Algorithmic Learning Theory
Spring 2017
Lecture 1

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There are 4 section in the Lecture 1:

1. Overview
2. Fundamental Concepts
3. Distinction of Concepts
4. Bayesian Decision Rule

1 Overview

Brief of the Syllabus and the tools needed.

1.1 Textbooks

See *Syllabus* in *Sakai*.

1.2 Student Work

See *Syllabus* in *Sakai*.
No test, **BUT**:

1. Course note, in LaTeX
2. Student projects (Can be in small group)

1.3 Topics

See *Syllabus* in *Sakai*.

1.4 Tools

R & RStudio

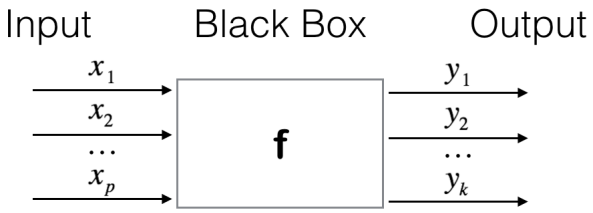
2 Fundamental Concepts

Introduction of basic concepts and ideas of machine learning.

2.1 Black Box

Definition by Wikipedia: *In science, computing, and engineering, a black box is a device, system or object which can be viewed in terms of its inputs and outputs (or transfer characteristics), without any knowledge of its internal workings.*

Consider a black box with a bunch of input and output,



Which consists of 3 parts, Input, Output and the Function f:

Input $x_1, x_2, ..., x_p$, also called independ vars, predicators, features and controls.
The input can be numerical or non-numerical(categorical), if input is in multi-class, it can be ordered(ordinal) or unordered.

Output $y_1, y_2, ..., y_k$, also called response, depend vars and target vars. For example,

f()	Output
regression	numerical
classification	categorical

2.2 Data

Definition by Wikipedia: Data(computing) *is any sequence of one or more symbols given meaning by specific act(s) of interpretation.*

Data is also called example and observation and usually organized by table, like this:

row	x_1	x_2	...	x_p	y_1	y_2	...	y_k
1	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
N	...	...	...	...	...	...	...	...

The data table can be regarded as $\hat{f}$, an estimation of f, which is considered an algorithm or a rule.
For most application scenarios, data is given or generated by an algorithm called generation method.
In a dataset, there may be some missing data(N/A) in the table.

2.3 Big Data

Data that can't be located in one computer.

"Big" large scale in 2 dimensions, the number of records/items(rows) and the number of features(columns).

More features is good? There may be some irrelevant or redundant features in dataset, so that feature selection is a major problem.

3 Distinctions

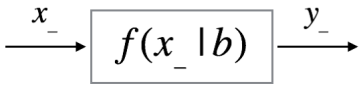
The distinctions between some concepts in machine learning.

3.1 Parametric vs Non-parametric

e.g. $x = height, y = weight, y = f(x)$

3.1.1 Parametric

e.g. Regression
Assumption: $y = b_0 + b_1 x$, b_0, b_1 are the parameters.
So, the problem can be showed like this:

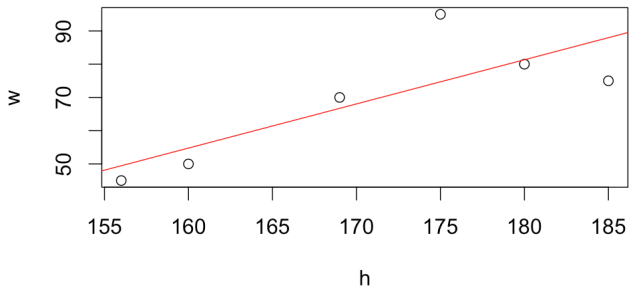


The work of machine learning is to get data and find the parameters.

Supposed we get the dataset like this:

W	H
w_1	h_1
w_2	h_2
...	...
w_n	h_n

Assume: $w = b_0 + b_1 h$
So, the regression plot is: $\hat{w} = \hat{b_0} + \hat{b_1} h$

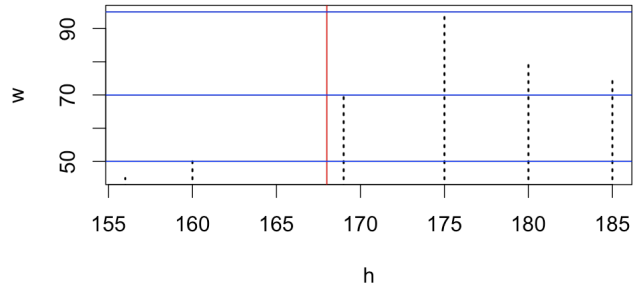


3.1.2 Non-parametric

e.g. kNN.(k-nearest neighbors algorithm)

if $k = 3$, find k near point of h_{new}, so,

$$w_{new} = \text{average(k closest points to } h_{new}) = \frac{w_1 + w_2 + w_3}{3}$$



In this case, the h_{new} is the red line, so, the w_1, w_2, w_3 are the w in blue lines.

3.1.3 Conclusion

Parametric Assumption: $y = b_0 + b_1 x$, b_0, b_1 are the parameters.

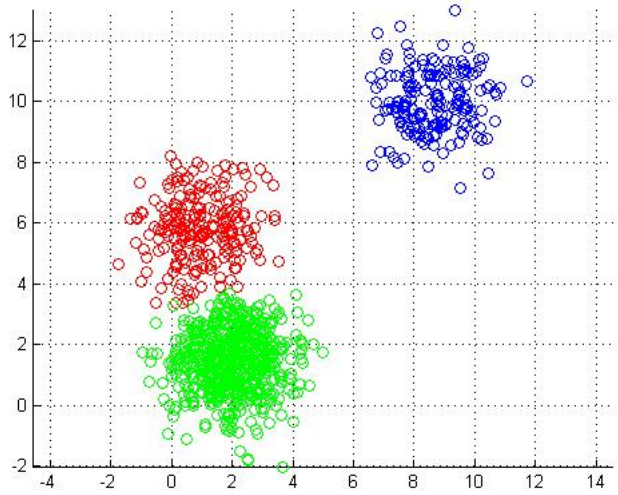
$$x_{new} \longrightarrow \boxed{\hat{f} = \hat{b}_0 + \hat{b}_1 x} \longrightarrow y_{new}$$

Non-parametric More flexible. No parameters to impose, no assumption.

3.2 Supervised vs Unsupervised(non-supervised)

Supervised Name x and y, the input and output are specific by labels.

Unsupervised Every column of data is x, no labels.
e.g k-means



When the number of dimensions increase, the difficulty of recognition increase, so that the selection of x is a major problem.

3.3 Online learning vs Offline(not online) learning

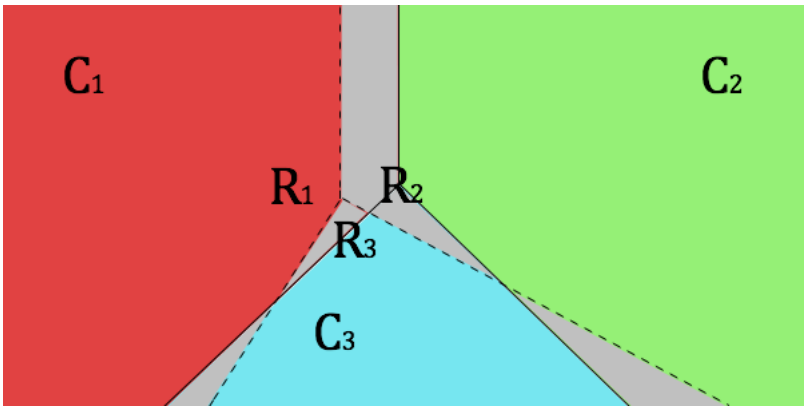
Online learning data becomes available in a sequential order and is used to update our best predictor for future data at each step.

Offline(not online) learning generate the best predictor by learning on the entire training data set at once.

3.4 Loss vs Overfitting

3.4.1 Loss for classification

Definition by Wikipedia a loss function or cost function is a function that maps an event or values of one or more variables onto a real number intuitively representing some "cost" associated with the event.



The true classification is f, and the classes are C_1, C_2, and C_3, which are divided by line and painted in red, green and blue. Supposed that we don't know the true classification.
The estimated classification is $\hat{f}$, and the classes are R_1, R_2, and R_3, divided by dotted line.
The mistake area, painted by gray, is the difference between estimation and true classification.
Loss function abstract the size of mistake area.

$$P_x(t) \rightarrow \text{Probability density function}$$

Misclassification rate:

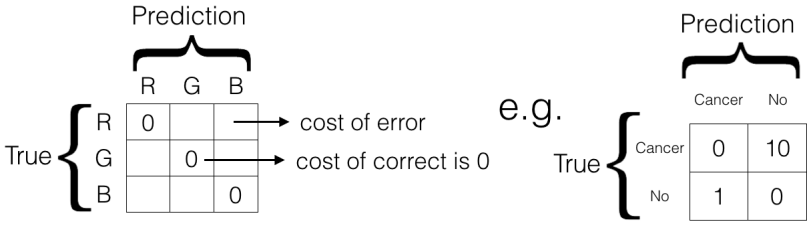
$$Pr_x[f(x) \neq \hat{f}(x)]$$

But misclassification rate above is not computable, because we have to know two function or classification.

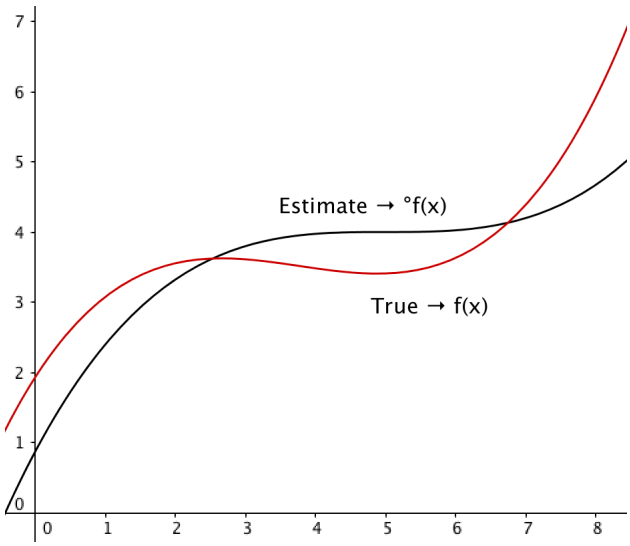
Empirical error/misclassification rate:

$$\frac{\text{Number of misclassified data}}{\text{Total data number}}$$

Loss matrix To show the cost of error in a matrix.



3.4.2 Loss for regression



Mean square error is used to eliminate the + and -.
Loss function: $E_x\left(\ell(f,\hat{f},x)\right)$
Error rate:

$$\int \ell(f,\hat{f},x)p(x)dx = \int \left(f(x)-\hat{f}(x)\right)^2 p(x)dx$$

Major of loss: $Max|f(x)-\hat{f}(x)|$
Empirical error:

$$\frac{\sum_{i=1}^{N}\left(f(x)-\hat{f}(x)\right)^2}{N}$$

3.4.3 Overfitting

Definition by Wikipedia In overfitting, a statistical model describes random error or noise instead of the underlying relationship. Overfitting occurs when a model is excessively complex, such as having too many parameters relative to the number of observations.

In this case, overfitting is that the $\hat{f}(x)$ performs in perfect result. Generally, overfitting is a bad phenomenon, because the data self may have error or miss relevant vars.

Validation set The technique to guard overfitting from complex model.

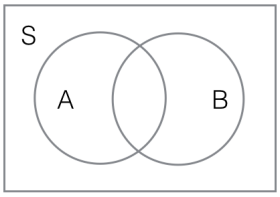
1. Set a set of data aside(validation set)
2. learning is done on the remaining data(training set)
3. Use validation set to predict response

4 Bayesian Decision Rule

Bayes Law: For Event A and Event B

$$0 \leq P[A] \leq 1, 0 \leq P[B] \leq 1$$

(A and B: $A \cap B$, A or B: $A \cup B$, not A: $\bar{A}$ or A^*)



If,

$$A \cap B \neq \emptyset, \tag{1}$$

So, according to (1),

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \Rightarrow P[A \cap B] = P[A|B] \times P[B], \tag{2}$$

And, the same with (2),

$$P[B|A] = \frac{P[B \cap A]}{P[A]} \Rightarrow P[B \cap A] = P[B|A] \times P[A] \tag{3}$$

Obviously, we have,

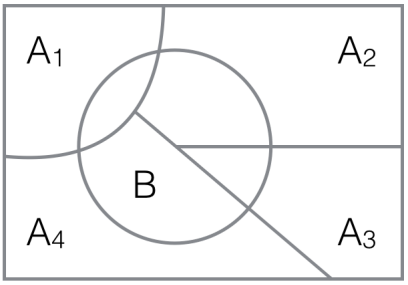
$$P[A|B]P[B] = P[B|A]P[A] \tag{4}$$

So,

$$P[A|B] = \frac{P[B|A] \times P[A]}{P[B]}$$

$$P[B|A] = \frac{P[A|B] \times P[B]}{P[A]}$$

e.g. If, a partition of A,



$$A_1 \cup A_2 \cup A_3 \cup A_4 = S, A_i \cap A_j = \emptyset, i, j = 1, 2, 3, 4$$

So,

$$P[B] = \sum_{i=1}^{4} P[B|A_i]P[A_i]$$