Week 3 Video Lecture Notes

A. Learning Outcomes and Key Terms - for categorical data analysis

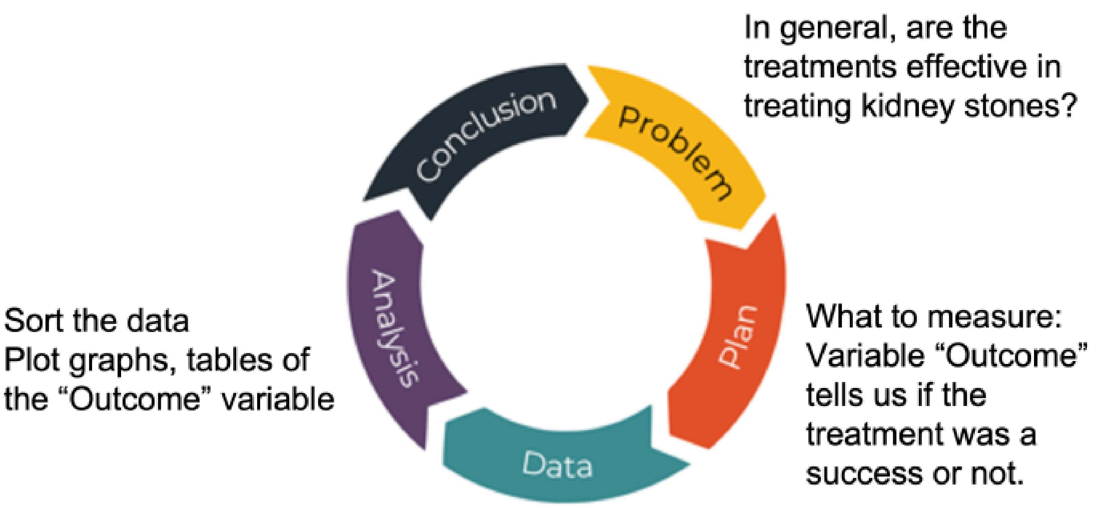
- EDA techniques and concepts for **categorical data**
- describe categorical variables using frequency and rates
- use and interpret contingency tables and bar graphs for categorical variables
- what is a conditional rate versus a joint rate?
- basic rule of rates, symmetry rule
- establish association between categorical variables

B. Understanding Rates

- using the kidney stones dataset `kidneystones.csv` throughout this chapter.
  - `Treatment` - nominal categorical (i.e. two categories $\implies$ X and Y)
  - `size` - ordinal categorical (i.e. small, large)
  - `Gender` - nominal categorical (i.e. two categories $\implies$ Male and Female)
  - `Outcome` - nominal categorical (i.e. two categories $\implies$ Success and Failure)
- When looking just at absolute numbers, there is a tendency to misinterpret the higher count to be better, even though the percentage of success for it may not be so.

Using PPDAC

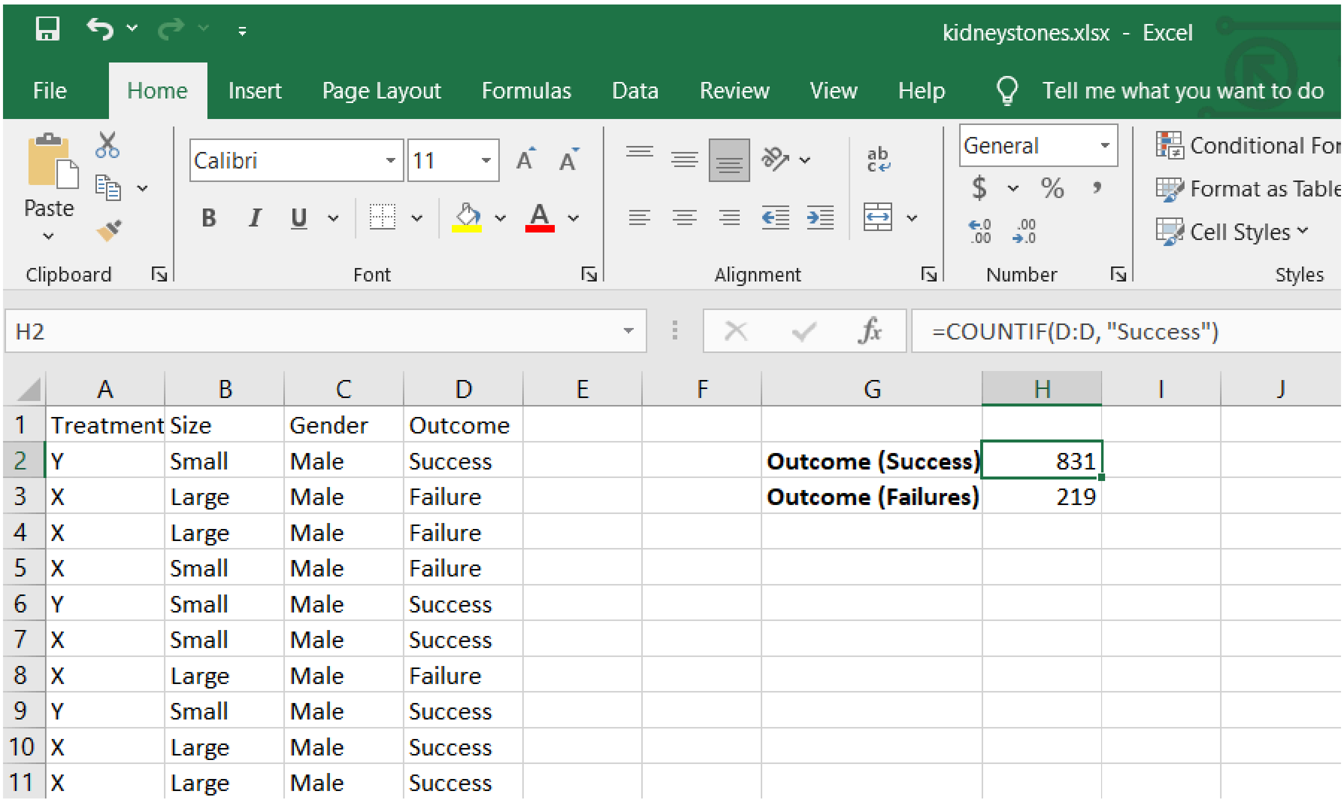
- Problem (may have more than one): *Do treatments provided to patients in general tend to be successful?*
- Plan (not conducting experiment, no need for measurement or quantification): Take a look at `outcome` **variable** to show us if the treatment was a success $\implies$ $\because$ this is an observational study .
- Data (reveal interesting trends)
- Analysis: sorting the data, plot graphs etc.
- Conclusion:
  - preliminary types of conclusions may lead us to ask more questions



1. Categorical Variables

> *def* Rate: a quantity or amount that can be represented through a fraction, proportion or percentage (measured as compared to something else)

Using example dataset:



	A	B	C	D	E	F	G	H
1	Treatment	Size	Gender	Outcome				
2	Y	Small	Male	Success			**Outcome (Success)**	831
3	X	Large	Male	Failure			**Outcome (Failures)**	219
4	X	Large	Male	Failure				
5	X	Small	Male	Failure				
6	Y	Small	Male	Success				
7	X	Small	Male	Success				
8	X	Large	Male	Failure				
9	Y	Small	Male	Success				
10	X	Large	Male	Success				
11	X	Large	Male	Success				

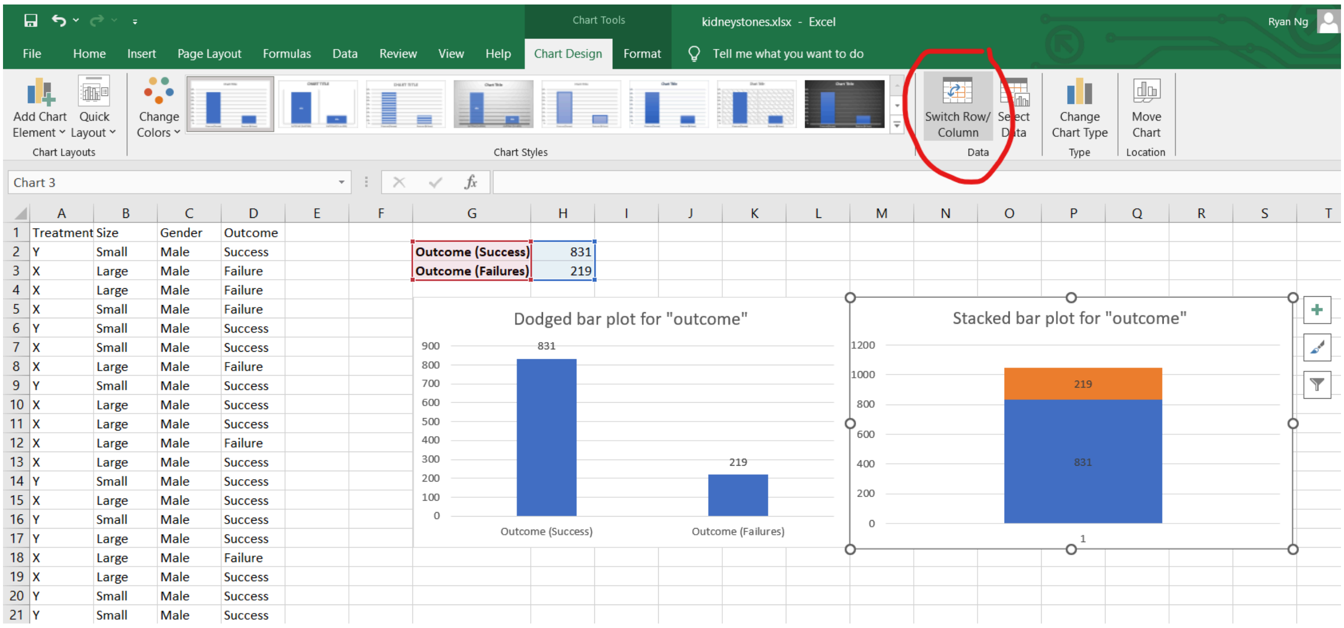
`Rate(Success)` = $\frac{831}{1050} = 0.791 \text{ or } 79.1\%$.

2. Tables and Plots

- allows us to visualize the data and come to the same conclusion

i. Single variable

- can use either a dodged bar plot or a stacked bar plot to measure the variable being explored.
- can normalize values as a percentage or fraction instead of just a count.



- can also use these plots for two variables
  - can normalize the y-axis to become 100% (transform to become a 100% bar plot)

ii. Two variables

using PPDAC chart to answer new question discovered

> *def:* A two-way contingency table is a cross-classification of observations by the levels of two discrete variables

- can make use of data to determine if treatment X or Y is better in giving a Successful outcome?
  - make use of a `2x2` data/contingency table (not to be confused with a *two-way relative frequency table*)
    - dependent variable (outcome) as the row headers of the table (horizontal)
    - independent variable as the column headers of the table (vertical)

3. Marginal, Conditional & Joint rates

Marginal Rate

> *def*: Marginal rate - how the numbers in the margin of the table relate to (change in respect with) categorical variables

- to calculate, take the **row or column total** (depending on the question) and divide it by the **grand total**.

> *Formulae:*
>
> - Row Marginal rate: $\frac{Row\ Total}{Grand\ Total}$
> - Column marginal rate: $\frac{Column\ Total}{Grand\ Total}$

Conditional Rate

> *def:* Conditional rate - consider one part of the population and "ignore" the others (provided based on a **given condition**)

> *Formulae*:
>
> - *In General*: $rate(Y \mid X) = \frac{rate(Y \wedge X)}{rate(X)}$
> - $Conditional\ rate = \frac{Joint\ count\ of\ Y \wedge X}{Marginal\ Count\ of\ X}$

- total number of participants / size of EITHER control OR treatment group will function as the denominator of the conditional rate.
  - conditional rate because only certain margins or conditions are taken into account

Joint Rate

i.e. based on "filtering out" both the independent and dependent variables

- looking at **all observations** as the base / total (as the denominator)

> *Formulae:*
> $Joint\ rate = \frac{Joint\ count\ of\ X \wedge Y}{Grand\ Total}$

4. Normalization and Parity

- normalization makes it such that in an experiment (in this case) comparing two dependent variables, we can make it such that the **discrepancy in the sizes** of the treatment and control groups are addressed
  - can be through the calculation of rates *instead of using absolute numbers* which might provide a false representation of the success of either treatment.

Workflow:

1. Compares the success rates of treatments X and Y
2. Given a treatment, what is the success rate? (calculate and normalize for both treatments in question, in this case X and Y).
3. Make a fair comparison (i.e. use some similar scale
   1. Treatment X, ~77 out of 100 patients found success
   2. Treatment Y, ~83 out of 100 patients found success (positively associated with the success of the treatment)
4. Conclusion

Calculate the percentages across all rows (limit focus to one row at a time)

- use of marginal percentages

Table with row percentages

Treatment \ Outcome	Success (row %)	Failure (row %)	Row Total (row %)
X	542 (77.4%)	158 (22.6%)	$\boxed{700}$ (100%)
Y	289 (82.6%)	61 (17.4%)	350 (100%)
Column Total	831 (79.1%)	219 (20.9%)	1050 (100%)

Conclusions drawn

- Treatment Y is positively associated to the success of the treatment (tend to see that Treatment Y and successes go hand-in-hand).
- Treatment X is negatively associated to the success of the treatment.
- Associative relationship between the Treatments and their outcomes.

C. Association

> *def* Association: there is a relationship between some variables -- the independent variable (i.e. the treatment type) and the dependent variable (i.e. the outcome of the treatment)

- how two variables are related to each other
- use of the term association when we don't know if the y variable is entirely based on the x variable.
  - use of rates to determine that one of the dependent `x` variables resulted in a better `y` variable or outcome.
- association is NOT causation!

Types of Conditional rates

Case	Remarks	
Rate(A | B) = Rate(A | NB)	*Not* Associated / Association is absent	
Rate(A | B) < Rate(A | NB)	Positive association	Presence of A, when B is present is stronger than when B is absent
Rate(A | B) > Rate(A | NB)	Negative Association	Presence of A when B is present is weather than when B is absent

Assumptions
A: Successful Treatment (kidney stone size was reduced)
NA: Failed Treatment (kidney stone size roughly the same or worse, bigger)
B: Treatment X was administered on the patient
NB: Treatment Y was administered on the patient

Notes:

- `x | y` is read as "x given y"
- `A` and `B` represent the **first** dependent (outcome) and independent variables respectively.
- `NA` and `NB` represent the **second** dependent and independent variables respectively.

Misconceptions when establishing association

Exercise
`rate(A | B) < rate(A | NB)`

- x neg associated w success
- y pos associated w success

`rate(NA | B) < rate(NA | NB) ≡ rate(A | B) < rate(A | NB)`

- x pos associated w failure
- y neg associated w failure

Given x pos associated w failure ≡ x neg associated w success

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D. Rules that govern rates

1. Symmetry Rule

> Notation:
> $rate(X|Y) < trichotomy\_operator > rate(X|NY) \iff rate(B|A) < trichotomy\_operator > rate(B|NA)$

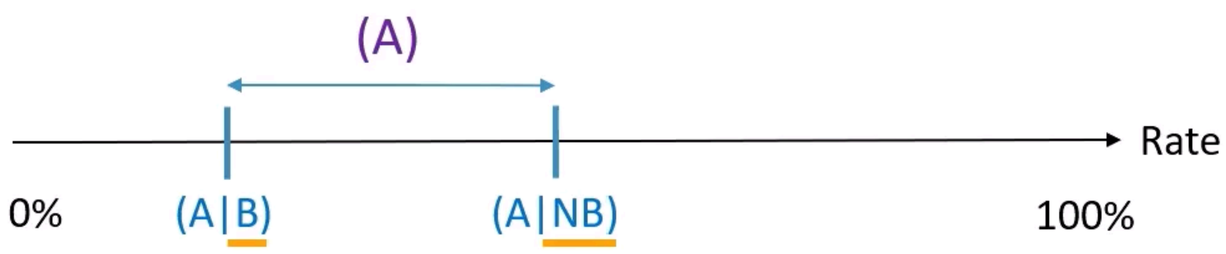
- first statement must hold for second statement to hold AND second statement must hold for first statement to hold.

Consequences of the symmetry rule
checking for association (use either one)

1. $rate(A|B) \neq rate(A|NB)$
2. $rate(B|A) \neq rate(B|NA)$
   The above imply that the variables are either positively or negatively associated

2. Basic rule on rates

- The overall rate (`rate(A)`) will always lie between `rate(A|B)` and `rate(A|NB)`



Three Consequences of the basic rule on rates

> `rate(A | B) ≤ rate(A) ≤ rate(A | NB)` or vice versa

- As `rate(B)` approaches 100%, `rate(A)` gets closer and closer to `rate (A | B)` as compared to `rate(A | NB)` (should still fulfil the above criteria)

> $rate(B) = 50\% \implies rate(A) = \frac{[rate(A \mid B) + rate(A \mid NB)]}{2}$

- if the `rate(B)` is exactly 50%, then the `rate(A)` is exactly halfway between the boundaries of `rate(A | B)` and `rate(A | NB)`.

> $rate(A|B) = rate(A|NB) \implies rate(A) = rate(A|B) = rate(A|NB)$

- If there is equality between A given B and A given not B, then the overall rate of A would also be the same value