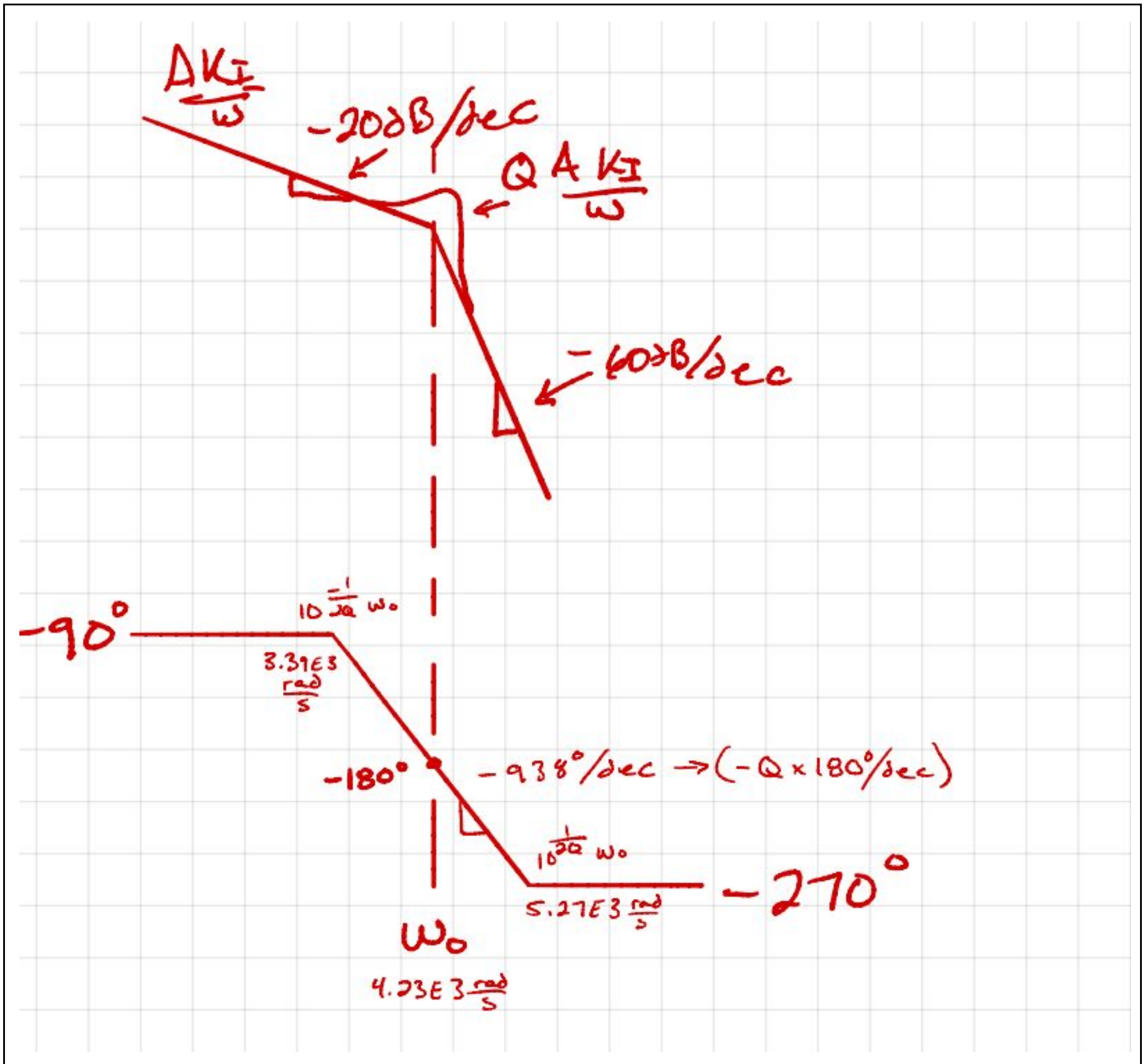
Magnitude and Phase of Uncompensated Loop Gain

Task 1a.ii:



Magnitude and Phase of Compensator

Task 1a.iii:



Magnitude and Phase of Compensated Loop Gain

K_I in the figure above is determined by $K_I = \frac{1}{RC}$. Taking values for R and C from Figure 15.4 in the lab assignment $K_I = 454.5$.

The unity gain crossover frequency occurs when the magnitude is 1 or 0 dB. This occurs when $\frac{AK_I}{\omega} = 1$. Solving for ω, we find that the unity gain crossover frequency occurs at $\frac{K_I}{2}$. Using the values of R and C from Figure 15.4, this crossover frequency is 227.25 rad/s or approximately 36 Hz.

The phase margin is approximately 90 degrees.

The crossover frequency occurs when the phase is -180° and this frequency is equal to $\omega_0 = 4.23 * 10^3\ rad/s$ or approximately 673 Hz.

The gain margin is the difference between unity gain and the gain at -180° phase shift. The gain margin is determined by $\frac{QAK_I}{\omega}$. Again, using the values for L and C specified from Figure 15.4, this value is -0.280 or -11 dB. Since the gain margin is the difference between the unity gain and this value, the gain margin is then 11 dB.

Task 1b:

$$T(s) = \frac{K_I T_0}{s[1+\frac{s}{Q\omega_0}+(\frac{s}{\omega_0})^2]} = \frac{K_I 0.5}{s[1+\frac{s}{22.03E3}+(\frac{s}{4.22E3})^2]}$$

Using 1/(1+T) by using a calculator, the denominator polynomial is:

$$Qs^3 + \omega_0 s^2 + \omega_0{}^2 Qs + \omega_0{}^2 QT_0 K_I = 5.2124s^3 + 4226s^2 + 93.09E6s + 46.54E6\ * K_I$$

	s^4					
	s^3	5.21	93.03E6			
	s^2	4266	45.54E6(Ki)			
Ki<1673	s^1	93.03E6- 55.62E6(Ki)	0			
Ki>0	s^0	45.54E6(Ki)				

Routh-Hurwitz Table to Determine Stable Range of Ki

The stable range of Ki is 0<Ki<1673.

The actual value of Ki used is determined by $\frac{1}{RC}$ and is equal to 454.5.

Task 2a:

The closed loop gain:

$$T(s) = -G_c(s)G_{pwm}G_{vd}(s)H(s)$$

Where

$$G_c(s) = \frac{1}{R_c C_c s}$$

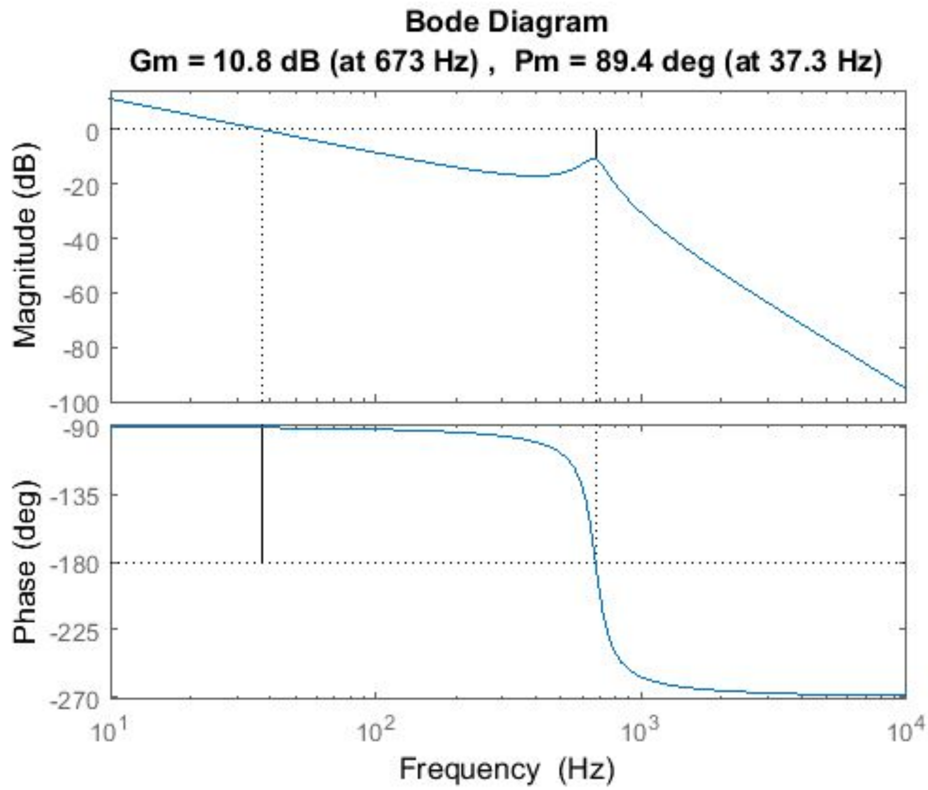
$$G_{pwm} = \frac{1}{V_m}$$

$$G_{vd} = \frac{V_g}{1+s(r_L C+\frac{L}{R})+s^2(LC)}$$

$$H(s) = \frac{R_b}{R_b+R_a}$$

Utilizing the values from the schematic: $R_c = 100k\Omega$, $C_c = 22nF$, $R = 25\Omega$, $C = 100\mu F$, $L = 560\mu H$, $V_g = 10V$, $R_a = R_b = 1k\Omega$. The pulse width modulation output voltage was previously measured to be $V_m = 9.73V_{pp}$ and we will be modeling inductor losses more accurately with $r_L = 230m\Omega$.

Here is the bode plot that the closed loop gain produces:



Closed Loop Gain Bode Plot

The unity gain crossover frequency is 37.3 Hz.

The phase margin is 89.4 degrees.

The -180 phase crossover frequency is 673 Hz.

The gain margin is 10.8 dB.

Task 2b:

The closed loop input to output voltage transfer function:

$$G_{vg-cl} = \frac{G_{vg}(s)}{1+T(s)}$$

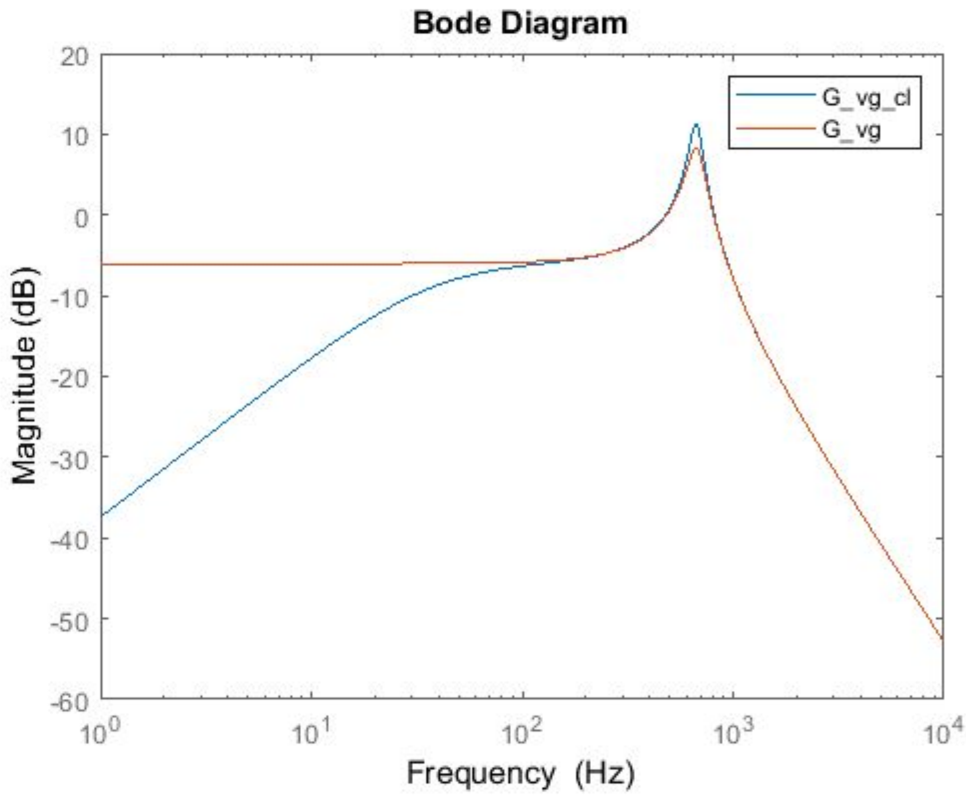
Where

$$G_{vg}(s) = \frac{D}{1+s(r_L C+\frac{L}{R})+s^2(LC)}$$

T(s) will be the previously defined closed loop gain expression from above. Also, the duty ratio was previously established to be $D = .5V$.

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_c = 100e03;
C_c = 22e-09;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
D = .5;     % Duty Ratio
% System Transfer Functions
G_vg = D/(1+s*(C*r_L+(L/R))+s^2*(L*C));     % IV to OV TF
G_c = 1/(R_c*C_c*s);                        % Compensator TF
G_pwm = 1/V_m;                              % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));   % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                          % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Input Voltage to Output Voltage (IV to OV) Transfer Function
G_vg_cl = G_vg/(1+T);
% Create Plot
bodemag(G_vg_cl,G_vg)                % Bode Plot
legend('G_{vg_cl}','G_{vg}')         % Create Legend
h = gcr;
h.AxesGrid.Xunits = 'Hz';            % Display Frequency in Hz
h.AxesGrid.TitleStyle.FontSize=12;  % Increase Font Size
h.AxesGrid.XLabelStyle.FontSize=12; % Increase Font Size
h.AxesGrid.YLabelStyle.FontSize=12; % Increase Font Size
xlim([1,10e3])                       % Range X-Axis 1Hz to 10kHz
```

Matlab Code for Input Voltage to Output Voltage Transfer Functions



Bode Plot for Input Voltage to Output Voltage Transfer Functions (1Hz to 10kHz)

Task 2c:

$$-Z_{out-cl} = \frac{Z_{out}}{1+T(s)}$$

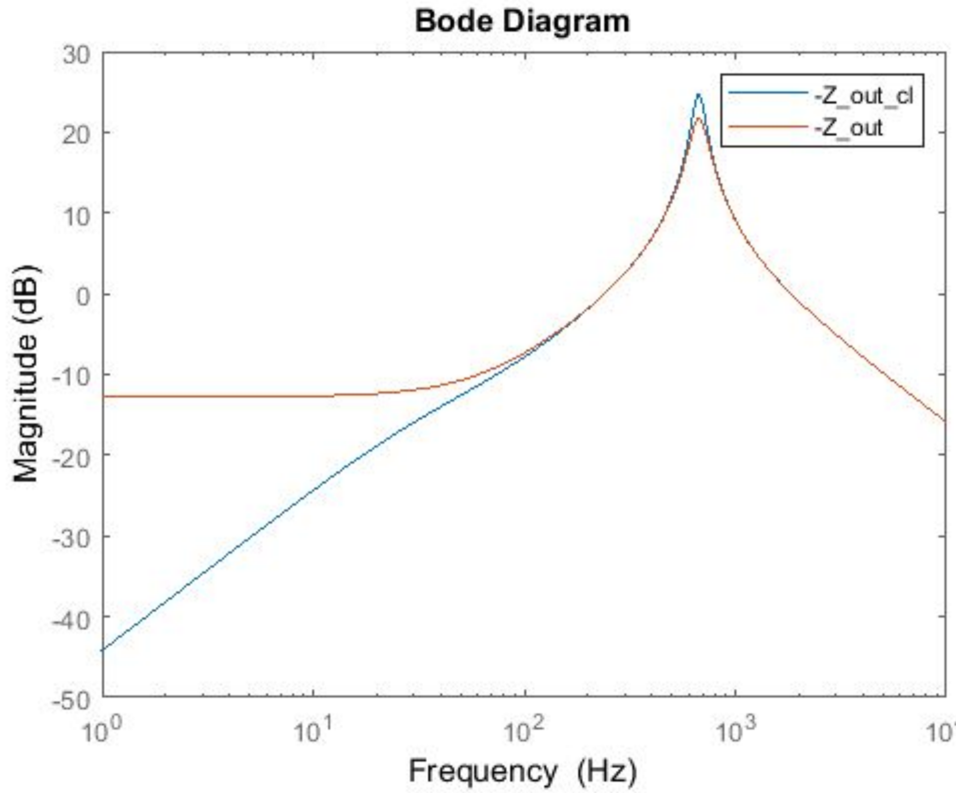
Where

$$-Z_{out} = -\frac{r_L(1+\frac{sL}{r_L})}{1+s(r_L C+\frac{L}{R})+s^2(LC)}$$

T(s) will be the previously defined closed loop gain expression from above.

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_c = 100e03;
C_c = 22e-09;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
% System Transfer Functions
Z_out = -(r_L+s*L)/(1+s*(C*r_L+(L/R))+s^2*(L*C));% OC to OV TF
G_c = 1/(R_c*C_c*s);                            % Compensator TF
G_pwm = 1/V_m;                                  % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));       % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                              % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Output Current to Output Voltage (OC to OV) Transfer Function
Z_out_cl = Z_out/(1+T);
% Create Plot
bodemag(Z_out_cl,Z_out)              % Bode Plot
legend('-Z_{out_cl}','-Z_{out}')     % Create Legend
h = gcr;
h.AxesGrid.Xunits = 'Hz';            % Display Frequency in Hz
h.AxesGrid.TitleStyle.FontSize=12;  % Increase Font Size
h.AxesGrid.XLabelStyle.FontSize=12; % Increase Font Size
h.AxesGrid.YLabelStyle.FontSize=12; % Increase Font Size
xlim([1,10e3])                       % Range X-Axis 1Hz to 10kHz
```

Matlab Code for Output Current to Output Voltage Transfer Functions

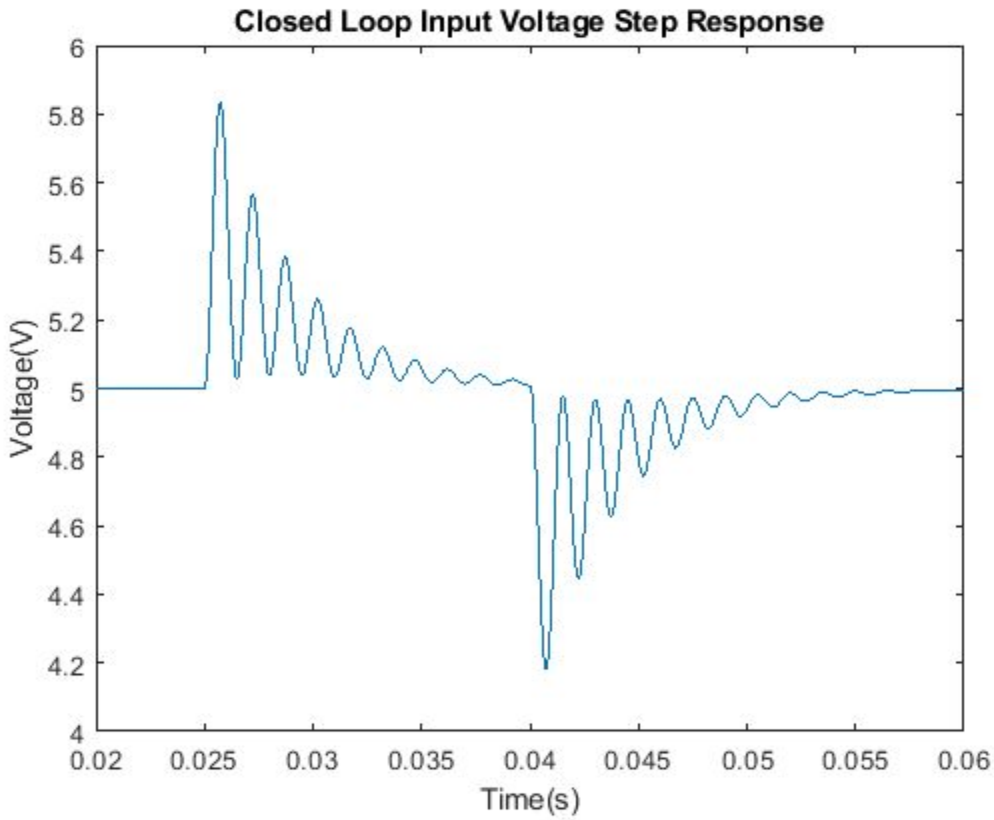


Bode Plot for Output Current to Output Voltage Transfer Functions (1Hz to 10kHz)

Task 3a:

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_c = 100e03;
C_c = 22e-09;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
D = .5;     % Duty Ratio
V = D*V_g;  % Voltage from Duty Cycle (Switching)
% System Transfer Functions
G_vg = D/(1+s*(C*r_L+(L/R))+s^2*(L*C));     % IV to OV TF
G_c = 1/(R_c*C_c*s);                        % Compensator TF
G_pwm = 1/V_m;                              % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));   % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                          % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Input Voltage to Output Voltage (IV to OV) Transfer Function
G_vg_cl = G_vg/(1+T);
% Create Plot
t = linspace(0.02, 0.06, 1000);
u = zeros(size(t));
ind = find(t>=0.025 & t<=0.04); % Step between 0.025<t<0.04
V_g_diff = 1;
u(ind) = u(ind) + V_g_diff; % Form Input Vector Containing Step
y = lsim(G_vg_cl, u, t);    % Simulate the Step Response
plot(t, y+V);               % Add Steady State Voltage to Response and Plot
title('Closed Loop Input Voltage Step Response');
xlabel('Time(s)');
ylabel('Voltage(V)');
del_v = max(y) - min(y)     % Peak-to-Peak Output Voltage Deviation
SSE = y(ind(end))           % Steady State Error
```

Matlab Code for Closed Loop IV to OV TF Step Response



Closed Loop IV to OV TF Step Response

```
del_v =

    1.6525


SSE =

    0.0081
```

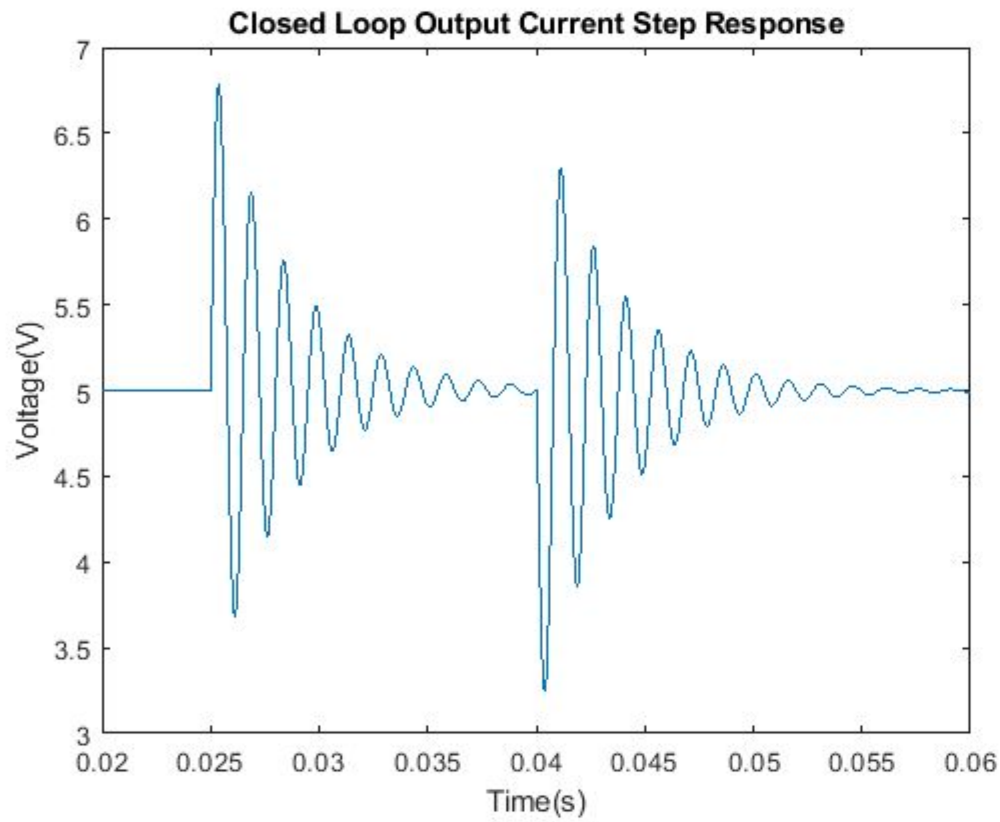
Peak-to-Peak Max Voltage Deviation and Steady State Error of Step Response

Task 3b:

```
s = tf('s');
% System Component Values
C = 100e-06;
R = 25;
r_L = 230e-03;
L = 560e-06;
R_c = 100e03;
C_c = 22e-09;
R_b = 1e03;
R_a = 1e03;
% System Design/Measured Values
V_m = 9.73; % PWM Peak-to-Peak Voltage
V_g = 10;   % Input Voltage
V = .5*V_g;
% System Transfer Functions
Z_out = -(r_L+s*L)/(1+s*(C*r_L+(L/R))+s^2*(L*C));% OC to OV TF
G_c = 1/(R_c*C_c*s);                              % Compensator TF
G_pwm = 1/V_m;                                    % Pulse Width Modulator TF
G_vd = V_g/(1+s*(C*r_L+(L/R))+s^2*(L*C));         % Duty Ratio to OV TF
H = R_b/(R_b+R_a);                                % Feedback TF
% Closed Loop Gain Transfer Function
T = G_c*G_pwm*G_vd*H;
% Closed Loop Output Current to Output Voltage (OC to OV) Transfer Function
Z_out_cl = Z_out/(1+T);
% Create Plot
Io_1 = V/25;               % Load Current Before Step (25 ohm load)
Io_2 = V/5;                % Load Current After Step (5 ohm load)
Io_diff = Io_2 - Io_1;     % Current Step
t = linspace(0.02, 0.06, 1000);
u = zeros(size(t));
ind = find(t>=0.025 & t<=0.04); % Step between 0.025<t<0.04
u(ind) = u(ind) + Io_diff;  % Form Input Vector Containing Step
y = lsim(-Z_out_cl, u, t);  % Simulate the Step Response
plot(t,y+V)                 % Add Steady State Voltage to Response and Plot
title('Closed Loop Output Current Step Response');
xlabel('Time(s)');
ylabel('Voltage(V)');
del_v = max(y) - min(y) % Peak-to-Peak Output Voltage Deviation
SSE = y(ind(end))       % Steady State Error
```

used negative sign twice

Matlab Code for Closed Loop OC to OV TF Step Response inverted plot

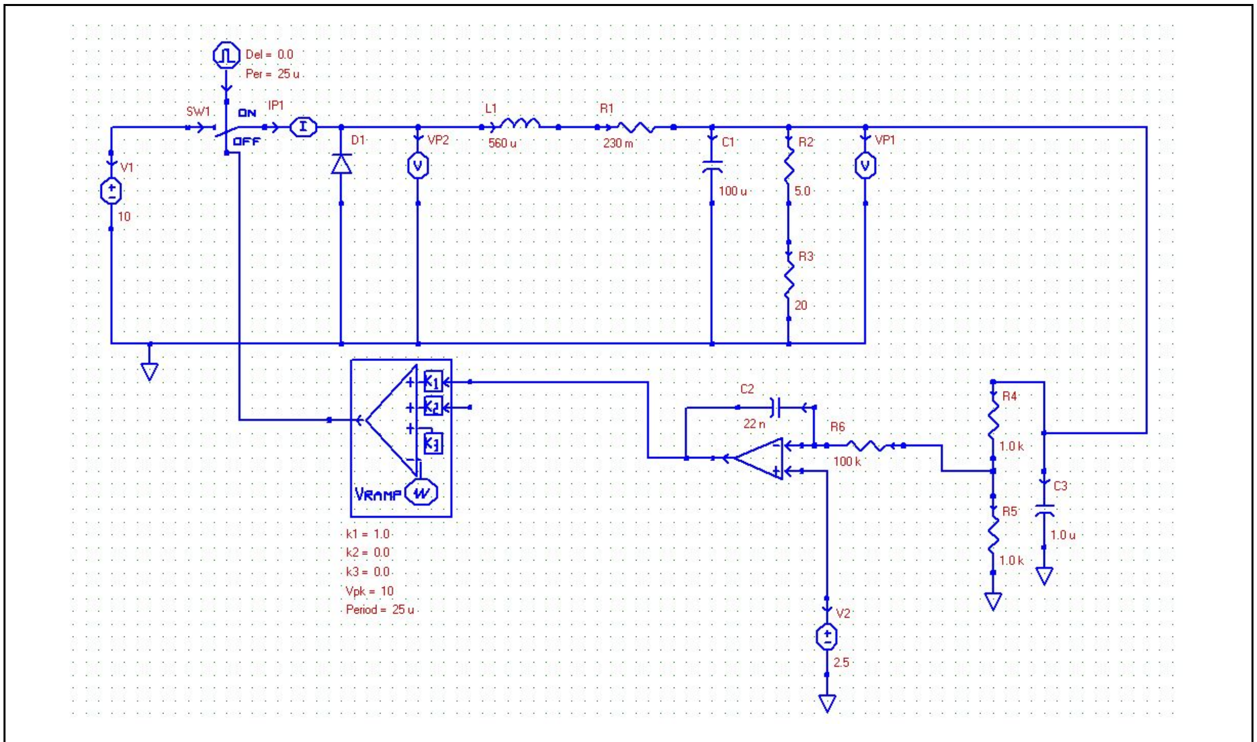


Closed Loop OC to OV TF Step Response

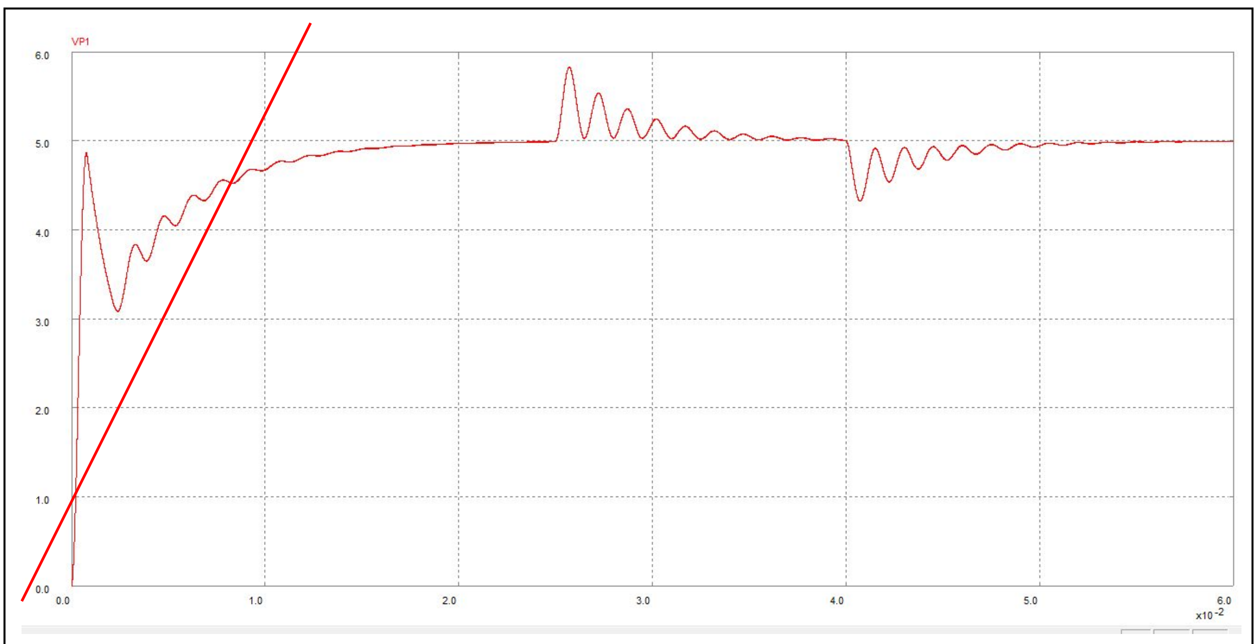
```
del_v =

    3.5401


SSE =

    0.0028
```

Peak-to-Peak Max Voltage Deviation and Steady State Error of Step Response

Task 4a:



PECS schematic for simulation of step response of Gvg_CL



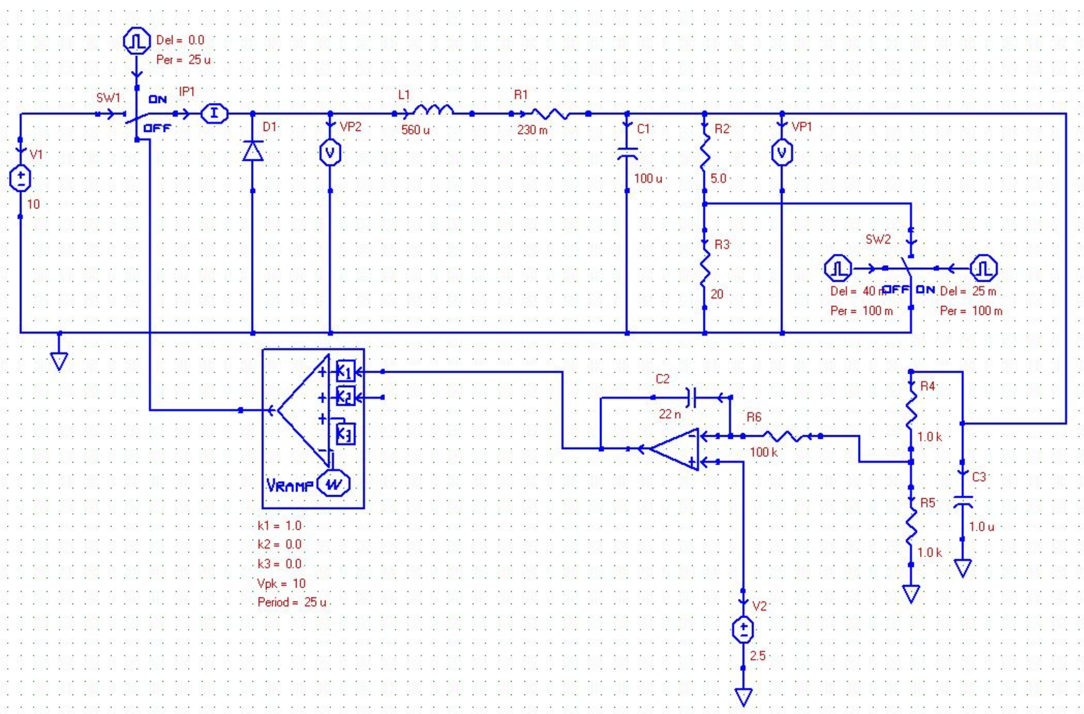
Input voltage step response

for next lab do not include start

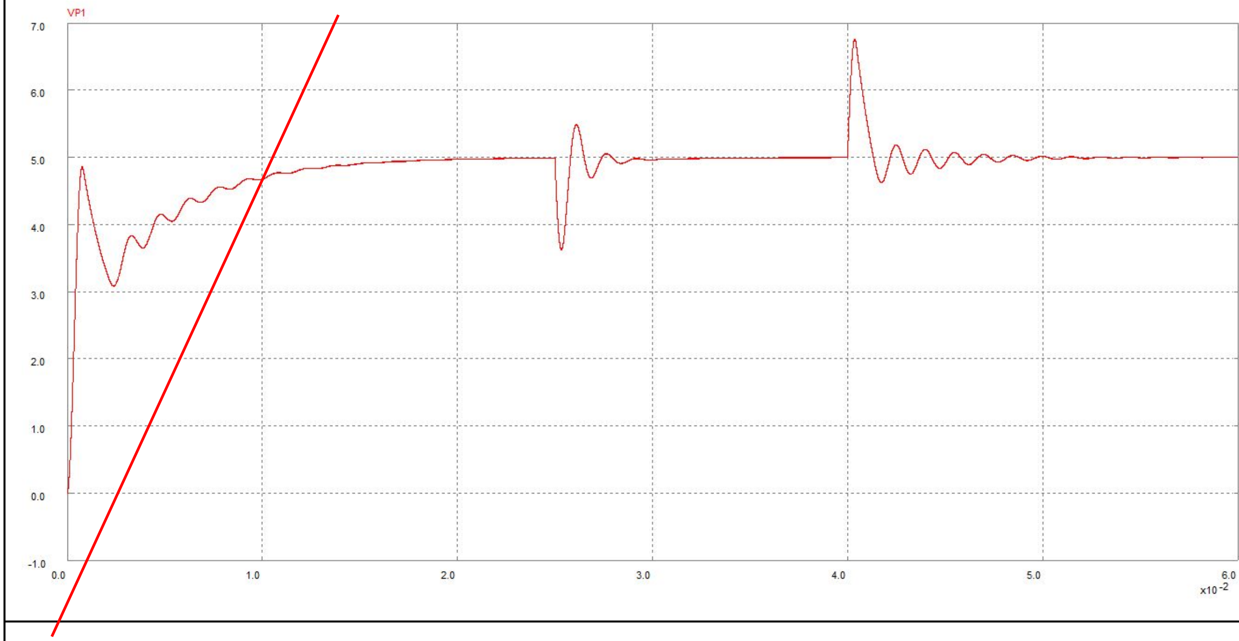
$$\Delta V = 5.823 - 4.322 = 1.501V$$

$$SSE = 5.024 - 4.996 = 28mV$$

Task 4b:



PECS schematic for simulation of step response of -Zout_CL



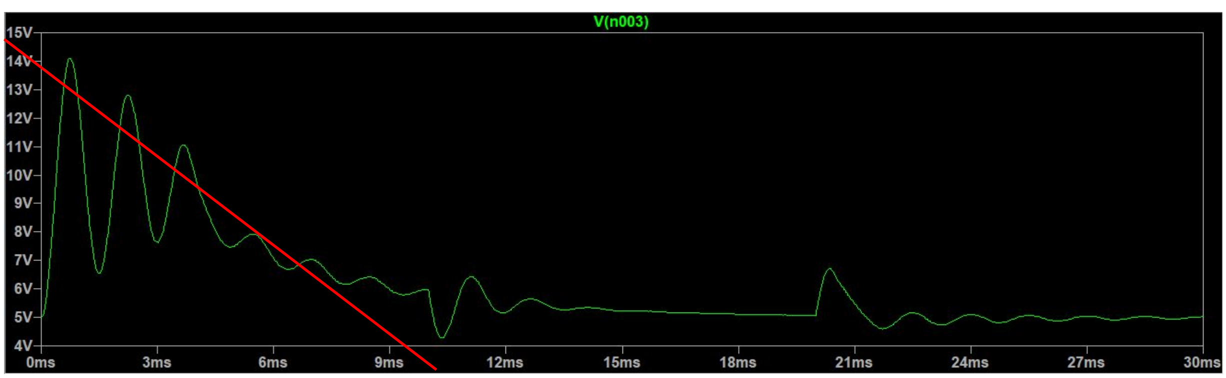
Output current step response

$\Delta V = 6.762 - 3.648 = 3.114V$

$SSE = 5.000 - 4.995 = 5mV$

12. Task 5b: Nic

Do not include start for next lab



LTspice screenshot for step response of -Zout_CL

$\Delta V = 6.711 - 4.305 = 2.406V$

$SSE = 5.079 - 5.013 = 66mV$

Task 6a:

	Phase Margin (Degrees)	Unity gain crossover Frequency (hz)	Gain Margin(dB)	Phase Crossover Frequency(hz)
Asymptotes	90	36	11	673
matlab	89.4	37.3	10.8	673

Task 6b:

	Matlab Simulation	PECS Simulation	Lab(LTSpice)
V_g :Step ΔV	1.6525V	1.501V	N/A
V_g :Step SSE	.0081V	28mV	N/A
i_{out}:Step ΔV	3.5401V	3.114V	2.406V
i_{out}:Step SSE	.0028V	5mV	66mV

Observations on results:

These results seem to match fairly close to one another. The Largest difference arises in the various SSE associated with each metric, as well as the LTSpice simulations. This can most likely be accounted for in slight differences in how each simulation is set up. There are more assumptions made in the Matlab transfer functions versus how PECS and LTSpice solve the full circuits. PECS has a higher level look at the compensator and comparator, while LTspice uses more defined models with various parasitics and impedances that will affect the SSE.

missing task 7

Lab 5 Grading Sheet

1. Task 1:
   (i) Sketch of magnitude and phase response of uncompensated loop gain__/1
   (ii) Sketch properly annotated__/1
2. Task 1a.ii:
   (i) Sketch of magnitude and phase response of compensator__/1
   (ii) Sketch properly annotated__/1
3. Task 1a.iii:
   (i) Sketch of magnitude and phase response of compensated loop gain (i.e. above two combined)__/1
   (ii) Sketch properly annotated__/2
   (iii) From sketch determine, unity gain crossover frequency__/1
   (iv) From sketch determine, phase margin__/1
   (v) From sketch determine, crossover frequency__/1
   (vi) From sketch determine, gain margin__/1
4. Task 1b:
   (i) Expression for compensated loop gain, T__/1
   (ii) Expression for denominator polynomial of $1/(1+T)$ or $T/(1+T)$ __/1
   (iii) Stable range of K_i __/1
   (iv) Actual value of K_i used__/1
5. Task 2a:
   (i) Matlab *margin* plot__/1
   (ii) unity gain crossover frequency, phase margin, -180 crossover frequency, gain margin __/1
6. Task 2b:
   (i) Expression for G_{vg_CL}__/1
   (ii) Matlab code to produce *bodemag* plots of G_{vg_CL} and (open loop)G_{vg} /1
   (iii) Matlab *bodemag* plots of G_{vg_CL} and (open loop) G_{vg} __/1
   (iv) (iii) with requested frequency range__/1
7. Task 2c:
   (i) Expression for $-Z_{out\_CL}$ __/1
   (ii) Matlab code to produce *bodemag* plots of $-Z_{out\_CL}$ and(open loop)$-Z_{out}$ /1
   (iii) Matlab *bodemag* plots of $-Z_{out\_CL}$ and (open loop) $-Z_{out}$ __/1
   (iv) (iii) with requested frequency range __/1
8. Task 3a:

(i) Full Matlab code for simulation of step response of G_{vg_CL} ______ /1
(ii) Step response obtained from the Matlab code ______ /1
(iii) Δv ______ /1
(iv) SSE ______ /1

9. Task 3b:

(i) Full Matlab code for simulation of step response of $-Z_{out\_CL}$ ______ /1
(ii) Step response obtained from the Matlab code ___0.5___ /1
(iii) Δv ______ /1
(iv) SSE ______ /1

10. Task 4a:

(i) PECS schematic for simulation of step response of G_{vg_CL} ______ /1
(ii) Step response obtained from PECS ______ /1
(iii) Δv ______ /1
(iv) SSE ______ /1

11. Task 4b:

(i) PECS schematic for simulation of step response of $-Z_{out\_CL}$ ______ /1
(ii) Step response obtained from PECS ______ /1
(iii) Δv ______ /1
(iv) SSE ______ /1

12. Task 5b:

(i) Lab screen shot of step response of $-Z_{out\_CL}$ ______ /1
(ii) Δv ______ /1
(iii) SSE ______ /1

13. Task 6a:

(i) Loop stability margins and frequencies table ______ /1

14. Task 6b:

(i) Δv and SSE summary table ______ /1
(ii) Observations on results ______ /2

15. Task 7:

(i) Step response difference with proportional and lead compensator with the integral compensator ___0___ /2

16. Task 8: Optional compensator – proportional compensator: if submitted grade with identical grading sheet ______ /2

17. Task 9: Optional compensator – lead compensator: if submitted grade with identical grading sheet ______ /2

Report: ______ /5

Total: ___52.5___ /55