Math 3002: Problem Set 5

1. Find a function which satisfies the given conditions.

   (a)
   $$\begin{cases} 2y'' - 18y' + 36y = 0 \\ y(0) = 2 \\ y'(0) = 3 \end{cases}$$

   (b)
   $$\begin{cases} u'' - 6u = 0 \\ u(0) = 0 \\ u'(0) = \sqrt{2} \end{cases}$$

   (c)
   $$\begin{cases} 3f'' - 12f' + 12f = 0 \\ f(0) = -1 \\ f'(0) = 0 \end{cases}$$

   (d)
   $$\begin{cases} 8y'' + 18y = 0 \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$$

2. Given some real number k, solve the equation

$$y'' + k^2 y = 0$$

Suppose we want solutions with $y(0) = y(1) = 0$. For which k do such solutions exist? Using the solutions you found (for different k), write function $f(x)$ such that

- $f(0) = f(\pi) = 0$
- $f(x) > 0$ for all $x \in (0, \pi)$
- $f(x)$ is 'almost constant' between $\pi/3$ and $2\pi/3$

3. Find all solutions to

$$y''' - 3y'' - 6y' + 8y = 0$$

Check that your purported solutions are actually solutions (you do not need to prove that you have found *all* solutions).