MATH 3002 - Practice Exam 1

Name: _____

Instructions: Write cleanly, show all work. Explain any trick questions.

1. Find the general solution to the following differential equations:

   (a)

   $$2xy' + y = 0$$

   (b)

   $$\cos(x)y' - \sin(x)y = \cos(x)$$

   (c)

   $$y' = 3xy - 3x^2$$

2. Solve the initial value problem

$$\begin{cases} \frac{dx}{dt} = 1 + x^2 \\ x(0) = 1 \end{cases}$$

What is the maximum domain where this solution is defined and continuous?

3. Find the general form of the solution to

$$\frac{dy}{dx} = -\frac{1 + (xy + 1)e^{xy}}{x^2 e^{xy}}$$

Write the solution so that y is a function of x. What is the domain of this function (this will depend on a parameter)?

4. Solve the following initial value problems

(a)

$$\begin{cases} y'' + 4y' - 12y = 0 \\ y(0) = 1 \\ y'(0) = -2 \end{cases}$$

(b)

$$\begin{cases} y'' + 4y' + 4y = 0 \\ y(0) = 3 \\ y'(0) = 0 \end{cases}$$

(c)

$$\begin{cases} y'' - 2y' + 3y = 0 \\ y(0) = 1 \\ y'(0) = \sqrt{2} \end{cases}$$