Math 3002: Problem Set 7

1. Find all eigenvectors for the following matrices:

   (a)
   $$\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 2 & 4 \\ 5 & 1 \end{pmatrix}$$

   (c) How is this one different?
   $$\begin{pmatrix} 5 & 1 \\ -1 & 3 \end{pmatrix}$$

   (d)
   $$\begin{pmatrix} 3 & 5 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

   (e) (Compare to part (c))
   $$\begin{pmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

2. Show that for a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant $ad - bc$ is the product of the eigenvalues. (Hint: write the characteristic polynomial of this matrix, then write an expression for a quadratic with roots p and q). Does this argument work for $n \times n$ matrices?

3. Use row-reduction to find an inverse for the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ -2 & -3 & -1 \end{pmatrix}$$

(Hint: you can either do the row reductions and then interpret each row reduction as a matrix multiplication, or you can do it all at once by row reducing the 'augmented matrix' $\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ -2 & -3 & -1 & 0 & 0 & 1 \end{pmatrix}$ so that the first three columns are the identity, then the last three columns will be the inverse). Check that your answer is really the inverse.