Chapter 1 - Financial Asset Prices and Returns

Finance 5330: Financial Econometrics

Tyler J. Brough

Department of Finance and Economics

UtahStateUniversity

Section 1.1: What is Financial Econometrics?

- No simple definition
- Data analysis for finance/economics (we will be reading Tukey's paper later)
- Empirical implementation of financial models (ex: CAPM)
- Methods of estimation and inference
- Forecasting, policy analysis, academic understanding of financial market phenomena
- Draws on finance/economics, probability, statistics, applied math
- Connections to ML & AI
- Basics: Finance Theory + Data Analysis

Section 1.2: Financial Assets

- Fixed income
- Equity
- Derivatives
- Cash flows generated from these securities/contracts (streams of cash flows)

- Cash: claim on stream of services that it can secure by virtue of its role as a medium of exchange
  - Ludwig von Mises on the evolution of money
  - Cash is a kind of derivative security that derives its value from the opportunity cost of goods and services
  - Exchange rates between currencies (foreign exchange markets are the largest financial markets in the world)

Fixed-Income Securities

- Two streams of cash flows
  - Stream of coupon payments made at regular fixed intervals
  - The eventual return of principal at maturity
  - Financial innovation is a major factor in these markets
  - (the original term fixed-income came from the simplest forms. Subsequent forms are much more sophisticated)
- Money Markets
  - Short-term, very liquid
    - Treasury bills: simplest form of government debt (3,6,9 month maturities | pure discount bonds)
  - Eurodollar deposits: deposits of US banks held in financial institutions outside the US denominated in USD.

- Bond markets
  - Government bonds (e.g. US Treasury) (often zero-coupon or pure discount)
  - Corporate bonds (e.g. CAT)
    - Typically coupon-paying bonds
- Equity Securities
  - Common stock: give the owner an equity stake in the assets of the company and it's earnings
    - (call option on assets w/ $K =$ face value of liabilities)
  - Dividends: payments representing distribution of company earnings
  - Dividend yield: $ amount per share, or as a percentage of current market price

- Derivative contracts
  - "Derive" their value from a reference asset (called the underlying or spot)
  - Payoff based on an underlying asset (commodities, stocks, interest, rates, etc)

- Two main classes:
  - (1) options:
    - Call: the right, but not the obligation, to buy an underlying asset at a predetermined price (strike) at a predetermined time (expiry)
    - Put: the right, but not the obligation, to sell an underlying asset at a strike price at expiry
  - (2) futures: specify delivery of an asset (or cash value equivalent) at a time known as maturity for an agreed upon price (payable at maturity)
    - Long futures (person buying / taking delivery)
    - Short futures (person selling / making delivery)

Section 1.3: Equity Prices & Returns

- Prices: represent the most basic type of data for financial econometrics
  - Quoted bid/ask prices
  - Historical transaction prices
  - Frequency
    - High: intraday (sub-second time resolution)
    - Medium: daily, weekly,
    - Low: monthly, quarterly, annual

Hayek on the Informational Role of Prices

- AER 1945 article titled "The Use of Knowledge in Society"
- Prices convey information
  - CME closing due to flooding (grain elevators would not quote prices)
  - Roll on weather and OJ futures
  - Armen Alchian on stocks who were contractors to build the first atomic bomb
- Prices are what we want to work with, but they present very challenging statistical problems (nonstationarity)
  - Project II focuses on one particular econometric technique (cointegration and error-correction) that powerfully overcomes these problems

Section 1.3.2 Returns

- Dollar returns

$$\$R_{kt} = P_t - P_{t-k}$$

- where P_t is the price at time t and P_{t-k} is the price at $t - k$ (k is the length of the period)
- Problems:
  - Not scale-free (still in $ terms)
  - Not proportional to initial investment

- Simple returns

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

- where $\frac{P_t}{P_{t-1}}$ is the price relative quotient (or gross return)

- Note that:

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

- Represents the value of investing \$1 at $t-1$ from perspective of t

- Holding period return $R_t(k)$ is given by

$$\begin{aligned}
R_t(k) &= \frac{P_t}{P_{t-1}} - 1 \\
&= \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \cdots \times \frac{P_{t-k+2}}{P_{t-k+1}} \times \frac{P_{t-k+1}}{P_{t-k}} - 1 \\
&= (1 + R_t) \times (1 + R_{t-1}) \times \cdots \times (1 + R_{t-k+2}) \times (1 + R_{t-k+2}) \times (1 + R_{t-k+1}) - 1 \\
&= \left[\prod_{j=0}^{k-1} (1 + R_{t-j}) \right] - 1
\end{aligned}$$

Ex: If the frequency is monthly, then the simple return for a holding period of 1 year is given by

$$R_t(12) = \left[\prod_{j=0}^{11}(1 + R_{t-j})\right] - 1$$

- The annualized return $R_t(12) = (1 + R_t)^{12} - 1$

Log Returns

- The log return of an asset is defined as

$$r_t = \log(1 + R_t) = \log P_{t-} - \log P_{t-1}$$

- Log returns are continuously compounded returns
- Note: $\log_e(\cdot) = \ln(\cdot)$ (natural logarithm)
- Loenhard Euler (1707 - 1783)

$$e \equiv \lim_{s\to\infty}(1 + \frac{1}{s})^s \approx 2.71828$$

- So called *natural expotential* (and *natural log*) because e is the natural limit of this quantity
- Demo in Julia

- The limit formula represents the value of an account at the end of the year that started with \$1 and paid 100% interest per year but with the interest compounded continuously over time rather than at discrete intervals (i.e. at every infinitesimal increment of time)

- If m is the compounding period and r_t the return, then

$$P_t = P_{t-1}\left(1 + \frac{r_t}{m}\right)^m$$

- Continuous compounding is produced when $m \to \infty$

$$P_t = P_{t-1}\lim_{m\to\infty}\left(1 + \frac{r_t}{m}\right)^m$$

- Let $s = \frac{m}{r_t}$, then we can rewrite this as

$$
\begin{aligned}
P_t &= P_{t-1} \lim_{s \to \infty} \left[\left(1 + \frac{1}{s} \right)^{s r_t} \right] \\
&= P_{t-1} \left[\lim_{s \to \infty} \left(1 + \frac{1}{s} \right)^{s} \right]^{r_t} \\
&= P_{t-1} e^{r_t}
\end{aligned}
$$

- Note: $\ln(\cdot)$ is the inverse of $\exp(\cdot)$, so taking logs gives us

$$
\begin{aligned}
e^{r_t} &= \frac{P_t}{P_{t-1}} \\
\ln\left(e^{r_t}\right) &= \ln\left(\frac{P_t}{P_{t-1}}\right) \\
r_t &= \ln\left(P_t\right) - \ln\left(P_{t-1}\right)
\end{aligned}
$$

- (So we've come full circle)

- Log returns are especially useful for multiperiod calculations because products become sums
- Example:

$$
\begin{aligned}
R_t(2) &= \log(P_t) - \log(P_{t-2}) \\
&= [\log(P_t) - \log(P_{t-1})] + [\log(P_{t-1}) - \log(P_{t-2})] \\
&= r_t + r_{t-1}
\end{aligned}
$$

- The k-period return (by extension) is

$$
\begin{aligned}
r_t(k) &= \log(P_t) - \log(P_{t-k}) \\
&= r_t + r_{t-1} + \cdots + r_{t-(k-1)} \\
&= \sum_{j=0}^{k-1} r_{t-j} \quad \text{(i.e. sum of single-period log returns)}
\end{aligned}
$$