

# 3

C H A P T E R

# Hedging Strategies Using Futures



Many of the participants in futures markets are hedgers. Their aim is to use futures markets to reduce a particular risk that they face. This risk might relate to fluctuations in the price of oil, a foreign exchange rate, the level of the stock market, or some other variable. A *perfect hedge* is one that completely eliminates the risk. Perfect hedges are rare. For the most part, therefore, a study of hedging using futures contracts is a study of the ways in which hedges can be constructed so that they perform as close to perfect as possible.

In this chapter we consider a number of general issues associated with the way hedges are set up. When is a short futures position appropriate? When is a long futures position appropriate? Which futures contract should be used? What is the optimal size of the futures position for reducing risk? At this stage, we restrict our attention to what might be termed *hedge-and-forget* strategies. We assume that no attempt is made to adjust the hedge once it has been put in place. The hedger simply takes a futures position at the beginning of the life of the hedge and closes out the position at the end of the life of the hedge. In Chapter 18 we will examine dynamic hedging strategies in which the hedge is monitored closely and frequent adjustments are made.

The chapter initially treats futures contracts as forward contracts (that is, it ignores daily settlement). Later it explains an adjustment known as “tailing” that takes account of the difference between futures and forwards.

## 3.1 BASIC PRINCIPLES

When an individual or company chooses to use futures markets to hedge a risk, the objective is usually to take a position that neutralizes the risk as far as possible. Consider a company that knows it will gain \$10,000 for each 1 cent increase in the price of a commodity over the next 3 months and lose \$10,000 for each 1 cent decrease in the price during the same period. To hedge, the company's treasurer should take a short futures position that is designed to offset this risk. The futures position should lead to a loss of \$10,000 for each 1 cent increase in the price of the commodity over the 3 months and a gain of \$10,000 for each 1 cent decrease in the price during this period. If the price of the commodity goes down, the gain on the futures position offsets the loss on the rest of the company's business. If the price of the commodity

goes up, the loss on the futures position is offset by the gain on the rest of the company's business.

### Short Hedges

A *short hedge* is a hedge, such as the one just described, that involves a short position in futures contracts. A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future. For example, a short hedge could be used by a farmer who owns some hogs and knows that they will be ready for sale at the local market in two months. A short hedge can also be used when an asset is not owned right now but will be owned at some time in the future. Consider, for example, a US exporter who knows that he or she will receive euros in 3 months. The exporter will realize a gain if the euro increases in value relative to the US dollar and will sustain a loss if the euro decreases in value relative to the US dollar. A short futures position leads to a loss if the euro increases in value and a gain if it decreases in value. It has the effect of offsetting the exporter's risk.

To provide a more detailed illustration of the operation of a short hedge in a specific situation, we assume that it is May 15 today and that an oil producer has just negotiated a contract to sell 1 million barrels of crude oil. It has been agreed that the price that will apply in the contract is the market price on August 15. The oil producer is therefore in the position where it will gain \$10,000 for each 1 cent increase in the price of oil over the next 3 months and lose \$10,000 for each 1 cent decrease in the price during this period. Suppose that on May 15 the spot price is \$80 per barrel and the crude oil futures price for August delivery is \$79 per barrel. Because each futures contract is for the delivery of 1,000 barrels, the company can hedge its exposure by shorting (i.e., selling) 1,000 futures contracts. If the oil producer closes out its position on August 15, the effect of the strategy should be to lock in a price close to \$79 per barrel.

To illustrate what might happen, suppose that the spot price on August 15 proves to be \$75 per barrel. The company realizes \$75 million for the oil under its sales contract. Because August is the delivery month for the futures contract, the futures price on August 15 should be very close to the spot price of \$75 on that date. The company therefore gains approximately

$$\$79 - \$75 = \$4$$

per barrel, or \$4 million in total from the short futures position. The total amount realized from both the futures position and the sales contract is therefore approximately \$79 per barrel, or \$79 million in total.

For an alternative outcome, suppose that the price of oil on August 15 proves to be \$85 per barrel. The company realizes \$85 per barrel for the oil and loses approximately

$$\$85 - \$79 = \$6$$

per barrel on the short futures position. Again, the total amount realized is approximately \$79 million. It is easy to see that in all cases the company ends up with approximately \$79 million.

### Long Hedges

Hedges that involve taking a long position in a futures contract are known as *long hedges*. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

Suppose that it is now January 15. A copper fabricator knows it will require 100,000 pounds of copper on May 15 to meet a certain contract. The spot price of copper is 340 cents per pound, and the futures price for May delivery is 320 cents per pound. The fabricator can hedge its position by taking a long position in four futures contracts offered by the COMEX division of the CME Group and closing its position on May 15. Each contract is for the delivery of 25,000 pounds of copper. The strategy has the effect of locking in the price of the required copper at close to 320 cents per pound.

Suppose that the spot price of copper on May 15 proves to be 325 cents per pound. Because May is the delivery month for the futures contract, this should be very close to the futures price. The fabricator therefore gains approximately

$$100,000 \times (\$3.25 - \$3.20) = \$5,000$$

on the futures contracts. It pays  $100,000 \times \$3.25 = \$325,000$  for the copper, making the net cost approximately  $\$325,000 - \$5,000 = \$320,000$ . For an alternative outcome, suppose that the spot price is 305 cents per pound on May 15. The fabricator then loses approximately

$$100,000 \times (\$3.20 - \$3.05) = \$15,000$$

on the futures contract and pays  $100,000 \times \$3.05 = \$305,000$  for the copper. Again, the net cost is approximately \$320,000, or 320 cents per pound.

Note that, in this case, it is clearly better for the company to use futures contracts than to buy the copper on January 15 in the spot market. If it does the latter, it will pay 340 cents per pound instead of 320 cents per pound and will incur both interest costs and storage costs. For a company using copper on a regular basis, this disadvantage would be offset by the convenience of having the copper on hand.<sup>1</sup> However, for a company that knows it will not require the copper until May 15, the futures contract alternative is likely to be preferred.

The examples we have looked at assume that the futures position is closed out in the delivery month. The hedge has the same basic effect if delivery is allowed to happen. However, making or taking delivery can be costly and inconvenient. For this reason, delivery is not usually made even when the hedger keeps the futures contract until the delivery month. As will be discussed later, hedgers with long positions usually avoid any possibility of having to take delivery by closing out their positions before the delivery period.

We have also assumed in the two examples that there is no daily settlement. In practice, daily settlement does have a small effect on the performance of a hedge. As explained in Chapter 2, it means that the payoff from the futures contract is realized day by day throughout the life of the hedge rather than all at the end.

### 3.2 ARGUMENTS FOR AND AGAINST HEDGING

The arguments in favor of hedging are so obvious that they hardly need to be stated. Most companies are in the business of manufacturing, or retailing or wholesaling, or providing a service. They have no particular skills or expertise in predicting variables such as interest rates, exchange rates, and commodity prices. It makes sense for them to

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<sup>1</sup> See Section 5.11 for a discussion of convenience yields.

hedge the risks associated with these variables as they become aware of them. The companies can then focus on their main activities—for which presumably they do have particular skills and expertise. By hedging, they avoid unpleasant surprises such as sharp rises in the price of a commodity that is being purchased.

In practice, many risks are left unhedged. In the rest of this section we will explore some of the reasons for this.

## Hedging and Shareholders

One argument sometimes put forward is that the shareholders can, if they wish, do the hedging themselves. They do not need the company to do it for them. This argument is, however, open to question. It assumes that shareholders have as much information as the company's management about the risks faced by a company. In most instances, this is not the case. The argument also ignores commissions and other transaction costs. These are less expensive per dollar of hedging for large transactions than for small transactions. Hedging is therefore likely to be less expensive when carried out by the company than when it is carried out by individual shareholders. Indeed, the size of futures contracts makes hedging by individual shareholders impossible in many situations.

One thing that shareholders can do far more easily than a corporation is diversify risk. A shareholder with a well-diversified portfolio may be immune to many of the risks faced by a corporation. For example, in addition to holding shares in a company that uses copper, a well-diversified shareholder may hold shares in a copper producer, so that there is very little overall exposure to the price of copper. If companies are acting in the best interests of well-diversified shareholders, it can be argued that hedging is unnecessary in many situations. However, the extent to which managers are in practice influenced by this type of argument is open to question.

## Hedging and Competitors

If hedging is not the norm in a certain industry, it may not make sense for one particular company to choose to be different from all others. Competitive pressures within the industry may be such that the prices of the goods and services produced by the industry fluctuate to reflect raw material costs, interest rates, exchange rates, and so on. A company that does not hedge can expect its profit margins to be roughly constant. However, a company that does hedge can expect its profit margins to fluctuate!

To illustrate this point, consider two manufacturers of gold jewelry, SafeandSure Company and TakeaChance Company. We assume that most companies in the industry do not hedge against movements in the price of gold and that TakeaChance Company is no exception. However, SafeandSure Company has decided to be different from its competitors and to use futures contracts to hedge its purchase of gold over the next 18 months. If the price of gold goes up, economic pressures will tend to lead to a corresponding increase in the wholesale price of jewelry, so that TakeaChance Company's gross profit margin is unaffected. By contrast, SafeandSure Company's profit margin will increase after the effects of the hedge have been taken into account. If the price of gold goes down, economic pressures will tend to lead to a corresponding decrease in the wholesale price of jewelry. Again, TakeaChance Company's profit margin is unaffected. However, SafeandSure Company's profit margin goes down. In extreme conditions,

**Table 3.1** Danger in hedging when competitors do not hedge.

<i>Change in gold price</i>	<i>Effect on price of gold jewelry</i>	<i>Effect on profits of TakeaChance Co.</i>	<i>Effect on profits of SafeandSure Co.</i>
Increase	Increase	None	Increase
Decrease	Decrease	None	Decrease

SafeandSure Company's profit margin could become negative as a result of the "hedging" carried out! The situation is summarized in Table 3.1.

This example emphasizes the importance of looking at the big picture when hedging. All the implications of price changes on a company's profitability should be taken into account in the design of a hedging strategy to protect against the price changes.

### Hedging Can Lead to a Worse Outcome

It is important to realize that a hedge using futures contracts can result in a decrease or an increase in a company's profits relative to the position it would be in with no hedging. In the example involving the oil producer considered earlier, if the price of oil goes down, the company loses money on its sale of 1 million barrels of oil, and the futures position leads to an offsetting gain. The treasurer can be congratulated for having had the foresight to put the hedge in place. Clearly, the company is better off than it would be with no hedging. Other executives in the organization, it is hoped, will appreciate the contribution made by the treasurer. If the price of oil goes up, the company gains from its sale of the oil, and the futures position leads to an offsetting loss. The company is in a worse position than it would be with no hedging. Although the hedging decision was perfectly logical, the treasurer may in practice have a difficult time justifying it. Suppose that the price of oil at the end of the hedge is \$89, so that the company loses \$10 per barrel on the futures contract. We can imagine a conversation such as the following between the treasurer and the president:

- President: This is terrible. We've lost \$10 million in the futures market in the space of three months. How could it happen? I want a full explanation.
- Treasurer: The purpose of the futures contracts was to hedge our exposure to the price of oil, not to make a profit. Don't forget we made \$10 million from the favorable effect of the oil price increases on our business.
- President: What's that got to do with it? That's like saying that we do not need to worry when our sales are down in California because they are up in New York.
- Treasurer: If the price of oil had gone down...
- President: I don't care what would have happened if the price of oil had gone down. The fact is that it went up. I really do not know what you were doing playing the futures markets like this. Our shareholders will expect us to have done particularly well this quarter. I'm going to have to explain to them that your actions reduced profits by \$10 million. I'm afraid this is going to mean no bonus for you this year.

**Business Snapshot 3.1** Hedging by Gold Mining Companies

It is natural for a gold mining company to consider hedging against changes in the price of gold. Typically it takes several years to extract all the gold from a mine. Once a gold mining company decides to go ahead with production at a particular mine, it has a big exposure to the price of gold. Indeed a mine that looks profitable at the outset could become unprofitable if the price of gold plunges.

Gold mining companies are careful to explain their hedging strategies to potential shareholders. Some gold mining companies do not hedge. They tend to attract shareholders who buy gold stocks because they want to benefit when the price of gold increases and are prepared to accept the risk of a loss from a decrease in the price of gold. Other companies choose to hedge. They estimate the number of ounces of gold they will produce each month for the next few years and enter into short futures or forward contracts to lock in the price for all or part of this.

Suppose you are Goldman Sachs and are approached by a gold mining company that wants to sell you a large amount of gold in 1 year at a fixed price. How do you set the price and then hedge your risk? The answer is that you can hedge by borrowing the gold from a central bank, selling it immediately in the spot market, and investing the proceeds at the risk-free rate. At the end of the year, you buy the gold from the gold mining company and use it to repay the central bank. The fixed forward price you set for the gold reflects the risk-free rate you can earn and the lease rate you pay the central bank for borrowing the gold.

Treasurer: That's unfair. I was only...

President: Unfair! You are lucky not to be fired. You lost \$10 million.

Treasurer: It all depends on how you look at it...

It is easy to see why many treasurers are reluctant to hedge! Hedging reduces risk for the company. However, it may increase risk for the treasurer if others do not fully understand what is being done. The only real solution to this problem involves ensuring that all senior executives within the organization fully understand the nature of hedging before a hedging program is put in place. Ideally, hedging strategies are set by a company's board of directors and are clearly communicated to both the company's management and the shareholders. (See Business Snapshot 3.1 for a discussion of hedging by gold mining companies.)

### 3.3 BASIS RISK

The hedges in the examples considered so far have been almost too good to be true. The hedger was able to identify the precise date in the future when an asset would be bought or sold. The hedger was then able to use futures contracts to remove almost all the risk arising from the price of the asset on that date. In practice, hedging is often not quite as straightforward as this. Some of the reasons are as follows:

1. The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.

2. The hedger may be uncertain as to the exact date when the asset will be bought or sold.
3. The hedge may require the futures contract to be closed out before its delivery month.

These problems give rise to what is termed *basis risk*. This concept will now be explained.

## The Basis

The *basis* in a hedging situation is as follows:<sup>2</sup>

$$\text{Basis} = \text{Spot price of asset to be hedged} - \text{Futures price of contract used}$$

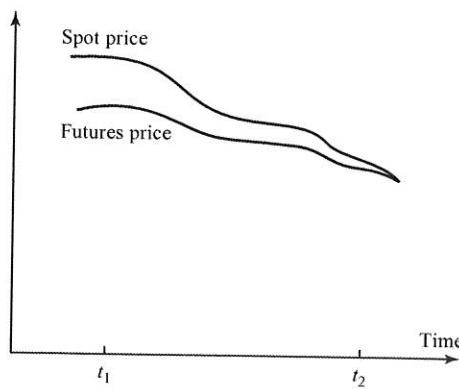
If the asset to be hedged and the asset underlying the futures contract are the same, the basis should be zero at the expiration of the futures contract. Prior to expiration, the basis may be positive or negative. From Table 2.2, we see that, on May 26, 2010, the basis was negative for gold and positive for short maturity contracts on soybeans.

As time passes, the spot price and the futures price for a particular month do not necessarily change by the same amount. As a result, the basis changes. An increase in the basis is referred to as a *strengthening of the basis*; a decrease in the basis is referred to as a *weakening of the basis*. Figure 3.1 illustrates how a basis might change over time in a situation where the basis is positive prior to expiration of the futures contract.

To examine the nature of basis risk, we will use the following notation:

- $S_1$ : Spot price at time  $t_1$
- $S_2$ : Spot price at time  $t_2$
- $F_1$ : Futures price at time  $t_1$
- $F_2$ : Futures price at time  $t_2$
- $b_1$ : Basis at time  $t_1$
- $b_2$ : Basis at time  $t_2$ .

**Figure 3.1** Variation of basis over time.



<sup>2</sup> This is the usual definition. However, the alternative definition  $\text{Basis} = \text{Futures price} - \text{Spot price}$  is sometimes used, particularly when the futures contract is on a financial asset.

We will assume that a hedge is put in place at time  $t_1$  and closed out at time  $t_2$ . As an example, we will consider the case where the spot and futures prices at the time the hedge is initiated are \$2.50 and \$2.20, respectively, and that at the time the hedge is closed out they are \$2.00 and \$1.90, respectively. This means that  $S_1 = 2.50$ ,  $F_1 = 2.20$ ,  $S_2 = 2.00$ , and  $F_2 = 1.90$ .

From the definition of the basis, we have

$$b_1 = S_1 - F_1 \quad \text{and} \quad b_2 = S_2 - F_2$$

so that, in our example,  $b_1 = 0.30$  and  $b_2 = 0.10$ .

Consider first the situation of a hedger who knows that the asset will be sold at time  $t_2$  and takes a short futures position at time  $t_1$ . The price realized for the asset is  $S_2$  and the profit on the futures position is  $F_1 - F_2$ . The effective price that is obtained for the asset with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

In our example, this is \$2.30. The value of  $F_1$  is known at time  $t_1$ . If  $b_2$  were also known at this time, a perfect hedge would result. The hedging risk is the uncertainty associated with  $b_2$  and is known as *basis risk*. Consider next a situation where a company knows it will buy the asset at time  $t_2$  and initiates a long hedge at time  $t_1$ . The price paid for the asset is  $S_2$  and the loss on the hedge is  $F_1 - F_2$ . The effective price that is paid with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

This is the same expression as before and is \$2.30 in the example. The value of  $F_1$  is known at time  $t_1$ , and the term  $b_2$  represents basis risk.

Note that basis risk can lead to an improvement or a worsening of a hedger's position. Consider a short hedge. If the basis strengthens (i.e., increases) unexpectedly, the hedger's position improves; if the basis weakens (i.e., decreases) unexpectedly, the hedger's position worsens. For a long hedge, the reverse holds. If the basis strengthens unexpectedly, the hedger's position worsens; if the basis weakens unexpectedly, the hedger's position improves.

The asset that gives rise to the hedger's exposure is sometimes different from the asset underlying the futures contract that is used for hedging. This is known as cross hedging and is discussed in the next section. It leads to an increase in basis risk. Define  $S_2^*$  as the price of the asset underlying the futures contract at time  $t_2$ . As before,  $S_2$  is the price of the asset being hedged at time  $t_2$ . By hedging, a company ensures that the price that will be paid (or received) for the asset is

$$S_2 + F_1 - F_2$$

This can be written as

$$F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$

The terms  $S_2^* - F_2$  and  $S_2 - S_2^*$  represent the two components of the basis. The  $S_2^* - F_2$  term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract. The  $S_2 - S_2^*$  term is the basis arising from the difference between the two assets.

## Choice of Contract

One key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:

1. The choice of the asset underlying the futures contract
2. The choice of the delivery month.

If the asset being hedged exactly matches an asset underlying a futures contract, the first choice is generally fairly easy. In other circumstances, it is necessary to carry out a careful analysis to determine which of the available futures contracts has futures prices that are most closely correlated with the price of the asset being hedged.

The choice of the delivery month is likely to be influenced by several factors. In the examples given earlier in this chapter, we assumed that, when the expiration of the hedge corresponds to a delivery month, the contract with that delivery month is chosen. In fact, a contract with a later delivery month is usually chosen in these circumstances. The reason is that futures prices are in some instances quite erratic during the delivery month. Moreover, a long hedger runs the risk of having to take delivery of the physical asset if the contract is held during the delivery month. Taking delivery can be expensive and inconvenient. (Long hedgers normally prefer to close out the futures contract and buy the asset from their usual suppliers.)

In general, basis risk increases as the time difference between the hedge expiration and the delivery month increases. A good rule of thumb is therefore to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge. Suppose delivery months are March, June, September, and December for a futures contract on a particular asset. For hedge expirations in December, January, and February, the March contract will be chosen; for hedge expirations in March, April, and May, the June contract will be chosen; and so on. This rule of thumb assumes that there is sufficient liquidity in all contracts to meet the hedger's requirements. In practice, liquidity tends to be greatest in short-maturity futures contracts. Therefore, in some situations, the hedger may be inclined to use short-maturity contracts and roll them forward. This strategy is discussed later in the chapter.

### **Example 3.1**

It is March 1. A US company expects to receive 50 million Japanese yen at the end of July. Yen futures contracts on the CME Group have delivery months of March, June, September, and December. One contract is for the delivery of 12.5 million yen. The company therefore shorts four September yen futures contracts on March 1. When the yen are received at the end of July, the company closes out its position. We suppose that the futures price on March 1 in cents per yen is 0.7800 and that the spot and futures prices when the contract is closed out are 0.7200 and 0.7250, respectively.

The gain on the futures contract is  $0.7800 - 0.7250 = 0.0550$  cents per yen. The basis is  $0.7200 - 0.7250 = -0.0050$  cents per yen when the contract is closed out. The effective price obtained in cents per yen is the final spot price plus the gain on the futures:

$$0.7200 + 0.0550 = 0.7750$$

This can also be written as the initial futures price plus the final basis:

$$0.7800 + (-0.0050) = 0.7750$$

The total amount received by the company for the 50 million yen is  $50 \times 0.00775$  million dollars, or \$387,500.

### **Example 3.2**

It is June 8 and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November. Oil futures contracts are currently traded for delivery every month on the NYMEX division of the CME Group and the contract size is 1,000 barrels. The company therefore decides to use the December contract for hedging and takes a long position in 20 December contracts. The futures price on June 8 is \$68.00 per barrel. The company finds that it is ready to purchase the crude oil on November 10. It therefore closes out its futures contract on that date. The spot price and futures price on November 10 are \$70.00 per barrel and \$69.10 per barrel.

The gain on the futures contract is  $69.10 - 68.00 = \$1.10$  per barrel. The basis when the contract is closed out is  $70.00 - 69.10 = \$0.90$  per barrel. The effective price paid (in dollars per barrel) is the final spot price less the gain on the futures, or

$$70.00 - 1.10 = 68.90$$

This can also be calculated as the initial futures price plus the final basis,

$$68.00 + 0.90 = 68.90$$

The total price paid is  $68.90 \times 20,000 = \$1,378,000$ .

## **3.4 CROSS HEDGING**

In Examples 3.1 and 3.2, the asset underlying the futures contract was the same as the asset whose price is being hedged. *Cross hedging* occurs when the two assets are different. Consider, for example, an airline that is concerned about the future price of jet fuel. Because jet fuel futures are not actively traded, it might choose to use heating oil futures contracts to hedge its exposure.

The *hedge ratio* is the ratio of the size of the position taken in futures contracts to the size of the exposure. When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a hedge ratio of 1.0. This is the hedge ratio we have used in the examples considered so far. For instance, in Example 3.2, the hedger's exposure was on 20,000 barrels of oil, and futures contracts were entered into for the delivery of exactly this amount of oil.

When cross hedging is used, setting the hedge ratio equal to 1.0 is not always optimal. The hedger should choose a value for the hedge ratio that minimizes the variance of the value of the hedged position. We now consider how the hedger can do this.

## Calculating the Minimum Variance Hedge Ratio

The minimum variance hedge ratio depends on the relationship between changes in the spot price and changes in the futures price. Define:

- $\Delta S$ : Change in spot price,  $S$ , during a period of time equal to the life of the hedge
- $\Delta F$ : Change in futures price,  $F$ , during a period of time equal to the life of the hedge.

We will denote the minimum variance hedge ratio by  $h^*$ . It can be shown that  $h^*$  is the slope of the best-fit line from a linear regression of  $\Delta S$  against  $\Delta F$  (see Figure 3.2). This result is intuitively reasonable. We would expect  $h^*$  to be the ratio of the average change in  $S$  for a particular change in  $F$ .

The formula for  $h^*$  is:

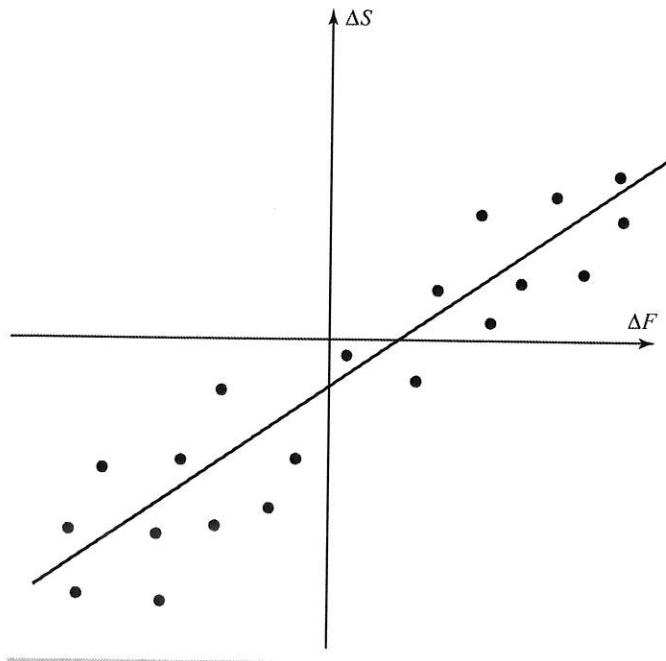
$$h^* = \rho \frac{\sigma_S}{\sigma_F} \quad (3.1)$$

where  $\sigma_S$  is the standard deviation of  $\Delta S$ ,  $\sigma_F$  is the standard deviation of  $\Delta F$ , and  $\rho$  is the coefficient of correlation between the two.

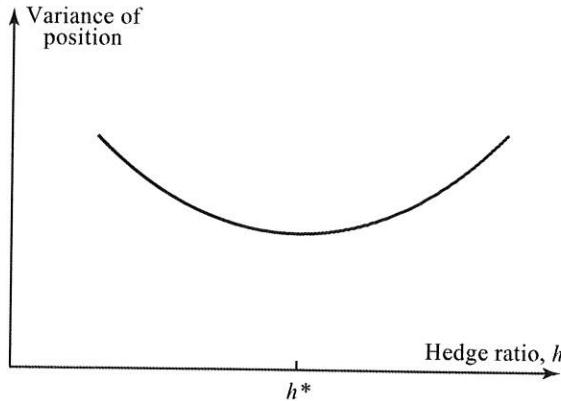
Equation (3.1) shows that the optimal hedge ratio is the product of the coefficient of correlation between  $\Delta S$  and  $\Delta F$  and the ratio of the standard deviation of  $\Delta S$  to the standard deviation of  $\Delta F$ . Figure 3.3 shows how the variance of the value of the hedger's position depends on the hedge ratio chosen.

If  $\rho = 1$  and  $\sigma_F = \sigma_S$ , the hedge ratio,  $h^*$ , is 1.0. This result is to be expected, because in this case the futures price mirrors the spot price perfectly. If  $\rho = 1$  and  $\sigma_F = 2\sigma_S$ , the

**Figure 3.2** Regression of change in spot price against change in futures price.



**Figure 3.3** Dependence of variance of hedger's position on hedge ratio.



hedge ratio  $h^*$  is 0.5. This result is also as expected, because in this case the futures price always changes by twice as much as the spot price. The *hedge effectiveness* can be defined as the proportion of the variance that is eliminated by hedging. This is the  $R^2$  from the regression of  $\Delta S$  against  $\Delta F$  and equals  $\rho^2$ .

The parameters  $\rho$ ,  $\sigma_F$ , and  $\sigma_S$  in equation (3.1) are usually estimated from historical data on  $\Delta S$  and  $\Delta F$ . (The implicit assumption is that the future will in some sense be like the past.) A number of equal nonoverlapping time intervals are chosen, and the values of  $\Delta S$  and  $\Delta F$  for each of the intervals are observed. Ideally, the length of each time interval is the same as the length of the time interval for which the hedge is in effect. In practice, this sometimes severely limits the number of observations that are available, and a shorter time interval is used.

### Optimal Number of Contracts

To calculate the number of contracts that should be used in hedging, define:

- $Q_A$ : Size of position being hedged (units)
- $Q_F$ : Size of one futures contract (units)
- $N^*$ : Optimal number of futures contracts for hedging.

The futures contracts should be on  $h^* Q_A$  units of the asset. The number of futures contracts required is therefore given by

$$N^* = \frac{h^* Q_A}{Q_F} \quad (3.2)$$

Example 3.3 will show how the results in this section can be used by an airline hedging the purchase of jet fuel.<sup>3</sup>

<sup>3</sup> Heating oil futures are more actively traded than jet fuel futures contracts. For an account of how Delta Airlines used heating oil futures to hedge its future purchases of jet fuel, see A. Ness, "Delta Wins on Fuel," *Risk*, June 2001: 8.

**Example 3.3**

An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging. We suppose that Table 3.2 gives, for 15 successive months, data on the change,  $\Delta S$ , in the jet fuel price per gallon and the corresponding change,  $\Delta F$ , in the futures price for the contract on heating oil that would be used for hedging price changes during the month. In this case, the usual formulas for calculating standard deviations and correlations give  $\sigma_F = 0.0313$ ,  $\sigma_S = 0.0263$ , and  $\rho = 0.928$ .

From equation (3.1), the minimum variance hedge ratio,  $h^*$ , is therefore

$$0.928 \times \frac{0.0263}{0.0313} = 0.7777$$

Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil. From equation (3.2), the optimal number of contracts is

$$\frac{0.7777 \times 2,000,000}{42,000} = 37.03$$

or, rounding to the nearest whole number, 37.

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**Table 3.2** Data to calculate minimum variance hedge ratio when heating oil futures contract is used to hedge purchase of jet fuel.

Month <i>i</i>	Change in heating oil futures price per gallon (= $\Delta F$ )	Change in jet fuel price per gallon (= $\Delta S$ )
1	0.021	0.029
2	0.035	0.020
3	-0.046	-0.044
4	0.001	0.008
5	0.044	0.026
6	-0.029	-0.019
7	-0.026	-0.010
8	-0.029	-0.007
9	0.048	0.043
10	-0.006	0.011
11	-0.036	-0.036
12	-0.011	-0.018
13	0.019	0.009
14	-0.027	-0.032
15	0.029	0.023

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### Tailing the Hedge

When futures are used for hedging, a small adjustment, known as *tailing the hedge*, can be made to allow for the impact of daily settlement. In practice this means that equation (3.2) becomes<sup>4</sup>

$$N^* = \frac{h^* V_A}{V_F} \quad (3.3)$$

where  $V_A$  is the dollar value of the position being hedged and  $V_F$  is the dollar value of one futures contract (the futures price times  $Q_F$ ). Suppose that in Example 3.3 the spot price and the futures price are 1.94 and 1.99 dollars per gallon, respectively. Then  $V_A = 2,000,000 \times 1.94 = 3,880,000$  while  $V_F = 42,000 \times 1.99 = 83,580$ , so that the optimal number of contracts is

$$\frac{0.7777 \times 3,880,000}{83,580} = 36.10$$

If we round this to the nearest whole number, the optimal number of contracts is now 36 rather than 37. The effect of tailing the hedge is to multiply the hedge ratio in equation (3.2) by the ratio of the spot price to the futures price. Theoretically, the futures position used for hedging should then be adjusted as the spot price and futures price change, but in practice this usually makes very little difference.

If forward contracts rather than futures contracts are used, there is no daily settlement and equation (3.2) should be used.

## 3.5 STOCK INDEX FUTURES

We now move on to consider stock index futures and how they are used to hedge or manage exposures to equity prices.

A *stock index* tracks changes in the value of a hypothetical portfolio of stocks. The weight of a stock in the portfolio at a particular time equals the proportion of the hypothetical portfolio invested in the stock at that time. The percentage increase in the stock index over a small interval of time is set equal to the percentage increase in the value of the hypothetical portfolio. Dividends are usually not included in the calculation so that the index tracks the capital gain/loss from investing in the portfolio.<sup>5</sup>

If the hypothetical portfolio of stocks remains fixed, the weights assigned to individual stocks in the portfolio do not remain fixed. When the price of one particular stock in the portfolio rises more sharply than others, more weight is automatically given to that stock. Sometimes indices are constructed from a hypothetical portfolio consisting of one of each of a number of stocks. The weights assigned to the stocks are then proportional to their market prices, with adjustments being made when there are stock splits. Other indices are constructed so that weights are proportional to market capitalization (stock price  $\times$  number of shares outstanding). The underlying portfolio is then automatically adjusted to reflect stock splits, stock dividends, and new equity issues.

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<sup>4</sup> See Problem 5.23 for an explanation of equation (3.3).

<sup>5</sup> An exception to this is a *total return index*. This is calculated by assuming that dividends on the hypothetical portfolio are reinvested in the portfolio.

## Stock Indices

Table 3.3 shows futures prices for contracts on a number of different stock indices on May 26, 2010.

The *Dow Jones Industrial Average* is based on a portfolio consisting of 30 blue-chip stocks in the United States. The weights given to the stocks are proportional to their prices. The CME Group trades two futures contracts on the index. The one shown is on \$10 times the index. The other (the Mini DJ Industrial Average) is on \$5 times the index.

The *Standard & Poor's 500 (S&P 500) Index* is based on a portfolio of 500 different stocks: 400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions. The weights of the stocks in the portfolio at any given time are proportional to their market capitalizations. The stocks are those of large publicly held companies that trade on NYSE Euronext or Nasdaq OMX. The CME Group trades two futures contracts on the S&P 500. The one shown is on \$250 times the index; the other (the Mini S&P 500 contract) is on \$50 times the index.

The *Nasdaq-100* is based on 100 stocks using the National Association of Securities Dealers Automatic Quotations Service. The CME Group trades two futures contracts. The one shown is on \$100 times the index; the other (the Mini Nasdaq-100 contract) is on \$20 times the index.

The *Russell 1000 Index* is an index of the prices of the 1,000 largest capitalization stocks in the United States. The *US Dollar Index* is a trade-weighted index of the values of six foreign currencies (the euro, yen, pound, Canadian dollar, Swedish krona, and Swiss franc).

As mentioned in Chapter 2, futures contracts on stock indices are settled in cash, not by delivery of the underlying asset. All contracts are marked to market to either the opening price or the closing price of the index on the last trading day, and the positions are then deemed to be closed. For example, contracts on the S&P 500 are closed out at the opening price of the S&P 500 index on the third Friday of the delivery month.

**Table 3.3** Index futures quotes as reported by exchanges on May 26, 2010.

	Open	High	Low	Settlement	Change	Volume	Open interest
<b>Dow Jones Industrial Average, \$10 times index (CME Group)</b>							
June 2010	10080	10160	9921	9921	-104	533	12,353
Sept. 2010	10080	10085	9864	9864	-104	2	225
<b>S&amp;P 500, \$250 times index (CME Group)</b>							
June 2010	1080.0	1089.5	1060.5	1061.2	-11.8	43,076	296,397
Sept. 2010	1084.0	1085.5	1057.5	1057.0	-11.9	6,850	26,966
Dec. 2010	1074.0	1081.9	1052.9	1053.1	-11.8	7	4,326
<b>Nasdaq-100, \$100 times index (CME Group)</b>							
June 2010	1827.0	1850.0	1788.0	1791.5	-24.0	2,350	20,674
<b>Russell 1000, \$100 times index (ICE)</b>							
June 2010	595.4	601.0	586.4	585.7	-5.8	1,214	19,275
<b>US Dollar Index, \$1000 times index (ICE)</b>							
June 2010	86.725	87.480	86.625	87.244	0.356	37,321	35,401
Sept. 2010	87.270	87.800	87.050	87.584	0.356	74	2,533

## Hedging an Equity Portfolio

Stock index futures can be used to hedge a well-diversified equity portfolio. Define:

$V_A$ : Current value of the portfolio

$V_F$ : Current value of one futures contract (the futures price times the contract size).

If the portfolio mirrors the index, the optimal hedge ratio,  $h^*$ , equals 1.0 and equation (3.3) shows that the number of futures contracts that should be shorted is

$$N^* = \frac{V_A}{V_F} \quad (3.4)$$

Suppose, for example, that a portfolio worth \$5,050,000 mirrors the S&P 500. The index futures price is 1,010 and each futures contract is on \$250 times the index. In this case  $V_A = 5,050,000$  and  $V_F = 1,010 \times 250 = 252,500$ , so that 20 contracts should be shorted to hedge the portfolio.

When the portfolio does not exactly mirror the index, we can use the capital asset pricing model (see the appendix to this chapter). The parameter beta ( $\beta$ ) from the capital asset pricing model is the slope of the best-fit line obtained when excess return on the portfolio over the risk-free rate is regressed against the excess return of the index over the risk-free rate. When  $\beta = 1.0$ , the return on the portfolio tends to mirror the return on the index; when  $\beta = 2.0$ , the excess return on the portfolio tends to be twice as great as the excess return on the index; when  $\beta = 0.5$ , it tends to be half as great; and so on.

A portfolio with a  $\beta$  of 2.0 is twice as sensitive to movements in the index as a portfolio with a beta 1.0. It is therefore necessary to use twice as many contracts to hedge the portfolio. Similarly, a portfolio with a beta of 0.5 is half as sensitive to market movements as a portfolio with a beta of 1.0 and we should use half as many contracts to hedge it. In general,

$$N^* = \beta \frac{V_A}{V_F} \quad (3.5)$$

This formula assumes that the maturity of the futures contract is close to the maturity of the hedge.

Comparing equation (3.5) with equation (3.3), we see that they imply  $h^* = \beta$ . This is not surprising. The hedge ratio  $h^*$  is the slope of the best-fit line when changes in the portfolio are regressed against changes in the futures price of the index. Beta ( $\beta$ ) is the slope of the best-fit line when the return from the portfolio is regressed against the return for the index.

We illustrate that this formula gives good results by extending our earlier example. Suppose that a futures contract with 4 months to maturity is used to hedge the value of a portfolio over the next 3 months in the following situation:

Value of S&P 500 index = 1,000

S&P 500 futures price = 1,010

Value of portfolio = \$5,050,000

Risk-free interest rate = 4% per annum

Dividend yield on index = 1% per annum

Beta of portfolio = 1.5.

One futures contract is for delivery of \$250 times the index. As before,  $V_F = 250 \times 1,010 = 252,500$ . From equation (3.5), the number of futures contracts that should be shorted to hedge the portfolio is

$$1.5 \times \frac{5,050,000}{252,500} = 30$$

Suppose the index turns out to be 900 in 3 months and the futures price is 902. The gain from the short futures position is then

$$30 \times (1010 - 902) \times 250 = \$810,000$$

The loss on the index is 10%. The index pays a dividend of 1% per annum, or 0.25% per 3 months. When dividends are taken into account, an investor in the index would therefore earn  $-9.75\%$  over the 3-month period. Because the portfolio has a  $\beta$  of 1.5, the capital asset pricing model gives

$$\begin{aligned} & \text{Expected return on portfolio} - \text{Risk-free interest rate} \\ &= 1.5 \times (\text{Return on index} - \text{Risk-free interest rate}) \end{aligned}$$

The risk-free interest rate is approximately 1% per 3 months. It follows that the expected return (%) on the portfolio during the 3 months when the 3-month return on the index is  $-9.75\%$  is

$$1.0 + [1.5 \times (-9.75 - 1.0)] = -15.125$$

The expected value of the portfolio (inclusive of dividends) at the end of the 3 months is therefore

$$\$5,050,000 \times (1 - 0.15125) = \$4,286,187$$

It follows that the expected value of the hedger's position, including the gain on the hedge, is

$$\$4,286,187 + \$810,000 = \$5,096,187$$

Table 3.4 summarizes these calculations together with similar calculations for other values of the index at maturity. It can be seen that the total expected value of the hedger's position in 3 months is almost independent of the value of the index.

The only thing we have not covered in this example is the relationship between futures prices and spot prices. We will see in Chapter 5 that the 1,010 assumed for the futures price today is roughly what we would expect given the interest rate and dividend we are assuming. The same is true of the futures prices in 3 months shown in Table 3.4.<sup>6</sup>

## Reasons for Hedging an Equity Portfolio

Table 3.4 shows that the hedging procedure results in a value for the hedger's position at the end of the 3-month period being about 1% higher than at the beginning of the 3-month period. There is no surprise here. The risk-free rate is 4% per annum, or 1% per 3 months. The hedge results in the investor's position growing at the risk-free rate.

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<sup>6</sup> The calculations in Table 3.4 assume that the dividend yield on the index is predictable, the risk-free interest rate remains constant, and the return on the index over the 3-month period is perfectly correlated with the return on the portfolio. In practice, these assumptions do not hold perfectly, and the hedge works rather less well than is indicated by Table 3.4.

**Table 3.4** Performance of stock index hedge.

Value of index in three months:	900	950	1,000	1,050	1,100
Futures price of index today:	1,010	1,010	1,010	1,010	1,010
Futures price of index in three months:	902	952	1,003	1,053	1,103
Gain on futures position (\$):	810,000	435,000	52,500	-322,500	-697,500
Return on market:	-9.750%	-4.750%	0.250%	5.250%	10.250%
Expected return on portfolio:	-15.125%	-7.625%	-0.125%	7.375%	14.875%
Expected portfolio value in three months including dividends (\$):	4,286,187	4,664,937	5,043,687	5,422,437	5,801,187
Total value of position in three months (\$):	5,096,187	5,099,937	5,096,187	5,099,937	5,103,687

It is natural to ask why the hedger should go to the trouble of using futures contracts. To earn the risk-free interest rate, the hedger can simply sell the portfolio and invest the proceeds in risk-free instruments such as Treasury bills.

One answer to this question is that hedging can be justified if the hedger feels that the stocks in the portfolio have been chosen well. In these circumstances, the hedger might be very uncertain about the performance of the market as a whole, but confident that the stocks in the portfolio will outperform the market (after appropriate adjustments have been made for the beta of the portfolio). A hedge using index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market. This will be discussed further shortly. Another reason for hedging may be that the hedger is planning to hold a portfolio for a long period of time and requires short-term protection in an uncertain market situation. The alternative strategy of selling the portfolio and buying it back later might involve unacceptably high transaction costs.

### Changing the Beta of a Portfolio

In the example in Table 3.4, the beta of the hedger's portfolio is reduced to zero so that the hedger's expected return is almost independent of the performance of the index. Sometimes futures contracts are used to change the beta of a portfolio to some value other than zero. Continuing with our earlier example:

$$\begin{aligned} \text{S\&P 500 index} &= 1,000 \\ \text{S\&P 500 futures price} &= 1,010 \\ \text{Value of portfolio} &= \$5,050,000 \\ \text{Beta of portfolio} &= 1.5 \end{aligned}$$

As before,  $V_F = 250 \times 1,010 = 252,500$  and a complete hedge requires

$$1.5 \times \frac{5,050,000}{252,500} = 30$$

contracts to be shorted. To reduce the beta of the portfolio from 1.5 to 0.75, the

number of contracts shorted should be 15 rather than 30; to increase the beta of the portfolio to 2.0, a long position in 10 contracts should be taken; and so on. In general, to change the beta of the portfolio from  $\beta$  to  $\beta^*$ , where  $\beta > \beta^*$ , a short position in

$$(\beta - \beta^*) \frac{V_A}{V_F}$$

contracts is required. When  $\beta < \beta^*$ , a long position in

$$(\beta^* - \beta) \frac{V_A}{V_F}$$

contracts is required.

### Locking in the Benefits of Stock Picking

Suppose you consider yourself to be good at picking stocks that will outperform the market. You own a single stock or a small portfolio of stocks. You do not know how well the market will perform over the next few months, but you are confident that your portfolio will do better than the market. What should you do?

You should short  $\beta V_A/V_F$  index futures contracts, where  $\beta$  is the beta of your portfolio,  $V_A$  is the total value of the portfolio, and  $V_F$  is the current value of one index futures contract. If your portfolio performs better than a well-diversified portfolio with the same beta, you will then make money.

Consider an investor who in April holds 20,000 IBM shares, each worth \$100. The investor feels that the market will be very volatile over the next three months but that IBM has a good chance of outperforming the market. The investor decides to use the August futures contract on the S&P 500 to hedge the market's return during the three-month period. The  $\beta$  of IBM is estimated at 1.1. Suppose that the current futures price for the August contract on the S&P 500 is 900. Each contract is for delivery of \$250 times the index. In this case,  $V_A = 20,000 \times 100 = 2,000,000$  and  $V_F = 900 \times 250 = 225,000$ . The number of contracts that should be shorted is therefore

$$1.1 \times \frac{2,000,000}{225,000} = 9.78$$

Rounding to the nearest integer, the investor shorts 10 contracts, closing out the position in July. Suppose IBM falls to \$90 and the futures price of the S&P 500 falls to 750. The investor loses  $20,000 \times (\$100 - \$90) = \$200,000$  on IBM, while gaining  $10 \times 250 \times (900 - 750) = \$375,000$  on the futures contracts.

The overall gain to the investor in this case is \$175,000 because IBM did not go down by as much as a well-diversified portfolio with a  $\beta$  of 1.1. If the market had gone up and IBM went up by more than a portfolio with a  $\beta$  of 1.1 (as expected by the investor), then a profit would be made in this case as well.

### 3.6 STACK AND ROLL

Sometimes the expiration date of the hedge is later than the delivery dates of all the futures contracts that can be used. The hedger must then roll the hedge forward by closing out one futures contract and taking the same position in a futures contract with

**Table 3.5** Data for the example on rolling oil hedge forward.

Date	Apr. 2011	Sept. 2011	Feb. 2012	June 2012
Oct. 2011 futures price	68.20	67.40		
Mar. 2012 futures price		67.00	66.50	
July 2012 futures price			66.30	65.90
Spot price	69.00			66.00

a later delivery date. Hedges can be rolled forward many times. The procedure is known as *stack and roll*. Consider a company that wishes to use a short hedge to reduce the risk associated with the price to be received for an asset at time  $T$ . If there are futures contracts 1, 2, 3, ...,  $n$  (not all necessarily in existence at the present time) with progressively later delivery dates, the company can use the following strategy:

- Time  $t_1$ : Short futures contract 1
- Time  $t_2$ : Close out futures contract 1  
Short futures contract 2
- Time  $t_3$ : Close out futures contract 2  
Short futures contract 3  
⋮
- Time  $t_n$ : Close out futures contract  $n - 1$   
Short futures contract  $n$
- Time  $T$ : Close out futures contract  $n$ .

Suppose that in April 2011 a company realizes that it will have 100,000 barrels of oil to sell in June 2012 and decides to hedge its risk with a hedge ratio of 1.0. (In this example, we do not make the “tailing” adjustment described in Section 3.4.) The current spot price is \$69. Although futures contracts are traded with maturities stretching several years into the future, we suppose that only the first six delivery months have sufficient liquidity to meet the company’s needs. The company therefore shorts 100 October 2011 contracts. In September 2011, it rolls the hedge forward into the March 2012 contract. In February 2012, it rolls the hedge forward again into the July 2012 contract.

One possible outcome is shown in Table 3.5. The October 2011 contract is shorted at \$68.20 per barrel and closed out at \$67.40 per barrel for a profit of \$0.80 per barrel; the March 2012 contract is shorted at \$67.00 per barrel and closed out at \$66.50 per barrel for a profit of \$0.50 per barrel. The July 2012 contract is shorted at \$66.30 per barrel and closed out at \$65.90 per barrel for a profit of \$0.40 per barrel. The final spot price is \$66.

The dollar gain per barrel of oil from the short futures contracts is

$$(68.20 - 67.40) + (67.00 - 66.50) + (66.30 - 65.90) = 1.70$$

The oil price declined from \$69 to \$66. Receiving only \$1.70 per barrel compensation for a price decline of \$3.00 may appear unsatisfactory. However, we cannot expect total compensation for a price decline when futures prices are below spot prices. The best we

**Business Snapshot 3.2** Metallgesellschaft: Hedging Gone Awry

Sometimes rolling hedges forward can lead to cash flow pressures. The problem was illustrated dramatically by the activities of a German company, Metallgesellschaft (MG), in the early 1990s.

MG sold a huge volume of 5- to 10-year heating oil and gasoline fixed-price supply contracts to its customers at 6 to 8 cents above market prices. It hedged its exposure with long positions in short-dated futures contracts that were rolled forward. As it turned out, the price of oil fell and there were margin calls on the futures positions. Considerable short-term cash flow pressures were placed on MG. The members of MG who devised the hedging strategy argued that these short-term cash outflows were offset by positive cash flows that would ultimately be realized on the long-term fixed-price contracts. However, the company's senior management and its bankers became concerned about the huge cash drain. As a result, the company closed out all the hedge positions and agreed with its customers that the fixed-price contracts would be abandoned. The outcome was a loss to MG of \$1.33 billion.

can hope for is to lock in the futures price that would apply to a June 2012 contract if it were actively traded.

In practice, a company usually has an exposure every month to the underlying asset and uses a 1-month futures contract for hedging because it is the most liquid. Initially it enters into ("stacks") sufficient contracts to cover its exposure to the end of its hedging horizon. One month later, it closes out all the contracts and "rolls" them into new 1-month contracts to cover its new exposure, and so on.

As described in Business Snapshot 3.2, a German company, Metallgesellschaft, followed this strategy in the early 1990s to hedge contracts it had entered into to supply commodities at a fixed price. It ran into difficulties because the prices of the commodities declined so that there were immediate cash outflows on the futures and the expectation of eventual gains on the contracts. This mismatch between the timing of the cash flows on hedge and the timing of the cash flows from the position being hedged led to liquidity problems that could not be handled. The moral of the story is that potential liquidity problems should always be considered when a hedging strategy is being planned.

## SUMMARY

This chapter has discussed various ways in which a company can take a position in futures contracts to offset an exposure to the price of an asset. If the exposure is such that the company gains when the price of the asset increases and loses when the price of the asset decreases, a short hedge is appropriate. If the exposure is the other way round (i.e., the company gains when the price of the asset decreases and loses when the price of the asset increases), a long hedge is appropriate.

Hedging is a way of reducing risk. As such, it should be welcomed by most executives. In reality, there are a number of theoretical and practical reasons why companies do not hedge. On a theoretical level, we can argue that shareholders, by holding well-diversified portfolios, can eliminate many of the risks faced by a company.

They do not require the company to hedge these risks. On a practical level, a company may find that it is increasing rather than decreasing risk by hedging if none of its competitors does so. Also, a treasurer may fear criticism from other executives if the company makes a gain from movements in the price of the underlying asset and a loss on the hedge.

An important concept in hedging is basis risk. The basis is the difference between the spot price of an asset and its futures price. Basis risk arises from uncertainty as to what the basis will be at maturity of the hedge.

The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure. It is not always optimal to use a hedge ratio of 1.0. If the hedger wishes to minimize the variance of a position, a hedge ratio different from 1.0 may be appropriate. The optimal hedge ratio is the slope of the best-fit line obtained when changes in the spot price are regressed against changes in the futures price.

Stock index futures can be used to hedge the systematic risk in an equity portfolio. The number of futures contracts required is the beta of the portfolio multiplied by the ratio of the value of the portfolio to the value of one futures contract. Stock index futures can also be used to change the beta of a portfolio without changing the stocks that make up the portfolio.

When there is no liquid futures contract that matures later than the expiration of the hedge, a strategy known as stack and roll may be appropriate. This involves entering into a sequence of futures contracts. When the first futures contract is near expiration, it is closed out and the hedger enters into a second contract with a later delivery month. When the second contract is close to expiration, it is closed out and the hedger enters into a third contract with a later delivery month; and so on. The result of all this is the creation of a long-dated futures contract by trading a series of short-dated contracts.

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### Practice Questions (Answers in Solutions Manual)

- 3.1. Under what circumstances are (a) a short hedge and (b) a long hedge appropriate?
- 3.2. Explain what is meant by *basis risk* when futures contracts are used for hedging.
- 3.3. Explain what is meant by a *perfect hedge*. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.
- 3.4. Under what circumstances does a minimum variance hedge portfolio lead to no hedging at all?
- 3.5. Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk.
- 3.6. Suppose that the standard deviation of quarterly changes in the prices of a commodity is \$0.65, the standard deviation of quarterly changes in a futures price on the commodity is \$0.81, and the coefficient of correlation between the two changes is 0.8. What is the optimal hedge ratio for a 3-month contract? What does it mean?
- 3.7. A company has a \$20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index futures price is currently standing at 1080, and each contract is for delivery of \$250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?
- 3.8. In the corn futures contract traded on an exchange, the following delivery months are available: March, May, July, September, and December. Which of the available contracts should be used for hedging when the expiration of the hedge is in (a) June, (b) July, and (c) January.
- 3.9. Does a perfect hedge always succeed in locking in the current spot price of an asset for a future transaction? Explain your answer.

- 3.10. Explain why a short hedger's position improves when the basis strengthens unexpectedly and worsens when the basis weakens unexpectedly.
- 3.11. Imagine you are the treasurer of a Japanese company exporting electronic equipment to the United States. Discuss how you would design a foreign exchange hedging strategy and the arguments you would use to sell the strategy to your fellow executives.
- 3.12. Suppose that in Example 3.2 of Section 3.3 the company decides to use a hedge ratio of 0.8. How does the decision affect the way in which the hedge is implemented and the result?
- 3.13. "If the minimum variance hedge ratio is calculated as 1.0, the hedge must be perfect." Is this statement true? Explain your answer.
- 3.14. "If there is no basis risk, the minimum variance hedge ratio is always 1.0." Is this statement true? Explain your answer.
- 3.15. "For an asset where futures prices are usually less than spot prices, long hedges are likely to be particularly attractive." Explain this statement.
- 3.16. The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?
- 3.17. A corn farmer argues "I do not use futures contracts for hedging. My real risk is not the price of corn. It is that my whole crop gets wiped out by the weather." Discuss this viewpoint. Should the farmer estimate his or her expected production of corn and hedge to try to lock in a price for expected production?
- 3.18. On July 1, an investor holds 50,000 shares of a certain stock. The market price is \$30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index futures price is currently 1,500 and one contract is for delivery of \$50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?
- 3.19. Suppose that in Table 3.5 the company decides to use a hedge ratio of 1.5. How does the decision affect the way the hedge is implemented and the result?
- 3.20. A futures contract is used for hedging. Explain why the daily settlement of the contract can give rise to cash-flow problems.
- 3.21. An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price." Discuss the executive's viewpoint.
- 3.22. Suppose that the 1-year gold lease rate is 1.5% and the 1-year risk-free rate is 5.0%. Both rates are compounded annually. Use the discussion in Business Snapshot 3.1 to calculate the maximum 1-year gold forward price Goldman Sachs should quote to the gold-mining company when the spot price is \$1,200.
- 3.23. The expected return on the S&P 500 is 12% and the risk-free rate is 5%. What is the expected return on an investment with a beta of (a) 0.2, (b) 0.5, and (c) 1.4?

## Further Questions

- 3.24. A company wishes to hedge its exposure to a new fuel whose price changes have a 0.6 correlation with gasoline futures price changes. The company will lose \$1 million for each 1 cent increase in the price per gallon of the new fuel over the next three months. The new fuel's price changes have a standard deviation that is 50% greater than price changes in gasoline futures prices. If gasoline futures are used to hedge the exposure, what should the hedge ratio be? What is the company's exposure measured in gallons of the new fuel? What position, measured in gallons, should the company take in gasoline futures? How many gasoline futures contracts should be traded? Each contract is on 42,000 gallons.
- 3.25. A portfolio manager has maintained an actively managed portfolio with a beta of 0.2. During the last year, the risk-free rate was 5% and equities performed very badly providing a return of  $-30\%$ . The portfolio manager produced a return of  $-10\%$  and claims that in the circumstances it was a good performance. Discuss this claim.
- 3.26. The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate a minimum variance hedge ratio.

Spot price change	+0.50	+0.61	-0.22	-0.35	+0.79
Futures price change	+0.56	+0.63	-0.12	-0.44	+0.60
Spot price change	+0.04	+0.15	+0.70	-0.51	-0.41
Futures price change	-0.06	+0.01	+0.80	-0.56	-0.46

- 3.27. It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index futures price is currently 1,000 and each contract is on \$250 times the index.
- What position should the company take?
  - Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?
- 3.28. A fund manager has a portfolio worth \$50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next 2 months and plans to use 3-month futures contracts on the S&P 500 to hedge the risk. The current level of the index is 1,250, one contract is on 250 times the index, the risk-free rate is 6% per annum, and the dividend yield on the index is 3% per annum. The current 3-month futures price is 1,259.
- What position should the fund manager take to hedge all exposure to the market over the next 2 months?
  - Calculate the effect of your strategy on the fund manager's returns if the index in 2 months is 1,000, 1,100, 1,200, 1,300, and 1,400. Assume that the 1-month futures price is 0.25% higher than the index level at this time.
- 3.29. It is now October 2010. A company anticipates that it will purchase 1 million pounds of copper in each of February 2011, August 2011, February 2012, and August 2012. The company has decided to use the futures contracts traded in the COMEX division of the CME Group to hedge its risk. One contract is for the delivery of 25,000 pounds of copper. The initial margin is \$2,000 per contract and the maintenance margin is \$1,500 per contract. The company's policy is to hedge 80% of its exposure. Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the

company's needs. Devise a hedging strategy for the company. (Do not make the "tailing" adjustment described in Section 3.4.)

Assume the market prices (in cents per pound) today and at future dates are as in the following table. What is the impact of the strategy you propose on the price the company pays for copper? What is the initial margin requirement in October 2010? Is the company subject to any margin calls?

<i>Date</i>	<i>Oct. 2010</i>	<i>Feb. 2011</i>	<i>Aug. 2011</i>	<i>Feb. 2012</i>	<i>Aug. 2012</i>
Spot price	372.00	369.00	365.00	377.00	388.00
Mar. 2011 futures price	372.30	369.10			
Sept. 2011 futures price	372.80	370.20	364.80		
Mar. 2012 futures price		370.70	364.30	376.70	
Sept. 2012 futures price			364.20	376.50	388.20

## APPENDIX

### CAPITAL ASSET PRICING MODEL

The capital asset pricing model (CAPM) is a model that can be used to calculate the expected return from an asset during a period in terms of the risk of the return. The risk in the return from an asset is divided into two parts. *Systematic risk* is risk related to the return from the market as a whole and cannot be diversified away. *Nonsystematic risk* is risk that is unique to the asset and can be diversified away by choosing a large portfolio of different assets. CAPM argues that the return should depend only on systematic risk. The CAPM formula is<sup>7</sup>

$$\text{Expected return on asset} = R_F + \beta(R_M - R_F) \quad (3A.1)$$

where  $R_M$  is the return on the portfolio of all available investments,  $R_F$  is the return on a risk-free investment, and  $\beta$  (the Greek letter beta) is a parameter measuring systematic risk.

The return from the portfolio of all available investments,  $R_M$ , is referred to as the *return on the market* and is usually approximated as the return on a well-diversified stock index such as the S&P 500. The beta ( $\beta$ ) of an asset is a measure of the sensitivity of its returns to returns from the market. It can be estimated from historical data as the slope obtained when the excess return on the asset over the risk-free rate is regressed against the excess return on the market over the risk-free rate. When  $\beta = 0$ , an asset's returns are not sensitive to returns from the market. In this case, it has no systematic risk and equation (3A.1) shows that its expected return is the risk-free rate; when  $\beta = 0.5$ , the excess return on the asset over the risk-free rate is on average half of the excess return of the market over the risk-free rate; when  $\beta = 1$ , the expected return on the asset equals to the return on the market; and so on.

Suppose that the risk-free rate  $R_F$  is 5% and the return on the market is 13%. Equation (3A.1) shows that, when the beta of an asset is zero, its expected return is 5%. When  $\beta = 0.75$ , its expected return is  $0.05 + 0.75 \times (0.13 - 0.05) = 0.11$ , or 11%.

The derivation of CAPM requires a number of assumptions.<sup>8</sup> In particular:

1. Investors care only about the expected return and standard deviation of the return from an asset.
2. The returns from two assets are correlated with each other only because of their correlation with the return from the market. This is equivalent to assuming that there is only one factor driving returns.
3. Investors focus on returns over a single period and that period is the same for all investors.
4. Investors can borrow and lend at the same risk-free rate.
5. Tax does not influence investment decisions.
6. All investors make the same estimates of expected returns, standard deviations of returns, and correlations between returns.

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<sup>7</sup> If the return on the market is not known,  $R_M$  is replaced by the expected value of  $R_M$  in this formula.

<sup>8</sup> For details on the derivation, see, for example, J. Hull, *Risk Management and Financial Institutions*, 2nd edn. Upper Saddle River, NJ: Pearson, 2010, Chap. 1.

These assumptions are at best only approximately true. Nevertheless CAPM has proved to be a useful tool for portfolio managers and is often used as a benchmark for assessing their performance.

When the asset is an individual stock, the expected return given by equation (3A.1) is not a particularly good predictor of the actual return. But, when the asset is a well-diversified portfolio of stocks, it is a much better predictor. As a result, the equation

$$\text{Return on diversified portfolio} = R_F + \beta(R_M - R_F)$$

can be used as a basis for hedging a diversified portfolio, as described in Section 3.5. The  $\beta$  in the equation is the beta of the portfolio. It can be calculated as the weighted average of the betas of the stocks in the portfolio.