Assignment 1
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Question 1

a.

First, prove that $\emptyset \in S$.
Obviously, we have $\emptyset \in E = \{1, 2, ..., \mathrm{n}\}$, and $a(\emptyset) = \sum_{j \in \emptyset} = 0 \leqslant b \in \mathbb{R}_+$
So, $\emptyset \in S$.
Second, prove that $X \subseteq Y \in S \Rightarrow X \in S$

$$Y \in S \Rightarrow a(Y) = \sum_{j \in Y} a_j \leqslant b$$

$$X \subseteq Y \Rightarrow a(X) = \sum_{j \in X} a_j = \sum_{i \in Y} a_i - \sum_{j \in Y \setminus X} a_j \leqslant \sum_{j \in Y} a_j \leqslant b$$

So, this difines an independence system $\mathcal{F} \subseteq 2^E$

b.

$$\mathrm{n} = 6 \Rightarrow S \subseteq E = \{1, 2, 3, 4, 5, 6\}$$

And we have,

$$a = (1, 1, 1, 4, 4, 5), \mathrm{b} = 8 \Rightarrow a(S) = \sum_{j \in S} a_j \leqslant 8$$

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8

Question 9

Question 10

$$\mathcal{B} + \mathcal{F}$$