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Business School
Newark and New Brunswick

Understanding Opaque Selling from an Inventory-control Perspective

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Opaque Selling

- **Normal Market**

  Yellow T-shirt: $10

  Pink T-shirt: $10

- **With O-good**

  Yellow T-shirt: $10

  Pink T-shirt: $10

  Random T-shirt: **$9**

Lower price for customers

Seller decides what to deliver

Opaque Or Not Opaque?

“Hmmm… It sounds interesting and attractive, but … ”

Is it beneficial?

How does O-good help business?

How to ration products for O-good?

Pricing Strategy? Inventory Strategy?

……

Basic Assumption: Symmetric

Two similar regular products, X-good and Y-good, only few differences, e.g. different color, and **with the same**

- Selling price p_R
- Holding cost h
- Ordering cost c
- Lost sales punishment l
- Poisson arrival rate λ
- Ordering together, with a fixed cost K

Name	X**-good**	O**-good**	Y**-good**
Price	p_R	p_O	p_R
Inventory	x		y

Basic Assumption: Replenishment Policy

Without X-Y distinction,

- Best replenishment policy:

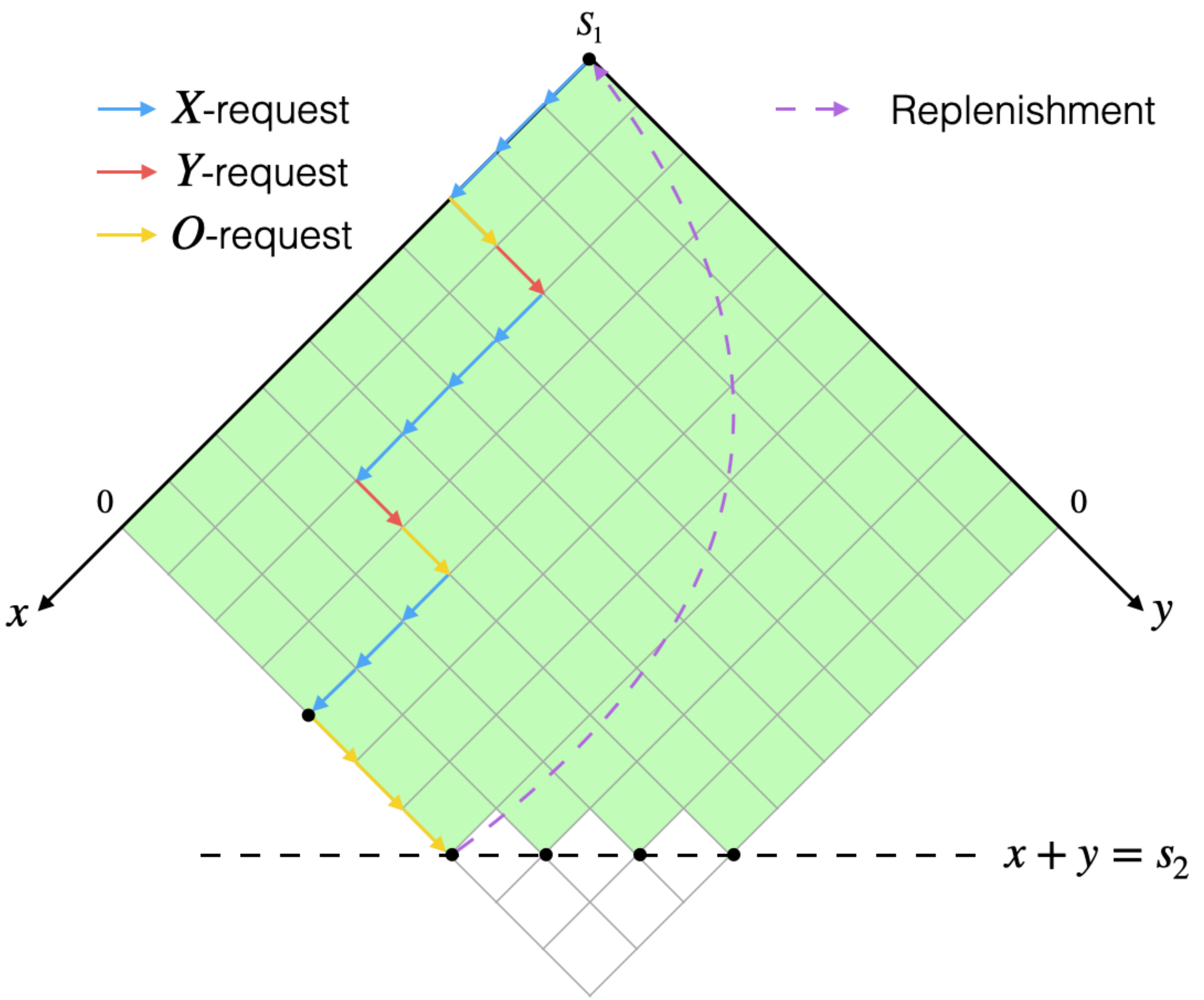
  $(\mathscr{R}, \mathscr{Q})$-policy

- Adjusted: $(s_2, 2 \times S_1)$-policy

  $s_2 = \mathscr{R}, 2 \times S_1 = \mathscr{R} + \mathscr{Q}$.

  When $x + y \leq s_2$,

  order both x and y to S_1



Seller Model: Cycle-wise Profit Rate

Given buyers' behaviors β, we can define …

- At any inventory level (x, y), till the next replenishing moment,
    - $f(x, y)$ — total average profit
    - $g(x, y)$ — total average duration
- Seller's profit generation rate

$$r \equiv \frac{f(S_1, S_1)}{g(S_1, S_1)}$$

Seller Model: Monotonicity

At any fixed total inventory $s = x + y$, and given buyers' behaviors β, we have

Monotonicity Property 1

*With reasonable pricing p_O, the cycle-wise profit is **decreasing** in the inventory difference $d = |x - y|$.*

Monotonicity Property 2

*The cycle-wise duration is **increasing** in d.*

Seller Model: Balancedness

Takeaway (Balancedness): The seller prefers **more balanced inventory** layouts between the X- and Y-goods

$$\Downarrow$$

A Balance-inducing Rationing Strategy

Takeaway (Rationing): The optimal rationing strategy for O-good is —always giving out the product with **the higher inventory level, which promotes balancedness.**

Seller Model: O-good Increases Profit Rate

With a reasonable pricing p_O*,*

$$r'(\beta_O) \geq \left[\frac{(p_O - p_R) \cdot (2S_1 - s_2)}{\gamma(2S_1, 0 \mid \beta_O)}\right] \wedge 0$$

Takeaway (Beneficial): When p_O catches up to p_R, **the long-run profit rate increases** with the O-good demand ratio β_O

Seller Model: Seller's Behavior

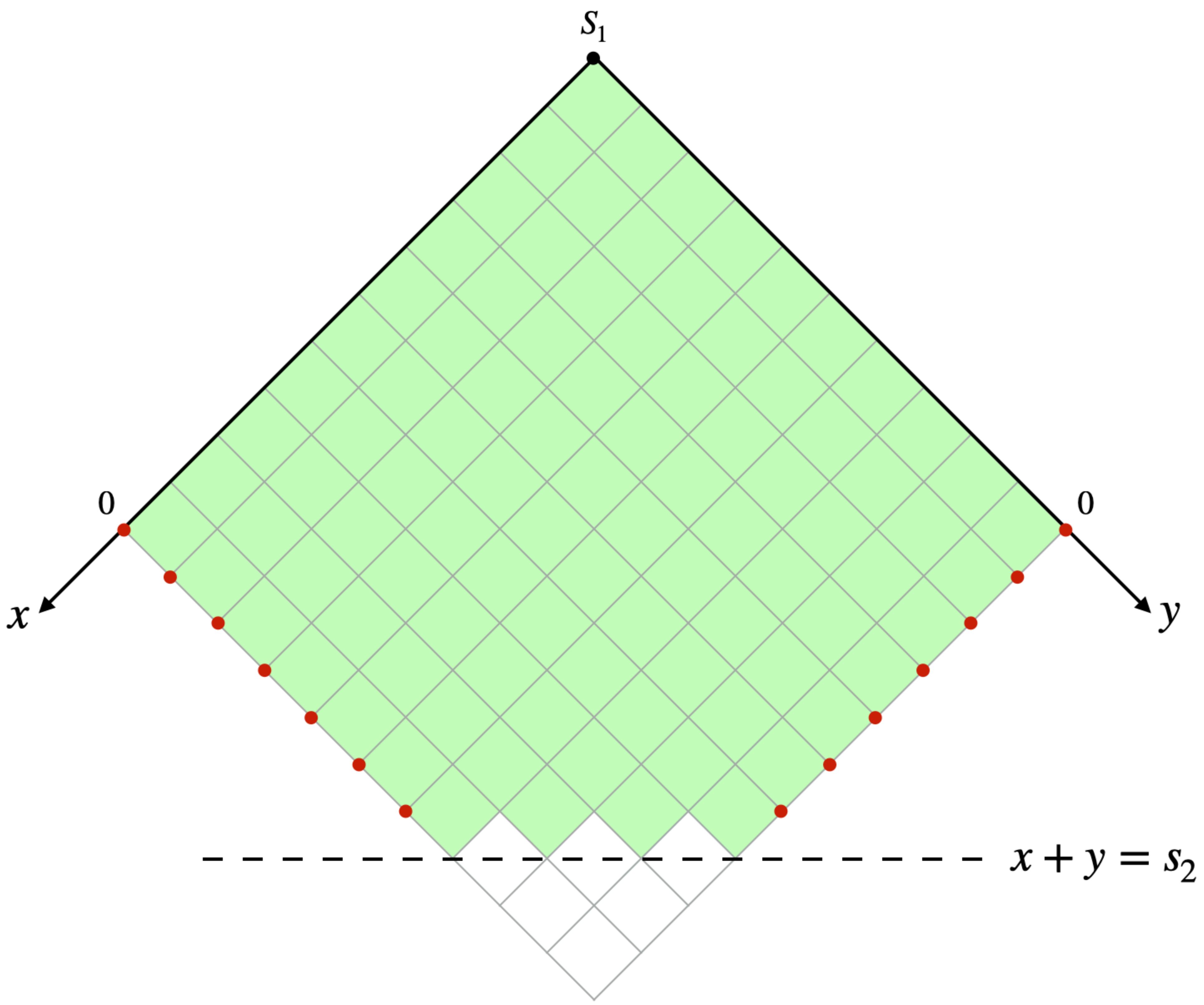
The chance that an regular request to be granted

$$\theta_R = \Pr(X \geq 1) = \Pr(Y \geq 1)$$

can be obtained through steady-state analysis.

θ_R is a function of (s_2, S_1), but varies slowly

$$\theta_R \approx 0.9$$



Estimation: **A Good** $(s_2, 2 \times S_1)$

Plugging θ into DP formulas, we have

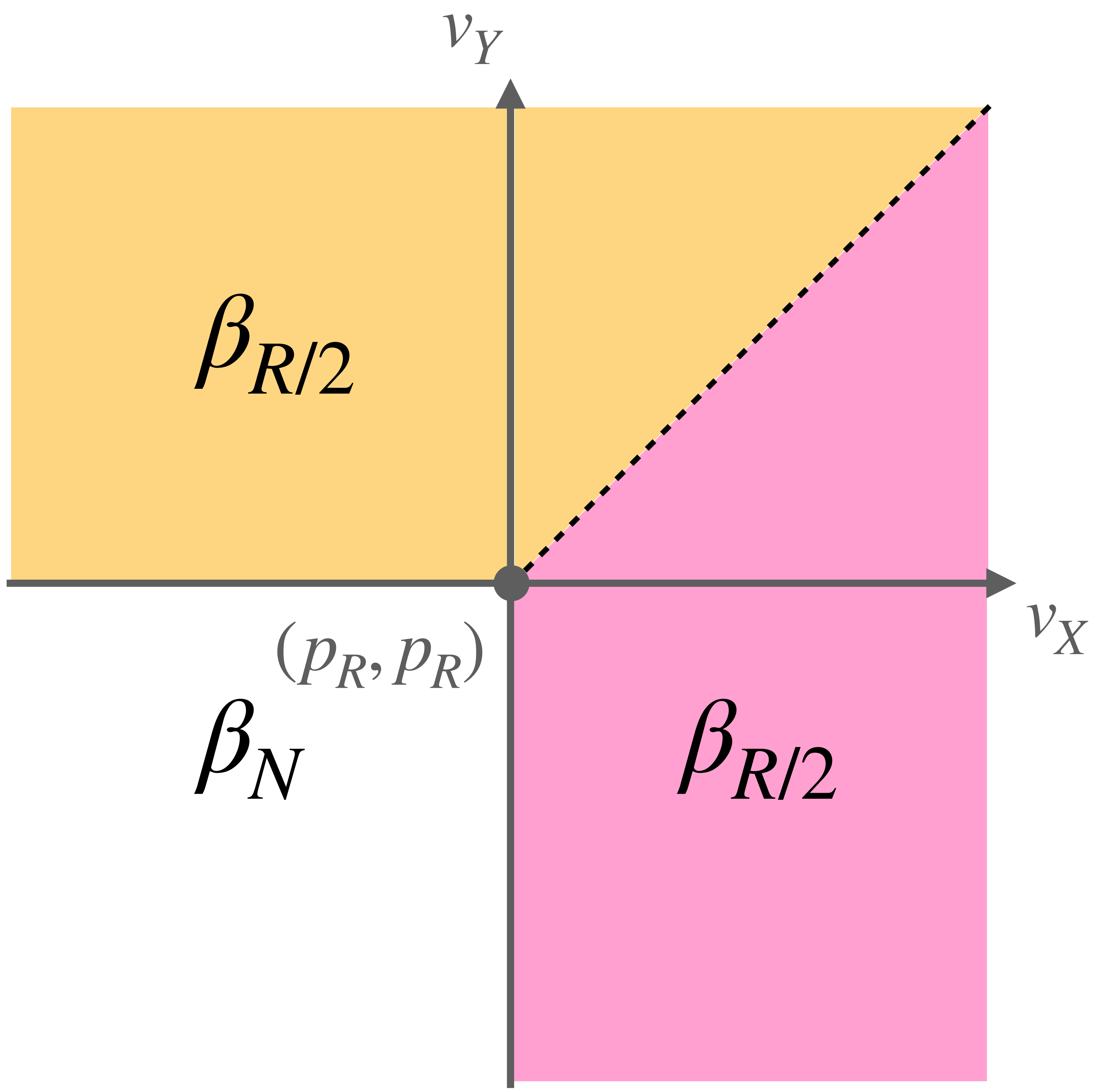
$$r(s_2, \mathcal{Q}) = P(s_2) - C(\mathcal{Q}), \qquad \text{where } \mathcal{Q} \equiv 2S_1 - s_2.$$

The revenue function $P(s_2)$ is **decreasing** in s_2; and $C(\mathcal{Q})$ follows the **EOQ** structure. With the effective arrival rate $\tilde{\lambda}$, we have

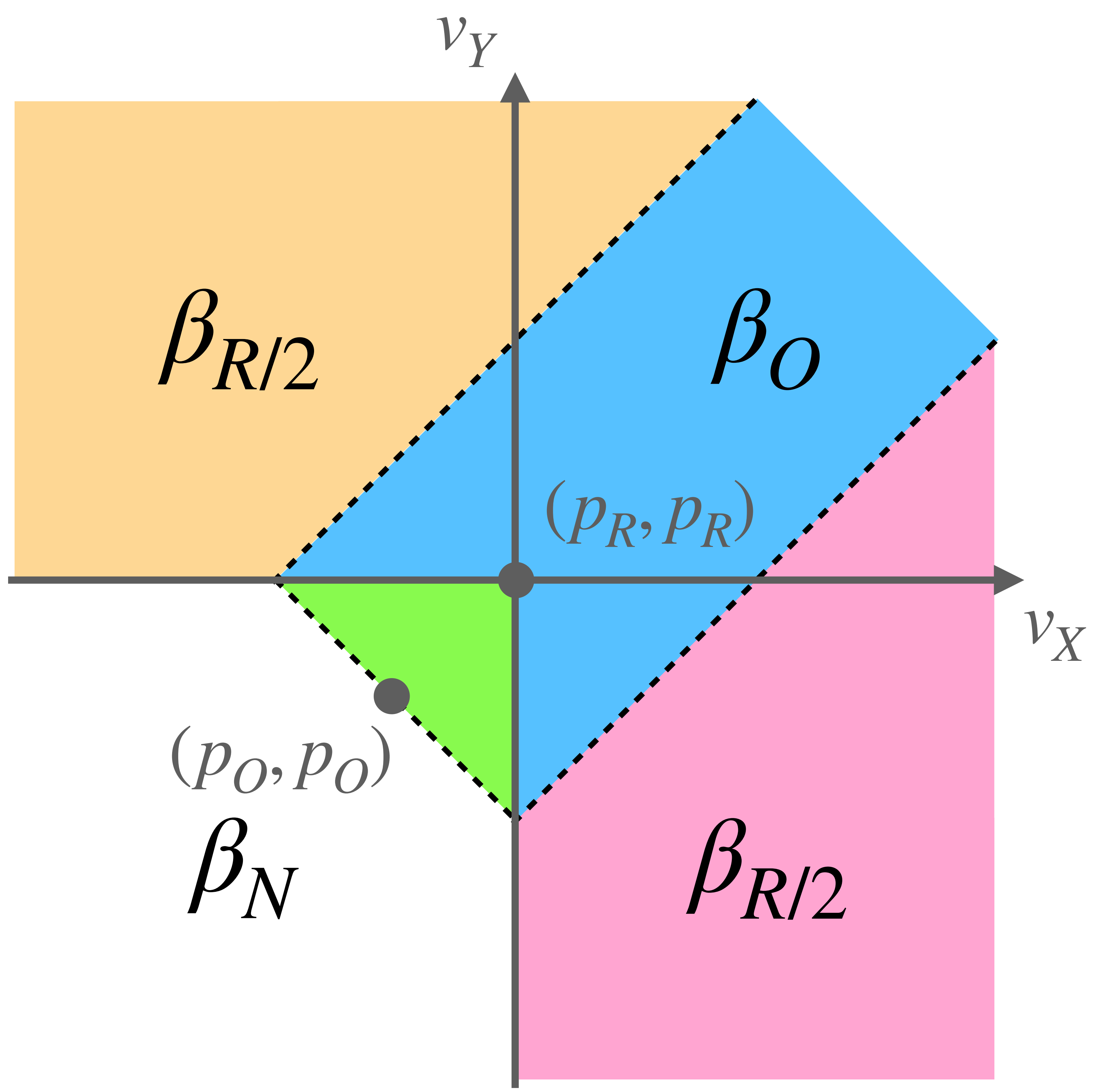
$$2S_1 - s_2 \simeq \mathcal{Q}^* \equiv \sqrt{\frac{2\tilde{\lambda}K}{h}}$$

Takeaway (Inventory): **EOQ** still works for the O-good scenario.

Strategic Buyers: Overall Behaviors



Without O-good



With O-good

O-good May Be Harmful!

Strategic Buyers: Equilibrium Existence

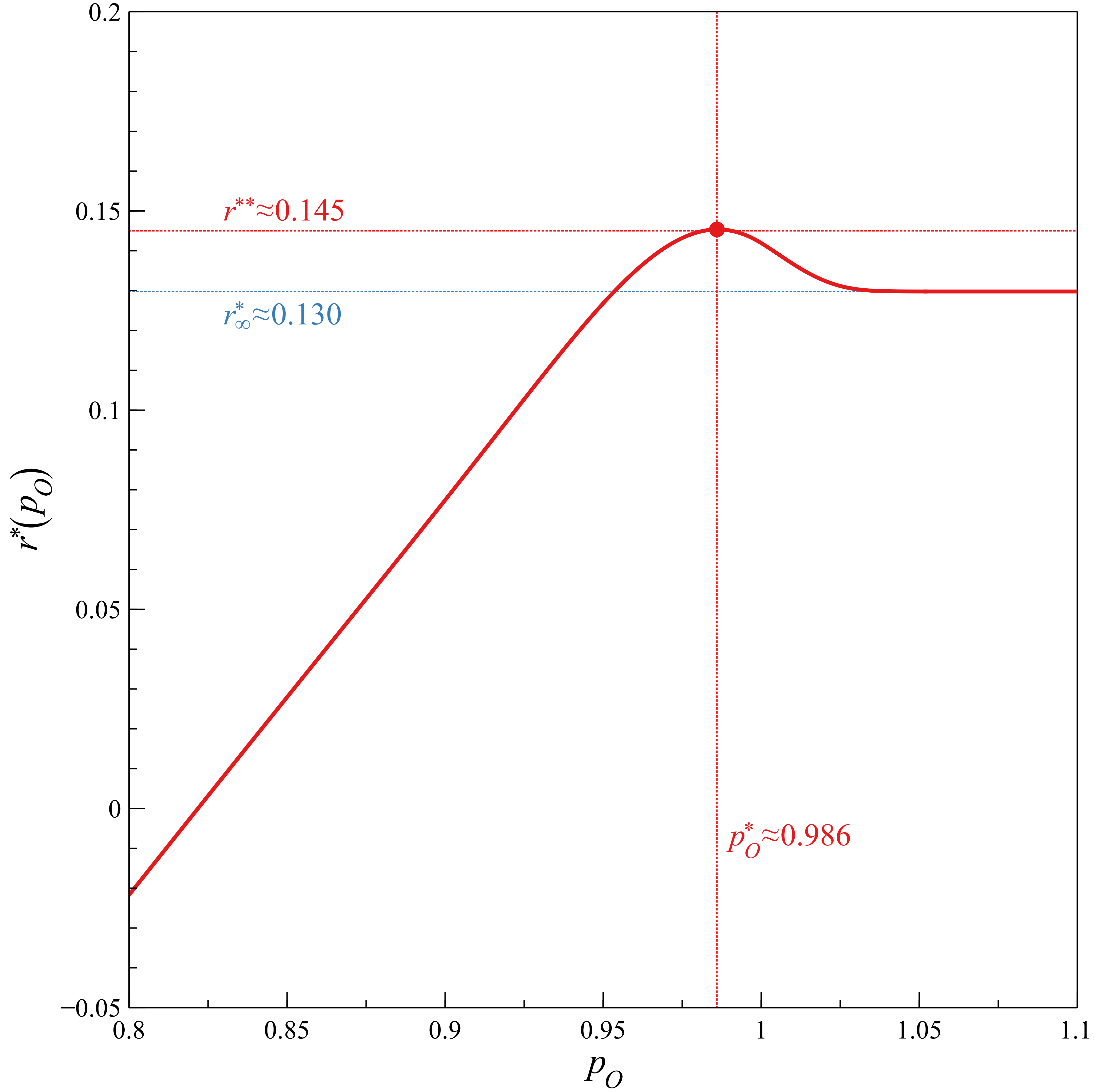
The earlier $\beta \to \theta$ process defined by Markov chain and steady-state analysis is continuous, so is the current $\theta \to \beta$ process

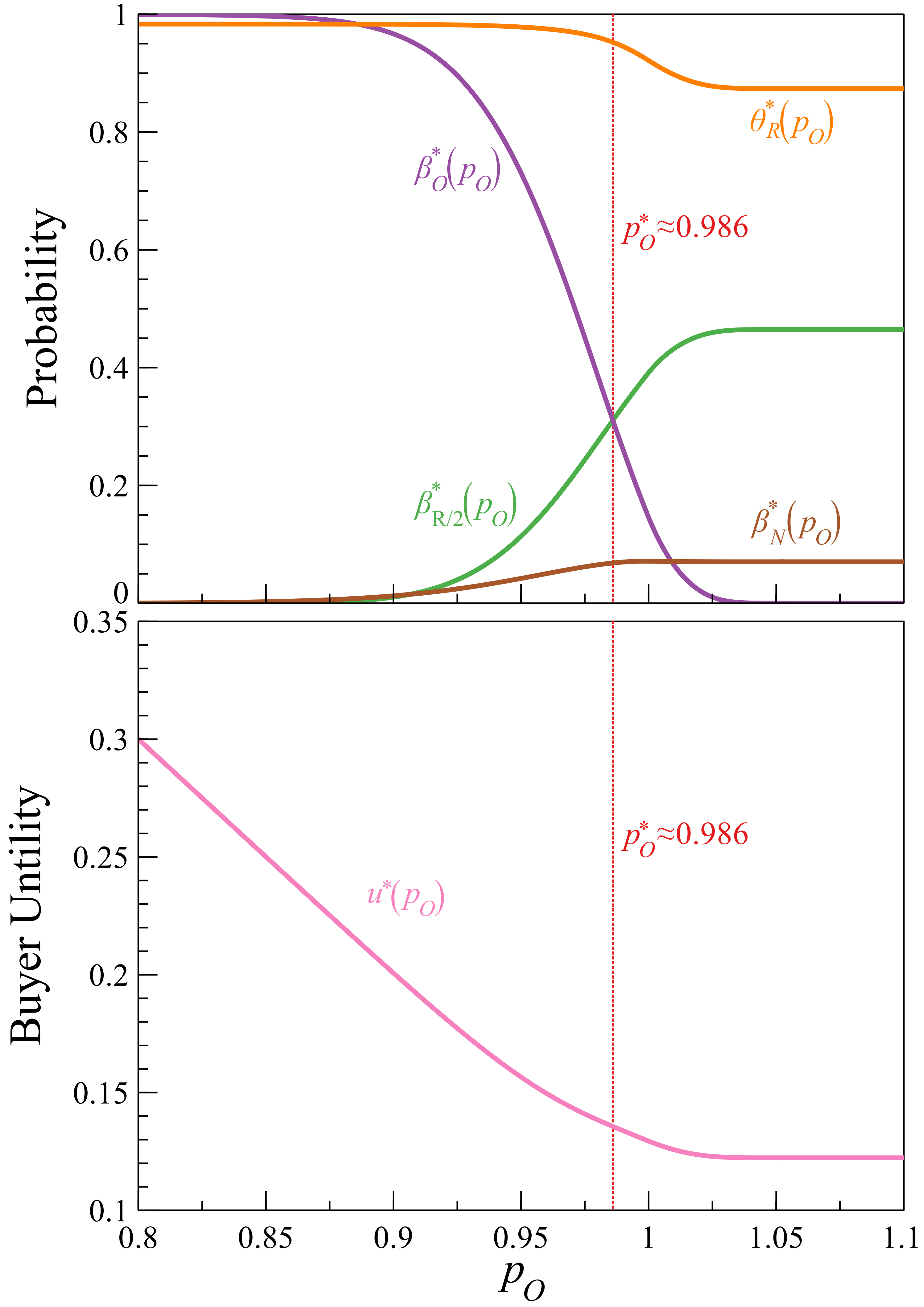
By Brouwer's fixed point theorem, there should exist (β^*, θ^*) such that

$$\beta^* \to \theta^* \to \beta^* \to \theta^* \to \cdots$$

This provides an endogenous generation of buyer behaviors $(\beta_{R/2}, \beta_O)$ that can move with parameters such as $p_O, \rho, \sigma, S_1, s_2, \ldots$

Simulation: Equilibrium





Summary

- The seller prefers **more balanced inventory**
- The optimal rationing strategy is—always giving out the **higher-inventory-level** product
- Under a reasonable pricing range, the **long-run profit rate increases** with the O-good demand ratio
- **EOQ** still works for the -good scenario.
- Buyers' behavior and the seller's behavior will reach an **equilibrium**, which enables numerical study of the pricing strategy.

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Thanks!

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