Russell/Frege correspondence 1902

Dear Colleague,

…[N]ow this view seems doubtful to me because of the following contradiction. Let *w* be the predicate: to be a predicate that cannot be predicated of itself. Can *w* be predicated of itself? From each answer its opposite follows. Therefore we must conclude that *w* is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection . . . does not form a totality.

Dear Colleague,

. . . Your discovery of the contradiction caused me the greatest surprise and, I would almost say, consternation, since it has shaken the basis on which I intended to build arithmetic… [The matter is] all the more serious since, with the loss of my Rule V, not only the foundations of my arithmetic, but also the sole possible foundations of arithmetic, seem to vanish… In any case your discovery is very remarkable and will perhaps result in a great advance in logic, unwelcome as it may seem at 1st glance. (van Heijenoort 1967: 127–8)

Frege to Russell

Jena
13 November 1904

Dear Colleague,

. . . Mont Blanc with its snowfields is not itself a component part of the thought that Mont Blanc is more than 4,000 metres high . . .

Russell to Frege

Ivy Lodge
Tilford, Farnham
12 December 1904

Dear Colleague,

. . .. Concerning sense and *Bedeutung*, I see nothing but difficulties which I cannot overcome ... I believe that in spite of all its snowfields Mont Blanc itself is a component part of what is actually asserted in 'Mont Blanc is more than 4,000 metres high'. . . .

Russell on Definite Descriptions

The Principles of Mathematics view (1903)

- Sentences express *Russellian (or "singular") propositions,* which are complexes of objects ("terms"), properties ("concepts"), and relations.
- Name-predicate sentences express ⟨ **property: object** ⟩ propositions:
  - "Venus is a planet" expresses ⟨ **being a planet: Venus** ⟩
- Properties characteristically, but not always, *occur predicatively* in such propositions*:*
  - In ⟨ **is a property: being a planet** ⟩, the property of *being a planet* occurs *not* predicatively, but as an object the proposition is about.
  - That is, there is an important logical distinction between different *positions* things fill in propositions: object positions and predicative positions.

The Principles of Mathematics view (1903)

- ○
- But what about sentences that have, where proper names would fit syntactically, “denoting phrases” like “all dogs,” “some dog,” “no dogs,” or “the dog”?
- Here there does not seem to be any *particular* dog that the expressed proposition is *about*.
- “On Denoting” is Russell’s “aha” moment, where he completely abandons and replaces his *Principles of Mathematics* treatment of denoting phrases.

PofM account of Denoting Phrases

These phrases express, and provide *into the "thing place" in a proposition* a special element called a *denoting complex*. (He uses "concept," not "complex," in *PofM*, which he seems to have forgotten two years later in "On Denoting.")

- "The dog" expresses the denoting complex **[the: being a dog]**
- "The dog is hungry" expresses 〈 **being hungry: [the: being a dog]** 〉

PofM account of Denoting Phrases

- “The dog is hungry” expresses 〈 **being hungry: [the: being a dog]** 〉

But here is the crucial twist: this proposition does not attribute the property of being hungry *to* the denoting complex itself (no denoting complex is hungry), *despite* the fact that the complex sits in the object position in the proposition.

Instead, because there’s a denoting complex in the object position, the proposition requires that that property holds of the thing that has the property of being a dog.

In the case of the denoting complexes expressed by “all dogs,” “no dog,” “some dog,” and so on, the requirement is correspondingly different.

PofM account of Denoting Phrases

If we say that a subsentential expression *refers* to an object when it makes the expressed proposition *about* that object *by* placing that object in the proposition's object position, …

Then denoting phrases do not *refer* to things at all, because, while they do place denoting complexes in propositional object positions, these propositions are not strictly speaking *about* those denoting complexes at all.

Nor are are denoting phrases like "the dog," "some dog," and so on, strictly speaking about any dogs, because no dog or dogs are constituents of the expressed propositions.

PofM account of Denoting Phrases

Instead, in Russell’s terminology, such denoting complexes *denote* dogs, and so in an important but *indirect* way, the propositions are about dogs.

When a particular object *is* “the dog” we’re talking about, the denoting phrase, and the denoting concept it expresses, denotes *that very dog* (even though the dog is not itself in the proposition we express).

What do other denoting phrases (“all,” “some,” etc.) denote? Here’s what Russell says about that...

PofM account of Denoting Phrases

“A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept.” (*PofM,* p. 53)

“(1) *All a’s* denotes a_1 and a_2 and …

(2) *Every a* denotes a_1 and denotes a_2 and …

(3) *Any a* denotes a_1 or a_2 …, where *or* has the meaning that it is irrelevant which we take.

(4) *An a* denotes a_1 or a_2 …, where *or* has the meaning that no one in particular must be taken…

(5) *Some a* denotes a_1 or denotes a_2 …, where it is not irrelevant which is taken, but on the contrary some one particular *a* must be taken.” (59)

PofM account of Denoting Phrases

The key point is that denoting complexes are devices of *aboutness redirection*. Their function in propositions is to make the propositions concern things other than themselves -- things that do not occur in the propositions.

The (Notoriously Baffling) Gray’s Elegy Argument

I attempt a close reading of it in a document in the Readings folder. tl;dr:

- If a denoting complex in an object place in a proposition makes that proposition *about* something else (by denoting it), then there can be no propositions (directly) *about* that denoting concept. E.g., the proposition

  $\langle$ being a denoting complex: [the: being a red planet] $\rangle$

  would be *false* despite predicating a property that the thing in the object place in fact has. But surely that denoting complex *is* a denoting complex.

- So if the *PofM* account is correct, denoting complexes, by their nature, cannot be referred to or thought about directly. There *are* no propositions that are strictly speaking *about* them. This is already weird, but there’s more...

But can’t DCs at least be thought about indirectly?

Sure, they can be *denoted* by a “higher order” denoting complex, as here:

> **⟨ being a DC: [the: being the DC expressed in English by “the red planet”] ⟩**

But it is intolerable to think that *this* is our primary way of being acquainted with that DC. Anyone who knows only that some DC or other is expressed in English by “the red planet” can think about that DC *that* way. We who *understand* the phrase must have a way of thinking of the DC that is more direct than this.

Russell’s conclusion: DCs can’t be thought about directly, nor does there seem to be any good answer as to how we who understand them think about them indirectly. We should seek a theory that doesn’t lead to such difficulties.

A better way to think about DCs indirectly?

Here is an attempted solution to these “difficulties” that does not abandon the general approach of *PofM* (as “On Denoting” does!).

As our higher-order DC to denote **[the: being a red planet]**, instead of “whichever DC is expressed in English by ‘the red planet’,” consider “the DC of the *the* kind containing the property *being a red planet.*” That expresses the following DC:

> **[the: being a DC of the *the* kind & containing the property <u>being a red planet</u>]**

This doesn’t diminish the oddness of there being things that there are no propositions about (directly), but it might defeat Russell’s claim that it’s *also* a mystery how we understanders might think about them by denoting them.

Is the Gray's Elegy argument fair against Frege?

Does Frege even hold the views that make for the problems Russell claims to find?

Russell says that he does:

> [Frege] distinguishes, in a denoting phrase, two elements, which we may call the *meaning* and the *denotation*. . . . I shall, however, not repeat the grounds in favor of this theory, as I have urged its claims elsewhere (*loc. cit.*), and am now concerned to dispute those claims. . . .
>
> The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong.

Is the Gray’s Elegy argument fair against Frege?

Not entirely, because on Frege’s view, propositional aboutness is *always* indirect: the nominatum referred to by a name or description is determined by the mode of presentation it expresses. So there is nothing exceptional about the indirect aboutness that comes with definite descriptions.

To Russell’s question, how (under what modes of presentation) do we think about and “denote” other modes of presentation that we understand, Frege *does* owe an answer, because he holds that we *do* think *about* modes of presentation, for instance, in propositional attitude sentences like “Hammurabi doubted whether Hesperus was Phosphorus.”

And the issue seems to get more urgent for Frege with multiple embeddings, like “Some say Hammurabi wondered whether Hesperus was Phosphorus.”

(Additional) Problems for Frege:

- Predicts "*nonsense"* where there's really *falsity*:
  - On Frege's view "one would suppose that 'the King of France is bald' ought to be nonsense; but it is not nonsense, since it is plainly false."
  - Similarly "Ferdinand is *my only living son*" is false, not "nonsense", if Ferdinand is dead (and hence "my only living son" has no denotation).
- Doesn't explain uses of descriptions in *hypothetical* contexts, such as 'If there is exactly one *u*, then the *u* is a *u.*' Russell: "This proposition ought to be *always* true, since the conclusion is true whenever the hypothesis is true… Now if *u* is *not* a unit class, "the *u*" seems to denote nothing; hence our proposition would seem to become nonsense as soon as *u* is not a unit class."

Russell might well have added...

Pronouns can occur *bound* inside definite descriptions.

> **Everyone** remembers [the teacher **they** have learned the most from]

Here, the description doesn't have a nominatum when the pronoun contained in it is not used *deictically* (as when pointing at a specific person) but *anaphorically* (being bound by the quantifier "Everyone"). But that, contra Frege, does not prevent the sentence from having a truth value.

Meinong

Meinong serves in “On Denoting” as a second example of how we might take definite descriptions to contribute propositional items sitting in object position.

Meinong distinguishes between barely *being* and genuinely *existing*.

- Zeus and the largest prime number *are* but don’t *exist*.

Advantages:

- Gives a simple semantic account of “Zeus does not exist:” he doesn’t!
- Explains what our talk and thought “about Zeus” are about: Zeus! We can talk about things that don’t exist, as well as those that do.

Russell’s complaint

These non-existent objects "infringe the law of noncontradiction":

- The existent present King of France both exists and doesn’t.
- The round square is round and (because square) not round.

Russell’s diagnosis is that Meinong’s key mistake is the same as the one he ascribes to the *PofM* account (and misascribes to Frege), namely, the unsupported assumption that all subject-predicate sentences express propositions of the same *logical form*, i.e., that they have something in the property position and something in the object position.

The New Theory of Descriptions

Key features of the view:

- Descriptions introduce *quantificational structure* as well as propositional constituents (notice the plural) into the propositions expressed by sentence in which they occur.
- Unlike names and predicates, they do not express single propositional constituents that sit in a single place in the expressed proposition.

Logically Complex Propositions

Remember that ⟨ being a planet: Venus ⟩ is the Russellian (or singular) proposition that is true just in case Venus is a planet.

Russell also believed in logically complex propositions: if P and Q are propositions, so for instance is ⟨ Not: ⟨ Or ; P, Q ⟩ ⟩.

Propositional Functions

For Russell, in addition to complete propositions, there are *propositional functions*, which have the full structure of a proposition, but one or more elements is absent from its slot. We diagram these using *variables* in those positions:

〈 being a planet: x 〉

Russell calls this a function because it yields a proposition if you “feed” it an object, such as Venus, as its argument.

Propositional functions are not true or false, but we can say that they are true or false *of* or *for* objects taken to be values of the variable.

Quantificational Propositions

In addition to logical propositional operators such as And, Or, Not, and If, there are quantificational operators. Russell chose to work with just one, namely All. The (complete) proposition:

$\langle$ All x: $\langle$ Or: $\langle$ being a planet: x $\rangle$, $\langle$ being angry, x $\rangle$ $\rangle$

is true just in case the contained propositional function is true of every object (and so it is in fact false).

In thinking about Russellian propositions, however, we will use the now standard language of first order logic to be a simpler tool for diagramming them:

$$\forall x \,(Planet(x) \lor Angry(x))$$

The New Theory of Indefinite descriptions

Russell:

C(a dog) means [C(x) and x is canine] is not always false

In our notation:

$\sim(\forall x)\sim[\ C(x)\ \&\ x \text{ is canine}\ ]$

Equivalently:

$(\exists x)[\ C(x)\ \&\ Canine(x)\ ]$

Definite Descriptions

"C(the F)" means:

> It is not always false of x that [F(x) and C(x), and it is always true of y that [if F(y) then y is identical with x]]

That is,

$$\sim(\forall x)\sim(\ F(x)\ \&\ C(x)\ \&\ (\forall y)(F(y) \rightarrow y=x)\ )$$

or, equivalently:

$$(\exists x)(\ F(x)\ \&\ (\forall y)(\ F(y) \rightarrow y=x)\ \&\ C(x)\ )$$

In slightly more ordinary terms: something is <u>unique in being an F</u>, and it is a C as well.

Puzzles Resolved

How does this not attribute a silly attitude to King George?

George IV wondered whether the author of Waverley was Scott.

Even though Scott was indeed the AoW, this does not mean:

$$\text{Wondered}(\ \text{George}_4,\ \text{Identity}(\ \text{Scott},\ \text{Scott}\ )\ )$$

Instead, it means:

$$\text{Wondered}(\ \text{George}_4,\ \exists x\ (\ x \text{ is uniquely AoW } \&\ x = \text{Scott}\ )\ )$$

Puzzles Resolved 2

Is “The present King of France is bald” true or not?

It is not true, because it means (where PKF means present King of France):

> There is something uniquely PKF and also bald.

How about, “The present King of France is not bald”?

This *can* mean the (true) negation of the previous proposition. But it is more likely to mean this *false* proposition:

> There is something uniquely a PKF and also not bald.

This *ambiguity* is actually a successful *prediction* of Russell’s new theory.

Primary (wide-scope) and Secondary (narrow-scope) occurrences:

There is an ambiguity in many sentences containing descriptions, which is not only explained but predicted by Russell's theory.

Consider a sentence of the form "It's not the case that C(the F)."

Since "C(the F)" means (in isolation) that there is something that is uniquely F and also C, our sentence *can* mean the negation of that proposition:

$$\sim \exists x \; (\; x \text{ is uniquely } F \;\&\; C(x) \;)$$

Primary (wide-scope) and Secondary (narrow-scope) occurrences:

But because “It’s not the case that C(the F)” is also a complete sentence, we can also apply Russell’s theory of descriptions directly to it rather than to the contained sentence “C(the F)”. Doing so, we get:

$$\exists x(\ x \text{ is uniquely } F\ \&\ {\sim}C(\ x\ )\ )$$

This predicts the two readings of “The PKF is bald.”

It also predicts two readings of “George IV wondered if <u>the AoW</u> was Scott”

What are the two meanings predicted by Russell?

George IV wondered if the AofW was Scott.

For Russell (as for Frege), “wondered” expresses a *propositional attitude relation*, a relation between an agent and a proposition. Both disambiguations entail that George bears the wondering relation to a proposition. But in one case the wondered proposition is specified, and in the other case it is described:

- Wondered(George_4, ∃x(x is uniquely AoW & x = Scott))
- ∃x(x is uniquely AoW & Wondered(George_4, x = Scott))

Ordinary and Logically Proper Names:

Russell held that we can understand propositions that are directly about a thing only if we know that thing “by acquaintance.”

If we merely know that there is a unique F, but are not acquainted with the thing, then we can know that there is a proposition about the thing that is uniquely F, predicating G of it, but we cannot grasp that proposition itself. (We cannot have *de re* thoughts about the thing, to use current philosophical jargon.)

Ordinary and Logically Proper Names:

Sentences involving an ordinary proper name, according to Russell, clearly express propositions -- things we can believe or disbelieve -- whether or not the name stands for anything, and certainly whether or not we are acquainted with that object. So those propositions must really not be directly about the object (if anything) the name stands for. Instead, he holds, ordinary proper names are denoting expressions; they are *abbreviated definite descriptions*.

A *logically proper* name, in contrast, (by definition) does serve to express propositions in which the thing it stands for occurs (singular propositions, in contemporary terminology).

Why think ordinary names are not logical names?

1. I certainly believe that Cheney is a poor shot. (Assumption)

2. To believe that p is to be belief-related to the proposition expressed by “p”. (Assumption)

3. Therefore, I am certainly belief-related to a proposition, call it q, which is expressed by “Cheney is a poor shot”. (From 1 and 2)

4. It is *not* certain that the name “Cheney” denotes an individual. (Assumption)

5. So, I am not certainly belief-related to a proposition containing an individual denoted by the name “Cheney.” (From 4)

8. Therefore, q is not a proposition containing an individual denoted by the name “Cheney.”

Generalized Quantifiers

What are the supposed chief semantic advances of the “On Denoting View”?

1. Denoting phrases, including definite descriptions, do not contribute single entities to the propositions expressed by sentences in which they occur.

2. The propositions expressed by sentences containing denoting phrases have a quantificational character as opposed to directly concerning individual things.

Russell is fond of emphasizing (1), but arguably it is (2) that is the more fundamental -- and which has been retained in many contemporary semantic theories.

Generalized Quantifiers

The structure of the proposition expressed by "C(the F)" can be taken to be something like:

< [The x: F]; C(x) >

This is a proposition not of the term-predicate form, but of the quantificational form (there will also be negated forms, conditional forms, …). A proposition of the quantificational form consists of a quantifier component (playing the distinctive quantificational role) and a predicate component. The quantifier component is composed of a determiner component (in this case The) and a predicate component (F).

Referential vs. Attributive (Keith Donnellan)

Two uses of “the murderer” in the very same sentence (but in different circumstances) seem to have rather different import:

We come upon Smith’s mutilated corpse and say “Smith’s murderer is insane”.

This (attributive) use of the description could be paraphrased, “Smith’s murderer, whoever that might be”.

At the trial, where we both assume Jones is guilty of Smith’s murderer, Jones is acting bizzarre. I say, “Smith’s murderer is insane.”

This (referential) use could be paraphrased “that guy, i.e., Smith’s murderer”.

Key question: are these different uses signs of an ambiguity--that there are two different things for “Smith’s murderer” to mean?