

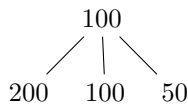
# Mathematical Finance M551

## Section 9.1 Lecture Notes

### Indiana University East

In this section, we provide an example of when a no-arbitrage cost for an option may not be unique.

**Example:** Consider a stock with initial price 100. After one period, the stock may either be 200, 100, or 50. So we now allow for the possibility that the stock price stays the same (notice the difference with the model given in 5.1). Suppose we wish to price a call option with strike price 150 (i.e. we have the option to buy stock for 150 after 1 period. Assume that interest rate is 0 for simplicity.



By the arbitrage theorem, we know arbitrage will not be present provided there exist probabilities  $P_{50}, P_{100}, P_{200}$  such that the expected gain when purchasing a stock or option is 0. That is

$$E[G_s] = 0 \quad \text{and} \quad E[G_c] = 0.$$

Where  $G_s$  denotes the gain at time 1 from buying the stock and  $G_c$  " " buying the option.

Hence,

$$G_s = \begin{cases} 100 & \text{if } S(1) = 200 \\ 0 & \text{if } S(1) = 100 \\ -50 & \text{if } S(1) = 50 \end{cases}$$

Which implies,

$$E[G_s] = 100P_{200} + 0P_{100} + (-50)P_{50}$$

If we let  $C$  be the price of the option,

then

$$G_c = \begin{cases} 50 - C & \text{if } S(1) = 200 \\ -C & \text{if } S(1) = 100 \quad \text{or } S(1) = 50 \end{cases}$$

So,

$$E[G_c] = (50 - C)P_{200} - CP_{50} - CP_{100} = 50P_{200} - C$$

For no arbitrage, we must have  
 $0 = E[G_s]$  and  $0 = E[G_c]$ , thus

$$E[G_s] = E[G_c]$$

This implies

$$P_{200} = \frac{1}{2}P_{50} \quad \text{and} \quad C = 50P_{200}$$

but  $P_{200} = \frac{1}{2}P_{50} \implies P_{200} \leq \frac{1}{3}$  (Since  $P_{50} + P_{100} + P_{200} = 1$ ). It follows that  
 $0 \leq C \leq \frac{50}{3}$ .

So the noarbitrage price  $C$  in this case can be any value in the interval  $[0, \frac{50}{3}]$ ,  
hence it is not unique!