Waveform Uncertainty Notation and Equations

1) Definition of the discontinuity correction frequency:

$$\frac{\partial^2}{\partial f^2}\left(\Delta A_\mu(f_{COR};\theta)\right) = -10^{-6}$$

2) Definition of amplitude difference between models μ:

$$\Delta A_\mu(f;\theta) = \begin{cases} \frac{|\mu_{IMR}(f;\theta)| - |\mu_{EOB}(f;\theta)|}{|\mu_{IMR}(f;\theta)|} & f \leq f_{COR} \\ \Delta A_\mu(f_{COR};\theta) & f > f_{COR} \end{cases}$$

3) Definition of raw phase difference between models μ:

$$\Delta\phi_\mu(f;\theta) = \begin{cases} \tan^{-1}\left(\frac{\mathrm{Im}[\mu_{IMR}(f;\theta)]}{\mathrm{Re}[\mu_{IMR}(f;\theta)]}\right) - \tan^{-1}\left(\frac{\mathrm{Im}[\mu_{EOB}(f;\theta)]}{\mathrm{Re}[\mu_{EOB}(f;\theta)]}\right) & f \leq f_{COR} \\ \Delta\phi_\mu(f_{COR};\theta) & f > f_{COR} \end{cases}$$

4) Definition of residual phase difference between models μ:

$$\Delta\Phi_\mu(f;\theta) = \begin{cases} \Delta\phi_\mu(f;\theta) - (2\pi t_0 f + \phi_0) & f \leq f_{COR} \\ \Delta\Phi_\mu(f_{COR};\theta) & f > f_{COR} \end{cases}$$

5) Chebyshev polynomial series approximation of model amplitude difference:

$$\Delta A_\mu(f;\theta) \approx \Delta A_T(f; a, f_{COR}, \Delta A_\mu(f_{COR};\theta)) = \begin{cases} \sum_{i=0}^{N-1} a_i T_i(f) & f \leq f_{COR} \\ \Delta A_\mu(f_{COR};\theta) & f > f_{COR} \end{cases}$$

6) Chebyshev polynomial series approximation of model residual phase difference:

$$\Delta\Phi_\mu(f;\theta) \approx \Delta\Phi_T(f; b, f_{COR}, \Delta\Phi_\mu(f_{COR};\theta)) = \begin{cases} \sum_{i=0}^{N-1} b_i T_i(f) & f \leq f_{COR} \\ \Delta\Phi_\mu(f_{COR};\theta) & f > f_{COR} \end{cases}$$

7) Definition of model amplitude uncertainty:

$$\delta A_\mu(f) = \sqrt{\frac{\sum_{i=1}^{N}\left(\Delta A_\mu(f;\theta_i) - \overline{\Delta A_\mu}(f)\right)}{N}}$$

8) Definition of model residual phase uncertainty:

$$\delta\Phi_\mu(f) = \sqrt{\frac{\sum_{i=1}^{N}\left(\Delta\Phi_\mu(f;\theta_i) - \overline{\Delta\Phi_\mu}(f)\right)}{N}}$$

9) Likelihood function of a waveform model with added waveform uncertainty parameters:

$$\mathcal{L}(h|\theta,\alpha,\beta) = \prod_j \frac{1}{2\pi P(f_j)} \exp\left(-2\Delta f \frac{|h(f_j) - \mu(f_j;\theta)\left(1 + \Delta A_\delta(f_j;\{f_n,\alpha_n\})\right)\exp\left[i\Delta\Phi_\delta(f_j;\{f_n,\beta_n\})\right]|^2}{P(f_j)}\right)$$

10) Definitions of α and β parameters from waveform uncertainties:

$$\alpha_n \sim \mathcal{N}(0, \delta A(f_n)) \qquad \beta_n \sim \mathcal{N}(0, \delta\Phi(f_n))$$

Notation Key	
Symbol	Description
Δ	Waveform Difference
δ	Waveform Uncertainty
A	Waveform Amplitude
ϕ	Waveform Phase
Φ	Residual Waveform Phase
μ	Waveform Model
IMR	IMRPhenomPv2_NRTidalv2
EOB	SEOBNRv4T_surrogate
θ	Source Parameters
f	Frequency
f_{COR}	Discontinuity Correction Frequency
T_n	Chebyshev Polynomial of the First Kind
N	Number of Terms
ϕ_0	Overall Phase Shift
t_0	Overall Time Shift
$\mathcal{L}$	Likelihood
h	Gravitational Wave Strain
P	Power Spectral Densities (PSD)
Δf	Frequency Bin Spacing
a	Amplitude Difference Chebyshev Coefficients
b	Phase Difference Chebyshev Coefficients
α_n	Amplitude Uncertainty Spline Parameters
β_n	Phase Uncertainty Spline Parameters
f_n	Frequency Nodes
$\mathcal{N}$	Normal Distribution