MATH 3002: PROBLEM SET 1

Exercise 1. *Just to practice some integrals, compute the following:*

(1) $\int x\cos(x)dx$
(2) $\int (2x+1)^2 dx$
(3) $\int \arctan(x)dx$

Exercise 2. *Compute*

$$\int xe^x dx,$$

(hint: integrate by parts) then compute

$$\int x^2 e^x dx$$

Can you write a general formula for

$$\int x^n e^x$$

?

Exercise 3. *We've mentioned in class a couple of times that the indefinite integral(/antiderivative) $\int e^{x^2}$ cannot be written in terms of simpler function. Let $E(x) = \int_0^x e^{t^2} dt$, so $E(x)$ is the antiderivative of e^{x^2} such that $E(0) = 0$.*

Find

$$\int xe^{x^2}$$

in terms of $E(x)$, and then

$$\int x^2 e^{x^2}$$

Would you want to write a general form for

$$\int x^n e^{x^2} dx$$

?

Assuming you said no, what is different between this question and the last question? If you said yes, write out a general form.

Exercise 4. *Show all work (i.e., do not just cite a formula) (integrate when possible):*

(1) *Find a function $y(x)$ such that $y' = x^2 y$ and $y(1) = 3$.*
(2) *Find a function $y(x)$ such that $x \cdot y' + y = 0$ and $y(1) = 2$. Show the solution is unique, or show it is not.*

Exercise 5. *Recall that we solved any first order linear homogeneous differential equation by noticing the derivative relation $\frac{d}{dx} \log(y) = \frac{y'}{y}$*

Compute $\frac{d}{dx}\left(y^2\right)$
Use this to solve the equation

$$y \cdot y' - x^3 = 0$$

Warning: here by 'solve' I mean find the general form of solutions. Make a remark about the constant of integration.