Assignment 1
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Question 1

a.

First, prove that $\emptyset \in S$.
Obviously, we have $\emptyset \in E = \{1, 2, ..., \mathrm{n}\}$, and $a(\emptyset) = \sum_{j \in \emptyset} = 0 \leqslant b \in \mathbb{R}_+$
So, $\emptyset \in S$.
Second, prove that $X \subseteq Y \in S \Rightarrow X \in S$

$$Y \in S \Rightarrow a(Y) = \sum_{j \in Y} a_j \leqslant b$$

$$X \subseteq Y \Rightarrow a(X) = \sum_{j \in X} a_j = \sum_{i \in Y} a_i - \sum_{j \in Y \setminus X} a_j \leqslant \sum_{j \in Y} a_j \leqslant b$$

So, this difines an independence system $\mathcal{F} \subseteq 2^E$

b.

According to the defination, we have

$$\mathrm{n} = 6 \Rightarrow S \subseteq E = \{1, 2, 3, 4, 5, 6\}$$

$$a = (1, 1, 1, 4, 4, 5), \mathrm{b} = 8 \Rightarrow a(S) = \sum_{j \in S} a_j \leqslant 8$$

According to the defination of *rank*, we have

$$r(X) = \underset{F \in \mathcal{F}}{max} |F \cap X| = \underset{B \in \mathcal{B}_X}{max} |B|, \text{ and } \rho(X) = \underset{B \in \mathcal{B}_X}{min} |B|$$

We can find that

$$\mathcal{B}_X = \big\{\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 3, 6\}, \{4, 5\}\big\}$$

So that we have $r(X) = 4, \rho(X) = 2$

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8

Question 9

Question 10

$$\mathcal{B} + \mathcal{F}$$