Math 3002: Problem Set 11

1. Solve the following differential equations using the Laplace transform:

   (a)
   $$\begin{cases} y' + y = \sin(t) \\ y(0) = 1 \end{cases}$$

   (b) (Hint: compute the Laplace transform of $\sin(t) - t\cos(t)$.)
   $$\begin{cases} y'' + y = \sin(t) \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$$

   (c)
   $$\begin{cases} y^{(4)} - y = 0 \\ y(0) = 1 \\ y'(0) = 0 \\ y''(0) = 0 \\ y^{(3)} = 0 \end{cases}$$

   (d)
   $$\begin{cases} y'' - 5y' + 4y = e^{-5t} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

2. Solve the following system of differential equations by taking the Laplace transform of each, then solving the resulting system of algebraic equations.

   $$\begin{aligned} x'(t) &= 2x(t) + y(t) \\ y'(t) &= 2x(t) + 3y(t) \end{aligned}$$

   subject to the initial conditions $x(0) = 1$ and $y(0) = 1$.