Functions and Graphs

1. Let $f(x) = \frac{1}{\sqrt{x-8}}$

   (a) Use the limit definition of the derivative to find $f'(x)$.

   (b) Use the idea of linear approximation to estimate $f(9.2)$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{x}{x^2 - 4}$$

   Find all the significant features (domain and range, any intercepts, end behavior/limits at infinity, asymptotes, intervals of increasing or decreasing, any inflection points, intervals of upwards and downwards concavity), and use this information to sketch a graph of f.

Related Rates

3. Two cars start moving from the same point. One travels north at 25 mi/h, and the other travels west at 60 mi/h. After two hours, how quickly is the distance between the cars increasing?

4. Gas is escaping from a spherical balloon at a rate of 2 ft^3/min. How fast is the surface area changing when the radius is 12 ft?

5. Oil is leaking from a tanker in the Gulf of Mexico. The oil spreads outward in a circular film on the water. If the area of the spill increases by 1200π m^2/min, how quickly is the radius of the spill changing when the total area is $40,000\pi$ m^2?

Optimization

6. Find the point on the curve
$$y = 2 + \sqrt{x+3}$$
which is nearest to the point $(4, 2)$.

7. We are making boxes. We have decided each box has a square base, and we will use $1200cm^2$ of cardboard to construct each box. What dimensions will give the largest volume of box?

8. Find the area of the largest shaded rectangle that can be inscribed in a semicircle of radius 6 (see figure below). **Hint:** The graph of the semicircle is $y = \sqrt{36 - x^2}$, so a vertex of the rectangle has coordinates $(x, \sqrt{36 - x^2})$.