Mathematical Finance M551
Section 9.4 Lecture Notes
Indiana University East

Suppose G denotes the present value gain from some investment. So if the interest rate is r, intial payment/investment is C, and X is the return after one period, then

$$G = \frac{x}{1+r} - C$$

Def: The value at risk (VAR) of an investment is the value $|V|$ such that there is only a 1% chance that the loss from the investment will be greater than $|v|$. Hence v satisfies

$$P(G < v) = 0.01$$

(*We ussually interpret v in terms of its magnitude, hence we will discuss v in terms of $|v|$. That is, we would not say the VAR is -15. We would say the VAR is* 15. *That is, There is a* 1% *chance the investment will return an amount 'below' -15. This is the same as saying there is a* 1% *chance our loss will be greater than* 15).

VAR is important because it can be used as a measure of performance. We would prefer an incestment with small VAR:

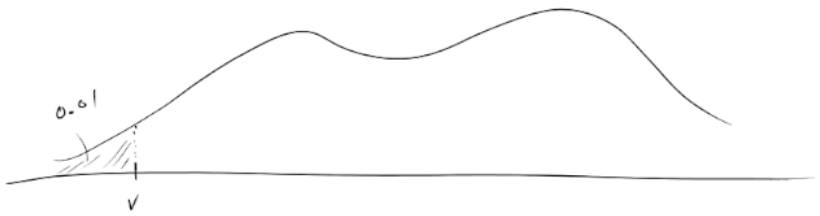

Figure 1: figure 1

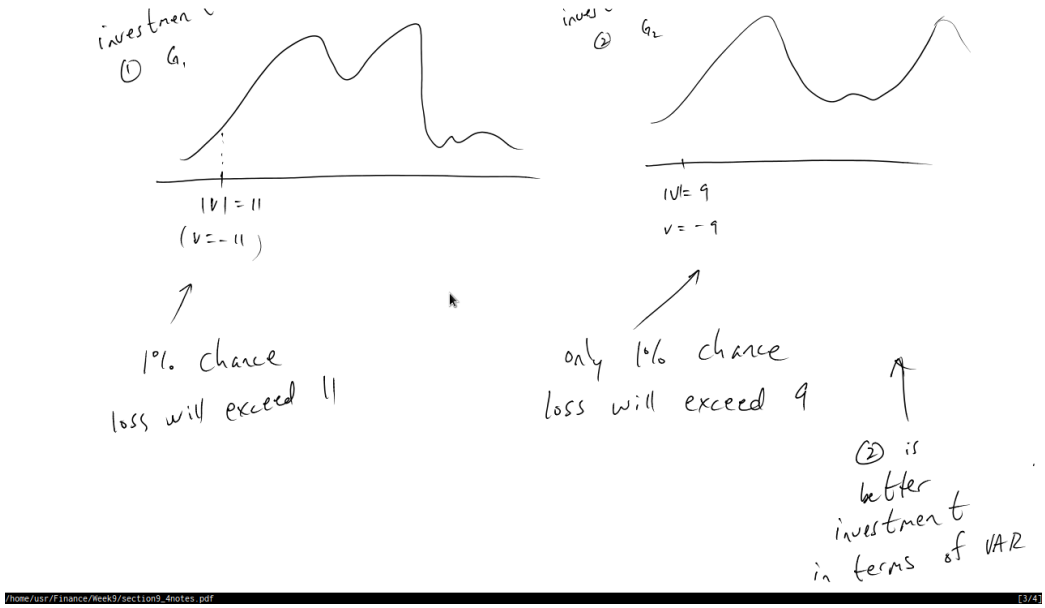

Figure 2: figure 1

Example Suppose that the gain G from an investment is a normal random varible with mean μ and standard dev σ. Let's determine the value at risk $|v|$.

$$\begin{aligned} 0.01 &= P(G < v) \\ &= P(\frac{G-\mu}{\sigma} < \frac{v-\mu}{\sigma}) \\ &= P(z < \frac{v-\mu}{\sigma}) \end{aligned}$$

From table 2.1 we see that $P(z < -2.33) = 0.01$, hence $\frac{v-\mu}{\sigma} = -2.33$ which implies $v = -2.33\sigma + \mu = \mu - 2.33\sigma$

Hence there is a 1% chance we will incur a loss from our investment greater than $|v| = |\mu - 2.33\sigma|$.