1 First Concepts

Definition 1. *A **function** is a rule assigning a unique output (also called value) to a given input (also called point) from some domain of allowed inputs.*

Definition 2. *The **domain** of a function is the set of all inputs for which the function is defined. Sometimes we write* $\mathrm{dom}(f)$ *for the domain of a function* f*.*

Definition 3. *The **range** of a function is the set of all possible outputs which the functions gives. Sometimes we write* $\mathrm{ran}(f)$ *for the range of a function* f*.*

Definition 4. *The **composition** of two functions* f *and* g*, written* $f \circ g$*, is the function given by*

$$f \circ g(x) = f(g(x))$$

That is, for a given input x*, first apply the function* g *to get an output* $g(x)$*. Then use this as the input for the function* f*, to get the output* $f(g(x))$*.*

Definition 5. *The **inverse** of a function* f*, written as* f^{-1}*, is a function so that*

$$f \circ f^{-1}(x) = x,$$

WARNING!: One needs to be careful about the domains here.

Definition 6. *The **graph** of a function* f *is the collection of all ordered pairs* $(x, f(x))$*, where* x *is in the domain of* f*, plotted on a Cartesian coordinate system.*

2 Kinds of Functions

Definition 7. *A **polynomial** is a function of the form*

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

where $a_0, a_1, ..., a_n$ *are some real numbers (called 'coefficients').*

If $a_n = 0$*, we could ignore it and add up fewer things. The largest natural number* n *with* $a_n \neq 0$ *is called the **degree** of the polynomial.*

For instance

$$p(x) = 3x^4 - 2x^2 + \sqrt{7}x + 7$$

is a polynomial of degree 4.

Definition 8. *A polynomial of degree* 1 *is called a **linear function**.*

Definition 9. *A polynomial of degree* 2 *is called **quadratic**, a polynomial of degree* 3 *is called **cubic**, a polynomial of degree* 4 *is called **quartic**, ...*

Definition 10. *A **power** function is a function of the form*

$$f(X) = x^a$$

where a is some real number.

*If $a = -1$, this is called the **reciprocal** function, which has domain all non-zero numbers, and is given by the formula*

$$f(x) = x^{-1} = \frac{1}{x}$$

If a is negative, say $a = -3$, we take the reciprocal of the positive version:

$$f(X) = x^{-3} = \frac{1}{x^3}$$

Definition 11. *A **rational** function is a function of the form one polynomial divided by another:*

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + ... + b_1 x + b_0}$$

Definition 12. *An **algebraic** function is a function which can be built (by adding, multiplying, dividing, and composing) from polynomials and fractional power functions (x^a with a a fraction).*

Functions which are not algebraic are called 'transcendental'.

Definition 13. *An **exponential** function is a function of the form*

$$f(x) = b^x$$

*The number b is called the **base** of the exponential.*

Definition 14. *A **logarithmic** function is an inverse of an exponential function. We write*

$$g(x) = \log_b(x)$$

for the inverse of $f(x) = b^x$.

Definition 15. *A **trigonometric** function is any of the following six functions:* $\sin(x)$, $\cos(x)$, $\tan(x)$, $\csc(x)$, $\sec(x)$, *and* $\cot(x)$ *(in order, these are 'sine', 'cosine', 'tangent', 'cosecant', 'secant', and 'cotangent').*

Recall these are related by $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\csc(x) = \frac{1}{\sin(x)}$, $\sec(x) = \frac{1}{\cos(x)}$, *and* $\cot(x) = \frac{1}{\tan(x)}$