## Name:

# **Assignment 1 (Due: 09/19/19)**

# Q1: Data

Classify the following attributes as discrete or continuous. Also classify them as qualitative (nominal or ordinal) or quantitative (interval or ratio). Some cases may have more than one interpretation, so briefly indicate your reasoning if you think there may be some ambiguity.

**Example**: Age in years. **Answer**: Discrete, quantitative, ratio

- Speed of a vehicle measured in mph.

## Continuous, Quantitative, Ratio

- Altitude of a region.

#### Continuous, Quantitative, Ratio

- Intensity of rain as indicated using the values: no rain, intermittent rain, incessant rain.

#### Discrete, Qualitative, Ordinal

- Brightness as measured by a light meter.

#### Continuous, Quantitative, Ratio

- Barcode number printed on each item in a supermarket.

#### Discrete, Qualitative, Nominal

#### Q2: Random Variables

The number of **thousands** of miles, X, a car can be driven before it breaks down is an exponential random variable with parameter  $\lambda=1/30$ . Suppose the car has already been driven 20,000 miles without breaking down. What is the probability it can be driven for at least another 30,000 miles? Repeat the question assuming that X is a uniform random variable with parameters 0 and 60,000.

### Since the memoryless property of exponential distribution, we have

$$P[X \ge (20+30) \, | \, X \ge 20] = P(X \ge 30) = \int_{30}^{\infty} \lambda e^{-\lambda x} dx = [-e^{-\lambda x}] \, |_{30}^{\infty} = e^{-1}$$

If X is a uniform random variable with params 0 and 60000.

$$P[X \ge (20+30) \mid X \ge 20] = \frac{P(X \ge 50)}{P(X \ge 20)} = \frac{1 - \frac{5}{6}}{1 - \frac{2}{6}} = \frac{1}{4}$$

# Q3: Sampling & Measures

In Sample Variance, we have the formula  $S^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{n-1}$ , why n-1 here?

Degree of freedom. Since  $\mu$  is given, only n-1 variables are free to vary in this calculation.

# Q4: Sampling Distribution of the Mean

Given a normal distribution with  $\mu=100$  and  $\sigma=10$ , if you select a sample of n=25, what is the probability that  $\bar{X}$  is

- 1. less than 95?
- 2. between 95 and 97.5?
- 3. above 102.2?
- 4. There is a 65% chance that  $\bar{X}$  above what value?

$$(Z_{2.5} = 0.9938, Z_{1.25} = 0.8944, Z_{1.1} = 0.8643, Z_{0.38} = 0.6480, Z_{0.39} = 0.6517)$$

1. 
$$Z = \frac{95 - 100}{10/\sqrt{25}} = -2.5, Z_{-2.5} = 1 - Z_{2.5} = 0.0062$$

2. 
$$Z = \frac{97.5 - 100}{10/\sqrt{25}} = -1.25, Z_{-1.25} = 1 - Z_{1.25} = 0.1056, Z_{-1.25} - Z_{-2.5} = 0.0994$$

3. 
$$Z = \frac{102.2 - 100}{10/\sqrt{25}} = 1.1, Z_{1.1} = 0.8643, P = 1 - Z_{1.1} = 0.1357$$

**4.** 
$$Z_{0.39} = 0.6517, Z_{-0.39} = 0.3483. \frac{x - 100}{10/\sqrt{25}} = -0.39 \Rightarrow x = 99.22$$