Our grounds for "2+1=3" are not in trusting our senses

- a priori arithmetic knowledge
- how do we know it? in part thru proof
  - how can this tell us truths about abstract objects?

also abstractness: what *is* the number 2?

- not the numeral "2" (linguistic object/symbol)
  - "II" stands for the same number
- "+": the addition <u>function</u> $\neq$ itself
  - signs are kind of arbitrary – chosen by convention, not forbiddable
    - Quine: <u>mention</u> vs. <u>use</u>
      - "2" vs. 2
      - "Michael" starts with a "M", vs.
      - Michael is hungry

predicates and relations tie to some sort of meaning

- "is the father of" } these "get" at certain actual
- "is w/in a mile of PA" } properties

Frege: looking at arithmetical knowledge → arithmetical language
"stepping back"

- "2+3=5": some perceptual-like insight into an abstract realm...?
- assimilate these arithmetic ideas to the domain of logical tautologies
  - "If A, then either A or B" ← consequent; ← antecedent
  - complex conditional expression
    - the overall content is determined <u>functionally</u> by the content of the antecedent/consequent
      - by convention: not by any mysterious way
        - ↳ logical consequence of the terminology, a priori (w/o trusting our senses)
        - but: that it can't be false isn't about me – can't be a contingent fact either – <u>necessary</u> fact about the <u>world</u> (in the case of the tautology above)
      - truth functionality (from logic)
      - ordinary English use of "if" is richer than the logical meaning (but this is parenthetical)
        - expressing causal relationship btwn. P & Q
    - not just a list of words: certain syntactic function
      - functionality is how complexification of syntax happens
        - happens at the level of the <u>content</u> of those sentences

$(\forall x)(\underbrace{\text{Man}(x) \Rightarrow \text{Mortal}(x)})$

- ↳ true or false? <u>neither</u> – b/c x doesn't stand for anything
- semantically guaranteed truth given what man & mortal <u>mean</u>
  - not a logical truth

numbers

- numerosity of sets } a lot easier when you're looking
- positions in sequences } at them in terms of numbers
  - ↳ true – but not particularly fancy
- Frege: developing account of classes, sets of objects, etc. that reveal them to be just as simple/topic neutral as logical argumentation
  - stepping back: how does sequence talk/number talk work?
    - stepping back: how does language work in general?
      - believables/judgeables/deniables → contents!
        - sentence content determined compositionally

Frege's "proper names": have the job of introducing an object as the thing being talked about

- "the tallest member of the philosophy dept."
  - $\underbrace{\text{Michael}}$ $\underbrace{\text{is hungry}}$.
    - ↓  ↓ f_{hungry}

f_{hungry} ( ) = content ("Michael is hungry")

not how you think of the argument that settles the answer (e.g. "Michael" vs. "Michael Bratman")

functions are "extensionals"

"="

<u>quantitative</u> vs. <u>qualitative</u> identity

share qualities / be alike: 2 different 8-balls are qualitatively identical

→ the same individual: what "=" means in arithmetic

"A = B" expresses the idea that the two nominations are actually the same thing — numerical identity

the Object model: names "contribute" the objects they stand for as their content

distinguish from "A is taller than B": that a certain relation holds between those objects

→ in identity sentences there's an exception: names express or stand for "their own selves"

in the

Begriffsschrift model

"A = B" → these names corefer: they have the same content

they're names of the same thing ("Superman" and "Clark" corefer)

"A" "B" (the names)

not the objects being referred to – it's talk about language (metalinguistic)

$f_{coreference}$

→ consequence that, e.g., the Spanish translation of an English sentence expresses different content

under the object model: at the <u>level of content</u>, "A = A" means the same as "A = B"

→ this is crazy! clearly "A=B" is (can be) epistemically useful – the object model conflates sentence contents that are clearly different

→ even though the truth value predictions are correct

"the object referred to by A is the same object as the one referred to by B"

vs.

"the names 'A' and 'B' corefer to the same object" (Begriffsschrift model)

→ how can we properly understand identity knowledge as non-trivial?

<u>and</u> not just linguistic! but arithmetical or etc.

==what is the worry about the (new) Begriffsschrift model?==