

1. The position of a particle as a function of time is given by

$$\vec{r} = (-5 + 2t^2)\hat{i} - (t + 6t^3)\hat{j}$$

where r is in meters and t is in seconds.

- (a) What is the particle's displacement between $t_1 = 2\text{ s}$ and $t_2 = 3\text{ s}$?
 (b) Determine the particle's instantaneous velocity and acceleration as a function of time.
 (c) Evaluate velocity and acceleration at $t = 2\text{ s}$.

a) $\Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$

$$\vec{r}(2) = \underbrace{(-5 + 2(2)^2)}_{3i} \hat{i} - \underbrace{(2 + 6(2)^3)}_{50j} \hat{j}$$

$$\vec{r}(3) = \underbrace{(-5 + 2(3)^2)}_{13i} \hat{i} - \underbrace{(3 + 6(3)^3)}_{165j} \hat{j}$$

$$\Delta \vec{r} = \begin{bmatrix} 13 \\ -165 \end{bmatrix} - \begin{bmatrix} 3 \\ -50 \end{bmatrix} = \begin{bmatrix} 10 \\ -115 \end{bmatrix}$$

$$\Delta \vec{r} = 10\hat{i} - 115\hat{j}$$

b) $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$

$$\vec{v}(t) = \left[\frac{d}{dt}(-5 + 2t^2) \right] \hat{i} - \left[\frac{d}{dt}(t + 6t^3) \right] \hat{j} = \begin{bmatrix} 4t \\ -18t^2 - 1 \end{bmatrix}$$

$$\vec{v}(t) = 4t - 18t^2 - 1 \text{ m/s}$$

c) $\vec{v}(2) = \underbrace{4(2)}_8 \hat{i} - \underbrace{18(2)^2 - 1}_{72 - 1} \hat{j} \text{ m/s}$

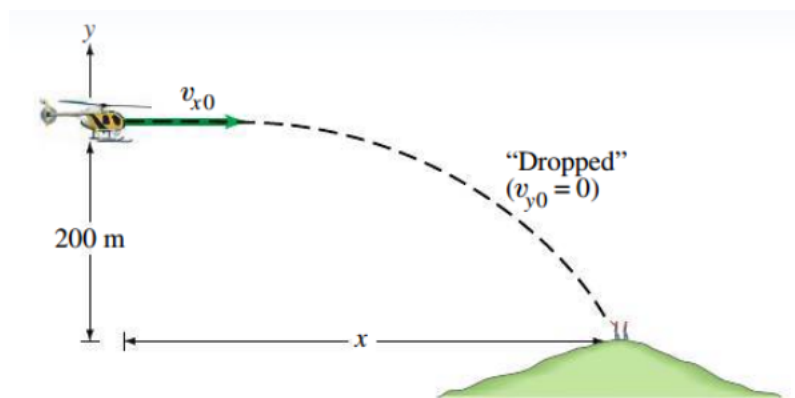
$$\vec{v}(2) = 8\hat{i} - 73\hat{j} \text{ m/s}$$

$$\vec{a}(t) = \left[\frac{d}{dt}(4t) \right] \hat{i} - \left[\frac{d}{dt}(-18t^2 - 1) \right] \hat{j} = \begin{bmatrix} 4 \\ -36t \end{bmatrix}$$

$$\vec{a}(2) = 4\hat{i} - 36(2)\hat{j} =$$

$$\vec{a}(2) = 4\hat{i} - 72\hat{j} \text{ m/s}^2$$

2. A rescue helicopter wants to drop a package of supplies to isolated mountain climbers on a rocky range 200 m below. If the helicopter is traveling horizontally with a speed of 70 m/s, (a) How many seconds before the plane is directly overhead should the supplies be dropped? (b) How far in advance of the recipients (horizontal distance) must the package be dropped?



$$a) y = v_{oy}t + \frac{1}{2}gt^2 \quad y = 200\text{ m} \quad v_{oy} = 0 \quad g = 9.8\text{ m/s}^2$$

$$200 = \frac{1}{2}(9.8)t^2$$

$$\frac{200}{4.9} = \frac{4.9t^2}{4.9} \quad \sqrt{t^2} = \sqrt{40.81}$$

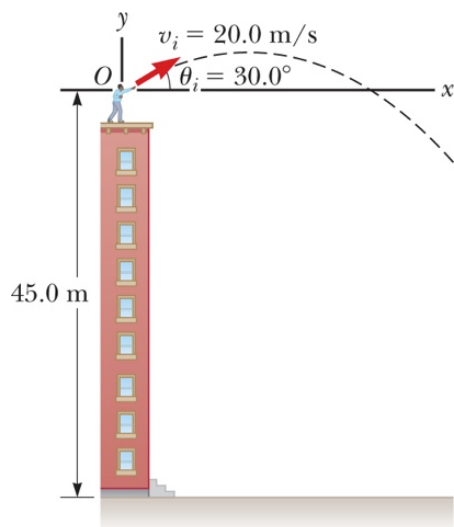
$$t = 6.39\text{ s}$$

$$b) x = v_x \cdot t$$

$$x = 70\text{ m/s} \cdot 6.39\text{ s}$$

$$x = 447.21\text{ m}$$

3. A person throws a stone on top of a 45.0 m high building with the angle 30° up from the horizontal direction with the velocity 20.0 m/s. A) How long does it take the stone to reach the ground? B) How far away from the bottom of the building will the stone hit? (C) What is the speed of the stone just before it strikes the ground?



$$V_{iy} = V \sin(\theta)$$

$$V_{ix} = V \cos(\theta)$$

a) $V_{iy} = 20 \text{ m/s} \cdot \sin(30^\circ) = 10 \text{ m/s}$

$$0 = 45 + 10t - \frac{1}{2}(9.8)t^2$$

$$0 = 45 + 10t - 4.9t^2$$

$$-4.9t^2 + 10t + 45 = 0$$

$$t = \frac{-10 \pm \sqrt{(10)^2 - 4(-4.9)(45)}}{-2(-4.9)} \rightarrow t = \frac{-10 \pm \sqrt{982}}{-9.8}$$

$$t = 4.22 \text{ s}$$

b) $x = v_{ix} \cdot t$

$$v_{ix} = 20 \cos(30^\circ) = 17.32 \text{ m/s}$$

$$x = 17.32 \text{ m/s} \cdot 4.22 \text{ s}$$

$$x = 73.06 \text{ m}$$

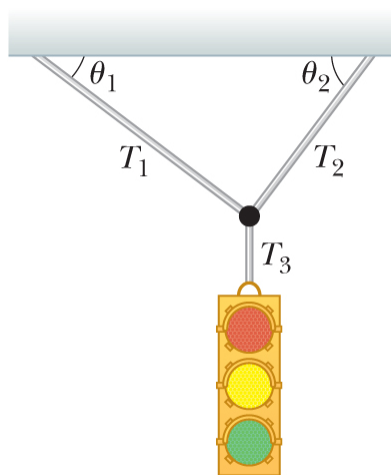
c) $V_f = \sqrt{V_x^2 + V_y^2}$

$$V_x = 17.32 \quad V_y = 10 - 9.8t = -31.36$$

$$\sqrt{(17.32)^2 + (-31.36)^2}$$

$$V_f = 35.81 \text{ m/s}$$

4. A 20 kg traffic light hangs from a cable tied to two cables fastened to a support as the figure below with $\theta_1 = 37^\circ$ and $\theta_2 = 53^\circ$. Upper cables will break if the tension in them exceeds 150 N. Will any of the cables break?



$$\theta_1 = 0.6458 \text{ rad} \quad \theta_2 = 0.925 \text{ rad}$$

$$T_3 = m \cdot g = 20 \cdot 9.8 = 196 \text{ N}$$

$$T_1 \cos(\theta_1) = T_2 \cos(\theta_2)$$

$$T_2 = T_1 \cdot \frac{\cos(\theta_1)}{\cos(\theta_2)}$$

$$T_2 = T_1 \cdot \frac{\cos(37^\circ)}{\cos(53^\circ)} = T_1 \cdot \frac{0.7986}{0.6018} = T_1 = 1.327$$

$$T_1 \sin(\theta_1) + T_2 \sin(\theta_2) = T_3$$

$$T_1 (0.6018 + 1.327 \cdot 0.7986) = 196$$

$$T_1 = \frac{196}{1.6618} = 117.96$$

$$T_2 = 117.96 \cdot 1.327 = 156.53$$