FIN 5330 : Financial Econometrics

Notes on Chapter 01 - Financial Asset Prices and Returns

Sect. 1.1 What is Financial Econometrics?

- No simple definition
- Data Analysis for Finance/Economics (we will be reading Tukey's Paper later)
- Empirical implementation of financial models (Ex: CAPM)
- Methods of estimation & Inference
- Forecasting, policy analysis, academic understanding of financial market phenomena
- Draws on finance/econ, probability, statistics, applied math
- Connections with ML + AI
- Basics : Finance Theory + Data Analysis

Section 1.2 – Financial Assets

- fixed income
- Equity
- Derivatives

} cash flows generated from these securities/contracts (streams of cash flows)

Cash : claim on stream of services that it can secure by virtue of its role as a medium of exchange

- Ludwig von Mises on the evolution of money
- Cash is a kind of derivative security that derives its value from the opportunity cost of goods and services
- Exchange rates between currencies
  (Foreign exchange markets are the largest financial markets)

Fixed-Income Securities (Debt)

- Two streams of cash flows
  - stream of coupon payments made at regular fixed intervals
  - The eventual return of principal at maturity
  - Financial innovation is a major factor in these markets
    - (the original term fixed-income came from the simple st forms. Subsequent forms are much more sophisticated)

- Money Markets
  - Short-term, very liquid
    - Treasury Bills: simplest form of government debt (3, 6, 9 month maturities) (Pure discount bonds)
    - Eurodollar Deposits: deposits of US banks held in financial institutions outside the US denominated in USD.

- Bond Markets
    - Government bonds (e.g. US Treasury) (often zero-coupon/discount)
    - Corporate bonds (e.g. CAT)
        - Typically coupon-paying bonds

- Equity Securities
    - Common Stock: give the owner an equity stake in the assets of the company and it's earnings
      (call option on assets w/ K = Face Value of Liabilities)
        - Dividends: payments representing distribution of company earnings
        - Dividend Yield: \$ amount per share, or as a percentage of current market price

- Derivative Securities

  - "Derive" their value from a reference asset
  - Payoff based on an underlying asset (commodities, stocks, interest rates, etc)
  - Two main classes:
    - (1) options:
      - Call: the right, but not obligation, to buy an underlying asset at a predermined price (strike) at a predermined time (expiry)
      - Put: the right, but not the obligation, to sell an underlying asset at a strike price at expiry
    - (2) Futures: specify delivery of an asset (or ~~cash~~ cash value equivalent) at a time known as maturity for an agreed upon price (payable at maturity)

- Long futures (person buying / taking delivery)
- Short futures (person selling / making delivery)

Section 1.3 - Equity Prices & Returns

- Prices : represent the most basic type of data for financial econometrics
    - Quoted bid/ask prices
    - Historical transaction prices
    - Frequency :
        - High-frequency - intraday (sub-second time resolution)
        - Medium frequency - daily
        - Low-frequency - weekly, monthly, annual, etc.

- F. A. Hayek on the Informational Role of Prices
    - AER 1945 "The Use of Knowledge in Society"
    - Prices convey information
        - CME closing due to flooding (grain elevators)
        - Roll + OJ futures
        - Armen Alchian + stocks who built the Atomic Bomb

- Prices are what we want to work with, but they present very challenging statistical problems (non-stationarity)
    - Project II focuses on one particular econometric technique (Cointegration) that powerfully overcomes this problem

- 1.3.2 Returns

- Dollar Returns

$$\$R_{kt} = P_t - P_{t-k}$$

where P_t is the price at time t

and P_{t-k} is the price at $t-k$

(K is the length of the period)

- Not scale-free (still in $ terms)
- Not proportional to initial investment

- Simple Returns

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

where $\frac{P_t}{P_{t-1}}$ is the

price relative quotient

(or Gross return)

- Note that $\underbrace{1+R_t}_{\text{simple gross return}} = \underbrace{\frac{P_t}{P_{t-1}}}_{\text{price relative (prel)}}$

  - Represents the value of investing \$1 at t-1 from perspective of t

- Holding period return $R_t(k)$ is given by

$$R_t(k) = \frac{P_t}{P_{t-k}} - 1$$

$$= \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \cdots \times \frac{P_{t-k+2}}{P_{t-k+1}} \times \frac{P_{t-k+1}}{P_{t-k}} - 1$$

$$= (1+R_t) \times (1+R_{t-1}) \times \cdots \times (1+R_{t-k+2}) \times (1+R_{t-k+1}) - 1$$

$$= \left[\prod_{j=0}^{k-1} (1+R_{t-j})\right] - 1$$

Ex: If the frequency is monthly, then the simple return for a holding period of 1 year is given by

$$R_t(12) = \left[\prod_{j=0}^{11} (1 + R_{t-j}) \right] - 1$$

$$\text{Annualized } R_t(12) = (1 + R_t)^{12} - 1$$

- Log Returns
  - The log return of an asset is defined as

$$r_t = \log(1 + R_t) = \log(P_t) - \log(P_{t-1})$$

  - Log returns are continuously compounded returns
  - Note : $\log_e = \ln$ (natural logarithm)
  - Leonhard Euler (1707 – 1783)

$$e \equiv \lim_{s \to \infty} \left(1 + \frac{1}{s}\right)^s \approx 2.71828$$

  - So called "natural log" b/c e is the natural limit of this quantity (Demonstrate in Julia)

- The limit formula represents the value of an account at the end of the year that started with \$1 and paid 100% interest per year but with the interest compounded continuously over time rather than at discrete intervals (i.e. at every infinitesimal increment of time)

- If m is the compounding period and r_t the return, then

$$P_t = P_{t-1}\left(1 + \frac{r_t}{m}\right)^m$$

  - Continuous compounding is produced when $m \to \infty$

$$P_t = P_{t-1} \lim_{m \to \infty}\left(1 + \frac{r_t}{m}\right)^m$$

- Let $s = m/r_t$, then we can rewrite this as

$$P_t = P_{t-1} \lim_{s \to \infty} \left[\left(1 + \frac{1}{s}\right)^{s r_t} \right]$$

$$= P_{t-1} \left[\lim_{s \to \infty} \left(1 + \frac{1}{s}\right)^{s} \right]^{r_t}$$

$$= P_{t-1} e^{r_t}$$

- Note: ln is the inverse of e^x, so taking logs gives us

$$e^{r_t} = \frac{P_t}{P_{t-1}}$$

$$\ln(e^{r_t}) = \ln\left(P_t / P_{t-1}\right)$$

$$r_t = \ln(P_t) - \ln(P_{t-1})$$

(so we've come full circle!)

- Log returns are especially useful for multiperiod calculations b/c products become sums

  - Example:

~~$r_t(2) = \log$~~

$$r_t(2) = \log(P_t) - \log(P_{t-2})$$

$$= \underbrace{[\log(P_t) - \log(P_{t-1})]}_{r_t} + \underbrace{[\log(P_{t-1}) - \log(P_{t-2})]}_{r_{t-1}}$$

$$= r_t + r_{t-1}$$

- The k-period return (by extension) is

$$r_t(k) = \log P_t - \log P_{t-k}$$

$$= r_t + r_{t-1} + \cdots + r_{t-(k-1)}$$

$$= \sum_{j=0}^{k-1} r_{t-j} \quad \text{(i.e. sum of single-period log returns)}$$