Non-Autonomous Ricker's Map in $\mathbb{R}^n$

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Using our results (O. Merino, M.R.S. Kulenovic) we can prove facts about the more robust **non-autonomous model**:

$$(x_{n+1}, y_{n+1}) = F_n(x_n, y_n) = (x_n e^{\rho_n - x_n - \alpha_n y_n}, y_n e^{s_n - \beta_n x_n - y_n})$$

where $\rho_n \to \rho$, $s_n \to s$, $\alpha_n \to \alpha$, $\beta_n \to \beta$.

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10. $\mathbb{1}$ denotes the vector $(1, 1, \dots, 1)$
11. A **Kolmolgorov Map** is a map of the form $F(x) = \mathrm{diag}(x)\, G(x)$ where $G \colon \mathbb{R}^n_+ \to \mathrm{int}\, \mathbb{R}^n_+$ is continuous.

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Theorem 2

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note: In 2 dimensions, the set Z°_ℓ is either $\mathbb{X}^\circ$ or $\mathbb{Y}^\circ$.

2-Dimensional Ricker's Map

The map associated with the 2 dimensional Ricker's model is

$$F(x,y) = \left(xe^{\rho - x - \alpha y}, ye^{s - \beta x - y}\right)$$

which can be written as $F(\mathbf{x}) = diag(\mathbf{x})G(\mathbf{x})$ where $\mathbf{x} = (x, y)$ and $G(x,y) = \left(e^{\rho - x - \alpha y}, e^{s - \beta x - y}\right)$.

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2. F is bounded: $F(\mathbb{R}^2_+) \subset [o, (e^{\rho - 1}, e^{s-1})]$
3. (Baigent et al.) If $0 < \rho, s < 2$ and $\alpha < \frac{\rho}{s} < \frac{1}{\beta}$ then the interior equilibrium exists and is globally asymptotically stable, while if either $\alpha < \frac{\rho}{s}$ and $\frac{\rho}{s} > \frac{1}{\beta}$ or $\alpha > \frac{\rho}{s}$ and $\frac{\rho}{s} < \frac{1}{\beta}$, one of the axial points is globally attracting.

Proposition 2

Assume $0 < \rho, s < 1$ and $G_t \rightrightarrows G$. Then the following statements are true for the equation $x_{t+1} = F_t(x_t)$ with $F_t(\mathbf{x}) = \mathbf{x}G_t(\mathbf{x})$

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2. $\alpha < \frac{\rho}{s} > \frac{1}{\beta}$ and $x_0 \in \text{int}(\mathbb{R}^2_+) \cup \mathbb{X}^\circ \implies x_t \to (\rho, 0)$

Proposition 2

Assume $0 < \rho, s < 1$ and $G_t \rightrightarrows G$. Then the following statements are true for the equation $x_{t+1} = F_t(x_t)$ with $F_t(\mathbf{x}) = \mathbf{x}G_t(\mathbf{x})$

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2. $\alpha < \frac{\rho}{s} > \frac{1}{\beta}$ and $x_0 \in \text{int}(\mathbb{R}^2_+) \cup \mathbb{X}^\circ \implies x_t \to (\rho, 0)$
3. $\alpha > \frac{\rho}{s} < \frac{1}{\beta}$ and $x_0 \in \text{int}(\mathbb{R}^2_+) \cup \mathbb{Y}^\circ \implies x_t \to (0, s)$

Proof of Proposition 2

Proof of Proposition 2

Claim 1

Let $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ defined by $\phi(v) = ve^{c-v}$, where $0 < c < 1$ is fixed. Then $\forall 0 < \varepsilon < 1 - c, \exists 0 < \varepsilon' < \varepsilon$ and $\tau = \tau(\varepsilon) \in \mathbb{N}$ such that $f^{\circ\tau}(\mathbb{R}_+) \subset [0, c + \varepsilon]$ and $f([0, c + \varepsilon]) \subset [0, c + \varepsilon']$.

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Claim 2

$\forall \varepsilon > 0$ s.t. $p(\varepsilon) := (\rho + \varepsilon, s + \varepsilon) \ll 1, \exists \varepsilon' \in (0, \varepsilon)$ and $\tau = \tau(\varepsilon) \in \mathbb{N}$ s.t. $F^{\circ\tau}(\mathbb{R}^2_+) \subset [o, p(\varepsilon)]$ and $F([o, p(\varepsilon)]) \subset [o, p(\varepsilon')]$

Proof of Proposition 2

Proof.

To prove part (2) of Proposition 2, assume $\alpha < \frac{\rho}{s} > \frac{1}{\beta}$. Since $s < \frac{\rho}{\alpha}$, there exists $\varepsilon > 0$ such that and $s + \varepsilon < \frac{\rho}{\alpha}$. For this choice of ε, the conditions for theorem 2 have been met:

1. **$G_t \rightrightarrows G$ on $[o, r]$.** $\checkmark$

Proof of Proposition 2

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1. $G_t \rightrightarrows G$ **on** $[o, r]$**.** ✓
2. $\bar{x} = (\rho, 0)$ **is a global attractor for** F **relative to** $Z_j^\circ \cup$ **int**$([o, r])$**.** ✓

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1. **$G_t \rightrightarrows G$ on $[o, r]$.** ✓
2. **$\bar{x} = (\rho, 0)$ is a global attractor for F relative to $Z_j^\circ \cup \mathrm{int}([o, r])$.** ✓
3. $\gamma > 1$ **where** $\gamma_i = \min\{G_i(x) : x \in [o, r] \cap Z_i\}$ **and** $\gamma = \min\{\gamma_i : i \neq j\}$ needs to be proved

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4. $\exists r' \in \mathbb{R}^n$ **s.t.** $o \ll r' \ll r$ **and** $F([o, r]) \subset [o, r']$ ✓

Proof of Proposition 2

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To prove part (2) of Proposition 2, assume $\alpha < \frac{\rho}{s} > \frac{1}{\beta}$. Since $s < \frac{\rho}{\alpha}$, there exists $\varepsilon > 0$ such that and $s + \varepsilon < \frac{\rho}{\alpha}$. For this choice of ε, the conditions for theorem 2 have been met:

1. $G_t \rightrightarrows G$ **on** $[o, r]$. ✓
2. $\bar{x} = (\rho, 0)$ **is a global attractor for** F **relative to** $Z_j^\circ \cup$ **int**$([o, r])$. ✓
3. $\gamma > 1$ **where** $\gamma_i = \min\{G_i(x) : x \in [o, r] \cap Z_i\}$ **and** $\gamma = \min\{\gamma_i : i \neq j\}$ needs to be proved
4. $\exists r' \in \mathbb{R}^n$ **s.t.** $o \ll r' \ll r$ **and** $F([o, r]) \subset [o, r']$ ✓
5. $F_t([o, r]) \subset [0, r]$ ✓

$\square$

Proof of Part 3

To prove the third condition, notice $\min\{\gamma_i : i \neq j\}$ has to be γ_2 since $i = 1, 2$ and $j = 1$. Hence, since $s + \varepsilon < \frac{r}{\alpha}$, we have

$$\gamma = \gamma_2 = \min\{e^{r-\alpha y} : y \in [o, p(\varepsilon)] \cap \mathbb{Y}^\circ\} > e^{r-\alpha(s+\varepsilon)} > 1$$

This completes the proof of part (2) of proposition 2.

Figure 1

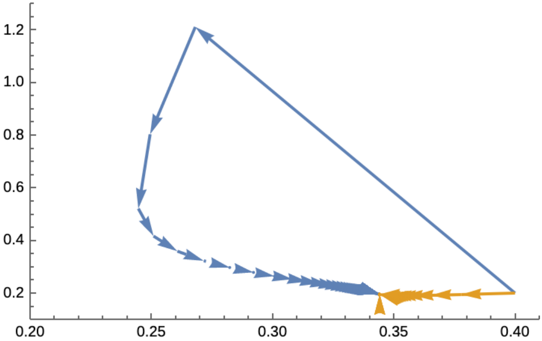

Figure: Ricker's map with $\alpha = 2/7$, $\beta = 3/7$, $\rho = 2/5$, and $s = 12/35$. Rickers non-autonomous map with $\alpha_n = \frac{2n^2}{7n^2+20}$, $\beta_n = \frac{3n^8}{7n^8+2}$, $\rho_n = \frac{2n^2}{5n^2+2}$, and $s_n = \frac{12n^3+18n+2}{35n^3+1}$

Figure 2

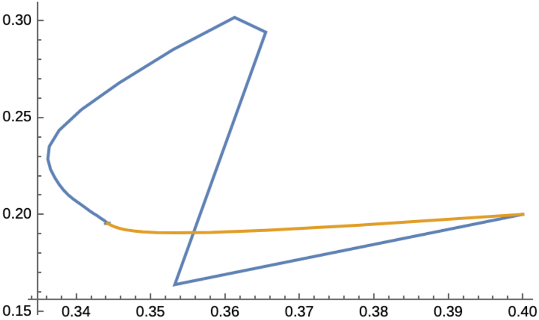

Figure: Ricker's map with $\alpha = 2/7$, $\beta = 3/7$, $\rho = 2/5$, and $s = 12/35$. Rickers non-autonomous map with $\alpha_n = \frac{2n^2+\cos(n)}{7n^2+20}$, $\beta_n = \frac{3n^8}{7n^8+2\cos(n)}$, $\rho_n = \frac{2e^{n^2}+\sin(n)}{5e^{n^2}+2}$, and $s_n = \frac{12n^3+18n+2\sin(n)}{35n^3+1}$

Figure 2

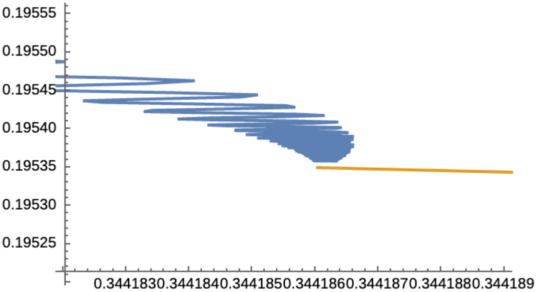

Figure: Figure 2 zoomed in on the positive equilibrium point.