Laboratory #2: System Identification of a 2nd Order System through Step Response

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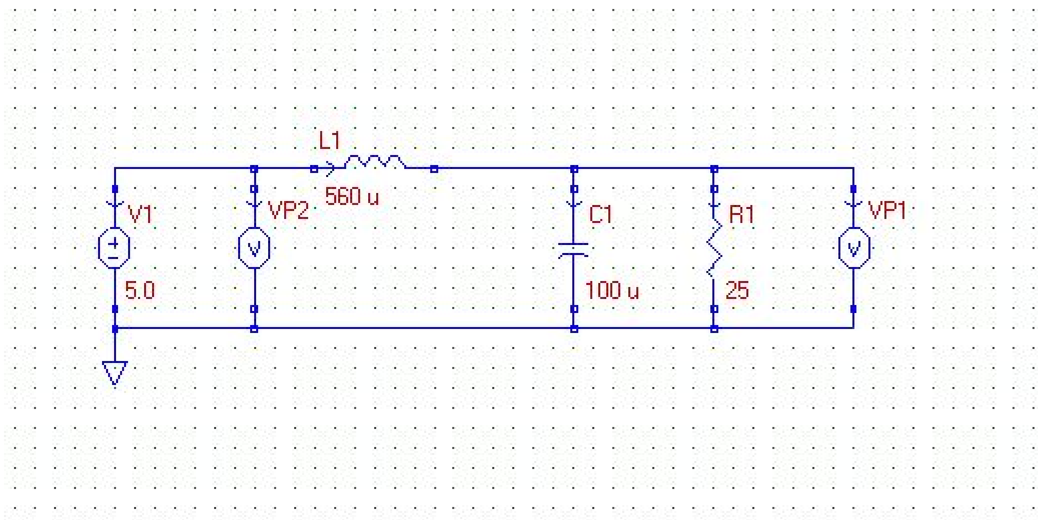
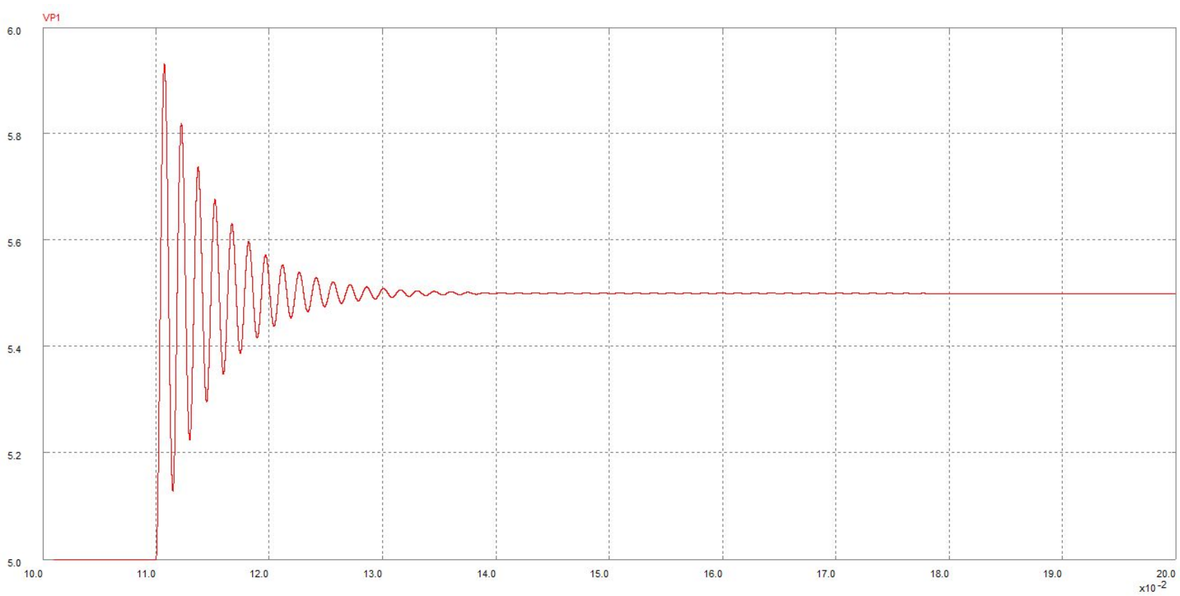


The figure above shows our PECS schematic of an LCR network used for this lab.



The figure above shows the voltage (VP1) across the 25 Ω resistor from our PECS schematic.

Overshoot	c'_{max}	5.931 V
Steady State Voltage	c'_{final}	5.5 V

As the table above shows, our max overshoot is 5.931 V and the steady state voltage is 5.5 V.

$$c_{max} = c'_{max} - c'_0$$
$$c_{max} = 0.931\ V$$
$$c_{final} = c'_{final} - c'_0$$
$$c_{final} = 0.5\ V$$

$$\%OS = \frac{Cmax - Cfinal}{Cfinal}\ x\ 100 \Rightarrow \frac{0.931-0.5}{0.5}\ x\ 100 = 86.2\%$$

$$\zeta = \frac{-ln(\%OS/100)}{\sqrt{\pi^2 + ln^2(\%OS/100)}} \Rightarrow \frac{-ln(86.2/100)}{\sqrt{\pi^2 + ln^2(86.2/100)}} = 0.047$$

0.12942 19.42

T's = ~~0.1168s~~

Ts = T's - step time $\Rightarrow \cancel{0.1168} - 0.11 = \cancel{6.8ms}$

$$K = \frac{\Delta c}{\Delta v} = \frac{Cfinal - Co}{Vfinal - Vo} \Rightarrow \frac{5.5-5.0}{5.5-5.0} = 1$$

<u>LCR transfer function derivation:</u>

$$Z_L = sL \text{ and } Z_C = \frac{1}{sC}$$

Using KCL and the relations of the impedances above we get the following:

$$\frac{V_o - 0}{R} + \frac{V_o - 0}{Z_C} + \frac{V_o - V_i}{Z_L} = 0$$

Isolate input and output.

$$V_o\left(\frac{1}{R} + \frac{1}{Z_C} + \frac{1}{Z_L}\right) = \frac{V_i}{Z_L}$$

Combine impedances.

$$V_o\left(\frac{Z_L Z_C + RZ_L + RZ_C}{RZ_C Z_L}\right) = \frac{V_i}{Z_L}$$

Divide for transfer function.

$$\frac{V_o}{V_i} = \frac{RZ_C}{Z_L Z_C + RZ_L + RZ_C}$$

Now looking only at the right hand side of the transfer function, plug in the impedance values.

$$\frac{R\frac{1}{sC}}{sL\frac{1}{sC} + RsL + R\frac{1}{sC}}$$

Multiply by sC.

$$\frac{R}{sL+RsLsC+R}$$

Simplify.

$$\frac{R}{s^2RLC+sL+R}$$

Divide by R.

$$\frac{1}{s^2LC+s\frac{L}{R}+1}$$

Now we have the desired transfer function.

$$\frac{1}{s^2LC+s\frac{L}{R}+1} = \frac{K}{a_1s^2+a_2s+1} = \frac{K}{(\frac{s}{\omega_n})^2+2\xi\frac{s}{\omega_n}+1}$$

From the relationships of the transfer functions above K = 1.

$$a_1 = \frac{1}{\omega_n{}^2} = LC$$

Therefore, $\omega_n = \frac{1}{\sqrt{LC}}$.

$$a_2 = 2\xi\frac{1}{\omega_n} = \frac{L}{R}$$

Plug in ω_n .

$$2\xi(\sqrt{LC}) = \frac{L}{R}$$

Therefore, $\xi = \frac{L}{2R\sqrt{LC}}$.

Transfer Function Parameters	PECS Simulation Derived Values	Symbolic Transfer Function	Evaluated Transfer Function
ξ	.047	$\frac{L}{2R\sqrt{LC}}$	.047
ω_n	~~12515.6~~ 4380	$\frac{1}{\sqrt{LC}}$	4225.77
K	1	1	1

```matlab
% Component values
L = 560e-06;
C = 100e-06;
R = 25;

% Transfer function parameters
K = 1;      % DC gain
a1 = L*C;   % Coefficient of s^2 of denominator
a2 = L/R;   % Coefficient of s^1 of denominator

% Tranfer function
tf_LCR = tf(K, [a1, a2, 1]);

% t is a vector of 1000 time values linearly spaced between 0 and .2
t = linspace(0, .2, 1000);
% Create step function
u = 5*ones(length(t), 1);
% Create the time for step excitation
step_time = .11;
% n accounts for all time after excitation
n = find(t >= step_time);
% The input after the excitation time initiates
u(n) = u(n) + .5;  % A 10% step

% Simulation of the LCR network
y = lsim(tf_LCR,u,t);

% The output response
figure(1)
plot(t,y)
title('Output response including the large-signal start-up transient')

% Isolate the small-signal step response
c_prime_0 = y(n(1)-1); % Initial output before the step
ys = y(n) - c_prime_0; % Small signal output response
ts = t(n) - step_time; % Small signal response times

% Small signal step response
figure(2)
plot(ts, ys)
title('Step response')

% Small signal step response metrics
stepinfo(ys,ts)
```

LCR Low-Pass Filter Matlab Code

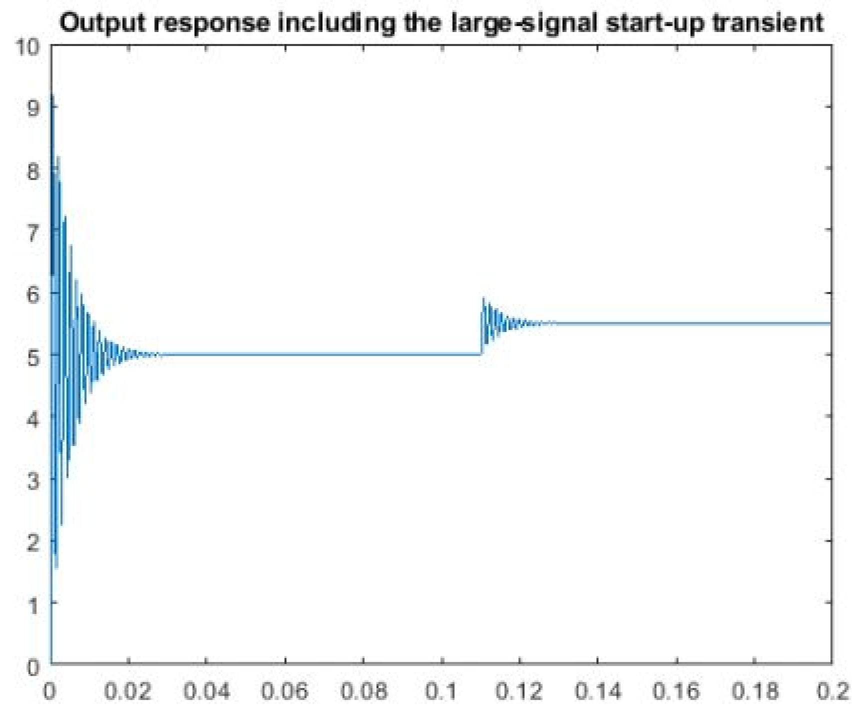


Fig. 1 Output Response LCR Filter

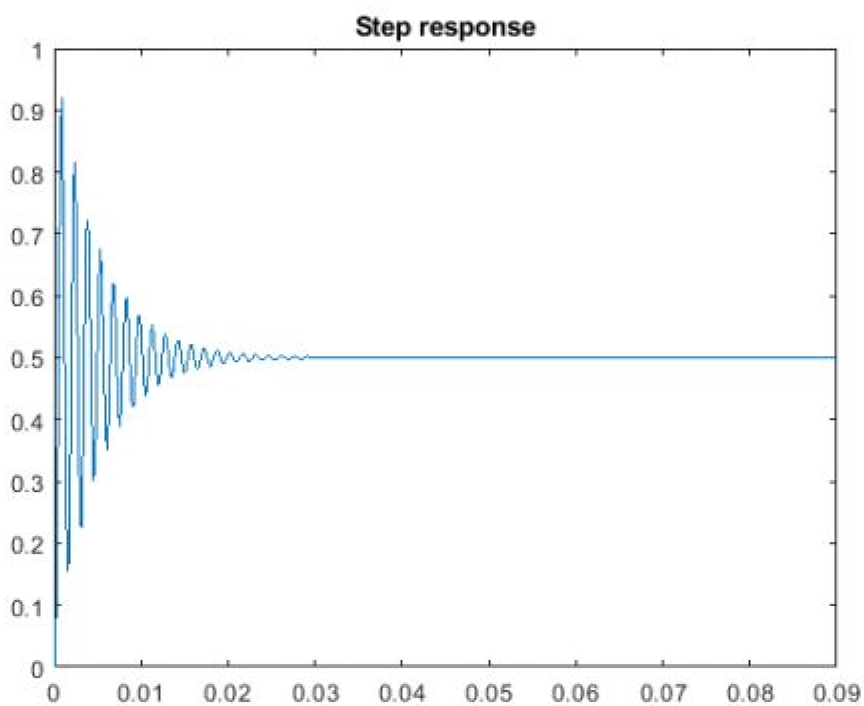


Fig. 2 Step Response of LCR Filter

```
    RiseTime: 2.9428e-04
SettlingTime: 0.0188
 SettlingMin: 0.1539
 SettlingMax: 0.9187
   Overshoot: 83.7408
  Undershoot: 1.2966e-07
        Peak: 0.9187
    PeakTime: 9.1091e-04
```

Output of Matlab stepinfo function

Response Feature	From PECS	From Matlab
%OS	86.2	83.74
T_S (taken from plots when V<0.39)	~~8ms~~ 19.42	~~7.5ms~~ 19.5

Derivation of transfer function with rL included:

Adding in Series Resistor with inductor called r_L Leads to voltage divider equation

$$\frac{V_o}{V_{IN}} = \frac{R}{CLRs^2+(Cr_L R+L)s+r_L+R}$$

Transfer function with simplification applied:

$$\frac{V_o}{V_{IN}} = \frac{R}{CLRs^2+(Cr_L+L)s+R} = \frac{1}{CLs^2+(Cr_L+\frac{L}{R})s+1}$$

Appreciable change in DC gain?

No change in DC gain after approximation r_L +R=R.

Appreciable change in undamped natural frequency?

No change in undamped natural frequency.

Appreciable change in damping factor?

This factor is what gets changed the most from transfer function above

$$2\xi \frac{1}{\omega_n} = Cr_L + \frac{L}{R}$$

$$\xi = \frac{CRr_L + L}{2R\sqrt{LC}}$$

Lab 2 Grading Sheet

1. Task 1: Your PECS schematic________________ /1
2. Task 4:
   (i) Plot________________ /1
   (ii) c'_{max}________________ /1
   (iii) c'_{final}________________ /1
3. Task 5:
   (i) c_{max}________________ /1
   (ii) c_{final}________________ /1
4. Task 6:
   (i) %OS________________ /2
   (ii) ζ________________ /2
5. Task 7:
   (i) T'_s_______0_______ /1
   (ii) Ts_______0_______ /1
6. Task 8: K________________ /2
7. Task 9:
   (i) LCR transfer function derivation________________ /2
   (ii) K as a function of elements________________ /1
   (iii) *a1* as a function of elements________________ /1
   (iv) *a2* as a function of elements________________ /1
   (v) completed table_______5.5_______ /6
8. Task 10: Your complete Matlab code________________ /2
9. Task 11:
   (i) Matlab plot 1________________ /1
   (ii) Matlab plot 2________________ /1
10. Task 12:
    (i) Output of Matlab *stepinfo* function________________ /1
    (ii) Completed table_______1_______ /2
11. Task 13:
    (i) Derivation of transfer function with rL included________________ /2
    (ii) Transfer function with simplification applied________________ /1
    (iii) Appreciable change in DC gain?________________ /1
    (iv) Appreciable change in undamped natural frequency?________________ /1
    (v) Appreciable change in damping factor?________________ /1

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Total: _______41.5_______ /45