Neural Network Training

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review: Learning in Perceptron

Learning in perceptron

- Perceptron rule
- Delta rule (Gradient Descent)

review: Learning in Perceptron

Perceptron algorithm

- Cycle through the training instances
- Only update W on misclassified instances
- If instance misclassified:
  - If instance is positive class (positive misclassified as negative)

$$W = W + X_i$$

  - If instance is negative class (negative misclassified as positive)

$$W = W - X_i$$

Perceptron

Delta rule (Gradient Descent)

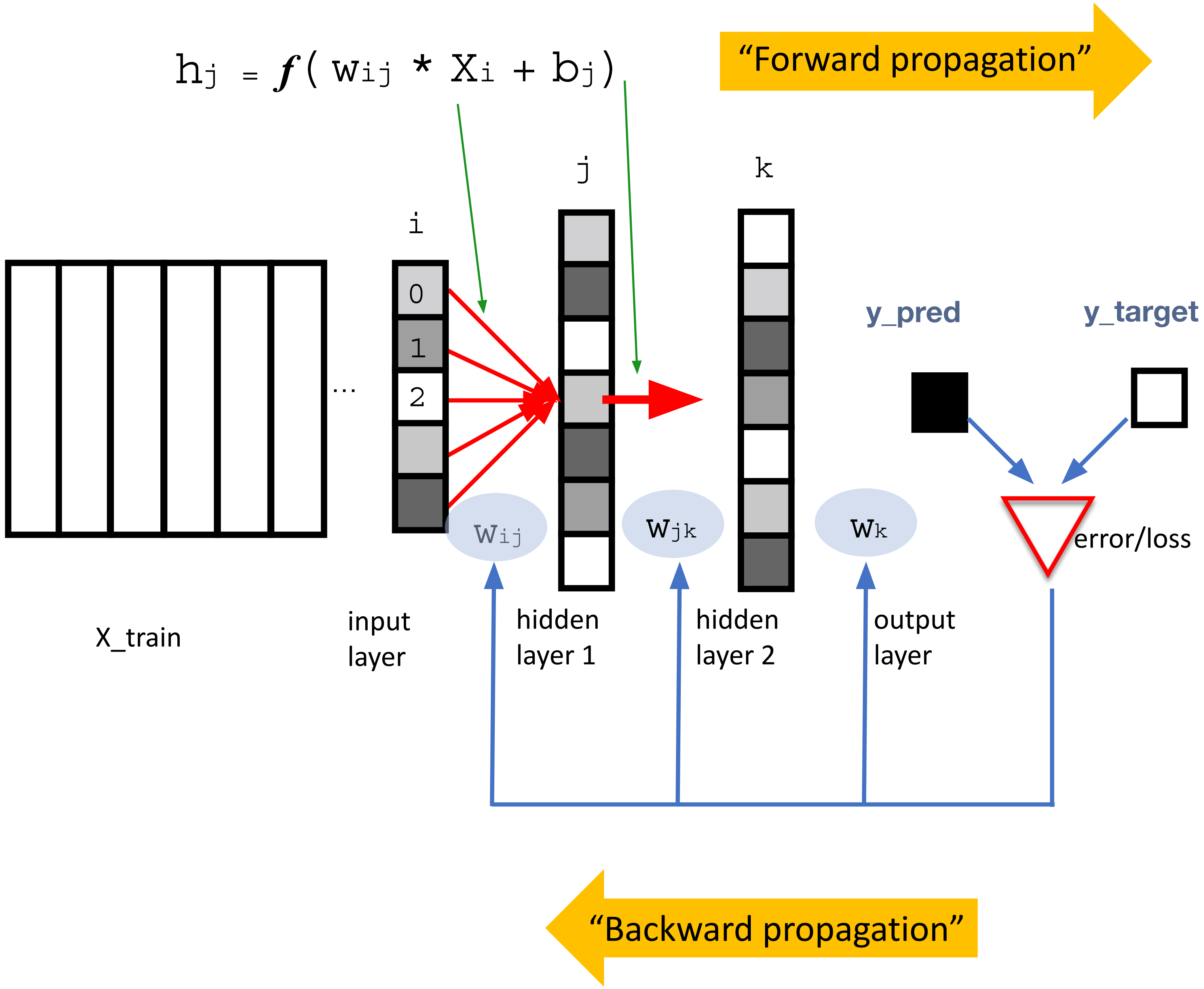
$$\omega_j \leftarrow \omega_j - \alpha \frac{\partial \mathcal{L}}{\partial \omega_j}$$

$$\mathcal{L} = \tfrac{1}{2}(\hat{y}_i - y_i)^2$$

$$\hat{y}_i = \textstyle\sum_j \omega_j X_{ij}$$

$$\omega_j \leftarrow \omega_j - \alpha(\hat{y}_i - y_i) X_{ij}$$

How Neural Network Training Works



Weight Update in deep neural nets

$$W_{ij} \leftarrow W_{ij} - \alpha \frac{\partial \mathcal{L}}{\partial W_{ij}}$$

Weight Update Rule

Loss Function

Gradient (Chain Rule)

Back Propagation

Chain Rule Reminder

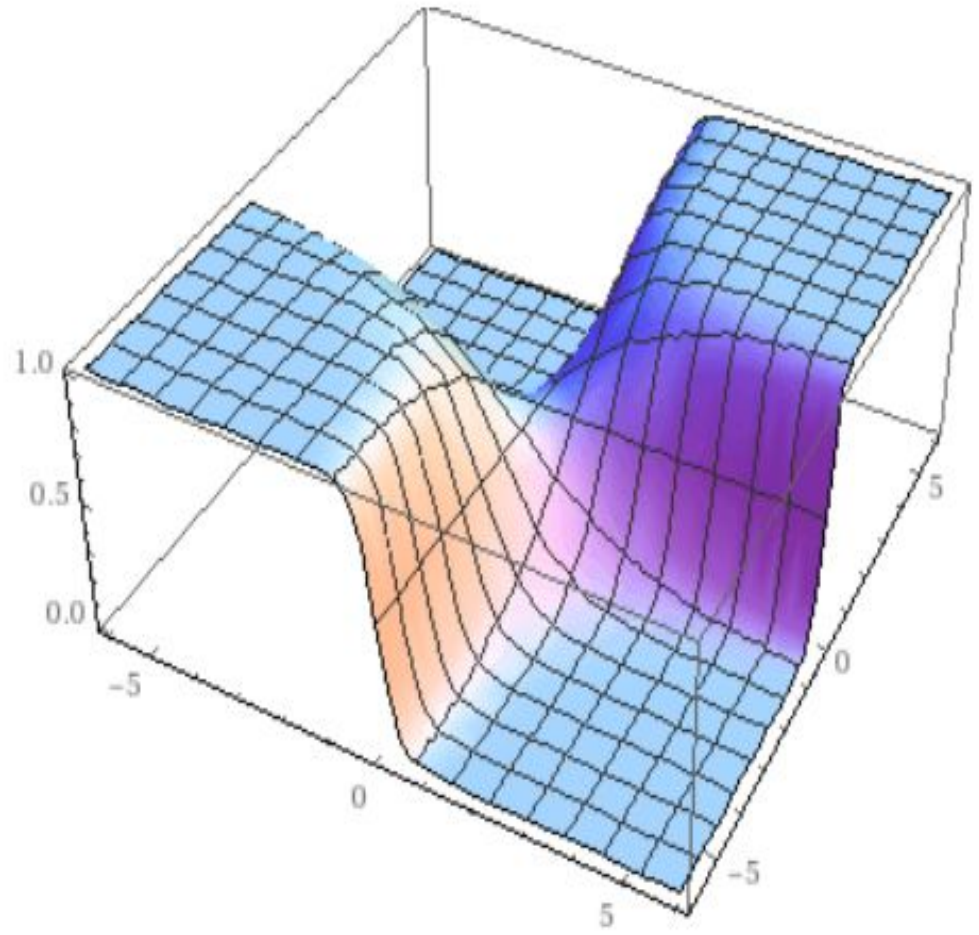
$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x}$$

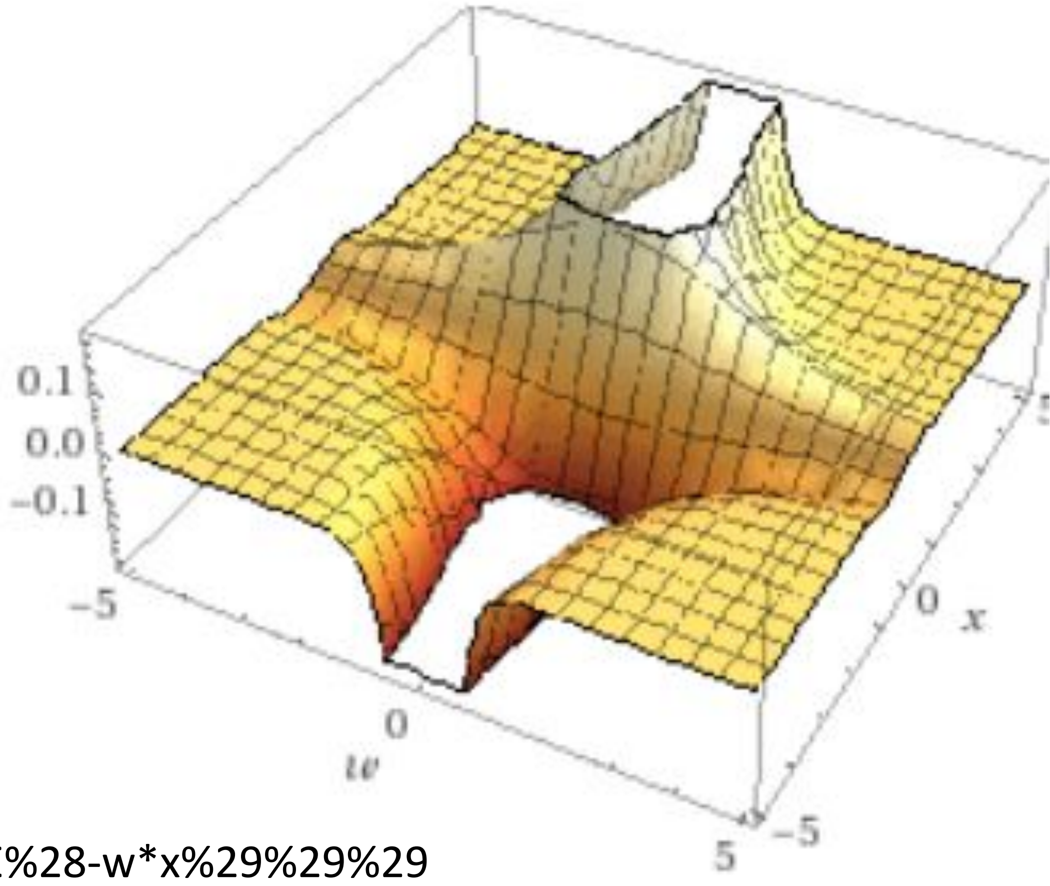
Example: gradient of sigmoid

Chain Rule Reminder

$$\frac{\partial}{\partial w}\left(\frac{1}{1+e^{-w\,x}}\right) = \frac{x\,e^{-w\,x}}{\left(e^{-w\,x}+1\right)^2}$$

$$\frac{1}{1+e^{-w\,x}}$$





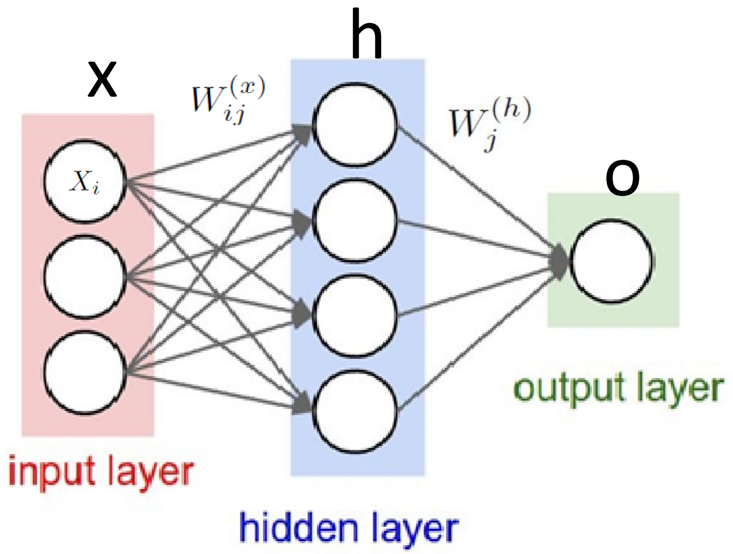
https://www.wolframalpha.com/input/?i=d%2Fdw+%281%2F%281%2Be%5E%28-w*x%29%29%29

Calculating Gradient- Chain Rule

$$\mathcal{L} = -\frac{1}{m}\sum_{s}^{m} y_s \log a_s + (1-y_s)\log(1-a_s)$$

$$W_{ij} \leftarrow W_{ij} - \alpha\frac{\partial \mathcal{L}}{\partial W_{ij}}$$

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x}$$



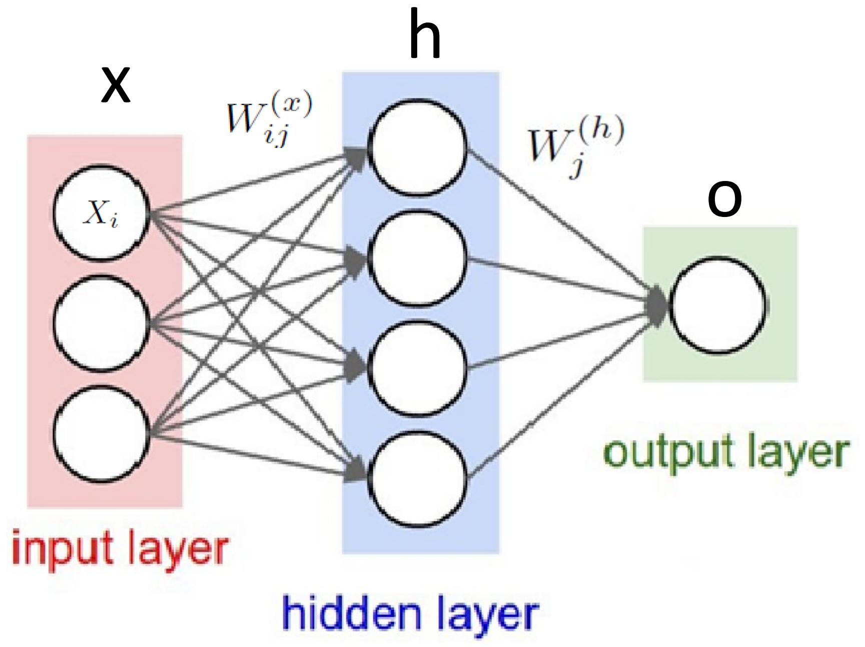
$$\frac{\partial \mathcal{L}}{\partial W_{ij}^{(x)}} = \frac{\partial \mathcal{L}(a^{(o)})}{\partial a^{(o)}}\frac{\partial a^{(o)}}{\partial W_{ij}^{(x)}}$$

$$= \left(\frac{y}{a^{(o)}} - \frac{1-y}{1-a^{(o)}}\right)\frac{\partial a^{(o)}}{\partial W_{ij}^{(x)}}$$

$$\frac{\partial a^{(o)}}{\partial W_{ij}^{(x)}} = \frac{\partial \sigma(z^{(o)})}{\partial z^{(o)}}\frac{\partial z^{(h)}}{\partial W_{ij}^{(x)}}$$

$$= \frac{\partial \sigma(z^{(o)})}{\partial z^{(o)}}\frac{\partial z^{(o)}}{\partial W_{ij}^{(x)}} = \sigma(z^{(o)})(1-\sigma(z^{(o)}))\frac{\partial z^{(o)}}{\partial W_{ij}^{(x)}}$$

Calculating Gradient- Chain Rule



$$\frac{\partial z^{(o)}}{\partial W_{ij}^{(x)}} = \sum_j^{n^{(h)}} W_j^{(h)} \frac{\partial a_j^{(h)}}{\partial W_{ij}^{(x)}}$$

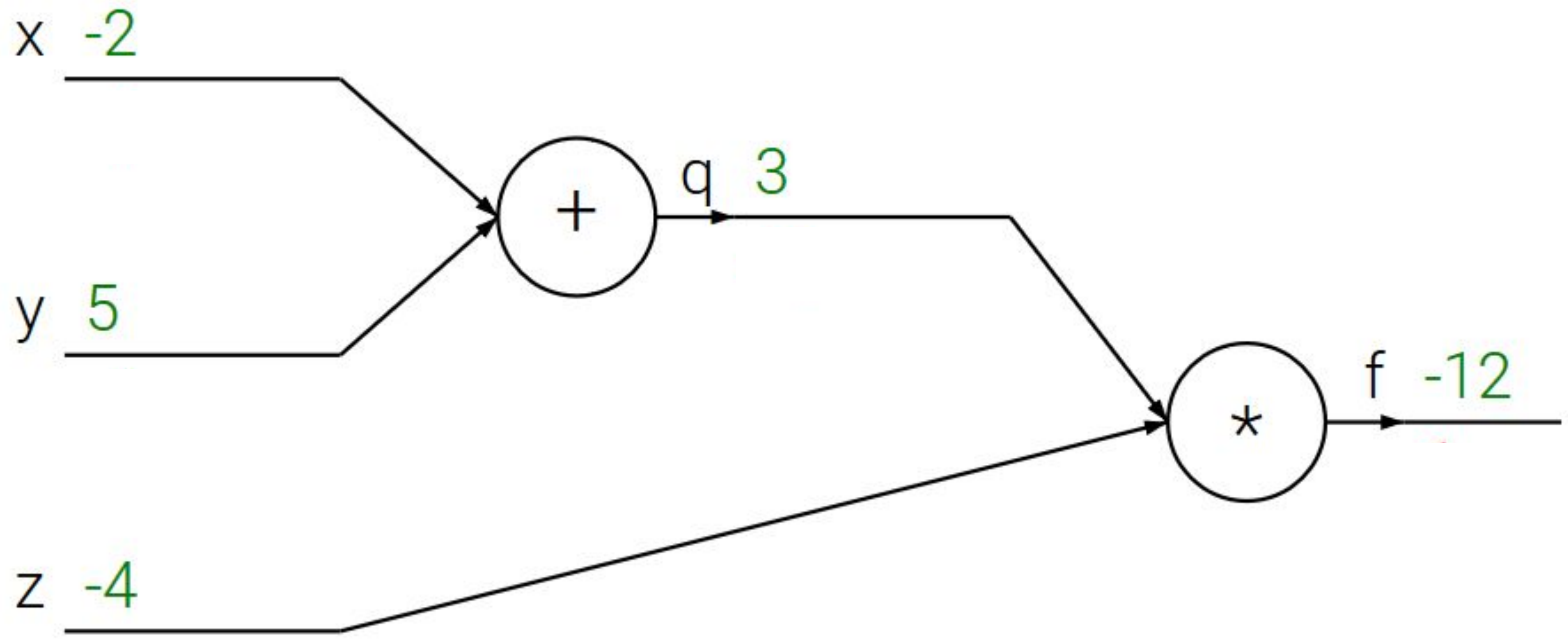
$$= \sum_j^{n^{(h)}} W_j^{(h)} \frac{\partial \sigma(z_j^{(h)})}{\partial W_{ij}^{(x)}} = \sum_j^{n^{(h)}} W_j^{(h)} \sigma(z_j^{(h)})(1-\sigma(z_j^{(h)})) \frac{\partial (z_j^{(h)})}{\partial W_{ij}^{(x)}} = \sum_j^{n^{(h)}} W_j^{(h)} \sigma(z_j^{(h)})(1-\sigma(z_j^{(h)}))X_i$$

$$\frac{\partial \mathcal{L}}{\partial W_{ij}^{(x)}} = \left(\frac{y}{a^{(o)}} - \frac{1-y}{1-a^{(o)}}\right) \sigma(z^{(o)})(1-\sigma(z^{(o)})) \sum_j^{n^{(h)}} W_j^{(h)} \sigma(z_j^{(h)})(1-\sigma(z_j^{(h)}))X_i$$

Back Propagation- Computation Graph

q = x+y

f = q*z



Back Propagation- Computation Graph

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

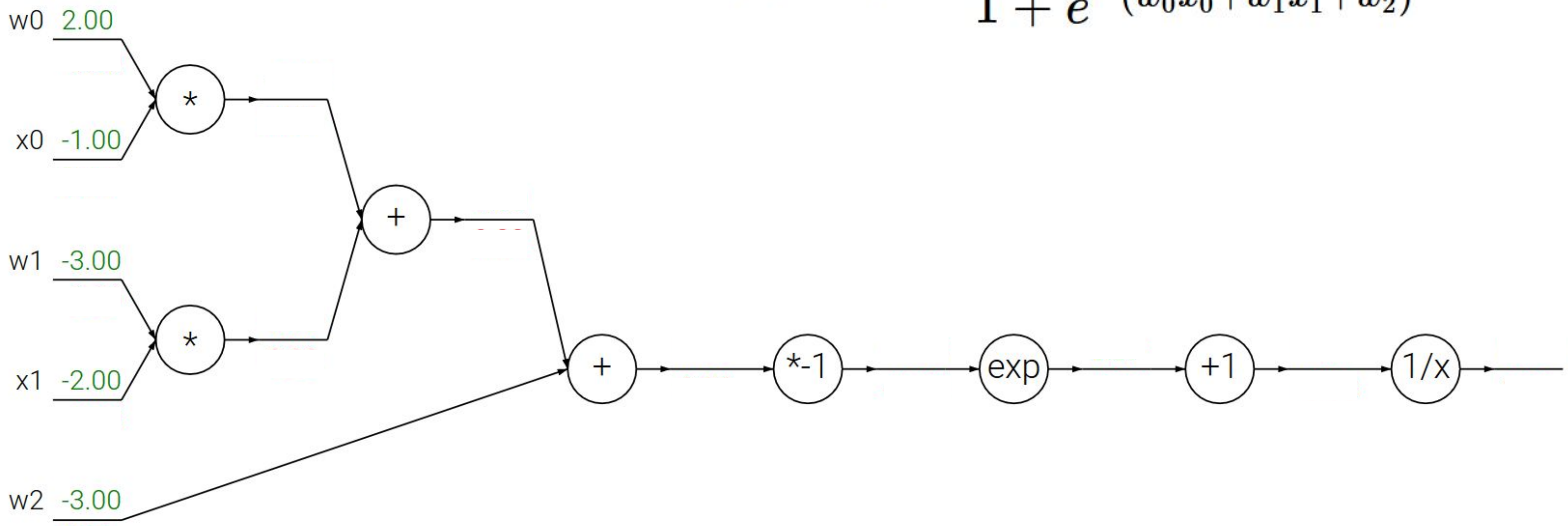


Image borrowed from Stanford cs231n

Back Propagation- Computation Graph

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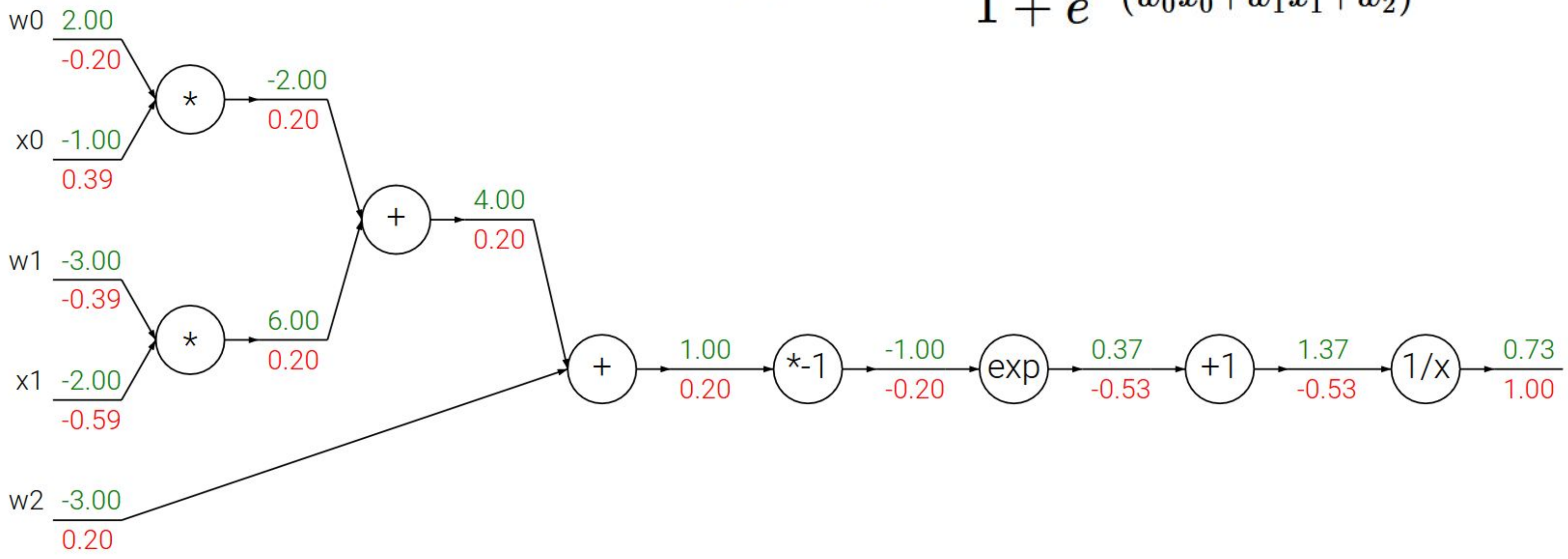
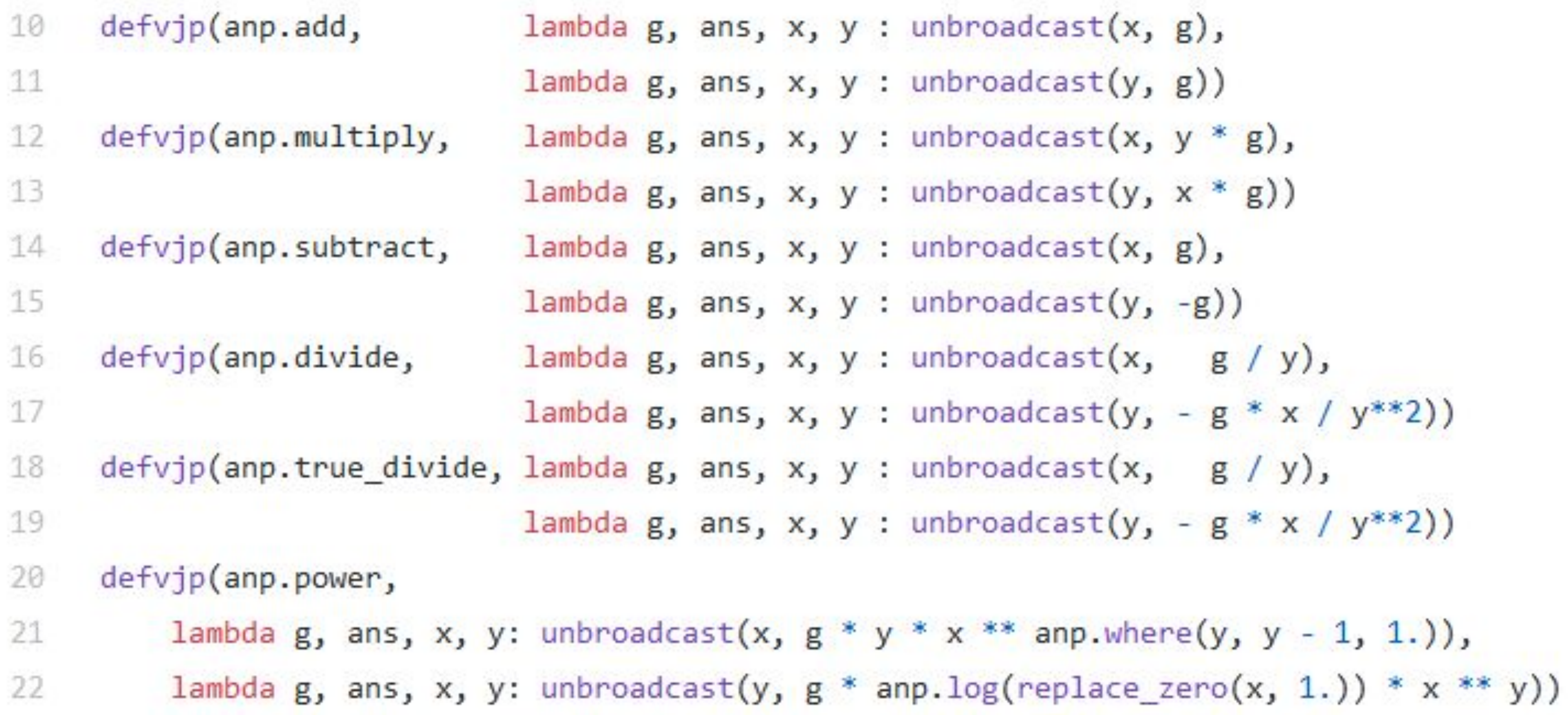


Image borrowed from Stanford cs231n

How does the computer perform differentiation?

Automatic Differentiation (Autodiff)



```
defvjp(anp.add,         lambda g, ans, x, y : unbroadcast(x, g),
                        lambda g, ans, x, y : unbroadcast(y, g))
defvjp(anp.multiply,    lambda g, ans, x, y : unbroadcast(x, y * g),
                        lambda g, ans, x, y : unbroadcast(y, x * g))
defvjp(anp.subtract,    lambda g, ans, x, y : unbroadcast(x, g),
                        lambda g, ans, x, y : unbroadcast(y, -g))
defvjp(anp.divide,      lambda g, ans, x, y : unbroadcast(x,   g / y),
                        lambda g, ans, x, y : unbroadcast(y, - g * x / y**2))
defvjp(anp.true_divide, lambda g, ans, x, y : unbroadcast(x,   g / y),
                        lambda g, ans, x, y : unbroadcast(y, - g * x / y**2))
defvjp(anp.power,
    lambda g, ans, x, y: unbroadcast(x, g * y * x ** anp.where(y, y - 1, 1.)),
    lambda g, ans, x, y: unbroadcast(y, g * anp.log(replace_zero(x, 1.)) * x ** y))
```

https://github.com/mattjj/autodidact/blob/master/autograd/numpy/numpy_vjps.py

https://www.cs.toronto.edu/~rgrosse/courses/csc321_2018/slides/lec10.pdf