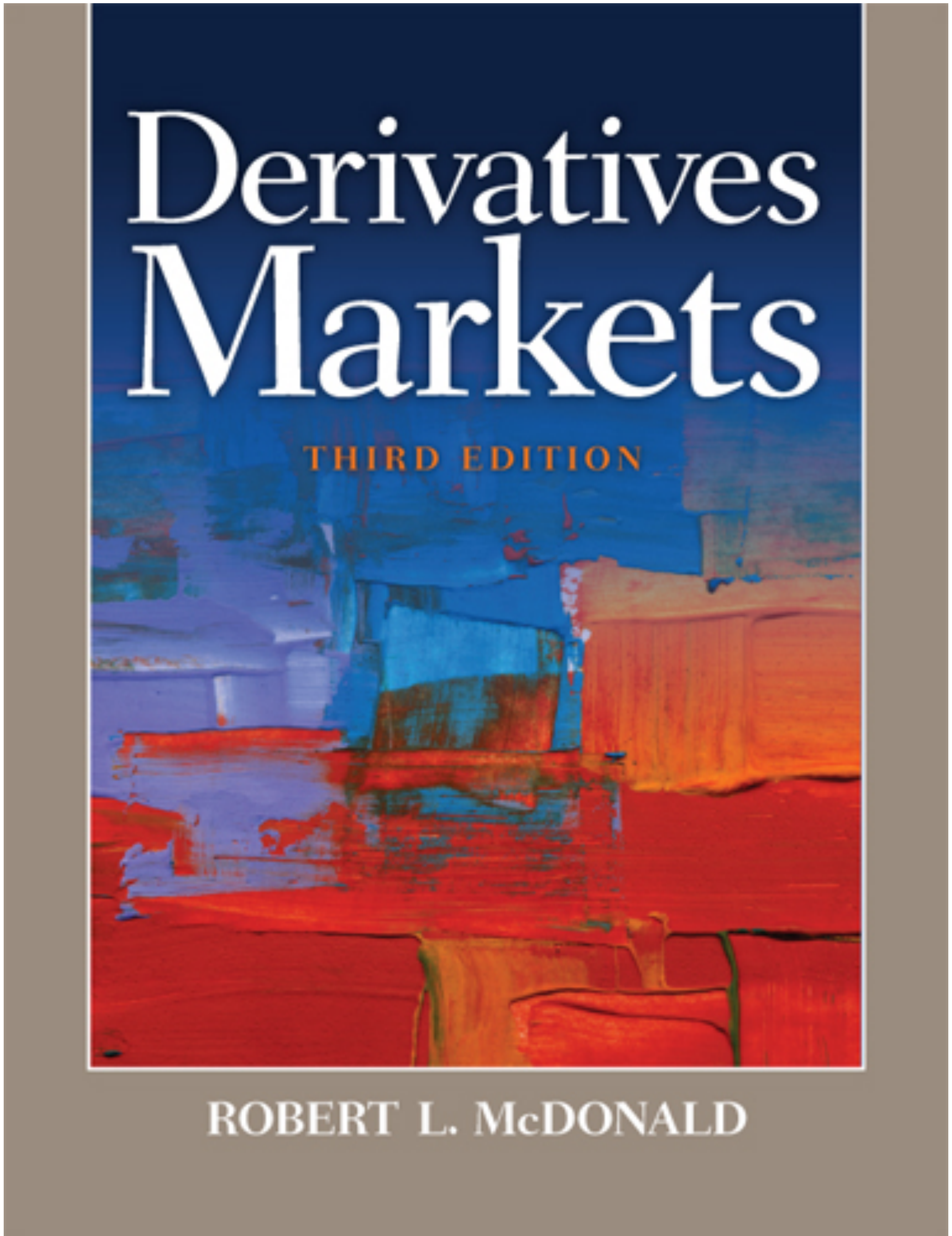


Chapter 5

Financial Forwards and Futures

Introduction

- Financial futures and forwards
  - On stocks and indexes
  - On currencies
  - On interest rates
- How are they used?
- How are they priced?
- How are they hedged?

Alternative Ways to Buy a Stock

- Four different payment and receipt timing combinations
  - Outright purchase: ordinary transaction
  - Fully leveraged purchase: investor borrows the full amount
  - Prepaid forward contract: pay today, receive the share later
  - Forward contract: agree on price now, pay/receive later
- Payments, receipts, and their timing

TABLE 5.1 Four different ways to buy a share of stock that has price S_0 at time 0. At time 0 you agree to a price, which is paid either today or at time T. The shares are received either at 0 or T. The interest rate is r.

Description	Pay at Time	Receive Security at Time	Payment
Outright purchase	0	0	S_0 at time 0
Fully leveraged purchase	T	0	S_0e^{rT} at time T
Prepaid forward contract	0	T	?
Forward contract	T	T	$? \times e^{rT}$

Pricing Prepaid Forwards

- If we can price the *prepaid* forward (F^P), then we can calculate the price for a forward contract

$$F = \text{Future value of } F^P$$

- Three possible methods to price prepaid forwards
  - Pricing by analogy
  - Pricing by discounted present value
  - Pricing by arbitrage
- For now, assume that there are no dividends

Pricing Prepaid Forwards (cont'd)

- Pricing by analogy
  - In the absence of dividends, the timing of delivery is irrelevant
  - Price of the prepaid forward contract same as current stock price
  - $F^P_{0,T} = S_0$ (where the asset is bought at $t = 0$, delivered at $t = T$)
- Pricing by discounted preset value
  (α: risk-adjusted discount rate)
  - If expected t=T stock price at t=0 is $E_0(S_T)$, then $F^P_{0,T} = E_0(S_T)e^{-\alpha T}$
  - Since t=0 expected value of price at t=T is $E_0(S_T) = S_0 e^{\alpha T}$
  - Combining the two, $F^P_{0,T} = S_0 e^{\alpha T} = S_0$

Pricing Prepaid Forwards (cont'd)

- Pricing by arbitrage
  - Arbitrage: a situation in which one can generate positive cash flow by simultaneously buying and selling related assets, with no net investment and with no risk → free money!!!
  - If at time t=0, the prepaid forward price somehow exceeded the stock price, i.e., $F^P_{0,T} > S_0$, an arbitrageur could do the following

TABLE 5.2 Cash flows and transactions to undertake arbitrage when the prepaid forward price, $F^P_{0,T}$, exceeds the stock price, S_0.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy stock @ S_0	$-S_0$	$+S_T$
Sell prepaid forward @ $F^P_{0,T}$	$+F^P_{0,T}$	$-S_T$
Total	$F^P_{0,T} - S_0$	0

  - Since, this sort of arbitrage profits are traded away quickly, and cannot persist, at equilibrium we can expect: $F^P_{0,T} = S_0$

Pricing Prepaid Forwards (cont'd)

- What if there are dividends? Is $F^P_{0,T} = S_0$ still valid?
  - No, because the holder of the forward will not receive dividends that will be paid to the holder of the stock ➔ $F^P_{0,T} > S_0$

$$F^P_{0,T} = S_0 - PV \text{ (all dividends paid from t=0 to t=T)}$$

  - For discrete dividends D_{t_i} at times t_i, $i = 1, \ldots, n$
    - The prepaid forward price: $F^P_{0,T} = S_0 - \Sigma^n_{i=1} PV_{0,t_i}(D_{t_i})$
    - For continuous dividends with an annualized yield δ, the prepaid forward price is $F^P_{0,T} = S_0 e^{-\delta T}$

Pricing Prepaid Forwards (cont'd)

- Example 5.1
  - XYZ stock costs $100 today and is expected to pay a quarterly dividend of $1.25. If the risk-free rate is 10% compounded continuously, how much does a 1-year prepaid forward cost?
  - $F^{p}_{0,1} = \$100 - \Sigma^{4}_{i=1} \$1.25 e^{-0.025 i} = \$95.30$

Pricing Prepaid Forwards (cont'd)

- Example 5.2
  - The index is \$125 and the dividend yield is 3% continuously compounded. How much does a 1-year prepaid forward cost?
  - $F^{P}_{0,1} = \$125e^{-0.03} = \$121.31$

Pricing Forwards on Stock

- Forward price is the future value of the *prepaid* forward price
  - No dividends

$$F_{0,T} = FV(F^{P}_{0,T}) = FV(S_0) = S_0 e^{rT}$$

  - Continuous dividends

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Pricing Forwards on Stock (cont'd)

- Forward premium
  - The difference between current forward price and stock price
  - Can be used to infer the current stock price from forward price
  - Definition
    - Forward premium = $F_{0,T} / S_0$
    - Annualized forward premium = $(1/T)\, ln\,(F_{0,T} / S_0)$

Creating a *Synthetic* Forward

- One can offset the risk of a forward by creating a *synthetic* forward to offset a position in the actual forward contract
- How can one do this? (assume continuous dividends at rate δ)
  - Recall the long forward payoff at expiration: $= S_T - F_{0,\,T}$
  - Borrow and purchase shares as follows

TABLE 5.3 Demonstration that borrowing $S_0e^{-\delta T}$ to buy $e^{-\delta T}$ shares of the index replicates the payoff to a forward contract, $S_T - F_{0,T}$.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0e^{-\delta T}$	$+\ S_T$
Borrow $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-\ S_0e^{(r-\delta)T}$
Total	0	$S_T - S_0e^{(r-\delta)T}$

  - Note that the total payoff at expiration is same as forward payoff

Creating a *Synthetic* Forward (cont’d)

- The idea of creating synthetic forward leads to following
  - Forward = Stock – zero-coupon bond
  - Stock = Forward – zero-coupon bond
  - Zero-coupon bond = Stock – forward
- Cash-and-carry arbitrage: Buy the index, short the forward

TABLE 5.6 Transactions and cash flows for a cash-and-carry: A market-maker is short a forward contract and long a synthetic forward contract.

Transaction	Cash Flows: Time 0	Cash Flows: Time T (expiration)
Buy tailed position in stock, paying $S_0e^{-\delta T}$	$-S_0e^{-\delta T}$	$+S_T$
Borrow $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_0e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0e^{(r-\delta)T}$

Creating a *Synthetic* Forward (cont'd)

- Cash-and-carry arbitrage with transaction costs
  - Trading fees, bid-ask spreads, different borrowing/lending rates, the price effect of trading in large quantities, make arbitrage harder
  - Suppose
    - Bid-ask spreads: for stock $S^b < S^a$, and for forward $F^b < F^a$
    - Cost k of transacting forward
    - Interest rate for borrowing and lending are $r^b < r^l$
    - No dividends and no time T transaction costs for simplicity
  - Arbitrage possible if
    - $F^b > F^+ = (S_0^a + 2k)e^{r^b T}$
    - $F^a < F^- = (S_0^b - 2k)e^{r^l T}$

Other Issues in Forward Pricing

- Does the forward price predict the future price?
  - According the formula $F_{0,T} = S_0 e^{-(r-\delta)T}$ the forward price conveys no additional information beyond what S_0, r, and δ provides
  - Moreover, the forward price underestimates the future stock price

- Forward pricing formula and cost of carry
  - Forward price =

$$\text{Spot price} + \underbrace{\text{Interest to carry the asset} - \text{asset lease rate}}_{\text{Cost of carry, } (r-\delta)S}$$

Futures Contracts

- Exchange-traded “forward contracts”
- Typical features of futures contracts
  - Standardized, with specified delivery dates, locations, procedures
  - A clearinghouse
    - Matches buy and sell orders
    - Keeps track of members’ obligations and payments
    - After matching the trades, becomes counterparty
- Differences from forward contracts
  - Settled daily through the mark-to-market process ➔ low credit risk
  - Highly liquid ➔ easier to offset an existing position
  - Highly standardized structure ➔ harder to customize

Example: S&P 500 Futures

- Notional value: $250 x Index
- Cash-settled contract
- Open interest: total number of buy/sell pairs
- Margin and mark-to-market
  - Initial margin
  - Maintenance margin (70 – 80% of initial margin)
  - Margin call
  - Daily mark-to-market

Example: S&P 500 Futures (cont'd)

- Futures prices versus forward prices
  - The difference negligible especially for short-lived contracts
  - Can be significant for long-lived contracts and/or when interest rates are correlated with the price of the underlying asset

Example: S&P 500 Futures (cont'd)

- Mark-to-market proceeds and margin balance for 8 long futures contracts

TABLE 5.8 Mark-to-market proceeds and margin balance over 10 weeks from long position in 8 S&P 500 futures contracts. The last column does not include additional margin payments. The final row represents expiration of the contract.

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance($)
0	2000.00	1100.00	—	220,000.00
1	2000.00	1027.99	−72.01	76,233.99
2	2000.00	1037.88	9.89	96,102.01
3	2000.00	1073.23	35.35	166,912.96
4	2000.00	1048.78	−24.45	118,205.66
5	2000.00	1090.32	41.54	201,422.13
6	2000.00	1106.94	16.62	234,894.67
7	2000.00	1110.98	4.04	243,245.86
8	2000.00	1024.74	−86.24	71,046.69
9	2000.00	1007.30	−17.44	36,248.72
10	2000.00	1011.65	4.35	44,990.57

Example: Nikkei 225 Futures

- Contract specifications

FIGURE 5.2

Specifications for the Nikkei 225 index futures contract.

Underlying	Nikkei 225 Stock Index
Where traded	Chicago Mercantile Exchange
Size	$5 × Nikkei 225 Index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based upon opening Osaka quotation of the Nikkei 225 index on the second Friday of expiration month

Uses of Index Futures

- Why buy an index futures contract instead of synthesizing it using the stocks in the index? Lower transaction costs
- Asset allocation: switching investments among asset classes
- Example: invested in the S&P 500 index and temporarily wish to temporarily invest in bonds instead of index. What to do?
  - Alternative #1: sell all 500 stocks and invest in bonds
  - Alternative #2: take a short forward position in S&P 500 index

TABLE 5.10 Effect of owning the stock and selling forward, assuming that $S_0 = \$100$ and $F_{0,1} = \$110$.

	Cash Flows		
Transaction	**Today**	**1 year, $S_1 = \$80$**	**1 year, $S_1 = \$130$**
Own stock @ \$100	−\$100	\$80	\$130
Short forward @ \$110	0	\$110 − \$80	\$110 − \$130
Total	−\$100	\$110	\$110

Uses of Index Futures

- Cross-hedging with perfect correlation

TABLE 5.11 Results from shorting 509.09 S&P 500 index futures against a $100m portfolio with a beta of 1.4.

S&P 500 Index	Gain on 509 Futures	Portfolio Value	Total
900	33.855	72.145	106.000
950	27.491	78.509	106.000
1000	21.127	84.873	106.000
1050	14.764	91.236	106.000
1100	8.400	97.600	106.000
1150	2.036	103.964	106.000
1200	−4.327	110.327	106.000

- Cross-hedging with imperfect correlation
- General asset allocation: futures overlay
- Risk management for stock-pickers

Currency Contracts

- Widely used to hedge against changes in exchange rates
- Example

FIGURE 5.3

Specifications for EUR/USD futures contract.

Underlying	Euro currency
Where traded	Chicago Mercantile Exchange
Size	125,000 euro
Months	March, June, September, December (six consecutive contracts)
Trading ends	9:16A.M. on the second business day prior to the third Wednesday of the month
Delivery	Cash-settlement

Currency Contracts: Pricing

- Currency prepaid forward
  - Suppose you want to purchase ¥1 one year from today using $\$x_0$
  - $$F^P_{0,T} = x_0 e^{-r_y T}$$
    - Where x_0 is current (\$/ ¥) exchange rate, and r_y is the yen-denominated interest rate
    - Why? By deferring delivery of the currency one loses interest income from bonds denominated I that currency
- Currency forward
  - $$F_{0,T} = x_0 e^{(r - r_y)T}$$
    - r is the \$-denominated domestic interest rate
    - $F_{0,T} > x_0$ if $r > r_y$ (domestic risk-free rate exceeds foreign risk-free rate)

Currency Contracts: Pricing (cont'd)

- Example 5.4
  - ¥-denominated interest rate is 2% and current (\$/ ¥) exchange rate is 0.009. To have ¥1 in one year one needs to invest today
    - $0.009/¥ \times ¥1 \times e^{-0.02} = \$0.008822$
- Example 5.5
  - ¥-denominated interest rate is 2% and \$-denominated rate is 6%. The current (\$/ ¥) exchange rate is 0.009. The 1-year forward rate is
    - $0.009e^{0.06-0.02} = 0.009367$

Currency Contracts: Pricing (cont'd)

- Synthetic currency forward: borrowing in one currency and lending in another creates the same cash flow as a forward contract
- Covered interest arbitrage: offset the synthetic forward position with an actual forward contract

TABLE 5.13 Synthetically creating a yen forward contract by borrowing in dollars and lending in yen. The payoff at time 1 is ¥1 − \$0.009367.

	Cash Flows			
	Year 0		Year 1	
Transaction	\$	¥	\$	¥
Borrow $x_0 e^{-r_y}$ dollar at 6% (\$)	+0.008822	—	−0.009367	—
Convert to yen @ 0.009 \$/¥	−0.008822	+0.9802	—	—
Invest in yen-denominated bill (¥)	—	−0.9802	—	1
Total	0	0	−0.009367	1

Eurodollar Futures

- Contract Specifications

FIGURE 5.4

Specifications for the Eurodollar futures contract.

Where traded	Chicago Mercantile Exchange
Size	3-month Eurodollar time deposit, $1 million principal
Months	March, June, September, December, out 10 years, plus 2 serial months and spot month
Trading ends	5 A.M. (11 A.M. London) on the second London bank business day immediately preceding the third Wednesday of the contract month.
Delivery	Cash settlement based on 100 − British Banker's Association Futures Interest Settlement Rate for 3-Month Eurodollar Interbank Time Deposits. (This is a 3-month rate annualized by multiplying by 360/90.)

TABLE 5.4 Demonstration that going long a forward contract at the price $F_{0,T} = S_0 e^{(r-\delta)T}$ and lending the present value of the forward price creates a synthetic share of the index at time T.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Long one forward	0	$S_T - F_{0,T}$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_0 e^{(r-\delta)T}$
Total	$-S_0 e^{-\delta T}$	S_T

TABLE 5.5 Demonstration that buying $e^{-\delta T}$ shares of the index and shorting a forward creates a synthetic bond.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Buy $e^{-\delta T}$ units of the index	$-S_0 e^{-\delta T}$	$+S_T$
Short one forward	0	$F_{0,T} - S_T$
Total	$-S_0 e^{-\delta T}$	$F_{0,T}$

TABLE 5.7 Transactions and cash flows for a reverse cash-and-carry: A market-maker is long a forward contract and short a synthetic forward contract.

	Cash Flows	
Transaction	**Time 0**	**Time T (expiration)**
Short tailed position in stock, receiving $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_T$
Lend $S_0e^{-\delta T}$	$-S_0e^{-\delta T}$	$+S_0e^{(r-\delta)T}$
Long forward	0	$S_T - F_{0,T}$
Total	0	$S_0e^{(r-\delta)T} - F_{0,T}$

FIGURE 5.1

Specifications for the S&P 500 index futures contract.

Underlying	S&P 500 index
Where traded	Chicago Mercantile Exchange
Size	$250 × S&P 500 index
Months	March, June, September, December
Trading ends	Business day prior to determination of settlement price
Settlement	Cash-settled, based upon opening price of S&P 500 on third Friday of expiration month

TABLE 5.9 Actual and theoretical S&P 500 futures prices, December 16, 2010. The S&P 500 index closed at 1242.87 on December 16, and the dividend yield was 1.89%. Theoretical prices are computed using equation (5.6).

Maturity	Closing Price	T-bill Yield	Theoretical Price	LIBOR	Theoretical Price
Spot	1242.87				
March 2011	1238.50	0.13%	1237.41	0.30%	1237.94
June 2011	1233.60	0.19%	1232.35	0.46%	1234.02

Rates and dividend yield from Bloomberg.

TABLE 5.12 Dollar cost of foreign currencies.

Date	Currency	June	Sept.	Dec.	Mar.	June	Sept.
June 6, 2007	Euro	1.351	1.355	1.358	1.361	1.363	1.365
	100 Yen	0.828	0.837	0.847	0.856	0.865	0.873
	Sterling	1.992	1.99	1.987	1.984	1.98	1.975
June 2, 2010	Euro	1.224	1.225	1.226	1.228	1.229	1.231
	100 Yen	1.085	1.087	1.089	1.092	1.095	1.098
	Sterling	1.464	1.465	1.465	1.466	1.466	1.467

Source: CMEGroup via Datastream.

TABLE 5.13 Synthetically creating a yen forward contract by borrowing in dollars and lending in yen. The payoff at time 1 is ¥1 − $0.009367.

	Cash Flows			
	Year 0		Year 1	
Transaction	$	¥	$	¥
Borrow $x_0 e^{-r_y}$ dollar at 6% ($)	+0.008822	—	−0.009367	—
Convert to yen @ 0.009 $/¥	−0.008822	+0.9802	—	—
Invest in yen-denominated bill (¥)	—	−0.9802	—	1
Total	0	0	−0.009367	1

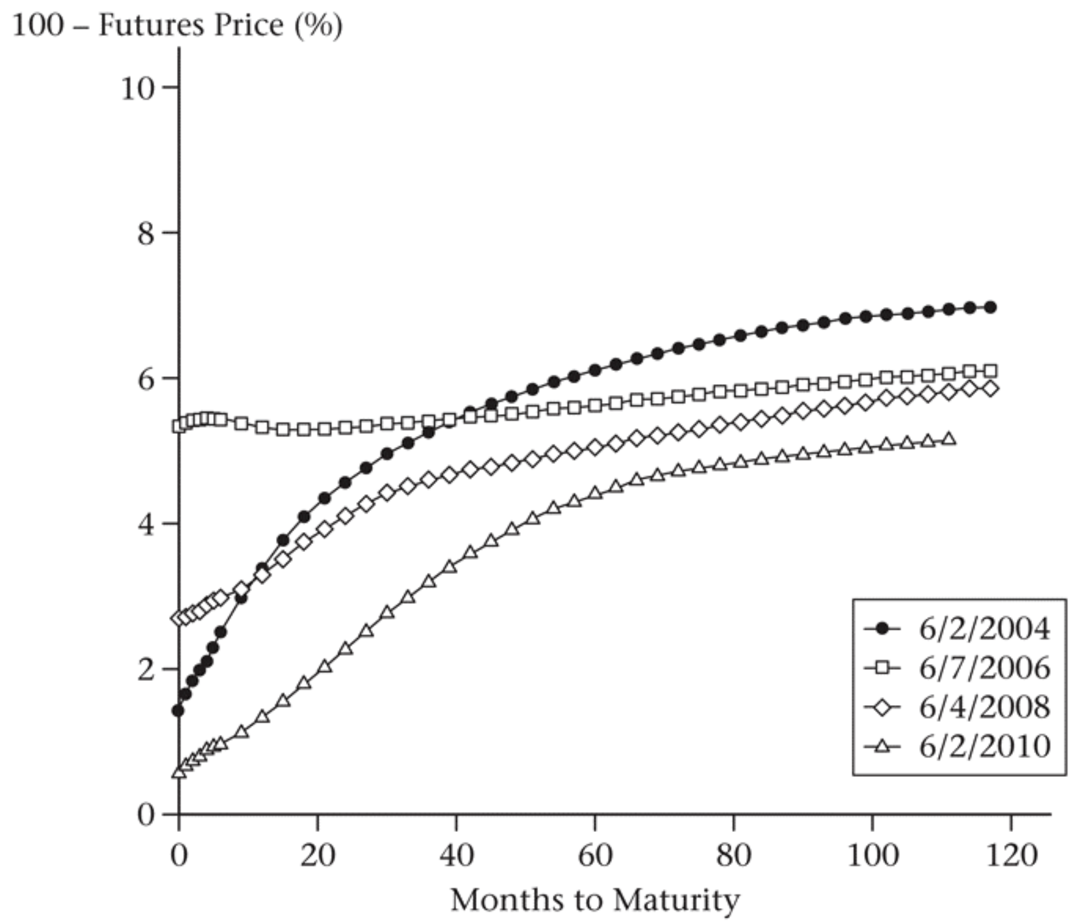
FIGURE 5.4

Specifications for the Eurodollar futures contract.

Where traded	Chicago Mercantile Exchange
Size	3-month Eurodollar time deposit, $1 million principal
Months	March, June, September, December, out 10 years, plus 2 serial months and spot month
Trading ends	5 A.M. (11 A.M. London) on the second London bank business day immediately preceding the third Wednesday of the contract month.
Delivery	Cash settlement based on 100 – British Banker's Association Futures Interest Settlement Rate for 3-Month Eurodollar Interbank Time Deposits. (This is a 3-month rate annualized by multiplying by 360/90.)

FIGURE 5.5

Eurodollar futures price strip for four dates.



Source: Datastream.

TABLE 5.14 Synthetic equivalents assuming the asset pays continuous dividends at the rate δ.

Position	Synthetic Equivalent
Long forward	$=$ Buy $e^{-\delta T}$ shares of stock $+$ Borrow $S_0 e^{-\delta T}$
Bond paying $F_{0,T}$	$=$ Buy $e^{-\delta T}$ shares of stock $+$ Short forward
Synthetic stock	$=$ Long forward $+$ Lend $e^{-rT} F_{0,T}$

TABLE 5.15 Marking-to-market proceeds and margin balance from long position in the S&P 500 futures contract, where hedge is adjusted on a weekly basis.

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance ($)
0	1979.34	1100.00	—	217,727.21
1	1981.62	1027.99	−72.01	75,446.43
2	1983.91	1037.88	9.89	95,131.79
3	1986.20	1073.23	35.35	165,372.88
4	1988.49	1048.78	−24.45	117,001.17
5	1990.79	1090.32	41.54	199,738.33
6	1993.09	1106.94	16.62	233,055.86
7	1995.39	1110.98	4.04	241,377.01
8	1997.69	1024.74	−86.24	69,573.25
9	2000.00	1007.30	−17.44	34,813.80
10	2000.00	1011.65	4.35	43,553.99