Edwards, Ryan
399067099

# Assignment 4

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## Theoretical questions

1. Illustrate that if a new email from the address $a \in A$, it always gets through (1 pts).

   *Proof.* Consider the email address: redw764@aucklanduni.ac.nz

   In binary, this is equivalent to:

   01110010 01100101 01100100 01110111 00110111 00110110 00110100 01000000 01100001 01110101 01100011 01101011 01101100 01100001 01101110 01100100 01110101 01101110 01101001 00101110 01100001 01100011 00101110 01101110 01111010

   Let this be represented as an integer $x$ such that $x \in A$

   When the hash table is constructed, the function will set $B[h(x)] = 1$

   When an email is received, the email filter will apply the hash function $h(x)$

   By definition:

   - If $B[h(x)] = 1$ the email will go through
   - If $B[h(x)] = 0$ the email is considered spam

   Because $B[h(x)] = 1$ was set when the hash table was constructed, and because a hash function will always return the same output for $h(x)$, the lookup function will return 1 and the email will go through.

   □

2. Given any position $0 \leq i < n$, what is the probability that $B[i] = 1$ (2 pts).

   As there is a universal hashing function for integers

   $$h_{ab}(x) = ((ax + b) \mod p) \mod n$$

   As $n = 8,000,000,000$ the probability of a collision is $Pr_h[h(x) = h(y)] \leq \frac{1}{n} \leq \frac{1}{8\text{B}}$

   Therefore, the probability of $B[i] = 0$ after 1 insert is $\geq 1 - \frac{1}{8\text{B}}$

   The probability of $B[i] = 0$ after 1B inserts is $\geq (1 - \frac{1}{8\text{B}})^{1\text{B}}$

   Thus, the the probability that $B[i] = 1$ is simply

   $$\leq 1 - (1 - \frac{1}{8\text{B}})^{1\text{B}} \lessapprox 0.11750$$

3. Given a spam email from the address $a' \notin A$, what is the probability that it gets through (2 pts)

   As, by definition, a universal hashing function will uniformly distribute a new hashed value, the probability of collision is the probability that $B[i] = 1$.

   Thus, the probability of $B[h(a')] = 1$ is $\lessapprox 0.11750$