ESTIMATION OF THE OPTIMAL FUTURES HEDGE

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Abstract—Standard approaches to designing a futures hedge often suffer from two major problems. First, they focus only on minimizing risk, so no account is taken of the impact on expected return. Second, in estimating the hedge ratio, no allowance is made for time variation in the distribution of cash and futures price changes. This paper describes a technique for estimating the optimal futures hedge that corrects these problems, and illustrates its use in hedging Treasury bonds with T-bond futures.

I. Introduction

THE recent broad expansion of futures markets has been accompanied by substantial interest in the theory and practice of hedging. Yet, several very important aspects of optimal hedging are frequently neglected in practical hedge design. For example, textbooks and practitioners both typically suggest that the appropriate hedge ratio can be found by regressing historical series of realized price changes or returns from the item being hedged on those for the futures contract to be used.[1] Choosing hedge ratios in this way suffers from two major shortcomings. First, it is assumed that the objective is to minimize risk, not to maximize expected utility, which also depends on expected return. Second, the joint distribution of cash and futures price changes, and therefore the hedge ratio, is estimated incorrectly, since there is no adjustment for the fact that it varies substantially over time. This paper describes a technique for estimating an optimal futures hedge that corrects these problems, and illustrates its use in hedging 20 year Treasury bonds with T-bond futures.

The first of these problems has been recognized in the literature on futures hedging.[2] For example, Heifner (1972) and Rolfo (1980) compute utility maximizing hedge ratios for cattle feeding and cocoa futures. The literature on financial futures, following Ederington (1979), focuses on constructing minimum risk hedges, however. Furthermore, the problem of time-varying distributions has not been considered.

To emphasize the importance of time variation in the distribution of asset returns, table 1 presents sample estimates for annualized one-month returns on Treasury bills (the riskless rate), 20 year Treasury bonds, and the nearby T-bond futures contract. Data sources and methods of calculation are described fully in the appendix.

The table reveals that the return to holding Treasury bonds was often below the riskless rate or even negative. In fact, long-term bonds earned less than T-bills, on average, from 1978 through 1984. Standard statistical techniques such as regression tacitly take the sample mean as an estimate of the expected return, which is clearly inappropriate in this case. Time variation in the volatility of returns is also obvious. But, while the mean returns and variances move substantially, the correlation of the cash and futures return is nearly constant over time.

The next section of the paper discusses the theory of the optimal hedge when the joint probability distribution of returns is known. Section III describes the problem of obtaining estimates of this distribution from historical data. The focus is on estimating the ex-ante distribution. We restrict the ex-ante expected return on long-term bonds to exceed the riskless rate (so the expected risk premium is always positive), and allow for time variation in the return covariance matrix. We then present the results of our estimation procedure. Section IV contains in-sample and post-sample

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[1] See, for example, the discussion in Ederington (1979), Kolb (1985), Schwartz, Hill and Schneeweis (1986), or Figlewski (1986). Other approaches, such as computing the hedge ratio from the duration of a bond position, or using the beta of a stock portfolio to be hedged, attempt to do essentially the same thing.

[2] The extensive literature on the optimal futures hedge includes Telser (1955), Johnson (1960), Stein (1961), Ward and Fletcher (1971) and, more recently, Anderson and Danthine (1981) and Stulz (1984).

results comparing the optimal hedge with those constructed by other methods. The final section offers a few conclusions.

II. The Optimal Futures Hedge

In this section we discuss the optimal futures hedging decision for an investor with a fixed (long) position in a commodity or financial instrument, assuming that the ex-ante distribution of returns is known. That is, we focus on the choice of what hedge ratio to use in hedging a given cash position. This abstracts from complications that arise when the investor also holds the riskless asset or other risky assets in his portfolio. Extension of our approach to more general cases is not difficult.

If the investor hedges a fraction h of the cash position in the futures market and leaves a fraction $(1-h)$ unhedged, the expected return on the hedged position, $E[R_h]$, is given by

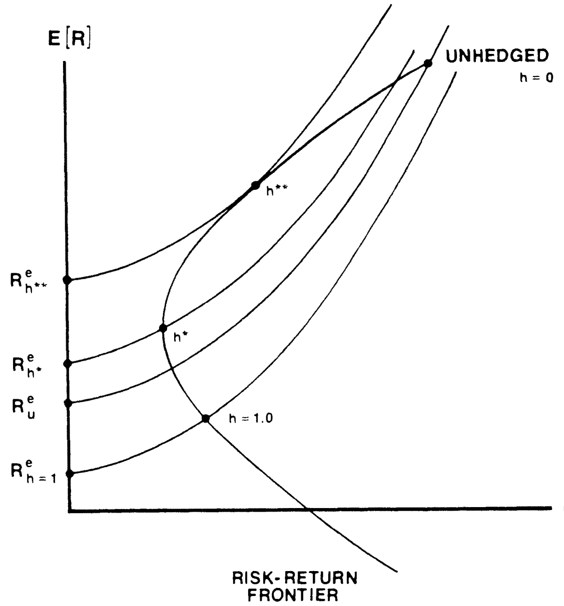
$$E[R_h] = E[R_c] - hE[R_f] \tag{1}$$

where R_c is the rate of return on the cash position and R_f is the rate of return on the futures position. The variance of the return to the hedged portfolio is

$$\sigma_h^2 = \sigma_c^2 + h^2\sigma_f^2 - 2h\sigma_{cf}. \tag{2}$$

As the hedge ratio varies, the expected return and the variance of the return changes. This is depicted by the risk-return frontier in figure 1.

FIGURE 1.—RISK AND RETURN FOR VARIOUS HEDGE STRATEGIES



The return on a hedged position will normally be exposed to risk caused by unanticipated changes in the relative price between the position being hedged and the futures contract. Because of this "basis risk," no hedge ratio can completely eliminate risk. The one that minimizes risk can be computed by setting the derivative of (2) with respect to h equal to zero, giving $h^* = \sigma_{cf}/\sigma_f^2$. The point corresponding to h^* is marked on the figure. It is commonly the case that h^* is less than 1.0 (i.e., the fully hedged position), so that the hedge ratio that minimizes risk in the absence of basis risk turns out to be dominated by h^* when basis risk is taken into account.[3]

An estimate of h^* can be easily derived from historical data. The coefficient of the univariate regression of R_c on R_f will equal the realized h^* over the sample period. But, when there is time variation in the joint distribution of (R_c, R_f), a

[3] The expression for h^* can be written $h^* = \rho\sigma_c/\sigma_f$. Thus, if the futures have the same or higher price volatility than the cash (as is typically the case), h^* can be no greater than the correlation between them, which will be less than 1.0.

TABLE 1.—REALIZED MEANS AND STANDARD DEVIATIONS OF ONE MONTH HOLDING PERIOD RETURNS
(in percent, at annual rates)

	T-Bills		20 yr. T-Bonds		Bond Futures		Correlation of
Year	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Cash & Futures
1978	6.93	.31	−1.29	4.84	−7.84	4.94	.961
1979	9.69	.22	−0.60	12.82	−9.85	10.50	.989
1980	10.59	.84	−2.61	22.50	−14.45	21.85	.986
1981	13.65	.48	−1.25	18.70	−17.87	21.13	.966
1982	10.01	.69	40.46	13.36	22.10	12.87	.979
1983	8.38	.08	0.37	11.86	−5.06	13.23	.991
1984	9.08	.20	15.51	12.93	6.23	12.93	.984
1985	7.04	.14	30.47	13.19	24.24	13.29	.994

regression employing past data will not correctly estimate the *current* risk minimizing hedge ratio.

A more severe problem with this approach is that risk minimization without any regard to the effect on expected returns cannot be optimal. In equilibrium, risky securities are priced to earn an expected premium over the riskless rate. Hedging away the risk must also hedge away the expected return to bearing that risk. This is the cost of hedging, and the reward to risk bearing in the futures market. Only a totally risk averse investor can make an optimal hedging decision without taking the impact on both risk and return into account.[4]

The optimal futures hedge is the one that maximizes expected utility. Given the investor's utility function, indifference curves can be generated and the true optimal hedge ratio, denoted h^{**} in figure 1, can be computed by setting the investor's subjective marginal rate of substitution between risk and return equal to the slope of the risk-return possibility curve.

Different hedge positions can be compared directly by examining their certainty-equivalent returns. The certainty-equivalent return is the point at which the indifference curve passing through the hedged position in question intersects the expected return axis. Four such points are plotted on the figure. Naturally, the highest certainty-equivalent return corresponds to the optimal hedge h^{**}. The figure also plots the certainty equivalent return associated with the risk minimizing hedge, the unhedged position, and the fully hedged position ($h = 1.0$). It is clear that if $h^* < 1$, every investor would find the overhedged $h = 1$ position dominated by the risk minimizing hedge h^*, but otherwise, the ordering of these three alternatives depends upon the shape of the indifference curves.

As is clear from this analysis, the optimal hedge ratio is a function of both the risk-return possibility curve that is available in the market and the investor's utility function. Implementation of this hedging strategy involves two steps: first, estimation of the joint distribution of returns, which is needed to construct the risk-return frontier, and then optimization, by finding the hedge ratio that maximizes the investor's expected utility. We now turn to the problem of estimation of this joint distribution. We will then use the estimates to analyze the optimal hedge strategy for an investor in Treasury bonds with log utility.

III. Estimation

Estimation of the optimal Treasury bond hedge is complicated by the fact that the probability distribution governing returns on Treasury bonds and bond futures varies substantially over time, as was shown in table 1. One clear reason to expect changing means is that the riskless interest rate varied over a range of more than 10 percentage points during this period. If one thinks of the expected return to a long-term bond as being made up of the current riskless rate plus a risk premium, time variation in mean holding-period returns on long-term bonds seems inevitable. This suggests that a good first step in dealing with time-varying returns is not to try to estimate the mean directly, but rather to use past data to estimate the risk premium and then add it to the current level of the risk-free rate.

Unfortunately, that approach does not work in this case since, as mentioned in the introduction, the ex-post premium was negative, on average, during the estimation period.[5] Nevertheless, since we have a strong prior belief that the market is made up of risk averse investors and the true expected excess return to holding long-term bonds cannot be negative, we want to impose this constraint in the estimation. Accordingly, we assume the risk premium is equal to the average return difference between long-term Treasury bonds and one-month T-bills from 1926 to 1977, as calculated by Ibbotson and Sinquefield (1982). Thus, the expected one-month holding-period return on a twenty-year bond is equal to the Treasury bill rate plus a constant premium of 90 basis points at an annual rate (or 7.5 basis points monthly). The expected one-month return to a long position in Treasury bond futures is also assumed constant and equal to the same 90 basis points annually. This is consistent with the futures contract being

[4] Real world hedgers are very aware of this tradeoff. They may hedge partially or selectively, and remain exposed to market risk on part of their position or part of the time. In many cases, potential hedgers decide that hedging is not attractive for them because it is "too expensive."

[5] If realizations of the risk premium were predominantly positive, it would be possible to use a moving parameter technique such as Kalman filtering to obtain estimates of moving means.

priced by a carrying cost relationship relative to the underlying bond.[6]

For convenience, we define the deviations of the monthly returns to 20-year bonds (the cash position) and the Treasury bond futures around their means (specified a priori) as

$$\epsilon_c(t) = R_c(t) - r(t) - .075$$

and

$$\epsilon_f(t) = R_f(t) - .075,$$

where $r(t)$ is the return to one-month Treasury bills, the riskless nominal rate.

Although we are forced to constrain time variation in expected returns, we allow the return covariance matrix to vary over time by using Engle's (1982) model of autoregressive conditional heteroskedasticity (ARCH).[7] In the ARCH model, the variance of a zero-mean random variable at time t, conditional on information at time $t-1$, is assumed to be a function of the past values of that random variable and some unknown set of parameters. We specify a third-order linear ARCH for each of the variance processes,[8]

$$E_{t-1}\epsilon_c(t)^2 = a_0 + a_1^2\epsilon_c(t-1)^2 + a_2^2\epsilon_c(t-2)^2 + a_3^2\epsilon_c(t-3)^2, \quad (3a)$$

$$E_{t-1}\epsilon_f(t)^2 = b_0 + b_1^2\epsilon_f(t-1)^2 + b_2^2\epsilon_f(t-2)^2 + b_3^2\epsilon_f(t-3)^2, \quad (3b)$$

where $E_{t-1}(\cdot)$ is defined as the expectation conditional on information available at $t-1$, and the parameters, the a's and b's, are squared to insure that the in-sample estimates of the variances will always be positive.[9]

While it is straightforward to model the variances of the returns, the covariance between the return on the 20-year bonds and the Treasury bond futures must be dealt with somewhat differently since unrestricted ARCH estimates of the covariance would not force the estimated covariance matrix to be positive definite each period. In order to assure positive definiteness, we constrain R_c and R_f to have a constant correlation coefficient throughout the sample, so changes in the covariance are due only to changes in the standard deviations. Given that the year-by-year estimate of this correlation only varies from 0.96 to 0.99, and given the large changes in the standard deviations of returns reported in table 1, assuming the correlation coefficient to be constant is unlikely to introduce noticeable errors. The covariance of returns is then,

$$E_{t-1}\epsilon_c(t)\epsilon_f(t) = \rho\left[E_{t-1}\epsilon_c(t)^2 E_{t-1}\epsilon_f(t)^2\right]^{1/2} \quad (3c)$$

where ρ is the correlation between expected returns on cash and futures. As long as ρ is less than one in absolute value, the third equation (3c), insures that the covariance matrix of expected returns will be positive definite.

In summary, the procedure we use for estimating the joint distribution of returns has two stages. First we calculate the difference of realized returns from their ex-ante means, which we assume to be equal to the riskless rate plus a time-invariant premium. Then we estimate the three-equation ARCH model, (3). This yields time varying estimates of the covariance matrix of the two returns, R_c and R_f, assuming that their correlation is constant.

Before estimating the ARCH model, it is useful to test whether autoregressive conditional heteroskedasticity is present in the bivariate system. To do this we construct a Wald test based on three ordinary least squares regressions of $\epsilon_c(t)^2$, $\epsilon_f(t)^2$, and $\epsilon_c(t)\epsilon_f(t)$ on lags of $\epsilon_c(t)^2$, lags of $\epsilon_f(t)^2$, and lags of the cross product, $\epsilon_c(t)\epsilon_f(t)$. The test examines whether all the coefficients, excluding the three constants, are simultaneously zero when the three equations are estimated as seemingly unrelated regressions.[10] With an alternative hypothesis that the return process is a bivariate third-order ARCH, the test statistic of

[6] Since buying the bond, selling futures and holding for delivery should yield a riskless position earning the riskless rate of interest, and holding the bond unhedged has an expected return equal to the riskless rate plus the risk premium, the short sale of futures must have an expected return equal to the negative of the risk premium.

[7] Kraft and Engle (1983) extend Engle's (1982) analysis to the multivariate case.

[8] A sixth-order ARCH specification was also examined but it yielded implausible parameter estimates.

[9] Engle proves that a p^{th} order linear ARCH process is covariance stationary if and only if each of the coefficients is nonnegative and the coefficients on the lagged squared ϵ's sum to less than one.

[10] This is a variant of a test proposed by Kraft and Engle (1983). In their test, the alternative hypothesis assumes that there is no contemporaneous covariance between $\epsilon_c(t)$ and $\epsilon_f(t)$, an assumption that is clearly inappropriate in this application.

interest is distributed as $\chi^2(27)$ under the null hypothesis that both series are homoskedastic with constant nonzero contemporaneous correlation. We obtain a test statistic of 70.51, which leads us to reject the null hypothesis of homoskedasticity at the 0.1% level.

The specification of the covariance equation (3c), implies that the three equations contain nonlinear constraints. As such, the system (3) was estimated simultaneously using nonlinear least squares. The available data begin in October 1977 and end in May 1986. This period is divided into an estimation period, from January 1978 to December 1983, and a post-sample period to examine model performance, from January 1984 to May 1986. The results are presented in table 2. Unfortunately, the nonnegativity constraint is binding on one of the parameters, b_1, the first lag in the futures equation.

The fitted values from the estimates in table 2 are estimates of the ex-ante covariance matrix of returns for the hedging problem. In the following section these are used to compute hedge positions under several alternative assumptions.

IV. The Optimal Hedge

Using the estimates of the covariance matrix of returns, we can construct the optimal hedge for an investor holding a position in long-term bonds once the utility function is specified. We assume that the investor has log utility, which implies relative risk aversion equal to one.[11] At every time period t, the investor chooses the hedge ratio $h(t)$ to maximize the expected value of the logarithm of terminal wealth.

The problem can be written as

$$\max_{h(t)} \int_{R_f} \int_{R_c} \log\left[1 + R_c(t) - h(t)R_f(t)\right] \times f_t(R_c, R_f)\, dR_c\, dR_f, \tag{4}$$

where $f_t(R_c, R_f)$ is the bivariate normal density with mean $(r(t) + .075, .075)$ and covariance matrix determined by the ARCH model. The solution to (4), $h^{**}(t)$, is found by numerical methods.[12] This yields a value for expected utility, $U^{**}(t)$, which can then be used to determine the certainty-equivalent return, $R^e(h^{**}(t)) = \exp[U^{**}(t)]$. (The certainty-equivalent returns are the points at which the indifference curves meet the vertical axis of figure 1.)

Consistent with the data and estimation of the ex-ante distribution of returns, we examine one-month hedges over the period January 1978 to May 1986. An essential characteristic of actual hedging problems is that parameters must be forecasted from past data. This is the rationale for splitting the data sample into two parts, before and after January 1984, with the latter part used only for post-sample tests.

Figure 2 plots the monthly values of the risk minimizing hedge, h^*, and the optimal hedge h^{**}.[13] Table 3 shows the average values for the relevant parameters in the two periods. Recall that $E[R_f]$ is not estimated. All returns are averages of

[11] The choice of utility function in this example is arbitrary. One with a higher degree of risk aversion would lead to larger optimal hedge ratios, while a less risk averse utility function would yield smaller ones. Under risk neutrality, the investor increases expected return by taking a *long* position in futures along with his long cash position. However, for no utility function allowing a tradeoff between risk and return would the minimum risk hedge ratio be optimal.

[12] We solve the integral in (4) by replacing the normal density function $f_t(R_c, R_f)$ by a multinomial approximation. The function is evaluated at 81 points, corresponding to realizations of $-4, -3, \ldots, +3, +4$ standard deviations away from the mean for each of the variables. Tests with a finer grid showed that this was adequate to determine the optimal hedge ratio to within 0.01.

[13] Even if there were no risk premium on long-term bonds, so that all investors would choose h^*, the substantial variation exhibited by h^* indicates that the issues addressed here are important in practice.

TABLE 2.—ESTIMATES OF THE BIVARIATE ARCH MODEL

Variance of Cash Return				
	$a_0 = 16.026$	$a_1^2 = .083$	$a_2^2 = .039$	$a_3^2 = .078$
	(3.93)	(.037)	(.081)	(.083)
Variance of Futures Return				
	$b_0 = 14.981$	$b_1^2 = 0.00^{a}$	$b_2^2 = .041$	$b_3^2 = .162$
	(3.99)		(.080)	(.080)
Covariance of Cash and Futures				
	$\rho = .981$			
	(.008)			

Note: Numbers in parentheses are asymptotic standard errors.
[a] The constraint was binding and the coefficient was restricted to equal zero.

FIGURE 2.—OPTIMAL AND RISK MINIMIZING HEDGE RATIOS

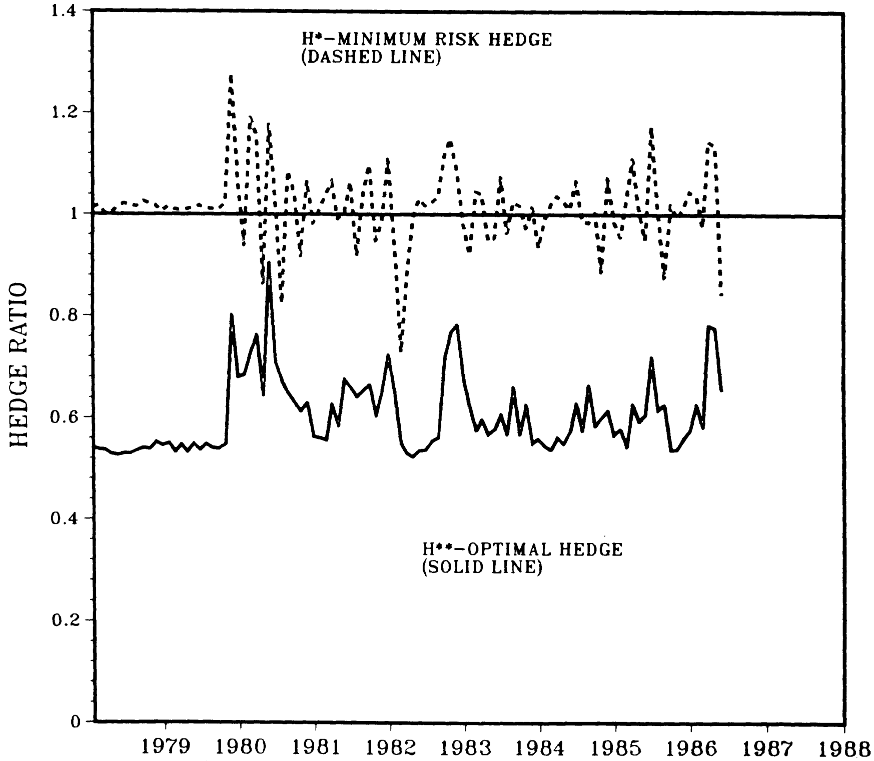


TABLE 3.—AVERAGE HEDGE PERFORMANCE

Date	In Sample 1/78–12/83	Post Sample 1/84–6/86	Pre 10/79 1/78–9/79	Post 10/79 10/79–9/82	Post 10/82 10/82–5/86
r	9.88	7.96	7.97	11.69	8.04
$(ER_c - r) = ER_f$	0.90	0.90	0.90	0.90	0.90
σ_c	15.36	15.29	14.09	16.02	15.38
σ_f	14.94	14.88	13.64	15.60	14.97
ρ	0.98	0.98	0.98	0.98	0.98
Ex Ante					
$R^e(h = 0)$	9.61	7.71	7.90	11.32	7.77
$R^e(h = 1)$	9.87	7.96	7.97	11.67	8.03
$R^e(h = .97)$	9.89	7.98	7.99	11.69	8.06
h^*	1.01	1.01	1.01	1.02	1.01
$R^e(h^*)$	9.86	7.95	7.96	11.67	8.03
h^{**}	0.61	0.61	0.54	0.64	0.61
$R^e(h^{**})$	10.04	8.13	8.17	11.83	8.20
Ex Post					
R_c	5.85	24.74	1.35	7.18	19.36
R_f	−5.49	17.37	−6.15	−8.52	12.37
$R(h = 1)$	11.34	7.38	7.50	15.70	6.99
$R(h = .97)$	11.19	7.84	7.34	15.47	7.32
$R(h^*)$	11.26	7.15	7.57	16.19	6.27
$R(h^{**})$	8.89	13.82	4.64	12.33	11.36

Source: Calculations described in text.

one-month returns measured at an annual rate. As measured by these averages, the two periods appear quite similar. But the path of interest rates was quite different in the two periods of the sample, rising in the first and falling in the second.

When making a hedging decision, the investor does not know what will happen to interest rates over the hedge horizon. The ex-ante effectiveness of the different hedging strategies, therefore, must be evaluated in terms of expected utility, or certainty-equivalent return.

In the six-year in-sample period, the investor with log utility would have viewed holding cash bonds unhedged and earning an expected 90 basis points over the one-month T-bill rate as being, on average, equivalent to a riskless 9.61%. The minimum risk hedge ratio, based on the estimated joint probability distribution of cash and futures returns averaged 1.01, and yielded a certainty-equivalent return of 9.86%.[14] The optimal hedge, taking both the risk and return into account, led to much smaller hedges, with h^{**} averaging 0.61 from 1978 to 1983, less than two-thirds the average minimum-variance hedge. Meanwhile, the certainty-equivalent return averaged 18 basis points better than h^* and 43 basis points better than not hedging at all. (These differences may seem small, but they should be compared in magnitude with the 90 basis point risk premium for bearing the risk on these long-term bonds.) Only in the case of the optimal hedge, is the certainty-equivalent return in excess of the riskless rate.

In the post-sample period, we use the same estimates of the parameters reported in table 2, but construct the ex-ante joint return density from realized values of R_c, r, and R_f. From January 1984 to May 1986, the utility maximizing hedge ratio again averaged 0.61. The comparisons of h^{**}, h^*, and $h = 0$ are also similar. The investor with log utility would have been worst off not hedging at all, and would have gained an average certainty-equivalent return of 42 basis points by choosing h^{**}. Again, only the optimal hedge yields a certainty-equivalent return in excess of the riskless rate.

[14] An alternative procedure is to use a naive hedge based on the simple univariate regression of R_c on R_f and a constant. Over the estimation period this yields $h_N^* = 0.97$ and a certainty-equivalent return of 9.89.

The table also contains ex-post results, which show the realized return to different hedges given the actual behavior of yields over each month. Prior to 1981, yields on both long- and short-term instruments rose dramatically and then retreated somewhat, ending the period well above their initial levels. The result was that realized returns on long-term bonds were well below those on T-bills. The cash bond return averaged only 5.85% and the futures price declined on average over this period. Given these facts, it is evident that any short position in bond futures increased returns on an overall hedged position. Clearly, the larger the hedge ratio, the higher the ex-post return. This brings out an important practical point about hedge evaluation. Accurate hedge design can increase *expected* utility by a significant, but fairly modest amount. It should pay off over the long run. But relative performance ex-post is entirely dominated by the direction of price change in the underlying asset. If the market rises, the smaller the hedge ratio, the better. If the market falls, short futures are profitable, and the larger the hedge ratio, the greater the profit.

The numbers reported in table 3 naturally depend upon the assumptions we made about the investor's utility function and the expected excess return on bonds and bond futures contracts. To get an idea of how sensitive the quantitative results are to changing these assumptions, we computed optimal hedges with different values for the first post-sample period, January 1984. In that month, the risk-free interest rate was 8.46% and the expected standard deviation for bond returns was about 14%.

Under the original assumptions, the investor used a hedge ratio of $h^{**} = 0.55$ to create a certainty equivalent return of 8.66%. An investor with a greater relative risk aversion of 3.0 would regard the unhedged position as being equivalent to a risk free 6.39%. His h^{**} would be 0.86, giving a certainty equivalent for his optimal hedge of 8.52%. On the other hand, an investor with log utility who expected the bond risk premium to be only half of what we have assumed would consider not hedging to be equivalent to 7.92% risk free and would use an h^{**} of 0.78 to increase that to a certainty equivalent of 8.51%. Lower risk aversion or higher expected premia would move h^{**} and the certainty equivalent return on the optimal hedge in the other direction.

V. Concluding Remarks

This paper has presented an approach to hedging with futures contracts which takes into consideration several important factors that are seldom given adequate attention: First, that a true optimal hedge is one in which the investor's subjective marginal rate of substitution between risk and return is equal to their marginal rate transformation within the hedged position. And second, that in computing this optimum, one should estimate the statistical relation between the cash position and the futures contract in a manner that allows for the fact that it varies substantially over time.

As an illustration, we described how the optimal hedge ratios could be estimated for 20-year Treasury bonds that would be held for one-month periods. Since the data exhibit wide variation over time in the mean returns as well as variation in the return covariance matrix, allowing for a time-varying distribution was quite important. The estimates we obtained from an autoregressive conditional heteroskedasticity (ARCH) specification of the joint distributions were used to obtain a time series of expected utility maximizing hedge ratios for a logarithmic investor.

As expectations about risk and return changed, the optimal hedge ratio varied over a sizable range, from 0.52 to over 0.91. Over both the in-sample and post-sample periods, an investor with log utility would have preferred the utility maximizing hedge to a variance-minimizing hedge by a certainty-equivalent return of, on average, 20 basis points. Furthermore, only the optimal hedge yielded a certainty equivalent return in excess of the riskless nominal rate.

APPENDIX

Data Sources

All data refer to the return from the end of the previous month to the end of the current month. May 1978, for example, refers to the return over the month of May 1978. The data came from the following sources:

Riskless Nominal Rate (r): One Month Treasury Bill Rate from Salomon Brothers *Analytical Record of Yields and Yield Spreads*, June 1986, part IV, table 1.

Cash Return (R_c): One Month Holding Period Return on 20 Year Treasury Bonds. The series of bond prices was constructed from the series of yields to maturity on 20 year Treasuries in the Salomon Brothers *Analytical Record of Yields and Yield Spreads*, June 1986, part I, table 1, assuming an 8% coupon rate. The cash return is the percentage change in the price plus one month's accrued coupon interest.

Futures Return (R_f): The futures price data come from the Interactive Data Corporation. The futures return for a month is computed from the change in the closing prices over the month from the last trading day of the previous month. This timing matches that of the cash return series. To compute a realized rate of return on the future, the futures price change is divided by the initial price of the hypothetical 20 year 8% bond at the beginning of the month. The contract used is the nearest to expiration that is not in the delivery month. Contracts expire in March, June, September and December so, for example, the June contract is used to compute returns in March, April and May.

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