

1. We discuss the derivative of a product.

- (a) Recall the product rule: if we have a function F which is the product of two functions f and g , so $F(x) = f(x) \cdot g(x)$, then the derivative is given by the formula:

- (b) Suppose now P is the product of three different functions,

$$P(x) = f(x) \cdot g(x) \cdot k(x)$$

Using the function $F(x)$, rewrite $P(x)$ to be the product of two functions, and apply the product rule to get a formula for $P'(x)$ in terms of $F(x)$ and $k(x)$. Use this and part (a) to give a formula for $P'(x)$ in terms of $f(x)$, $g(x)$, $k(x)$, $f'(x)$, $g'(x)$, and $k'(x)$.

- (c) If we define $G(x) = g(x) \cdot k(x)$, how can we rewrite $P(x)$ as a product of two functions in a different way? What does the product rule say in this case? Does this match with your answer in part (b)?

2. Compute the following derivatives:

(a) If $q(x) = x^3 \cdot \sin(x) \cdot \ln(x)$, then $q'(x) =$

(b) If $k(t) = 2^t \cdot \ln(t) \cdot \tan(t)$, then $k'(t) =$

3. Here we give a proof of the quotient rule, using the power rule, the product rule, and the chain rule.

(a) Suppose that $b(x) = \frac{1}{k(x)}$. Rewrite this using negative exponents, then use the chain rule to compute $b'(x)$ in terms of $k(x)$ and $k'(x)$.

(b) Suppose we have a quotient $f(x) = \frac{g(x)}{k(x)}$. Rewrite this as a product, using $b(x)$ from the previous part, and use the product rule to get a formula for $f'(x)$ in terms of $g(x)$, $g'(x)$, $k(x)$, and $k'(x)$.

4. Compute the following derivatives:

(a) If $c(t) = \frac{t \cdot \cos(t)}{t^2 + 1}$, then $c'(t) =$

(b) If $R(x) = \frac{\ln(x) - \cos(x)}{e^x}$, then $R'(x) =$