Math 3002: Problem Set 4

1. Check that the following differential equations are exact, and solve them.

   (a)
   $$\frac{dy}{dx} = \frac{2xe^y + \sin(x)y^2}{2\cos(x)y - x^2e^y}$$

   (b)
   $$y' = \frac{y^2 - 3x^2}{4y^3 - 2xy}$$

   (c)
   $$\frac{dy}{dx} = -\frac{2x\sin(y) - \sin(x)}{x^2\cos(y)}$$

   (d)
   $$y' = -\frac{x^2 + y^2}{2y(x + y)}$$

   (e) Graph some of the solutions. Are any of them interesting?

2. Check that these differential equations are not exact, but can be made exact by multiplying by the given integrating factor.

   (a)
   $$3xy - y^2 + (x^2 - xy)\frac{dy}{dx} = 0$$
   Integrating factor $\mu(x, y) = x$

   (b)
   $$6xy + 5(x^2 + y)\frac{dy}{dx} = 0$$
   Integrating factor $\mu(x, y) = y^{\frac{2}{3}}$

3. Consider the differential equation
   $$\frac{dy}{dx} = \frac{y}{x}$$

   (a) Solve this differential equation.

   (b) Rewrite this equation as $-y + x\frac{dy}{dx} = 0$, and multiply everything by $\frac{1}{x^2+y^2}$. Is the resulting equation exact?

   (c) Consider the path $\gamma : [0, 2\pi] \to \mathbb{R}^2$ given by
   $$\gamma(t) = (\cos(t), \sin(t)),$$
   then integrate the vector field $(M(x, y), N(x, y))$, where M and N are the coefficients from the previous part.

   (d) Is there any contradiction between the previous parts and what we discussed in class?