Week 2 Video Lecture Notes

A. Lesson Objectives and Key Terms

- understand the concept and usage of the remainder of the summary statistics (S.S.):
  - standard deviation
  - variance
  - Inter-Quartile Range (IQR)
- differentiate between design of studies (experiments and observational studies)
  - understand merits, feasibility and ethical concerns of each

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B. Standard Deviations, Medians and IQRs

Sample Variance & Standard Deviation

- are measures of dispersion

I. Sample Variance

$$s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n-1}$$

- `n` is the number of data points; x_1 to x_n is the values contained within the set of inputs (values of numerical `x` in the data set).

II. Standard Deviation

- provides a way to *quantify* the "spread" of data about the mean.
- formula of S.D. is derived via the *variance* (σ^2), particularly using the square root operation.
- S.D. value of zero -> there is no spread; S.D. value > 0 -> there is some sort of spread in the sample.
- *formulae:* $\frac{1}{n-1}\ldots$

Properties

- formula for Sample S.D. = $\sqrt{variance}$

$$s_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n-1}}$$

  - Intuition on the S.D. formula -> might make sense to:
    - take difference between each value and mean
    - add up the differences to get the `"total spread"`
    - divide by total number of points to get `"average spread"`
    - can't do this as average spread will be `0` ❌, ∵ `+ve` and `-ve` values might cancel each other out

Population S.D. formula

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}}$$

- standard deviation is:
  - **non-negative** (i.e. ≥ 0).
  - **adding a constant** k to a dataset changes the mean $\bar{x}$, but **doesn't change** the standard deviation s_x.
    - will only "shift" everything by constant k

- **multiplying** all data points by constant c results in the S.D. being multiplied by the **absolute value of** c *(i.e.* $s_{xNew} = s_{xOld} \cdot |c|$*)*.

Example of S.D. calculation (explicit calculation)
Qn. Consider a simple sample data set x of just `3` points. Given that $x = \{1, 4, 7\}$, find the S.D. value of this dataset.

1. Using the formula, $s_x = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots+(x_n-\bar{x})^2}{n-1}}$ we need to first obtain the mean of the dataset.
2. Also given $\bar{x} = \frac{1}{n}\sum x_n$ and $n = 3$

$$\bar{x} = \frac{1+4+7}{3} = 4$$

$$s_x = \sqrt{\frac{(1-4)^2+(4-4)^2+(7-4)^2}{3-1}}$$

$$\iff s_x = \sqrt{\frac{(1-4)^2+(4-4)^2+(7-4)^2}{3-1}}$$

$$\therefore s_x = 3.$$

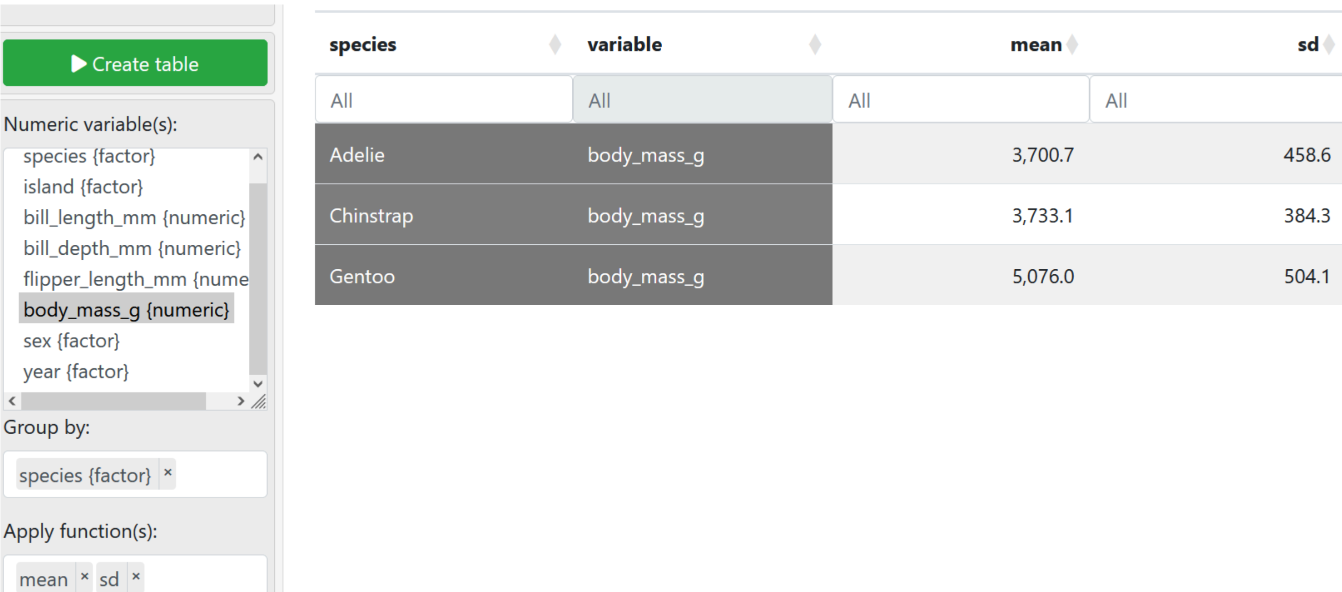
Understanding EDA, mean and S.D. through Palmer Penguins dataset

Palmer Penguins Intro

- consists of 3 species - Chinstrap, Gentoo and Adelie
- data from 342 penguins with various data points (i.e. species, bill length, bill depth, flipper length, mass, gender etc.)

Question to answer from the dataset
How similar are these penguins? -- compare:

1. characteristics like behaviours, habitats and living environments
2. r/s between two or more variables
3. feeding habits across species?
4. mass of the penguins - are males heavier than females within each species?
5. flipper length across species?



species	variable	mean	sd
All	All	All	All
Adelie	body_mass_g	3,700.7	458.6
Chinstrap	body_mass_g	3,733.1	384.3
Gentoo	body_mass_g	5,076.0	504.1

Why is it that the Adelie and Chinstrap (species) have almost the same mean mass but yet the S.D. for the Adelie species is higher?

- due to what?
  - gender
  - age, or other factors not inside the dataset?
  - location

Comparing spread btwn variables

- when considering to factor in the spread btwn variables, we also need to consider spread relative to the mean (a.k.a. coefficient of variation)

$$coefficient\ of\ \sigma^2 = \frac{s_x}{\bar{x}} = \frac{\sigma}{\mu} \mid \bar{x} \neq 0$$

- larger coefficient of variance (i.e. = $\frac{s_y}{\bar{y}} > \frac{s_z}{\bar{z}} \implies$ spread of $y >$ spread of z).

Median

- *definition*: median of set of values in a dataset is the **middle value** after arranging the values of the dataset in ascending or descending order.
    - sort column -> find middle value
- 50th percentile of the data
- formula
  - note: n is the number of element in the set and so $\frac{n+1}{2}$ gives the middle element for odd cases.

$$Med(X) = \begin{cases} X\left[\frac{n+1}{2}\right] & if\ n\ is\ odd. \\ \frac{X\left[\frac{n}{2}\right]+X\left[\frac{n+1}{2}\right]}{2} & if\ n\ is\ even. \end{cases}$$

Overall vs Subgroup Medians

- subgroup mean would *NOT* lie closer to the group with the larger proportion $\implies$ knowing the median of the subgroup does not tell one about overall median

R/s btwn Mean and Median

- For roughly symmetric distributions, $\bar{x} \approx Med(X)$.

Quartiles and Interquartile Range (IQR)

- Quartiles allow use to defined another notion/type of dispersion measurement via IQR.
    - generally use software(s) for this computation

	in terms of percentile	# element
Q_1 (first quartile)	25th	$\frac{n+1}{4}$ th
Q_2 (second quartile / median)	50th	$\frac{n+1}{2}$ th or $\frac{2n+1}{4}$ th*
Q_3 (third quartile)	75th	$\frac{3(n+1)}{4}$ th

Formula:

1. Median: $Med(X) = Q_2$ (see above for full formula)
2. Interquartile Range: $IQR(X) = Q_3 - Q_1$
3. Quartile Deviation = $\frac{Q_3-Q_1}{2}$

Similarities btwn IRQ and S.D. (in terms of properties)

1. $IQR(X)\ must\ be \geq 0, \because Q_3 \geq Q_1$
2. $\forall x$, given c is a constant, $x + c$ does not result in $\triangle IQR(X)$, for $\pm c$
3. Multiplying all data points by constant k results in $IQR(X)$ being multiplied by $|k|$.
    1. i.e. $IQR(X)_{new} = IQR(X)_{old} \cdot k$

Deciding which pairs of S.S. to use

- $\bar{x}$ & s_x or Median & IQR -> depends on the distribution of data points.

    - symmetrical vs non-symmetrical data

Mode

- value that appears the most frequent for a particular feature/variable in a dataset
    - "peak" of the distribution

C. Study Designs

- in study design, we focus on Research Questions that examine a r/s btwn two variables

Exemplar Question: Does **drinking coffee** help students to **pass the maths exam**?

- Dependent: passing the maths exam
- Independent: drinking coffee

Steps:

1. Take a census or sample of the target population (i.e. students who drink coffee and students who don't drink coffee)
2. Conduct the study
    1. Experimental
    2. Observational

1. Experimental Studies

> *def:* intentionally manipulates one variable in an attempt to cause an effect on another variable

- can be also termed "controlled experiment"

goal: provide a cause-effect relationship btwn the two variables

- researcher may hypothesize the relationship using the independent and dependent variables

Groups within the experiment
(a) Treatment Group

- Coffee group $\implies$ drinks exactly one cup of coffee every day, for a month (should also make sure that it is around the same time)
    - the "treatment" in this case is coffee

(b) Control Group

- No Coffee group $\implies$ not drink **any coffee** (at all) for a month
- Control is needed because it provides a baseline for comparison w the Treatment group
    - control group might be the same or even outperform the treatment group (in the exam) in this case
- in some cases, might receive some "baseline" treatment elements (**placebo**) to **reduce bias**
    - since bias is an effect of "leaving the control group alone"
    - provided with a substitute to what the treatment group has been provided
- both groups will then take the maths exam (provide experimental results for benchmarking)

Random Assignment

- makes group assignment is **completely unrelated** to participant's background characteristics (ensures that treatment and control groups are similar in every way other them receipt of the treatment)
- **required** because there may be other dependent variables affecting the results of the independent variable

- the coffee-exam r/s case $\implies$ other factors like revision time (shorter vs longer), IQ of subjects, age of subjects etc. etc.
- *if no random assignment done*: effect of confounding of 3rd party variables may be apparent
- helps to remove the effects of other dependent variables to make the treatment and control group largely similar in terms of other factors/variables
  - aim is to not have any deterministic human discretion $\implies$ **reduce selection bias**
- is an *impartial* procedure using ***chance*** and is highly effective
  - each piece of paper has an equal chance of being picked out (random draw *without replacement*)
  - Steps:
    1. randomly draw subjects until about half ($\approx 50\%$) of the subjects have been "removed" or grouped into the treatment group
    2. the other half of the main group form the control group
  - helps to create similar treatment & control groups in terms of other factors (i.e. revision time, IQ, age etc.) $\implies$ allows for similar distributions
- treatment and control groups *can have different sizes* but as long as *groups are quite large*

"Random" connotations

- actual meaning: has a strict meaning related to an **impartial chance** mechanism
- connotation / association: often interchangeable with "haphazard" -> researcher must ensure other experimental variables are not the case

Placebo

> *Definitions*
>
> 1. Placebo: Treatment with no active ingredients and no effects
> 2. Placebo effect: response observe when subject receive placebo treatment but **still show some positive effects.**, even if the treatment has no effect

Blinding

- somewhat like blindfolding the subjects
- blinded subjects don't know which group they belong to (treatment or control)
  - can add a placebo ("substitute for the treatment" taken by the control group) to help make the blinding more effective
  - helps to prevent subject's own beliefs and in turn behaviours from affecting the results of the experiment(al study)
- Returning to coffee-and-substitute example
  - Each subject won't know if they're in the treatment or control group -> are blind to how the test or control might look like (treatment and placebo should smell and taste the same)
  - Each subject is provided with a drink every morning
- assessors marking the test also need to be blinded to avoid biases (being more lenient to one group compared to the other)

> *def:* Double-blinding experiment occurs when both the subjects and assessors (of the experiment) are blinded.

For controlled experiment w both double-blinding and random assignment -> can enable experimental results to show causality (ref generalisability).

2. Observational Studies

> *def*: Observational study involves observing individuals and measures **variables of interest.**

- helps to eliminate ethical issues associated with the experiment (i.e. ethical to inject low doses of virus consent provided?)
  - just record data based on real-world cases (don't force or incentivize participation)
- researcher **does NOT attempt** to **directly** manipulate one variable to cause an effect on another variable.
- there may be logistical challenges when conducting the randomized experimental study

- observe subjects (association versus causation)

Advantages

- Better external validity than experimental studies

Disadvantages

- $\therefore$ observational studies do not provide convincing evidence supporting a cause-effect relationship.
- have weaker internal validity than experimental studies

Groups within the experiment

- Will still use the terms treatment group (i.e. smokers) and control group (non-smokers) even though no actual "treatment" is applied from the researcher's end.

  Treatment group $\iff$ exposure; Control Group $\iff$ non-exposure

3. Experimental vs Observational Studies

> *def* Confounder: variable that influences both independent and dependent variable.

Experimental Studies	Observational Studies
Assigned by researcher (should have only one independent variable -> random assignment + double-blinding)	Decided by subjects themselves (usually lifestyle choices)
Can provide cause-and-effect relationship (CAUSATION)	Cannot provide cause-and-effect relationship (can only corroborate and expand on other studies)
	Can still provide evidence on association; can show correlation/relationship

- we wish that we as researchers can do an experiment all the time -> might not be ethical (people might not respond well, governing their lifestyle choices)

D. Generalisability of Studies

Even if an experiment is:

- well-designed
- no ethical issues
- has double-blinding **and** random assignment,

We still might not be able to generalize the results to the entire population / everyone (recall generalisability has 3 other criteria and a total of 4 criterion) $\implies$ still have to consider the other factors.

E. Additional Notes (may not be tested)

- Research conclusions / goals may be broadly categorized into three categories – *descriptive, causal or predictive*

> *def:* An **inference** is using what is observed to learn more about what is not observed.

1. Descriptive Inference

- summarize and visualize data (to better understanding a phenomenon)
  - using observed facts
- systematic description of a data set

> *Example Problem /Research Question:* How well do students perform across countries?

2. Causal Inference

- quantifies the effect of **one variable on another variable**
  - goal: understand how one variable affects another (independent variable affecting depending variable)

> *Example:* What is the effect of public schools on reading and math scores on the PISA test?

3. Predictive Inference

- forecast data points **outside of the population sample**
  - use of observe facts to create the forcase

> *Example:* What will the distribution of education look like in India in 10 years?